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Abstract

The statistics that summarise probability density functions (pdfs) implied from option prices can be used to assess market expectations about future uncertainty, asymmetry and the probability of extreme movements in asset prices. A time-series analysis of these statistics for equity index and interest rate markets provides some stylised facts about the behaviour of these elements of market expectations, their historical distribution, similarity and relative stability. Relationships between them and movements in underlying asset prices are considered. Cross-asset and cross-country comparisons and the information content of the implied pdfs for future macroeconomic and financial variables are also assessed.

Key words: Options, implied probability density functions (pdfs), summary statistics, implied volatility, implied asymmetry, market expectations.

JEL classification: G13, G19.

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Summary

Financial markets can provide policymakers with timely information about *aggregate* market expectations of future asset prices and returns. Options, which give investors the right, without obligation, to buy/sell assets in the future, possess information about the likelihood that market participants attach to alternative future outcomes for asset prices. The previous decade has seen much development in the methods of extraction of distributions of the probabilities that market participants attach to future asset prices from options prices. Time series of the statistics that summarise these 'option-implied distributions' can be examined to consider the behaviour of market views over time.

The focus of this paper is on the properties of these summary statistics for option-implied probability density functions (pdfs). These statistics provide us with various measures of aggregate expected uncertainty, asymmetry (or balances of risk) and expectations of extreme movements. We estimate a daily time series of option-implied pdfs (in terms of logarithmic changes in asset prices) and their summary statistics for various equity indices (FTSE 100 and S&P 500) and interest rates (3-month sterling Libor and Eurodollar). The series begin in 1985 for S&P 500 and three-month eurodollar interest rate futures; 1987 for three-month sterling interest rate futures; and in 1992 for FTSE 100.

We found that markets reacted to, but did not predict, the major episodes of financial crisis since the mid-1980s. The implied summary statistics were found to be highly persistent suggesting the impact of shocks on market views does not die away quickly. A shock to market beliefs can be expected to persist for about 60 weeks for equity indices and 30 weeks for interest rates. Interestingly, there was little extra information to be gleaned from the implied pdf summary statistics, as opposed to non pdf based measures such as the 'at-the-money' implied volatility and 'risk reversal', about views of expected uncertainty and asymmetry. But this was not the case for measures of expectations of extreme movements in asset returns where the statistics from the implied pdfs differed from other standard market measures/indicators of expectations of extreme market moves.

Potential relations were investigated between the estimated summary statistics, both within and across asset classes, and between UK and US markets. Implied uncertainty about equity returns was found to significantly influence absolute equity returns and tends to lead perceptions about asymmetry and extreme equity index movements. In contrast, implied uncertainty for interest rates was found to both influence, and respond to, changes in interest rates. Internationally, expected uncertainty was found to be strongly correlated between the United Kingdom and the United States, for both equity and interest rate markets. Implied balances of risk about future US interest rates were found to influence those of UK interest rates. And uncertainty about US equity returns tended to influence implied views about balances of risk and expectations of extreme moves in UK equity returns.

Finally, we related the summary statistics to other financial and economic variables such as output, investment, inflation, aggregate equity market earnings, corporate spreads (an indicator of the prospects for corporate default) and the slope of the yield curve (an indicator for the

market outlook for economic activity and/or expectations of future inflation) . The slope of the yield curve had a causal effect on interest rate uncertainty, and, in the United States, corporate credit spreads tended to lead implied uncertainty about equity returns. There was no incremental predictive power in option-implied summary statistics for economic variables beyond that in past values of the macroeconomic variables themselves, and past returns on the underlying financial asset. However, the data sample we examined is relatively short, covering just one business cycle in the case of the United States. Similarly for the United Kingdom, data for FTSE 100 implied pdfs were only available from 1992. Ideally, a more complete assessment of the information content of options prices for future economic conditions would require a data sample covering a number of business cycles.

1 Introduction

Economic agents act on the basis of what they expect to happen in the future, so it is useful to have some indication of what these expectations are and how agents are expected to respond to various economic situations. However, beyond survey-based measures, the information available about expectations has been scant. Further, we have had even less information on how uncertain agents are about their expectations, or about the balance of risks that surround their expected outcomes.

Financial markets provide policymakers with some information about *aggregate* financial market expectations for a range of financial and economic variables. Option markets have the potential to inform our view further by allowing market participants to efficiently trade risk. In doing so, option prices reveal the likelihood that market participants attach to different future outcomes. Such information is implicit in the prices that they are willing to pay for securities whose pay-offs are dependent upon the realisation of these outcomes. The past decade or so has seen much attention focused on developing this idea. The focus of that work was on estimating and examining the distributions of the probabilities that, in the aggregate market view, are attached to different levels of financial variables occurring. The statistics that summarise these distributions can then be used to examine expectations and market views around these expectations.

Many central banks, in particular, use probability density functions (pdfs) that are implied by option prices to assess market participants’ expectations of various asset prices, like interest rates, foreign exchange rates, equity prices and commodities. A list of examples of central bank research and use of implied pdfs can be found in Bliss and Panigirtzoglou (2002).

A number of studies have used implied pdfs to examine how markets anticipated or reacted to major events. Leahy and Thomas (1996) studied the behaviour of the Canadian dollar before and after the Quebec sovereignty referendum using a mixture of both two and three lognormal distributions. They found that in the run-up to the vote uncertainty about the Canadian dollar increased significantly and in particular, on the day just before the referendum, the distribution was trimodal. They relate the three modes to three different outcomes: no surprise, victory and defeat of the sovereignty proposal.

Melick and Thomas (1997) also applied the mixture of three lognormals methodology to crude oil option prices during the Gulf crisis. Campa, Chang and Refalo (1998) studied the response of Brazilian real exchange rate expectations in the period 1994-97. Coutant, Jondeau and Rockinger (1999) found that participants in the options market for Pibor interest rate futures anticipated the French snap election of 1997 and continued to reflect substantially higher uncertainty a month after the election.

Gemmill and Saflekos (1999) studied the behaviour of FTSE 100 implied pdfs during market crashes and British general elections since 1987. They found little to suggest that these crashes were anticipated. Instead, the market reacted to these crashes with increased volatility and placed more weight on downward, relative to upward, movements in prices. They also tested the hypothesis that a bimodal distribution is appropriate during general elections. They found a bimodal pattern in the 1987 election but not in the 1992 or 1997 elections.

Other papers have studied the information contained in option prices in predicting future asset price movements and/or realised distributions. Christensen and Prabhala (1998) found that implied volatility predicts future volatility. Weinberg (2001) used daily data from 1988 to 1989 for S&P 500, US dollar/Japanese yen and US dollar/Deutschemark exchange rates to find that uncertainty, implied from a cross-section of option prices, predicts future realised uncertainty (without dominating implied volatility that is derived from just one option price). He also found that there is no predictive ability of the implied skew, but as the author points out this may be because of risk premia or measurement errors in the realised skew. Shiratsuka (2001) found that, in the case of the Nikkei 225 stock price index, implied pdfs are useful in forecasting the subsequent realised distribution for future index level fluctuations. However, when compared with an equivalent historical distribution, the distribution implied from option prices was not found to be superior in this regard. Also, in considering the information content of the Nikkei implied pdfs for future index level changes, the implied pdf summary statistics – implied volatility and skew – are found to lead, as well as respond to, index level changes. However, the paper questions the usefulness of this result for monetary policy makers by showing that these results are highly dependent on macroeconomic and financial conditions.

We make two contributions to the existing literature. First we provide a comprehensive examination of the properties of alternative measures of uncertainty, asymmetry[(1)] and extreme movement tendency[(2)] obtained from summary statistics for option-implied pdfs. We assess the degree of information, if any, that these measures can provide about aggregate market expectations of future economic and financial variables. Second, we look across many different assets over long sample periods (that span many different events). In addition we use data that are unique in that they deal with the time-to-maturity effect.

Our analysis of the properties of the series focuses on the behaviour over time of these measures for equity indices and interest rates in the United Kingdom and the United States. This provides us with average values over a large historical period, the uncertainty and the degree of mean reversion of beliefs about risk and the expected balances of risk for these assets. We also look at differences in the stability of the various measures, which have implications for their potential use. We then investigate the potential relations between the estimated summary statistics, both within and across asset classes. Further, we examine cross-country relationships between the summary statistics.

A number of channels have been posited whereby volatility may affect real economic variables such as output and investment. Fundamental models of equity valuation suggest that equity prices depend on expectations of future variables such as earnings or dividends. These variables are dependent on the economic growth/conditions that are expected to prevail and the extent to

[(1)] In this paper, by asymmetric market expectations, we mean differences in the probabilities attached to outcomes above and below the most likely outcome (mode).
[(2)] By extreme movement tendency we mean the probabilities attached to unusually large movements. This determines the 'fatness' of the tails of the distribution of market expectations.

which agents are uncertain about future economic conditions may be reflected in their expectations of uncertainty about future interest rates and equity returns. The theory of irreversible investment suggests that in the presence of greater uncertainty about the cost of finance, investment may be delayed. Further, high equity market volatility may deter investors from holding equities thus raising the cost of equity finance for firms in the short term and hence lowering investment. Alternatively, excessive volatility in equity markets may result in firms ignoring short-term equity market developments when considering long-term investment decisions. Under this scenario, expectations of future volatility in the short term would not have any effect on investment decisions. We seek to address these issues by formally considering the predictive power of the implied summary statistics for future macroeconomic variables. Finally we examine the relationship between expected uncertainty over future equity returns and aggregate measures of the spread between yields on corporate and risk-free debt. Corporate spreads have been used as an indicator of default risk/creditworthiness of the corporate sector. Higher uncertainty about the ability of firms to meet their debt obligations should reflect concern about the future solvency of the firm resulting in greater uncertainty about future equity returns.

Section 2 addresses the implied pdf estimation procedure and describes the input and output data for the process. Section 3 will then consider the historical behaviour of these implied statistics and pdfs. The relationships between the various types of summary statistics and between the summary statistics on different assets will be examined in Section 4. The information content of the summary statistics for future macroeconomic and financial outcomes will be assessed in Section 5. Section 6 concludes.

2 Estimation and data issues

The methodology for estimating the implied pdfs is based on a cubic smoothing spline interpolation of the implied volatility smile in delta space(3) described in Bliss and Panigirtzoglou (2002). After fitting(4) the smoothed implied volatility function, 20,000 points along the curve are converted into call prices (using the Black-Scholes formula). The second derivative of the call price function with respect to the strike price is then numerically estimated to produce the implied density function according to Breeden and Litzenberger (1978).

The option contracts used in this paper have fixed expiry dates, that is, the time to maturity changes with time. This is a problem for our study as the summary statistics extracted each day correspond to different horizons and so are not directly comparable. To deal with this problem, we constructed constant-horizon probability density functions as described in Clews, Panigirtzoglou and Proudman (2000). The technique involves the interpolation (using the smoothing cubic spline method) of implied volatilities of a specific delta across different

(3) The interpolation of the implied volatility smile in delta space has the advantage that far out-of-the-money options are grouped together in the tails allowing for more shape near the centre of the distribution where more trading occurs.
(4) The smoothing parameter controls the trade-off between smoothness and goodness-of-fit. After experimenting with different values we chose a value of 0.99 for all contracts used in this study.

horizons.[5] We can then construct the implied volatilities of an artificial contract with a specified maturity (for example, three months), the density of which is derived using the smoothed implied volatility methodology described in the previous paragraph.

Our input data set includes daily call and put option and futures prices on all traded (quarterly) contracts, between various start dates and April 2002, for the FTSE 100 index and S&P 500 futures options and three-month sterling (short sterling) and eurodollar (Libor) interest rate futures options. The FTSE 100 and short sterling options contracts are all traded on Euronext.liffe (LIFFE).[6] Daily settlement prices from LIFFE were obtained from 1987 for short sterling futures options and from 1992 for FTSE 100 index options. The associated value of the underlying was the futures price reported by LIFFE. Option contracts on eurodollar futures and the S&P 500 futures are traded on the Chicago Mercantile Exchange (CME). Daily settlement prices for these option contracts were obtained from 1983 for S&P 500 and from 1985 for eurodollar.[7] The associated values of the underlying were the settlement prices of the futures contracts maturing on or just after the option expiry dates. The risk-free rates used are the British Bankers' Association's 11am fixings for short sterling and eurodollar Libor rates reported by Bloomberg.

Only at and out-of-the-money call and put option prices were used because there is usually more trading in these, rather than, in-the-money options (see Bliss and Panigirtzoglou (2002)). Option prices that violated the monotonicity or convexity properties were discarded.[8][9] Option prices for which an implied volatility was impossible to compute or with deltas smaller than 0.01 or greater than 0.99 (far out-of-the-money options with usually little or no trading) were also discarded. A pdf was produced for a given cross-section of option prices only if there were at least three remaining strikes with a minimum delta of at most 0.25 and maximum delta of at least 0.75. This ensures that the remaining strikes span a sufficiently large area of the density function.

After the screening process described above, we fit the available implied volatilities of a given maturity cross-section in delta space with the smoothing spline method. However, it is necessary to extrapolate the spline beyond the range of available data. Since the spline method extrapolates linearly outside the available range (resulting sometimes in negative or implausible large implied volatilities) we force the spline to extrapolate horizontally. This is done by introducing three pseudo strikes above and below the available range with implied volatilities equal to that of the respective available extreme strikes.

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(5) The smoothing parameter for the interpolation of particular delta implied volatilities across different maturities was 0.9999, that is, there was almost a perfect fit of the implied volatilities in the maturity spectrum.

(6) FTSE 100 European-style contracts are used. Short sterling futures options contracts are American in nature but due to the premium-payment on exercise system for these contracts and the non dividend paying nature of the underlying asset, the short sterling contract trades as if it were a European-style option and we treat it as so. See Bahra (1997, pages 28-29), for further details.

(7) The CME contracts are American. We correct for the early exercise premium using the Barone-Adesi and Whaley (1987) approximation.

(8) Monotonicity requires that the call (put) prices are strictly decreasing (increasing) with respect to the exercise price.

(9) Convexity requires that a butterfly spread at a particular strike (formed by selling two call options at this strike and buying the two adjacent call options) is positive. In a continuum of strikes, this is equivalent to require that the call and put price functions are convex.

We apply this methodology to the cross-sections of all available maturities and we derive an implied volatility smile, as a function of delta, for each maturity. For each delta we use the same cubic smoothing spline method to interpolate these particular delta implied volatilities across all available maturities. From this interpolation we pick the value that corresponds to the desired constant horizon maturity[(10)] (ie three or six months). We repeat this for nine different deltas, from 0.1 to 0.9. The result is the implied volatility smile of an artificial contract with a specified maturity, the density of which is derived using the smoothed implied volatility methodology described in Bliss and Panigirtzoglou (2002).

The output from the estimation process consists of daily pdfs of logarithmic returns[(11)] for each of the four assets, over a three-month and six-month horizon. A number of summary statistics relating to market expectations of uncertainty, asymmetry and extreme movement (fatness of tails) were produced for each asset and maturity. These are shown in Table 1.

The derived pdfs are risk-neutral, that is, they represent the probabilities of a hypothetical investor who does not require any compensation for risk. As a result, the pdfs and their summary statistics are different from actual expectations of market participants. The difference between the two is ignored in this paper, as it is beyond its scope.

Table 1: Implied pdf summary statistics

Uncertainty measures	*Asymmetry measures*	*Extreme movement risk measures*
at-the-money (atm) volatility	skewness	kurtosis
standard deviation	(mean-mode)/standard deviation	standardised strangle
	(mean-median)/standard deviation	probability of extreme movement
	standardised risk reversal	
	probability asymmetry	

All of the summary statistic data used in this study are weekly averages. A full definition for each of the above statistics is provided below:

i. *atm volatility*: it is obtained from options contracts with a strike price coinciding with the underlying futures (at-the-money). It is related to the standard deviation of the logarithmic change of the underlying per unit of time.
ii. *standard deviation*: the square root of the variance of the implied pdf.
iii. *skewness*: the third central moment of the implied pdf standardised by the third power of the standard deviation. It provides a measure of asymmetry. It measures the relative probabilities (weighted by cubic distances) above and below the mean outcome. That the

(10) In the case that the desired maturity is not within the range of available maturities we have to extrapolate. No extrapolation is performed if the desired maturity is more than 1.25 months apart from the shortest or longest available maturity.
(11) Returns are logarithmic changes in the futures prices over the relevant horizon. For interest rates they are logarithmic changes in the implied interest rates calculated as 100 minus the futures price.

(cubic) distance from the central outcome (mean) weights these probabilities is of particular importance. The difference between the unweighted probabilities above and below the mean has the opposite sign to that of skewness. For example, a pdf with positive asymmetry has a mean that is above the median and the mode. But the median divides the density into two parts of equal 50% probability mass. Therefore, in this case, the unweighted probability above the mean is smaller than that below the mean.

iv. *(mean-mode)/standard deviation*: the difference between the mean and the mode of the implied pdf standardised by the standard deviation.

v. *(mean-median)/standard deviation*: the difference between the mean and the median of the implied pdf standardised by the standard deviation.

vi. *kurtosis*: the fourth central moment of the implied pdf standardised by the fourth power of the standard deviation. It provides a measure of the degree of 'fatness' of the tails of the implied pdf. Fatter tails in pdfs are usually associated with a greater degree of 'peakedness' in the centre of the pdf.

vii. *standardised risk reversal*: provides a further measure of asymmetry that is independent of the shape of the tails of the implied pdf. It is given by the difference between the 25-delta call and 75-delta call implied volatilities, divided by the atm (50-delta) volatility. It reflects the slope of the volatility smile. Since it is divided by the atm volatility it adjusts for changes in uncertainty. A lognormal pdf has a risk reversal equal to zero, that is, the risk reversal shows the asymmetry of the implied pdf in excess of the benchmark lognormal pdf.

viii. *probability asymmetry*: a measure of asymmetry that is independent of the shape of the tails of the implied pdf. It is obtained by taking the difference between the probabilities of an upward and downward movement of the logarithmic returns of the underlying asset in excess of one standard deviation, relative to the mode. Since the standard deviation is used, this asymmetry measure adjusts for changes in uncertainty.

ix. *standardised strangle*: a measure of the degree of 'fatness' of tails of the implied pdf that is independent of the shape of the tails of the implied pdf. It is obtained by taking the difference between the average of the 25 and 75-delta call implied volatilities and the atm volatility, divided by the atm volatility. It provides a measure of the degree of curvature of the volatility smile. Since it is divided by the atm volatility it adjusts for changes in uncertainty. A lognormal pdf has a strangle equal to zero, that is, the strangle shows the degree of 'fatness' of tails of the implied pdf in excess of the benchmark lognormal pdf.

x. *probability of extreme movements*: a measure of the degree of 'fatness' of tails and 'peakedness' of the implied distribution. It is obtained by taking the sum of two probabilities: the probability of the logarithmic change in the underlying asset price lying more than two standard deviations from the mode; and the probability of lying one half standard deviation either side of the mode.[12] As the standard deviation is used, this extreme movement measure adjusts for changes in uncertainty.

[12] We are grateful to Robert Tompkins for suggesting this measure.

3 Historical behaviour of implied pdf summary statistics

We begin by examining the historical properties of the implied pdf summary statistics. Initially we present the time-series plots of the series, focusing in particular on the behaviour of the series during periods of financial crisis and recession. The historical distributions of the summary statistics will then be considered followed by an analysis of the stationarity of the series. We consider the implied uncertainty, implied asymmetry and implied extreme movement tendency series in turn for all securities.

3.1 Implied measures of uncertainty

Implied volatilities can be used to provide an estimate of the degree of uncertainty in the market regarding expected future levels of equity indices and interest rates. In the subsections that follow we analyse the historical behaviour, distributions and time-series properties of the implied volatility series of the estimated implied pdfs of FTSE 100 and S&P 500 indices; and eurodollar and short sterling futures.

3.1.1 Historical behaviour of implied uncertainty

Time-series plots of the atm volatility and implied standard deviations for the equity index and interest rate pdfs are shown in Charts A1.2, A1.4, A1.6 and A1.8. The implied pdf standard deviations have been transformed[13] to make them comparable with the atm volatility.

Comparing the atm volatilities and standard deviations for the four assets, it is clear from these that, in each case, the two series are very closely related – with correlations of 0.99. Given that the estimation of the implied pdf and hence the implied standard deviation uses implied volatilities at a whole range of exercise prices, whereas the atm volatility is based on the implied volatility at a single strike, this result is surprising. It implies that there is little marginal information (ie beyond that in atm volatility) to be gained from the standard deviation of the implied pdfs about market uncertainty about future movements in equity indices and short-term interest rates. This is further confirmed in Table A1.2 which reports the differences between implied volatilities and standard deviations. Although the average differences are statistically significant, they are small. The equity index implied volatilities are, on average, lower than implied standard deviations by about 1%. For interest rates, the average differences are about 0.5%.

Turning to the behaviour of the implied volatilities over their respective sample periods, it is clear from Charts A1.1 and A1.7 that the 1987 crash had a huge influence on both expected equity and interest rate uncertainty in the United States. This was more pronounced for S&P 500 where implied volatility increased dramatically at the beginning of the crisis (to a level of more than 60%), reaching levels that have not, to date, been repeated. These above-average levels of uncertainty were sustained in both cases for at least three months following the crash. Uncertainty then remained relatively stable until the onset of the 1990 US recession.

[13] The variance of the implied pdf was transformed by taking the ratio of the standard deviation and the square root of the time to maturity, expressed as a fraction of a year.

The intensification of the Middle East crisis during the summer of 1990 saw the S&P 500 implied volatility increase during the second half of 1990, peaking at a level of 25%, much lower than the peak of the 1987 crash. Interest rate implied volatility, while itself volatile during this period, averaged around 15%, a relatively low level for both eurodollar and short sterling. The 1992 ERM crisis saw uncertainty in both interest rate markets increase to levels of about 35%, similar to the eurodollar implied volatility peak during the 1987 crash. This level represents a sample high for short sterling and this high degree of uncertainty continued into early 1993. The interest rate crisis also had an impact on the expected uncertainty of equity market returns in the United Kingdom with FTSE 100 implied volatility increasing from 12% in June 1992 to 20% by mid-September. The effect on S&P 500 uncertainty was however muted.

The mid-1990s represented a period of relative stability for asset market uncertainty. Although affected slightly by the Latin American (peso) crisis, the FTSE 100 and S&P 500 implied volatilities between 1993 and mid-1997 averaged around 14% and 13% respectively. This compares with their full sample averages of 18% and 17% respectively. Bond market turmoil saw UK, and US, interest rate uncertainty remain volatile during 1994 before embarking on a downward trend until the onset of the LTCM crisis in 1998.

The financial crises towards the end of the 1990s prompted a change in both the level and variation of market expected equity and, to a lesser extent, interest rate uncertainty. As expected, equity market uncertainty rose sharply during the Asian and Russian/LTCM crises, the later being more pronounced. In the case of the S&P 500, the levels of market expected uncertainty experienced during the two crises were similar to those experienced in the period immediately following the 1987 crash. The period since the Russian crisis has seen market uncertainty about future UK and US equity returns fluctuate considerably around average levels of about 20%. The turmoil surrounding the NASDAQ crash in March 2000 together with the worsening global economic environment since the end of 2000 sustained these levels into 2001 peaking again at a level of about 30% immediately after the 11 September attacks.

Market uncertainty surrounding expected future interest rates increased only slightly during the Asian crisis, temporarily arresting the downward trend that had generally been in place following the ERM crisis. The impact of the Russian crisis in 1998 was more pronounced, with a doubling of interest rate implied volatility. Given the responses of the respective monetary authorities to the Russian crisis, this was to be expected. Between late 1998 and late 2000, interest rate uncertainty had fallen. It moved sharply higher towards the end of 2000/beginning of 2001 reflecting expectations of the loosening of monetary policies in the United Kingdom and the United States as a response to the economic slowdown. They peaked sharply following the September 2001 terrorist attacks, at about 25% for short sterling and a sample high of 40% for eurodollar. This is because interest rates had fallen to 40-year lows in both the United Kingdom and the United States and, as a result, the proportional movements in interest rates were very high relative to the past.

3.1.2 Historical distributions of implied uncertainty

To consider the historical properties of the implied uncertainty measures, summary statistics for the implied volatility series themselves are presented in Table A1.1. The patterns of the implied volatilities of the two equity indices are very similar. Comparing the means and standard deviations of the FTSE 100 and S&P 500 implied volatility series, it is evident that the average level and volatility of market uncertainty in the two markets has not differed significantly. Both distributions exhibit positive asymmetry with skews greater than one. This suggests that market expectations of equity uncertainty in the past tended to be higher more often than lower, relative to that level of uncertainty observed most frequently. Both implied volatility series have highly kurtotic distributions (in excess of that of a normal distribution (3.0)) indicating that, historically, the market was prone to large changes in its view on the uncertainty of future equity returns. The excess skewness and kurtosis is further confirmed when the distribution of the S&P 500 implied volatility is examined. The skewness for the three-month S&P 500 implied volatility is double, and the kurtosis triple, that of the corresponding FTSE 100 statistics. This is because of the extreme values that the implied volatility displayed during the 1987 crash. It is interesting that if we restrict the S&P 500 sample to the same as the FTSE 100 (that is from 1992 to 2002) the skewness and kurtosis of the S&P 500 volatility become significantly lower than that of the FTSE 100. So it is the period before 1992 (which includes the 1987 crash) that mainly determines the properties of the historical distribution of S&P 500 implied volatility. Moreover, market participants were more likely to revise upwards or by a large amount the uncertainty about FTSE 100 returns rather S&P 500 returns for their common period. These findings have interesting implications for the debate on volatility in equity markets, suggesting that the manner in which changes to expectations occur may be a factor in generating high volatility in equity market expected returns and/or risk premia and, therefore, in equity prices.

The historical distributions of interest rate implied volatilities are also comparable. The means and standard deviations are slightly higher for eurodollar as a result of the extreme values of eurodollar implied volatility during September 2001. The six-month interest rate volatilities are also higher than those of the three-month horizon, reflecting a positive, on average, slope of the short interest rate volatility term structure. Both eurodollar and short sterling volatility distributions exhibit positive skews but less than that found for the corresponding distributions of expected equity return uncertainty. They are also highly kurtotic with short sterling displaying less kurtosis than eurodollar. This suggests that market uncertainty about future US interest rates in the past has been more likely to reach higher levels than uncertainty over future UK interest rates.

The fact that implied volatilities for both equity indices and interest rates display more skewness and kurtosis than that of the normal distribution can be possibly justified by their zero lower bound, that is, they may exhibit lognormality. We test for this in Table A1.3. For equity indices we reject the hypothesis that the historical distributions of implied volatility are lognormal. In particular, they are more positively skewed than the lognormal distribution. Looking at the period 1992-2002 for S&P 500, the distribution for implied volatility is closer to lognormal in terms of skewness but has a kurtosis significantly lower than that of a lognormal. This is in contrast to the whole sample distribution, which exhibits more skewness and kurtosis than a lognormal distribution.

For interest rates however, we fail to reject the hypothesis of lognormality. These results have implications for volatility models by providing support to lognormal volatility models for interest rates but not for equity indices.

3.1.3 Time-series properties of implied uncertainty

To consider the stability of alternative indicators of market expectations of uncertainty we consider some time-series properties of the implied volatility series. In particular we look at the autocorrelation functions for the series and conduct unit root tests to examine the stationarity of the series.

The autocorrelation functions for the FTSE 100, S&P 500, eurodollar and short sterling atm volatilities are shown in Table A1.4. These suggest that the implied volatility series are highly persistent with significant autocorrelations up to 30 weeks for interest rates and up to 60 weeks for equity indices. This means that shocks to equity index uncertainty can be more persistent than those to interest rate uncertainty.

The high first-order autocorrelations indicate that all four series are close to being non-stationary. Unit root tests for the daily series were carried out and the results from these tests are presented in Table A1.5. The unit root testing procedure involved applying Phillips-Perron[14] test to each of the daily series over their respective sample periods. Since the results were insensitive to the truncation lag, it was chosen to be 20 for all series to capture possibly long lag effects.

The results from these tests rejected the hypothesis of non-stationarity at 5% confidence level for all series apart from three-month FTSE 100 and six-month eurodollar volatility, which were rejected at a 10% level, and the six-month FTSE 100 volatility. Given the low power of stationarity tests, a 10%, rather than 5%, confidence level may be more appropriate. The failure to reject the hypothesis of non-stationarity for six-month FTSE 100 could be the result of the relatively short sample period for which the FTSE 100 implied volatility is available, 1992–2002. For example, the changes in implied volatility that occurred with the onset of the financial crises at the end of the 1990s have an important bearing on the time-series properties of the FTSE 100 implied volatility over this period. Indeed looking at the series in isolation one could suggest that this period marked a structural break on market expectations of uncertainty about future FTSE 100 returns. However an examination of the S&P 500 series which includes the effects of the 1987 crash, for example, allows us to put the events of the late 1990s in proper perspective. There we see that, in light of the movements during and after the crash in 1987, the change in implied volatility following the Asian and Russian crises was not that unusual and did not represent a structural break in the series. Furthermore if we restrict the sample period for the S&P 500 to the FTSE 100 sample period, we also fail to reject the null hypothesis of non-stationarity for the S&P 500 implied volatility. These observations, together with a high

[14] See Phillips and Perron (1988).

degree of correlation between the FTSE 100 and S&P 500 implied volatility series, suggest that the finding of non-stationarity for the FTSE 100 implied volatility series is likely to reflect a short sample period.

The time-series analysis of the implied volatility series thus presents an interesting picture of the behaviour of market expectations of uncertainty about future equity returns and interest rate levels. It appears that these expectations have very long memories, with shocks impacting on financial markets continuing to have a significant influence on the series up to several months later. However, on a longer-term basis these shocks do eventually die out, as market expectations of future uncertainty tend to revert to their long-term averages.

3.2 Implied measures of asymmetry

We now turn our attention to the asymmetry of market expectations of future changes in equity indices and interest rates. That is, we examine how market perceptions of the balance of risk around central expectations have behaved over time. As previously, we begin our analysis with a look at the time-series plots of these measures, followed by an examination of their historical distributions and time-series properties.

3.2.1 Historical behaviour of implied asymmetry

We focus our attention on the skewness statistic of each of the pdfs, which are shown in Charts A2.1-A2.4. The decision to examine just the skewness of the implied pdfs in considering the asymmetry of market expectations is justified by an examination of the correlations between the series, and the relative stability of the series. Tables A2.2-A2.5 show the correlations between the various measures of asymmetry and it is clear that, for each set of implied pdfs, the asymmetry measures are very similar in terms of movement and trend.

We also look at the stability of the series. Table A2.6 shows the stability ratios for each of the five asymmetry measures, as measured by the ratio of the average absolute weekly change in the statistic to the average absolute level of the statistic. The resulting ratios show that the skewness of the implied pdf together with the risk reversals have been the more stable statistics for both equity indices and interest rates.

One noticeable feature from these correlations is the near-perfect correlation between the implied skew and risk-reversal measures. This result is analogous to the finding earlier for the atm-implied volatility and standard deviation of the implied equity pdfs. In this case the risk reversal is based on the implied volatilities at just two exercise prices – those corresponding to 25 and 75 delta. The implied skew measure however incorporates information at all available strikes so that once again it seems that the gain from estimating an implied pdf to examine the degree of asymmetry in market expectations is minimal relative to the information contained in just two points.

Turning to the behaviour of the asymmetry series we see that the 1987 crash had an immense effect on the market perception of the risks surrounding future expected equity returns. Having averaged close to zero in the 1985 to mid-1987 period, the turmoil of the 1987 crash resulted in a sudden drop in skewness. This jump clearly marks a structural break in the series. During the fourteen years since then, the implied pdfs have remained consistently negatively skewed. That the market, since 1987, never seems to have attributed more probability to the upside risk relative to the downside risks is perhaps more surprising, especially in light of the magnitude of the gains experienced in equity markets in the latter half of the 1990s.

There are a number of potential explanations for the consistently negative skewness since the 1987 crash. It can be considered as an insurance premium that investors are prepared to pay to protect against large equity market declines. It can also be attributed to liquidity effects or irrationality among investors. However, the latter factors are less convincing given that the negative skewness has persisted for a long time.[15]

The effect of the 1987 crash on eurodollar skewness was small with the balance of risks to interest rate changes remaining almost symmetric.

Events between the end of 1989 and mid-1991 also had a marked effect on the S&P 500 implied skewness series. The mini-crash of October 1989 produced another sharp downward jump in skewness. This drop was sustained into the first quarter of 1990, with the emergence of uncertainty about the macroeconomic prospects of most industrialised countries. Implied skewness stayed low for the rest of the year. The outbreak of hostilities in the Middle East together with the end of the recession during the first quarter of 1991 then saw implied skewness rise back quickly, similar to the movements of expected uncertainty experienced at the time.

The interest rate skew turned negative in mid-1990 during the UK and US recessions. In the United States, the negative skew, evident at the three-month horizon only, was short-lived however. In contrast, the negative interest rate skew in the United Kingdom was more pronounced (especially at the three-month horizon) and persistent. That is, on remaining negative throughout 1991, short sterling skew turned slightly positive for a short time in 1992, around the ERM crisis, before subsequently returning to negative levels (consistent with the post-ERM loosening of monetary policy). The ERM crisis had little effect on eurodollar skewness. The bond market turmoil of 1993-94 and the perceived high likelihood of large increases in US interest rates saw eurodollar skewness peaking at a high of 0.93. Short sterling skewness also peaked at a level of 0.78.

Internationally, the implied equity index skewness measures behaved differently during the mid-1990s, showing a weak correlation across markets and responding in differing degrees to events such as the bond market turmoil and Latin American crises of 1993/95. This changed in early 1997, with persistently less negative skews for both markets accompanying significant increases in equity prices at the time. The effects of the Asian and Russian crises reversed this move as market participants revised down their assessment of the risk of increases, relative to,

[15] In relating subjective and risk-neutral implied pdfs, Jackwerth (2000) finds that the negative skew of equity index pdfs leads to unorthodox (negative at some levels of wealth) estimates of risk aversion. But this conclusion is heavily dependent on the assumptions that he makes about the shape of the unobservable subjective pdf. Other studies like those of Ait-Sahalia and Lo (2000) and Bliss and Panigirtzoglou (2003) reach different conclusions about risk preferences by using different assumptions.

falls in the two equity indices. Of the two crises, the Russian crisis had the more pronounced negative effect on interest rate skewness, with markets anticipating the large falls in interest rates that followed the crisis. Eurodollar interest rate skew reached a sample low of –1.3 around that time (August 1998).

The increased comovement in international equity index skews that began in early 1997 has, to a good extent, continued to the end of our sample (2002), possibly reflecting the international nature of the shocks during the post-1996 period. Having remained very negative for nearly two years after the 1997 crisis, the implied equity index skews both became less negative during the second half of 1999 and into 2000. Uncertainty regarding global economic prospects may have resulted in a slight reversal of these movements during August 2000 as financial markets at that time considered the prospect of a slowdown in US economic growth. Despite falls in equity prices and negative corporate earnings news during 2001, changes in the equity asymmetry measures during 2001 were noticeably smaller than those of the S&P 500 during the 1990 US recession for example. Further, the 11 September attack had a small negative effect on asymmetry measures (relative to 1997 Asian crisis for example). One reason for this may be the seemingly prevalent market view at the time that (i) the US slowdown would be short-lived and (ii) given the already large falls in equity prices, the likelihood of further falls had diminished.

The 2000-01 economic slowdown had a very significant effect on interest rate skewness. Short sterling and eurdollar skews decreased substantially – turning negative – up to the end of 2000 in response to an expected economic slowdown and monetary policy loosening. They then rose rapidly – turning neutral/slightly positive again – during 2001 and 2002 as historically low interest rates may have led to changes in the perceived risk of further downward, relative to upward, moves. The 11 September attack led to only a small decline in skew, in line with what happened to equity index asymmetry.

3.2.2 Historical distributions of implied asymmetry

Looking at the distribution for the time series of the implied skews themselves, Table A2.1 confirms a negative average asymmetry for the implied equity pdfs. The full-sample S&P 500 implied asymmetry has a less negative mean and is more positively skewed and kurtotic than the FTSE 100 equivalent. This appears to suggest that market views of asymmetry (or balances of risk) for US equities are less negative than those for FTSE 100, on average, and subject to bigger fluctuations. However this reflects the effect of the 1987 crash which is not included in the 1992-2002 FTSE 100 sample period: the properties of the S&P 500 and FTSE 100 implied asymmetries are comparable in their common sample. Although the historical distributions of implied asymmetry display skews of different sign, these are relatively close to zero. The difference in the degree of kurtosis of the historical distributions is more marked, with the FTSE 100 implied asymmetry being less kurtotic. This means that the FTSE 100 implied asymmetry has been less prone to large movements relative to that of the S&P 500.

Interest rate asymmetries have been close to zero on average. Eurodollar asymmetry was more prone to large movements than short sterling.

3.2.3 *Time-series properties of implied asymmetry*

The autocorrelation patterns of implied asymmetry (measured by skewness) are similar to those for implied volatility. The equity index skew has been more persistent than that of interest rates. There are significant autocorrelations of about 0.4-0.5 for up to 60 lags in equity index skews. The autocorrelations of interest rate skews die out more quickly reaching the 0.3-0.4 level in 30 lags. Shocks to the series in the past continue to have a significant positive influence on the series up to six months later. Thus market expectations regarding the relative likelihood of up/downward movements have quite a persistent memory.

We easily reject the hypothesis of non-stationarity for most assets, though with the exception of the FTSE 100. As in the case of implied volatility, failure to reject the non-stationarity hypothesis for FTSE 100 skew could be due to the shorter sample available for FTSE 100 (see S&P 500 skew unit root test in Table A2.8 for the period 1992-2002).

3.3 *Implied measures of extreme movements or 'fatness of tails'*

The third set of summary statistics for the implied pdfs that we consider provides indications of market expectations of substantial movements in the underlying asset price. We are interested in the concentration of probability or the degree of 'fatness' in the tails of the implied pdfs. A large degree of probability in the tails of an implied pdf (ie a fatter-tailed pdf, relative to a Normal pdf) indicates that the market attaches significant probability to the chance that the underlying asset price may experience a relatively large change over the time horizon of the implied pdf.

3.3.1 *Historical behaviour of implied extreme movement measures*

The correlations of the three extreme movement measures are shown in Tables A3.2-A3.5. These show the kurtosis and probability of extreme movement measures to be very highly correlated for all assets. But they are much less correlated with the standardised strangle. The strangle is based on the implied volatilities at just three exercise prices (those at 50, 75 and 25 delta) as opposed to the other fatness of tails measures, which employ implied volatilities at all available exercise prices. Notwithstanding the greater degree of noise in the standardised strangle, the fact that the strangle is not highly related to either of the other two measures suggests there is more information in the two pdf-based measures of fatness of tails. Moreover, the low correlation between the strangle and the pdf-based measures suggests that the fatness of tails can be affected by deep out-of-the-money strikes. This finding contrasts with what we found for pdf and non pdf based measures of uncertainty and asymmetry. Given the limited reliability and availability of these strikes, fatness of tails measures are potentially less reliable than implied uncertainty or asymmetry measures.

A stability measure for each of the series is presented in Table A3.6. These show that the standardised strangle exhibits a lot more noise than the other two measures. This is consistent across all maturities and assets. We choose to focus on the implied kurtosis for the reminder of this study.

Once again the 1987 crash had a fundamental impact on market expectations about extreme movements in equity prices. During the period between 1985 and the 1987 crash, the implied kurtosis statistic averaged about 3.0 (similar to that of a normal distribution) and was remarkably stable. Following the equity market declines in 1987, the market seems to have markedly reassessed the probability attached to large equity price movements in subsequent years. That is, a structural break in the kurtosis measure occurred at the time of the crash: despite an unusually sharp fall initially, there was an upward jump in the level of the statistic together with a noticeable increase in the volatility of the series.

Market expectations of extreme movements in equity prices became more volatile in response to the events of the 1989-91 period. The mini-crash of October 1989 produced a jump in the level of kurtosis. Averaging around this level for a number of months subsequently, kurtosis then continued to move higher, exceeding 5.0 (a sample high for the S&P 500) by the end of the first quarter in 1990 before dropping back. These movements probably reflected a heightening concern at the time regarding the outlook for economic activity and inflation and their implications for equity prices, following two relatively strong years of macroeconomic and asset price performance after 1987.

Several events during the mid-1990s influenced market expectations of large movements for both the FTSE 100 and S&P 500. The 1994 bond market turmoil, the Asia crisis of 1997 and the Russian crisis of 1998 caused the most significant impacts. Kurtosis fell significantly during 1999 and fluctuated around a level of 3.5 for 2000 and 2001. The 2001/2002 economic slowdown and the 11 September attack had little effect on kurtosis.

The kurtosis of implied interest rate density functions also spiked during the episodes mentioned above. For short sterling, the ERM crisis had a more significant impact on kurtosis than for eurodollar, reflecting the large response of UK interest rates to the crisis. Short sterling kurtosis reached its sample high of just over 5.0 during late 1996, possibly reflecting a growing expectation of a tightening of monetary policy in the United Kingdom. The bond market turmoil of 1993-94 had a more significant impact on eurodollar kurtosis compared to short sterling, and the eurodollar kurtosis also peaked at its highest level (about 5.0) at that time.

3.3.2 Historical distributions of implied extreme movement measures

The historical distributions of the implied kurtosis statistic are summarised in Table A3.1. The historical distribution of the FTSE 100 and S&P 500 implied kurtosis series are similar in all respects for their common sample, 1992-2002.

The main difference between the historical distributions of the S&P 500 kurtosis measure in the whole sample and the 1992-2002 subsample is the higher skewness for the subsample, due to the period before the 1987 crash.

Eurodollar and short sterling implied kurtosis also had comparable distributions, both with large positive skew and a high degree of kurtosis. This suggests that perceptions of the risk of extreme interest rate changes have tended to be revised up more often than down (relative to the sample modal perception). Further, these revisions have tended to be greater than those observed for the equity indices.

3.3.3 Time-series properties of implied extreme movement measures

Once again we begin our time-series analysis with a look at the autocorrelation functions of the implied kurtosis series. These are presented in Table A3.7 and show that the series have exhibited different patterns. Equity indices have had more persistent kurtosis than interest rates, as with the uncertainty and asymmetry measures.

The results from the stationarity tests show that the assumption of non-stationarity is generally rejected for the implied kurtosis statistics. The FTSE 100 case is again an exception, most likely due to the smaller sample size available.

4 Analysis of implied pdf summary statistics relationships

This section considers potential relationships between the various implied pdf summary statistics. We examine issues such as whether movements in implied volatility, asymmetry and extreme movement measures are related; whether changes in one element of market expectations lag or lead or necessarily imply a change in another. Further, we are interested in the extent to which measures of market expectations might possess information with regard to future changes in the underlying asset prices or level. Having already discussed some episodes during which market expectations reacted strongly to financial events/crises what, on average, can we say about this type of behaviour? In the following subsections we initially focus on equity and interest rate implied pdfs separately while also investigating cross-country relationships within each asset class. We then complete the section with an assessment of the links, if any, domestically and internationally, between equity and interest rate expectations.

To examine these issues we employ a framework that focuses on contemporaneous correlations followed by Granger causality tests. The causality tests are used to formally assess whether changes in one variable lag/lead changes in another. To implement these tests we estimate bivariate vector autoregressions (VARs) for a specified pair of variables and then test for zero restrictions on the relevant coefficients in each equation. In the absence of clear guidelines regarding the choice of the lag order for VARs in the literature, the choice of lag for each VAR was first made using a pre-test based on the system Akaike Information Criterion (AIC) and Schwartz Criterion (SC) and then followed by an LM test for serial correlation. In circumstances where the order of the VAR is different to that suggested by the AIC and SC (or indeed where these statistics differ), the number of lags was augmented to ensure white-noise residuals.(16) The analysis is based on average weekly observations from the estimated series and the results of the tests are summarised in tables in the appendix.

(16) The VAR models were estimated with OLS, that is, the coefficient standard errors were not adjusted for autocorrelation. We check the significance of the VAR causality results by also conducting bivariate tests with Newey-West autocorrelation-robust standard errors.

4.1 Interest rate implied summary statistics interactions and relations with underlying levels

We begin by comparing the relationships of eurodollar and short sterling implied summary statistics and underlying level changes individually. Table A4.1 shows the correlations between interest rate level changes, implied volatility, skewness and kurtosis for three and six-month eurodollar and short sterling. Two strong associations emerge from this table. The first is a strong positive correlation between implied volatility and the absolute value of level changes in underlying interest rates. This correlation is consistent for the two interest rates at both maturities. In contrast, the association between interest rate implied volatilities and raw interest rate level changes is relatively weak. This suggests that the size of interest rate level changes, rather than the direction, is more important from a volatility perspective. No further significant associations were found between the two measures of interest rate level changes and either skewness or kurtosis. The second association of interest is a strong positive correlation between skewness and kurtosis. Again this is consistent between eurodollar and short sterling at both three and six-month horizons. This is the only strong contemporaneous relationship over the respective sample periods between the summary statistics for interest rate implied probability distributions. A weak and variable-across-horizon contemporaneous association exists between eurodollar skew and implied volatility but not for short sterling. The remaining correlations between the other summary statistics are insignificant.

To assess the dynamic interactions between the implied summary statistics themselves and their relationship with underlying interest rate changes we estimated vector autoregressions in implied volatility, skew, kurtosis and interest rate level changes for short sterling and eurodollar individually. Table A4.3 reports the probability values from Granger causality test results for each bivariate relationship. These show that the dynamic relationships between the summary statistics themselves and between the summary statistics and underlying rate changes are generally consistent with the contemporaneous associations identified above. Both implied volatility and the absolute value of underlying interest rate changes were found to significantly lag and lead each other, each at the 1% level of significance. This two-way relationship, together with the strong contemporaneous association, suggests that while implied volatility reacts to past and current developments in underlying interest rates, it also possesses some predictive power for future interest rate changes. This interaction between implied volatility and absolute underlying interest rate changes is similar for both short sterling and eurodollar and for each horizon.

In addition, the causality tests suggest that there is a significant dynamic relationship between skewness and kurtosis. However, in contrast to the implied volatility/underlying level relationship, this relationship is less consistent in that it differs between short sterling and eurodollar in terms of the direction of impact and also differs across maturity in terms of significance. Short sterling skew significantly leads kurtosis whereas for eurodollar it is kurtosis that appears to lead skewness. Further, the causality tests suggest that only the eurodollar six-month skew/kurtosis relationship is significant.

The weak contemporaneous association between eurodollar skew and implied volatility noted above is not significant in a dynamic context. However, the Granger causality tests suggest that a dynamic relationship does exist between short sterling implied volatility and skewness. That is, implied volatility significantly leads skewness at a 5% and 10% level of significance for the three and six-month horizons respectively.

The extent to which the summary statistics for each of the short-term interest rate implied pdfs are related internationally is also of interest. That is, we want to examine the importance of market expectations of risks to future interest rate levels in overseas markets for equivalent domestic market expectations. Contemporaneous correlations between the three-month eurodollar and short sterling implied statistics, shown in Table A4.2, show a strong positive association between short sterling and eurodollar implied volatilities. Further, both implied volatilities are weakly correlated with absolute interest rate level changes overseas. The two implied skews are also weakly positively correlated internationally. The dynamic significance of these relationships is assessed using Granger causality tests based on a single VAR in all of the eurodollar and short sterling three-month implied summary statistics and absolute level changes. Table A4.4 shows the probability values from these tests. Encouragingly, the results outlined above, where we considered the short sterling and eurodollar relationships/interactions separately, are again confirmed by this joint interest rate analysis. In addition to these we find both implied volatilities to be very strongly related dynamically. Interestingly, there appears to be a feedback between the two: both eurodollar and short sterling implied volatilities lag/lead each other at 1% levels of significance. Further, the influence of overseas interest rate level developments seems significant with eurodollar absolute level changes leading short sterling implied volatility, again at a 1% level of significance. The causality tests also imply that the implied skews for eurodollar and short sterling are dynamically related. However, while eurodollar skew significantly leads its short sterling counterpart at almost the 1% level of significance, the reverse relationship is only marginally significant at a 10% level of significance. This suggests that directional risks for interest rates globally is relatively more important for short sterling than for eurodollar.

This analysis has highlighted many important features in the relations between the different elements of market expectations that we infer from interest rate implied pdfs. It suggests that uncertainty about future interest rate levels, while reacting to current developments in underlying markets, also contains information about future changes in interest rate levels. However uncertainty appears to reflect developments in, and the perceived risks for, international interest rate markets as well as domestic market factors. Other relationships of importance are those between market expectations of the balance of risk and expectations of extreme level movements on the one hand; and that between uncertainty and future balances of risk expectations, for short sterling, on the other. However these relationships vary in terms of direction and significance. These results suggest that implied probability distributions for short-term interest rates are informative for policymakers about future developments in interest rates. And they emphasise the importance of considering all aspects of market expectations simultaneously – both overseas and domestically – in assessing perceived developments in market expectations about future interest risks and levels.

4.2 *Equity index implied summary statistics interactions and relations with underlying levels*

The correlations between S&P 500 and FTSE 100 implied summary statistics and underlying level changes are shown in Table A4.5. As for interest rates, there is a high positive correlation between implied volatility and the absolute value of the (logarithmic) changes in underlying equity indices. This correlation is consistent across both equity indices and maturities. In contrast, the association between equity implied volatilities and level changes is relatively weak. This suggests that the size of equity index returns, rather than the direction, is more important from a volatility perspective. No further significant associations were found between the two measures of equity index returns and either skewness or kurtosis. The second association of interest from Table A4.5 is a strong negative correlation between skewness and kurtosis. Again this is consistent between S&P 500 and FTSE 100 at both three and six-month horizons. This is in contrast to the strong positive relation between interest rate skewness and kurtosis. The other strong contemporaneous relationship is the negative correlation between FTSE 100 volatility and skewness.

To assess the dynamic interactions between the implied summary statistics themselves and their relationship with underlying equity returns, we estimated vector autoregressions in implied volatility, skew, kurtosis and absolute equity index level changes. Table A4.7 reports the probability values from Granger causality test results for each bivariate relationship. Equity index implied volatility was found to significantly lead equity returns at a 1% level of significance for both indices and maturities. There is thus a one-way relationship between volatility and absolute returns as opposed to the two-way relationship apparent for interest rates. This suggests that volatility possess some predictive power for future equity returns, with perhaps the equity risk premium being the likely channel for this strong one-way causality.

In addition, the causality tests suggest that there is a significant one-way causality from volatility to skewness and kurtosis. Again, this is common to both indices and is consistent across maturity. This result implies that equity uncertainty causes revisions to market participants' perceptions about asymmetry and extreme equity index movements and that there is no feedback from higher order moments to the second moment of equity return implied distributions.

A significant two-way dynamic relationship between skew and kurtosis was found for the S&P 500. This two-way relationship, together with the strong contemporaneous association, implies that while the fatness of tails of implied equity return distributions can be revised based on current and past values of expected asymmetry, it can also influence the balance of risks that people attach to future equity movements. The fact that this relationship was not significant for the FTSE 100 may be related to the different sample used.

We also examine the importance of market expectations of risks to future equity index levels in overseas markets for equivalent domestic market expectations. Table A4.6 shows a strong contemporaneous positive correlation between S&P 500 and FTSE 100 implied volatilities, in line with the strong correlation between eurodollar and short sterling implied volatilities. Further, the returns of the two equity indices are strongly correlated confirming strong linkages between the two markets. There are also strong associations between S&P 500 volatility and FTSE 100 kurtosis and skew, with uncertainty about S&P 500 returns negatively correlated with FTSE 100 skewness and positively correlated with FTSE 100 kurtosis.

The dynamic significance of these relationships is assessed using Granger causality tests based on a single VAR in all of the S&P 500 and FTSE 100 implied summary statistics and absolute returns. Table A4.8 shows the probability values from these tests. There is a significant relationship between the implied volatilities of the two indices. Interestingly, there appears to be feedback between the two. The causality tests also imply that there was a significant one-way causality from S&P 500 implied volatility to FTSE 100 skew and kurtosis. This one-way relationship, together with the strong contemporaneous association, suggests that higher than second order moments of FTSE 100 implied distributions react to current and past changes in S&P 500 uncertainty.

This analysis has highlighted many important features in the relations between the different elements of market expectations that we infer from equity index implied pdfs. They suggest that uncertainty about future equity returns contains some information about future changes in equity indices as well as higher order moments (skew and kurtosis) of future returns distributions. Implied asymmetry and fatness of tails seem to affect each other through a two-way dynamic relationship. Uncertainties regarding returns for the two equity markets are strongly related, both contemporaneously and dynamically, suggesting that there are strong linkages between the two markets between second order, as well as first order, moments. Finally the significance of S&P 500 uncertainty in affecting asymmetry and fatness of tails of future FTSE 100 return distributions shows the importance of US market developments for the balance of risks and risk of extreme movements of equity markets in the United Kingdom.

5 The information content of implied pdfs for future macroeconomic and financial variables

We might expect these summary statistics to possess some information for the future levels of economic variables. The previous section considered the relationship between the implied pdf summary statistics and the changes in financial asset prices underlying them. This section looks at the information content of the summary statistics for macroeconomic variables. That is, we investigate the links between expected interest rate and equity uncertainty and future economic conditions. Our analysis focuses on variables that proxy for future output growth, investment growth and inflation. We also relate the statistics to other financial variables such as corporate spreads and real earnings growth.

Table A5.1 presents the results from regressions undertaken to assess the information content of the equity and interest rate implied volatilities with respect to output, inflation, investment and equity market earnings. The equations estimated are regressions of twelve month ahead growth rates for the specified economic variable on the previous non-overlapping twelve-month growth rate and twelve-month lagged average implied volatility and returns (logarithmic level changes) on the underlying futures price. The lagged underlying returns were included following the results from the previous section, which highlighted the influence that returns have on the summary statistics. This allows us to assess the usefulness of the summary statistics in predicting future economic conditions in excess of movements in underlying equity prices or interest rates. Monthly industrial production figures for the United Kingdom and the United States are used to proxy output. Inflation is given by the twelve-month growth rate in the retail prices index for the United Kingdom and consumer price index for the United States. A monthly investment series was constructed from real gross fixed capital formation figures for the United Kingdom and the United States. Equity market earnings were estimated by taking the product of the level of the Datastream 'US Total Market Index' with the inverse of its price earnings ratio. All growth rates were calculated by taking the change in the logarithm of each series.

Overall the regressions provide little support for the hypothesis that implied volatility adds marginal information for future macroeconomic outcomes in addition to past values of the macroeconomic variables themselves and past returns on the underlying asset. The coefficients on S&P 500 atm volatility in each of the regressions are negative suggesting that higher volatility may be an indicator of lower future industrial production, investment and real earnings. However, none of the atm volatility coefficients are significantly different from zero at 10% levels of significance. Past growth rates for each of the macroeconomic variables were all highly significant and past returns on the S&P 500 index had significant predictive power for future US industrial production growth.

The results for interest rate implied volatilities also suggest that these measures have little information content for future macroeconomic variables. Once again the signs of the coefficients on eurodollar and short sterling atm volatilities are as expected – higher volatility/uncertainty indicating lower future industrial production and investment and a greater change in future inflation. Tests of significance using Newey-West corrected t-statistics fail to reject the hypotheses that the coefficients on the atm volatilities are equal to zero at 10% levels of significance in all of the regressions.

For completeness, we now consider the information content of the other measures of expectations that we obtain from implied pdfs – implied asymmetry and kurtosis. We limit our analysis to the S&P 500 implied statistics in this respect as again we consider the FTSE 100 sample to be too short. Similar regressions to those for the implied volatilities were carried out at the same frequency and the results are given in Table A5.2. The sample for these regressions is again restricted to 1988-2002. These average measures appear to have little predictive power for future output, investment or real earnings growth. Thus the third and fourth order moments of the implied distribution do not seem to possess any information relevant to the outlook for future economic conditions for policymakers.

Next we consider the relationships between the implied volatilities and credit spreads and the slope of the yield curve. Credit spreads for the United Kingdom and the United States are constructed by taking the difference between the yield to maturity on Merrill Lynch AA, A and BBB (7-10 years' maturity) corporate bond indices for the United States and the United

Kingdom, respectively, over the corresponding ten-year swap rate.(17) The spread of long over short-term default-free rates, or the yield curve slope, provides an indication of the market's outlook for economic activity and/or expectations of future inflation. We consider the absolute value of the yield curve slope, which provides an indication of the market outlook for changes in the economic environment. One would expect that increased (reduced) uncertainty regarding the future economic environment would be associated with higher (lower) uncertainty over future interest rates. The yield curve slope is estimated by taking the difference between five-year interest rate swaps and three-month eurodollar and sterling Libor rates. Our analysis differs to that for the other macroeconomic variables in that these variables are more forward looking and are available at similar frequency to the implied volatilities. Thus we use contemporaneous correlations followed up with causality tests in assessing these relationships. The causality tests are conducted by estimating a three variable vector autoregression between the relevant yield curve slope, implied volatility and return on the underlying futures asset.

The implied volatility/yield curve slope correlations are strong and positive: 0.53 for the eurodollar/US slope and the 0.36 for short sterling/UK slope. Statistical tests to formally assess the significance of the relationships (Table A5.3) show that lagged absolute values of the yield curve slope possess some information for future interest rate uncertainty in excess of that present in lagged values of uncertainty itself and futures returns for both the United Kingdom and the United States. The slope variable also appears to be significantly related to future returns on the underlying asset while the relationships between the implied volatilities and the underlying returns are similar to those set out previously.

The correlations between credit spreads and implied volatilities are shown in Table A5.4. With the exception of the spread on US AA-rated debt, the US spreads are reasonably highly correlated with the S&P 500 implied volatility. The correlations for the United Kingdom are actually negative between the volatility and spread variables. Such a counterintuitive association may be in part due to the composition of the UK indices, which include both domestic and overseas issuers in sterling.

The relationship between uncertainty about future equity returns in the United States and expectations of future corporate credit worth is significant (Table A5.5). That is, lagged values of US BBB and A-rated corporate spreads were found to possess some predictive power for future levels of S&P 500 atm volatility. The hypothesis that the relationship between corporate spreads in the United Kingdom and FTSE 100 atm volatility is statistically significant can be rejected. Thus, insofar as US credit spreads reflect expected future corporate default rates, changes in market expectations of future equity return uncertainty appear to follow changes to market sentiment over the expected likelihood of future corporate defaults. This finding is similar to those above with implied volatilities appearing to react to, as opposed to forecast, innovations in financial variables.

(17) The Merrill Lynch credit spread indices are affected by compositional/basket changes (due to up/downgrades for example). This is less of an issue for US indices as the number of issues is much larger.

6 Conclusions

This paper focuses on implied pdfs obtained from options on S&P 500 index futures, FTSE 100 index and eurodollar and short sterling futures and their properties. Its contribution lies in a comprehensive analysis of the properties of alternative measures of uncertainty, asymmetry and extreme movement tendency from option-implied pdfs. In doing so we consider the potential of these measures to inform policymakers regarding future economic and financial outcomes. The data used span relatively long sample periods and are adjusted to deal with the time-to-maturity effect.

The time-series analyses provided a number of interesting results. The fundamental impact of the crash of October 1987 was clear with equity return uncertainty reaching record levels and the crash producing one-time jumps in the levels of expected asymmetry and in the tendency for extreme market movements. Eurodollar uncertainty peaked at a sample high in September 2001 reflecting the uncertainty about future interest rate movements in the United States and record low interest rate levels at that time. Short sterling volatility reached its maximum during the 1992 ERM crisis. The effect of the 11 September attacks on equity risks was modest in comparison to the October 1987 crash or the Russian 1998 crisis.

Most interest rate and equity implied pdf summary statistics were found to be stationary although highly persistent. This suggests that, while slow to adjust to shocks in the short-term market, expectations of future uncertainty, asymmetry and extreme movements revert to their long-run averages outside of very exceptional circumstances. A shock to market beliefs can be expected to persist for a long period of about 60 weeks for equity indices and 30 weeks for interest rates. This persistence is for example evident during the financial crises of 1987, 1992 (ERM), 1997 (Asia), 1998 (LTCM) where uncertainty, negative asymmetry and fatness of tails jumped up and these higher levels lasted for several months.

Comparing implied pdf with non pdf based measures of expected future uncertainty, asymmetry and extreme movements, we found that with the exception of the extreme movement measures, there was little extra information to be gained from pdf-based measures over their non pdf based counterparts. This result for implied uncertainty is consistent with that of Weinberg (2001). Concerning the stability of the different summary statistics, the risk reversal and skewness were the most stable among the asymmetry measures while kurtosis and probability of extreme movements were the most stable fatness of tails measures.

Turning to the historical distributions of the implied summary statistics themselves, implied volatilities are positively skewed and highly kurtotic. This means that market participants are more likely to revise uncertainty upwards rather than downwards and there is a high probability, relative to a normal distribution, that revisions will be large. For the same sample, the FTSE 100 implied volatility distribution is more skewed and kurtotic than S&P 500. A lognormal model for implied volatility is more appropriate for interest rates than it is for equity indices.

Full sample asymmetry measures for S&P 500 were highly positively skewed and kurtotic. However, when restricted to the 1992-2002 sample, both FTSE 100 and S&P 500 asymmetry measure distributions were symmetric with close-to-normal fatness of tails. Finally expected extreme movement measures were very highly positively skewed and kurtotic for interest rates but less so for equity indices. Turning to interest rates, balances of risk (skews) were almost symmetric on average.

Implied volatilities were contemporaneously positively related to the absolute changes of interest rates or equity indices. There was also a two-way lead/lag relation between implied volatilities and absolute changes of interest rates and a one-way dynamic effect from implied volatilities to absolute equity returns. That is, equity uncertainty can be an important variable in forecasting absolute equity returns with the equity risk premium being a possible channel.

There was a two-way dynamic relationship between eurodollar and short sterling as well as between S&P 500 and FTSE 100 implied volatilities. Moreover, eurodollar skew led its short sterling counterpart and S&P 500 uncertainty led higher moments of future FTSE 100 distributions. For equity indices, we found that equity uncertainty tended to lead perceptions about asymmetry and extreme equity index movements.

In both the United Kingdom and the United States, interest rate uncertainty was related to the absolute value of the slope of the yield curve. In particular, the slope of the yield curve has a Granger causal effect on interest rate uncertainty. One reason for this may be that high yield-curve slopes can be associated with large expected movements in interest rates in an upturn or a downturn of the cycle, during which there is less certainty about the size of interest rate movements. Equity volatility changes in the United States were found to lag changes in corporate spreads, a finding that is consistent with Merton (1974).

Our analysis of the information content of these expectational variables found little evidence to suggest that the implied summary statistics had any incremental predictive power in relation to future macroeconomic and financial variables over the sample period considered. However, the sample period available was relatively short, covering, at most, two business cycles in the United States, while equity option data for the United Kingdom was only available from 1992. Despite their poor forecasting performance, these implied summary statistics could still provide useful information about the expectations of financial market participants beyond the ‘point’ expectations embodied in futures prices.

This study on the summary statistics of implied pdfs can be extended by comparing them with the measures of realised uncertainty, asymmetry or fatness of tails to look at their forecasting performance, and consider possible systematic biases related to risk premia.

Appendix 1: Measures of uncertainty and their properties

Table A1.1: Historical distributions of implied volatilities

	Sample	Mean	Max	Min	Standard deviation	Skew	Kurtosis
S&P 500 three-month	83-02	0.170	0.647	0.095	0.053	2.017	13.896
S&P 500 six-month	83-02	0.171	0.502	0.096	0.048	1.304	7.030
S&P 500 three-month	92-02	0.171	0.379	0.095	0.052	0.573	2.967
S&P 500 six-month	92-02	0.175	0.367	0.096	0.049	0.544	2.806
FTSE 100 three-month	92-02	0.182	0.415	0.107	0.056	1.118	4.380
FTSE 100 six-month	92-02	0.185	0.406	0.119	0.053	1.187	4.444
Eurodollar three-month	85-02	0.152	0.449	0.050	0.061	1.195	5.710
Eurodollar six-month	85-02	0.176	0.428	0.074	0.061	1.202	5.292
Short sterling three-month	88-02	0.128	0.343	0.054	0.044	1.061	4.973
Short sterling six-month	88-02	0.148	0.330	0.065	0.044	0.665	3.636

Table A1.2: Difference between implied volatilities and standard deviations

	Sample	Mean	t-statistic	p-value
S&P 500 three-month	83-02	-0.012	-40.474	0
S&P 500 six-month	83-02	-0.013	-38.351	0
FTSE 100 three-month	92-02	-0.011	-24.896	0
FTSE 100 six-month	92-02	-0.015	-23.543	0
Eurodollar three-month	85-02	-0.005	-67.777	0
Eurodollar six-month	85-02	-0.005	-59.904	0
Short sterling three-month	88-02	-0.003	-45.995	0
Short sterling six-month	88-02	-0.003	-33.266	0

Table A1.3: Log implied volatility

	Sample	Skew	Kurtosis	Jarque-Bera normality test	p-value
S&P 500 three-month	83-02	0.425	3.369	35.812	0
S&P 500 six-month	83-02	0.319	2.926	17.286	0
S&P 500 three-month	92-02	0.056	1.925	25.396	0
S&P 500 six-month	92-02	0.090	1.912	26.449	0
FTSE 100 three-month	92-02	0.415	2.569	19.152	0
FTSE 100 six-month	92-02	0.579	2.615	32.665	0
Eurodollar three-month	85-02	-0.170	2.965	4.280	0.117
Eurodollar six-month	85-02	0.095	2.972	1.357	0.507
Short sterling three-month	88-02	0.056	2.683	3.501	0.173
Short sterling six-month	88-02	-0.121	2.635	5.903	0.052

Table A1.4: Autocorrelations for three-month implied volatilities

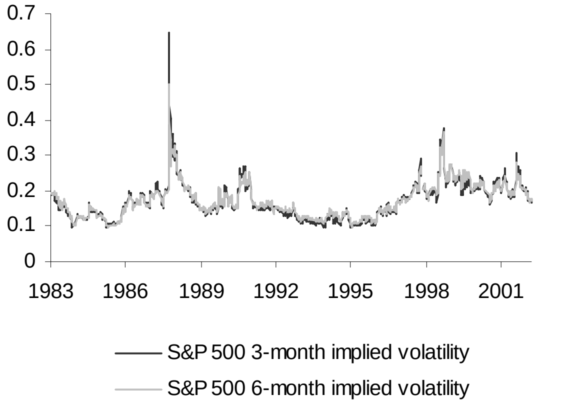
	Lag length =	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60
	Sample															
S&P 500	83-02	0.94	0.88	0.83	0.80	0.77	0.76	0.74	0.72	0.70	0.69	0.56	0.50	0.44	0.37	0.30
FTSE 100	92-02	0.97	0.94	0.90	0.87	0.83	0.80	0.78	0.76	0.75	0.73	0.61	0.50	0.51	0.44	0.34
Eurodollar	85-02	0.97	0.93	0.90	0.87	0.84	0.81	0.78	0.75	0.73	0.70	0.47	0.36	0.32	0.22	0.12
Short sterling	88-02	0.95	0.90	0.86	0.82	0.79	0.76	0.73	0.70	0.67	0.64	0.41	0.28	0.25	0.18	0.06

Table A1.5: Unit root tests for implied volatilities

	Sample	Phillips-Perron statistic		Sample	Phillips-Perron statistic
S&P 500 three-month	83-02	-5.009***	Eurodollar three-month	85-02	-3.362**
S&P 500 six-month	83-02	-3.813***	Eurodollar six-month	85-02	-2.674*
S&P 500 three-month	92-02	-1.960	Short sterling three-month	88-02	-4.333***
S&P 500 six-month	92-02	-1.529	Short sterling six-month	88-02	-3.505***
FTSE 100 three-month	92-02	-2.635*			
FTSE 100 six-month	92-02	-2.210			

Rejection of the unit root hypothesis is denoted as
*** at 1% significance level; ** at 5% significance level; * at 10% significance level

Chart A1.1



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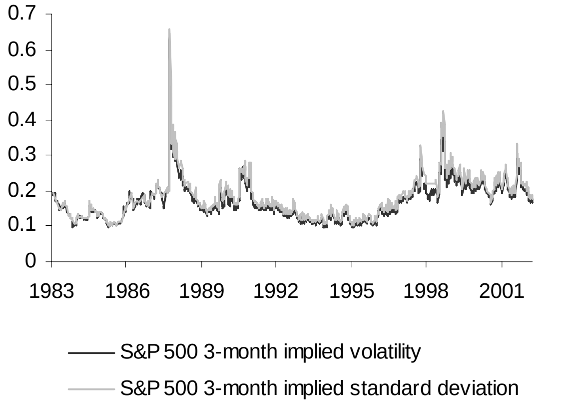

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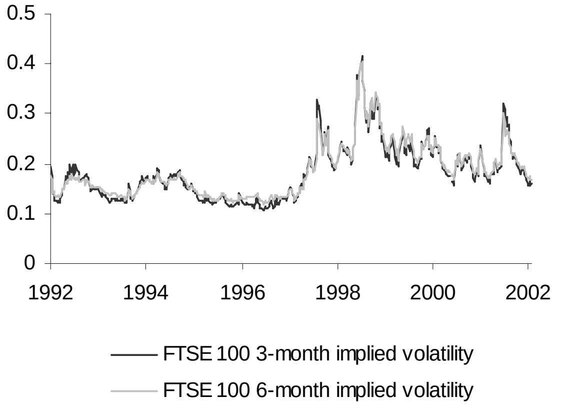

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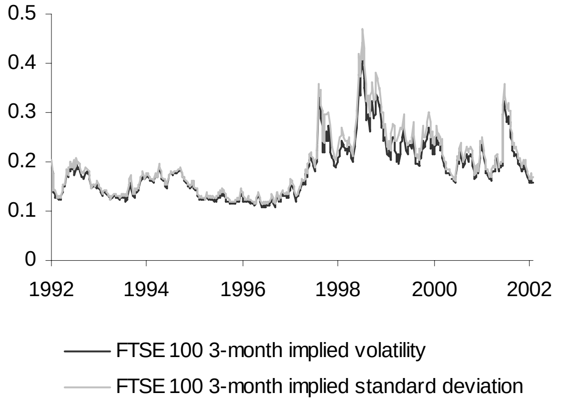

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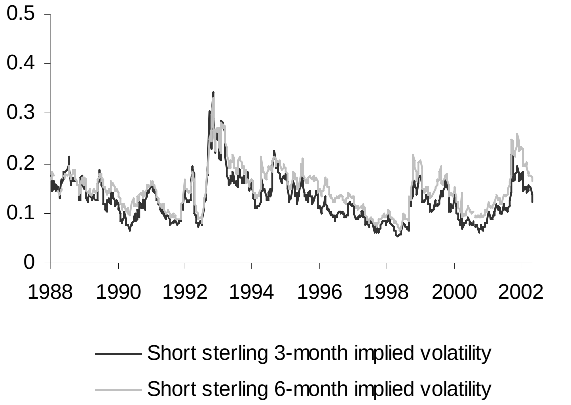

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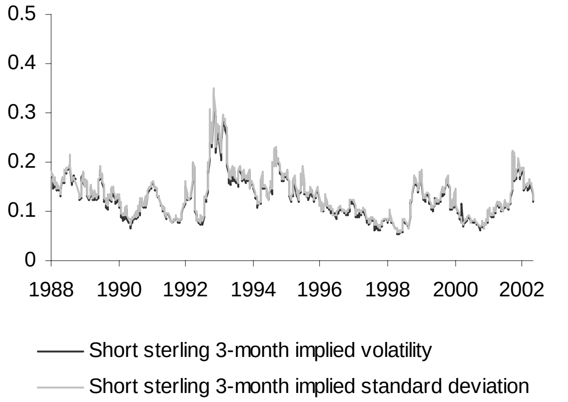
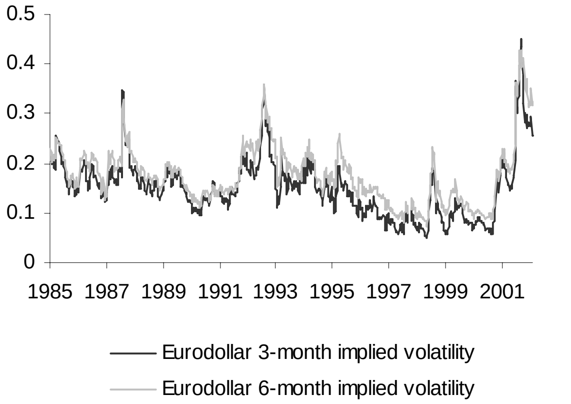


Chart A1.7



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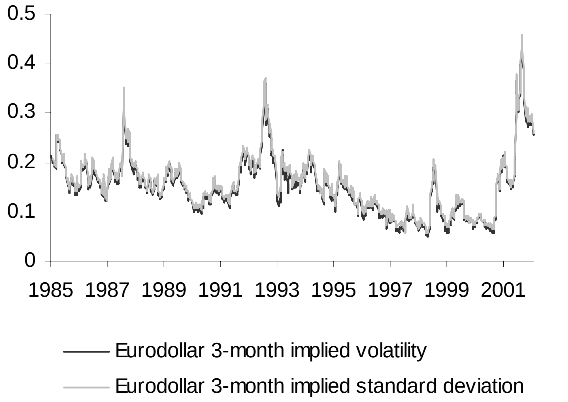


Appendix 2: Measures of asymmetry and their properties

Table A2.1: Historical distributions of implied asymmetry (skew)

	Sample	Mean	Max	Min	Standard deviation	Skew	Kurtosis
S&P 500 three-month skew	83-02	-0.805	0.255	-1.511	0.378	1.056	3.123
S&P 500 six-month skew	83-02	-0.837	0.653	-1.356	0.353	1.387	4.238
S&P 500 three-month skew	92-02	-0.981	-0.452	-1.376	0.168	0.120	2.758
S&P 500 six-month skew	92-02	-0.984	-0.545	-1.356	0.151	0.039	2.859
FTSE 100 three-month skew	92-02	-0.734	-0.204	-1.307	0.245	-0.159	2.072
FTSE 100 six-month skew	92-02	-0.754	-0.245	-1.381	0.281	-0.243	1.915
Eurodollar three-month skew	85-02	0.024	1.000	-1.294	0.334	-0.398	4.077
Eurodollar six-month skew	85-02	0.000	0.687	-1.076	0.288	-0.823	4.814
Short sterling three-month skew	88-02	-0.004	0.979	-0.762	0.304	0.317	2.958
Short sterling six-month skew	88-02	0.001	0.905	-0.598	0.237	0.683	4.158

Table A2.2: Correlation matrix of S&P 500 three-month asymmetry measures (sample 1983-2002)

skewness	1				
(mean-mode)/stddev	0.928	1			
(mean-median)/stddev	0.989	0.953	1		
standardised risk reversal	0.996	0.931	0.991	1	
probability asymmetry	0.911	0.993	0.939	0.908	1

Table A2.3: Correlation matrix of FTSE 100 three-month asymmetry measures (sample 1992-2002)

skewness	1				
(mean-mode)/stddev	0.837	1			
(mean-median)/stddev	0.988	0.869	1		
standardised risk reversal	0.996	0.845	0.991	1	
probability asymmetry	0.795	0.991	0.833	0.800	1

Table A2.4: Correlation matrix of eurodollar three-month asymmetry measures (sample 1985-2002)

skewness	1				
(mean-mode)/stddev	0.927	1			
(mean-median)/stddev	0.993	0.950	1		
standardised risk reversal	0.999	0.928	0.993	1	
probability asymmetry	0.928	1.000	0.951	0.929	1

Table A2.5: Correlation matrix of short sterling three-month asymmetry measures (sample 1988-2002)

skewness	1				
(mean-mode)/stddev	0.957	1			
(mean-median)/stddev	0.995	0.966	1		
Standardised risk reversal	1.000	0.959	0.995	1	
probability asymmetry	0.958	1.000	0.967	0.959	1

Table A2.6: Stability of asymmetry measures

	sample	skew	(mean-mode) /stdev	(mean-median)/ stddev	standardised risk reversal	probability asymmetry
S&P 500 three-month	83-02	0.051	0.076	0.075	0.056	0.073
S&P 500 six-month	83-02	0.041	0.060	0.063	0.046	0.064
FTSE 100 three-month	92-02	0.053	0.078	0.075	0.060	0.077
FTSE 100 six-month	92-02	0.042	0.072	0.066	0.048	0.066
Eurodollar three-month	85-02	0.391	0.440	0.412	0.395	0.427
Eurodollar six-month	85-02	0.284	0.346	0.315	0.283	0.333
Short sterling three-month	88-02	0.336	0.375	0.351	0.338	0.364
Short sterling six-month	88-02	0.272	0.343	0.306	0.279	0.333

Table A2.7: Autocorrelations for three-month implied asymmetry (skew)

	Lag length =	1	2	3	4	5	10	20	30	40	50	60
	Sample											
S&P 500	83-02	0.86	0.80	0.77	0.73	0.72	0.68	0.62	0.56	0.54	0.49	0.43
FTSE 100	92-02	0.93	0.91	0.89	0.88	0.86	0.78	0.66	0.63	0.64	0.63	0.57
Eurodollar	85-02	0.91	0.84	0.78	0.74	0.71	0.62	0.41	0.28	0.15	0.04	-0.03
Short sterling	88-02	0.93	0.89	0.86	0.84	0.82	0.72	0.55	0.46	0.31	0.19	0.06

Table A2.8: Unit root tests for implied asymmetry (skew)

	Sample	Phillips-Perron statistic		Sample	Phillips-Perron statistic
S&P 500 three-month	83-02	-3.386**	Eurodollar three-month	85-02	-5.735***
S&P 500 six-month	83-02	-4.918***	Eurodollar six-month	85-02	-3.619***
S&P 500 three-month	92-02	-2.625*	Short sterling three-month	88-02	-3.765***
S&P 500 six-month	92-02	-2.203	Short sterling six-month	88-02	-3.486***
FTSE 100 three-month	92-02	-1.876			
FTSE 100 six-month	92-02	-1.340			

Rejection of the unit root hypothesis is denoted as
*** at 1% significance level; ** at 5% significance level; * at 10% significance level

Chart A2.1

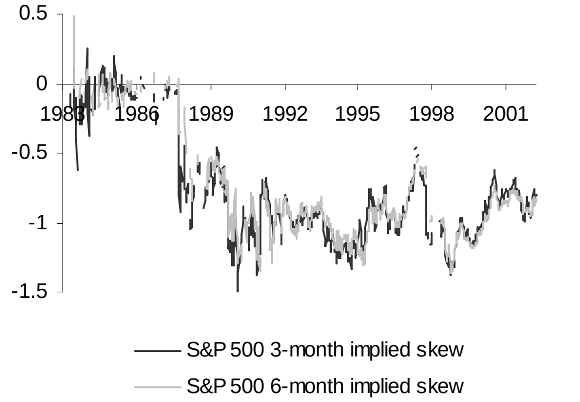


Chart A2.2

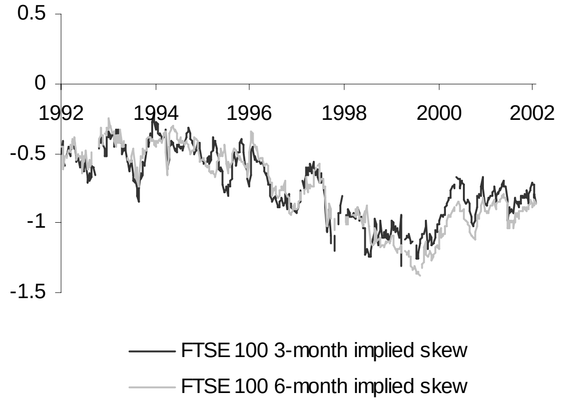
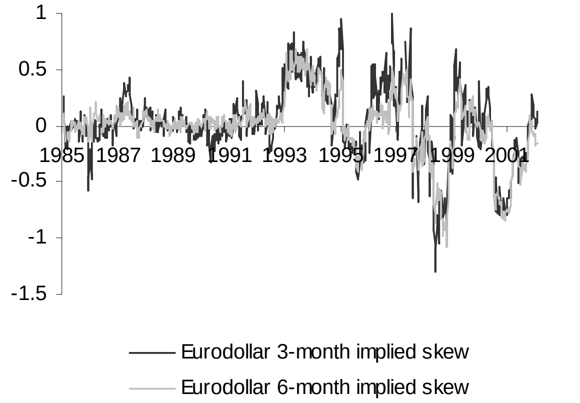


Chart A2.3



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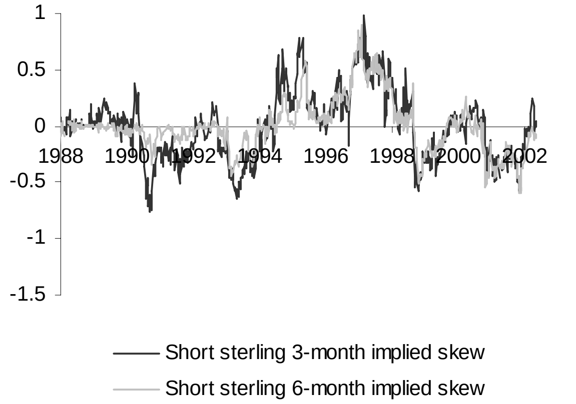


Appendix 3: Measures of fatness of tails and their properties

Table A3.1: Historical distributions of fatness of tails (kurtosis)

	Sample	Mean	Max	Min	Standard deviation	Skew	Kurtosis
S&P 500 three-month kurtosis	83-02	3.760	5.010	2.974	0.416	-0.200	2.333
S&P 500 six-month kurtosis	83-02	3.721	4.574	2.872	0.370	-0.379	2.657
S&P 500 three-month kurtosis	92-02	3.940	4.631	3.260	0.280	0.095	2.466
S&P 500 six-month kurtosis	92-02	3.862	4.574	3.329	0.246	0.342	2.920
FTSE 100 three-month kurtosis	92-02	3.573	4.522	3.055	0.306	0.446	2.418
FTSE 100 six-month kurtosis	92-02	3.555	4.537	3.059	0.348	0.586	2.518
Eurodollar three-month kurtosis	85-02	3.318	4.648	3.019	0.215	1.938	7.871
Eurodollar six-month kurtosis	85-02	3.231	4.228	2.967	0.177	1.747	7.059
Short sterling three-month kurtosis	88-02	3.232	4.325	2.989	0.154	1.621	7.770
Short sterling six-month kurtosis	88-02	3.139	4.041	2.956	0.140	1.951	9.036

Table A3.2: Correlation matrix of S&P 500 three-month fatness of tails measures (sample 1983-2002)

Kurtosis	1		
Standardised strangle	0.445	1	
Probability of extreme movements	0.970	0.321	1

Table A3.3: Correlation matrix of FTSE 100 three-month fatness of tails measures (sample 1992-2002)

Kurtosis	1		
Standardised strangle	0.674	1	
Probability of extreme movements	0.974	0.555	1

Table A3.4: Correlation matrix of eurodollar three-month fatness of tails measures (sample 1992-2002)

Kurtosis	1		
Standardised strangle	0.572	1	
Probability of extreme movements	0.970	0.578	1

Table A3.5: Correlation matrix of short sterling three-month fatness of tails measures (sample 1992-2002)

Kurtosis	1		
Standardised strangle	0.524	1	
Probability of extreme movements	0.954	0.513	1

Table A3.6: Stability of fatness of tails measures

	sample	kurtosis	standardised strangle	probability of extreme movements
S&P 500 three-month	83-02	0.015	0.155	0.017
S&P 500 six-month	83-02	0.014	0.195	0.016
FTSE 100 three-month	92-02	0.013	0.173	0.017
FTSE 100 six-month	92-02	0.011	0.171	0.016
Eurodollar three-month	85-02	0.021	0.222	0.021
Eurodollar six-month	85-02	0.012	0.174	0.014
Short sterling three-month	88-02	0.018	0.253	0.019
Short sterling six-month	88-02	0.010	0.253	0.014

Table A3.7: Autocorrelations of three-month fatness of tails (kurtosis)

	Lag length =	1	2	3	4	5	10	20	30	40	50	60
	Sample											
S&P 500	83-02	0.88	0.81	0.77	0.74	0.71	0.65	0.58	0.49	0.49	0.43	0.35
FTSE 100	92-02	0.91	0.90	0.87	0.86	0.84	0.75	0.63	0.57	0.57	0.56	0.49
Eurodollar	85-02	0.87	0.76	0.69	0.66	0.63	0.50	0.35	0.25	0.18	0.09	0.11
Short sterling	88-02	0.82	0.71	0.63	0.61	0.58	0.37	0.21	0.12	0.05	-0.14	-0.10

Table A3.8: Unit root tests for fatness of tails (kurtosis)

	Sample	Phillips-Perron statistic		Sample	Phillips-Perron statistic
S&P 500 three-month	83-02	-3.279**	Eurodollar three-month	85-02	-7.041***
S&P 500 six-month	83-02	-2.240	Eurodollar six-month	85-02	-4.307***
S&P 500 three-month	92-02	-2.661*	Short sterling three-month	88-02	-7.247***
S&P 500 six-month	92-02	-2.442	Short sterling six-month	88-02	-4.943***
FTSE 100 three-month	92-02	-1.297			
FTSE 100 six-month	92-02	-1.188			

Rejection of the unit root hypothesis is denoted as
*** at 1% significance level; ** at 5% significance level; * at 10% significance level

Chart A3.1

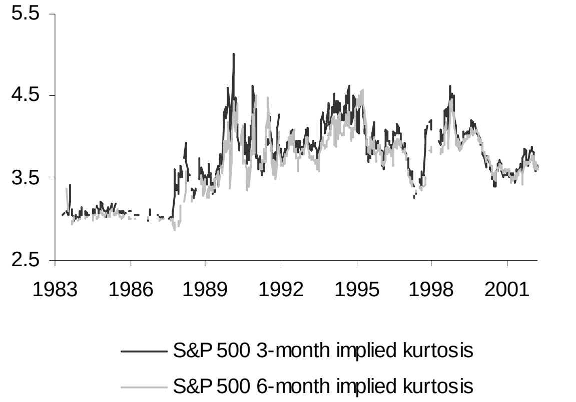


Chart A3.2

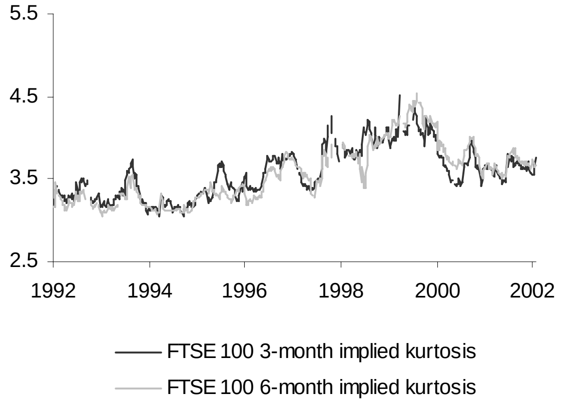


Chart A3.3

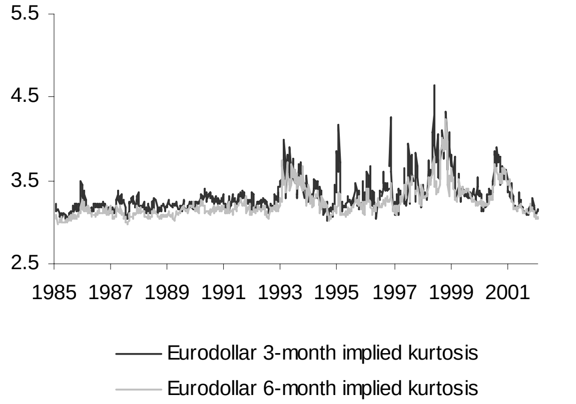
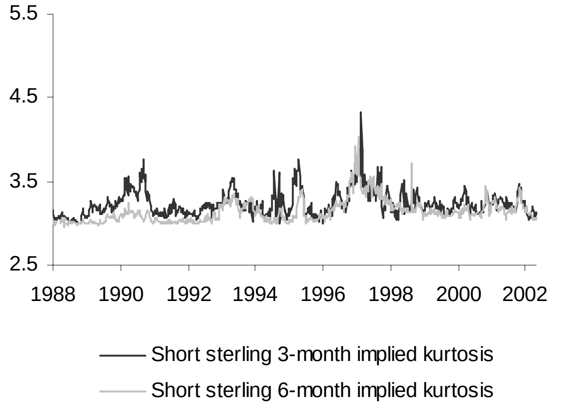


Chart A3.4



Appendix 4: Relationships between summary statistics

Table A4.1: Interest rate level changes and implied summary statistics correlation matrices

Short sterling three-month	***1988-2002***				
Implied volatility	1.000				
Kurtosis	-0.079	1.000			
Skew	0.031	0.812	1.000		
Change short sterling level	-0.093	0.077	0.137	1.000	
Absolute change short sterling level	0.399	-0.085	-0.024	-0.276	1.000
Short sterling six-month	***1988-2002***				
Implied volatility	1.000				
Kurtosis	0.077	1.000			
Skew	0.104	0.904	1.000		
Change short sterling level	-0.082	0.020	0.075	1.000	
Absolute change short sterling level	0.370	-0.009	-0.009	-0.265	1.000
Eurodollar three-month	***1985-2002***				
Implied volatility	1.000				
Kurtosis	0.016	1.000			
Skew	0.261	0.592	1.000		
Change eurodollar level	-0.093	0.077	0.144	1.000	
Absolute change eurodollar level	0.482	-0.061	0.056	-0.226	1.000
Eurodollar six-month	***1985-2002***				
Implied volatility	1.000				
Kurtosis	0.360	1.000			
Skew	0.412	0.804	1.000		
Change eurodollar level	-0.036	0.098	0.114	1.000	
Absolute change eurodollar level	0.437	0.105	0.129	-0.036	1.000

Table A4.2: Correlations between three-month short sterling implied statistics & eurodollar implied summary statistics

Sample	1988-2002				
	eurodollar implied volatility	eurodollar kurtosis	eurodollar skew	eurodollar level changes	eurodollar absolute level changes
Short sterling implied volatility	0.665	0.242	0.334	0.026	0.249
Short sterling kurtosis	-0.191	0.135	0.268	0.096	-0.136
Short sterling skew	-0.106	0.069	0.318	0.155	-0.108
Change short sterling level	-0.100	0.023	0.119	0.187	-0.101
Absolute change short sterling level	0.281	0.005	0.036	-0.002	0.181

Table A4.3: Probability values from short sterling and eurodollar Granger causality tests of significance based on multivariate VARs

Short sterling three-month	***1988-2002***			
VAR(5)	*Dependent variable*			
Independent variables	*absolute level changes*	*implied volatility*	*kurtosis*	*skew*
absolute level changes	-	0.000	0.387	0.927
implied volatility	0.000	-	0.418	0.082
kurtosis	0.669	0.284	-	0.111
skew	0.660	0.145	0.023	-

Short sterling six-month	***1988-2002***			
VAR(4)	*Dependent variable*			
Independent variables	*absolute level changes*	*implied volatility*	*kurtosis*	*skew*
absolute level changes	-	0.003	0.843	0.396
implied volatility	0.000	-	0.176	0.013
kurtosis	0.981	0.965	-	0.503
skew	0.759	0.829	0.044	-

Eurodollar three-month	***1985-2002***			
VAR(4)	*Dependent variable*			
Independent variables	*absolute level changes*	*implied volatility*	*kurtosis*	*skew*
absolute level changes	-	0.000	0.600	0.848
implied volatility	0.000		0.810	0.464
kurtosis	0.381	0.685	-	0.426
skew	0.519	0.683	0.472	-

Eurodollar six-month	***1985-2002***			
VAR(6)	*Dependent variable*			
Independent variables	*absolute level changes*	*implied volatility*	*kurtosis*	*skew*
absolute level changes	-	0.000	0.445	0.731
implied volatility	0.000	-	0.591	0.880
kurtosis	0.485	0.075	-	0.000
skew	0.606	0.803	0.869	-

Table A4.4: Probability values from joint eurodollar/short sterling implied summary statistics and level changes tests of causality

VAR(6)	Dependent variable: short sterling absolute level changes	short sterling implied volatility	short sterling kurtosis	short sterling skew	eurodollar absolute level changes	eurodollar implied volatility	eurodollar kurtosis	eurodollar skew
Independent variables								
Short sterling absolute level changes	-	0.000	0.707	0.942	0.330	0.002	0.899	0.401
Short sterling implied volatility	0.000	-	0.616	0.018	0.415	0.220	0.720	0.227
Short sterling kurtosis	0.239	0.235	-	0.242	0.388	0.233	0.388	0.012
Short sterling skew	0.363	0.371	0.111	-	0.751	0.255	0.864	0.095
Eurodollar absolute level changes	0.243	0.001	0.628	0.096	-	0.075	0.198	0.688
Eurodollar implied volatility	0.934	0.000	0.639	0.594	0.000	-	0.304	0.957
Eurodollar kurtosis	0.769	0.387	0.879	0.471	0.693	0.930	-	0.436
Eurodollar skew	0.656	0.905	0.659	0.018	0.748	0.552	0.721	-

Table A4.5: Equity returns and implied summary statistics correlation matrices

S&P 500 three-month	***1988-2002***				
Implied volatility	1.000				
Kurtosis	-0.194	1.000			
Skew	0.014	-0.968	1.000		
S&P 500 level changes	-0.148	0.075	-0.007	1.000	
S&P 500 absolute level changes	0.414	-0.074	-0.002	-0.075	1.000
S&P 500 six-month	***1988-2002***				
Implied volatility	1.000				
Kurtosis	-0.250	1.000			
Skew	0.051	-0.952	1.000		
S&P 500 level changes	-0.120	0.137	-0.060	1.000	
S&P 500 absolute level changes	0.408	-0.105	0.018	-0.090	1.000
FTSE 100 three-month	***1992-2002***				
Implied volatility	1.000				
Kurtosis	0.611	1.000			
Skew	-0.679	-0.980	1.000		
FTSE 100 returns	-0.132	0.023	0.012	1.000	
FTSE 100 absolute returns	0.385	0.146	-0.187	0.000	1.000
FTSE 100 six-month	***1992-2002***				
Implied volatility	1.000				
Kurtosis	0.616	1.000			
Skew	-0.710	-0.978	1.000		
FTSE 100 returns	-0.107	0.036	0.008	1.000	
FTSE 100 absolute returns	0.377	0.125	-0.182	-0.029	1.000

Table A4.6: Correlations between three-month S&P 500 implied statistics & FTSE 100 implied summary statistics

Sample	1988-2002				
	FTSE 100 implied volatility	FTSE 100 kurtosis	FTSE 100 skew	FTSE 100 level changes	FTSE 100 absolute level changes
S&P 500 implied volatility	0.853	0.746	-0.814	-0.127	0.305
S&P 500 kurtosis	0.089	-0.106	0.153	0.049	0.060
S&P 500 skew	-0.270	-0.040	0.003	-0.011	-0.122
S&P 500 level changes	-0.101	0.038	-0.001	0.672	-0.056
S&P 500 absolute level changes	0.402	0.302	-0.338	-0.074	0.447

Table A4.7: Probability values from S&P 500 and FTSE 100 Granger causality tests of significance based on multivariate VARs

S&P 500 three-month	***1988-2002***			
VAR(4)	Dependent variable			
Independent variables	absolute level changes	implied volatility	kurtosis	skew
absolute level change	-	0.301	0.510	0.338
implied volatility	0.000	-	0.013	0.001
kurtosis	0.601	0.104	-	0.009
skew	0.487	0.184	0.007	-
S&P 500 six-month	***1988-2002***			
VAR(6)	Dependent variable			
Independent variables	absolute level changes	implied volatility	kurtosis	skew
absolute level changes	-	0.121	0.521	0.310
implied volatility	0.001	-	0.061	0.006
kurtosis	0.911	0.123	-	0.004
skew	0.947	0.237	0.037	-
FTSE 100 three-month	***1992-2002***			
VAR(4)	Dependent variable			
Independent variables	absolute level changes	implied volatility	kurtosis	skew
absolute level changes	-	0.261	0.520	0.479
implied volatility	0.000	-	0.047	0.064
kurtosis	0.248	0.527	-	0.923
skew	0.464	0.912	0.645	-
FTSE 100 six-month	***1992-2002***			
VAR(7)	Dependent variable			
Independent variables	absolute level changes	implied volatility	kurtosis	skew
absolute level changes	-	0.202	0.169	0.161
implied volatility	0.001	-	0.023	0.011
kurtosis	0.798	0.186	-	0.425
skew	0.904	0.827	0.414	-

Table A4.8: Probability values from joint FTSE 100 / S&P 500 implied summary statistics and level changes tests of causality

Sample	1992-2002							
	Dependent variable							
VAR(5)	FTSE 100 absolute level changes	FTSE 100 implied volatility	FTSE 100 kurtosis	FTSE 100 skew	S&P 500 absolute level changes	S&P 500 implied volatility	S&P 500 kurtosis	S&P 500 skew
Independent variables								
FTSE 100 absolute level changes	-	0.235	0.547	0.531	0.024	0.523	0.579	0.212
FTSE 100 implied volatility	0.006	-	0.772	0.845	0.069	0.007	0.063	0.182
FTSE 100 kurtosis	0.193	0.745	-	0.955	0.859	0.335	0.032	0.048
FTSE 100 skew	0.423	0.953	0.874	-	0.745	0.697	0.119	0.097
S&P 500 absolute level changes	0.943	0.055	0.329	0.644	-	0.055	0.260	0.065
S&P 500 implied volatility	0.477	0.075	0.004	0.005	0.074	-	0.049	0.006
S&P 500 kurtosis	0.562	0.223	0.291	0.071	0.992	0.055	-	0.079
S&P 500 skew	0.677	0.324	0.290	0.110	0.955	0.084	0.775	-

Appendix 5: Relationships between summary statistics and other financial or macroeconomic variables

Table A5.1: Results from regressions on the information content of implied volatilities for selected macroeconomic variables

(Sample: 1988 – 2002)

Regression equations: $y_{t+12,t} = c + ay_t + bx_t + dr_t + e_t$

(r = underlying futures price returns)

x variable = S&P 500 implied volatility

y variable	a	b	d	R^2
Growth US Ind. Production	0.325	-0.09	0.06	0.21
	[0.04]	[0.103]	[0.002]	
Growth US Investment	0.497	-0.062	0.13	0.29
	[0.004]	[0.155]	[0.502]	
Growth US Earnings	0.275	-0.11	-0.126	0.11
	[0.032]	[0.626]	[0.172]	

x variable = eurodollar implied volatility

y variable	a	b	d	R^2
Growth US Ind. Production	0.577	-0.015	-0.04	0.23
	[0.003]	[0.799]	[0.004]	
Growth US Investment	0.588	-0.01	-0.02	0.33
	[0.000]	[0.765]	[0.007]	
Annual change US Inflation	-0.29	0.02	0.008	0.09
	[0.05]	[0.52]	[0.12]	

x variable = short sterling implied volatility

y variable	a	b	d	R^2
Growth UK Ind. Production	0.341	0.0733	-0.05	0.43
	[0.005]	[0.204]	[0.000]	
Growth UK Investment	0.493	0.06	-0.016	0.19
	[0.008]	[0.504]	[0.47]	
Annual change UK inflation	-0.342	0.089	0.026	0.09
	[0.248]	[0.118]	[0.181]	

Note:
(1) Values in square brackets refer to probability values for 't statistics' for the corresponding coefficients.
(2) Regressions were estimated using Newey-West procedure.

Table A5.2: Results from regressions on the information content of implied asymmetry and kurtosis for macroeconomic variables

(Sample period 1988 – 2001)

Regression equations: $y_{t+12,t} = c + ay_t + bx_t + dr_t + e_t$

(r = underlying futures price returns)

x variable = S&P 500 implied asymmetry

y variable	a	b	d	R^2
Growth US Ind. Production	0.351	-0.0008	0.028	0.20
	[0.046]	[0.942]	[0.01]	
Growth US Investment	0.548	0.014	0.015	0.31
	[0.01]	[0.296]	[0.552]	
Growth US Earnings	0.32	-0.01	-0.13	0.12
	[0.04]	[0.85]	[0.14]	

x variable = S&P 500 implied kurtosis

y variable	a	b	d	R^2
Growth US Ind. Production	0.36	0.005	0.06	0.20
	[0.05]	[0.66]	[0.006]	
Growth US Investment	0.31	-0.0009	0.065	0.18
	[0.06]	[0.92]	[0.003]	
Growth US Earnings	0.32	0.03	-0.12	0.13
	[0.03]	[0.47]	[0.17]	

Note:
(1) Values in square brackets refer to probability values for 't statistics' for the corresponding coefficients.
(2) Regressions were estimated using Newey-West procedure.

Table A5.3: Probability values for causality tests from bivariate VARs between interest rate implied volatilities, yield curve slopes and futures returns

	Dependent variable		
Eurodollar (VAR(5))	ATM Volatility	Returns	US Slope
ATM Volatility	-	0.55	0.67
Returns	0.07	-	0.32
US Slope	0.02	0.00	-

	Dependent variable		
Short Sterling (VAR(12))	ATM Volatility	Returns	UK Slope
ATM Volatility	-	0.00	0.18
Returns	0.00	-	0.73
UK Slope	0.04	0.06	-

Table A5.4: Correlations between equity implied volatilities and corporate spreads

	AA	A	BBB
S&P ATM volatility	0.08	0.30	0.45
FTSE 100 ATM vol.	-0.62	-0.49	-0.30

Table A5.5: Probability values for causality tests from VAR between implied volatility, futures returns and credit spreads

	Dependent variable		
S&P 500 (VAR(6))	ATM Volatility	Returns	A
ATM Volatility	-	0.11	0.12
Returns	0.001	-	0.18
A-rated spread	0.06	0.002	-
S&P 500 (VAR(5))	ATM Volatility	Returns	BBB
ATM Volatility	-	0.04	0.17
Returns	0.000	-	0.91
BBB-rated spread	0.08	0.01	-
FTSE 100 (VAR(6))	ATM Volatility	Returns	A
ATM Volatility	-	0.26	0.82
Returns	0.15	-	0.08
A-rated spread	0.26	0.24	-
FTSE 100 (VAR(3))	ATM Volatility	Returns	BBB
ATM Volatility	-	0.04	0.75
Returns	0.36	-	0.21
BBB-rated spread	0.34	0.27	-

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