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Erlend Nier, Jing Yang, Tanju Yorulmazer and
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Abstract

Systemic risk is a key concern for central banks charged with safeguarding overall financial stability. In this paper we investigate how systemic risk is affected by the structure of the financial system. We construct banking systems that are composed of a number of banks that are connected by interbank linkages. We then vary the key parameters that define the structure of the financial system — including its level of capitalisation, the degree to which banks are connected, the size of interbank exposures and the degree of concentration of the system — and analyse the influence of these parameters on the likelihood of contagious (knock-on) defaults. First, we find that the better capitalised banks are, the more resilient is the banking system against contagious defaults and this effect is non-linear. Second, the effect of the degree of connectivity is non-monotonic, that is, initially a small increase in connectivity increases the contagion effect; but after a certain threshold value, connectivity improves the ability of a banking system to absorb shocks. Third, the size of interbank liabilities tends to increase the risk of knock-on default, even if banks hold capital against such exposures. Fourth, more concentrated banking systems are shown to be prone to larger systemic risk, all else equal. In an extension to the main analysis we study how liquidity effects interact with banking structure to produce a greater chance of systemic breakdown. We finally consider how the risk of contagion might depend on the degree of asymmetry (tiering) inherent in the structure of the banking system. A number of our results have important implications for public policy, which this paper also draws out.

Key words: Networks, financial stability, contagion, liquidity risk.

JEL classification: C63, C90, G28.

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Summary

Systemic risk is a key concern for central banks charged with safeguarding overall financial stability. Systemic risk arises when there is the potential for multiple banks to fail and to impose costs on the financial system and ultimately on the economy as a whole. The costs of systemic failure have been estimated to be large.

A number of determinants of systemic failure, including the role of government safety nets have been studied extensively in the literature. However, relatively little is known about how the *structure* of a banking system, including the degree to which banks are connected to each other through bilateral exposures may affect its susceptibility to systemic breakdown. How does the size and distribution of exposures, between banks determine the resilience of the system as a whole? How does the potential for interbank exposures to transmit shocks from one bank to another interrelate with the aggregate amount of capital available to cushion shocks? In the presence of interbank exposures, are more concentrated banking systems with a small number of large banks, more or less susceptible to systemic breakdowns than systems that comprise a large number of smaller banks? And, are ‘tiered’ systems, where a small number of first-tier banks coexist with a fringe of smaller banks, more or less susceptible to systemic breakdown than systems that are more uniform?

This paper studies these questions drawing on recent advances in the study of networks. A banking system is represented by a set of nodes (banks) that are connected by directed links (interbank exposures) with a certain predefined probability. The ‘weight’ of these links (the size of interbank exposures) determines the capacity for losses to flow from one bank to another. Capital and deposits are introduced as the first and ultimate recipients of any losses incurred. In this set-up we simulate the extent of contagious (knock-on) defaults arising from losses being transmitted through interbank exposures for a wide variety of banking systems that differ in their underlying structural characteristics.

The analysis suggests that increasing the size of interbank liabilities tends to increase the risk of knock-on default relatively sharply once the size of liabilities moves beyond a certain threshold. Moreover, this is the case even if banks hold capital against such exposures in the same way as they hold capital against other credit exposures. This means that capital requirements alone may not be adequate to protect the system against knock-on defaults. Further, the simulations show that the effect on the likelihood of systemic breakdown of varying the probability of connection and hence the resulting density of interbank connections (connectivity) is ambiguous and depends on the initial level. For a low initial degree of connectivity, an increase in connectivity unambiguously increases the risk of knock-on defaults. When connectivity is already high, however, a further increase in connectivity tends to help dissipate losses across the system and can thus make the system more resilient to shocks.

The potential for knock-on defaults depends on the aggregate level of capital in the system. The lower the aggregate amount of capital in the system the greater is the potential for knock-on defaults, for any given level of connectivity. Moreover, there is an interaction between the level of capital and the systemic risk created by varying the degree of connectivity. In particular, low aggregate capital increases the potential for interbank connections to transmit shocks through

the system. This means that aggregate shocks that might be able to draw down the capital cushion of the system as a whole pose a risk in turning the system from one where connections dissipate shocks to one where connections tend to work as shock transmitters.

The results suggest that when there are few banks, the potential for one failure to entail knock-on defaults is greater, all else equal. This means that more concentrated banking systems are prone to larger systemic risk, all else equal. Further, the risk of contagion (knock-on defaults) is shown to depend on the degree of asymmetry (tiering) inherent in the structure of the banking system. However, the results suggest that tiering does not necessarily lead to greater scope for knock-on defaults. The reason is that the effect of connectivity on the likelihood of systemic breakdown is shown to be ambiguous. When large banks are connected to many other banks this increases the scope for shock transmission. But it could equally lead to better absorption of the initial shock.

1 Introduction

Systemic risk is a key concern for central banks charged with safeguarding overall financial stability. Systemic risk arises when there is the potential for multiple banks to fail and to impose costs on the financial system and ultimately on the economy as a whole. The costs of systemic failure have been estimated to be large; see Hoggarth, Reis and Saporta (2002) for an estimate of the cost of 33 systemic banking crises over the past 25 years. The failure or weakness of multiple banks at the same time arises through four main mechanisms: (i) direct bilateral exposures between banks; (ii) correlated exposures of banks to a common source of risk; (iii) feedback effects from endogenous fire-sale of assets by distressed institutions; (iv) informational contagion.⁽¹⁾

A number of determinants of systemic failure, including the role of regulation and government safety nets and their effect on bank risk-taking incentives have been studied extensively in the literature, eg Demirguc-Kunt and Detragiache (2002) and Barth, Caprio and Levine (2006). However, relatively little is known about how the *structure* of a banking network may affect its susceptibility to systemic breakdown. First, how does the risk of systemic breakdown relate to the number of institutions that comprise the banking system? In other words, are more concentrated systems (such as the Netherlands and Sweden) more prone to systemic risk than less concentrated systems (such as Italy and Germany)? Second, are systems where a small number of large money-centre banks coexist with a fringe of smaller banks more or less susceptible to systemic breakdown? In other words, does asymmetry ('tiering') matter? Third, how do interbank exposures and banking system capital (and its regulation) interact? In other words, is capital regulation adequate in addressing the risk of systemic breakdown that arises in systems where banks are linked to each other?

In studying these questions we focus on the role of direct interbank connections as a source of systemic risk and study the potential for knock-on defaults that are created by such exposures. In so doing, we think of common exposures to aggregate risk as affecting all banks in the same way and thus as *pre-determining* the aggregate amount of capital in the system that is available to cushion the effect of any further idiosyncratic bank default. In addition, we study in some detail the feedback effects arising through fire-sales of banking assets on banks' valuations. This mechanism represents a special case of aggregate risk that may arise *after* defaults occur.

Throughout we assume perfect information and thus abstract from informational contagion, which has been studied extensively in the literature, eg Chen (1999). In practice, informational contagion may amplify the mechanisms studied in this paper, as it is based on imperfect information on either direct linkages or common exposures across banks. This may cause funding problems for banks even when they are fundamentally sound or in any case before they have reached the point of insolvency. Finally, throughout, we assume that the initial shock affects the *solvency* (asset side) of an institution, rather than its liquidity (liability side). While pure liquidity shocks, as exemplified by bank runs that are associated with multiple equilibria have been the focus of an influential literature, eg Diamond and Dybvig (1983), the historical evidence has shown that severe liquidity problems are not usually random events but that there is a close connect between banking panics and the state of the economy, eg Calomiris and Gorton (1991). In other words, severe liquidity problems are typically triggered by adverse news about the *solvency* of banking institutions, as modelled by Allen and Gale (1998).

⁽¹⁾ De Bandt and Hartmann (2000) provide a comprehensive survey on systemic risk.

The approach followed to address our questions is to model banking systems that consist of a number of banks (known as *nodes*) and that are interlinked with each other through interbank exposures with a certain predefined probability. Building on Eboli (2004) we define the weight of these links (the size of interbank exposures) and introduce capital and deposits as the first and ultimate recipients of any losses incurred. In this set-up we simulate the extent of contagious defaults arising from losses being transmitted through interbank exposures for a wide variety of banking systems that differ in their underlying structural characteristics.

Our approach complements a growing empirical literature which analyses contagion risk through interbank connections for particular countries. By construction, these studies are unable to draw conclusions on the *generic* relationship between contagion risk and the characteristics of the financial system. By contrast, our approach conceptualises the main characteristics of a financial system using network theory and allows us to vary continuously the key parameters of the network. In particular, we investigate how the resilience of the interbank network to shocks relates to the following key parameters of the system: the capacity of banks to absorb shocks (capital); the size of interbank exposures; the degree of connectivity; and the degree of concentration of the banking sector. In doing so, for the most part we assume that the probability of an interbank exposure linking any two banks is identical and independent across (ordered) pairs of banks. This assumption is made mainly for tractability and is relaxed in an extension, where we study more realistic ‘tiered’ banking systems.⁽²⁾

The remainder of this paper is organised as follows. Section 2 reviews in more detail the related theoretical and empirical literature on interbank contagion. Section 3 introduces our model of the banking system and describes the way contagion effects are simulated. Section 4 presents comparative statics exercises to study how the resilience of the network is affected by its structural characteristics (capitalisation, size of interbank exposures, connectivity and concentration). Section 5 introduces fire-sale of assets as an additional mechanism that operates alongside direct linkages to create knock-on defaults. Section 6 extends the analysis to asymmetric or ‘tiered’ structures. Section 7 summarises the findings and concludes.

2 Related literature

2.1 Theoretical economics literature

The economics literature has made some important advances in modelling interbank relationships. In particular, as has been pointed out by Bhattacharya and Gale (1987), the interbank market may act as a device for co-insurance against uncertain liquidity shocks, and can thus be a mechanism to absorb shocks. Interbank exposures may also provide incentives for peer-monitoring, see Flannery (1996) and Rochet and Tirole (1996), and may thus improve market discipline. However, such exposures can also serve as a channel through which problems in one bank can spread to another. Allen and Gale (2000) use simple examples to show that the extent of fragility of the system depends on the structure of these interbank linkages. If each bank is connected to all other banks, ie the structure is ‘complete’, then the shock to one bank can be absorbed within the system since each bank bears a small share of the shock. However, if banks have few counterparties, the spillover effect can be large and can magnify financial difficulties. While the study by Allen and Gale (2000) provides valuable insights into stability of

⁽²⁾ This extension relaxes the assumption of *identical* probabilities, by allowing the probability of connection between a first-tier and a second-tier bank to be greater than that between any two second-tier banks, but maintains the assumption of *independence*. For many social networks a high degree of clustering of links means that the independence assumption is difficult to justify. However, empirical research on the structure of real-world banking systems implies that clustering coefficients are relatively low for banking networks. Independence may therefore be a more defensible assumption. See Section 6 for further discussion.

interbank markets, their model has only four banks and both the network structures employed and the financial structure of banks are too simplistic to be sure that the intuitions generated generalise to real-world financial systems.

2.2 Empirical studies of interbank contagion

The importance of the structure of interbank linkages has recently been recognised by central banks charged with safeguarding overall financial stability and there has been a surge of empirical research on the importance of such linkages as a channel of contagion. These studies use data on the structure of banking markets, including data on interbank exposures – as far as available – for a number of countries.⁽³⁾ Recent examples are Sheldon and Maurer (1998) for Switzerland, Furfine (1999) for the United States, Upper and Worms (2004) for Germany, Wells (2002) for the United Kingdom and Boss, Elsinger, Summer and Thurner (2004) for Austria. This line of research is valuable in providing insights on the empirical importance of interbank contagion for real-world networks. However, the results are invariably driven by the particulars of the banking system under study and cannot therefore provide easily generalisable insights into the drivers of systemic risk. In particular, these studies are silent as regards the influence of key parameters, such as net worth, the size of interbank exposures, connectivity and concentration for the resilience of the banking system. Moreover, the use of maximum entropy techniques in cases where data on interbank exposures are patchy means that in the limit, some of these papers assume that all banks are connected to each other bank with probability one. In other words, the empirical literature assumes maximum diversification or a ‘complete structure’ in the sense of Allen and Gale (2000), without offering a quantifiable appreciation of how this might drive the results.⁽⁴⁾

2.3 Networks literature

This paper, as well as much of the available empirical research is based on advances in network theory that at first were made outside of the mainstream economics literature and that have only recently started to be employed by economists. In a seminal paper, Erdős and Rényi studied one of the earliest theoretical models of a network: the random graph (Erdős and Rényi (1959)). A number of versions of this model have since been developed that differ in the probability law governing the distribution of links between nodes. In the most commonly studied and simplest version, each possible link between any two nodes is present with an independent and identical probability p , which has become known as the Erdős-Rényi probability.

Network theory as a branch of mathematics has more recently been applied to a number of phenomena in economics and the social sciences, as well as in biology. One set of problems that have been studied widely is the transmission of shocks through networks. An example is how contagious diseases can be transmitted through a network of nodes from an initially small set of infected nodes. The literature focuses on how transmission might depend on the probability distribution that governs whether any two nodes have contact with each other, see Newman (2003) for a survey. An example from the social sciences is herding behaviour and informational cascades, where the state of one node affects the likelihood of that same state being taken by

⁽³⁾ These studies are based on an influential paper by Eisenberg and Noe (2001) who investigate defaults of firms belonging to a single clearing mechanism, where the obligations of all firms within the system are determined simultaneously in a fashion consistent with the priority of debt claims and the limited liability of equity. Using a fixed-point theorem, Eisenberg and Noe show that a *clearing payment vector* that clears the obligations of the members of the clearing system always exists and also, under mild regularity conditions, is unique.

⁽⁴⁾ Mistrulli (2005) shows that for the Italian banking system the use of maximum entropy techniques underestimates contagion risk relative to an approach that uses information on actual bilateral exposures. Lelyveld and Liedorp (2006) show that for the Netherlands, regulatory data on large exposures provides a good approximation to comprehensive survey data.

connected nodes, eg Watts (2002). As has first been noted by Grassberger (1983), a number of these problems have analogues in the physics of flow or percolation processes.

Eboli (2004) points out that default dynamics related to contagion through interbank links can similarly be related to the physics of flow networks. In his model, nodes (banks) are connected to a ‘source’ where the initial shock is generated and every node is assigned a ‘sink’ where the losses are directed to – the bank’s net worth or capital. When losses reach a node, they are absorbed by the sink or, if they are large enough, losses create a first default and flow further through the network via links. The framework we develop in this paper builds on Eboli (2004) and identifies external assets as the source of shocks and adds depositors to the model as the ultimate sink (recipient of loss). Moreover, we define a probability distribution that governs whether or not a bank is exposed to another bank through an interbank exposure. This means that we can study varying degrees of connectivity, ranging from non-connected to complete structures in a systematic way. We start by positing identical and independent probabilities. While this specification is not considered to be a highly realistic model of most real-world networks, it is often used as a first approximation because of its tractability, Watts (2002). We follow this approach for much of the analysis. We consider an extension to a more realistic probability law in an extension (Section 6).

3 A model of banking systems and a simulation tool

In order to study system resilience we develop a simple model of a real-world banking system that captures the key features relevant for modelling knock-on defaults. We also devise a simulation engine that (a) is able to construct banking systems according to the underlying exogenous parameters of the model and that (b) allows us to study the resilience of the realised banking system to shocks and how resilience to shocks depends on the key parameters of the system.

3.1 Constructing banking systems

As in Eboli (2004), we think of a banking system as a network of nodes, where each node represents a bank and each link represents a directional lending relationship between two nodes. Importantly, the banking system needs to obey bank-level as well as aggregate balance sheet restrictions.

The network is based on the following two exogenous parameters that describe the random graph: the number of nodes, N , and the probability p_{ij} that a bank i has lent to another bank j . The probability p_{ij} is assumed to be equal across all (ordered) pairs (i, j) .⁽⁵⁾ For simplicity, we denote this probability, also referred to as the Erdős-Rényi probability, by p . Our simulation engine then delivers realisations of this graph that conform to the specified parameters and that exhibit a set number (Z) of realised links. See Figure A1 in the Appendix for a realisation of a network with 25 banks and $p = 0.2$.

For any realisation of the random graph, we populate the individual banks’ balance sheets in a manner consistent with bank-level and aggregate balance sheet identities. To describe this in more detail we introduce some notation. For the purpose of clarity, lower-case letters are used for variables at the individual bank level, capital letters for variables at the aggregate level and Greek letters for ratios.

⁽⁵⁾ This allows for links to go in both directions. If this is the case we do not allow the exposures to be netted.

An individual bank's assets, denoted by a , includes external assets (investors' borrowing), denoted by e , and interbank assets (other banks' borrowing), denoted by i . Thus, for bank i , we have $a_i = e_i + i_i$, where $i = 1, \dots, N$.

A bank's liabilities, denoted by l , are composed of net worth of a bank, denoted by c , customer deposits, denoted by d , and interbank borrowing, denoted by b . Hence for bank i , we have $l_i = c_i + d_i + b_i$, where $i = 1, \dots, N$. And as a balance sheet identity, we have $a_i = l_i$ for $i = 1, \dots, N$. An example of a bank's balance sheet as generated by the simulator is also shown in Figure A2 in the Appendix.

We construct balance sheets for individual banks in a number of steps. First, we decide on the total *external assets* of the banking system, denoted by E . These external assets represent total loans made to ultimate investors and thus relate to the total size of the flow of funds from savers to investors through the banking system. Next, we decide on the percentage of *external assets* in total assets (A) at the *system* level, denoted by $\beta = E/A$.

Note that the aggregate assets of the whole banking industry can be written as $A = E + I$, where I represents the aggregate size of interbank exposures. Hence, for a given aggregate amount of external assets E , the aggregate ratio of external assets in total assets, denoted by β , delivers both the size of total assets A and the aggregate size of interbank exposures I . That is, we have $A = E/\beta$ and $I = \theta A$, where $\theta = (1 - \beta)$ is the percentage of interbank assets in total assets.

Dividing the total interbank assets by the total number of links Z , we arrive at the bank-level size (the weight) of any directional link, denoted by w , which determines how much one bank lends to another. That is, we have $w = I/Z$. Hence, using w and the structure of the network, we can calculate i_i and b_i .

The size of each bank's external assets is slightly trickier to determine. In particular for any bank to be able to operate we require that its external assets are no less than its *net* interbank borrowing, that is, we have $e_i \geq b_i - i_i$. We fulfil this constraint by applying the following two-step algorithm.

First, for each bank, we fill up the bank's external assets such that its external assets plus interbank lending equal its interbank borrowing.⁽⁶⁾ That is, $\tilde{e}_i + i_i = b_i$, where \tilde{e}_i is the level of bank i 's external assets we got after this first step.

Second, whatever is left in aggregated external assets is evenly distributed among all banks. Note that total external assets equal E . Hence, a total of $\left(E - \sum_{i=1}^N \tilde{e}_i\right)$ units of external assets have not been allocated to individual banks' balance sheets, yet. In the second step we distribute this amount equally among all N banks. Now, let $\hat{e}_i = \left[\frac{\left(E - \sum_{i=1}^N \tilde{e}_i\right)}{N}\right]$, which is the amount to be allocated to each individual bank. Hence, we have $e_i = \tilde{e}_i + \hat{e}_i$.

⁽⁶⁾ This constraint can become difficult to meet if the percentage of external asset is too low. Since the distribution of links is stochastic, some banks may be assigned interbank borrowing much larger than interbank lending. When the total amount of external assets is low, there may not be enough assets to go round to close all balance sheet gaps opened up in this way. To avoid this difficulty we make sure that external assets are at least 30% of the total assets.

Hence, we complete the asset side of the bank balance sheet as well as interbank borrowing b on the liability side. The determination of the remaining two components, net worth c and deposits d , on the liability side is relatively straightforward. *Net worth* is set as a fixed proportion γ of total assets at bank level, that is, $c_i = \gamma a_i$. And, consumer deposits take up the remainder to meet the bank's balance sheet identity, that is, $d_i = a_i - c_i - b_i$. This completes the construction of the banking system and of each constituent bank's balance sheet.

Note that all banking systems constructed in this way can be conveniently described by the following set of structural parameters $(\gamma, \theta, p, N, E)$, where, recall, γ denotes net worth as a percentage of total assets, θ is the percentage of interbank assets in total assets, p is the probability of any two nodes being connected, N is the number of banks and E is total external assets of the banking system.

3.2 Shocks and shock transmission

In this paper we study the consequences of an idiosyncratic shock hitting one of the banks in the system and relate this to the structural parameters of the system. Throughout, the shock is to the value of a bank's external assets and can be thought of as resulting from operational risk (fraud) or credit risk.⁽⁷⁾ While for credit risk in particular, aggregate or correlated shocks affecting all banks at the same time may also be relevant in practice, idiosyncratic shocks are a cleaner starting point for studying knock-on defaults due to interbank exposures and liquidity effects. In our model, aggregate shocks amount to reducing the net worth of all banks at the same time. If this shock is big enough to bring down any bank, in our set-up this will typically lead to all banks defaulting with little scope for further analysis. We therefore think of aggregate shocks as potentially affecting all banks' net worth to a point where none of the banks are yet in default. We then study the consequences of idiosyncratic shocks for any given positive aggregate net worth.⁽⁸⁾

For any given realisation of the banking system, we shock one bank at a time, by wiping out a certain percentage of its external assets (the 'source' of the shock). Let s_i be the size of the initial shock. This loss is first absorbed by the bank's net worth c_i , then its interbank liabilities b_i and last its deposit d_i , as the ultimate 'sink'. That is, we assume priority of (insured) customer deposits over bank deposits which, in turn, take priority over equity (net worth). If the bank's net worth is not big enough to absorb the initial shock, the bank defaults and the residual is transmitted to creditor banks through interbank liabilities. And in case these liabilities are not large enough to absorb the shock, some of the losses are born by depositors. Formally, if $s_i > c_i$, then the bank defaults. If the residual loss $(s_i - c_i)$ is less than the amount bank b_i that bank i has borrowed from the interbank market, then all the residual loss $(s_i - c_i)$ is transmitted to creditor banks. However, if $(s_i - c_i) > b_i$, then all of the residual cannot be transmitted to creditor banks and depositors receive a loss of $(s_i - c_i - b_i)$.

All creditor banks receive an equal share of the residual shock, which in turn, is first absorbed by their net worth. If the net worth is larger than the shock transmitted, the creditor bank

⁽⁷⁾ Fraud is a relatively common cause of bank default. Iyer and Peydro-Alcalde (2006) provide a recent event study that highlights the importance of interbank linkages in transmitting losses from fraudulent activity through the system.

⁽⁸⁾ As we do, much of the existing empirical literature on interbank linkages focuses on the consequences of a shock to a single bank when tracing through the consequences of that shock for other banks in the system, eg Sheldon and Maurer (1998), Furfine (1999), Upper and Worms (2004), Wells (2002). A recent exception is Elsinger, Lehar and Summer (2006), who study the relative importance of correlations and interbank contagion as a source of systemic risk. We leave further analysis of the interplay between correlated shocks and interbank contagion for future research.

withstands the shock. Otherwise, the creditor bank defaults and again the residual loss is transmitted through interbank liabilities first, and in case these liabilities are not large enough to absorb the shock, are borne by depositors. The part that is transmitted through the interbank channel may cause further rounds of contagious default. The transmission continues down the chain until the shock is completely absorbed.

Formally, let k be the number of creditor banks and let bank j be one of those banks that have lent to bank i , the bank that has been hit by the initial shock. Hence, if $(s_i - c_i) < b_j$, then bank j receives a loss of $s_j = \left\lceil \frac{(s_i - c_i)}{k} \right\rceil$. If $s_j \leq c_j$, then bank j withstands the shock. Otherwise, the creditor bank defaults and again the residual loss is transmitted through interbank liabilities, and so forth.

4 Default dynamics for banking systems – simulations

Recall that all banking systems we can construct can be described by the following set of structural parameters $(\gamma, \theta, p, N, E)$. In this section we describe simulation experiments that are designed to study how the resilience of the system is affected by these key parameters: net worth as a percentage of total assets (γ), the percentage of interbank assets in total assets (θ), the probability of any two nodes being connected (p) and the number of banks (N). Throughout we keep the size of aggregate external banking system assets (E) fixed (at 100000). That is, we keep constant the total loan supply to the real sector and focus on the structure of the banking system for a given total size of intermediated funds. We mainly report on *comparative statics* experiments in which we vary one parameter at a time and briefly comment on exercises where two parameters are varied simultaneously as a way of exploring the robustness of our main results.

In our *benchmark experiment*, the exogenous parameters take the following values: number of nodes ($N = 25$), Erdős-Rényi probability ($p = 0.2$), percentage of interbank assets in total assets ($\theta = 20\%$) and net worth as a percentage of total assets ($\gamma = 5\%$). In each comparative statics exercise we vary one of these parameters while keeping the other four fixed. Table A summarises the values of the benchmark parameters of the model.

Table A: Summary of benchmark parameters of the model

Parameter	Description	Benchmark value	Range of variation
E	Total external assets	100000	Fixed
N	Number of banks in the network	25	10 to 25
p	Erdős-Rényi probability of connection between any two nodes	0.2	0 to 1
θ	Percentage of interbank assets to total assets	20%	0% to 50%
γ	Percentage of net worth to total assets	5%	0% to 10%

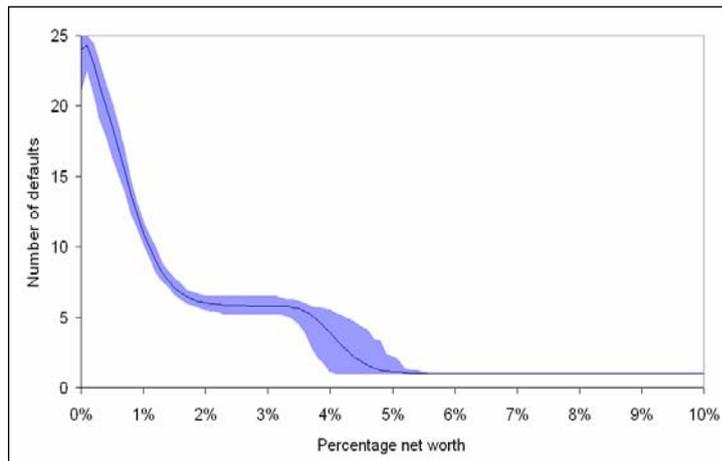
Recall that our parameters describe a random graph, rather than a realisation of this random graph. Thus, a first step in the experiment is to generate one such realisation. In the second step, for each realisation of the network, we shock one bank at a time and in this way successively shock each bank in the system. Throughout, unless explicitly stated otherwise, the shock applied to each bank is calibrated to wipe out all external assets of the bank. For each bank that is shocked, we count the overall (first and later-round) number defaults. We then take the average number of defaults across all banks for each realisation of the system. For each set of parameters

$(\gamma, \theta, p, N, E)$ we repeat this exercise for 100 draws of the network and report averages across realisations.

4.1 Bank capitalisation and contagion

In a first experiment, we investigate the effect of banks' net worth (γ) on the resilience of the banking system.

Chart 1: Impact of bank capitalisation



Note: Number of defaults as a function of the percentage of net worth to total assets (γ). Based on 100 runs for each parameter constellation $(\gamma, \theta, p, N, E)$. Parameter values as in Table A (except for γ).

Chart 1 reports the results. The bold line represents the average of 100 experiments and the coloured area shows the range over which 95% of the observations lie. Chart 1 illustrates a weakly monotonic and negative relationship between bank capitalisation and contagion. As we can see from the chart, when net worth is set at 0% of assets, all banks fail. As net worth increases, the number of failures decreases and for net worth sufficiently large only the first bank defaults and all other banks are able to withstand the shock.

However, contagion does not decrease *linearly* in bank capitalisation. For high levels of net worth only the first bank defaults and much of the shock is absorbed by the bank's net worth. When the net worth decreases to a certain level, second-round defaults occur because the first bank, as well as its creditors, is less cushioned against external shocks. More of the loss is transmitted to creditors and creditors are less well protected, increasing the likelihood of second-round defaults. When net worth continues to fall, but does not fall enough to generate further rounds of contagious default, one observes that the number of defaults stays constant for the range of 4% to 1% of net worth in the benchmark case. This is because the net worth of candidates for a third-round default remains big enough to absorb the shock received. However, as soon as the level of net worth falls below 1%, multiple rounds of default are triggered and, therefore, the number of defaults increases sharply.

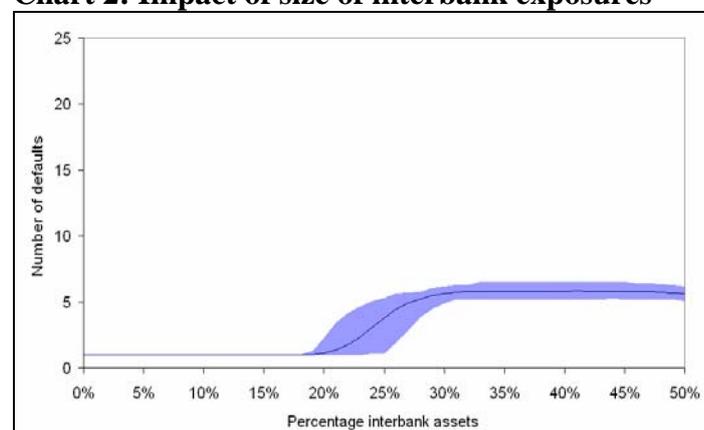
While the reasons for banking systems to become undercapitalised are not explicitly modelled here, low aggregate capitalisation can be the result of a prior aggregate shock that weakens the aggregate net worth of the banking system as a whole. Our analysis therefore suggests that such aggregate shocks may greatly increase the scope for contagious or knock-on default resulting from one of the banks failing. Low aggregate capitalisation may alternatively have structural

reasons and be the result of an extensive safety net which reduces individual banks' incentive to keep adequate capital buffers, see Baumann and Nier (2003) for evidence on this effect. Our analysis suggests that such incentive effects of safety nets may greatly increase the potential for systemic breakdowns.

4.2 Size of interbank exposures and contagion

In a second simulation exercise, we examine the effect of the size of interbank lending/borrowing (θ) on the number of failures. Note that we hold the *absolute* size of external assets constant, while increasing total assets by increasing the percentage of interbank assets in total assets (θ). Given that the *number* of links is fixed and determined by the Erdős-Rényi probability, an increase in interbank assets implies an increase in the size of each interbank lending relationship, that is, an increase in the weight (w) of the link. But also, since net worth is a fixed proportion of total assets, an increase in interbank assets is by construction associated with an increase in total net worth.

Chart 2: Impact of size of interbank exposures



Note: Number of defaults as a function of the percentage of interbank assets in total assets (θ). Based on 100 runs for each parameter constellation (γ, θ, p, N, E). Parameter values as in Table A (except for θ).

Chart 2 shows the results. An increase in the percentage of interbank assets has two distinct and potentially opposing effects on the resilience of the banking system. First, it implies an increase in the transmission of the shock to interbank creditors and a decrease of the size of the shock that is borne by customer deposits. That is, it leads to enhanced shock propagation. Second, since by assumption banks hold capital against interbank exposures, an increase in the size of the interbank market results in an increase in overall net worth. This has the potential to dampen contagion effects.

Indeed, as we observe from Chart 2, for very low levels of interbank assets, there is no contagion effect from the first-round default. This is because most of the loss is absorbed by customer deposits and any losses that are transmitted to creditor banks are small enough to be absorbed by their net worth. When the proportion of interbank assets rises beyond a threshold (18% in Chart 2) we start to see second-round defaults. This is because more of the shock is transmitted to creditor banks and because their net worth does not rise fast enough to be able to absorb the losses received. Thus we see second-round defaults increasing quickly from zero to five. We do not see the overall number of defaults rise beyond six in this experiment for any realistic proportion of interbank assets. This is because when interbank assets rise, this raises the net worth of the first bank as well as its creditor banks relative to the defaulted external assets of

the first bank. That is, more of the loss starts being absorbed by the net worth of the first bank as well as its creditor banks and less is passed on to candidates for a third-round default.

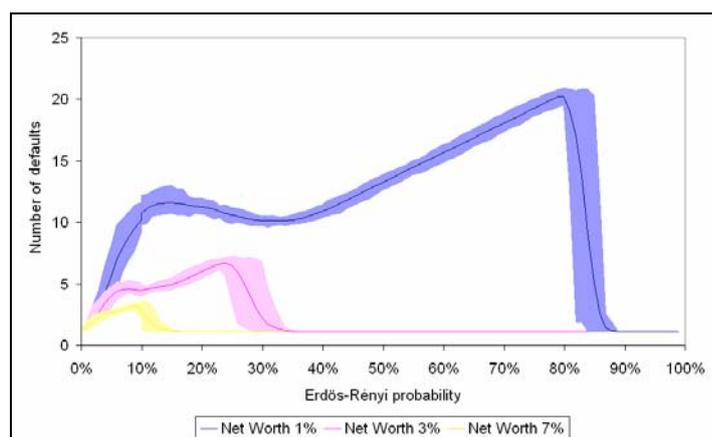
In sum, we show that increases in the size of interbank liabilities may lead to an increase in the threat of contagion. Interestingly, we show that this effect is at work – if mitigated – even if banks hold capital against interbank exposures.

4.3 Connectivity and contagion

We next investigate the effect of connectivity on the resilience of the banking system. We also check how the relationship changes for different levels of net worth. The results from this experiment thus give us a first helpful insight into the interplay between connectivity and net worth.

In Chart 3, on the horizontal axis, we have the Erdős-Rényi probability p , where, as p increases, on average banks become more connected. The blue line and the blue range represent networks with net worth equal to 1% of total assets, whereas, the red and yellow ones represent networks with net worth equal to 3% and 7% of total assets, respectively.

Chart 3: Impact of likelihood of interbank exposure



Note: Number of defaults as a function of the probability of connectedness (p) for different values of percent of net worth (γ). Based on 100 runs for each parameter constellation (γ, θ, p, N, E). Parameter values as in Table A (except for p and γ).

Interbank connections have two opposing effects. On the one hand, they may act as channels to propagate shocks to the whole system, that is, they act as ‘shock-transmitters’. On the other hand, through these interbank linkages, shocks can be shared and absorbed by other bank’s net worth, that is, interbank linkages may act as ‘shock-absorbers’.

In Chart 3, we see that each of these two mechanisms dominate over different ranges, generating an M-shaped graph. First, for very low levels of connectivity (p close to zero), an increase in connectivity reduces system resilience, since connectivity increases the chance of shock transmission. For higher levels of connectivity, increases in connectivity may decrease or increase system resilience. But when connectivity is sufficiently high, further increases in connectivity unambiguously decrease contagion as the shock absorption effect starts to dominate and the initial shock is spread over more and more banks, each able to withstand the shock received.

Chart 3 also documents an interesting interdependence between the effects of connectivity and net worth on contagion. In systems with undercapitalised banks (1% net worth), only a very small portion of the shock is absorbed by net worth and the remainder is transmitted to other banks. In those cases, as connectivity increases (high p), the number of failures keeps increasing since, due to low levels of net worth, interbank linkages act as shock-transmitters, rather than shock-absorbers. This leads us to conclude that undercapitalised banking systems are fragile and even more so when connectivity is high.

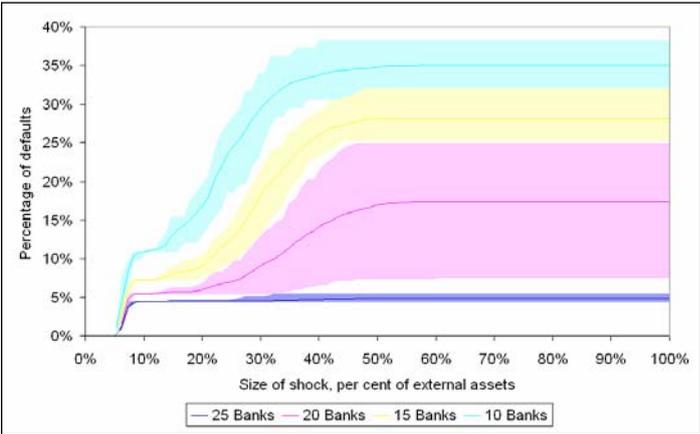
However, for better capitalised banking systems, the effect is reversed. While interlinkages still transmit shocks from one bank to another, high levels of net worth act as a cushion and increase the capacity of interbank linkages to act as shock-absorbers, increasing the resilience of the system. This leads us to conclude that well-capitalised banking systems are resilient to shocks, even more so when connectivity is high.

In sum, low aggregate capital increases the potential for interbank connections to transmit shocks through the system. This means that shocks that might be able to draw down the capital cushion of the system as a whole pose a risk in turning the system from one where connections dissipate shocks to one where connections tend to work as shock-transmitters.

4.4 Concentration and contagion

Finally, we investigate the effect of concentration of the banking system on its resilience to contagion resulting from interbank linkages. To do so, we vary the number of banks (N) in the banking sector from 10 to 25. As in the previous simulations we keep the aggregate size of banks’ external assets constant. Thus, the banking system with 10 (25) banks is the most (least) concentrated, for a given size of the economy. In these experiments, we apply shocks that vary as a percentage of banks’ external assets.

Chart 4: Impact of concentration



Note: Percentage of defaults as a function of the shock size as a percentage of external assets for different N . Based on 100 runs for each shock size. Parameter values (γ, θ, p, E) are as in the benchmark experiment, Table A.

Our first finding is that, regardless of concentration, as the size of the shock, relative to the shocked bank’s external assets, increases, the fraction of defaulted banks increases. Moreover, for the same percentage of shock (relative to the external assets), we observe a higher percentage of failures, that is, a higher risk of contagion, the more concentrated the banking system is, all else being equal. The intuition for this is simple: since total assets are fixed, as the

number of banks decreases, each bank becomes big enough to make a significant impact on the rest of the system.

However, one may ask whether this positive relationship between concentration and the fraction of defaults is partly driven by the fact that the absolute size of the shock also increases when concentration increases? The answer is that while this is a factor, it is not the only one driving the result. The other is that when the system is more concentrated, interbank connections have an enhanced capacity to lead to a breakdown of the entire system. To see this, consider the following numerical example: take the experiment with 25 banks (dark blue line in Chart 4). A shock that wipes out 100% of the external assets of a bank results in only 4% of banks defaulting, which translates into one bank default. Note that, since the aggregate size of external assets is 100000, the shock size applied in this experiment is 4000. We now compare this experiment to the one with ten banks and a shock at 40% of external assets. Note that this shock also amounts to an absolute size of 4000. With ten banks this shock generates a bigger impact with 30% of banks defaulting in total, which translates into three banks (30% x 10).

In sum, the same shock in terms of total external assets leads to only one bank defaulting in the system with low concentration (system with 25 banks), whereas 30% of the banking system defaults in the case of the concentrated system (three banks out of ten). This shows that even for a given shock size, and all else equal, a more concentrated banking system is more vulnerable to systemic risk.

As regards the impact of the four key parameters on system resilience we have undertaken further analysis that varies more than one parameter at the time. Some of these are shown in Figures A5-A8 in the Appendix. Overall these experiments show that the findings presented in this section hold not only when our benchmark experiment is considered as a starting point, but in a neighbourhood of this benchmark configuration that is large relative to the overall parameter space.

5 Liquidity risk

In this section we extend the analysis to incorporate liquidity effects. These arise when failed banks' assets need to be sold but there is limited overall liquidity in the market to absorb the assets. Limited liquid funds on the part of potential buyers for these assets result in cash-in-the-market pricing, as modelled by Allen and Gale (1994, 1998). This means that the price of banking assets becomes a decreasing function of the aggregate amount of banking assets that need to be liquidated. Moreover, when claims on banks are traded in the market, in equilibrium all banks' assets are revalued in the way that reflects the price of assets sold, as in Cifuentes, Ferrucci and Shin (2005).

We capture such liquidity effects by positing the following inverse demand function for banking assets:

$$P(x) = \exp(-\alpha x) \tag{1}$$

Here P denotes the price of banking assets and x denotes the aggregate amount of assets sold. The parameter α measures the speed at which the price for banking assets declines with the amount of assets sold. Thus, it can easily be thought of as a measure of the *illiquidity* of the market for banking assets. In particular, note that the elasticity of the inverse demand function is

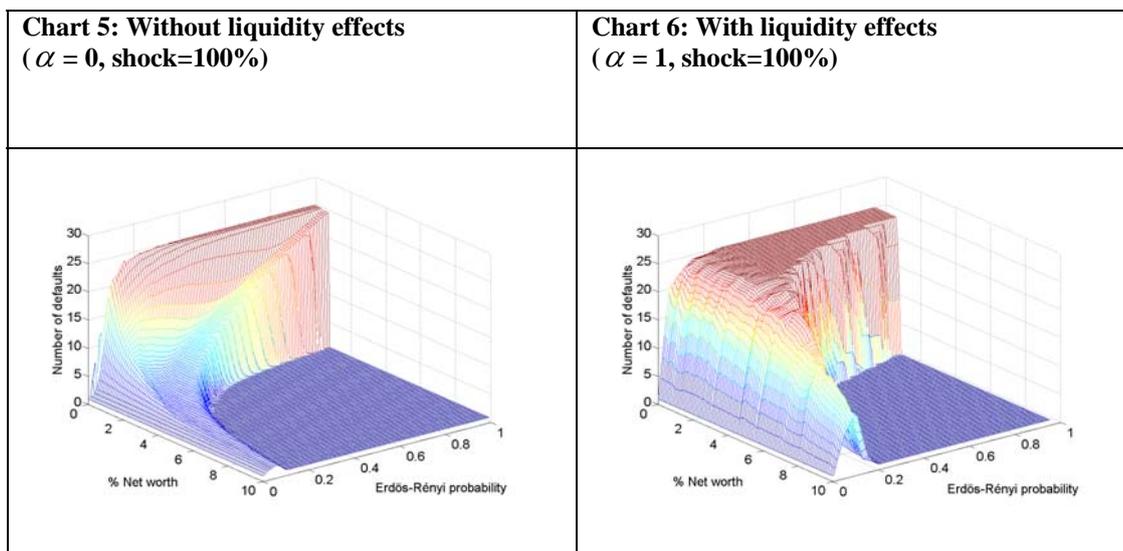
$$e(x) = \frac{dP(x)}{dx} \frac{x}{P(x)} = -\alpha x \quad (2)$$

This implies that the semi-elasticity of this function is $(e(x)/x) = \alpha$, which we interpret as the illiquidity of the market for banking assets. Also note that $P(0) = 1$ and $\lim_{x \rightarrow \infty} P(x) = 0$. In other words, the price of banking assets is (normalised to) unity if no assets are sold and approaches 0 as x goes to infinity.

To study default dynamics with liquidity effects, we modify our algorithm as follows. If the shock to the external assets of the first bank is high enough so that this bank is insolvent as a result of the shock, all remaining external assets of this bank (if any) are sold in the market. This means that the price for external assets declines from $P = 1$ to some $P' < 1$, which depends on the amount of external assets that need to be liquidated. The first bank thus suffers an additional loss of $(1 - P')$ per unit of external assets sold. Therefore, the total loss suffered by this bank is the sum of the initial loss of external assets arising from the shock and the loss arising from liquidating the remaining external assets. This total loss at the first bank is transmitted to other banks in the system through interbank exposures to that bank. However, other banks suffer an additional loss since these banks revalue their assets to reflect the fall in price from 1 to P' . At this stage, losses incurred by other banks may trigger second-round defaults, which again lead to losses being spread through interbank linkages and depressed asset prices as a result of forced sales. The default process continues in this way until either all banks default or at any round the resulting follow-on losses are insufficient to cause a further round of defaults.

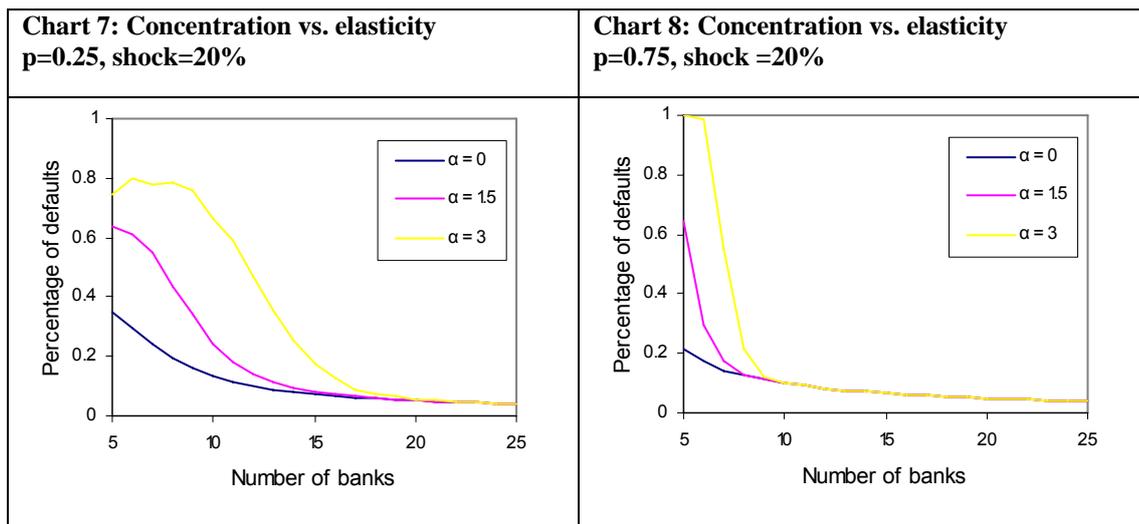
The following experiment illustrates the default dynamics with and without liquidity effects. In Chart 5, we show the effect of connectivity on the number of defaults for a range of values for net worth and in the absence of liquidity effects, ie $\alpha = 0$. Note how this experiment generalises the findings in the previous section (Chart 3). For any level of net worth, we find a non-linear 'M-shaped' relationship between connectivity and default. Also, as net worth is reduced, larger and larger increases in connectivity are needed to prevent knock-on defaults. Perhaps most interestingly, in the limit as net worth approaches zero, the total breakdown of the banking system cannot be contained by ever larger degrees of connectivity. Indeed, when net worth is zero, the number of defaults is lower than 25 only for levels of connectivity less than $p = 0.2$ and is lowest (equal to 1) for $p = 0$.

Chart 6 shows the same experiment but with an active asset price channel ($\alpha = 1$). First, observe that with the liquidity channel active, the number of defaults is never less than in the case when illiquidity is zero. In other words, illiquidity increases contagious default for any level of connectivity. Second, the region where we observe a total breakdown (25 defaults) turns from a thin area associated with very low levels of net worth to a contagion plateau where a total breakdown happens even for relatively sizable levels of net worth. Third, liquidity effects smooth out the 'M-shaped' non-linearity between connectivity and defaults. The intuition is that all banks are affected by illiquidity effects in the same way. This means that non-linearities arising from threshold effects associated with interbank links become less pronounced. Finally, increases in connectivity are even less powerful than in the case without liquidity effects in preventing the complete breakdown of the system. Even for relatively sizable values of net worth this can be prevented only when connectivity is small (less than 0.2).



Note: Number of defaults as a function of percentage of net worth and probability p , without and with liquidity effects, respectively. Based on 100 experiments with benchmark parameter values as in Table A (except for p and γ).

In the previous section we showed that for a given shock size, and all else being equal, a more concentrated banking system is more vulnerable to systemic risk (Chart 4). We now investigate how this relationship is affected by the presence of illiquidity effects. Chart 7 demonstrates the following: first, it confirms the positive relationship between the level of concentration and contagion when there are no illiquidity effects ($\alpha = 0$). Second, when illiquidity effects are stronger ($\alpha \in \{1.5, 3\}$), the effect of increasing concentration (reducing N) on fragility tends to become more pronounced. In other words, more concentrated systems are fragile in particular when markets are illiquid. The intuition is that in a concentrated system, the default of one of the large banks requires the liquidation of a large part of the banking system. This can quickly drive down market valuations for the remaining banks, exacerbating asset price contagion for concentrated systems relative to non-concentrated systems. Chart 8 confirms that the same logic applies for banking systems that are highly connected ($p = 0.75$) than for those systems where connectivity is moderate, as shown in Chart 7, where $p = 0.25$.



Note: Percentage of defaults as a function of number of banks N and elasticity α , for low and high connectivity, respectively. Results for 100 runs for each parameter constellation. Parameter values for $(\gamma, \theta, p, N, E)$ as in Table A (except for N and p).

6 Extension to networks with tiering

In the previous sections, we analysed networks that were homogenous (*ex ante*), in the sense that each bank had the same probability of being connected to another bank. But we know that many real-world networks do not conform to this description. In particular, as has been pointed out by Albert and Barabasi (1999), while for random graphs the degree distribution (ie the distribution of links) is strongly peaked about the mean, the degree distribution for real-world networks is highly skewed. Boss, Elsinger, Summer and Thurner (2003) confirm that the network structure of the Austrian interbank market is scale-free and that therefore there are ‘very few banks with many interbank linkages whereas there are many with only few links’. Boss *et al* (2003) explain that the Austrian banking markets exhibit ‘tiering’, where first-tier or head-institutions are connected to second-tier or sub-institutions as well as between each other, while there is limited connectivity between sub-institutions. Similarly, ‘tiered’ structures with highly connected first-tier banks and a second tier of ‘peripheral’ banks are found in many other banking markets, including the United Kingdom, Harrison *et al* (2005). Boss *et al* (2003) also point out that highly connected first-tier banks are typically larger in size, that is, the degree distribution corresponds to the size distribution across banks in the banking system.⁽⁹⁾

To model tiered structures, we group the N banks in the network into ‘large’ (m out of N) and ‘small’ (n out of N) banks, where $(m + n) = N$. We allow for the large banks to have a higher probability of being connected. In particular, we denote the probability of a directed link⁽¹⁰⁾ to

⁽⁹⁾ As has first been pointed out by Watts and Strogatz (1998), some real-world networks display a high degree of *clustering* or *network transitivity*, while the random graph model does not. High clustering refers to a situation where if nodes A, B and C form part of a larger network, and if A is connected to B and B is connected to C, the likelihood of A being connected to C is higher than in the random graph. For many social networks, clustering coefficients are several orders of magnitude higher than would be implied by the random graph model, see Newman (2003). However, interestingly, Boss *et al* (2003) found relatively small clustering coefficients for the Austrian interbank market. They point out that in the context of the interbank market a small clustering coefficient is a reasonable result: ‘While banks might be interested in some diversification of interbank links, keeping a link is also costly. If for instance two small banks have a link with their head institution, there is no reason for them to additionally open a link among themselves’. In keeping with this logic we found that for the tiered structures analysed in this section, clustering coefficients were only marginally higher than implied by the random graph.

⁽¹⁰⁾ As in the case of the random graph (homogenous case), there can be links going in both directions.

exist from a small bank i to another bank j (small or large) as p_s , while we denote this probability by p_l for a large bank, with the natural assumption that $p_l \geq p_s$.

Recall that in homogenous networks the probability of being connected was denoted by p . Therefore, for $p_l = p_s = p$, the system corresponds to a homogeneous network. The other limiting case is $m = 1$ and $p_l = 1$, so that all of the small banks are connected to the one large bank. This limiting case is known in the literature as a *star formation*. Our simulation engine allows us to move continuously from one extreme to the other.

Note that as we increase p_l , keeping p_s constant, the number of connections would increase. To establish a clean comparison with the homogeneous network, we prefer to adjust p_s in such a way that the overall number of connections stays the same and is comparable to a homogenous network. In other words, in the tiered network, the unconditional probability for a node to be connected to another should be equal to the Erdős-Rényi probability of the homogenous network. This is achieved as follows.

For a given probability p_l , we find the probability p_s for small banks being connected in such a way that the total number of links in the homogeneous and tiered networks stays the same. In particular, in a homogeneous network with N banks and probability of connection p , the number of links is given by $p[N(N-1)]$. In the tiered network, the number of links can be calculated as:

$$p_s (N - m)^2 + p_l m(N - 1) + p_l (N - m)m \quad (3)$$

By equating the two expressions and solving for p_s , we find the value of s that gives us the same number of links for the homogeneous and the tiered networks as:

$$p_s = \frac{p_l m + p_l m^2 - 2 p_l m N - p + N p}{(m - N)^2} \quad (4)$$

We also define the percentage of total external assets assigned to large banks, denoted by k . Thus, each large bank has (k / m) percent of the total external assets in their portfolio. The case where the large and the small banks are equally large is $k = \left[\frac{m}{N - m} \right]$.

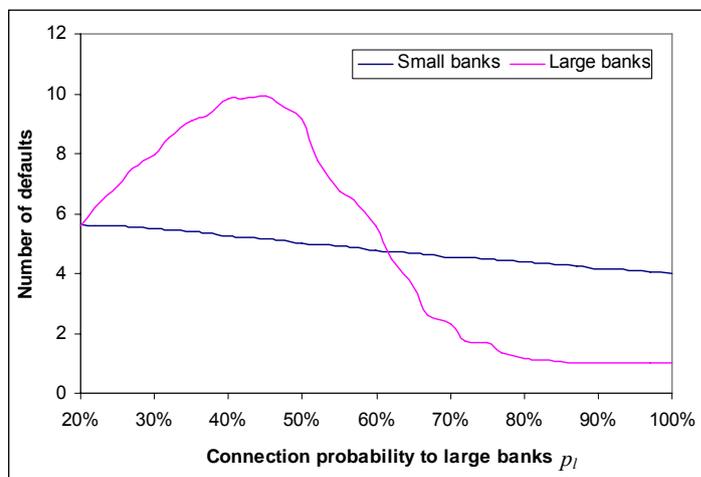
To investigate the effect of tiering, we analyse a network with 25 banks, where only one bank is large, that is $N = 25$ and $m = 1$. We let the probability of being connected for large banks, denoted by p_l , take values between 20% and 100%. To focus on centrality rather than differences in size⁽¹¹⁾ we let $k = 1/24$. Hence, the only difference to the homogeneous network is that one bank has more connections to the other banks.

Chart 9 displays the average number of defaults over 100 runs, when the shock is applied to the large bank and when shocks are applied to the small banks. Note that when $p_l = 20\%$, we have a homogeneous network, that is, $p = p_s = p_l$ and the number of defaults for that value matches the value obtained in the experiment with homogeneous network (around 5.7 defaults). As we increase p_l , the number of defaults increases until p_l reaches around 42%, where the number of defaults peaks at 10. From then on, the number of defaults start to decrease, because as the large

⁽¹¹⁾ The basic insights do not change materially where the percentage of external assets assigned to first-tier banks is bigger. The main difference is that a larger first-tier bank allows for a larger absolute shock to hit the system.

bank becomes more connected, the shock received by the large bank is distributed among more of the smaller banks, and therefore, the individual shocks that small banks receive is smaller and can be absorbed by the small banks' net worth.

Chart 9: Tiered networks



Note: Number of defaults as a function of the connection probability p_l of the large bank for small and large banks, for 100 experiments for each parameter constellation. Parameter values are $(\gamma, \theta, N, E) = (0.05, 0.3, 25, 100000)$.

We conducted a number of additional experiments on tiered structures. Overall we found that the main results obtained for homogenous networks (as regards net worth, interbank exposures, connectivity and concentration) carry over to tiered structures, while tiering is able to generate a number of additional results. We leave a more in-depth analysis for future research.

7 Conclusions

This paper analysed how systemic risk depends on the structure of the banking system. We applied network theory to construct banking systems and then analysed the resilience of the system to contagious defaults. In the homogenous case and in the absence of liquidity effects, for a given amount of external assets and therefore for a given size of intermediated flows, four parameters are required to describe the banking system as a random graph. These are net worth, the size of the interbank market, the degree of connectivity and the concentration of the system.

We analysed the degree of system resilience with respect to each of these parameters. From the point of view of central banks that are charged with the analysis and mitigation of systemic risk, the following four key results emerged.

First, we showed how the level of net worth affects the scope for contagious defaults. We found that decreases in net worth increase the number of contagious defaults and that this effect is non-linear. For high levels of net worth, the system is immune to contagious defaults. But when net worth falls, once capitalisation reaches a lower threshold, a further decrease in net worth leads to sharp increases in the risk of a systemic breakdown, where a large number of banks default as a result of contagion. Moreover, while common shocks to all banks in the system were not explicitly analysed, it is clear that common shocks are one mechanism that may lead to such decreases in overall net worth. Our results thus suggest that common shocks and interlinkages may interact in a way that may lead to sharp increases in systemic risk.

Second, increases in the size of interbank liabilities tend to increase the risk of knock-on default. Interestingly, this is the case even in a world where banks hold capital against interbank assets in the same way as they hold capital against external assets. This suggests capital requirements that are a function of the asset side of banks' balance sheets may not protect against the threat of systemic breakdown that arises when banks are interlinked through interbank liabilities.

Third, contagion is shown to be a non-monotonic function of the number of interbank connections, all else equal. When the level of connectivity is low, an increase in the number of links increases the chance of contagious defaults. However, when connectivity is already high, a further increase in the number of links increases the capacity of the system to withstand shocks. Moreover, these effects are shown to interact with the level of capital. For less capitalised systems, an increase in connectivity tends to be associated with an increase in contagious defaults, while for better capitalised systems connectivity tends to work as a shock absorber, dissipating the effect of the shock across a larger number of banks, each able to withstand the shock. In sum, this means that aggregate shocks that might be able to draw down the capital cushion of the system as a whole pose a risk in turning the system from one where connections dissipate shocks to one where connections tend to work as shock-transmitters.

Fourth, all else being equal, more concentrated banking systems tend to be more prone to systemic breakdown. The intuition is that for a given degree of connectivity, when the system is made up of a small number of banks, when one bank is hit by an idiosyncratic shock, interbank contagion is able to affect a large fraction of the banks in the system. By contrast, in a less concentrated system, contagion may stop before it spreads to a large fraction of banks. This finding is important from the point of view of central banks charged with the analysis of system risk. In concentrated systems, the failure of a large bank has adverse effects, not merely due to its size, but also because the failure is more likely to affect a large fraction of other banks in the system.

We extended the analysis to incorporate liquidity effects. When distressed banks' assets are sold, if there is limited liquidity within the financial system, this may result in cash-in-the-market prices. Thus, the more assets are liquidated, the lower their price. We show that the presence of liquidity effects increases the chance of systemic breakdown for any given aggregate capitalisation and any given degree of connectivity between banks. Moreover, we find that more concentrated banking systems are particularly vulnerable to the presence of such liquidity effects. We find that more concentrated banking systems tend to be more fragile even in the absence of liquidity effects. However, the less liquid is the market for failed banking assets the more concentration increases the risk of systemic breakdown.

Finally we extended the analysis to *ex-ante* heterogeneous (tiered) structures. We found that tiered structures are not necessarily more prone to systemic risk. Whether they are or not, depends on the degree of centrality – that is the number of connections to the central node, such that, as the degree of centrality increases, contagious defaults first increase, but then start to decrease, as the number of connections to the central node start to lead to greater dissipation of the shock. This implies that banking systems that show a high degree of tiering, ie where there is a first tier of money-centre banks and a second tier of smaller banks that depend on the first-tier banks – are not necessarily more fragile than more homogenous banking systems.

Appendix: A.1 Constructing interbank networks using the simulator

Figure A1: A banking structure with 25 banks and $p = .20$

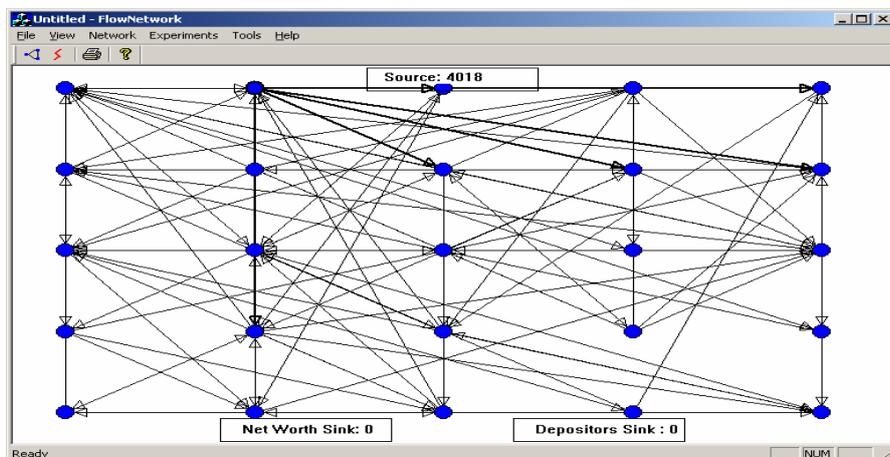


Figure A2: A bank's balance sheet

LIABILITIES		ASSETS	
Net Worth 258.00		External Assets 4018.52	
Deposits 3528.24			
Borrowed 1388.89		Lent 1157.41	
Total: 5175.93		Total: 5175.93	

Bank number: 1
Assets : 4018
Lent : 1157
TOTAL ASSETS : 5175
Net worth: 258
Depositors : 3528
Borrowed : 1388
TOTAL LIABILITIES : 5174
Received loss: 0
Absorbed loss: 0
Residual loss: 0
Depositors Sink loss: 0

A.2 Experiment to illustrate shock transmission

Here, we provide an experiment to illustrate the dynamics of a shock (Figures A3 and A4). In this example, the received shock is $s_i = 4008$, of which 226 get absorbed by the bank's net worth, 617 of which is transmitted to creditors in the interbank market and 3164 of which flow into the depositor sink (see Figure A3).

Figure A3: Shock absorption

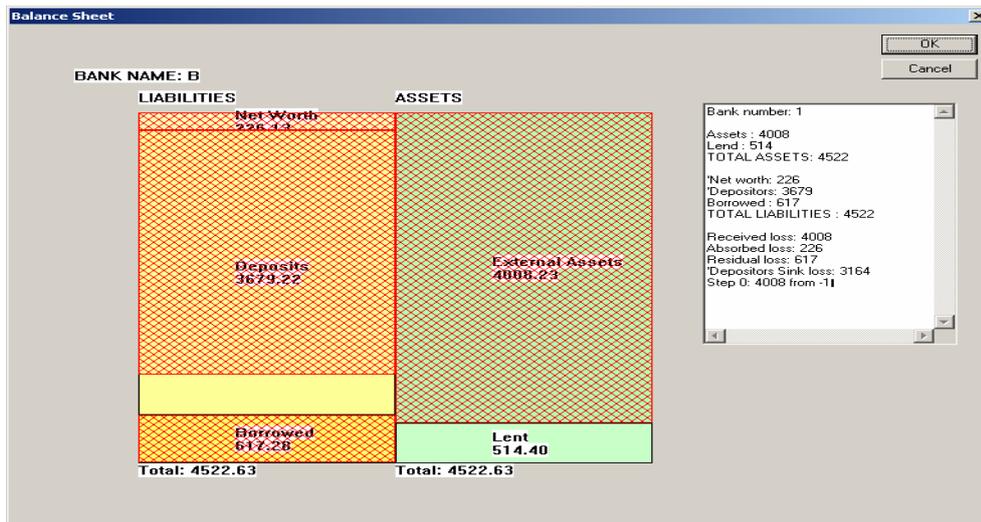
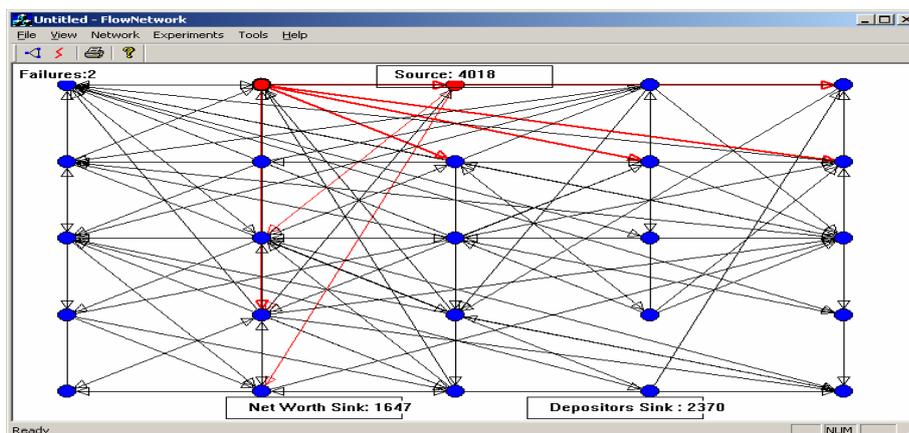


Figure A4 illustrates the transmission process. The nodes coloured red reflect the defaulted banks. In this experiment, an initial shock of 4018 caused one initial bank default which led to the default of one other bank through contagion. Other banks are also affected by the default of the first defaulted bank as well as by the second-round default, but their net worth is sufficient to withstand the shock received.

Figure A4: Shock transmission in a banking system



A.3 Multivariate analysis – simulation results

Figures. A5-A8 show the results of experiments where two key parameters are varied at the same time.

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Figure A5: Impact of net worth and interbank exposures

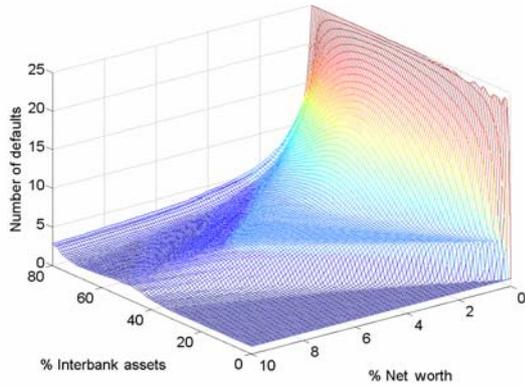


Figure A6: Net worth and connectivity

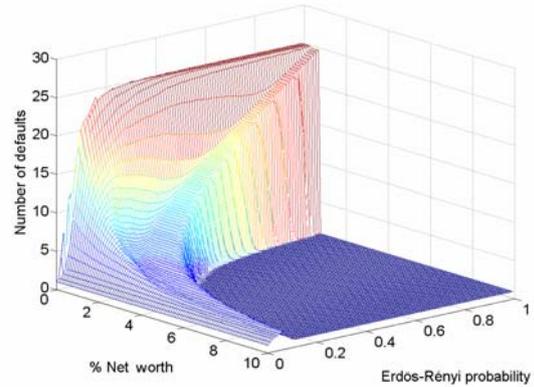


Figure A7: Impact of connectivity and interbank assets

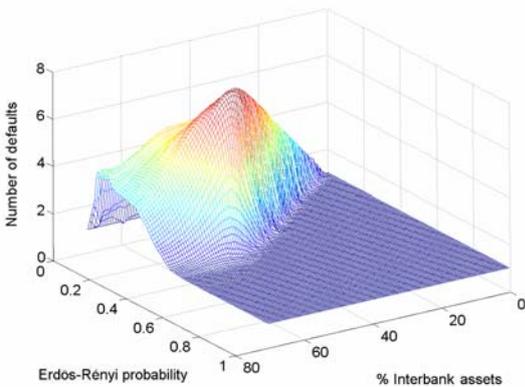
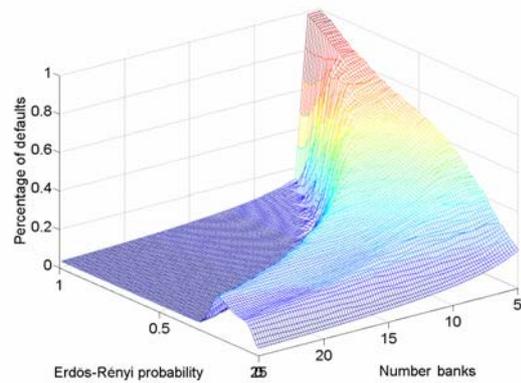


Figure A8: Impact of connectivity and concentration



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