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Dealing with country diversity: challenges for the IMF credit union model

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Gregor Irwin,[1] Adrian Penalver,[2] Chris Salmon[3] and Ashley Taylor[4]

Abstract

We develop a model in which countries can protect themselves against shocks by subscribing to a credit union that shares the key features of the International Monetary Fund, or by self-insuring through accumulating reserves. We assess the impact of the increasing heterogeneity of the Fund's membership on the political equilibrium Fund size and hence its effectiveness as a credit union. We find the Fund's existing lending framework is well suited to a world in which its members have homogeneous interests, but as the membership has become more heterogeneous the Fund is increasingly unlikely to provide financing on a sufficient scale to meet the demands of higher-risk members, leading them to rely more heavily on self-insurance. We conclude that the framework governing the Fund's lending operations may no longer be appropriate.

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Summary

This paper assesses the efficiency of the International Monetary Fund's (IMF's) lending framework using a simple, yet novel theoretical model of the IMF as a credit union, in which the membership decides collectively by a vote on the size of the Fund and hence the amount of crisis lending it can provide. This decision, in turn, impacts on individual country choices over the amount of self-insurance to hold in the form of reserves. The equilibrium Fund size and individual country reserve choices are analysed under three different characterisations of the Fund's decision-making processes – unconstrained majority voting, constrained majority voting, and qualified majority voting with an agenda setter. The welfare implications in each case are assessed and we consider how the existence of spillovers between countries affects the outcome.

In all cases the analysis suggests the present IMF lending framework may no longer be appropriate. It may well have been during the first two to three decades of the Fund's existence, when almost all countries were potential Fund debtors and had broadly homogenous interests, but the analysis suggests it is much less well suited to the current situation in which members differ sharply in their economic characteristics and needs. In particular, we find that with an increasingly heterogeneous membership, in terms of crisis probabilities, the decisions over the size of the Fund are likely to be driven by members with a relatively low crisis probability. Consequently, the Fund is increasingly unlikely to provide financing on a sufficiently large scale to meet the demands of higher-risk members, leading them to rely more heavily on self-insurance. The analysis suggests that increasing the size of the Fund may be Pareto improving, but only if the financial burden is distributed so that those who benefit most – that is, the countries which have the highest crisis probability – pay the most. This would constitute a significant change in the financing of the Fund's lending operations.

The main message of the paper is that the framework governing the Fund's lending operations may no longer be appropriate. An alternative approach may be needed: one which takes into account that creditor and debtor countries have different interests, but which also takes into account the moral hazard consequences of large-scale lending.

1 Introduction

The creation of the International Monetary Fund (IMF) in 1946 was a political solution to the economic challenge of ensuring international monetary co-operation. The IMF was placed at the apex of a monetary system based on fixed-but-adjustable exchange rate pegs, with responsibility for managing the system. Importantly, the IMF was provided with financial resources so that it could ease the adjustment burden for countries experiencing temporary macroeconomic disequilibria. This lending role was necessary to ensure that countries had the appropriate incentives to eschew non co-operative behaviour and abide by the Fund's rulings.

Since 1946 the international monetary system, the Fund's oversight role in relation to it, and the composition of the Fund's membership have all changed markedly. The system of fixed-but-adjustable exchange rate pegs broke down in the 1970s. Since then floating exchange rate regimes have become much more widespread. The focus of the Fund has since shifted from managing the system to surveillance over the system, to ensure that policymakers take informed decisions, cognisant of the policy challenges faced by other countries and the economic linkages between countries. The membership of the Fund has more than quadrupled since its inception, expanding from a club of 44 industrialised countries in 1946, to become a near-universal institution with 185 members in 2007.

But during the same period the basic framework governing the Fund's lending operations has undergone much less change. It is still essentially a type of credit union into which countries pay a quota (or subscription) to become a member. Countries experiencing an adverse economic shock are entitled, under certain restrictions, to draw down their quota and temporarily borrow money from the Fund. The drawing (or access) right of each member is proportional to the size of its quota. Importantly, the overall size of the Fund, which determines how much crisis lending is available in aggregate, is voted on by the membership every five years.

This paper seeks to assess the efficiency of this credit union model, given the existing political decision-making process, and in the light of the changes in the Fund's membership that have occurred over its lifetime. We do this using a simple, yet novel theoretical model of the IMF as a credit union, in which the membership decides collectively by a vote on the size of the Fund and hence the amount of crisis lending it provides. This decision, in turn, impacts on individual country choices over the amount of self-insurance to hold in the form of reserves. The equilibrium Fund size and individual country reserve choices are analysed under three different characterisations of the decision-making processes – unconstrained majority voting, constrained majority voting, and qualified majority voting with an agenda setter. The welfare implications in each case are assessed and we consider how the existence of spillovers between countries affects the outcome.

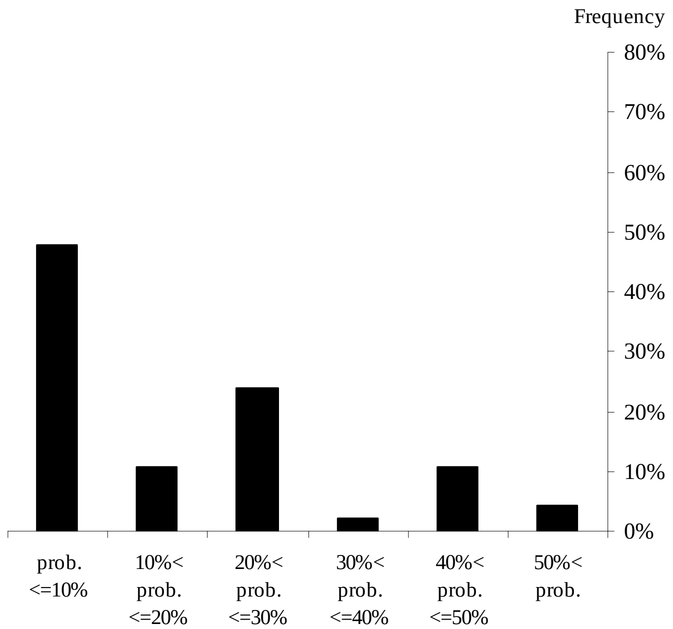
In all cases the analysis suggests the current lending framework of the Fund may no longer be appropriate. It may well have been during the first two to three decades of the Fund's existence, when almost all countries were potential Fund debtors and had broadly homogenous interests, but the analysis suggests it is less well suited to the current situation in which members differ sharply in their economic characteristics and needs.[1] In particular, we find that with an increasingly heterogeneous membership, in terms of crisis probabilities, the decisions over the size of the Fund are likely to be driven by members with a relatively low crisis probability. Consequently, the Fund is increasingly unlikely to provide financing on a sufficient scale to meet the demands of higher-risk members, leading them to rely more heavily on self-insurance. The analysis suggests that increasing the size of the Fund may be Pareto improving, but only if the financial burden is distributed so that those who benefit most – that is, the countries which have the highest crisis probability – pay the most. This would constitute a significant change in the financing of the Fund's lending operations.

Our analysis is consistent with some of the more striking recent global economic developments, although clearly there are other potential explanations. First, it predicts that as the Fund's members become more diverse, then those countries most at risk of experiencing a payments imbalance will increasingly self-insure and hold more reserves. Second, it suggests that the development of intra-regional coinsurance mechanisms, such as the Chaing Mai initiative, are a natural artefact of the increasingly diverse interests of the Fund's membership, to the extent that the second-round spillover effects of crises are stronger within than across regions. Finally, the analysis is consistent with the trend increase in the average size of financial assistance granted by the Fund that has been observed over the past 30 years, and with the concerns about moral hazard that this trend has generated.

The increase in the diversity of the Fund membership is illustrated in Charts 1 and 2 which show

[1]Between 1944 and 1977 industrialised and developing countries turned to the Fund for financial assistance. During that period five of the future G7 members borrowed from the Fund, some repeatedly so. (The US and Germany are the exceptions.) But since 1977 the membership has become bifurcated.

Chart 1: Annual probabilities of borrowing from the Fund: 1951-65 (for members at end-1950)



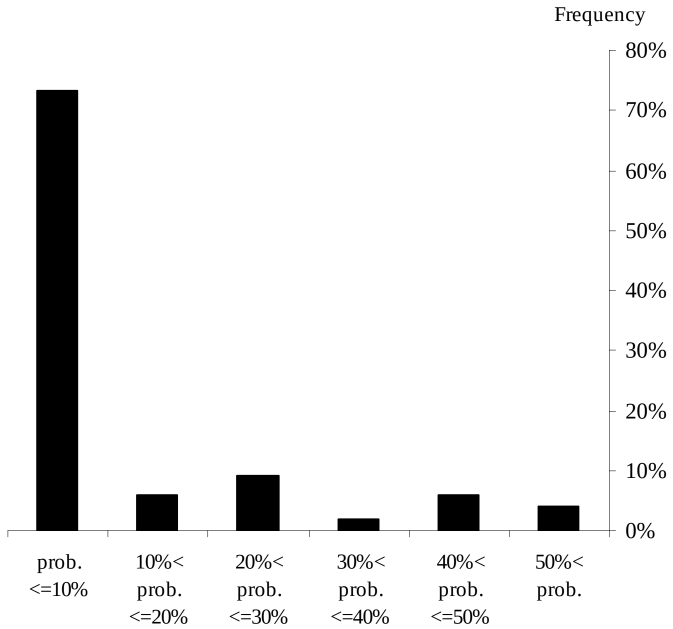
Sources: IMF *International Financial Statistics* (*IFS*) and authors' calculations.
Notes: Probability indicates the annual average probability of country making a purchase from the IMF General Resources Account, excluding reserve tranches, over the period. The sample of member countries is those for which *IFS* indicates non-zero quota at end-1950.

how Fund lending activity has become increasingly concentrated on a (albeit still large) subset of the membership. This indicates that over the lifetime of the Fund the mean crisis probability of the membership has fallen, but the median crisis probability has fallen even faster. The distribution of crisis vulnerabilities has therefore become more skewed.

Chart 3 shows how the Fund has shrunk relative to GDP since its formation. In the first two decades since its foundation IMF members pledged quotas that amounted to around 1% of their GDP. In real terms the Fund has shrunk since then, so that by the end of 2005, a much larger Fund membership pledged quotas that amounted to around 0.7% of their GDP. Compared with world trade and capital flows the decline in quotas has been even starker, particularly during the past two decades. For example, Chart 3 illustrates that total IMF quotas have fallen from around 4%-5% of world merchandise trade (exports plus imports) in the period up to 1970 to 1.3% in 2005.

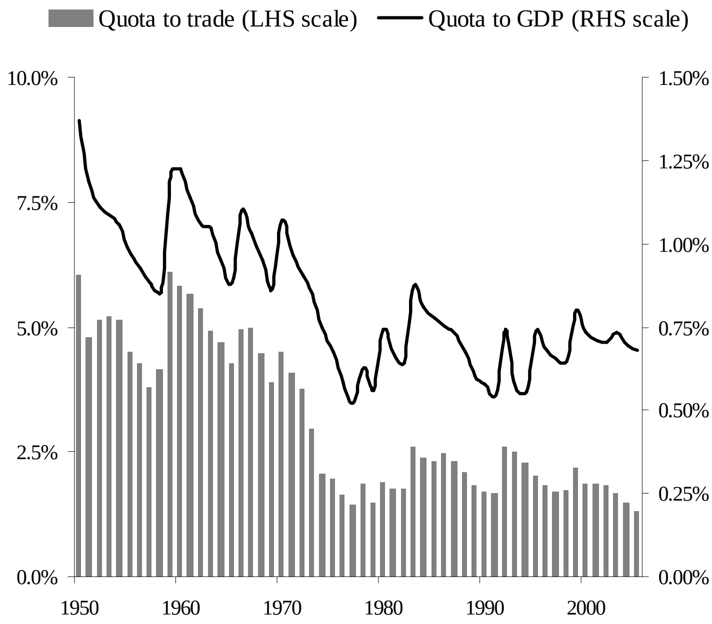
The increasing tendency towards self-insurance is quantified in Chart 4. As is well known the

Chart 2: Annual probabilities of borrowing from the Fund: 1992-2006 (for members at end-1991)



Sources: IMF *International Financial Statistics* (*IFS*) and authors' calculations.
Notes: Probability indicates the annual average probability of country making a purchase from the IMF General Resources Account, excluding reserve tranches, over the period. The sample of member countries is those for which *IFS* indicates non-zero quota at end-1950.

Chart 3: Average IMF quotas relative to member GDP and trade



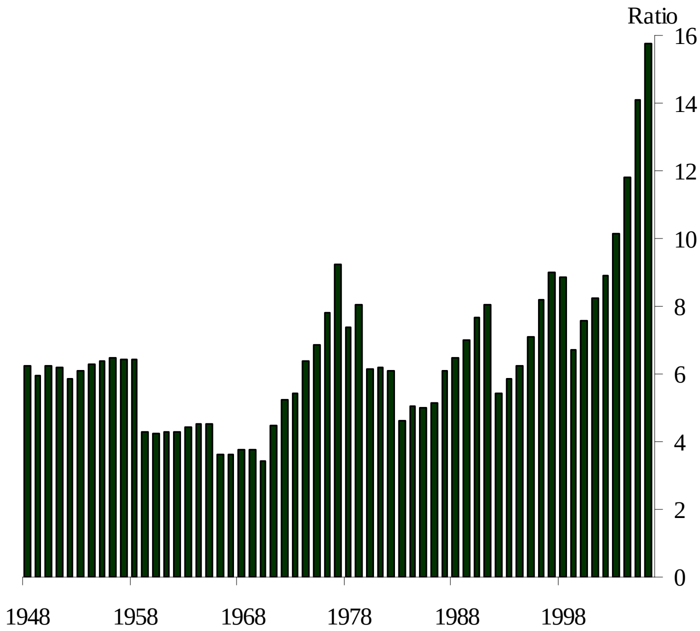
Sources: IMF *International Financial Statistics* (*IFS*) and authors' calculations.
Notes: Each series is calculated as a weighted average (by denominator) for those members for whom both the quota and the denominator are available in each year. Trade is merchandise trade (sum of exports and imports).

real value of reserve holdings has increased sharply over recent years and total reserves to IMF quota at end-2006 were three and a half times that at 1971, when the Bretton Woods system broke down. Most of that increase has taken place since the Asian crisis of 1997 and has been driven by countries in that region.

Finally, Table A shows how the average amount of borrowing from the Fund has changed over time. The weighted average ratio of borrowing to quota across the membership has remained stable over the past 40 years at around 6%. However over this period the fraction of the total IMF quota accounted for by those countries that borrow has shrunk from just under 50% to around 20%. Thus the average ratio of borrowing to quota for those countries that borrow has increased substantially. Changes to the distribution of crisis probabilities over this time, and in particular a decrease in the mean vulnerability, can help to explain this rise in ‘exceptional access’.

The main message of this paper is that the framework governing the Fund’s lending operations may no longer be appropriate. The existing credit union model was appropriate for a world in which the interests of the membership were homogenous. This may no longer be the case. The Fund comprises of increasingly heterogeneous countries. As a result, based on our model, the

Chart 4: Ratio of reserves to IMF quotas



Sources: IMF *International Financial Statistics* (*IFS*) and authors' calculations.
Note: The ratio is that of world reserves including gold to total IMF quotas.

amount of crisis lending that is available from the Fund is likely to be suboptimally low, abstracting from concerns about moral hazard, increasing the reliance of members on economically inefficient self-insurance. An alternative approach may be needed: one which takes into account that creditor and debtor countries have different interests, but which also takes into account the moral hazard consequences of large-scale lending.

1.1 Modelling approach and related literature

The model employed to analyse a country's choice over Fund size is a simple one-period, partial equilibrium investment model. A country's demand for 'insurance' is motivated by the possibility that its final investment output may be reduced following a crisis with countries varying in their likelihood of suffering a crisis. As in reality, the insurance options available to each country include self-insurance via reserves and also subscription to an international credit union mechanism (the IMF) which entitles a country to access pooled resources in the event of a crisis.[2] Clearly in a world with a full set of contingent contracts there is no need for an external party such as the IMF to mitigate the costs of crises. However, despite the substantial

[2]Cordella and Levy Yeyati (2006) provide a review of various other insurance-type mechanisms which, in theory, are potentially available to countries, for example capital controls, private insurance via contingent credit lines and regional swap arrangements.

Table A: Borrowing from the Fund

	End-period quota share of borrowers:[1]	Average ratio of borrowing to quota[2]	
		all members:[3]	borrowers only:[4]
1948-1960	47%	1.7%	20.6%
1961-1970	44%	5.8%	29.7%
1971-1980	44%	6.9%	52.2%
1981-1990	36%	6.4%	57.2%
1991-2000	33%	6.4%	60.9%
2001-2006	17%	6.0%	172.9%

Sources: *IMF International Financial Statistics* (*IFS*) and authors' calculations.

(1) Borrowers defined as those making at least one purchase from the General Resources Account, excluding reserve tranches, during the period.

(2) Weighted by quota.

(3) Sum of purchases over sum of quotas across all country-year observations in each period.

(4) Sum of purchases over sum of quotas across all country-year observations in which purchases made during each period.

development of international financial markets since the Fund's establishment, it remains the case that a range of market failures, such as inability to enforce sovereign debt claims across international borders, limit the ability of countries to insure against lower consumption states. Many developing countries continue to be excluded from world capital markets (either being quantity or price-rationed) particularly in crisis times when they most need the finance.

The analysis below draws on insights and approaches developed in a number of related literatures. For example, a clear analogy can be drawn with an individual's demand for private and public provision of health insurance. A country's crisis probability can be compared to the likelihood of an individual falling ill. Furthermore, an individual's choice of private insurance cover is often made conditional on the level of public insurance. So, in the model below, the political choice over the size of the credit union is taken before countries choose their level of self-insurance through reserve cover. In a similar manner Gouveia (1997) analyses the supplemental purchase of private health insurance above the level of public insurance and determines the political equilibrium level of provision of the latter by majority voting.

The political economy of risk-sharing across individuals in different countries via social insurance has been analysed in some detail in the context of fiscal federations. Motivated by developments within the European Union, recent papers, such as Persson and Tabellini (1996a, 1996b) and Alesina and Perotti (1998), examine the determination and characteristics of federal and state-level social insurance policies under various institutional arrangements. Many of the issues raised in such analyses, for example participation constraints on membership of the union, are of interest to our analysis. However, our focus is less on redistributive transfers and our level of analysis is the country rather than the region.

In this sense our approach is closely linked to recent political economy analysis of international organisations, in particular Alesina *et al* (2001, 2005). The focus of these papers is the provision of public goods with externalities by an international union. Clearly this differs somewhat from the credit union qualities of the IMF. Nevertheless the papers provide rich insights into the process of union formation, enlargement and decision-making (under both majority and qualified majority voting) which are in many ways applicable to the IMF.

While our modelling approach draws closely on the insights of the above literature, we believe it to be one of the first to model formally the political economy of decision-making by the shareholders of the Fund and the trade-off faced by countries between self-insurance and IMF subscriptions.[3]

The structure of the rest of the paper is as follows. Section 2 sets out the basic one-period investment model which is the work-horse of our analysis. Section 3 derives the optimal self-insurance choices that countries would make in the absence of the Fund. Section 4 introduces the Fund into the model, and derives the size of the Fund preferred by individual members and the self-insurance choices that members make, contingent upon the Fund being a particular size. Section 5 considers the political equilibrium Fund size that arises under different assumptions, in particular regarding the voting process. Section 6 assesses the welfare implications of the political equilibrium. The final section draws together the conclusions.

2 Model set-up

The basic set-up is a one-period, partial equilibrium investment model in which returns are realised at the end of the period. Consumption then takes place. Demand for insurance is motivated by a country-specific potential for a crisis to hit immediately after the initial investment decision has been made. The key features are as follows.

[3]For example, while Chami *et al* (2004) provide a model of IMF lending and postulate an objective function for the IMF they do not consider the optimal level of Fund subscriptions from a political perspective nor the interaction between Fund size and self-insurance.

2.1 Country characteristics

We consider N equally sized countries who only differ in their probability of a crisis. Country i has a crisis probability π^i drawn from a commonly known distribution with support $[\pi^1, \pi^N]$ with $0 < \pi^1 < \pi^2 < ... < \pi^N < 1$. Two key variables in our analysis are the median crisis probability, π^m, and the mean crisis probability, μ. The stylised facts demonstrate that, as the membership of the Fund has become more heterogeneous, π^m has fallen relative to μ. Below we argue that this has important implications.

Differing crisis probabilities are the minimum degree of heterogeneity required for our analysis. We are deliberately abstracting from many other issues such as economic size, economic structure or geopolitical significance to focus solely on the issue of relative risk. We also make a number of simplifying assumptions. First, the realisations of the idiosyncratic crisis risks are assumed to be independent. Second, we abstract from the question of moral hazard by assuming that the crisis probability is not affected by a country's policy effort or by its level of insurance. Third, unlike the analysis of, for example, Gouveia (1997) in relation to health insurance, we restrict the analysis to countries of the same income levels.

2.2 Investment technology and insurance

Each country receives a unit endowment which it can invest in a project of type A, which yields an exogenous gross return of ρ_A if the country does not suffer a crisis. However, if a crisis occurs a proportion δ of this return is destroyed.[4] Countries can also place some of their endowment in technologies with insurance-like properties – self-insurance via reserves and/or access to payouts in the event of a crisis through subscription to the IMF. In this partial equilibrium model, and in common with related models of the international financial architecture, the returns on the various investment technologies are taken as exogenous and are assumed to be common across countries. Endogenising interest rates, for example through inclusion of a reserve asset in zero net supply, would not change the political analysis of Fund size choices in the sense that countries still take interest rates as given. The assumption of common returns across countries means that there is only one source of heterogeneity across countries, namely the probability of crisis, which facilitates the political economy analysis.

[4]As with project returns, we impose uniform crisis losses across countries. Allowing for δ^i varying by country would not add substantially to the analysis.

2.2.1 Reserves/self-insurance

If held through to the end of the period, reserves yield a certain return of ρ_R. However, in the event of a crisis, countries can switch their reserves into crisis investment projects. Assumption 1 below is sufficient to ensure that in a crisis all reserves are put into the new crisis investment project.[5]

Assumption 1: The gross return from the crisis investments, ρ_C, is greater than the simple reserve return, ρ_R.

One rationale for this assumption is that a crisis may lead to the destruction of the domestic capital stock (for example through the liquidation of capital stocks by foreign investors). This would increase the marginal product of capital, facilitating new investment opportunities with increased returns. An alternative interpretation is that reserves can yield a higher return than in the non-crisis state through mitigating the negative impact of a crisis on investment.

2.2.2 IMF crisis payouts

Membership of the IMF credit union has the advantage of allowing the country to access a greater potential pool of resources in the event of a crisis. Fund subscriptions (proportion g^i of the initial endowment) are repaid to members by the IMF with gross return ρ_F at the end of the period. For those countries hit by a crisis, a payout from the Fund of f^i is made immediately post-crisis which they must then pay back at the end of the period with gross interest rate ρ_F.[6] In reality the rate of remuneration paid by the Fund is lower than the rate at which IMF lending is repaid by crisis economies, with the wedge between the two helping to finance Fund expenses.[7] Adding an exogenous wedge between these two rates within the model would provide little additional insight. In terms of the relation between ρ_F and ρ_R, in the model they are both exogenous and unrelated although in reality they are linked in the sense that the IMF interest

[5] In Section 3 the implications of Assumption 1 for relative consumption levels in crisis and non-crisis states are discussed.

[6] We thus assume that states of nature are verifiable in determining the payout and that there are no contract enforcement problems in ensuring repayment. Furthermore, we assume that the payout is automatic. In reality, access to IMF resources above a member's reserve tranche has to be approved post-crisis. However this would require a second round of voting within the model. In addition, in practice, excluding exceptional access cases, access to resources beyond the reserve tranche for crisis economies can usually be assumed to be forthcoming, albeit with conditionalities.

[7] For example, for the week 7-13 May 2007, the adjusted rate of remuneration was 4.06% while the adjusted rate of charge was 5.5% (see www.imf.org/external/np/tre/sdr/burden/2007/050707.htm). The former is the interest rate on repayment to members on their remunerated reserve tranche while the latter is the interest rate on a member's outstanding credit to the Fund.

rates are related to those on widely held reserve assets. The IMF's basic rate of remuneration and rate of charge are based on the Special Drawing Rate (SDR) interest rate. This interest rate is a weighted average of short-term money market interest rates, namely the Eurepo rate and UK, Japanese and US short-term government bills. These assets, or similar longer-term securities, can be held as reserve assets providing a linkage between ρ_F and ρ_R.

Assumption 2 below is sufficient to ensure that in a crisis all payouts received from the Fund are put into the new crisis investment project.

Assumption 2: The gross return from the crisis investments, ρ_C, is greater than the return paid on Fund subscriptions, ρ_F.

The Fund payout is pinned down by the chosen subscription levels through the Fund's budget constraint. To keep the analysis tractable, as in the health insurance analysis of Gouveia (1997), we employ the expected budget constraint rather than employing the budget constraints for all realisations of nature. If the *ex-post* budget constraint is employed then an additional stage of voting on *ex-post* subscription increases would be required. Furthermore, in the long term, which is the focus of the static model, the Fund's expected budget constraint is likely to hold (ie the expected total crisis payouts equal the size of the Fund).[8] With country-specific subscription levels and payouts, the expected budget constraint is $\sum_{i \in \mathcal{H}} \pi^i f^i = \sum_{i \in \mathcal{H}} g^i$ where $\mathcal{H}$ is the set of H countries that are members of the Fund. With common subscription levels and payouts this simplifies to $\mu f = g$, where μ is the mean crisis probability of those countries within the Fund. Thus, for a given subscription level, the higher is μ, the lower the crisis payouts are. While this does not fit with the Fund's formal access limits relative to quota, it does seem broadly consistent with trends of actual disbursements over time. Since the 1970s the proportion of members (by quota share) accessing Fund resources has fallen, as illustrated in Charts 1 and 2, which can be thought of as representing a fall in the mean crisis probability. At the same time those borrowing have accessed increasing funds relative to quota, as illustrated in Table A.

2.3 Preferences

Countries maximise their expected utility from consumption at the end of the period:

$$W^i = \pi^i u(c_c) + (1 - \pi^i) u(c_n) \qquad \textbf{(1)}$$

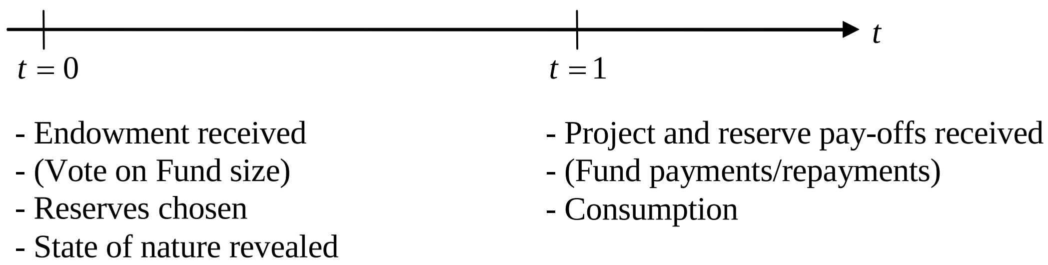
[8]For sufficiently large numbers of countries this assumption can also be justified through appeal to the law of large numbers.

where c_c is consumption in the crisis state and c_n is consumption in the non-crisis state. The state-specific utility functions are event independent with $u'(c) > 0$, $u''(c) < 0$ and standard Inada conditions $\lim_{c \to \infty} u'(c) = 0$ and $\lim_{c \to 0} u'(c) = \infty$.

It could be that countries also care about outcomes in other countries for a variety of reasons. For example, crises overseas may spill over to home consumption via trade and financial flows, or there may be concerns for others' consumption for political or altruistic reasons. For simplicity, we ignore possible spillover effects in the main body of this paper. However, in Appendix A we extend the analysis to incorporate spillover effects and assess their likely impact. In particular, we demonstrate that if a high weight is placed on spillovers from countries with high crisis probabilities, then this is likely to result in a larger Fund size, other things being equal, as lower crisis probability countries perceive that IMF crisis payouts offer greater benefits.

To recap, Chart 5 provides a summary timeline of the model. As an initial reference point, in the following section we derive a country's optimal reserve choice in a world with no Fund.

Chart 5: Timeline (features present with Fund indicated in parentheses)



3 World with no Fund

With no Fund (denoted ^{nf}), country i chooses reserves, $b^{i,nf}$, and the residual level of investment in project A, equal to $1 - b^{i,nf}$, to maximise expected utility subject to the constraint that reserves are between zero and one. The consumption level in the crisis state is:

$$c_c^{i,nf} = \rho_A(1-\delta)(1-b^{i,nf}) + \rho_C b^{i,nf}$$

In the crisis state, loss-adjusted returns of $\rho_A(1-\delta)$ are earned on the investment in project A, with returns of ρ_C earned on the funds placed in reserves which are then transferred to the crisis investment project. In the non-crisis state, consumption is

$$c_n^{i,nf} = \rho_A(1-b^{i,nf}) + \rho_R b^{i,nf}$$

In this state the full return of ρ_A is earned on the investment in project A and ρ_R is earned on reserve holdings.[9] The constrained optimisation problem for country i is the following Lagrangean:

$$\max_{b^{i,nf}} \mathcal{L} = \pi^i u(c_c^{i,nf}) + (1-\pi^i)u(c_n^{i,nf}) + \theta_1 b^{i,nf} - \theta_2(b^{i,nf} - 1) \qquad \textbf{(2)}$$

Assumption 3 below is required to ensure that the crisis loss incurred on the return on the initial investment project, δ, is high enough so that some countries choose to invest in reserves.[10]

Assumption 3: $\delta > 1 - \rho_C/\rho_A$

Proposition 1 In a world with no Fund, optimal reserve holdings are increasing in a country's crisis probability. Countries with crisis probability below $\underline{\pi}^{b,nf}$ hold zero reserves while countries with crisis probability above $\bar{\pi}^{b,nf}$ put all their endowment in reserves.

Proof. See Appendix B.1.

As expected, under Proposition 1, the higher a country's crisis probability, the higher the insurance via reserves. This is illustrated in Chart 6. The level of the reserves is also non-decreasing in the severity of the crisis, captured by δ, and depends on the returns on both the investment projects and reserves.[11]

4 Country choice of optimal Fund size and reserves

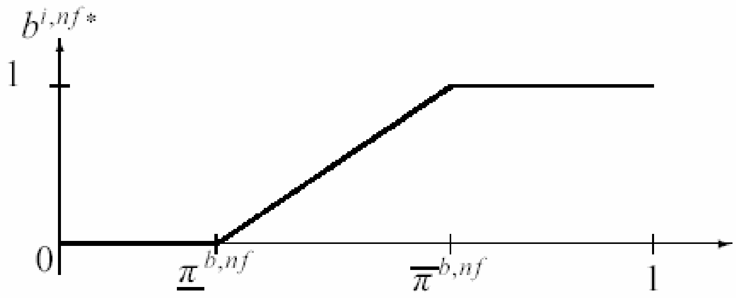
We now analyse how countries choose their overall level of insurance when both reserves and Fund membership are available (denoted ^{f}). As countries are assumed to be identical in size, and

[9] The assumption that $\rho_C > \rho_R$ could potentially give rise to a country's consumption crisis level being higher than in the non-crisis state. This is, however, only the case if the crisis probability is high enough (for log utility we require the condition $\pi^i > \frac{(\rho_A - \rho_R)}{\delta\rho_A + (\rho_C - \rho_R)}$).

[10] This assumption is required to obtain positive interior values for reserve holdings (see Appendix B.1).

[11] The sensitivity of the optimal reserve choice to these returns depends on the concavity of the utility function. With log utility the level of reserves increases with ρ_R and ρ_C and falls with ρ_A. As would be expected, with a constant relative risk aversion (CRRA) utility function, the level of reserves also varies with the coefficient of relative risk aversion. Interestingly, the sensitivity of reserves to the degree of relative risk aversion depends on the level of the crisis probability. For small (large) crisis probabilities the level of reserves rises (falls) with the level of risk aversion. The intuition is that if the crisis probability is sufficiently high then, for given reserves, the higher risk aversion reduces overall marginal expected utility with respect to reserves, as it has a greater negative effect on expected marginal utility in the crisis state than the positive effect on marginal utility in the non-crisis state. Thus, for such crisis probabilities, higher risk aversion leads to falling optimal reserves.

Chart 6: Optimal choice of reserves with no Fund



given the present institutional arrangement at the Fund which links quotas to measures of economic size, we consider the case in which there are common subscription levels and common payouts in the event of a crisis for all Fund members. Reserves supplement the crisis payouts available from membership of the Fund.[12] In this section we derive two key elements of the solution to our problem, before determining the political equilibrium provision of Fund coinsurance in the next section. These elements are (1) the amount of self-insurance that each country will choose, contingent upon a given Fund size, and (2) the Fund size that each prefers, given that quotas are allocated uniformly to all members.

4.1 Country choice of reserves for given Fund size

For a given Fund size, countries must decide whether to supplement their crisis consumption insurance from IMF membership with additional reserves. Consumption in the non-crisis state differs from the no-Fund world through the effect of the initial Fund subscription g which receives a return ρ_F. With common Fund subscription rates, non-crisis consumption is given by

$$\begin{aligned} c_n^{i,f} &= \rho_A(1 - b^{i,f*}(g) - g) + \rho_R b^{i,f*}(g) + \rho_F g \\ &= \rho_A - (\rho_A - \rho_R) b^{i,f*}(g) - (\rho_A - \rho_F) g \end{aligned}$$

[12]The situation is similar to the problem analysed by Crémer and Palfrey (2000) and Alesina *et al* (2005) in the context of federal public goods provision, in which decisions over country policy are made following a decision on provision at the federal level.

In the crisis state, the Fund payout can be invested in the crisis investment technology, but must be paid back at return ρ_F, so crisis consumption is

$$\begin{aligned} c_c^{i,f} &= (1-\delta)\rho_A(1-b^{i,f*}(g)-g)+\rho_C b^{i,f*}(g)+((\rho_C-\rho_F)/\mu+\rho_F)g \\ &= (1-\delta)\rho_A+\Gamma b^{i,f*}(g)+\Omega g \end{aligned}$$

where $\Gamma \equiv \rho_C-(1-\delta)\,\rho_A$ and $\Omega \equiv (\rho_C-\rho_F)/\mu+\rho_F-(1-\delta)\,\rho_A$.

Taking the Fund subscription level, g, as given, the constrained optimisation problem for country i is:

$$\max_{b^{i,f}} \mathcal{L} = \pi^i u(c_c^{i,f})+(1-\pi^i)u(c_n^{i,f})+\lambda_1 b^{i,f}-\lambda_3(b^{i,f}+g-1) \qquad \textbf{(3)}$$

Focusing on the non-trivial case in which $0<g<1$, countries choose to supplement their Fund insurance with reserves if their crisis probability is high enough.

Proposition 2 For $0<g<1$, countries will have positive additional reserve holdings if their crisis probability lies in the range $[\underline{\pi}^{b,f},\bar{\pi}^{b,f}]$. The preferred level of reserves is increasing in the crisis probability and decreasing in the size of the Fund, g. For $\pi^i<\underline{\pi}^{b,f}$ no reserves are held, while for $\pi^i>\underline{\pi}^{b,f}$ all the endowment is put into the Fund and reserves.

Proof. See Appendix B.2.

The cut-offs $\underline{\pi}^{b,f}$ and $\bar{\pi}^{b,f}$ depend on the returns on the investment projects and reserves and on the value of g. This proposition illustrates the substitution in the form of insurance which takes place as the Fund size increases. As g rises the level of supplemental reserves falls and the threshold for the crisis probability at which countries add further reserves increases.[13] This result is similar to that of Gouveia (1997) in which higher public provision of health insurance may reach 'choking levels', crowding out private provision.

[13]With CRRA preferences this crowding out is very apparent – the optimal reserve choice with the Fund is equal to the reserve choice in a no-Fund world minus an adjustment in proportion to the size of the Fund.

4.2 Country preferences over Fund size

We now consider individual country preferences over the size of the Fund and hence the uniform subscription level for all members. Country i derives its policy preference over g^i knowing how it would augment reserves in the second stage. Thus it chooses g^i by solving the following constrained optimisation problem:

$$\max_{g^i} \mathcal{L} = \pi^i u(c_c^{i,f}(g^i, b^{i,f*}(g^i))) + (1-\pi^i)u(c_n^{i,f}(g^i, b^{i,f*}(g^i))) \\ + \lambda_1 b^{i,f*}(g^i) + \lambda_2 g^i - \lambda_3(b^{i,f*}(g^i) + g^i - 1) \quad \textbf{(4)}$$

Define $\tilde{\mu} \equiv \{1 + \frac{(\rho_C-(1-\delta)\rho_A)(\rho_R-\rho_F)}{(\rho_C-\rho_F)(\rho_A-\rho_R)}\}^{-1}$.

Proposition 3 Individual country preference over Fund size:

(a) For $\mu < \tilde{\mu}$, all countries prefer a Fund size that is non-decreasing in their crisis probability, with interior solutions in the range $[\underline{\pi}^{g,f}, \bar{\pi}^{g,f}]$. The optimal supplemental reserve holding at the preferred Fund size is zero.

(b) For $\mu > \tilde{\mu}$, all countries prefer no Fund. The optimal reserve holdings are determined by Proposition 1.

(c) If $\mu = \tilde{\mu}$, all countries prefer either no Fund and zero reserves or a positive Fund size and positive reserves, with the choice increasing in the crisis probability.

Proof. See Appendix B.3.

Recall from the Fund's budget constraint that $\mu = g/f$, so $\tilde{\mu}$ can be viewed as a threshold value of the ratio of the initial subscription to the crisis payout. Under Proposition 3, provided the mean crisis probability is sufficiently low ($\mu < \tilde{\mu}$), each country would prefer to hold zero reserves and use the IMF to provide additional funds in the event of a crisis. This is because f is sufficiently greater than g so that the member gets high leverage out of the initial subscription. If the mean crisis probability is too high ($\mu > \tilde{\mu}$), each country would prefer a zero Fund size and the only insurance would be self-insurance. The intuition is that a high mean crisis probability reduces the gross expected utility gain from a given Fund subscription, since more countries are likely to share the fixed total pot for payouts. Note that $\tilde{\mu} \geq 1$ if $\rho_F \geq \rho_R$.

We focus on part (a) of Proposition 3 since our primary interest is in a world with a positive Fund size. With an interior solution under part (a), the level of Fund subscription $g^{i,f*}$ preferred by country i is increasing in that country's crisis probability and in the severity of the crisis (ie the size of δ).[14]

5 Political equilibrium

We have now determined that Fund members will have different preferences over g and in particular those with a higher vulnerability will tend to prefer a larger Fund size. How, then, is the actual Fund size determined? In this section we focus on the case where $\mu < \tilde{\mu}$ and consider whether there is a political equilibrium outcome when the size of the Fund is determined by a vote of the Fund's membership.

Under the Fund's Articles and Agreements (Article XII, Section XII), the vote allocation of each member for decisions of the Fund's Board of Governors or Executive Board is equal to 250 'basic votes' plus an extra vote for each 100,000 special drawing rights of its quota.[15] Thus, for larger countries the voting share is slightly less than the quota share while for smaller countries the voting share is above the quota share. Nevertheless, it is a reasonable approximation to equate quotas with voting shares.

As noted in the introduction, quotas – and hence, votes – reflect both economic size and openness, with the US holding the largest country quota. Allowing for bloc voting by larger members complicate the analysis of the political equilibrium and generally does not provide significant additional insights.[16] Consequently, unless otherwise stated, we make the simplifying assumptions of equal country size and hence equal voting shares.

[14]In general, the sensitivity of the Fund size to the other parameters is dependent upon the concavity of the utility function. With CRRA preferences again the choice of Fund size varies with the degree of risk aversion. As mentioned in relation to the reserve choice in the no-Fund world, the sign of this relationship depends on the crisis probability. For π^i low (high) enough the optimal Fund size increases (decreases) with the degree of risk aversion. For log utility, the optimal choice of Fund is increasing in ρ_C and decreasing in ρ_A. The optimal choice of Fund size falls with μ (for the reasons discussed above). The sensitivity of the preferred Fund size to ρ_F is ambiguous, depending on π^i. On the one hand a higher ρ_F increases consumption in the non-crisis state but, on the other hand, it increases repayments in the crisis state.

[15]The number of basic votes is currently under review.

[16]At the margin bloc voting is likely to give greater influence to the country whose bloc of votes straddles the critical threshold for either a majortity vote or a qualified-majority vote.

In the following subsection we consider the outcome when there is unconstrained majority voting, which allows us to employ the median-voter theorem to solve for a political equilibrium Fund size. In reality there may be a binding constraint on the decisions taken by a median voter. For example, it is possible that low crisis probability members may prefer to leave a Fund which they regard as being too large. Consequently, in Subsection 5.2 we consider when this participation constraint is likely to bind and the possible implications of this for the majority-voting equilibrium. Finally, although general decisions of the Fund are based on majority voting, the more important decisions actually require a qualified majority. For example, a revision to quotas (which is the policy choice variable in our model) requires an 85% majority. In the final subsection we consider how decisions might be taken under this scenario, assuming there is an agenda setter who determines the choices that are put to the vote by the membership of the Fund.

5.1 Unconstrained majority voting

Following the political economics literature, in order to generate equilibrium policies through pure majority rule, restrictions must be made on either the form of preferences over policy or the institutional framework.[17] In this subsection we take the former approach and check that the required conditions on preferences are satisfied. In doing so we follow a similar approach to the aforementioned literature on international political unions (see, for example, Alesina *et al* (2001, 2005)) and risk-sharing in federations (see, for example, Persson and Tabellini (1996a, 1996b)).

First we consider the case where all countries must be in the Fund and where they face a uniform subscription level and payout in the event of a crisis. The latter assumption reflects the current reality and adds tractability. Denote by π^m the crisis probability of the median country in the entire set of N countries.

With our single policy variable of the Fund subscription level, preferences exhibiting single-peakedness or the single-crossing property can generate a political equilibrium under pure majority rule (ie are sufficient for the median voter theorem to hold).[18] As in Persson and

[17] Pure majority rule is characterised by direct democracy, sincere voting and an open agenda.

[18] See Gans and Smart (1996) and Persson and Tabellini (2000). As detailed in Persson and Tabellini (2000, pages 22-23), with single-peaked preferences over different policy options, a Condorcet winner always exists (ie a policy which beats any other feasible policy in a pairwise vote) and is the median-ranked preferred policy. If all preferences exhibit the single-crossing condition then the policy preferences of voters can be ordered by their types. In this case again a Condorcet winner always exists, but is the preferred policy of the median-ranked individual by type.

Tabellini (1996b), the median voter theorem applies since the only source of heterogeneity among voters is the probability of a bad outcome (in our case a crisis, in their paper unemployment) which enters into preferences in a linear manner.

Proposition 4 Given $\mu < \tilde{\mu}$ and with countries voting over the size of a Fund to be applied uniformly to the entire set of N countries, the political equilibrium Fund size is determined by the median voter theorem and is the optimal choice of the country with the median crisis probability. This optimal choice, $g^*(\pi^m)$, is as defined in Proposition 3(a).

Proof. See Appendix B.4.

Thus, if $\pi^m \in [\underline{\pi}^{g,f}, \bar{\pi}^{g,f}]$ we have a positive Fund size which is increasing in the median country's crisis probability.[19]

Putting together Propositions 2 and 3 enables comparison of the levels of investment with and without the Fund. For $\mu < \tilde{\mu}$ and with a positive Fund size countries will hold supplemental reserve holdings if they have sufficiently high crisis probability $\pi^i > \underline{\pi}^{b,f}$. Adding Proposition 1 enables us to compare the crisis probability at which a country will begin to self-insure via reserves in the world with and without the Fund. Chart 7 illustrates the possible relative cut-off points for reserve holdings between the no-Fund and Fund worlds.[20]

Chart 8 provides a graphical example of the level of residual investment.[21] While investment is higher for lower crisis probability countries in the no-Fund world (as they are not forced to insure through a Fund subscription), countries begin to self-insure at a lower crisis probability in the no-Fund world than in the Fund world (ie $\underline{\pi}^{b,f} > \underline{\pi}^{b,nf}$). Similarly they reach the respective corner solutions for reserve holdings at a lower crisis probability in the no-Fund world ($\bar{\pi}^{b,f} > \bar{\pi}^{b,nf}$).

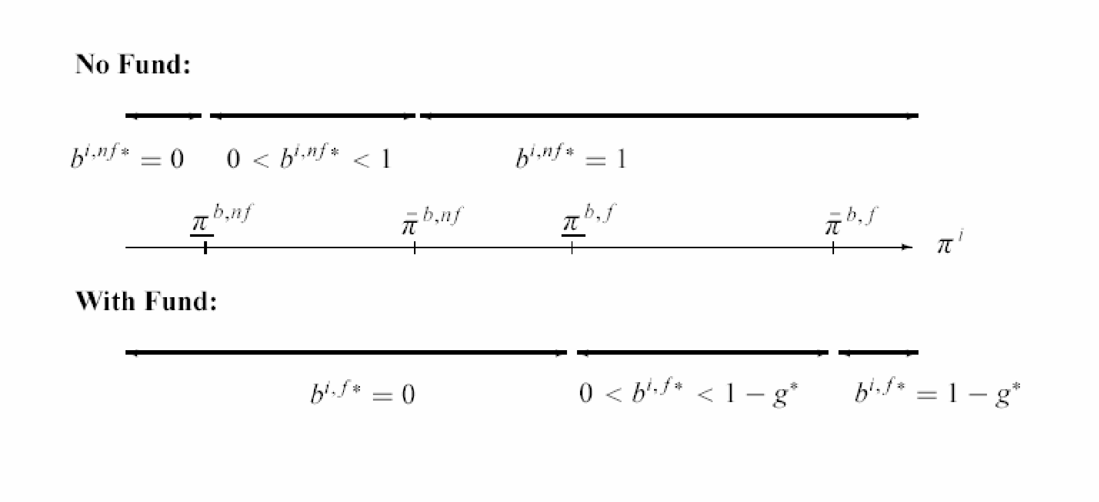
The key implication of this analysis is that it is π^m and not μ which drives the equilibrium Fund size. Therefore, if the distribution of crisis probabilities becomes more skewed, so that π^m falls relative to μ, as is consistent with the stylised facts, then we would expect that the Fund will

[19]For $\pi^m < \underline{\pi}^{g,f}$, the political equilibrium Fund size is zero, while for $\pi^m > \bar{\pi}^{g,f}$ the political equilibrium Fund size is one.

[20]This is one of two possible rankings of the cut-off points. It is ambiguous whether $\bar{\pi}^{b,nf} \gtrless \underline{\pi}^{b,f}$.

[21]We assume a constant relative risk aversion utility function with coefficient of relative risk aversion equal to 3, $\rho_C = 1.075$, $\rho_A = 1.05$, $\rho_R = \rho_F = 1.025$, $\mu = 0.025$, $\delta = 0.1$.

Chart 7: Optimal reserve holdings by crisis probability



decrease in size and so provide less coinsurance.

5.2 Constrained majority voting

So far we have assumed that all countries are members of a Fund whose subscription level is determined by the country with the overall median crisis probability. For simplicity we have ignored the potential for participation constraints to bind on Fund membership.[22] However, assuming that redistributive transfers between countries are not feasible, it may well be the case that for sufficiently high subscription levels some countries would be better off leaving the Fund.[23] Clearly this would affect the composition of the Fund's membership and hence the equilibrium size of the Fund.

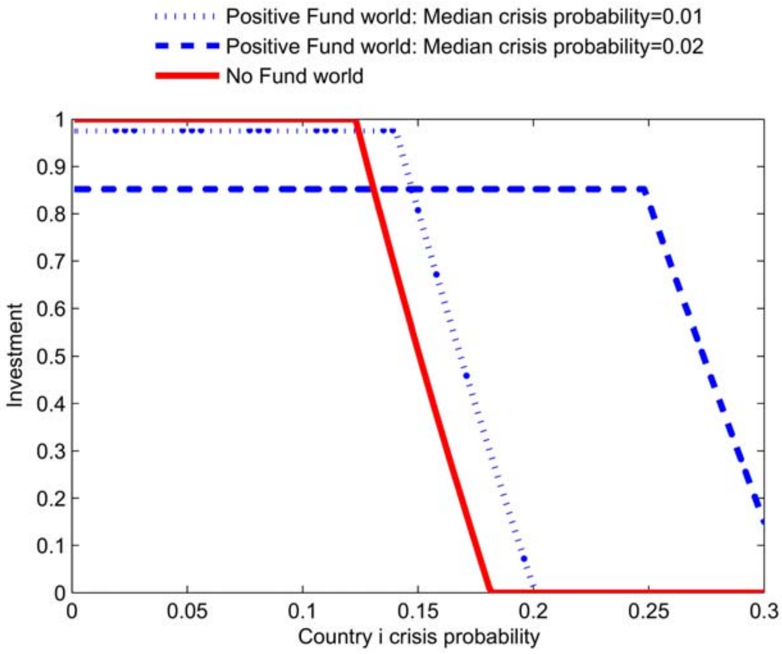
Denote the expected utility difference between being in a Fund of positive size and outside the Fund as $D^i(g, \pi^i) \equiv W^i(0 < g < 1, \pi^i) - W^i(g = 0, \pi^i)$.

Proposition 5 The expected utility difference between being in a Fund of given positive subscription level g and outside the Fund is increasing in a country's crisis probability $\partial D^i / \partial \pi^i > 0$. Any country with crisis probability π such that $0 < \pi < \hat{\pi}(g)$ strictly prefers not to be in the Fund.

[22] Such constraints are discussed in detail for international unions in Alesina *et al* (2001, 2005).

[23] Our game-theoretic focus is on the possibility of withdrawal from the Fund, rather than on entry to the Fund in the first place. This is because Fund membership is open to any country that satisfies the terms prescribed by the Board of Governors (which should be consistent with principles applied to existing members). Thus there is no formal vote on new membership.

Chart 8: Optimal investment with and without a Fund



Notes: Model with constant relative risk aversion utility. Parameter values: ρ_A = 1.05, ρ_C = 1.075, ρ_R =ρ_F = 1.025, μ = 0.025 and coefficient of relative risk aversion = 3.

Proof. See Appendix B.5.

Proposition 5 implies that, for a given Fund subscription level of g, if the lowest crisis probability in a Fund of all N countries, π^1, is low enough ($\pi^1 < \hat{\pi}(g)$) then there is at least one country who prefers not to be in such a Fund.[24] This is more likely to occur the greater is the difference between π^1 and π^m.

The key question is how does the median voter react to this participation constraint? The median voter has two options: either reduce g so as to keep the lower-bound member in the Fund, or allow the lower-bound member to drop out. In the latter case the putative median voter may lose its privileged position and be replaced by a new median voter as the size of the membership decreases. Which option is preferred is likely to depend on the shape of the distribution function for crisis probabilities in general, and in particular the crisis probability of the putative median voter relative to that of both the lower-bound member and the new median voter if countries drop out of the Fund. Moreover, depending on the distribution of country crisis probabilities, the dropping out process could continue until the Fund unravels and the Fund ceases to exist.[25] It is not possible to pin down whether this will be the case or, if not, what the equilibrium number of countries in the Fund will be, without specifying the exact distribution function for crisis probabilities and model parameters. However, we can characterise the nature of subscription levels for an equilibrium Fund with a stable number of members.

Suppose that the Fund initially has $\mathcal{M}$ members whose crisis probabilities, under Proposition 5, are ranked from π^{N-M+1} to π^N. The median country has crisis probability $\pi^m(\mathcal{M})$ and if unconstrained would set the Fund subscription level at $g^u(\pi^m(\mathcal{M}))$. If the participation constraint binds, ie $\pi^{N-M+1} < \hat{\pi}(g^u(\pi^m(\mathcal{M})))$, then the median voter could reduce the Fund subscription level to its constrained value of $g^c(\pi_{N-M})$ such that lower-bound member is indifferent between remaining in and leaving the Fund. Alternatively the putative median voter could allow those lower-bound members who would prefer not to be in a Fund of subscription level $g^u(\pi^m(\mathcal{M}))$ to drop out. This would result in a new Fund with median crisis probability country $\pi^m(\mathcal{R})$, where $\mathcal{R} < \mathcal{M}$ and $\pi^m(\mathcal{R}) > \pi^m(N)$, who would then go through the same process as above.

[24]We are implicitly assuming that there can be at most one Fund (ie we discount for now the possibility that countries at higher crisis probabilities may wish to leave the existing Fund to set up their own Fund with higher subscription levels).

[25]Concerns over participation constraints are also raised in Persson and Tabellini (1996b). They consider a second stage ratification vote which is imposed after the choice of federal policy which leads to repeal of the federal arrangement if either country rejects. In this case if there are large divergences in regional risk across the federation then the threat of secession leads to an upper bound on the level of federal insurance.

Given that, as a matter of fact, no low-crisis probability country has ever withdrawn from the Fund, it is interesting to consider under what conditions a stable Fund of $\mathcal{M}$ members is likely to exist. For $\frac{\partial \hat{\pi}(g^u(\pi^i))}{\partial \pi^i} > 0$ it can be shown that if the median country $\pi^m(\mathcal{M})$ of a Fund of $\mathcal{M}$ members has a crisis probability above a cut-off level (denoted by $\breve{\pi}(\mathcal{M})$) it will face a constrained choice of Fund subscription. With a standard log utility function it can be shown that $\frac{\partial \hat{\pi}(g^u(\pi^i))}{\partial \pi^i} > 0$.[26] Under this condition, an equilibrium Fund of $\mathcal{M}$ members can be characterised as follows:

- If $\pi^m(\mathcal{M}) < \breve{\pi}(\mathcal{M})$ then the median country's unconstrained choice of Fund subscription is the majority voting equilibrium. All $\mathcal{M}$ countries prefer to be in a Fund with subscription level $g^u(\pi^m(\mathcal{M}))$ than to be outside the Fund.
- If $\pi^m(\mathcal{M}) > \breve{\pi}(\mathcal{M})$ then, if $\mathcal{M}$ members are in the Fund in equilibrium, the only possible Fund subscription level is the constrained choice which satisfies $D^{N-M+1}(g^c(\pi^m(\mathcal{M})), \pi^{N-M+1}) = 0$. All $\mathcal{M}$ countries weakly prefer to be in the Fund.

Thus the observation that no low crisis probability countries have dropped out of the Fund is consistent with the model if there is either an unconstrained choice by a median country with sufficiently low crisis probability relative to $\breve{\pi}(\mathcal{M})$, or a constrained choice of a higher median crisis probability country. In the previous subsection we concluded that it is π^m and not μ which drives the equilibrium Fund size. We can now qualify and strengthen this conclusion: π^m drives the equilibrium Fund size, providing π^m is low enough; otherwise the median voter will be constrained and π^1 will determine an upper limit on the size of the Fund. Once again, if the distribution of crisis probabilities becomes more skewed, so that π^1 and π^m falls relative to μ, as is consistent with the stylised facts, then we would expect the Fund to provide less coinsurance in the political equilibrium.

5.3 Qualified-majority voting and agenda setting

As noted already, in practice revisions to quotas require the support of members holding at least 85% of the votes. What implications does this have for the political equilibrium?

[26] Since $\frac{\partial g^u(\pi^i)}{\partial \pi^i} > 0$ the condition $\frac{\partial \hat{\pi}(g^u(\pi^i))}{\partial \pi^i} > 0$ is equivalent to $\frac{\partial \hat{\pi}(g^u(\pi^i))}{\partial g^u} > 0$. With a log utility function this can be shown to hold for all $\hat{\pi}(g(\pi^i)) < \pi^i$. Note that if $\hat{\pi}(g(\pi^i))$ is low enough that the cut-off country would not hold any reserves in the no-Fund world then $\frac{\partial \hat{\pi}(g^u(\pi^i))}{\partial g^u} > 0$ holds for any general utility function.

In the related work of Alesina *et al* (2001), the authors define qualified-majority voting (QMV) as a situation where, in a union of N members, any policy changes require a majority Q, where $1 \geqslant Q/N \geqslant 1/2$. They find that under QMV no single policy outcome unambiguously emerges from voting against all alternatives. In the context of our model, if g_{N-Q} and g_Q are the Fund sizes preferred by the $N - Q$ and Q ranked countries (in increasing order of π) then the set of options $\{g_{N-Q}; \ldots; g_Q\}$ cannot be beaten under QMV by an alternative option.[27] The size of the potential winning set is an increasing function of the required majority Q.[28]

However, Alesina *et al* find that in this type of situation the ambiguity is resolved by an agenda setter who decides which alternatives are put to a vote. In the context of our model, if there is only one round of proposals, with no amendments allowed, then the agenda setter will make a proposal which maximises her expected utility subject to the incentive constraint that at least $Q - 1$ other countries weakly prefer the new Fund size to the *status quo*.

As the United States has a veto power on QMV decisions at the Fund (with more than $Q - N$ votes) it can block any proposal by an agenda setter with which it disagrees. This would seem to increase the *status quo* bias against any enlargement of the Fund. However, in the Fund's case the agenda setter is perhaps most likely to be the United States itself, given that it is the largest shareholder and perhaps also because the Fund is based in the United States, which may increase the political influence the United States can exert over the IMF's staff and its Executive Board. Suppose the United States is the sole agenda setter and that the United States is also the member with the lowest vulnerability to a crisis. In this scenario the initial size of the Fund will be chosen by the United States to maximise its expected utility. This will be preferred by the rest of the membership, compared with the option of no Fund. As the agenda setter the United States can prevent any other options from being put to the vote. Over time, assuming that the United States remains both the agenda setter and the member with the lowest crisis probability, it will be able to increase the Fund size, should it want to do so, as this will gain the consent of a sufficiently large majority of the membership, but it will not be able to reduce it, in the absence of a generalised reduction of crisis probabilities.

[27] In the case of the Fund, which requires a 85% qualified majority, with the current 185 members we have $Q = 158$ and $N - Q = 27$. So the optimal choices of Fund size chosen by countries (ranked by increasing crisis probability) 1 through to 27 and from 159 through to 185 can all be defeated by an alternative with an 85% qualified majority. The choices of countries 28 through to 158 cannot. For example, g_{28} cannot be beaten by g_{29} by the required 85% majority, as all countries from 29 to 185 would prefer g_{29}, providing a majority of only 84.9%.

[28] Assume that the alternative option is the maximum public good provision unanimously supported against no provision: then, the lower bound of the winning set is decreasing in the required majority moving from the simple majority voting level of g_m to the unanimity level of g_0. The upper bound is increasing in the required majority for Q small enough and decreasing for Q big enough.

Thus, the conclusion we reach again strengthens those of the previous subsections: if the member with the lowest crisis probability is the agenda setter, it is π^1 that drives the equilibrium Fund size. If the distribution of crisis probabilities becomes more skewed, so that π^1 falls relative to μ, we would expect the equilibrium Fund subscription level to fall.

In Appendix A we consider how the existence of spillovers might modify these conclusions. In particular, we show how spillovers can lead countries to prefer a larger Fund size, other things being equal. However, this should not detract from the key conclusion reached from the analysis of this section, which is essentially that the Fund size is likely to be driven by countries with crisis probabilities that are, perhaps considerably, below the mean for the membership as a whole. What it does demonstrate, however, is that even if these countries themselves have very low, or possibly zero crisis probabilities, the existence of spillovers can rationalise a revealed preference for a positive Fund size, even if this is still considerably below that which might be preferred by, for example, the member with a mean crisis probability.

6 Welfare analysis

In this section we consider the conditions under which there is scope for Pareto-improving changes in the Fund size and the associated subscriptions of members. The intention is to illustrate why the framework underpinning the IMF's lending operations might need reconsidering, rather than to advocate a particular new approach.

Consider a given interior political equilibrium choice of Fund subscription, g, with corresponding Fund size and crisis payout, f. Can we change the Fund size and *individual* subscriptions to make at least one country better off and no country worse off? Note that by framing the question in this way we introduce the possibility that members pay different subscriptions, although we assume they still receive equal payouts in the event of a crisis.

Revenue neutrality requires that $\Delta f = \frac{1}{\mu N} \sum_{j=1}^{N} \Delta g^j$. Denote the modified subscription level of country i as $g^{i'} = g + \Delta g^i$ with the modified Fund payout common across countries as $f' = f + \Delta f = g/\mu + \Delta f$. We first consider how the consumption of country i is affected in crisis and non-crisis states by Δg^i and Δf, before considering what this implies for that country's expected utility.

We can write the following general expressions for consumption in each state for country i, in terms of $g,\ f = g/\mu,\ \Delta g^i$ and Δf:

$$
\begin{aligned}
c_c^{i,f}(g^{i'},\ f', \pi^i) = (1-\delta)\,\rho_A\left[1 - b^{i,f*} - \left(g + \Delta g^i\right)\right] + \rho_C b^{i,f*} \\
+ \left(\rho_C - \rho_F\right)(g/\mu + \Delta f) + \rho_F\left(g + \Delta g^i\right)
\end{aligned}
\tag{5}
$$

$$
c_n^{i,f}(g^{i'},\ f', \pi^i) = \rho_A\left[1 - b^{i,f*} - \left(g + \Delta g^i\right)\right] + \rho_R b^{i,f*} + \rho_F\left(g + \Delta g^i\right) \tag{6}
$$

where $b^{i,f*}$ is a non-increasing function of the new Fund subscription of country i. It follows that for all values of π^i the following conditions hold:

$$
\begin{aligned}
&\frac{\partial c_c^{i,f}}{\partial \Delta g^i} = \frac{\partial c_c^{i,f}}{\partial g} - \frac{\left(\rho_C - \rho_F\right)}{\mu} \qquad && \frac{\partial c_n^{i,f}}{\partial \Delta g^i} = \frac{\partial c_n^{i,f}}{\partial g} \\
&\frac{\partial c_c^{i,f}}{\partial \Delta f} = \left(\rho_C - \rho_F\right) && \frac{\partial c_n^{i,f}}{\partial \Delta f} = 0
\end{aligned}
$$

Expected utility is defined as $W^i = \pi^i u(c_c^{i,f}) + (1-\pi^i)u(c_n^{i,f})$. Given the partial derivatives of consumption in each of the states, for all values of π^i we have:

$$
\begin{aligned}
\frac{\partial W^i}{\partial \Delta g^i} &= \frac{\partial W^i}{\partial g} - \frac{\left(\rho_C - \rho_F\right)}{\mu}\pi^i u'\left(c_c^{i,f}\right) \\
\frac{\partial W^i}{\partial \Delta f} &= \left(\rho_C - \rho_F\right)\pi^i u'\left(c_c^{i,f}\right)
\end{aligned}
$$

For country i, the change in welfare is equal to $\Delta W^i = W^i(g^{i'},\ f', \pi^i) - W^i(g,\ f, \pi^i)$. For small changes in payouts and subscription levels this can be approximated by:

$$
\Delta W^i = \Delta g^i \frac{\partial W^i}{\partial \Delta g^i} + \Delta f \frac{\partial W^i}{\partial \Delta f} \tag{7}
$$

Putting the above expressions into equation **(7)** yields:

$$
\Delta W^i = \Delta g^i \frac{\partial W^i}{\partial g} + \left(\Delta f - \frac{\Delta g^i}{\mu}\right)\left(\rho_C - \rho_F\right)\pi^i u'\left(c_c^{i,f}\right) \tag{8}
$$

This expression is useful to characterise the likely sign of ΔW^i since we know the range of π^i over which $\partial W^i/\partial g$ is positive or negative. From the Proof of Proposition 4 we know that $\partial W^i/\partial g$ is increasing in π^i. Denote by h the member for which $\partial W^h/\partial g = 0$. With unconstrained majority voting $h = m$, but with either constrained majority voting, or QMV with an agenda setter, as described in the previous section, then $h < m$. It follows that for $i < h$ then $\partial W^i/\partial g < 0$ and for $i > h$ then $\partial W^i/\partial g > 0$.

From equation **(8)** we can make the following inferences. First, all members would prefer to pay a higher subscription to bring about a (small) increase in the Fund size, providing the increase in their own subscription is not too high. This follows as the coefficient on Δf is necessarily positive, given $u'(c_c^{i,f}) > 0$, and so ΔW^i will be positive providing Δg^i is not too large. Conversely, all members would prefer a lower Fund size, providing their subscription falls sufficiently.

Second, starting from any given political equilibrium, there is no common increase or decrease in the Fund subscription that is Pareto improving. A common change in subscription requires that $\Delta g^i = \mu \Delta f$ for all i and so the second term in **(8)** is zero. Following an increase (decrease) in Fund size the first term is negative (positive) for any $i < h$, but positive (negative) for $i > h$, and so benefits high-risk (low-risk) members at the expense low-risk (high-risk) members.

Third, for member h it must be the case that $\Delta W^h > 0$, providing $\Delta g^h < \mu \Delta f$, that is, providing any increase (decrease) in subscription for member h is less (greater) than the average for the Fund membership as a whole.

Now suppose we restrict the scheme used to finance a change in the Fund size to be linear in the crisis probability, such that $\Delta g^i = k\pi^i$, where k is necessarily a monotonically increasing function of Δf.[29] Then we can write:

$$\Delta W^i = k\pi^i \left\{ \frac{\partial W^i}{\partial g} + \frac{1}{\mu} \left(\mu - \pi^i\right) \left(\rho_C - \rho_F\right) u' \left(c_c^{i,f}\right) \right\} \qquad \textbf{(9)}$$

If $\pi^h < \mu$, as we would expect under each of the political equilibria outlined in the previous section and which we assume in the remainder of this section, then it follows that a small increase in Fund size raises the welfare of all members with crisis probability $\pi^h < \pi^i < \mu$.[30] Conversely, a decrease in the Fund size reduces the welfare of these same members, and so is *never* Pareto improving.

Note that for $\pi^i < \pi^h$ an increase in the Fund size still improves the welfare of member i, providing π^i is sufficiently close to π^h. This is because in this case the negative term in **(9)** is of second-order magnitude, whereas the positive term is of first-order magnitude. Similarly, for $\pi^i > \mu$ an increase in the Fund size still improves the welfare of member i, providing π^i is sufficiently close to μ.

[29] If the increase in the Fund size is to be adequately financed we require that $k \sum_i \pi^i = \Delta f$.

[30] Similarly, in the unlikely case where $\pi^h > \mu$ a decrease (increase) in Fund size raises (reduces) the welfare of all members with crisis probability $\pi^h > \pi^i > \mu$.

By imposing some further restrictions we can show that a small increase in Fund size benefits all members for sure.

Proposition 6 If $\Delta g^i = k\pi^i$ then (a) $\rho_C \geq \rho_A$ is a sufficient, but not a necessary condition for a small increase in Fund size to improve the welfare of all members with $\pi^i < \pi^h$ and (b) $\rho_F \geq \rho_R$ is a sufficient, but not a necessary condition for a small increase in Fund size to improve the welfare of all members with $\pi^i > \mu$.

Proof. See Appendix B.6

Taken together, these results mean that under these assumptions small increases in the Fund size, financed by the rule $\Delta g^i = k\pi^i$, are necessarily Pareto improving. The key to this result is that the linear financing rule distributes the cost of the increase in Fund size so that those that benefit most from it – that is, those with a high crisis probability – pay proportionately more than those who benefit less.

7 Conclusions

This paper develops a simple one-period investment model in which countries can protect themselves against the risk of adverse shocks by subscribing to a credit union or by accumulating reserves. The financial structure of the credit union mimics that of the IMF, crucially in that its overall size is determined by a vote of the membership. We assume that countries are equal in all respects except in their vulnerability to a crisis. This allows us to isolate the impact of the increasing heterogeneity of the Fund's membership, in terms of vulnerability to a crisis, on its effectiveness as a provider of consumption smoothing over crisis states. Adding other aspects of country heterogeneity, eg size and returns, is clearly an important avenue to pursue in subsequent work.

Our simple model yields some useful insights. If we accept that IMF member countries in 1946 were broadly similar, our analysis suggests that the Fund's founding fathers created an institution that was fit for purpose. Moreover by giving members the opportunity to revisit the size of the Fund every five years, they created a mechanism to ensure that the size of the Fund could be modified so that it continued to provide the appropriate amount of crisis-state payouts for a homogeneous, but crisis-prone membership.

However, based on our model, this adjustment mechanism may no longer work so well. Nowadays the Fund's membership consists of creditor and would-be debtor groups. In our model the equilibrium choice of the size of the Fund is likely to be driven by the preferences of creditor countries with a relatively low crisis probability. We make essentially this same inference from each of the political equilibria identified in Section 5. Under unconstrained majority voting it is the median voter that is decisive and over the life of the Fund it is likely that the median crisis probability has fallen relative to the mean among its membership. If the median voter is constrained by a binding participation constraint then this will further limit the size of the Fund. Finally, if the Fund size is determined by an agenda setter with a low crisis probability, this also limits the size of the Fund in our model.

This result has several implications, each of which we can observe in the global economy, although clearly there are other potential explanations. First, high crisis probability countries are likely to increasingly turn to self-insurance and hold more reserves than before. Second, in those regions where second-round spillovers are larger than average, regional Funds are likely to develop to provide a 'second-line' of multi-lateral crisis insurance. Both of these features have been observed in Asia. And finally, the average size of Fund assistance to actual crisis countries is likely to increase as the proportion of countries at risk of crisis falls. This too has been observed, and to some extent should offset the incentive for risky countries to self-insure.

These results could be taken as implying the Fund should be increased in size. We would caution against rushing to this conclusion. For a start, without changing the structure of the Fund, such a conclusion risks wishing away the problem, which is rooted in the institutional constraints which limit the size of the Fund. But more fundamentally, our model does not take into account moral hazard – we have made no allowance in our model for a relationship between the crisis probability and the size of Fund assistance. This is potentially an important omission: as the debates of recent years have demonstrated, many commentators have been deeply concerned about the risk of moral hazard associated with large Fund financial programmes.

We draw a different conclusion: that the framework governing the Fund's lending operations may no longer be appropriate. The credit union model that underpins the Fund's structure made sense in 1946 when the Fund was comprised of similar countries. That may no longer be the case. An alternative approach may be needed: one which takes into account that creditor and debtor countries have different interests and which takes into account the moral hazard consequences of large-scale lending. The ongoing international debate about the strategic direction of the IMF could helpfully encompass this issue.

Appendix A: Spillovers

Cross-country crisis spillovers can be represented in a reduced-form by each country caring about the consumption of others. This formulation can pick up economic or geopolitical reasons why countries may care about the consumption levels of others. This leads to a modified expected utility function for country i of:

$$W^i = \pi^i u(c_c^{i,f}) + (1-\pi^i)u(c_n^{i,f}) + \sum_{j=1, j\neq i}^{N} \beta_j^i \left(\pi^j v(c_c^{j,f}) + (1-\pi^j)v(c_n^{j,f})\right) \qquad \textbf{(A-1)}$$

where country i cares about the expected consumption of another country j through the function $v(\cdot)$ with a weight β_j^i. This allows spillovers to be specific to country pairs. Note that with this formulation, *reserve* choices are not influenced by spillovers, as the reserves held by country i have no impact on the consumption of country j. Thus Propositions 1 and 2 are unaffected. However, the size of the Fund does impact on the consumption of all member countries. Consequently, preferences over Fund size are affected by the introduction of spillovers.

Consider Proposition 3 concerning a country's preference over the Fund size. What impact do spillovers have on individual country preferences over g? In order to proceed, we make two simplifying assumptions. First, assume that country i cares about country j's consumption in the same way that country j does (ie $v(\cdot)$ and $u(\cdot)$ are the same function). Second, let country i care about country j's consumption with a weight $\beta_j^i = \beta^i l(\pi^j) \leq 1$ for all i, j. Define $Z^i(g) = \pi^i u(c_c^{i,f}) + (1-\pi^i)u(c_n^{i,f})$. This is the expected utility country i receives from its own consumption alone. Note that in the absence of spillovers $W^i = Z^i(g)$. The new Lagrangean for the preferred choice of g^i is:

$$\max_{g^i} \mathcal{L} = Z^i(g^i) + \sum_{j=1, j\neq i}^{N} \beta^i l(\pi^j) Z^j(g^i) + \lambda_1 b^{i,f*}(g^i) + \lambda_2 g^i - \lambda_3(b^{i,f*}(g^i) + g^i - 1)$$

The first-order condition for the interior solution, g^{i*}, is:[31]

$$0 = Z^{i\prime}(g^i) + \beta^i \sum_{j=1, j\neq i}^{N} l(\pi^j) Z^{j\prime}(g^i) \qquad \textbf{(A-2)}$$

In Subsection 4.2 we showed that, in the model without spillovers, $g^{i*} \geq 0$ and $b^{i,f*}(g^{i*}) = 0$ providing $\mu < \tilde{\mu}$. In the model with spillovers a similar condition on the level of the mean crisis probability can be derived. Moreover, by using the same method as in the no-spillovers case we

[31] The second-order conditions are satisfied under weak assumptions on the concavity of the utility function.

can also show that the single-crossing condition holds and hence we can apply the median voter theorem.

The key question is how the magnitude of the spillovers, captured by the summary statistic β^i, impacts on the optimum choice of Fund subscription levels. We know from the proof of Proposition 4 that, holding g fixed, $Z^j(g)$ increases with j, and so if g^i is such that $Z^{i\prime}(g^i) = 0$ then $Z^{j\prime}(g^i) < 0$ for $j < i$ and $Z^{j\prime}(g^i) > 0$ for $j > i$. Consequently, we can reach two conclusions. First, if country i cares about consumption of all other countries equally, so that $l(\pi^j) = 1$ for all j, then this will bunch together country preferences over g and this bunching will be more pronounced as β^i increases for all i. To see this, consider the extreme case where $\beta^i = 1$ for all i, which means that all countries care about each others' consumption as much as they care about their own. In this situation the first-order condition **(A-2)** is identical for all Fund members and so accordingly is the preferred Fund size. We can therefore conclude that stronger spillovers are, other things being equal, likely to raise the political equilibrium Fund size, where this is driven by a member with a below-mean crisis probability, such as is the case in each of the political equilibria outlined in Section 5. Second, if countries only care about the consumption of other countries which have a *higher* crisis probability, so that $l(\pi^j) = 0$ for $j < i$ and $l(\pi^j) > 0$ for $j > i$, then stronger spillovers unambiguously raise the Fund size preferred by all countries. Under this assumption stronger spillovers will have an unambiguously positive impact on the political equilibrium Fund size.

Appendix B: Proofs

B.1 Proposition 1 – Choice of reserves in world with no Fund

From the first-order condition for the maximisation of equation **(2)** we obtain an implicit expression for the interior solution for the optimal reserve holdings for country i (satisfying the second-order condition):

$$\pi^i \Gamma u'(\rho_A(1-\delta)(1-b^{i,nf*}) + \rho_C b^{i,nf*}) = (1-\pi^i)(\rho_A - \rho_R)u'(\rho_R b^{i,nf*} + \rho_A(1-b^{i,nf*}))$$

where $\Gamma \equiv \rho_C - (1-\delta)\rho_A$. Using the implicit function theorem we can see that the partial derivative of $b^{i,nf*}$ with respect to π^i is strictly increasing for concave utility functions.

The first-order condition implies corner solutions with $b^{i,nf*} = 0$ for $\pi^i \in [\pi^1, \underline{\pi}^{nf})$ and $b^{i,nf*} = 1$ for $\pi^i \in (\bar{\pi}^{nf}, \pi^N]$ where:

$$0 < \underline{\pi}^{b,nf} \equiv [1 + \frac{\Gamma u'((1-\delta)\rho_A)}{(\rho_A - \rho_R)u'(\rho_A)}]^{-1} < \bar{\pi}^{b,nf}$$

$$\bar{\pi}^{b,nf} \equiv [1 + \frac{\Gamma u'(\rho_C)}{(\rho_A - \rho_R)u'(\rho_R)}]^{-1} < 1$$

B.2 Proposition 2 – Choice of reserves for given Fund size g

The non-median countries face the constrained optimisation problem expressed as the Lagrangean of equation **(3)**. The first-order condition with respect to $b^{i,f}$, given g, is as follows (with the second-order condition satisfied):

$$\pi^i \Gamma u'((1-\delta)\rho_A + \Gamma b^{i,f} + \Omega g)-$$

$$(1-\pi^i)(\rho_A - \rho_R)u'(\rho_A - (\rho_A - \rho_R)b^{i,f} - (\rho_A - \rho_F)g) + \lambda_1 - \lambda_3 = 0 \qquad \textbf{(B-1)}$$

where λ_1 and λ_3 are the Lagrange multipliers on the constraints $b^{i,f} \geq 0$ and $b^{i,f} + g^* \leq 1$ respectively. The interior solution for $b^{i,f*}$ is implicitly defined by **(B-1)** with $\lambda_1 = \lambda_3 = 0$. With a concave utility function, for the interior solution $\partial b^{i,f*}/\partial \pi^i > 0$ and $\partial b^{i,f*}/\partial g^* < 0$.

Turning to the corner solution with $b^{i,f} = 0$, the requirement that $\lambda_1 > 0$ implies the condition:

$$\pi^i < \underline{\pi}^{b,f} \equiv [1 + \frac{\Gamma u'((1-\delta)\rho_A + \Omega g^*)}{(\rho_A - \rho_F)u'(\rho_A - (\rho_A - \rho_F)g^*)}]^{-1}$$

If the median country holds no reserves then since $\partial b^{i,f*}/\partial \pi^i > 0$ it can be seen that $\pi^m < \underline{\pi}^{b,f}$.

For the other corner solution, $b^{i,f*} + g^* = 1$, $\lambda_3 > 0$ implies the condition:

$$\pi^i > \bar{\pi}^{b,f} \equiv [1 + \frac{\Gamma u'(\rho_C + g^*(\rho_C - \rho_F)(1/\mu - 1))}{(\rho_A - \rho_F)u'(\rho_R - g^*(\rho_R - \rho_F))}]^{-1}$$

B.3 Proposition 3 – Country choice of reserves and Fund size

First let us show the equivalence of conditions on mean crisis probability and on relative returns.

Using the definitions of Ω and Γ:

$$\frac{\Omega}{(\rho_A - \rho_F)} > \frac{\Gamma}{(\rho_A - \rho_R)} \Longleftrightarrow \Gamma(\rho_A - \rho_R) + (\rho_C - \rho_F)(\rho_A - \rho_R)(1/\mu - 1) > \Gamma(\rho_A - \rho_F)$$

$$\Longleftrightarrow \mu < \tilde{\mu} \quad \text{where} \quad \tilde{\mu} \equiv \{1 + \frac{(\rho_C - (1-\delta)\rho_A)(\rho_R - \rho_F)}{(\rho_C - \rho_F)(\rho_A - \rho_R)}\}^{-1}$$

Similarly $\frac{\Omega}{(\rho_A - \rho_F)} < \frac{\Gamma}{(\rho_A - \rho_R)} \Longleftrightarrow \mu > \tilde{\mu}$ and $\frac{\Omega}{(\rho_A - \rho_F)} = \frac{\Gamma}{(\rho_A - \rho_R)} \Longleftrightarrow \mu = \tilde{\mu}$

The first-order conditions of the optimisation problem represented in the Lagrangean of **(4)** with respect to the choice of $g^{i,f}$ is as follows (with second-order conditions satisfied and associated complementary slackness conditions applying):

$$\begin{aligned} &\pi^i \Omega u'(c_c^{i,f}) - (1-\pi^i)(\rho_A - \rho_F)u'(c_n^{i,f}) + \lambda_2 - \lambda_3 \\ &+ \frac{\partial b^{i,f*}}{\partial g^{i,f}}(\pi^i \Gamma u'(c_c^{i,f}) - (1-\pi^i)(\rho_A - \rho_R)u'(c_n^{i,f}) + \lambda_1 - \lambda_3) = 0 \end{aligned} \qquad \textbf{(B-2)}$$

However, substituting in from the first-order conditions for the optimal choice of reserves, given Fund size, we obtain the following condition.

$$\pi^i \Omega u'(c_c^{i,f}) - (1-\pi^i)(\rho_A - \rho_F)u'(c_n^{i,f}) + \lambda_2 - \lambda_3 = 0 \qquad \textbf{(B-3)}$$

The optimality conditions are thus specified by equations **(B-3)** and **(B-1)**.

Case 1: $\mu < \tilde{\mu} \Longleftrightarrow \frac{\Omega}{(\rho_A - \rho_F)} > \frac{\Gamma}{(\rho_A - \rho_R)}$

Rearranging the first-order conditions for the choice of Fund subscription levels and reserves, we obtain:

$$\frac{\pi^i \Omega u'(c_c^{i,f})}{(1-\pi^i)(\rho_A - \rho_F)u'(c_n^{i,f})} = 1 + \frac{\lambda_3 - \lambda_2}{(1-\pi^i)(\rho_A - \rho_F)u'(c_n^{i,f})}$$

$$\frac{\pi^i \Gamma u'(c_c^{i,f})}{(1-\pi^i)(\rho_A - \rho_R)u'(c_n^{i,f})} = 1 + \frac{\lambda_3 - \lambda_1}{(1-\pi^i)(\rho_A - \rho_R)u'(c_n^{i,f})}$$

Given $\mu < \tilde{\mu}$ and the above two expressions we obtain the condition that

$$\frac{\lambda_3 - \lambda_2}{(\rho_A - \rho_F)} > \frac{\lambda_3 - \lambda_1}{(\rho_A - \rho_R)}$$

Consider first interior solutions with positive investment (ie $b^{i,f} + g^{i,f*} < 1$, $\lambda_3 = 0$). The above inequality simplifies to $\frac{\lambda_1}{(\rho_A - \rho_R)} > \frac{\lambda_2}{(\rho_A - \rho_F)}$. If we have positive Fund choice ($\lambda_2 = 0$) then the choice of reserves is zero ($\lambda_1 > 0$). The interior solution for $g^{i,f*}$ is implicitly defined by the following expression:

$$\pi^i \Omega u'((1-\delta)\rho_A + \Omega g^{i,f*}) = (1-\pi^i)(\rho_A - \rho_F)u'(\rho_A - (\rho_A - \rho_F)g^{i,f*}) \qquad \textbf{(B-4)}$$

If the Fund choice is zero then the reserve choice must also be zero. The reserve choice cannot be positive – if it was then λ_2 would have to be negative which cannot be the case as the Lagrange multiplier is greater than or equal to zero. Thus the upper corner solution in this case must be that $g^* = 1$ and reserves are zero. We now consider the two corner solutions.

Consider the corner solution $g^{i,f*} = 0$. From the requirement $\lambda_2 > 0$, the condition for countries to choose zero Fund size in this case is:

$$\pi^i < \underline{\pi}^{g,f} \equiv [1 + \frac{\Omega u'((1-\delta)\rho_A)}{(\rho_A - \rho_F)u'(\rho_A)}]^{-1}$$

Now, consider the corner solution $g^{i,f*} = 1$. By similar analysis we obtain the condition that the crisis probability be high enough such that:

$$\pi^i > \bar{\pi}^{g,f} \equiv [1 + \frac{\Omega u'(\rho_F + ((\rho_C - \rho_F))/\mu)}{(\rho_A - \rho_F)u'(\rho_F)}]^{-1}$$

We thus have $0 < \underline{\pi}^{g,f} < \bar{\pi}^{g,f} < 1$.

Case 2: $\mu > \tilde{\mu} \Longleftrightarrow \frac{\Omega}{(\rho_A - \rho_F)} < \frac{\Gamma}{(\rho_A - \rho_R)}$

We follow a similar approach to Case 1. Using the first-order condition and the reserve relations we obtain:

$$\frac{\lambda_3 - \lambda_2}{(\rho_A - \rho_F)} < \frac{\lambda_3 - \lambda_1}{(\rho_A - \rho_R)}$$

Consider first interior solutions with positive investment (ie $b^{i,f} + g^{i,f*} < 1$, $\lambda_3 = 0$). The above inequality simplifies to $\frac{\lambda_1}{(\rho_A - \rho_R)} < \frac{\lambda_2}{(\rho_A - \rho_F)}$. A positive Fund choice would imply a negative Lagrange multiplier $\lambda_1 = 0$ and so this case cannot apply. If the choice of reserves is positive then the optimal Fund subscription level is zero. The binding cut-offs for the corner solutions of zero reserves and full reserve holdings are as in Section B.1. If reserves are zero then the optimal choice of Fund size is also zero.

Case 3: $\mu = \tilde{\mu} \Longleftrightarrow \frac{\Omega}{(\rho_A - \rho_F)} = \frac{\Gamma}{(\rho_A - \rho_R)}$

Substituting the reserve relation into **(B-3)** and **(B-1)** and simplifying we obtain:

$$\pi^i(\Omega - \Gamma)u'((1-\delta)\rho_A + \Gamma b^{i,f*} + \Omega g^{i*}) = (1-\pi^i)(\rho_R - \rho_F)u'(\rho_A - (\rho_A - \rho_R)b^{i,f*} - (\rho_A - \rho_F)g^{i*}) \quad \textbf{(B-5)}$$

This relation can only hold if $\rho_R > \rho_F$. Furthermore the reserve relation implies $\lambda_1 = \lambda_2$. The only consistent possibilities are that both $b^{i,f*}$ and $g^{i,f*}$ are zero or both are positive. Both are zero if $\pi^i < \left(1 + \frac{\Omega u'((1-\delta)\rho_A)}{(\rho_A - \rho_F)u'(\rho_A)}\right)^{-1}$. The other corner solution of $b^{i,f*} + g^{i,f*} = 1$ cannot hold.[32] Thus, if both are positive they are under-determined.

B.4 Proposition 4 – Political equilibrium choice over Fund size

Denote country i's expected utility from a Fund size of g as $W(g, \pi^i)$. Following Ashworth and Bueno de Mesquita (2006), 'increasing differences' in the return from changing policy is a sufficient condition for single-crossing. 'Increasing differences' holds if the return from changing policy is increasing in country type, ie

$$W(g, \pi^i) - W(g', \pi^i) \geq W(g, \pi^{i'}) - W(g', \pi^{i'})$$

$\forall g > g'$ and $\forall \pi^i > \pi^{i'}$. For small changes in π^i, this condition can be approximated by:

$$(\pi^i - \pi^{i'})[\frac{\partial W(g, \pi^i)}{\partial g} - \frac{\partial W(g, \pi^{i'})}{\partial g}] \geq 0$$

This is equivalent to $\frac{\partial^2 W(g,\pi^i)}{\partial g \partial \pi^i} \geq 0$. With $\frac{\partial W(g,\pi^i)}{\partial g} = \pi^i \Omega u'(c_c^f) - (1-\pi^i)(\rho_A - \rho_F)u'(c_n^f)$ we have,

$$\begin{aligned} \frac{\partial^2 W(g, \pi^i)}{\partial g \partial \pi^i} &= \Omega u'(c_c^f) + (\rho_A - \rho_F)u'(c_n^f) - \\ &\quad \frac{\partial b^{i,f}}{\partial \pi^i}[\pi^i \Gamma \Omega u''(c_c^f) + (1-\pi^i)(\rho_A - \rho_R)(\rho_A - \rho_F)u''(c_n^f)] \end{aligned}$$

[32] This can be seen from the condition for $\lambda_3 > 0$ obtained from **(B-3)**. Substituting in from **(B-5)** yields an inconsistent condition.

Thus $\frac{\partial^2 W(g,\pi^i)}{\partial g \partial \pi^i} \geq 0 \; \forall \pi^i > \pi^{i'}$. We therefore have strictly increasing differences. This is a sufficient condition for single-crossing and hence for the application of the median voter theorem.

B.5 Proposition 5 – Utility comparison in and outside the Fund

From the proof of Proposition 4, we have

$$W(g, \pi^i) - W(g', \pi^i) > W(g, \pi^{i'}) - W(g', \pi^{i'})$$

$\forall g > g'$ and $\forall \pi^i > \pi^{i'}$. Setting $g' = 0$ this equation tells us that the gain in welfare from being in the Fund compared to being outside is strictly increasing in a country's crisis probability.

Consider the extreme case where $\pi_1 = 0$. Such a country would hold zero reserves in both the Fund and no-Fund worlds. Expected utility of this country is just its utility in the non-crisis state and so $W(g, \pi_1 = 0) - W(0, \pi_1 = 0) = u(c_n^{1,f}) - u(c_n^{1,nf})$. In the no-Fund world non-crisis state consumption is $c_n^{1,f} = \rho_A$. Non-crisis state consumption in the Fund world is lower due to the required subscription to the Fund $c_n^{1,nf} = \rho_A - (\rho_A - \rho_F)g^*$. Thus the country is better off outside the Fund. The median country is better off in the non-zero Fund (since it had the option of choosing a zero Fund size). Given that the welfare difference between being in and outside the Fund is continuous and strictly increasing in the crisis probability we have a unique fixed point $\widehat{\pi}$ at which $W(g, \widehat{\pi}) = W(0, \widehat{\pi})$. So, provided that $\pi_1 \leq \widehat{\pi}$ we have at least one country who would be better off outside the Fund.

B.6 Proposition 6 – Pareto-improving increase in Fund size with a linear financing scheme

Partial differentiation of **(5)** and **(6)** gives:

$$\frac{\partial c_c^{i,f}}{\partial g} = \Gamma \left(1 + \frac{\partial b^{i,f*}}{\partial g}\right) + \left(\frac{1-\mu}{\mu}\right)(\rho_C - \rho_F) \qquad \textbf{(B-6)}$$

$$\frac{\partial c_n^{i,f}}{\partial g} = \rho_F - \rho_R - (\rho_A - \rho_R)\left(1 + \frac{\partial b^{i,f*}}{\partial g}\right) \qquad \textbf{(B-7)}$$

Using **(B-6)**, **(B-7)** and **(9)** the impact on expected utility of a small increase in Fund size can be written as:

$$\begin{aligned} \Delta W^i = k\pi^i \Big\{ &\left(1 + \tfrac{\partial b^{i,f*}}{\partial g}\right)\left[\pi^i \Gamma u'\left(c_c^{i,f}\right) - (1-\pi^i)(\rho_A - \rho_R)u'\left(c_n^{i,f}\right)\right] \\ &+ (1-\pi^i)\left[\left(\rho_C - \rho_F\right)u'\left(c_c^{i,f}\right) + (\rho_F - \rho_R)\,u'\left(c_n^{i,f}\right)\right]\Big\} \end{aligned} \qquad \textbf{(B-8)}$$

Consider first part (a) of the proposition. As $\pi^h < \underline{\pi}^{b,f}$ we know from Proposition 2 that $\partial b^{i,f*}/\partial g = 0$. Consequently **(B-8)** becomes:

$$\Delta W^i = k\pi^i \left\{\left[\pi^i \Gamma + (1-\pi^i)\left(\rho_C - \rho_F\right)\right] u'\left(c_c^{i,f}\right) - (1-\pi^i)\left(\rho_A - \rho_F\right) u'\left(c_n^{i,f}\right)\right\}$$

Given $c_n^{i,f} > c_c^{i,f}$ implies $u'\left(c_c^{i,f}\right) > u'\left(c_n^{i,f}\right)$, a sufficient, but not a necessary condition for $\Delta W^i > 0$ is that

$$\pi^i \Gamma + (1-\pi^i)\left(\rho_C - \rho_A\right) \geq 0$$

Given $\Gamma > 0$ a sufficient, but not a necessary condition for this to hold is that $\rho_C \geq \rho_A$, which proves part (a) of the proposition.

Now consider part (b) of the proposition. For $\pi^i > \bar{\pi}^{b,f}$ we know that $b^{i,f*} + g = 1$ and so $\partial b^{i,f*}/\partial g = -1$. In this case a positive marginal utility from consumption in each of the states is sufficient to ensure that $\Delta W^i > 0$. For $\mu < \pi^i < \bar{\pi}^{b,f}$ then from **(B-1)**, using the implicit function theorem:

$$1 + \frac{\partial b^{i,f*}}{\partial g} = -\frac{N}{D}$$

where

$$N = -\left[\pi^i \Gamma \left(\rho_C - \rho_F\right)\left(\frac{1-\mu}{\mu}\right) u''\left(c_c^{i,f}\right) - (1-\pi^i)(\rho_A - \rho_R)\left(\rho_F - \rho_R\right) u''\left(c_n^{i,f}\right)\right]$$
$$D = -\left[\pi^i \Gamma^2 u''\left(c_c^{i,f}\right) + (1-\pi^i)(\rho_A - \rho_R)^2 u''\left(c_n^{i,f}\right)\right] > 0$$

By substitution into **(B-8)** this becomes:

$$\begin{aligned} \Delta W^i = \tfrac{k\pi^i}{D} &\left\{\left[(1-\pi^i) D\left(\rho_C - \rho_F\right) - N\pi^i \Gamma\right] u'\left(c_c^{i,f}\right)\right. \\ &\left. + (1-\pi^i)\left[N(\rho_A - \rho_R) + D\left(\rho_F - \rho_R\right)\right] u'\left(c_n^{i,f}\right)\right\} \end{aligned} \qquad \textbf{(B-9)}$$

The concavity of the utility function means that $\rho_F \geq \rho_R$ is a sufficient, but not a necessary condition to ensure that each of the terms in square brackets in **(B-9)** is positive. This implies $\Delta W^i > 0$ for all $\pi^i > \mu$ and thus completes the proof.

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