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The real exchange rate and quality improvements

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Abstract

This paper studies how the real exchange rate might respond to product innovation (improvements in the quality of goods) as opposed to process innovation (increased efficiency in the production of goods). We develop a two-country dynamic stochastic general equilibrium model, where quality improvements affect both the demand and the supply side of the economy. We show that the real exchange rate defined in terms of prices per quality unit (quality-adjusted prices) does not always move in the same direction as that defined in terms of unit prices (quality-unadjusted prices), illustrating the importance of measuring quality correctly.

Key words: Product quality; real exchange rate; New Open Economy Macroeconomics.

JEL classification: F31; F41.

Summary

Much of the literature on the New Open Economy Macroeconomics (NOEM) focuses on technological progress that manifests itself through improvements in productivity, ie increased efficiency in the production of a given range of goods, which is also known as *process innovation*. A common finding in this literature is that a positive productivity shock in the home country tends to depreciate the real exchange rate. This is because a positive domestic productivity shock increases the supply of home relative to foreign goods, which then reduces the relative price of home goods causing a real depreciation. But technological progress can also come about via improvements in the quality of a given range of products, known as *product innovation*. We might envisage that this type of technological progress in the home country will cause the real exchange rate to appreciate. This is because higher quality goods, in general, can command higher prices, which then tend to increase the relative price of home goods leading to a real appreciation. The simple model presented in this paper aims to take the NOEM literature a step towards modelling this type of technological progress. In particular, we are interested in understanding the theoretical link between quality improvements and real exchange rates.

The real exchange rate is defined as the ratio of the two countries' price indices, expressed in a common currency. But in a world where goods become obsolete and are replaced due to quality improvements, the relevant real exchange rate is the real exchange rate measured in terms of quality-adjusted prices. In practice, price indices may not (fully) capture the quality improvements in goods, and the real exchange rate may consequently be miscalculated. Here, we examine the impact of quality improvements on two measures of the real exchange rate: the *quality-adjusted* and the *quality-unadjusted* measure. The former measure is calculated using price indices that aggregate prices per quality unit, and hence, by construction, it fully accounts for product quality. The latter measure, on the contrary, is calculated using price indices that aggregate unit prices only, and hence by construction, it fails to account for product quality.

Our analysis shows that a quality improvement can lead to either a depreciation or an appreciation of either measure of the real exchange rate depending on how costs of production are affected by the quality improvement. We also find that the real exchange rate defined in terms of unit prices does not always move in the same direction as the real exchange rate defined in terms of prices per quality unit, illustrating the importance of measuring quality correctly.

1 Introduction

Much of the literature on the New Open Economy Macroeconomics (NOEM) focuses on technological progress that manifests itself through improvements in productivity, ie increased efficiency in the production of a given range of goods, which is also known as *process innovation*. A common finding in this literature is that a positive productivity shock in the home country tends to depreciate the real exchange rate. This is because a positive domestic productivity shock increases the supply of home relative to foreign goods, which then reduces the relative price of home goods causing a real depreciation. But technological progress can also come about via improvements in the quality of a given range of products, known as *product innovation*. We might envisage that this type of technological progress in the home country will cause the real exchange rate to appreciate. This is because higher quality goods, in general, can command higher prices, which then tends to increase the relative price of home goods leading to a real appreciation. The simple model presented in this paper aims to take the NOEM literature a step towards modelling this type of technological progress. In particular, we are interested in understanding the theoretical link between quality improvements and real exchange rates.

The economics of quality improvements have been studied in the literature by industrial organisation and international trade economists. For instance Krugman (1979), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and Glass (2001) have looked at the impact of quality improvements on the rate of economic growth, welfare, and international trade. In doing so they have examined various issues related to market structure, the rate of product innovation, and creative destruction (where the new higher quality goods render the old lower quality goods obsolete). But, to our knowledge, this literature has so far not addressed issues related to the international transmission of product innovations and their impact on real exchange rates. In our opinion, the NOEM models are ideally placed to discuss such issues. Thus, here, we take a standard NOEM-type model and extend it by incorporating product quality. We then use this model to examine the implications of improvements in product quality on the real exchange rate. Given the early stages of this development, we aim to keep the modelling work as simple as possible. For this reason, we limit ourselves to analysing the effects of such an innovation and not its causes, and take quality improvements as exogenous.

As mentioned previously, we consider a standard two-country (country A and country B) NOEM

model with sticky prices and home bias in consumption where the economy is endowed with a single input, labour. To this, we add the new feature that goods in one country can be produced at a different quality level to those produced in the other country. Adding home bias to a model with sticky prices allows us to deviate from purchasing power parity (PPP) which, as well as being observed in the data, is necessary if we are to examine the implications of quality improvements on the real exchange rate. In our model, product innovations affect both the demand and the supply side of the economy. On the demand side, quality enters the model through consumers' preferences. Consumers prefer higher quality goods to lower quality goods and choose those goods that command the lowest quality-adjusted price (or price per quality unit). On the supply side, quality enters the model through its effect on production costs. We consider three different cases: one where an improvement in quality does not affect unit costs of production; one where it increases unit costs of production; and another where it decreases unit costs of production. Also, we assume that all producers will manufacture the higher quality product irrespective of its cost implications.

The real exchange rate is defined as the ratio of the two countries' price indices, expressed in a common currency. But in a world where goods become obsolete and are replaced due to quality improvements, the relevant real exchange rate is the real exchange rate measured in terms of quality-adjusted prices. In practice, price indices may not (fully) capture the quality improvements in goods, and the real exchange rate may consequently be miscalculated. Here, we examine the impact of quality improvements on two measures of the real exchange rate: the *quality-adjusted* and the *quality-unadjusted* measure. The former measure is calculated using price indices that aggregate prices per quality unit, and hence, by construction, it fully accounts for product quality. The latter measure, on the contrary, is calculated using price indices that aggregate unit prices only, and hence by construction, it fails to account for product quality.

Our analysis shows that, following a quality shock, the response of the real exchange rate essentially depends on how quality improvements affect production costs and that this response differs across the two measures. That is, after a quality shock in country *A*, the real exchange rate defined in terms of prices per quality unit depreciates (implying goods are becoming relatively more expensive in country *B* in terms of their prices per quality unit) if quality improvements do not affect unit costs of production and it depreciates even further if costs of production are lower for higher quality goods. However, if unit costs increase sufficiently as higher quality goods are

produced, then the quality-adjusted real exchange rate appreciates. The above conclusions will not hold if we instead take the quality-unadjusted real exchange rate as our reference. That is, the real exchange rate defined in terms of unit prices appreciates (implying goods are becoming relatively more expensive in country *A* in terms of their unit prices) if quality improvements do not affect unit costs of production and it appreciates even further if costs of production are higher for higher quality products. However, if unit costs decrease as higher quality goods are produced, then the quality-unadjusted real exchange rate depreciates. Finally, we compare the response of model variables to a quality shock with those to a positive productivity shock originating in country *A* and to a relative demand shock where global demand shifts towards country *A*'s goods. We find that the quality shock produces qualitatively similar results to the productivity shock, when the improvement in quality leads to a reduction in unit costs of production, and to the demand shock, when costs of production are the same regardless of the quality level of products.

The remainder of this paper is organised as follows. Section 2 sets out the structure of the model in detail. Section 3 presents impulse responses of the model to a temporary quality shock and compares these to a productivity shock and a relative demand shock. Section 4 concludes and offers some additional mechanisms by which quality could affect the real exchange rate, which constitute areas for future research.

2 The model

The world consists of two countries, country A and country B. In each period t, the world economy is inhabited by a continuum of infinitely lived producers/consumers indexed on the interval [0,1] with the fractions n and 1-n residing in country A and country B, respectively. Each household consumes goods, supplies labour for the production process, and holds financial assets. A continuum of goods are produced in the world economy by monopolistically competitive firms using only labour, with each firm producing a single differentiated tradable product. Each product has a quality level that is assumed to be the same across goods produced within a country but different to goods produced in the other country. Financial assets are made up of nominal, uncontingent bonds, and money balances. Households in both countries trade bonds domestically and internationally.

2.1 The demand side

Agents have perfect foresight, though they can be surprised by one-time unanticipated shocks. All individuals within a country have symmetric constraints and identical preferences over a real consumption index, C_A ; labour effort supplied in a competitive market, L_A^S ; and real money balances, M_A/P_A (where M_A denotes nominal money holdings and P_A is the consumption-based prices index CPI, to be shown later). At any time *t*, the typical country *A* resident *j* maximises the following intertemporal utility function which is separable in its three arguments:

$$U_{A,j,t} = \sum_{t=0}^{\infty} \beta^t [\log C_{A,j,t} + \log(1 - L_{A,j,t}^S) + \chi \log \frac{M_{A,j,t}}{P_{A,t}}]$$
(1)

where β , the subjective discount factor, is strictly between zero and one and χ , the weight of utility from money balances in overall utility, is greater than zero.

The consumption index for the representative country *A* consumer is a standard constant elasticity of substitution (CES) aggregator of domestic and foreign subindices:

$$C_{A,j,t} = \{ (n\alpha_A \delta_t^{\frac{1}{2}})^{\frac{1}{\theta}} C_{A,j,t}^{A} \stackrel{\frac{\theta-1}{\theta}}{+} + [(1 - n\alpha_A)\delta_t^{\frac{-1}{2}}]^{\frac{1}{\theta}} C_{A,j,t}^{B} \}^{\frac{\theta-1}{\theta}} \}^{\frac{\theta}{\theta-1}}$$

where $C_{A,j,t}^A$ denotes the consumption of goods produced in country A by country A households and $C_{A,j,t}^B$ denotes the consumption of goods produced in country B by country A households. Similarly for a representative country B consumer. That is:

$$C_{B,j,t} = \{ [(1 - (1 - n)\alpha_B)\delta_t^{\frac{1}{2}}]^{\frac{1}{\theta}} C_{A,j,t}^B \stackrel{\frac{\theta - 1}{\theta}}{+} [(1 - n\alpha_B)\delta_t^{-\frac{1}{2}}]^{\frac{1}{\theta}} C_{B,j,t}^B \stackrel{\frac{\theta - 1}{\theta}}{+} \}^{\frac{\theta}{\theta - 1}} \}$$

The parameter $\theta > 1$ is the intratemporal elasticity of substitution between country *A* and country *B* goods. The parameters α_A and α_B are our measures of preference bias. Note that we model home bias along the lines of Warnock (2003). That is, when there is a home bias in consumption, then for any given relative price, a country *A* consumer will always demand relatively more of its own goods than will a country *B* consumer. Similarly for a country *B* consumer. We discuss this point in more detail below. And in our setup, where we follow Spange and Zabczyk (2006), there is a home bias in consumption if $1 < n\alpha_A + (1 - n)\alpha_B$. As we will see, consumers' demand functions depend on this home bias parameter as well as relative prices. Finally, the consumption index is augmented by a demand shock parameter, δ_t , where an increase in δ shifts the world demand away from country *B* products towards country *A* products.

The consumption subindices for the representative country A consumer are:

$$C_{A,j,t}^{A} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\psi}} \int_{0}^{n} (q_{A,t}c_{A,j,t}^{A}(z))^{\frac{\psi-1}{\psi}} dz\right]^{\frac{\psi}{\psi-1}}$$
$$C_{A,j,t}^{B} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\psi}} \int_{n}^{1} (q_{B,t}c_{A,j,t}^{B}(z))^{\frac{\psi-1}{\psi}} dz\right]^{\frac{\psi}{\psi-1}}$$

where $C_{A,j,t}^A$ ($C_{A,j,t}^B$) denotes the time *t* unit consumption by country *A* households of a good *z* produced in country *A* (country *B*). The parameter $\psi > 1$ is the elasticity of substitution among brands produced within a country. The parameter ψ also represents the price elasticity of demand faced by each producer. The consumption subindices are standard CES aggregators. Following Grossman and Helpman (1991), the unit consumption of individual goods is multiplied by an exogenous quality parameter. We refer to this quantity as *quality-augmented* consumption. Hence, individuals in the model derive greater utility from merely consuming goods with higher levels of quality. The quality parameter is assumed to be the same across goods produced within a country but different from goods produced in the other country. Hence, $q_{A,t}$ denotes the quality level of all goods produced in country *A* at time *t*, whereas $q_{B,t}$ is the quality level of all goods produced in country *B* at time *t*. The CPI that corresponds to the above specification of preferences is then given by:⁽¹⁾

$$P_{A,t} = [n\alpha_A \delta_t^{1/2} (P_{A,t}^A)^{1-\theta} + (1 - n\alpha_A) \delta_t^{-1/2} (P_{A,t}^B)^{1-\theta}]^{\frac{1}{1-\theta}}$$

where $P_{A,t}^A$ is the price subindex for goods produced and consumed in country A and $P_{A,t}^B$ is the price subindex for goods produced in country B but consumed in country A, both expressed in units of country A's currency. Similarly, we let $p_{A,t}^A(z)$ denote the unit price of a good z produced and consumed in country A and $p_{A,t}^B(z)$ denote the unit price of good z produced in country B but consumed in country A, both expressed in units of country A, both expressed in units of country A's currency. Then, we have:

$$P_{A,t}^{A} = \left[\frac{1}{n} \int_{0}^{n} \left(\frac{p_{A,t}^{A}(z)}{q_{A,t}}\right)^{1-\psi} dz\right]^{\frac{1}{1-\psi}}$$
$$P_{A,t}^{B} = \left[\frac{1}{1-n} \int_{n}^{1} \left(\frac{p_{A,t}^{B}(z)}{q_{B,t}}\right)^{1-\psi} dz\right]^{\frac{1}{1-\psi}}$$

where $p_{A,t}^A(z)/q_{A,t}$ is the price per quality unit (or the *quality-adjusted* price) of a product produced and consumed in country A and where $p_{A,t}^B(z)/q_{B,t}$ is the price per quality unit of a product produced in country B and consumed in country A, both expressed in units of country A's currency. We assume that there are no impediments to trade and that firms do not engage in local currency pricing. Hence, the law of one price (LOOP) holds for each individual good: $p_{A,t}^B(z)/q_{B,t} = \xi_t p_{B,t}^B(z)/q_{B,t}$. The variable ξ_t is the time t nominal exchange rate, where a decrease in ξ_t reflects an appreciation of country A's currency. However, the presence of

⁽¹⁾ $P_{A,t}$ is the minimum expenditure needed to purchase one unit of the composite consumption index given prices.

non-identical consumer preferences across countries implies that purchasing power parity (PPP) does not hold (ie $P_{A,t} \neq \xi_t P_{B,t}$) except for the special case of no home bias case where $1 = n\alpha_A + (1 - n)\alpha_B$.

The optimal allocation of expenditure across domestic and imported brands yields the following *quality-augmented* demand functions for the representative country *A* consumer:

$$q_{A,t}c_{A,j,t}^{A}(z) = \left[\frac{p_{A,t}^{A}(z)/q_{A,t}}{P_{A,t}^{A}}\right]^{-\psi} \left(\frac{P_{A,t}^{A}}{P_{A,t}}\right)^{-\theta} \alpha_{A}C_{A,j,t}$$
$$q_{B,t}c_{A,j,t}^{B}(z) = \left[\frac{p_{A,t}^{B}(z)/q_{B,t}}{P_{A,t}^{B}}\right]^{-\psi} \left(\frac{P_{A,t}^{B}}{P_{A,t}}\right)^{-\theta} \left(\frac{1-n\alpha_{A}}{1-n}\right)C_{A,j,t}$$

We see that the quality-augmented demand functions for product z is now a function of the relative quality-adjusted price of the product as well as the aggregate demand at home. One can show that the total expenditure of the representative country A household on domestic good consumption is $\int_0^1 p_{A,t}^A(z)c_{A,j,t}^A(z)dz = P_{A,t}^A C_{A,t}^A$. Likewise, the household's total expenditure on imported goods satisfies $\int_0^1 p_{A,t}^B(z)c_{A,j,t}^B(z)dz = P_{A,t}^B C_{A,t}^B$. The representative country B household has analogous demand functions. That is:

$$q_{B,t}c_{B,j,t}^{B}(z) = \left[\frac{p_{B,t}^{B}(z)/q_{B,t}}{P_{B,t}^{B}}\right]^{-\psi} \left(\frac{P_{B,t}^{B}}{P_{B,t}}\right)^{-\theta} \alpha_{B}C_{B,j,t}$$
$$q_{A,t}c_{B,j,t}^{A}(z) = \left[\frac{p_{B,t}^{A}(z)/q_{A,t}}{P_{B,t}^{A}}\right]^{-\psi} \left(\frac{P_{B,t}^{A}}{P_{B,t}}\right)^{-\theta} \left[\frac{1-(1-n)\alpha_{B}}{n}\right]C_{B,j,t}$$

Notice that since LOOP holds in the model $p_{B,t}^A(z)/q_{A,t} = p_{A,t}^A(z)/\xi_t q_{A,t}$. The relative demands of a representative country *A* and country *B* consumers for their respective home goods are given by:

$$\frac{q_A c^A_{A,j,t}(z)}{q_B c^B_{A,j,t}(z)} = \frac{(1-n)\alpha_A}{1-n\alpha_A} \left[\frac{p^A_{A,t}(z)/q_{A,t}}{p^B_{B,t}(z)/q_{B,t}} \frac{P^B_{B,t}}{P^A_{A,t}}\right]^{-\psi} \left(\frac{P^A_{A,t}}{\xi_t P^B_{B,t}}\right)^{-\theta}$$

$$\frac{q_A c^A_{B,j,t}(z)}{q_B c^B_{B,j,t}(z)} = \left[\frac{1-(1-n)\alpha_B}{n\alpha_B}\right] \left[\frac{p^A_{A,t}(z)/q_{A,t}}{p^B_{B,t}(z)/q_{B,t}} \frac{P^B_{B,t}}{P^A_{A,t}}\right]^{-\psi} \left(\frac{P^A_{A,t}}{\xi_t P^B_{B,t}}\right)^{-\theta}$$

We can see that for $1 < n\alpha_A + (1 - n)\alpha_B$ there is a home bias in consumption: that is, for any given relative quality-adjusted price, a country *A* consumer will always demand relatively more of its own goods than a country *B* consumer will: $c_{A,j,t}^A(z)/c_{A,j,t}^B(z) > c_{B,j,t}^A(z)/c_{B,j,t}^B(z)$. Similarly for a country *B* consumer. As Warnock (2003) points out, this does not mean that a country *A* will always demand more home goods than foreign goods: with high enough relative quality-adjusted prices at home, a country *A* consumer, even with a home bias, will demand more imported goods than domestic goods.

The total quality-augmented demand for a good z produced in country A is obtained by adding the demands for that good coming from the two countries. Similarly for a good z produced in country

B. Then we have:

$$q_{A,t}Y_{A,t}^{D}(z) = \left[\frac{p_{A,t}^{A}(z)/q_{A,t}}{P_{A,t}^{A}}\right]^{-\psi} \left(\frac{P_{A,t}^{A}}{P_{A,t}}\right)^{-\theta} \underbrace{\left\{n\alpha_{A}\delta_{t}^{\frac{1}{2}}C_{A,t} + Q_{t}^{\theta}\frac{1-n}{n}[1-(1-n)\alpha_{B}]\delta_{t}^{\frac{1}{2}}C_{B,t}\right\}}_{Y_{A,t}^{DW}}$$

$$(2)$$

$$q_{B,t}Y_{B,t}^{D}(z) = \left[\frac{p_{B,t}^{B}(z)/q_{B,t}}{p_{B,t}^{B}}\right]^{-\psi} \left(\frac{P_{B,t}^{B}}{P_{B,t}}\right)^{-\theta} \underbrace{\left\{Q_{t}^{-\theta}\frac{n}{1-n}(1-n\alpha_{A})\delta_{t}^{\frac{-1}{2}}C_{A,t} + (1-n)\alpha_{B}\delta_{t}^{\frac{-1}{2}}C_{B,t}\right\}}_{Y_{B,t}^{DW}}$$
(3)

where, $Y_{A,t}^D(z) = nc_{A,j,t}^A(z) + (1-n)c_{B,j,t}^A(z)$ and $Y_{B,t}^D(z) = (1-n)c_{B,j,t}^B(z) + nc_{A,j,t}^B(z)$. The right-hand side variables $Y_{A,t}^{DW}$ and $Y_{B,t}^{DW}$ denote the world demand for the composite consumption good produced in country A and country B, respectively. Finally, $Q_t \equiv \xi_t P_{B,t}/P_{A,t}$ is the real exchange rate. An increase in Q_t corresponds to a real depreciation implying goods are becoming relatively cheaper in country A in terms of their prices per quality unit.

2.1.1 Household budget constraint and intertemporal choices

The representative household in country A enters each period t with domestic and foreign nominal bond holdings; $B_{A,j,t-1}^A$ and $B_{A,j,t-1}^B$, and holdings of domestic money; $M_{A,j,t-1}$, carried over from the previous period. It receives gross interest income on bond holdings, earns labour income and firm profits. The household allocates these resources between money balances and purchases of bonds to be carried to the next period, consumption, and lump-sum taxes. We assume that households can trade bonds domestically and internationally and that bonds are riskless. Further, country A bonds, issued by country A households, are denominated in country A's currency and country B bonds, issued by country B households, are denominated in country B's currency. Following Ghironi and Melitz (2005) we assume that agents must pay fees to domestic financial intermediaries in order to hold bonds.⁽²⁾ These fees are modelled as quadratic in the stock of bonds. Financial intermediaries are then assumed to rebate the revenues from bond-holding fees to domestic households.

The budget constraint of the representative country A household in units of country A's currency is

⁽²⁾ Here, international assets markets are incomplete as only risk-free bonds are traded across countries. As argued by Ghironi and Melitz (2005), in the absence of the cost of holding bonds, the incomplete market assumption would imply that the steady-state net foreign assets would be indeterminate and that the model would be non-stationary. Adding the cost of holding bonds pins down the steady state and ensures mean reversion in the long run. We have chosen realistic parameter values for the cost of adjusting bond holdings to ensure there is a very small impact on model dynamics.

then given by:

$$B_{A,j,t}^{A} + \xi_{t}B_{A,j,t}^{B} + \frac{\tau}{2}(B_{A,j,t}^{A})^{2} + \frac{\tau}{2}\xi_{t}(B_{A,j,t}^{B})^{2} + M_{A,j,t} + P_{A,t}C_{A,j,t} + P_{A,t}T_{A,j,t} = (1 + i_{A,t-1})B_{A,j,t-1}^{A} + (1 + i_{B,t-1})\xi_{t}B_{A,j,t-1}^{B} + P_{A,t}\Gamma_{A,t} + M_{A,j,t-1} + P_{A,t}\Pi_{A,j,t}(z) + W_{A,t}L_{A,j,t}^{S}$$

where $\frac{\tau}{2}(B_{A,j,t}^{A})^{2}$ is the per capita cost of holding country *A* bonds; $\frac{\tau}{2}\xi_{t}(B_{A,j,t}^{B})^{2}$ is the per capita cost of holding country *A* bonds; $\frac{\tau}{2}\xi_{t}(B_{A,j,t}^{B})^{2}$ is the per capita cost of holding country *B* bonds; and $P_{A,t}\Gamma_{A,t}$ is the rebate given to households by financial intermediaries and, in aggregate, it is equal to $\frac{\tau}{2}[(B_{A,j,t}^{A})^{2} + \xi_{t}(B_{A,j,t}^{B})^{2}]$. For simplicity, it is assumed that the cost parameter $\tau > 0$ is identical across holdings of home and foreign bonds.
 $i_{A,t-1}(i_{B,t-1})$ is the nominal interest rate on holdings of bonds between $t-1$ and t in country *A* (country *B*) and is known with certainty as of $t-1$. $P_{A,t}\Pi_{A,j,t}(z)$ denotes the total profits earned by the representative country *A* household *j* from producing product *z*. $W_{A,t}$ is the nominal wage rate.
 $T_{A,j,t}$ is a lump-sum tax (payable in the composite consumption good $C_{A,j,t}$). Since Ricardian equivalence holds in this setup, we can, without loss of generality, assume that the government runs a balanced budget every period. Further, we assume that there is no government spending and all the seignorage revenues are rebated to the public in the form of transfers. Then, in aggregate, we have: $P_{A,t}T_{A,t} + M_{A,t} - M_{A,t-1} = 0$. Similarly for the country *B* government.

The representative country A household maximises the utility function (1) subject to the above budget constraint. We can drop the j subscript (since agents are symmetric and so make identical choices in equilibrium) and then the optimal labour supply is given by:

$$L_{A,t}^S = 1 - \frac{C_{A,t}}{w_{A,t}}$$

where $w_{A,t} = W_{A,t}/P_{A,t}$ is the real wage rate. Hence, in equilibrium, marginal disutility of supplying labour should equal the marginal utility of consumption generated by the corresponding increase in labour income. Similarly for country *B*.

The Euler equations for bond holdings in country A are given by:

$$\frac{1}{C_{A,t}}(1+\tau B_{A,t}^{A}) = \beta(1+r_{A,t})\frac{1}{C_{A,t+1}}$$
(4)

$$\frac{1}{C_{A,t}}(1+\tau B_{A,t}^B) = \beta(1+r_{B,t})\frac{Q_{t+1}}{Q_t}\frac{1}{C_{A,t+1}}$$
(5)

The cost parameter τ now appears in the otherwise standard Euler equations. It ensures that there are zero holdings of bonds in the *unique* steady state so that economies return to their initial position after temporary shocks. In deriving the above equations, we use the familiar Fisher parity condition to link the nominal and the consumption-based real interest rate between *t* and *t*+1. That

is:

$$1 + r_{A,t} = (1 + i_{A,t}) \frac{P_{A,t}}{P_{A,t+1}}$$

The Euler equation for bond holdings in country *B* is given by:

$$\frac{1}{C_{B,t}}(1+\tau B_{B,t}^B) = \beta(1+r_{B,t})\frac{1}{C_{B,t+1}}$$
(6)

$$\frac{1}{C_{B,t}}(1+\tau B_{B,t}^A) = \beta(1+r_{A,t})\frac{Q_t}{Q_{t+1}}\frac{1}{C_{B,t+1}}$$
(7)

Equations (4)-(7) imply that:

$$\frac{1+r_{A,t}}{1+r_{B,t}} = \frac{Q_{t+1}}{Q_t} \frac{1+\tau B^A_{A,t}}{1+\tau B^B_{A,t}} = \frac{Q_{t+1}}{Q_t} \frac{1+\tau B^A_{B,t}}{1+\tau B^B_{B,t}}$$
(8)

Together with the Fisher parity condition the above equation implies that:

$$\frac{1+i_{A,t}}{1+i_{B,t}} = \frac{\xi_{t+1}}{\xi_t} \frac{1+\tau B^A_{A,t}}{1+\tau B^B_{A,t}} = \frac{\xi_{t+1}}{\xi_t} \frac{1+\tau B^A_{B,t}}{1+\tau B^B_{B,t}}$$
(9)

Equations (8) and (9) are the modified versions of the real and nominal uncovered interest rate parity conditions which say that, for agents to be indifferent between home and foreign bonds, the real and nominal interest rate differentials must be equal to the real and nominal depreciation adjusted for the cost associated with holding bonds.⁽³⁾ Finally, in aggregate, country *A* and country *B* bonds must be in zero net supply worldwide, that is:

$$nB_{A,t}^A + (1-n)B_{B,t}^A = 0$$
(10)

$$nB_{A,t}^B + (1-n)B_{B,t}^B = 0$$
(11)

The Euler equation for money demand function in country A is given by:

$$\frac{M_{A,t}}{P_{A,t}} = \chi C_{A,t} (\frac{1+i_{A,t}}{i_{A,t}-\tau B_{A,t}^A})$$

Similarly for country *B*. The above equation arises from the equilibrium condition that agents must be indifferent between consuming a unit of consumption good at time *t* or using the same amount to raise cash balances, enjoying the utility from money holdings at period *t*, and then converting the cash balances back to consumption in period t+1. Money market clearing requires that, in equilibrium, the domestic money supply must equal domestic money demand in each country.

2.2 The supply side

There is a continuum of monopolistically competitive firms each producing a single differentiated product with a unique quality level, which is assumed to be the same across products produced

⁽³⁾ Notice that when $\tau = 0$ equations (8) and (9) collapse to the standard real and nominal uncovered interest parity conditions.

within the country but different across products produced in the other country. Output supplied at time *t* by the representative country *A* producer is a linear function of labour demanded, $L_{A,t}^D$:

$$Y_{A,t}^{S}(z) = q_{A,t}^{\gamma-1} Z_{A,t} L_{A,t}^{D}(z)$$
(12)

where $Z_{A,t}$ is the economy-wide productivity parameter. Similarly, output supplied at time *t* by the representative country *B* producer is a linear function of $L_{B,t}^D(z)$ with productivity parameter $Z_{B,t}$ and quality parameter $q_{B,t}$. Following Hobijn (2002), the parameter γ , which we assume to be the same across countries, determines the impact of quality improvements on unit costs of production as discussed below.

The unit cost function associated with the above production function is given by:

$$mc_{A,t} = \frac{W_{A,t}}{q_{A,t}^{\gamma-1} Z_{A,t}}$$

Similarly for country *B*. As the equation shows, unit cost of production is an increasing function of wages and a decreasing function of total factor productivity. The impact of quality improvements on unit cost of production, however, will be governed by the parameter γ . If $\gamma = 1$, then quality improvements will have no impact on unit cost of production. If $\gamma < 1$, then the unit cost of production will be an increasing function of quality. In other words, it will be more costly to produce higher quality goods. And if $\gamma > 1$, then the unit cost of production will be a decreasing function of quality, where it will be less costly to produce higher quality goods. Put differently, if $\gamma = 1$, then the manufacturing of one unit of a product requires the same amount of labour input, regardless of the level of quality; if $\gamma < 1$, then a unit of the higher quality product requires more labour services to be manufactured, and if $\gamma > 1$, then a unit of the higher quality product can be manufactured with less labour input.

The total (real) profit of the representative country A producer is given by:

$$\Pi_{A,t}(z) = \frac{p_{A,t}^A(z)}{P_{A,t}} Y_{A,t}^S(z) - w_{A,t} L_{A,t}^D(z) - \Delta_{A,t}(z)$$
(13)

We model sticky prices along the lines of Rotemberg (1982). The variable $\Delta_{A,t}(z)$, where $\Delta_{A,t}(z) = \frac{\phi}{2} (\frac{p_{A,t}^A(z)}{p_{A,t-1}^A(z)} - 1)^2 \frac{p_{A,t}^A(z)}{P_{A,t}} Y_{A,t}^S(z)$, is the cost associated with adjusting nominal prices in country A. This cost is thought to capture the cost of purchasing new catalogues, changing price tags, etc. Following Ghironi (2002), $\Delta_{A,t}(z)$ increases with revenues, $\frac{p_{A,t}^A(z)}{P_{A,t}} Y_{A,t}^S(z)$, which are taken as a proxy for firm size. This structure implies that faster price movements are more costly to the firm. When $\phi = 0$, prices are flexible. When $\phi > 0$, due to costs involved in adjustment, nominal prices will be sticky. The firm chooses its price $p_{A,t}^A(z)$ and the amount of labour demanded $L_{A,t}^D(z)$ to maximise (13) subject to the constraints (2) and (12), and the market clearing condition $Y_{A,t}^D(z) = Y_{A,t}^S(z) = Y_{A,t}(z)$. The firm takes the quality level, the wage rate, the aggregate price indices, and the world aggregates as given. The first-order condition with respect to $p_{A,t}^A(z)$ yields the optimal pricing equation:

$$p_{A,t}^{A} = \Psi_{A,t} \frac{W_{A,t}}{Z_{A,t} q_{A,t}^{\gamma-1}}$$
(14)

which equates the unit price charged by the firm to the product of the unit cost and a mark-up, $\Psi_{A,t}$. Notice that we dropped the producer-specific index *z* since producers within a country are symmetric and so make identical choices in equilibrium. Equation (14) tells us that, *ceteris paribus*, if $\gamma = 1$, then the unit price and the mark-up charged by the firm is independent of the quality level of the product. If $\gamma > 1$, then the unit cost of production is lower for products with higher quality level and the suppliers will charge a lower unit price. And if $\gamma < 1$, then the unit cost of production is higher for products with higher quality level and the suppliers will charge a lower unit price. Similarly for the mark-up. That is, *ceteris paribus*, if $\gamma > 1$, then the suppliers will charge a lower mark-up.

The equation for the mark-up is given below. As shown, the mark-up depends on output demanded as well as on today's pricing decision and on current and future cost of adjusting the output price. That is:

$$\Psi_{A,t} = \psi\{(\psi - 1)[1 - \frac{\phi}{2}\eta_{A,t}^2] + \phi\Upsilon_{A,t}\}^{-1}$$

where

$$\eta_{A,t} \equiv p_{A,t}^A / p_{A,t-1}^A - 1$$

$$\Upsilon_{A,t} \equiv \frac{1}{2} \eta_{A,t}^2 + \eta_{A,t} [\eta_{A,t} + 1] - \beta \frac{Y_{A,t+1}^S}{Y_{A,t}^S} \frac{P_{A,t}}{P_{A,t+1}} \eta_{A,t+1} [\eta_{A,t+1} + 1]$$

 $\Upsilon_{A,t}$ reflects the firm's incentive to smooth prices over time. If $\phi = 0$, that is if prices are fully flexible, then $\Psi_{A,t} = \psi/(\psi - 1)$, collapses to the familiar constant mark-up. If $\phi \neq 0$, then prices are sticky and this will give rise to endogenous fluctuations of the mark-up.

Using the market clearing condition $Y_{A,t}^D = Y_{A,t}^S = Y_{A,t}$ together with equations (2) and (12), the labour demand of the representative country A firm can be written as:

$$L_{A,t}^{D} = \left[\frac{p_{A,t}^{A}/q_{A,t}}{P_{A,t}^{A}}\right]^{-\psi} \left(\frac{P_{A,t}^{A}}{P_{A,t}}\right)^{-\theta} \frac{Y_{A,t}^{DW}}{Z_{A,t}q_{A,t}^{\gamma}}$$

The representative firm's labour demand is a decreasing function of the relative price of the product and labour productivity. It is an increasing function of world demand for the composite

good produced in country A. The firm's labour demand increases (decreases) with an increase in $q_{A,t}$ if $\gamma < (>)\psi$. Labour market clearing conditions in country A requires that, in equilibrium, $L_{A,t}^D = L_{A,t}^S = L_{A,t}$. This implies that real wages will move to clear the labour market:

$$w_{A,t} = C_{A,t} \left[1 - \left(\frac{p_{A,t}^A/q_{A,t}}{P_{A,t}}\right)^{-\psi} \left(\frac{P_{A,t}^A}{P_{A,t}}\right)^{-\theta} \frac{Y_{A,t}^{DW}}{Z_{A,t}q_{A,t}^{\gamma}} \right]^{-1}$$

Similar results hold for country B.

2.3 Net foreign assets

The households' budget constraints evaluated at a symmetric equilibrium imply that the net foreign assets in country *A* and country *B* are given by:

$$\frac{B_{A,t}^{A}}{P_{A,t}} + Q_{t} \frac{B_{A,t}^{B}}{P_{B,t}} = (1 + i_{A,t-1}) \frac{B_{A,t-1}^{A}}{P_{A,t}} + (1 + i_{B,t-1}) Q_{t} \frac{B_{A,t-1}^{B}}{P_{B,t}} + \Pi_{A,t} + w_{A,t} L_{A,t}^{S} - C_{A,t}$$
(15)

and

$$\frac{B_{B,t}^{B}}{P_{B,t}} + \frac{B_{B,t}^{A}}{Q_{t}P_{A,t}} = (1+i_{B,t-1})\frac{B_{B,t-1}^{B}}{P_{B,t}} + (1+i_{A,t-1})\frac{B_{B,t-1}^{A}}{Q_{t}P_{A,t}} + \Pi_{B,t} + w_{B,t}L_{B,t}^{S} - C_{B,t}$$
(16)

Given bond market clearing conditions (10) and (11), if we multiply (16) by Q_t and subtract the resulting equation from (15), we obtain an expression for country *A*'s net foreign assets in terms of country *A*'s interest income and differences between profits, labour income, and consumption across countries as shown below:

$$\frac{B_{A,t}^{A}}{P_{A,t}} + Q_{t} \frac{B_{B,t}^{B}}{P_{B,t}} = (1 + i_{A,t-1}) \frac{B_{A,t-1}^{A}}{P_{A,t}} + (1 + i_{B,t-1}) Q_{t} \frac{B_{A,t-1}^{B}}{P_{B,t}} + (1 - n)(\Pi_{A,t} - Q_{t}\Pi_{B,t}) + (1 - n)(w_{A,t}L_{A,t}^{S} - Q_{t}w_{B,t}L_{B,t}^{S}) - (1 - n)(C_{A,t} - Q_{t}C_{B,t})$$

The current account in the two countries, which by definition equals changes in aggregate bond holdings, is given by the following expressions:

$$CA_{A,t} \equiv n\left(\frac{B_{A,t}^{A}}{P_{A,t}} - \frac{B_{A,t-1}^{A}}{P_{A,t}}\right) + nQ_{t}\left(\frac{B_{A,t}^{B}}{P_{B,t}} - \frac{B_{A,t-1}^{B}}{P_{B,t}}\right)$$
$$CA_{B,t} \equiv (1-n)\left(\frac{B_{B,t}^{B}}{P_{B,t}} - \frac{B_{B,t-1}^{B}}{P_{B,t}}\right) + (1-n)Q_{t}^{-1}\left(\frac{B_{B,t}^{A}}{P_{A,t}} - \frac{B_{B,t-1}^{A}}{P_{A,t}}\right)$$

Given bond market clearing conditions (10) and (11), we can show that one country's lending equals the other country's borrowing, $CA_{A,t} + Q_tCA_{B,t} = 0$, and world consumption equals world profits plus world labour income, $C_{A,t} + Q_tC_{B,t} = \prod_{A,t} + Q_t\prod_{B,t} + w_{A,t}L_{A,t}^S + Q_tw_{B,t}L_{B,t}^S$.

2.4 Model variables and equations

All in all we have 36 endogenous variables determined during time t: $P_{A,t}$, $P_{B,t}$, $p_{A,t}^A$, $p_{B,t}^B$, $B_{A,t}^A$, $B_{B,t}^B$, $B_{B,t}^B$, $B_{B,t}^A$, $i_{A,t}$, $i_{B,t}$, $r_{A,t}$, $r_{B,t}$, $C_{A,t}$, $C_{B,t}$, $Y_{A,t}^S$, $Y_{B,t}^D$, $Y_{B,t}^S$, $L_{A,t}^D$, $L_{A,t}^S$, $L_{B,t}^S$, $L_{B,t}^D$, $w_{A,t}$, $w_{B,t}$, $\pi_{A,t}$, $\pi_{B,t}$, $\Delta_{A,t}$, $\Delta_{B,t}$, $\psi_{A,t}$, $\psi_{B,t}$, $\Upsilon_{A,t}$, $\Upsilon_{B,t}$, $\eta_{A,t}$, $\eta_{B,t}$, Q_t , ξ_t . There are eight endogenous state variables that are predetermined as of time t: $i_{A,t-1}$, $i_{B,t-1}$, $p_{B,t-1}^A$, $B_{A,t-1}^A$, $B_{A,t-1}^B$, $B_{A,t-1}^B$, $B_{B,t-1}^B$. The 36 endogenous variables are determined by a system of 36 equations summarised in Table A.

2.5 The (symmetric) steady state

We denote steady-state levels of variables with over-bars. Given that the steady-state levels of the productivity, demand, and quality parameters are set to one, the steady-state levels of real variables are shown below:

$$\overline{r}_{A} = \overline{r}_{B} = \frac{1-\beta}{\beta}$$

$$\overline{w}_{A} = \overline{w}_{B} = \overline{\Psi}_{A} = \overline{\Psi}_{B} = \frac{\psi-1}{\psi}$$

$$\overline{\Delta}_{A} = \overline{\Delta}_{B} = 0$$

$$\overline{Y}_{A} = \overline{Y}_{B} = \overline{C}_{A} = \overline{C}_{B} = \overline{L}_{A} = \overline{L}_{B} = \frac{\psi-1}{2\psi-1}$$

$$\overline{\Pi}_{A} = \overline{\Pi}_{B} = \frac{\psi-1}{\psi(2\psi-1)}$$

$$\overline{Q} = 1$$

Given steady-state levels of money supply $\overline{M}_{A,t} = \overline{M}_{B,t} = \frac{\chi}{1-\beta}$, the steady-state levels of nominal variables are given by:

$$\bar{i}_A = \bar{i}_B = \frac{1-\beta}{\beta}$$
$$\bar{\xi} = 1$$
$$\bar{p}_A^A = \bar{p}_B^B = \bar{P}_A = \bar{P}_B = \frac{2\psi - 1}{\psi - 1}$$

3 Impulse responses

In this section we use our calibrated model to carry out some experiments. We provide intuition for the results of the model when country *A* experiences a temporary but prolonged improvement

in quality. We also compare the impact of a quality shock to that of a productivity and a relative demand shock. We consider the following benchmark parameter values that are standard in the literature: n = 0.5, $\beta = 0.99$ (this choice of β implies an annual steady-state real interest rate of 4%), $\psi = 10$ (this choice of ψ implies a steady-state mark-up of 10%), $\theta = 2$, $\tau = 0.01$, $\phi = 0$ (if prices are flexible) and $\phi = 200$ (if prices are sticky and this value of ϕ would imply that price inflation of 1% would generate a resource cost of 1% of per capita real output), $\alpha_A = \alpha_B = 1.5$ (the case of home bias in consumption), and $\chi = 0.004$. A period corresponds to a quarter in our exercise. We use standard log-linearisation techniques in order to analyse the dynamics of the model. Namely, we take a first-order Taylor approximation of model's equations around the non-stochastic steady state for which we solved in the previous section. The `above a variable denotes its percentage deviation from this steady state. The linearised model equations are shown in Table B.

3.1 Some definitions

Before we turn to the discussion of results, it is important to realise that in our analysis two definitions for the terms of trade and the real exchange rate emerge: the quality-adjusted versus the unadjusted measures.

• The quality-adjusted terms of trade in country A is the ratio of the price per quality unit of country A imports to the price per quality unit of country A exports (both expressed in terms of country A's currency). We denote this measure as $TOT_t \equiv \xi_t p_{B,t}^B q_{A,t}/p_{A,t}^A q_{B,t}$. An increase in TOT_t is a worsening in country A's terms of trade defined in terms of prices per quality unit.

• The quality-unadjusted terms of trade in country A is the ratio of the unit price of country A imports to the unit price of country A exports (both expressed in terms of country A's currency). We denote this measure as $TOT_{-q,t} \equiv \xi_t p_{B,t}^B / p_{A,t}^A$. An increase in TOT_t is a worsening in country A's terms of trade defined in terms of unit prices.

• The quality-adjusted real exchange rate is the consumption-based real exchange rate defined in terms of prices per quality unit. We previously denoted this measure $Q_t \equiv \xi_t P_{B,t}/P_{A,t}$, where price indices are based on prices per quality unit. An increase in Q_t represents a real depreciation of the quality-adjusted real exchange rate and implies that goods are becoming relatively cheaper in country A in terms of their prices per quality unit.

• The quality-unadjusted real exchange rate is the consumption-based real exchange rate defined in terms of unit prices. We denote this measure as $Q_{-q,t} \equiv \xi_t P_{-q,B,t}/P_{-q,A,t}$, where these price indices are now based on unit prices of products. That is:

$$P_{-q,A,t} = \left[n\alpha_A \delta^{\frac{1}{2}} (P^A_{-q,A,t})^{1-\theta} + (1 - n\alpha_A) \delta^{\frac{-1}{2}} (P^B_{-q,A,t})^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

where:

$$P^{A}_{-q,A,t} = \left[\frac{1}{n} \int_{0}^{n} (p^{A}_{A,t}(z))^{1-\psi} dz\right]^{\frac{1}{1-\psi}}$$
$$P^{B}_{-q,A,t} = \left[\frac{1}{1-n} \int_{n}^{1} (p^{B}_{A,t}(z))^{1-\psi} dz\right]^{\frac{1}{1-\psi}}$$

And similarly for $P_{-q,B,t}$. An increase in $Q_{-q,t}$ represents a real depreciation and implies that goods are becoming relatively cheaper in country A in terms of their unit prices.

3.2 A temporary quality shock in country A

Charts 1-4 show the response of model variables to a quality shock originating in country A. Here, we consider a temporary but prolonged 1% increase in the level of product quality in country A with persistence parameter 0.7, therefore the shock has no permanent impact. After the quality shock, we assume that the commodity space remains unchanged. That is, after the quality improvement, the new (higher quality) products completely replace the old (lower quality) products. We consider three cases: (1) when the quality improvement has no impact on unit costs of production, ie $\gamma = 1$; (2) when the quality improvement leads to an increase in unit costs of production, ie $\gamma < 1$; and (3) when the quality improvement leads to a decrease in unit costs of production, ie $\gamma > 1$.

Case 1. The quality shock has no impact on unit costs of production (Chart 1)

Under this scenario an improvement in quality does not affect unit costs of production. That is, $Z_{A,t}L_{A,t}$ units of effective labour are required to produce one unit of the product, regardless of the level of quality. An increase in quality in country *A* causes the quality-adjusted unit price in country *A* to fall relative to that in country *B*, and this leads to an increase in world demand for country *A*'s goods. The increase in demand allows profit-maximising producers in country *A* to increase their unit prices but quality-adjusted prices still fall since unit prices are sticky. Hence,

the CPI (defined in terms of quality-adjusted prices) in country A initially falls. The now lower quality-adjusted price in country A leads to higher quality-augmented consumption and output in country A. Consequently, country A firms increase their demand for labour leading to a rise in the real wage in country A. The mark-up charged by country A firms falls reflecting this rise in the real wage rate. With the shift in world demand towards country A's goods, production in country B falls, and so country B consumers become less well off as their production income is now lower. Country *B* consumers borrow from abroad to finance their increased consumption of country *A*'s goods and country A accumulates net foreign assets and the current account goes into surplus. Also, the increase in world demand for country A's goods leads to an appreciation in the nominal exchange rate. The real exchange rate defined in terms of quality-adjusted prices depreciates as goods in country A become relatively cheaper in quality-adjusted terms. Likewise, the terms of trade in country A defined in terms of quality-adjusted prices worsens. On the contrary, the real exchange rate (and the terms of trade in country A) defined in terms of unit prices appreciates (improves) reflecting the relative increase in the unit price of country A goods. Hence, there is a dichotomy between the responses of different measures of the real exchange rate and the terms of trade. These effects are reversed as the impact of the shock dissipates.

Case 2. The quality shock leads to an increase in unit costs of production (Chart 2)

Under this scenario, an improvement in quality does affect unit costs of production. Here, the supply-side effects of the quality shock operates similarly to a temporary negative productivity shock (opposite to the case discussed in Section 3.3 below). That is, one unit of the now higher quality product requires more labour input. For simplicity, we assume that all firms in country A will produce the now higher quality product, irrespective of its cost implications. Hence, as country A moves up the quality ladder, the marginal cost of production rises. To reflect this cost rise, profit-maximising firms increase their optimal unit price by more than they did under the previous scenario. As a result, the quality-adjusted price in country A still increase, but also by less than in the previous case. However, as γ decreases, quality improvements have an increasing effect on production costs. For instance, for $\gamma = -15$, the improvement in quality has such a substantial impact on the costs of production that the resulting increase in the unit price outweighs the increase in quality leading to an increase in quality-adjusted prices in country A. ⁽⁴⁾ As a result,

⁽⁴⁾ Notice that a 1% improvement in the level of quality, other things equal, will increase the quality-adjusted price in country A by 15%.

quality-augmented consumption and output in country *A* fall, as both country *A* and country *B* consumers demand less of the now more expensive goods produced by country *A*. Despite the fall in output, country *A* firms increase their demand for labour as now a unit production of higher quality good requires more labour services. Households also increase their supply of labour as they must now work more to afford a unit of their consumption. The real wage rate rises as the rise in demand for labour outweighs the rise in supply. The mark-up charged by country *A* firms decreases reflecting the increase in the real wage rate. Despite the rise in labour input, quality-augmented output supplied in country *A* falls. Country *A*'s current account goes into deficit and the nominal exchange rate initially depreciates as demand switches away from country *A* goods. The real exchange rate, defined in quality-adjusted and unadjusted terms, appreciates. Likewise, we see an improvement in both measures of the terms of trade in country *A*. These effects are reversed as the effect of the shock dissipates.

Case 3. Unit costs of production are lower for higher quality products (Chart 3)

Here, as country *A* moves up the quality ladder, the marginal cost of production actually falls. This is because one unit production of the now higher quality good requires less labour input. In this case, profit-maximising firms will reduce their optimal unit prices to reflect the fall in costs. As a result, the quality-adjusted price in country *A* falls by more than they do in the first case (ie production costs are independent of the level of quality). The higher fall in quality-adjusted prices under this scenario, causes quality-augmented consumption and output to rise further. The current account moves into a larger surplus position and the nominal exchange rate appreciates further. The real exchange rate, defined in both quality-adjusted and unadjusted terms, depreciates by more and the terms of trade in country *A*, defined in both quality-adjusted and unadjusted terms, worsens by a larger amount. These effects are reversed as the effect of the shock dissipates.

In Chart 4 we show the response of both measures of the real exchange rate to the quality shock under different values of γ . A few interesting points emerge. First, the figure shows that a quality improvement can lead to either a depreciation or an appreciation of either measure of the real exchange rate depending on how costs of production are affected by the quality improvement. Second, the real exchange rate defined in terms of unit prices does not always move in the same direction as the real exchange rate defined in terms of prices per quality unit, illustrating the importance of measuring quality correctly.

3.3 A temporary productivity shock in country A (Chart 5)

We now compare these findings to a typical labour productivity shock in country *A*. Chart 5 presents the responses to a temporary but prolonged 1% increase in productivity in country *A* with persistence parameter 0.7.

Country *A* is now temporarily endowed with more units of effective labour. All firms in country *A* experience a fall in their marginal cost of production and so they lower unit prices. Given that the level of quality has not changed in this scenario, quality-adjusted and unadjusted unit prices (along with other variables) are the same and so fall by the same amount. Hence, the CPI in country *A* falls initially reflecting this reduced cost of production. Consumption and output in country *A* rises reflecting the increased world demand for the now cheaper country *A* goods. Country *A* firms reduce their demand for labour in response to the shock and households decrease their supply of labour as they now can afford to take more leisure. Overall, labour input falls. The real wage rate in country *A* falls and the mark-up charged by country *A* firms increases. Since the productivity shock is temporary, households, in order to smooth consumption, will lower their current consumption by lending abroad. So, country *A* runs a current account surplus and accumulates net foreign assets. The nominal exchange rate appreciates at first reflecting the higher demand for country *A* worsens also by an equal amount. These effects are reversed as the effect of the shock dissipates.

The positive productivity shock is similar to the quality shock in terms of its qualitative impact on model variables when the improvement in quality substantially reduces units costs of production as in Case 3. This result is mainly due to our assumption that product quality is exogenous in the model and that producers manufacture the new higher quality products irrespective of its cost implications. Quantitatively speaking, we see that the quality shock in Case 3 has a greater impact on model's variables when compared to a positive productivity shock. This result is driven by our calibration of the parameter γ and the fact that the quality-adjusted variables in the case of a quality shock reflect the additional direct impact of an improvement in quality. However, when the quality shock does not affect unit production costs as in Case 1, the comparison of the response of model variables across these two types of shocks is not that systematic. For instance, although unit prices fall under the productivity shock, they rise following the quality shock. This is because the

quality shock results in incentives for producers to increase their unit prices in order to capture profit opportunities. But, quality-augmented consumption and output, and the CPI (defined in quality-adjusted prices) behave similarly across these two shocks. This is because, following the quality shock, the quality-adjusted price in country *A* falls despite the rise in unit prices, due to unit prices being sticky. And it is this divergence in the response of quality-adjusted and quality-unadjusted prices that results in a dichotomy in the response of the two different measures of the real exchange rate to a quality shock, a result which we do not find in the case of a productivity shock: we see a depreciation of the real exchange rate following an improvement in productivity and quality, but, in the latter case, it is only the real exchange rate defined in terms of quality-adjusted prices that depreciates while the other measure shows an appreciation.

3.4 A temporary relative demand shock towards country A's goods (Chart 6)

We now compare the findings in Subsection 3.2 to a worldwide preference shock towards country *A*'s goods. Chart 6 presents the responses to a temporary but prolonged 1% increase in relative demand towards country *A*'s goods with persistence parameter 0.7.

A shift in the worldwide preference towards goods produced in country *A* results in a rise in consumption and output in country *A* and an appreciation of the nominal exchange rate. As demand increases for country *A* goods, profit-maximising producers in country *A* increase their unit prices. Note that since quality has not changed in this scenario, quality-adjusted and unadjusted prices are the same (along with other variables). The CPI in country *A* falls reflecting the effects of cheaper imports from country *B* and home bias. The real exchange rate, defined in both adjusted and unadjusted terms, depreciates. Likewise, both measures of the terms of trade in country *A* improves.

Like a quality shock, the relative demand shock we consider here shifts the world demand towards country *A*'s goods. However, while an improvement in quality can have implications on costs of production the relative demand shock only affects the demand side of the economy with no effect on unit costs of production. Hence, comparing Case 1 under a quality shock to the relative demand shock, we see that the qualitative response of the model's variables are similar across the types of shocks, with the exception of the quality-adjusted price, which falls under a quality shock, and the quality-unadjusted real exchange rate, which appreciates under a quality shock.

4 Conclusion

This paper analyses the theoretical link between improvements in product quality and real exchange rates. To our knowledge, the NOEM literature has not yet studied the implications of this type of technological progress, and this paper aims to fill this gap. We consider a standard two-country (country A and country B) NOEM model with sticky prices and home bias in consumption where labour is the only factor of production. To this, we add the new feature that goods in one country can be produced at a different quality level to those produced in another country. In our analysis, exogenous quality improvements affect both the demand and the supply side of the economy. On the demand side, quality enters the model through consumers' preferences. On the supply side, quality enters the model through its effect on production costs. All producers are assumed to produce the new higher quality product, irrespective of its cost implications. The real exchange rate is defined as the ratio of the two countries' price indices, expressed in units of country A's currency. Two different definitions for the real exchange rate emerge in our analysis: the *quality-adjusted* and the *quality-unadjusted* real exchange rate. We measure the former using quality-adjusted prices (or prices per quality unit) and the latter using quality-unadjusted prices (or unit prices). Our analysis shows that, following a quality shock, the response of the real exchange rate essentially depends on how quality improvements affect production costs and that this response differs across the two measures. That is, after a quality shock in country A, the real exchange rate defined in terms of prices per quality unit depreciates (implying goods are becoming relatively more expensive in country *B* in terms of their prices per quality unit) if quality improvements do not affect unit costs of production and it depreciates even further if costs of production are lower for higher quality goods. However, if unit costs increase sufficiently as higher quality goods are produced, then the quality-adjusted real exchange rate appreciates. The above conclusions will not hold if we instead take the quality-unadjusted real exchange rate as our reference. That is, the real exchange rate defined in terms of unit prices appreciates (implying goods are becoming relatively more expensive in country A in terms of their unit prices) if quality improvements do not affect unit costs of production and it appreciates even further if costs of production are higher for higher quality products. However, if unit costs decrease as higher quality goods are produced, then the quality-unadjusted real exchange rate depreciates. Finally, we find that a quality shock produces qualitatively similar results, in terms of responses of model variables, to a positive productivity shock originating in country A, when the improvement in quality leads to a reduction in unit costs of production, and to a relative demand

shock where global demand shifts towards country *A*'s goods, when costs of production are the same regardless of the quality level of products.

Between 1996 Q1 and 2006 Q1, sterling's effective real exchange rate appreciated by over 22%. This real appreciation could have reflected a quality shock. Supposing the United Kingdom did start producing higher quality goods in the mid-late 1990s, then our model suggests an appreciation in both measures of the real exchange rate if quality improvements result in sufficiently higher costs of production. But if costs of production are independent of improvements in product quality, our analysis suggests the two measures of the real exchange rate would react differently – that is, the real exchange rate based on price indices that do not account for quality improvements would appreciate, while the real exchange rate calculated using price indices that do account for the quality level of products would depreciate. Crucially, it depends whether price indices are mismeasured in this way, which is hard to assess. But one possibility is that the United Kingdom did start producing higher quality goods, but that was not reflected in the real exchange rate due to those types of data issues.

There are a number of possible extensions to this work. One simplifying assumption in our analysis is that improvements in product quality are treated as exogenous and that producers always manufacture the new higher quality product, irrespective of its cost implications. We could alternatively consider a case in which an improvement in the quality of goods arises as a result of research and development activities, and thus where producers do undertake quality improvements if and when potential profits outweigh research costs. We could also study a case in which quality improvements occur in the intermediate good sector as opposed to the final goods sector. This would be particularly interesting since product innovations in the intermediate goods sector can in turn lead to process innovations in the final goods sector. And, consequently, this might imply a dichotomy between the behaviour of the real exchange rate calculated using producer prices and that calculated using consumer prices. We could also include a non-tradable goods sector and examine whether a quality shock brings about Balassa-Samuelson effects. Finally, there could be further empirical analysis of whether the United Kingdom did actually move up the quality ladder in the mid-late 1990s and, if it did, to estimate to what extent this contributed to the subsequent movements in the real exchange rate.

Tables and charts



Chart 1: The impact of a temporary positive quality shock originating in country A ($\gamma = 1$)



Chart 2: The impact of a temporary positive quality shock originating in country A ($\gamma=-15$)



Chart 3: The impact of a temporary positive quality shock originating in country A ($\gamma=15$)

Chart 4: The sensitivity of quality-adjusted and unadjusted real exchange rate to variations in γ





Chart 5: The impact of a temporary positive productivity shock originating in country A



Chart 6: The impact of a temporary relative demand shock towards country A goods

Abbreviations are used for consumer prices index (CPI), producer price (PP), import price (MP), mark-up (MUP), demand (D), supply (S), first-order condition (FOC), labour supply (LS), bonds (B), money balances (M), profits (PRF), price adjustment cost (PAC), net foreign assets (NFA), real exchange rate (RER), market clearing (MC), and Fisher parity condition (FPC).

Table A: Model equations

Table B: Log-linearised model equations

Id.	$\hat{P}_{A,t} = n\alpha_A(\hat{p}_{A,t}^A - \hat{q}_{A,t}) + (1 - n\alpha_A)(\hat{\xi}_t + \hat{p}_{B,t}^B - \hat{q}_{B,t}) + \frac{2n\alpha_{A-1}}{2(1-\theta)}\hat{\delta}_t$	PRF	$\hat{\Pi}_A = \frac{Y_A}{\overline{\Pi}_A} (\hat{p}_{A,t}^A - \hat{P}_{A,t} + \hat{Y}_{A,t}^S) - \frac{\overline{w}_A L_A}{\overline{\Pi}_A} (\hat{w}_{A,t} + \hat{L}_{A,t}^D)$
	$\begin{split} \hat{P}_{B,t} &= [1 - (1 - n)\alpha_B)](\hat{p}_{A,t}^A - \hat{q}_{A,t} - \hat{\xi}_t) + (1 - n)\alpha_B(\hat{p}_{B,t}^B - \hat{q}_{B,t}) + \frac{1 - 2(1 - n)\alpha_B}{2(1 - \theta)}\hat{\delta}_t\\ \hat{p}_{A,t}^A - \hat{P}_{A,t} &= \hat{\Psi}_{A,t} + \hat{\psi}_{A,t} - \hat{Z}_{A,t} - (\gamma - 1)\hat{q}_{A,t} \end{split}$	NFA	$\hat{\Pi}_{B} = \frac{Y_{B}}{\Pi_{B}} (\hat{\rho}_{B,t}^{B} - \hat{P}_{B,t} + \hat{Y}_{B,t}^{S}) - \frac{w_{B}L_{B}}{\Pi_{B}} (\hat{w}_{B,t} + \hat{L}_{B,t}^{D}) \\ \frac{1}{\bar{P}_{A}} \hat{B}_{A,t}^{A} + \frac{1}{\bar{P}_{B}} \hat{B}_{A,t}^{B} = \frac{1}{\beta \bar{P}_{A}} \hat{B}_{A,t-1}^{A} + \frac{1}{\beta \bar{P}_{B}} \hat{B}_{A,t-1}^{A}$
JP	$\hat{p}^B_{B,t} - \hat{P}_{B,t} = \hat{\Psi}_{B,t} + \hat{w}_{B,t} - \hat{Z}_{B,t} - (\gamma - 1)\hat{q}_{B,t}$ $\hat{\Psi}_{A,t} = rac{-\phi}{3i-1}[(1+eta)\hat{p}^A_{A,t} - \hat{p}^A_{A,t-1} - eta\hat{p}^A_{A,t-1}]$		$ + (1-n)\frac{\Pi_A}{\bar{C}^A}\hat{\Pi}_{A,t} - (1-n)\frac{\Pi_B}{\bar{C}^B}(\hat{\Pi}_{B,t} + \hat{Q}_t) + (1-n)\overline{w}_A(\hat{w}_{A,t} + \hat{L}^D_{A,t}) - (1-n)\overline{w}_B(\hat{w}_{B,t} + \hat{L}^D_{B,t} + \hat{Q}_t) $
	$\hat{\Psi}_{B,t} = \frac{-\hat{\phi}_{t-1}}{\psi^{-1}} [(1+\beta)\hat{p}_{B,t}^{B} - \hat{p}_{B,t-1}^{B} - \beta\hat{p}_{B,t+1}^{B}]$ $\hat{\eta}_{A,t} + \hat{\gamma}_{A,t}^{A} = -\theta(\hat{p}_{A,t}^{A} - \hat{q}_{A,t}) + \theta\hat{P}_{A,t} + n\alpha_{A}\hat{C}_{A,t} + \frac{1-n}{n} [1 - (1-n)\alpha_{B}](\hat{C}_{B,t} + \theta\hat{Q}_{t}) + \frac{1}{2}\hat{\delta}_{t-1}$	RER	$-(1-n)\hat{C}_{A,t} + (1-n)(\hat{C}_{B,t} + \hat{Q}_t)$ $\hat{Q}_t = \hat{\xi}_t + \hat{P}_{B,t} - \hat{P}_{A,t}$
	$\begin{split} \hat{q}_{B,t} + \hat{Y}_{B,t}^D &= -\theta(\hat{p}_{B,t}^B - \hat{q}_{B,t}) + \theta\hat{P}_{B,t} + \frac{n}{1-n}(1-n\alpha_A)(\hat{C}_{A,t} - \theta\hat{Q}_t) + (1-n)\alpha_B\hat{C}_{B,t} - \frac{1}{2}\hat{\delta}_t \\ \hat{Y}_{A,t}^S &= (\gamma - 1)\hat{q}_{A,t} + \hat{Z}_{A,t} + \hat{L}_{A,t}^D \end{split}$	MC	$\hat{Y}_A^S = \hat{Y}_A^D = \hat{Y}_{A,t}$ $\hat{Y}_B^S = \hat{Y}_B^D = \hat{Y}_{B,t}$
ST:D	$\hat{Y}_{B,t}^{S'} = (\gamma - 1)\hat{q}_{B,t} + \hat{Z}_{B,t} + \hat{L}_{B,t}^D$ $\hat{L}_{A,t}^S = rac{-\overline{CA}}{w \cdot L^A} (\hat{C}_{A,t} - \hat{w}_{A,t})$		$\hat{L}_A^S = \hat{L}_A^D = \hat{L}_{A,t}$ $\hat{L}_B^S = \hat{L}_B^D = \hat{L}_{B,t}$
)C:B	$\begin{split} \hat{L}_{B,t}^{S} &= \frac{-\overline{C} \hat{B}}{\overline{w_B} L_B} (\hat{C}_{B,t} - \hat{w}_{B,t}) \\ \hat{C}_{A,t+1} &+ \tau \overline{C}_A \hat{B}_{A,t}^A = \hat{C}_{A,t} + \hat{R} \hat{R}_{A,t} \end{split}$		$egin{array}{l} n \hat{B}^A_{A,t} + (1-n) \hat{B}^A_{B,t} = 0 \ n \hat{B}^A_{A,t} + (1-n) \hat{B}^B_{B,t} = 0 \end{array}$
DC:M	$\begin{split} \hat{C}_{A,t+1} + \tau \overline{C}_{A} \hat{B}_{A,t}^{A} = \hat{C}_{A,t} + \hat{R} R_{B,t} + (\hat{Q}_{t+1} - \hat{Q}_{t}) \\ \hat{C}_{B,t+1} + \tau \overline{C}_{B} \hat{B}_{B,t}^{A} = \hat{C}_{B,t} + \hat{R} R_{B,t} \\ \hat{C}_{B,t+1} + \tau \overline{C}_{B} \hat{B}_{B,t}^{A} = \hat{C}_{B,t} + \hat{R} \hat{R}_{A,t} + (\hat{Q}_{t} - \hat{Q}_{t+1}) \\ \hat{M}_{A,t} - \hat{P}_{A,t} = \hat{C}_{A,t} - \frac{\beta}{1-\beta} \hat{R}_{A,t} + \frac{1-\beta}{1-\beta} \hat{R}_{A,t} + \frac{1-\beta}{1-\beta} \hat{R}_{A,t}^{A} \end{split}$	FPC	$\hat{R}R_{A,t} = \hat{R}_{A,t} + \hat{P}_{A,t} - \hat{P}_{A,t+1}$ $\hat{R}R_{B,t} = \hat{R}_{B,t} + \hat{P}_{B,t} - \hat{P}_{B,t+1}$
	$\vec{M}_{B,t} - \vec{P}_{B,t} = \vec{C}_{B,t} - \frac{\beta}{1-\beta}\vec{R}_{B,t} + \frac{\beta}{1-\beta}\tau\vec{B}_{B,t}^B$		

[1] Abbreviations are used for consumer prices index (CPD), producer price (PP), import price (MP), mark-up (MUP), demand (D), supply (S), first-order condition (FOC), labour supply (LS), bonds (B), money balances (M), profits (PRF), price adjustment cost (PAC), net foreign assets (NFA), real exchange rate (RER), market clearing (MC), and Fisher parity condition (FPC). [2] $\hat{R}_{A,t} \cong (i_{A,t} - \bar{i}_A)$, $\hat{R}R_{A,t} \cong (r_{A,t} - \bar{i}_A)$.

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