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Efficient frameworks for sovereign borrowing

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Abstract

This paper presents a theoretical model of strategic default to assess how national and international policymakers should seek to influence the cost of default and the distribution of bargaining power in the event of a default. We find that, in the absence of restrictions on the parameter space, deadweight costs of default should be driven to zero. Moreover, if the debtor is risk-averse, there is an optimal division of bargaining power between the debtor and its creditors. Even with restrictions on the parameter space, marginally lower deadweight costs, possibly in some combination with greater creditor bargaining power, can always raise social welfare *ex ante*. However, once debt has been contracted, the debtor's trade-off between creditor bargaining power and deadweight costs changes fundamentally. In equilibrium, the deadweight costs of default may therefore tend to be too high, and the allocation of bargaining power inefficiently skewed towards the debtor. The challenge for policymakers is to find credible, time-consistent combinations of policies that can both reduce deadweight costs and shift bargaining power towards creditors.

Key words: Sovereign debt, default, restructuring.

JEL classification: F34, G15.

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Summary

There is no supranational authority that can enforce sovereign debt contracts. Consequently, the decision by a government to default on its debts is often as much a question of willingness to pay as it is of ability to pay. Debt restructurings, which change both the size and the timing of payments made to creditors, are therefore brought about through negotiation between the parties to the contract, rather than by court adjudication. When a sovereign decides whether to default it has to weigh the benefit against the cost. The main benefit comes in the form of a reduced repayment, which is often referred to in understated terms as a 'haircut' for creditors. The cost comes in a number of different forms, such as loss of reputation, or loss of current and future access to private capital markets. Moreover, sovereign default is often associated with costly currency crises and banking crises. The multiple costs of default, and the partial extent of the haircut that can be achieved in practice, both serve to limit the incentive that the sovereign has to default and underpin the very existence of sovereign debt markets.

National and international policymakers have some control *ex ante* over the size and form of the cost of default and the distribution of bargaining power in the event of a default. This paper presents a theoretical model of strategic default to assess how policymakers should exercise their control over these levers. We consider a world in which the sovereign issues fixed interest debt to finance an investment with uncertain returns. After both the productivity of the investment and the resulting income stream are known, the sovereign must decide whether to repay the debt in full, or to seek a restructuring. If the sovereign takes the second option we assume it must pay a deadweight cost, to reflect the loss of reputation and the economic disruption that ensues following a default. It must then negotiate over the size of the haircut, which is ultimately determined by the distribution of bargaining power. These factors – the deadweight cost and the distribution of bargaining power following a default – are the two key dimensions by which the 'framework for sovereign borrowing' is characterised in our model. We assess the welfare-maximising values for both these policy parameters.

We find that, if there are no restrictions on the distribution of the bargaining power, the deadweight costs of default should be driven to zero. Both deadweight costs and the need to settle with creditors can dissuade a debtor from defaulting. However, the latter is more efficient, as resources denied to the debtor are reallocated to creditors rather than being destroyed.

Assuming creditors are competitive and risk-neutral, this should benefit the debtor through lower interest rates. If the debtor is risk-averse, then in the event of a restructuring the optimal outcome requires bargaining power to be shared between the debtor and its creditors. This is because shifting bargaining power to creditors has two conflicting effects on the debtor's welfare. On the one hand, by dissuading default and lowering interest rates, it allows a sovereign to borrow more at a lower cost. But on the other, if creditors capture too much of the available resources after a default, the risk-sharing benefit of default is diminished, as creditors receive additional resources from the debtor when the latter needs them most. The optimal regime should balance this tension.

In constrained policy settings we find that, whenever welfare can be raised by marginally *increasing* the deadweight costs of default, welfare is also improved by shifting bargaining power to creditors. It follows that for any given value of the deadweight cost, if bargaining power is optimally allocated between the parties, it must be welfare-improving to reduce the deadweight cost. Moreover, starting from any situation where the welfare impact of marginally raising the deadweight cost is positive, there is always a step increase in the allocation of the bargaining power to creditors which is sufficient to ensure that the impact of raising the deadweight cost becomes negative. Taken together, these results mean that, so long as creditor bargaining power can be increased, *lower* deadweight costs can always raise social welfare *ex ante*.

The analysis shows that, once debt has been contracted, the debtor's trade-off between creditor bargaining power and deadweight costs changes fundamentally. With the interest rate on debt fixed, the incentives of the debtor change so that it no longer cares whether, after a default, resources are transferred to creditors or are wasted in the form of deadweight costs. There is therefore a need to design mechanisms that allow debtors to commit to the *ex-ante* optimal combination of policy parameters.

In sum, these results suggest that domestic and international policymakers should pay careful attention to the impact of their policies, not just on the deadweight costs of default, but also on the allocation of bargaining power in the event of a restructuring. The final result, in particular, suggests that in equilibrium the deadweight costs of default may tend to be too high, and the allocation of bargaining power inefficiently skewed towards the debtor. A challenge for all policymakers, therefore, is to find credible policies that can both reduce deadweight costs and shift bargaining power towards creditors. In due course this should raise welfare.

1 Introduction

There is no supranational authority that can enforce sovereign debt contracts. Courts in foreign jurisdictions sometimes rule that a sovereign in default has a legal obligation to repay its debts, but even so the principle of sovereign immunity severely curtails the ability of creditors to make good on their claims when the sovereign is unwilling to co-operate. This sets sovereign debt apart from corporate debt in two respects (Bedford *et al* (2005)). First, the decision to default (or to seek a restructuring pre-default) is a question of willingness to pay as much as ability to pay. Second, restructurings are brought about through negotiation between the debtor and its creditors, albeit usually indirectly and informally, rather than by court adjudication.[1]

When a sovereign decides whether to default on its external debts it has to weigh the benefit against the cost. The benefit comes in the form of a reduced repayment – the 'haircut' the sovereign obtains on the debt it owes to its creditors. This is the outcome of the negotiation process, which in turn depends on the relative bargaining strengths of the parties to the negotiation. Sturzenegger and Zettelmeyer (2007) find a considerable variation in the haircut achieved on external debt in recent restructurings, ranging from 13% on average in Uruguay in 2003, to around 50% in Russia in 1999 and 2000, and 74% in Argentina in 2005. The deadweight cost of default can come in a number of different forms, such as loss of reputation, or loss of current and future access to private capital markets. Moreover, sovereign default is often associated with costly currency crises and banking crises. The overall impact is often a significant and enduring output loss for the economy as a whole. De Paoli, Hoggarth, and Saporta (2006) estimate the median output loss from recent crises to be 6.9% of GDP in each crisis year, with the median length of crises eight years.[2] The multiple costs of default, and the partial extent of the haircut that can be achieved in practice, both serve to limit the incentive that the sovereign has to default. Dooley (2000) has gone so far as to conjecture that deadweight costs of default in particular are an *essential* component of sovereign debt markets: in their absence countries might never choose to repay their debts and, foreseeing this, creditors would never lend to them in the first place.

Domestic and international policymakers have some control *ex ante* over the size and form of the

[1] In mature domestic bankruptcy regimes corporate restructurings are often concluded in the 'shadow of the law', in order to preempt predictable court rulings. By so doing the parties concerned can avoid unnecessary legal costs. In sovereign restructurings there is no 'shadow of the law'.

[2] This is the cumulative difference per year between potential and actual output.

cost of default and the distribution of bargaining power in the event of a default. For example, sovereigns can raise deadweight costs by issuing debt in forms that are hard to restructure (Dooley (2000), Bolton and Jeanne (2005)), or difficult to default on selectively (Broner and Ventura (2005)). Sovereigns can also increase their creditors' bargaining power by placing some resources under foreign jurisdictions, or by entering into international treaties which bolster creditor rights.[3] The official sector can also influence deadweight costs and the distribution of bargaining power through 'architectural' innovations, such as encouraging the widespread inclusion of collective-action clauses in sovereign bond contracts (Haldane *et al* (2005)). By lending to a debtor in arrears the IMF can directly reduce the deadweight cost of default, but may also shift bargaining power away from creditors to the sovereign, as the latter has a less urgent need to re-access private international capital markets.

How should policymakers exercise their control over these levers? Should they seek to raise or lower the cost of default? Should they attempt to shift bargaining power to or from the sovereign in the event of a default? What impact will these measures have on the incentive to default in the first place and on welfare? Weak incentives – due to a low deadweight cost of default and/or high debtor bargaining power – will result in more frequent and larger defaults, leading creditors to require high interest rates. But strong incentives may cause the debtor undue hardship in bad times and result in insufficiently frequent defaults with suboptimally low haircuts. How can domestic and international policymakers strike a balance between these competing effects? Is Dooley right when he conjectures that deadweight costs are necessary to sustain sovereign debt markets?

This paper presents a stylised theoretical model of strategic external debt default to address these questions. We consider a world in which the sovereign issues plain vanilla debt to finance an investment with uncertain returns. After the productivity of the investment is known the sovereign must decide whether to repay the debt in full, or to seek a restructuring of the debt. If the sovereign takes the second course of action it must pay a deadweight cost, equal to a fixed proportion of the output from the investment, and negotiate over the size of the haircut, which is ultimately determined by the distribution of bargaining strength between the sovereign and its creditors, which we take to be exogenous. These factors – the deadweight costs and the distribution of bargaining power following a default – are the two key dimensions by which the

[3]For example, the 'Convention on the Settlement of Investment Disputes between States and Nationals of Other States', which is administered under the auspices of the World Bank's ICSID.

‘framework for sovereign borrowing’ is characterised in our model.[4] We then assess the *ex-ante* welfare-maximising values for these policy parameters, both with and without constraints on the parameter space. The analysis has implications for both domestic and international policymakers, insofar as they can influence these parameters *ex ante*.

The analysis produces five key results. The first main result is that, in the absence of restrictions on the parameters space, deadweight costs of default should be driven to zero. It is always better to tackle the enforcement problems of sovereign debt by giving creditors the power to bargain for some claim on the debtor’s resources after default. The simple intuition for this result is as follows. Both deadweight costs and the need to settle with creditors can dissuade a debtor from defaulting. But an *ex-post* transfer is more efficient, because the resources denied to the debtor go to creditors rather than being destroyed. Knowing that they will receive some payment in the event of default, competitive creditors demand a lower interest rate *ex ante*. With lower contractual interest rates to pay, debtors default less frequently, other things being equal, and have higher consumption when they do repay. Put another way, creditors in a competitive market never earn any surplus, so whatever framework maximises debtor surplus must also maximise social welfare. By the same token, debtors pay all the expected deadweight costs, so for a given level of lending it is optimal to reduce deadweight costs as far as possible. This result is germane to Dooley’s conjecture that deadweight costs are necessary to support the existence of sovereign external borrowing. Our simple framework demonstrates that sovereign borrowing can be supported in equilibrium in the absence of default costs, providing creditors have sufficient bargaining power post-default. Moreover, this is superior in welfare terms.

The second main result is that, if the debtor is risk-averse, then in the event of a restructuring there is an interior optimum allocation of bargaining power between the debtor and its creditors. This is because shifting bargaining power to creditors has two conflicting effects on the debtor’s welfare. On the one hand, by dissuading default and lowering interest rates, it allows a sovereign to borrow more at a lower cost. But on the other, if creditors capture too much of the available resources after a default, the risk-sharing benefit of default is diminished, as creditors receive additional resources from the debtor when the latter needs them most. The optimal regime should balance this tension.

[4]In practice, the distribution of bargaining power may be partly influenced by the extent to which creditors are able to threaten credibly to take actions which would impose additional costs on the debtor. We do not explicitly model this interconnection. Instead we take the distribution of bargaining power and the deadweight costs that *are* incurred in the event of a default as exogenously given, and consider the effect that varying each has on welfare.

In our specification, it is optimal to allocate creditors just enough bargaining power so they are willing to extend the amount of lending that maximises the expected consumption of the debtor (also equal to expected Gross National Product (GNP)). Returns to capital are diminishing, so the level of investment that maximises expected consumption is finite. Giving debtors a borrowing capacity beyond this level cannot raise expected consumption, and the required increase in creditor bargaining power must reduce the debtor's consumption when it defaults, which is when the marginal utility from consumption is highest. On the other hand, reducing borrowing capacity below the efficient level reduces the debtor's consumption for every realisation of productivity.

A related implication of this result is that *ex post* benchmarking a haircut against 'ability to pay' alone is an inappropriate test of the merits of a restructuring deal. *Ex-ante* efficiency, and in particular the desire to share risk, requires that creditors should not be able to bargain for *all* that a sovereign can repay in the event of a default.

The third and fourth results relate to constrained policy settings, when the policymaker does not have complete discretion over the size of the deadweight costs and the degree of creditor bargaining power. This analysis is important because in practice policymakers have only a limited ability to influence the size of each. Moreover, the factors which influence each are likely to be under the imperfect control of several parties. Consequently it is perhaps most useful for policymakers to understand the impact on welfare of marginal changes to either the deadweight cost or the allocation of bargaining power.

The third key result is that, for any arbitrary level of deadweight costs and creditor bargaining power, whenever welfare can be raised by *increasing* the deadweight costs of default, welfare is also improved by shifting bargaining power to creditors. Both measures discourage default, and can therefore increase credit supply. But they can also harm welfare by punishing debtors in bad states of the world. We show that whenever the former beneficial effect outweighs the latter adverse one for deadweight costs, such that it improves welfare to raise them, this must also be the case for increased creditor bargaining power. It then follows that for any given value of the deadweight cost, if bargaining power is optimally allocated between the parties, it must be welfare-improving to reduce the deadweight cost. Moreover, starting from any combination of the policy parameters for which the welfare impact of marginally raising the deadweight cost is positive, there is always a step increase in the allocation of the bargaining power to creditors which is sufficient to ensure that the impact of raising the deadweight cost becomes negative. Taken together, these results mean that, so long as creditor bargaining power can be increased, *lower* deadweight costs can always raise social welfare *ex ante*. Once again, this substantially qualifies Dooley's conjecture.

The fourth main result (obtained numerically) suggests that, if deadweight costs cannot be reduced to zero, the optimal degree of creditor bargaining power is generally lower than in the unconstrained case, and may even fall to zero. Intuitively, the two policy measures are imperfect substitutes, in that both dissuade default, and both punish debtors in bad states of the world. So a constrained policymaker should generally attempt to offset excessive deadweight costs by reducing creditor bargaining power.

The final main result is that, once debt has been contracted, the debtor's trade-off between creditor bargaining power and deadweight costs changes fundamentally. With the interest rate on outstanding debt fixed, the debtor no longer cares whether, after a default, resources are transferred to creditors or are wasted in the form of deadweight costs. The *ex-post* optimal combination of policy parameters diverges from the *ex-ante* optimal combination and the change in debtor incentives therefore gives rise to a time-consistency problem. With this, there is a need to design mechanisms that allow debtors to commit to resolving debt problems in ways that transfer resources to creditors rather than wasting them.

Taken together these results suggest that domestic and international policymakers should pay careful attention to the impact of their policies, not just on the deadweight costs of default, but also on the allocation of bargaining power in the event of a restructuring. The final result in particular suggests in equilibrium the deadweight costs of default may tend to be too high, and the allocation of bargaining power inefficiently skewed towards the debtor. A challenge for all policymakers, therefore, is to find credible, time-consistent combinations of polices that can both reduce deadweight costs and shift bargaining power towards creditors. In due course this should increase the amount of finance available to the debtor and raise welfare.

The remainder of the paper is structured as follows. The next subsection reviews the related literature in more detail. Subsection 1.2 then illustrates how policymakers can influence the key parameters in our model using the IMF's lending-into-arrears policy as an example. Section 2 sets out the model. Section 3 characterises optimal policy *ex ante*, assuming the policymaker is unconstrained, while Section 4 considers the case where the policymaker faces exogenous constraints on the parameter space. Section 5 highlights the tension between *ex-ante* and *ex-post* optimality, and the time-consistency problem that follows from it. Section 6 concludes.

1.1 Related literature

There is a large literature on the question of why sovereign debtors repay external debts. In the past, incentives to repay were often provided by the threat of force. Mitchener and Weidenmier (2005) document the use of 'supersanctions' – gunboat diplomacy, direct fiscal control and asset seizures – as a means of ensuring the repayment of sovereign debt during the gold standard era. Most modern explanations are less direct and fall into one of two categories. The first suggests that countries repay in order to avoid immediate economic costs of default, which might either be an inevitable consequence (eg Bolton and Jeanne (2005)), or voluntarily imposed by creditors and therefore a threat point in any negotiation (eg Bulow and Rogoff (1989)). The second category proposes that countries repay in order to avoid reputational costs, which may subsequently manifest themselves as poor financing terms when capital markets are re-accessed (eg Grossman and van Huyck (1985) and Cole, Dow and English (1995)).

There is also a literature on the risk-shifting aspect of the option to default. Several papers (eg van Wijnbergen (1990), Zame (1993), Zha (2001)) recognise that, when securities markets are incomplete, the option to default in some states of the world can raise welfare by providing partial insurance against bad states of the world.

But few papers address, even indirectly, the question of what environment domestic and international policymakers should aim for. The strategic default model of sovereign borrowing in Gai *et al* (2001) suggests that higher deadweight costs increase the amount of lending that is incentive compatible, but reduce the amount that is efficient through its negative effect on the expected productivity of investment. Dooley (2000) notes that sovereign borrowers may endogenously choose debt structures that are costly to restructure as a means of committing to repay their debts. Bolton and Jeanne (2005) find that a debtor may choose to issue debt with high restructuring costs in order to commit not to borrow excessively at a later date. Cavallo and Velasco (2006) find that a sovereign should only commit to limited creditor sanctions in the event of a default, using a model in which the sovereign enters into what is essentially an insurance contract with creditors, but in which payments by creditors are unpredictable and subject to sudden stops. Haldane *et al* (2005) examine the optimal threshold for collective-action clauses in sovereign bond contracts, given the effect this has on both the likelihood of a default and the restructuring outcome. They conclude that different debtors should choose different thresholds, depending on their degree of risk aversion and their creditworthiness.

Two papers touch directly on the class of policy questions we are trying to address. Hamann (2004) and Yue (2006) both compute and simulate infinite-horizon models of a small open economy with exogenous stochastic output, able to issue plain vanilla debt only. In Hamann's model, the sovereign can choose to default at the cost of a perpetual reduction in output and permanent exclusion from credit markets. He calibrates the model to Argentine data and finds that increasing the deadweight cost from 10% to 35% of output reduces to zero the incidence of default and raises welfare by around 7% of consumption. However, Hamann holds creditor bargaining power fixed across these experiments. Yue examines the impact of shifting post-default bargaining power from creditors to debtors. In a baseline parameterisation which includes unavoidable deadweight costs of default, she finds that welfare is maximised by giving debtors all of the bargaining power, such that a default implies complete repudiation of debt. However, Yue does not examine the sensitivity of these results to the size of the deadweight cost of default, which are held fixed.

Our paper does not attempt to match the data in the same way as these two papers do and our framework is essentially static. However, unlike these previous studies, the model we use incorporates both creditor bargaining power and variable deadweight costs of default. This enables us to assess the extent to which policies that redistribute bargaining power or vary deadweight costs are complements or substitutes and to identify both the globally-optimal and the constrained-optimal combinations of these 'policy parameters'. In so doing, our aim is to provide domestic and international policymakers with a better understanding of the trade-offs they face when assessing the merits of reforms that have been proposed for the framework for sovereign borrowing.

1.2 IMF lending into arrears

In this subsection we illustrate how policymakers can influence the policy parameters in our model using the IMF's lending-into-arrears (LIA) policy as an example. The provision of financing by the IMF to a sovereign debtor who is in arrears with private creditors can impact on the deadweight costs of default in a number of ways. In particular, financing can be used to offset the effect on exporters, who might otherwise find they are denied access to trade credit. Equally, IMF financing can be used to mitigate the impact of a default on the domestic banking sector, for fear that this would otherwise result in a credit crunch and so push the economy further into recession.

But IMF LIA also impacts on the distribution of bargaining power between a debtor in default and its creditors. Before 1989 it was IMF policy not to lend to members in arrears with their commercial bank creditors, who at that time provided the vast bulk of private lending to emerging market sovereigns. However, this had the unintended consequence of providing commercial banks with an effective veto over IMF lending decisions, hence giving a considerable boost to their bargaining position during a restructuring.

In 1989 the IMF introduced a new LIA policy which enabled it to lend to members in arrears on bank credit, subject to three conditions. The first condition was that the financing package be considered essential for the successful implementation of the debtor's adjustment program and therefore to help limit the scale of economic dislocation. The second and third conditions were, respectively, that restructuring negotiations had begun between the debtor and its creditors and that it was expected that a package to restore external sustainability would be agreed within a reasonable period.

In 1998, in order to reflect the changing nature of capital flows to emerging markets, the policy was amended to also allow the IMF to lend to members in arrears on international sovereign bonds. This raised concerns that not all creditors would be willing or able to enter into timely negotiations with a debtor, in part because of difficulties in co-ordinating among a more dispersed creditor base, but also because of fears that bondholders might have less of an interest in maintaining an ongoing relationship with a debtor in difficulty and hence attach a lower priority re-establishing the debtor's good financial standing. As a result, in 1999 the third condition was amended to instead require that the debtor be making 'good-faith efforts' to negotiate with creditors.[5]

The history of the LIA policy illustrates how the IMF can, through its actions, impact on both the

[5]In response to concerns that the good faith criterion was too vague, the policy was amended again in 2002 with the intention of providing greater procedural clarity.

deadweight cost of default and the distribution of bargaining power between debtors and creditors in the event that a restructuring is required. The fundamental rationale for IMF lending to a debtor in arrears is to reduce the deadweight cost or the 'economic dislocation' that can result. But the development of the policy also illustrates how the Fund has been sensitive to the potential impact of LIA on bargaining power and hence the outcome of restructuring negotiations. Prior to 1989 the concern was that the IMF's lending policy, and in particular its non-toleration of arrears, gave the whip hand in a restructuring negotiation to commercial bank creditors. Since then the evolution of the policy has seen the pendulum swing in the opposite direction, so that now the provision of finance to debtors in arrears is subject to only a very loosely defined requirement that the debtor engage in good-faith efforts to negotiate with creditors.

The model set out below helps to illustrate some of the trade-offs that must be faced when assessing this and other policies. In particular it helps to identify the conditions under which IMF financing that reduces deadweight costs is welfare-improving. It underscores the importance of designing the LIA policy in such a way as to ensure that the combined effect on deadweight costs and the distribution of bargaining power is beneficial. At the very least it illustrates that the impact of the LIA policy on the balance of bargaining power should not be neglected.

2 The model

We model an economy that has access to a production technology with uncertain returns. It has no resources of its own which it can use to invest in the project, so must borrow abroad to finance it. The country chooses how much to borrow, at an interest rate determined endogenously in the international capital market, so as to maximise the expected utility of consumption when the returns of the project are realised.

Crucially, there is no external commitment technology that can force the country to repay its debts. Once the returns on the project have been realised, the sovereign must choose whether to repay or default. If it defaults, we assume the sovereign loses a fraction α of output as a deadweight cost, and must relinquish a fraction β of the remainder to creditors in partial settlement of its debts. We assume that the parameters α and β can be set with credibility, either by the debtor itself or by some outside agency. We consider, in turn, the cases where the parameter space is unrestricted and where it is subject to exogenous restrictions which reflect the limited control that policymakers have over these parameters in practice.

The proportionality of the deadweight cost to output can be justified as, other things being equal, the absolute value of the deadweight cost is likely to be higher in larger economies. Alternatively we could specify the deadweight cost as a function of the size of the default, but this would add substantial algebraic complexity with little additional economic insight.

The parameter β is taken to represent the strength of creditor bargaining power. The assumption that the settlement creditors can bargain for is an increasing function of GDP, net of the deadweight cost, can be justified if we interpret creditors' bargaining power as arising from the credible threat of some economic damage, such as trade sanctions or the denial of future official financing or capital market access. These costs are likely to be increasing in the debtor's economic size. The impulse to settle with one's creditors may also come from the non-pecuniary 'diplomatic' costs of being seen to have treated creditors unfairly (see Sturzenegger and Zettelmeyer (2007)). Clearly, these costs are hard to quantify, but it seems likely a government's willingness to pay to avoid them will depend on the resources it has available.

Once the decision to default or not has been made, the country consumes the project's returns, net of any deadweight cost and the payment it must make to creditors. The capital markets anticipate this behaviour and demand an interest rate, r, which depends on the amount the country borrows, the probability of a default, and the amount creditors expect to receive in the event of a default. We assume creditors are competitive and risk-neutral, and therefore break even on average.

Perhaps the most fundamental assumption embedded in our model relates to the set of financial contracts that are available. The debtor can only finance investment with plain vanilla debt, with no *de jure* indexation to the productivity shock or any other state variable in the model. The option to default, as we will see, gives rise to some *de facto* conditioning of payment flows on other economic variables. But, particularly when the exercise of this option triggers a deadweight cost, we are left a long way from a first-best world in which risk-neutral creditors would assume all the risk of the project. However, the assumption that all debt is plain vanilla is realistic, given the rarity of GDP-linked sovereign bonds observed in practice. We do not attempt to model the underlying frictions that prevent such contracts from being written.

2.1 Production

The output of the project, y, depends on the amount of resources invested, L, an exogenous stochastic productivity shock, θ, and the deadweight cost, proportional to α, that is incurred in the event of default:

$$y = \begin{cases} \theta L^{\lambda} & \text{if no default} \\ (1-\alpha)\,\theta L^{\lambda} & \text{if default} \end{cases}$$

where $\theta \sim U\,[0, 1]$ and $\lambda \in (0, 1)$. The sovereign decides how much to borrow and invest before the value of productivity θ is realised.[6]

This particular production function has been chosen for analytical convenience: it can be shown that all of our main results go through with any well-behaved production function.[7] A uniform distribution is chosen for the productivity shock in order to make it possible to derive an analytical expression for the interest rate in Subsection 2.3 below.[8]

2.2 Consumption and default

After the productivity shock, θ, is realised, the debtor decides whether or not to default, so as to maximise consumption. The debtor consumes the project's returns net of any deadweight cost and less the payment it must make to creditors. In addition, the debtor consumes an exogenous amount equal to $\overline{c}$, which is interpreted as a non-stochastic endowment of goods that can be consumed, but not used for investment, nor pledged to creditors. It plays a largely technical role in the model.[9] However, economically it seems reasonable to assume that the debtor has access to some positive level of consumption even if it does not borrow anything. Using the superscripts ^{D} and ND to denote 'default' and 'no default' respectively, consumption in each of the states is given by

$$C^{ND}(\theta) = \overline{c} + \theta L^{\lambda} - rL \tag{1}$$

$$C^{D}(\theta) = \overline{c} + (1-\alpha)(1-\beta)\,\theta L^{\lambda} \tag{2}$$

We define $\widehat{\theta}$ as the critical value of the productivity shock at which the debtor is indifferent

[6] A more general framework might endogenise the productivity parameter θ. For example, productivity could be thought of as a function of the debtor's policy effort.

[7] We require only that $f(0) = 0$, $f'(0) = \infty$ and $f''(L) < 0$.

[8] The upper bound of the support of this distribution has no effect on the results: one could always rescale the production function to give the same result. However, setting the lower bound of the support at zero, as opposed to some strictly positive number, may have some effect on the results relating to optimal risk-sharing that we derive below.

[9] It allows us to evaluate the CES utility function at $L = 0$ and $\theta = 0$ when the coefficient of relative risk aversion $\gamma \geq 1$.

between default and full repayment, ie where $C^{ND} = C^{D}$. This yields

$$\widehat{\theta} = \frac{rL^{1-\lambda}}{\alpha + \beta - \alpha\beta} \quad \textbf{(3)}$$

This expression captures the main factors at play in an intuitive fashion. The probability of default (equal to $\widehat{\theta}$, given the assumed distribution for θ) is increasing in the contractual interest rate and the degree of indebtedness, and decreasing in the size of the deadweight cost of default and the degree of creditor bargaining power.

2.3 Equilibrium interest rate

The sovereign's payment to creditors in each of the states is given by

$$P^{ND} = rL \quad \textbf{(4)}$$

$$P^{D} = \beta(1-\alpha)\theta L^{\lambda} \quad \textbf{(5)}$$

Creditors expect to break even *ex ante*. With the risk-free interest rate normalised to unity, and given **(4)** and **(5)**, this implies

$$L = \left(1 - \widehat{\theta}\right) rL + \int_{0}^{\widehat{\theta}} \beta(1-\alpha)\theta L^{\lambda} d\theta \quad \textbf{(6)}$$

Substituting in the expression for $\widehat{\theta}$ gives a quadratic in r, the lower root of which[10] is

$$r(L) = \frac{1 - \sqrt{1 - 2L^{1-\lambda}\Omega}}{L^{1-\lambda}\Omega} \quad \textbf{(7)}$$

$$\text{where} \quad \Omega = \frac{(2\alpha + \beta - \alpha\beta)}{(\alpha + \beta - \alpha\beta)^{2}} \quad \textbf{(8)}$$

This expression only is real-valued if $L \leq \overline{L}$, where

$$\overline{L} = \left(\frac{1}{2\Omega}\right)^{\frac{1}{1-\lambda}} \quad \textbf{(9)}$$

We can interpret $\overline{L}$ as an upper borrowing limit. It is straightforward to show that $r(L) \geq 1$ and $\partial r(L)/\partial L > 0$ for all $L \in \left[0, \overline{L}\right]$, both of which are intuitively appealing conditions. If $L = \overline{L}$ then $r = 2$. Furthermore, if $L = \overline{L}$ and $\alpha = 0$ then $\widehat{\theta} = 1$, otherwise $\widehat{\theta} < 1$.

2.4 Equilibrium debt level

Given the terms on which it can borrow, $r(L)$, the debtor chooses borrowing, L, to maximise expected utility. With a utility function, $u(C)$, this is given by

$$E\left[U(L;\alpha,\beta)\right] = \int_{0}^{\widehat{\theta}} u\left(C^{D}\right) d\theta + \int_{\widehat{\theta}}^{1} u\left(C^{ND}\right) d\theta$$

[10] Both roots are a solution to the break-even condition, but only the lower root is a competitive equilibrium. If the prevailing interest rate were given by the higher root, individual lenders would find it profitable to offer credit at an infinitesimally lower interest rate, and the debtor would accept funds on these terms. This is not the case at the lower root.

Using Leibniz' rule, which allows us to eliminate terms in $\partial\widehat{\theta}/\partial L$, given $C^D = C^{ND}$ at $\widehat{\theta}$, the corresponding first-order condition is

$$\begin{aligned}\frac{\partial E\left[U\left(L;\alpha,\beta\right)\right]}{\partial L} &= \int_0^{\widehat{\theta}} \left(1-\alpha\right)\left(1-\beta\right)\theta\lambda L^{\lambda-1}u'\left(C^D\right)d\theta \\ &+ \int_{\widehat{\theta}}^{1}\left(\theta\lambda L^{\lambda-1}-\left(r+L\frac{\partial r}{\partial L}\right)\right)u'\left(C^{ND}\right)d\theta = 0\end{aligned} \tag{10}$$

We define $\widehat{L}\left(\alpha,\beta\right)$ as the expected utility maximising amount of lending as function of the policy parameters, α and β:

$$\widehat{L}\left(\alpha,\beta\right) = \arg\max E\left[U\left(L;\alpha,\beta\right)\right]$$

Note that when $L = \widehat{L}$ raising L must increase C^D and reduce C^{ND}, if the first-order condition is to be satisfied. A risk-averse debtor will be content to trade off a lower expected consumption if it can shift consumption from the state where marginal utility is low (ie the no-default state) to where it is high (the default state). Consequently we can infer that a risk-averse debtor will necessarily choose to borrow an amount higher than that which maximises expected consumption, if unconstrained by creditors.

3 Unconstrained optimum

We turn to the question of what values for the policy parameters, α and β, maximise the debtor's expected utility. By assumption, creditors are risk-neutral and always expect to break even, so we can ignore their utility when determining social welfare.

In this section we solve for the optimal values of the policy parameters when the policymaker has complete freedom to set both α and β over the set $[0, 1] \times [0, 1]$, with only weak restrictions on the utility function.[11] In the next section we consider the case where exogenous restrictions are placed on the policy parameter space. Our strategy is to establish as much as we can analytically, employing numerical methods to illustrate these results and to shed light on the remaining questions.

[11] In the risk-neutral case, we assume only that $u'(.) > 0$. In the risk-averse case, we add the condition that $u''(.) < 0$.

3.1 Optimal setting for α

We first consider the optimal setting for α, before examining the optimal setting for β in the next subsection. It is relatively straightforward to identify the optimal setting for α when there are no constraints on the policy parameter space. Equations **(1)**, **(2)** and **(3)** show that, taking r and L as given, raising β discourages default in the same way that raising α does, by reducing the debtor's consumption in the event of a default. In particular note that α and β enter symmetrically into both **(2)** and **(3)**. By contrast, equations **(4)** and **(5)** show that, taking r and L as given, α and β have opposing effects on the payment to creditors, specifically in the case of a default. We can therefore conclude that discouraging default by raising β dominates doing so by increasing α, as raising β entails an additional transfer of resources to creditors in the event of a default, which will reduce r in equilibrium and in so doing benefit the debtor. Consequently, it is always optimal for the debtor to trade a lower α for a higher β, and so welfare maximisation necessarily requires $\alpha = 0$, providing β is unrestricted. This is true regardless of how risk-averse is the debtor.

3.2 Optimal setting for β

We first assume the sovereign debtor is risk-neutral, before considering the case where the debtor is risk-averse. Define $P(\theta, L)$ as the *ex-post* payment from the debtor to creditors as a function of realised productivity and the amount of lending. When there are no deadweight costs ($\alpha = 0$) we can express the consumption of the debtor as follows:

$$C|_{\alpha=0} = \overline{c} + \theta L^{\lambda} - P(\theta, L)$$

Note that this is also equal to GNP in the debtor country: it is output, less factor payments abroad. The condition that creditors expect to break even *ex ante* means $E[P(\theta, L)] = L$ and so, once again assuming there are no deadweight costs, expected consumption/GNP is

$$E[C]|_{\alpha=0} = \overline{c} + \frac{L^{\lambda}}{2} - L \qquad \textbf{(11)}$$

The first-order condition tells us that this is maximised when $L = \widetilde{L}$ where

$$\widetilde{L} = \left(\frac{\lambda}{2}\right)^{\frac{1}{1-\lambda}} \qquad \textbf{(12)}$$

Proposition 1 If the sovereign debtor is risk-neutral, welfare is maximised when $\alpha = 0$ and $\beta \geq \lambda$.

Proof. See Appendix.

The intuition behind this result is as follows.[12] The creditors and the sovereign are both risk-neutral, so expected consumption/GNP is a sufficient statistic for social welfare. If $\alpha = 0$ this is maximised when $L = \widetilde{L}$ and this is feasible providing $\overline{L} \geq \widetilde{L}$. Given $\alpha = 0$ a comparison of **(9)** and **(12)** informs us this requires $\beta \geq \lambda$. If $\beta = \lambda$ the sovereign will choose to borrow $L = \widetilde{L} = \overline{L}$, and from **(3)**, given **(7)**, we can also infer that the sovereign will default with unit probability. If $\beta > \lambda$ the sovereign still chooses to borrow $L = \widetilde{L}$, even though $\widetilde{L} < \overline{L}$, as this remains the level of borrowing that maximises expected consumption (and hence expected utility). From **(11)**, given **(12)**, expected consumption is independent of β when L is not constrained: as β is increased beyond λ the default probability and consumption given default both fall, but these effects are completely offset by a lower interest rate when the sovereign chooses to repay. Consequently, the risk-neutral debtor is indifferent between any $\beta \geq \lambda$.

Proposition 2 If the sovereign debtor is risk-averse, welfare is maximised when $\alpha = 0$ and $\beta = \lambda$.

Proof. See Appendix.

When the sovereign is risk-averse it does not just care about expected consumption; it also cares about how consumption is distributed across different states of the world. Given $\alpha = 0$, for $\beta \leq \lambda$ it can be shown that the sovereign always exhausts its credit limit and defaults with unit probability, so the debtor's consumption is $C^D = \theta (1 - \beta) L^\lambda$ for all θ. It can be shown that the effect that raising β has in reducing consumption (in the certain event of default) for a given L is more than offset by the increase in the credit limit and the optimal choice of L. This is true for every realisation of θ and so higher β will be preferred by the debtor.

For $\beta > \lambda$, the sovereign is able to borrow more than $\widetilde{L}$ if it chooses. By definition, borrowing more than $\widetilde{L}$ reduces expected consumption. So, loosely speaking, the debtor can yield higher utility only if this delivers a better 'risk profile' (ie more consumption in bad states of the world and less in good states). However, it can be shown that the risk profile from any combination of higher β and higher L is always inferior to $\beta = \lambda$ with $L = \widetilde{L}$, so increasing β above λ must

[12] This result is related to the finding in Tanaka (2005) that IMF intervention to mitigate deadweight costs can be welfare-improving as long as debtors are required to reach a 'fair' settlement with their creditors.

reduce expected utility. Put another way, higher creditor bargaining power produces extra borrowing capacity but, given the larger share that goes to creditors in the event of default, consumption turns out to be lower in bad states of the world.

4 Constrained optima

It is implausible to regard domestic and international policymakers as having complete freedom to set the size of the deadweight cost and creditor bargaining power. Some loss of output in the event of default may be inevitable and there are likely to be limits to the amount of bargaining power that can be credibly promised to creditors. It is therefore salient to analyse the case where the policymaker's choice of α or β is constrained. We begin with a general result that holds at any point in the policy parameter space $\{\alpha, \beta\}$.

Proposition 3

$$\text{If} \quad \frac{dE\left[U\left(C(L;\alpha,\beta)\right)\right]}{d\alpha} > 0 \quad \text{then} \quad \frac{dE\left[U\left(C(L;\alpha,\beta)\right)\right]}{d\beta} > 0.$$

Proof. See Appendix.

In words, Proposition 3 means that if marginally increasing the deadweight cost raises welfare, marginally increasing creditor bargaining power will do so as well. This result can be explained as follows. Suppose to begin with that the debtor is not credit constrained. Raising the deadweight cost or increasing creditor bargaining power both reduce debtor welfare in the event of a default. Either of these changes can therefore only raise overall welfare if they have an offsetting positive effect on consumption when the borrower repays.[13] The channel through which this can occur is by lowering the interest rate. Raising creditor bargaining power always reduces the interest rate; increasing the deadweight cost only does so when these are sufficiently small in the first place, and creditor bargaining power is sufficiently weak. Whenever a higher deadweight cost reduces the interest rate sufficiently to increase expected utility, it is necessarily the case that giving bargaining power to creditors does so too. In the case in which the debtor is credit constrained, the interest rate is unaffected by the deadweight cost or bargaining power, but

[13]The Envelope Theorem allows us to ignore the effect of changes in the policy parameters on the optimal amount of lending. Leibniz' rule allows us to ignore the direct effect on the default probability, as in the region of the default barrier the debtor is indifferent between defaulting and not defaulting.

the credit limit is. The direct effect of a higher credit limit is always to raise expected utility, otherwise the constraint would not be binding. Raising the deadweight cost or increasing creditor bargaining power both lift the credit limit, but raising increasing creditor bargaining power does so more efficiently. Consequently, if the effect of a higher deadweight cost in raising the credit limit offsets the adverse effect it has on consumption in the event of a default, this will be true *a fortiori* for higher creditor bargaining power.

Corollary 4

$$\text{If} \quad \frac{dE\left[U\left(C(L;\alpha,\beta)\right)\right]}{d\beta} \leq 0 \quad \text{then} \quad \frac{dE\left[U\left(C(L;\alpha,\beta)\right)\right]}{d\alpha} < 0.$$

Taken together Proposition 3 and its Corollary have a number of implications. First, starting from *any* point in the policy parameter space either it is welfare-improving to reduce the deadweight cost in isolation, or there is some combination of lower deadweight cost and higher creditor bargaining power that is welfare-improving. Second, for any value of the deadweight cost, if creditor bargaining power is set optimally, so that the derivative of expected utility with respect to β is zero, then it is always welfare-improving to reduce the deadweight cost. Third, starting from any point in the policy parameter space at which derivative of expected utility with respect to α is positive, there is always a step increase in β which is itself welfare-improving, but which also in turn causes the sign of the derivative of expected utility with respect to α to become negative.

4.1 Numerical results

The policy problem does not yield an analytic solution in the case where α and/or β are constrained. To say any more about the nature of the constrained optima, we must assume a functional form for utility and evaluate the solution numerically. We adopt a standard CRRA utility function with parameter γ

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

Expected utility is therefore given by

$$E\left[U(L)\right] = \int_{\widehat{\theta}}^{1} \frac{\left[\theta L^{\lambda} - rL + \overline{c}\right]^{1-\gamma}}{1-\gamma} d\theta + \int_{0}^{\widehat{\theta}} \frac{\left[(1-\alpha)(1-\beta)\theta L^{\lambda} + \overline{c}\right]^{1-\gamma}}{1-\gamma} d\theta$$

for $\gamma \neq 1$. Our solution method is to analytically integrate the utility function, numerically evaluate it over the range $\left[0, \overline{L}\right]$, and search for the optimal value, L^{*}. This is repeated for a grid of points within the set $\{\alpha, \beta\} \in (0, 1) \times (0, 1)$, and a range of values for $\{\gamma, \lambda, \overline{c}\}$. Higher values of maximised utility $U(L^*(\alpha, \beta); \alpha, \beta)$ indicate superior policy settings.

Chart 1 shows the maximised value of utility as a function of α and β.[14] This confirms our earlier result that the global optimum is $\alpha = 0$, $\beta = \lambda$. Moreover, it confirms that there is only a particular set of circumstances for which the impact of a marginal decrease in α on expected utility is negative: this is only the case when both α and β are sufficiently low. Taken together with the analytical results of the previous subsection this suggests there are only restrictive circumstances under which it becomes suboptimal to reduce deadweight costs. This is only the case when both α and β are low *and* the policymaker is unable to increase β sufficiently in tandem with a decrease in α.

Chart 1 also shows that the optimal value of β is typically decreasing in α, and may even be zero for high enough α. Chart 2 plots the optimal choice of β as a function of α, as well as the values of utility, the credit limit and the default probability associated with it. If deadweight costs cannot be reduced to zero, it is optimal to give creditors less bargaining power than in the unconstrained case; *in extremis*, creditors might optimally have no bargaining power at all. This is because deadweight costs are a substitute for creditor bargaining power, albeit an inefficient one, in that both measures dissuade default. Note that, as deadweight costs increase, it becomes more expensive for the borrower to share risk with its creditors. Moreover, positive deadweight costs reduce the expected productivity of investment. For these reasons, the default probability falls.

5 Credibility and *ex-post* optimality

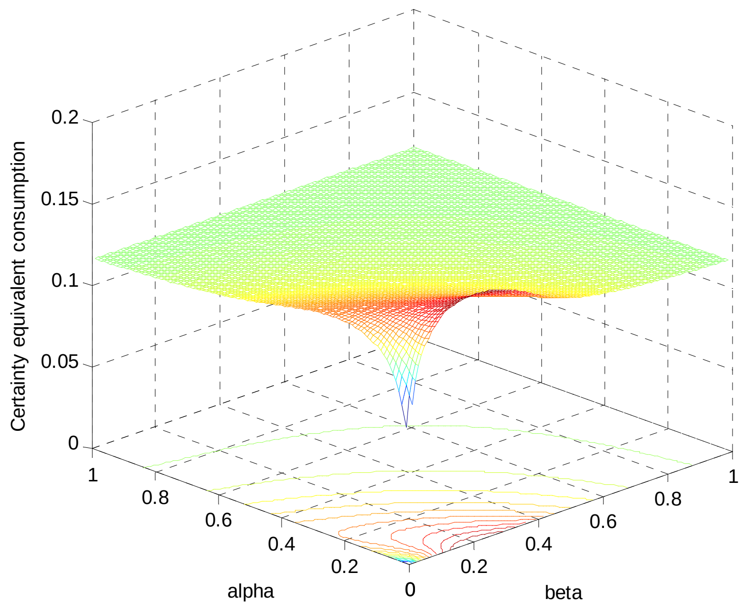
Up to this point we have assumed that the values of α and β, while potentially constrained, can nevertheless be credibly fixed in advance by the policymaker. Yet once debt has been contracted, the interest rate determined and the productivity realised, both the debtor and creditors will face incentives to deviate from this.

As before, consumption is given by

$$C = \max\left[C^{ND}, C^{D}\right] = \max\left[\theta L^{\lambda} - rL, (1-\alpha)(1-\beta)\theta L^{\lambda}\right]$$

[14]Utility is measured in units of certainty equivalent consumption. We set $\gamma = 1.5$, $\lambda = 0.2$ and $\bar{c} = 0.01$.

Chart 1: Maximised utility function



Charts 3 and 4 show consumption as a function of α and β, holding the interest rate and lending constant, for two different realisations of the productivity parameter θ.[15] *Ex post*, the debtor will seek to maximise this quantity. The charts illustrate the obvious point that debtors will seek to minimise both the deadweight cost and creditor bargaining power *ex post*. Note, however, that the policy setting that yields the highest attainable utility *ex post* (at $\alpha = 0$, $\beta = 0$) is also the worst policy configuration *ex ante*. This can be seen by comparing Charts 3 and 4 with Chart 1 at this particular combination of parameter values.

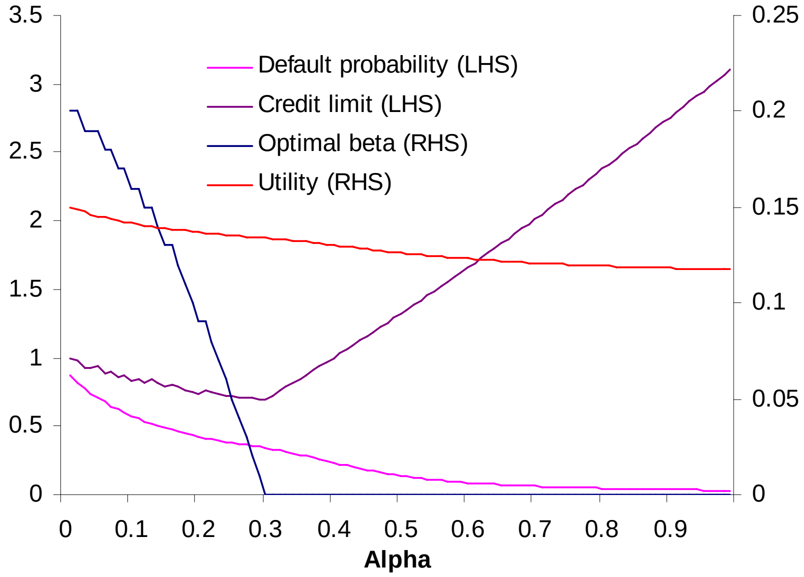
Ex post, the debtor is willing to trade the policy measures off against each other at a suboptimal rate. Once debt has been contracted, the relative inefficiency of deadweight costs – the fact that resources are wasted rather than transferred to creditors – is no longer internalised by the debtor.[16] The contour diagrams in Charts 5 and 6 illustrate this point.

Therefore, faced with an *ex-post* trade-off between deadweight costs and creditor bargaining power, debtors will choose 'too much' of the former, relative to the *ex-ante* optimum. For

[15] In these charts, the interest rate, lending and the policy parameters are set at the *ex-ante* optimal levels, ie $\alpha = 0$, $\beta = \lambda$, $\widehat{L} = \widetilde{L}$, $r = r\left(\widetilde{L}\right)$.

[16] This problem may be overcome to some extent if there is a repeated interaction between the debtor and creditors, as reputation concerns may constrain the debtor's actions. However, this effect is likely to be limited when the policymaker believes they are unlikely to be in office for much longer. Such fears are often justified in crisis situations (Frankel (2005)).

Chart 2: Optimal creditor bargaining power as a function of deadweight costs



example, faced with a choice between a pre-default restructuring (which one might characterise as relatively low α, high β) and a post-default restructuring (high α, low β), the debtor will be biased towards the latter, relative to the *ex-ante* optimum. This suggests that, to the extent that commitment problems prevent time-consistent choices of the policy parameters, we will observe an excessive incidence of deadweight costs.

Furthermore, considering the effect of marginal changes in the policy parameters, Charts 3 and 4 illustrate that these tensions are more likely to surface when realised productivity is low. When productivity is high, the debtor is *locally* indifferent to changes in the policy parameters over more of their range: intuitively, high output makes default less tempting, so small changes in the terms of default are less relevant.

6 Conclusions and policy implications

The model outlined above illustrates how sovereign borrowing can be supported in equilibrium by a combination of deadweight costs of default and/or the ability of creditors to bargain for a share of the debtor's post-default resources. The model allows us to explore the welfare implications of different combinations of deadweight costs and creditor bargaining power, which are the key 'policy parameters' of our analysis. We find that welfare is globally maximised when deadweight costs are zero and bargaining power is efficiently shared between the debtor and

Chart 3: *Ex-post* consumption, $\theta = 0.5$

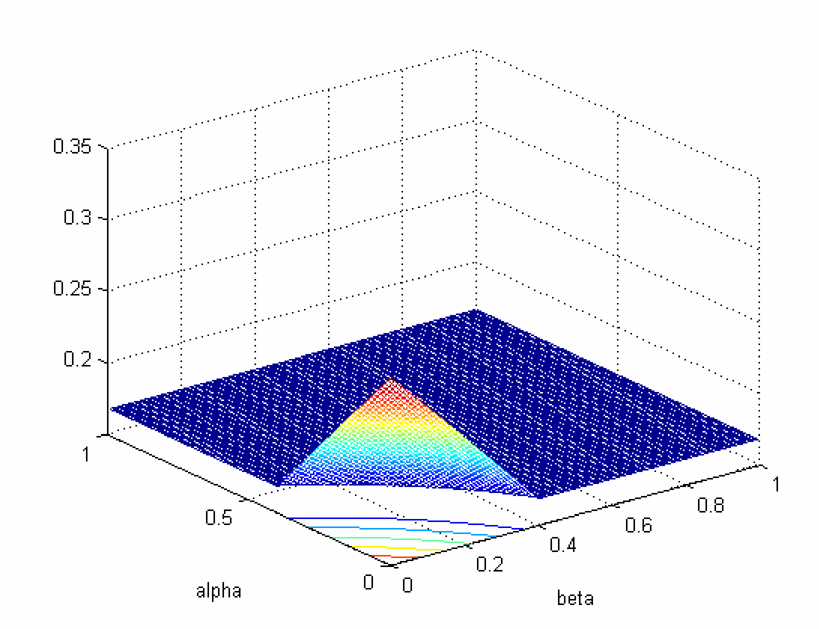


Chart 4: *Ex-post* consumption, $\theta = 1$

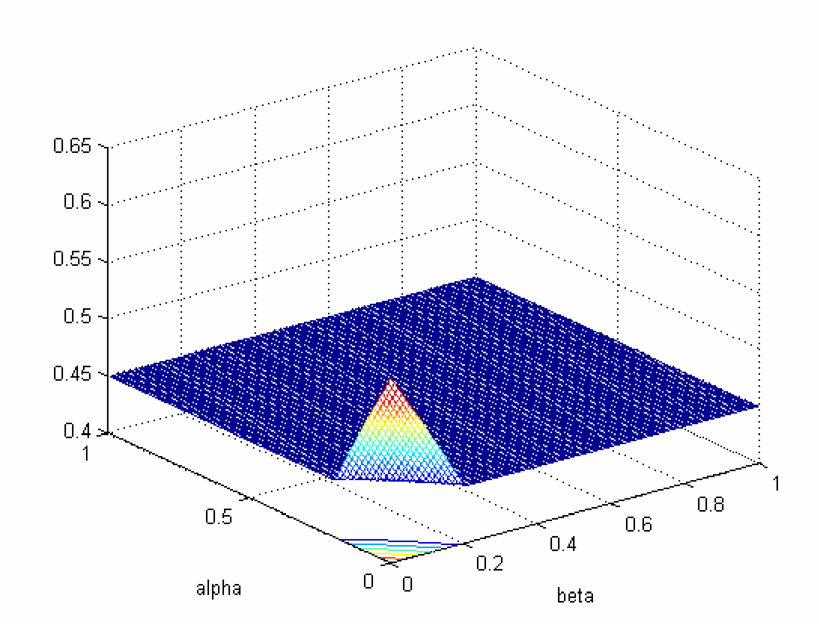


Chart 5: *Ex-ante* isoutility contours

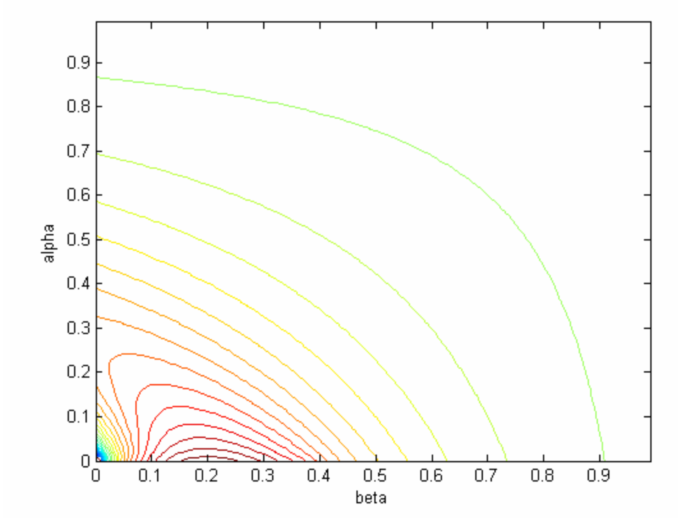
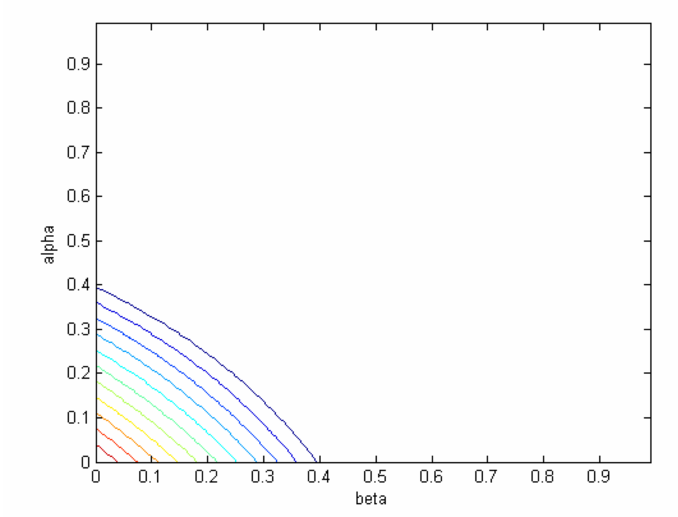


Chart 6: *Ex-post* isoutility contours



creditors.

How should domestic and international policymakers respond to these conclusions? No policymaker has complete control over the policy parameters. But sovereign debtors and the IMF, in particular, exert some influence over them. The model illustrates how policymakers should exercise this influence when they are partly constrained. Starting from *any* point in the policy parameter space either it is welfare-improving to reduce the deadweight cost in isolation – as is the case for most of the parameter space – or there is some combination of lower deadweight costs and higher creditor bargaining power that is welfare-improving. Moreover, if the welfare impact of a marginal reduction in deadweight costs is negative, there is always a step increase in the allocation of bargaining power to creditors that in turn means this welfare impact becomes positive.

The model also illustrates how difficult it might be to implement the optimal framework in practice, as commitments by the debtor are likely to be time-inconsistent: in the event of a default the debtor has an incentive to drive the policy parameters in the direction which is welfare-minimising from an *ex-ante* perspective. This suggests that some form of commitment technology is required so that the debtor can tie its hands. Alternatively, or in addition, the role of those outside agencies which also exert control over the key policy parameters, such as the IMF, becomes more important, to the extent that they are not subject to the same time-inconsistency problem.

The structure of the IMF's lending into arrears policy is potentially a key determinant of the policy parameters. Post-default balance of payments support can directly reduce deadweight costs of default, although how significant this impact is will depend on both the timing of the support and the particular circumstances faced by the debtor. The analysis suggests that reducing deadweight costs is either beneficial in its own right, or if not, it can be rendered beneficial if there is an accompanying shift in bargaining power from the debtor to creditors. In the future, as the LIA policy continues to evolve, the IMF should pay explicit attention to the impact the policy has on the balance of bargaining power in the event of a restructuring.

Appendix

Proof of Proposition 1

The debtor is risk-neutral, so maximising expected consumption will maximise expected utility. Expected consumption is given by

$$E[C] = \overline{c} + \int_0^{\widehat{\theta}} (1-\alpha)(1-\beta)\,\theta L^{\lambda} d\theta + \int_{\widehat{\theta}}^{1} (\theta L^{\lambda} - rL) d\theta$$

Given the equilibrium condition for the interest rate, expressed in equation **(6)**, this simplifies to

$$E[C] = \overline{c} + \left(1 - \alpha\widehat{\theta}^2\right) L^{\lambda}/2 - L$$

Expected consumption is maximised when $\alpha = 0$ and $L = \widetilde{L}$. From **(8)** and **(9)** it follows that when $\alpha = 0$ then $\widetilde{L} \leq \overline{L}$ if and only if $\beta \geq \lambda$. Consequently $\widetilde{L}$ is feasible and will be chosen providing $\beta \geq \lambda$. This completes the proof.

Proof of Proposition 2

The first step is to show that $\beta = \lambda$ dominates any configuration with $\beta < \lambda$. We know from Subsection 2.4 that if a risk-averse debtor is not credit constrained it will choose $L > \tilde{L}$. However, when $\beta < \lambda$, a comparison of **(12)** and **(9)** shows that $\tilde{L} > \overline{L}$, and so the debtor must be credit constrained. If the debtor is credit constrained then the default probability is unity and for all θ consumption is then given by

$$C^D = \theta(1-\beta)\left(\frac{\beta}{2}\right)^{\frac{\lambda}{1-\lambda}} + \overline{c} \qquad \textbf{(A-1)}$$

which is necessarily increasing in β when $\beta < \lambda$, for all realisations of θ. It follows that higher β is preferred by any agent with a non-decreasing utility function. This establishes that the optimal $\beta \geq \lambda$.

The next step is to show that $\beta = \lambda$ dominates any configuration with $\beta > \lambda$. Suppose, first, that the debtor is credit constrained. Then, once again, the default probability is unity and for all θ consumption is then given by **(A-1)**. This is necessarily decreasing in β when $\beta > \lambda$, for all realisations of θ. It follows that lower $\beta = \lambda$ is preferred by any agent with a non-decreasing utility function. This establishes that the optimal $\beta \ngtr \lambda$ if the debtor is credit constrained.

Suppose, instead, that the debtor is not credit constrained, and so $\widehat{L}(\beta) \leq \overline{L}$. We can evaluate the impact on expected utility of an increase in β using the Envelope Theorem, which allows us to ignore the effect of the change on the optimal amount of lending, and using Leibniz' rule, which allows us to eliminate terms in $\partial\widehat{\theta}/\partial\beta$, given $C^D = C^{ND}$ at $\widehat{\theta}$. This means

$$\frac{dE\left[U\left(C(\widehat{L}(\beta);\beta)\right)\right]}{d\beta} = -L^{\lambda}\int_0^{\widehat{\theta}} \theta u'\left(C^D(\theta)\right)d\theta - L\frac{\partial r}{\partial\beta}\int_{\widehat{\theta}}^1 u'\left(C^{ND}(\theta)\right)d\theta \qquad \textbf{(A-2)}$$

We want to show that **(A-2)** is negative when $\beta > \lambda$. Given the equilibrium interest rate, equation **(7)**, we can infer from the first-order condition **(10)** for L that

$$\begin{aligned} -L\frac{\partial r}{\partial\beta}\int_{\widehat{\theta}}^1 u'\left(C^{ND}(\theta)\right)d\theta &= \left(\frac{1-\beta}{\beta}\right)\left(\frac{\lambda}{1-\lambda}\right)L^{\lambda}\int_0^{\widehat{\theta}} \theta u'\left(C^D(\theta)\right)d\theta \qquad \textbf{(A-3)} \\ &\quad + \left(\frac{\lambda}{1-\lambda}\right)L^{\lambda}\int_{\widehat{\theta}}^1 \left(\theta\frac{\lambda}{\beta}-\hat{\theta}\right)u'\left(C^{ND}(\theta)\right)d\theta \end{aligned}$$

If $u(.)$ is strictly concave then the marginal utility from consumption must always be higher in default states than in non-default states, given that $C^D > C^{ND}$ for any of the values θ may take in each of the states. Consequently, given **(A-3)**, a sufficient condition for **(A-2)** to be negative is that

$$\left[1-\left(\frac{1-\beta}{\beta}\right)\left(\frac{\lambda}{1-\lambda}\right)\right]\int_0^{\widehat{\theta}} \theta d\theta > \left(\frac{\lambda}{1-\lambda}\right)\int_{\widehat{\theta}}^1 \left(\theta\frac{\lambda}{\beta}-\hat{\theta}\right)d\theta$$

After solving these integrals this sufficient condition simplifies to $L > \tilde{L}$, which we know will be chosen by a debtor that is not credit constrained. Consequently, **(A-2)** must be negative when $\beta > \lambda$. This establishes that the optimal $\beta \ngtr \lambda$ if the debtor is not credit constrained.

Taking these results together we have now established that the optimal $\beta \geq \lambda$ and that the optimal $\beta \ngtr \lambda$. Consequently, it must be the case that the optimal $\beta = \lambda$. This completes the proof.

Proof of Proposition 3

We first consider the case in which the debtor is not credit constrained. We can evaluate the effect of a marginal increase in α or β on expected utility using the Envelope Theorem, which allows us to ignore the effect of the changes on the optimal amount of lending, and Leibniz' rule, which allows us to eliminate terms in $\partial\widehat{\theta}/\partial\alpha$ and $\partial\widehat{\theta}/\partial\beta$, given $C^D = C^{ND}$ at $\widehat{\theta}$. This means:

$$\frac{dE\left[U\left(C(\widehat{L};\alpha,\beta)\right)\right]}{d\alpha} = -(1-\beta)L^{\lambda}\int_0^{\widehat{\theta}} \theta u'\left(C^D(\theta)\right)d\theta - L\frac{\partial r}{\partial\alpha}\int_{\widehat{\theta}}^1 u'\left(C^{ND}(\theta)\right)d\theta \quad \textbf{(A-4)}$$

$$\frac{dE\left[U\left(C(\widehat{L};\alpha,\beta)\right)\right]}{d\beta} = -(1-\alpha)L^{\lambda}\int_0^{\widehat{\theta}}\theta u'\left(C^D(\theta)\right)d\theta - L\frac{\partial r}{\partial\beta}\int_{\widehat{\theta}}^1 u'\left(C^{ND}(\theta)\right)d\theta \quad \textbf{(A-5)}$$

Comparing **(A-4)** and **(A-5)**, this part of the proof requires only that the following condition is satisfied

$$(1-\alpha)\frac{\partial r}{\partial\alpha} > (1-\beta)\frac{\partial r}{\partial\beta}$$

Given equation **(7)** it can be shown that this condition holds for all values α and β may take. This completes the proof for the case in which the credit constraint is not binding.

We now consider the case where the debtor is credit constrained. By substitution of **(8)** and **(9)** into **(7)** we can see that the interest rate is now fixed at $r = 2$ and is therefore independent of α and β. However, higher α or higher β will raise the credit limit and (at the margin) lead the debtor to borrow more. Consequently, **(A-4)** and **(A-5)** must now be modified as follows

$$\frac{dE\left[U\left(C(\overline{L};\alpha,\beta)\right)\right]}{d\alpha} = \frac{\partial E\left[U\left(C(\overline{L};\alpha,\beta)\right)\right]}{\partial\overline{L}}\frac{\partial\overline{L}}{\partial\Omega}\frac{\partial\Omega}{\partial\alpha} - (1-\beta)\overline{L}^{\lambda}\int_0^{\widehat{\theta}}\theta u'\left(C^D(\theta)\right)d\theta$$

$$\frac{dE\left[U\left(C(\overline{L};\alpha,\beta)\right)\right]}{d\beta} = \frac{\partial E\left[U\left(C(\overline{L};\alpha,\beta)\right)\right]}{\partial\overline{L}}\frac{\partial\overline{L}}{\partial\Omega}\frac{\partial\Omega}{\partial\beta} - (1-\alpha)\overline{L}^{\lambda}\int_0^{\widehat{\theta}}\theta u'\left(C^D(\theta)\right)d\theta$$

Note that

$$\frac{\partial E\left[U\left(C(\overline{L};\alpha,\beta)\right)\right]}{\partial\overline{L}}\frac{\partial\overline{L}}{\partial\Omega} \leq 0$$

Therefore, it is sufficient to show that

$$(1-\alpha)\frac{\partial\Omega}{\partial\alpha} > (1-\beta)\frac{\partial\Omega}{\partial\beta}$$

From equation **(8)** it can be shown that this condition is satisfied for all values α and β may take. This completes the proof.

Note that our specific assumption $y = \theta L^{\lambda}$ is not necessary for this proof: the above result holds for any well-behaved production function of the form $y = F(L)$.

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