BANK OF ENGLAND

Working Paper No. 342

That elusive elasticity and the ubiquitous bias: is panel data a panacea?

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February 2008

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Abstract

There is often assumed to be a unit elasticity of substitution between capital and labour. But estimates based on neoclassical capital demand equations frequently find a smaller value. Recent time-series work for the United States and Canada has suggested that, once the biases inherent in estimating cointegrating vectors are properly accounted for, the elasticity could indeed be close to 1. This paper investigates this possibility for the United Kingdom. First the analysis considers aggregate data where the estimated elasticity is close to 0.4. Then a unique industry-level data set for the United Kingdom is exploited in order to further pinpoint the estimated elasticity. Estimates using dynamic panel data methods are close to those from aggregate data, providing a robust statistical rejection of a unit elasticity in UK data.

Key words: Capital accumulation, cost of capital, panel data, cointegration.

JEL classification: C32, C33, E22.

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The views expressed in this paper are those of the author, and not necessarily those of the Bank of England. I am grateful to Nick Bloom, Robert Chirinko, Jenni Greenslade, George Kapetanios, Nick Oulton and conference participants at ‘A bright future at the age of 50 — the CES production function in theory and empirics of economic growth’ for valuable comments and discussion. This paper was finalised on 29 January 2008.

The Bank of England’s working paper series is externally refereed.

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ISSN 1749-9135 (on-line)

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Summary

The elasticity of substitution between capital and labour – a measure of the ease with which capital can be substituted for labour in the production process – is often assumed to be one. This is a standard simplifying assumption. But empirical studies frequently find that this elasticity takes a smaller value. Recent work, based on capital demand equations for the United States and Canada, has found that the elasticity may indeed be close to one – or perhaps even larger. The aim of this paper is to test whether applying a similar approach to UK data will yield similar results.

We start with a simple linear relationship between the optimal capital-output ratio and the real user cost of capital. But, because it is costly for firms to change the amount of capital they employ (for example because it takes time to learn how to use new machinery), we interpret this relationship as a long-run phenomenon. However, estimating a long-run relationship of this kind can lead to biased estimates. To ameliorate the influence of these biases analysis for the United States and Canada have applied methods based on the use of a single time series. In this paper we extend this approach in two important ways: first by exploiting variation across industries (panel estimation); and second by exploiting variation in the elasticity of substitution across different physical capital assets.

Given the flexibility of our theoretical framework, and the robustness of the different estimators we use, we are in a position to provide a sound statistical investigation of the possibility of a unit elasticity in UK data. So what do our results tell us? Estimates for the elasticity of substitution based on aggregate data are very similar to those found in previous studies for the United Kingdom: close to 0.4. Do these results simply reflect methodological differences in constructing UK data? By matching UK data as closely as possible to the data used in those studies we are able to eliminate this possibility. However aggregation biases could still affect our estimates. In addition, a single time series may not be enough to purge our estimates of the biases inherent in estimating this long-run relationship. To address these possibilities we use panel data. We find that, once we account for some of the problems commonly encountered when using dynamic panel methods, our estimates are close to the benchmark estimate using aggregate data. Thus we can provide a strong rejection of a unit elasticity of substitution between capital and labour in UK data.

1 Introduction and motivation

Under standard assumptions the absolute value of the elasticity of capital formation with respect to its price – the user-cost elasticity (UCE) – is the same as the elasticity of substitution between capital and labour, σ. Macro models frequently assume a unit elasticity; the Cobb-Douglas case. But, despite a daunting array of econometric methods, there is little empirical consensus as to the value of this key elasticity. The contribution of this paper is to provide new estimates for the United Kingdom using both aggregate and industry-level panel data.

Numerous estimation approaches have been taken to estimating the UCE. Comprehensive surveys – of the work done on US data – can be found in Auerbach and Hassett (1992) and Chirinko (1993). However recent studies have introduced an array of new approaches using microdata and policy experiments. For example, in two papers Chirinko *et al* (1999, 2004) use firm-level data to estimate σ. They implement a number of approaches and present estimates in the range of –0.06 to –0.56.[1] Tax changes have been considered in papers by Cummins, Hassett and Hubbard (1994,1996). They find UCE elasticities in the range of –0.4 to –1. And Guiso, Kashyap, Panetta and Terlizzese (2002) arrive at estimates close to one using an instrumental variables approach that exploits detailed data linking firms and banks.

However, the literature using time-series methods is closest to the work presented here. In particular Caballero (1994) and Schaller (2006) find that, once they account for the biases introduced by estimating the UCE in small samples, it is in the neighborhood of –1. We apply the approach of these authors to UK data, but extend it in two important ways. First we follow the insights of Tevlin and Whelan (2003) – namely that the asset composition of investment goods is important – and consider investment in different capital assets. Second, we consider capital accumulation at the industry level. In particular, we exploit data from an industry panel for the United Kingdom to estimate the UCE for different industries. In principle, panel data can be very helpful as a single time series may not cover a sufficient span to purge our estimates of the inherent biases.

In the next section we motivate the use of cointegration methods and discuss some of the properties of these estimators. In this paper we make use of sectoral-level investment data; these data are described in Section 3. Section 4 presents our results, and Section 5 concludes.

[1] Firm-level estimates are also presented by Caballero, Engel and Haltiwanger (1995) who find estimates in the range -0.01 to -2.0.

2 Theory and methods

In this section we outline how we can use single-equation cointegration methods to estimate the UCE and discuss the estimators that we employ. Our starting point is the neoclassical theory of capital demand. We then discuss why the data lend themselves to single-equation cointegration methods and the pitfalls in using these methods. In particular, the importance of finite sample biases. We provide a brief exposition of estimators which ameliorate the influence of these biases and their panel counterparts.

2.1 Estimating capital demand using single-equation methods

Following the seminal contribution of Jorgenson (1963), a variety of models lead to the following equation for the demand for capital,[2]

$$k_t^\star = \gamma - \sigma c_t \qquad \textbf{(1)}$$

Here $k^\star$ denotes the log of the optimal capital-output ratio; and c is the natural log of the real user cost of capital. Here σ is the elasticity of substitution between capital and labour and gives the size of the UCE. Adjustment costs imply that firms will not adjust straight to $k^\star$. Hence the actual capital-output ratio, k, will deviate from its optimal level,

$$k_t = k_t^\star + \upsilon_t \qquad \textbf{(2)}$$

Furthermore, if the log of the capital-output ratio and the real user cost are difference-stationary, $I(1)$, [3] the empirical counterpart to equation **(1)** will be the following triangular system,[4]

$$\begin{aligned} k_t &= \gamma - \sigma c_t + \upsilon_t \\ c_t &= c_{t-1} + u_t \end{aligned} \qquad \textbf{(3)}$$

Stationarity of υ implies that k and c are cointegrated. This follows from the desire of firms to bring k back to $k^\star$. Estimation of σ in **(3)** requires care, however, as simple linear estimators may be biased.

[2]One example is given in the appendix.

[3]We do not consider the question of whether c can feasibily be $I(1)$ in a large sample. This can be considered unlikely given the nature of the real cost of capital but nothing substantive rests on this assumption – we simply require the variance of the (log) level of the user cost to grow over time in our sample. This is easily satisfied in our data.

[4]For simplicity, we do not include a drift component at this point. This is for ease of exposition and is not maintained at the estimation stage.

2.1.1 *What is the source of these biases?*

Park and Phillips (1988) study the potential sources bias in the current setting. They demonstrate that, while the OLS estimator is (super-)consistent, its limiting distribution contains nuisance parameters arising from endogeneity and serial correlation. This is because the limiting distribution of the (stationary) errors above is a multivariate Brownian motion process. And elements of the long-run covariance matrix of this Brownian motion process appears in the limiting distribution of $\widehat{\sigma}$. So, while the OLS estimator is consistent, the distribution depends on the degree of long-run correlation in the error terms. Hence the OLS estimator can be biased in finite samples.

One way to provide intuition for this is to think of the bias in terms of the correlation between the user cost and the error term in equation **(3)**. Given $\{v_t\}$ and $\{u_t\}$ are stationary then the deviation of capital from its optimal level, v_t, and the change in the user cost, Δc_t, can be correlated because,

$$\begin{aligned} Cov(c_t, v_t) &= Cov(c_0 + \Delta c_1 + \Delta c_2 + \Delta c_3 + ... + \Delta c_t, v_t), \\ &= Cov(u_1 + u_2 + u_3 + ... + u_t, v_t) \text{ since } Cov(c_0, v_t) = 0 \text{ and } \Delta c_t = u_t, \\ &= Cov(u_t, v_t) \text{ as } \{v_t\} \text{ and } \{u_t\} \text{ are iid} \end{aligned} \tag{4}$$

Without the additional assumption that the covariance matrix of the errors is diagonal, $\widehat{\sigma}$ may be biased – even in this simple case. Given that a number of the components of the real user cost can be considered endogenous, this problem is likely to be important in this setting.[5]

The above problem can be ameliorated by adding the first difference of the user cost into the regression. The resultant error term will be orthogonal by construction. To see this note that after adding the first difference into the regression, the system becomes:

$$\begin{aligned} k_t &= \sigma c_t + \beta \Delta c_t + \eta_t, \quad t = 1, ...T, \\ \Delta c_t &= u_t \end{aligned} \tag{5}$$

Adding Δc_t means that $Cov(c_s, \eta_t) = 0$. To see this note that η is the least squares projection error $\eta_t \equiv v_t - \beta \Delta u_t = v_t - \beta \Delta c_t \Rightarrow Cov(\Delta c_s, \eta_t) = 0$ for all t and s. This means $Cov(c_s, v_t) = 0$ for all t and s.

[5] If we define the user cost, C, as $C_t \equiv (1-\tau)^{-1} p_t \left(R_t + \delta - \dot{p}_t / p_t \right)$ where τ is the effective tax rate, p is the relative price of capital, R is the risk-free rate and δ is the depreciation rate – then it is clear that certainly the relative price and possibly the tax rate and risk-free rate will be sensitive to the rate of capital accumulation.

This is the case of iid innovations. As alluded to above, endogeneity and adjustment costs are likely to mean that, rather than being iid, $\{\upsilon_t\}$ is (perhaps highly) persistent.[6] The same sort of correction can be applied in the case of serially correlated errors. If the error terms are serially correlated then biases may still occur – even if we estimate **(5)** because the error term can be correlated with the user cost at $t \neq s$. To correct for such biases we use the two-sided filter,

$$\eta_t \equiv \upsilon_t - \beta(L)\Delta c_t, \quad \beta(L) \equiv \sum_{j=-\infty}^{\infty} \beta_j L^j \tag{6}$$

In practice, because we have finite samples, we must truncate the filter at finite lag, p. In which case **(5)** becomes

$$\begin{aligned} k_t &= \sigma c_t + \beta \Delta c_t + \beta_{-1}\Delta c_{t+1} + \dots + \beta_{-p}\Delta c_{t+p} \\ &+ \beta_1 \Delta c_{t-1} + \dots + \beta_p \Delta c_{t-p} + \eta_t \end{aligned} \tag{7}$$

and, provided p lags and leads are sufficient to render Δc_t strictly exogenous, the user cost is also strictly exogenous.

The OLS estimator of σ in equation **(7)** is the Dynamic OLS estimator (DOLS) proposed by Saikkonen (1991) and Stock and Watson (1993). This is one way to correct for the biases inherent in single-equation estimates. Others have been proposed, and in what follows we shall make reference to the Fully Modified OLS (FM-OLS) estimator proposed by Phillips and Hansen (1990). This estimator attempts to estimate the bias using semi-parametric methods.

2.1.2 Are these biases quantitatively important?

It should be apparent that the importance of the problem will depend on the degree of contemporaneous correlation in the error terms and on the degree of serial correlation in the errors. Phillips and Hansen (1990) experiment with different values for degree of serial and cross correlation in the errors. Their results suggest that, while the static OLS estimator is biased, the bias is small in practice.

How can we reconcile this with the results in Caballero (1994), who shows that biases emanating from the same source can have much larger effects? He shows that, when trying to estimate a cointegrating vector of 1, the bias can be as large as 0.8 in sample sizes comparable to Phillips and Hansen (1990). The answer lies in the specification of the process for the errors. Phillips and Hansen study an MA(1) process. However, the dynamics in Caballero's model are driven by a

[6] This conjecture is supported by Monte Carlo evidence in Doms and Dunne (1998).

partial adjustment process. This process is much more persistent; indeed his adjustment parameter in one simulation is 0.025 (where the process becomes a random walk as this parameter approaches 0). In this case the dynamics will be close to MA(∞). As it is the sum of these MA terms that show up in the limiting distribution much larger biases can be present in heavily serially correlated processes (such as the capital adjustment process).[7]

2.2 Estimation in a panel setting

Estimation via single-equation methods are not the only solution to the problem estimating cointegrating vectors in the face of finite sample biases. Indeed Stock and Watson (1993) show that Johansen's (1991) systems estimator is asymptotically equivalent to FIML of the system and exhibits the least bias of any of the estimators considered in their Monte Carlo experiments (but the highest variance). However, we concentrate on the DOLS and FM-OLS estimators because they can be adapted in a reasonably straightforward way for use with our panel data.[8]

To see how this works in a panel setting, let us follow Pedroni (1995) and Phillips and Moon (1999) by extending the model of equation **(3)** to a panel setting,

$$\begin{aligned} k_{it} &= \gamma_i - \sigma c_{it} + \upsilon_{it}, \quad t = 1, .., T; \quad i = 1, ...N, \\ \Delta c_{it} &= \mu_i + u_{it} \end{aligned} \quad \textbf{(8)}$$

Note that, at this point, σ is assumed to be homogenous across N. We now define the error vector $\xi'_{it} = (\upsilon_{it}, u_{it})$ which is strictly stationary with $\mathbb{E}(\xi_{it}) = 0$. We pay special attention to the long-run covariance matrix of ξ_{it} which we define here as,

$$\boldsymbol{\Omega}_i = \sum_{k=-\infty}^{\infty} \mathbb{E}\left(\xi_{it}\xi'_{i,t+k}\right) \quad \textbf{(9)}$$

We partition $\boldsymbol{\Omega}_i$ as follows (but note that in our example all elements of partitioned matrix are scalars),

$$\boldsymbol{\Omega}_i = \begin{bmatrix} \boldsymbol{\Omega}_{\upsilon\upsilon i} & \boldsymbol{\Omega}_{u\upsilon i} \\ \boldsymbol{\Omega}_{u\upsilon i} & \boldsymbol{\Omega}_{\upsilon\upsilon i} \end{bmatrix} \quad \textbf{(10)}$$

[7] In Phillips and Hansen's example the sum of the MA terms is 0.8. When Caballero studies the adjustment process $\Delta K = \lambda\left(K_t^* - K_{t-1}\right)$ with $\lambda = 0.025$ the dynamics can be thought of as coming from an AR(1) process $K_t = c + (1-\lambda)K_{t-1} + \epsilon_t$. The equivalent MA representation is $K_t = (c/\lambda) + \epsilon_t + (1-\lambda)\epsilon_{t-1} + (1-\lambda)^2\epsilon_{t-2} + \cdots$. In which case the sum of the MA terms is $\sum_{j=0}^{\infty}\left|(1-\lambda)^j\right| = 40$.

[8] Given a large enough data set we would like to apply the panel Johansen-type ML estimator of Groen and Kleibergen (2003) but the single-equation methods we apply appear to be well behaved in smaller panels, such as the one we use.

note that this can be decomposed into, $\Omega_i = \Sigma_i + \Gamma_i + \Gamma_i'$, where

$$\Sigma_i = \mathbb{E}(\xi_{i0}\xi_{i0}') = \begin{bmatrix} \Sigma_{\upsilon\upsilon i} & \Sigma_{u\upsilon i} \\ \Sigma_{u\upsilon i} & \Sigma_{\upsilon\upsilon i} \end{bmatrix} \tag{11}$$

is the contemporaneous correlation matrix, and

$$\Gamma_i = \sum_{t=1}^{\infty} \mathbb{E}(\xi_{i0}\xi_{it}') = \begin{bmatrix} \Gamma_{\upsilon\upsilon i} & \Gamma_{u\upsilon i} \\ \Gamma_{u\upsilon i} & \Gamma_{\upsilon\upsilon i} \end{bmatrix} \tag{12}$$

is the sum of autocovariances. As shown in Pedroni (1995) the bias in the OLS estimator is due to long-run correlation between υ and u, which is captured by $\Omega_{u\upsilon}$. A sufficient condition for defining an asymptotically consistent estimator of σ in this panel setting is that the error term, ξ_i, satisfies a multivariate invariance principle (as in Phillips (1988)) such that the partial sum of the errors tends to a stochastic Brownian motion process as $T \to \infty$. This assumption places few restrictions on the time-series properties of ξ_i and allows for a broad class of stationary ARMA processes.

We are now in a position to define the estimators we shall use. We proceed with two panel estimators. One question here is whether $\sigma_i = \sigma$ for all i. If so, then pooling our data, as described in Kao and Chiang (2000) is appropriate. If not, then we should exploit between-group variation in the data. As pointed out in Pedroni (1995) this is important for three reasons. First, such estimators allow greater flexibility in the presence of heterogeneity of cointegrating vectors. In particular Pesaran and Smith (1995) show that an erroneous homogeneity assumption can lead to inconsistent estimates. Second, such 'between estimators' allow us to test hypothesis of the form $\mathbb{H}_0 : \beta_i = \beta_0$ for all i against $\mathbb{H}_1 : \beta_i \neq \beta_0$ rather than constraining all values to be one particular value *and then* testing if that value is the same as the hypothesised value. Third, Pedroni (1995) shows that test statistics have much less size distortion under the null hypothesis of heterogeneous cointegrating vectors. All this is particularly important in the current setting because it is clear that the UCE is likely to vary between industries.

We proceed with two such estimators: a mean-group DOLS estimator (the intuition for which was developed above; and Pedroni's (2001) grouped FM-OLS estimator.[9] The mean group

[9] Kao and Chiang (2000) provide some Monte Carlo evidence for homogenous cointegrating vectors in a panel setting. Their model has cross-sectional MA(1) dynamics – as in Phillips and Hansen (1990). They find that the pooled DOLS estimator does an excellent job of reducing the biases under a number of different parameterisations.

DOLS estimator $\widehat{\sigma}_{GDOLS}$ is taken from the following panel version of the DOLS equation,

$$\begin{aligned} k_{it} &= \sigma_i c_{it} + \beta \Delta c_{it} + \beta_{-1} \Delta c_{i,t+1} + ... + \beta_{-p} \Delta c_{i,t+p} + \beta_1 \Delta c_{i,t-1} \\ &+ ... + \beta_p \Delta c_{i,t-p} + \eta_{it}, \quad t = 1, .., T; \quad i = 1, ...N \end{aligned} \tag{13}$$

and is simply the average of the DOLS estimates for each industry, ie

$$\widehat{\sigma}_{GDOLS} = N^{-1} \sum_{i=1}^{N} \widehat{\sigma}_{DOLS,i} \tag{14}$$

where the DOLS estimator for each industry, $\widehat{\sigma}_{DOLS,i}$, is given by

$$\widehat{\sigma}_{DOLS,i} = \left[\sum_{i=1}^{N} \left(\sum_{t=1}^{T} z_{it} z_{it}' \right)^{-1} \left(\sum_{t=1}^{T} z_{it} \widetilde{k}_{it} \right) \right] \tag{15}$$

where z_{it} is a $2(p+1) \times 1$ vector of regressors, ie $z_{it} = [c_{it} - \overline{c_i},\ \Delta c_{i,t-p}, ..., \Delta c_{i,t+p}]$ and $\widetilde{k}_{it} = (k_{it} - \overline{k}_i)$. Consistency of this estimator follows from our invariance assumption about the process for the errors and is discussed in Phillips and Moon (1999).

The grouped FM-OLS is defined in a similar way and is given by,

$$\widehat{\sigma}_{GFMOLS} = N^{-1} \sum_{i=1}^{N} \widehat{\sigma}_{FMOLS,i} \tag{16}$$

where,

$$\widehat{\sigma}_{FMOLS,i} = \sum_{i=1}^{N} \left(\sum_{t=1}^{T} c_{it} - \overline{c}_i \right)^{-1} \left(\sum_{t=1}^{T} (c_{it} - \overline{c}_i) k_{it}^{\star} - T \widehat{\gamma}_i \right) \tag{17}$$

and

$$k_{it}^{\star} = \widetilde{k}_{it} - \frac{\widehat{\Omega}_{uvi}}{\widehat{\Omega}_{vvi}} \Delta c_{it} \tag{18}$$

$$\widehat{\gamma}_i = \widehat{\Gamma}_{uvi} + \widehat{\Omega}_{vui} - \frac{\widehat{\Omega}_{uvi}}{\widehat{\Omega}_{vvi}} \left(\widehat{\Gamma}_{vvi} - \widehat{\Omega}_{vvi} \right) \tag{19}$$

here the biases inherent in the OLS estimator are controlled for by the inclusion of $k_{it}^{\star}$ and $\widehat{\gamma}_i$.

3 Data

In this section we describe the data used in the estimation. We start by looking at the data we use for the aggregate regressions before turning to an outline of our industry-level data.

3.1 Aggregate data

We need three data series to perform the regressions described above for the business sector: value added, capital and the real cost of capital. National Accounts data are available for value added but we need to construct data for capital and the real cost of capital.

Our measure of capital is constructed as described in Oulton and Srinivasan (2003b) using the following formulae,

$$\ln\left[\frac{K_t}{K_{t-1}}\right] = \sum^i \overline{w}_{it} \ln\left[\frac{K_{it}}{K_{i,t-1}}\right], \quad \overline{w}_{it} \equiv \frac{\left(w_{it} + w_{i,t-1}\right)}{2}, \quad i = 1, ..., M \qquad \textbf{(20)}$$

where K_t is the volume of private sector capital services provided by all productive capital assets in period t. It is calculated as a Törnqvist aggregate of the capital services from M assets. Capital services provided by each asset i in period t, K_{it}, is proportional to the wealth stock at the end of the previous period, $A_{i,t-1}$, with a constant set to 1 such that,

$$K_{it} = A_{i,t-1} \qquad \textbf{(21)}$$

The wealth stock is computed as the sum of investment flows, such that,

$$A_{it} = \sum_{j=1}^{t}(1-\delta_i)^{t-j} I_{i,j} + (1-\delta_i)^t A_{i0} \qquad \textbf{(22)}$$

The starting stock, A_{i0}, (for 1948) is taken from Oulton (2001). The weights used in the aggregation of capital services are rental-price weights, w_{it}, given by,

$$w_{it} \equiv \frac{p_{it}^K K_{it}}{\sum^i p_{it}^K K_{it}} \qquad \textbf{(23)}$$

where p_{it}^K is the cost of capital defined by the standard Hall-Jorgensen formula,

$$C_t = p_t\left(R_{t+1} + \delta - \frac{p_{t+1}^K}{p_t^K}\right) \qquad \textbf{(24)}$$

Here p_t is relative price of capital goods: the business investment deflator divided by the GDP deflator. R_t is a measure of the real discount factor (our measure is a weighted cost of capital series, taken from Ellis and Price (2003)). Finally, δ is the depreciation rate and is given as a weighted average of the (assumed) depreciation rates on our M capital assets.

We also investigate how the UCE varies across different capital assets. To produce measures for different assets, the same procedures were followed but substituting the single asset equivalents of the measures above. In particular, given data for private sector investment in our individual assets (plant and machinery, buildings and ICT) we can build up capital stocks for each of these. And, for the cost of capital, we use the deflators for each asset and depreciation rates assumed for each asset. However arriving at consistent series for private sector investment by asset is not straightforward. These are not directly published statistics. Nominal investment in the ICT assets for 1992-2000 is extracted from the yearly Input-Output tables, published by the Office for National Statistics. For the period 1989-91 the data are obtained from earlier Supply and Use Tables, not published. Before 1989 the data are obtained from the Supply and Use Tables available for 1979 and 1984, and an interpolation method is used for intermediate years. Table A establishes unit roots in our data using the KPSS test and the DF-GLS test.

Table A: Unit root tests (Whole-economy data)

	KPSS test H_0: $I(0)$	DF-GLS test H_0: $I(1)$
Capital–output ratio for:		
Aggregate capital	1.61***	3.27
Equipment	1.24***	-0.38
Structures	1.51***	0.81
Vehicles	0.86***	-0.94
User-cost measure:		
Aggregate	1.35***	-0.81
Equipment	1.18***	-1.18
Buildings	0.62**	-1.87
Vehicles	0.51**	-1.28

DF-GLS notes: The 5% critical value for a DF-GLS is –1.94
KPSS notes: Long-run variance is estimated using a Bartlett Kernel with a lag window of 4. * (**,***) indicates rejection of the null at the 10% (5%, 1%) significance level.

3.2 Sectoral data

The principles above are applied for our sectoral data too. The data comes from the Bank of England industry data set (BEID), which contains data for 34 industries (31 market sector industries), from 1969 to 2000 (see Oulton and Srinivasan (2003a) for further details). For each industry, there are data on value added and, among other things, inputs of capital services. Capital services are available for three types of ICT and non-ICT assets (and, within these, for structures, plant, and vehicles). The real capital index is a rental-price weighted average – as above – of the growth rate of the different asset stocks and it is converted into a constant price series for capital services by setting real capital services in the base year, 2000. The cost of capital is given by,

$$C_{it} = p_{it}^K \left(R_{t+1} + \delta_i - \frac{p_{i,t+1}^K}{p_{it}^K} \right) \quad \textbf{(25)}$$

where p_{it}^K is the relative price of capital (investment deflator divided by the GDP value added deflator). Industry-level variation in this measure comes in the form of disaggregated prices and the depreciation rate; the rate of return are assumed not to vary by industry. In particular the depreciation rate is given by,

$$\delta_{it} = \sum_{j=1}^{M} r_{ijt}\delta_j \quad \textbf{(26)}$$

and is a weighted sum of depreciation weight on M capital assets, where the weights depend on the shares of each asset in total capital.

3.2.1 Unit root properties of the data

In order to establish the properties of our data set we perform unit root tests on the capital-output ratios. A number of tests are available here. We use that proposed by Im, Pesaran and Shin (2003, henceforth IPS).[10]

To construct the IPS test we must first estimate the following regression,

$$y_{it} = \alpha_i y_{i,t-1} + \delta_0 + \psi_t + \delta_1 t + \varepsilon_{it} \tag{27}$$

IPS show that the mean adjusted t-statistic for α_i is,

$$\widetilde{t}\text{-}bar_{NT} = N^{-1} \sum_{i=1}^{N} \widetilde{t}_{iT} \tag{28}$$

and has the following asymptotic distribution:

$$\sqrt{N}\, \frac{\widetilde{t}\text{-}bar_{NT} - \mu}{\sigma} \xrightarrow{d} N(0,1) \tag{29}$$

where μ and σ are estimates of the mean and variance of $\widetilde{t}\text{-}bar_{NT}$. This allows the development of a test for $\alpha_i = 1$ in all industries. The problem here is to find suitable estimates of μ and σ – this must be done using Monte Carlo methods and the values are taken from IPS (Table B). Table B displays the p-values for the test. As is clear we fail to reject the hypothesis that our capital-output ratios and user cost measures are $I(1)$ in (log) levels – even with a time trend. But we can reject this hypothesis at any level of significance for the first-differenced case implying our data are covariance stationary.

Table B: Im, Pesaran and Shin panel unit root tests

		Aggregate		Plant		ICT		Buildings	
		k_{it}	c_{it}	k_{it}	c_{it}	k_{it}	c_{it}	k_{it}	c_{it}
Levels	time–effects	0.73	0.83	0.97	0.40	0.97	0.99	0.99	0.88
	time–effects & trends	0.67	0.02	0.95	0.76	0.78	0.99	0.74	0.81
First	time–effects	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
differences	time–effects & trends	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: The table presents p–values from the Im, Pesaran and Shin test for unit roots in heterogeneous panels. Series are non-stationary under the null hypothesis. The test includes 1 lag. Sample: 1970–2000.

[10] We prefer the IPS test over, say, the Levin-Lin (1992) test because the IPS test allows for the possibility that the order of integration could be different in each of our groups and because the test has been shown to have greater power in small samples relative to the Levin-Lin test.

4 Results

As discussed in Section 2, our estimation results are based on empirical implementation of equation **(1)**. We apply the cointegrated system of **(3)** to aggregate and industry-level data.

4.1 Aggregate results

4.1.1 Results for aggregated capital

We start by estimating an aggregate relationship between the capital-output ratio and the user cost, as given by equation **(3)**. Given our priors, based on the discussion in Caballero (1994), a sufficient condition to proceed with estimation is that the variables are difference stationary.[11] This is easily satisfied in our data (see Table A). So we proceed by estimating this system using FM-OLS and DOLS, as discussed in Section 2. We repeat the estimation for the different capital assets discussed in Section 3. Our aggregate results are given in Table C. Our baseline, SOLS estimate for the σ is very close to -0.4, similar to other UK studies (see Ellis and Price (2003)). Such estimates will not, in general, be normally distributed (see discussion in Schaller (2006)) and, given the discussion above, will almost certainly be biased. We provide corrected standard errors but move onto estimation by DOLS and FM-OLS.

Table C: Target cost of capital elasticity (aggregate investment), private sector

Lags/ leads	0	5	9	13	17	21	25	29
FM–OLS	-0.38							
	(0.04)							
DOLS (lags)	-0.39	-0.38	-0.38	-0.36	-0.34	-0.33	-0.34	-0.36
	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)
DOLS (lags and leads)	-0.39	-0.37	-0.37	-0.35	-0.33	-0.30	-0.29	-0.27
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)

Notes: Sample: 1970 Q1 to 2005 Q2; all regressions include a constant. FM-OLS is Phillips-Hansen (1990) fully modified estimator using Bartlett kernel with up to 5 leads and lags; autocorrelation consistent standard errors are presented for DOLS estimates.

The FM-OLS are very similar to the static estimates. The estimates in Table C use the Bartlett kernel to estimate the asymptotic covariance matrix and hence the bias-adjustment term. But the result from the FM-OLS estimator is robust to the choice of kernel estimator and lag window

[11] In particular we do not perform cointegration tests. This is because, as noted in Caballero (1994), such tests will have low power in this instance as adjustment costs will imply that deviations from the optimal capital-output ratio will be persistent.

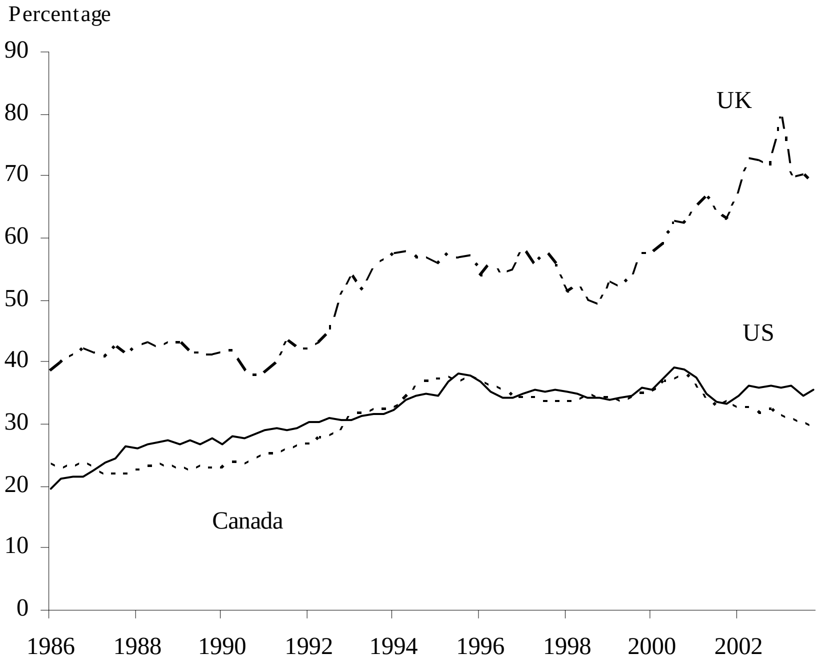
used in the estimation. One possibility is that the non-parametric correction for the biases inherent in the single-equation estimator are not sufficient. Monte Carlo evidence in Stock and Watson (1993) suggests that the DOLS estimator has the least RMSE in applications with significant serial correlation. So what does estimation of equation **(3)** via DOLS tell us? These results are also presented in Table C. Again, they suggest biases may not be a significant problem: as the table shows, adding up to 29 lags does not seem to alter our estimates significantly. On the face of it this seems puzzling: estimates in Caballero (1994) show evidence of sizable biases which disappear quickly as lags are added to the equation. This result is replicated for Canada by Schaller (2006).

What accounts for these differences? Either there is some inherent difference in the methodology applied here or the small-sample bias is not a problem in the UK data and the UCE is in the neighbourhood of -0.4. One issue may be data. Caballero (1994) uses the ratio of equipment capital to GDP on the left-hand side. Furthermore his user cost measure displays some differences: he used the T-bill rate as his measure of R and he uses the equipment deflator as his measure of the price of capital. What happens if we replicate Caballero's data as closely as possible using UK data? The results for equipment (plant and ICT assets) are shown in the top segment of Table D. As is clear, use of similar data does not resolve any puzzles, but rather exacerbates it. Using a similar number of observations, our UCE is now *smaller* and adding further lags drives the estimate towards zero. This process is not monotonic, however. Adding still further lags pushes the estimates back upwards; eventually approaching our baseline estimate of -0.4 – but only once we have added over 60 lags!

The puzzle deepens further when we consider the explanations of Schaller for the differences between the United States and Canada. Schaller observes that the UCE increases to around -1.5 when he adds further lags in his regressions. He puts the difference between his results and those of Caballero down to open-economy considerations: the user cost can be more plausibly thought of as exogenous in Canada as it has a higher proportion of imported investment goods than the United States. But, as shown in Chart 1, while the United States and Canada are reasonably similar in terms of the proportion of capital equipment they import, the UK imports a much higher proportion. By the logic in Schaller (2006) the UK results should be higher still.

Taken together the aggregate results are puzzling. First, our estimates of the UCE are far away from the unit elasticities found, using similar methods, for the United States and Canada. These

Chart 1: Percentage of imports in machinery and equipment investment



Sources: National Statistics; BEA, Statistics Canada.

Table D: DOLS estimates of the target cost of capital elasticity, private sector

Lags	0	5	9	13	17	21	25	29
Equipment (Caballero measure)								
DOLS (lags)	-0.29	-0.25	-0.21	-0.15	-0.10	-0.07	-0.03	-0.05
	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)
Plant & machinery								
DOLS (lags)	-0.42	-0.42	-0.42	-0.41	-0.38	-0.36	-0.35	-0.38
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
ICT assets								
DOLS (lags)	-1.23	-1.24	-1.26	-1.29	-1.33	-1.34	-1.38	-1.37
	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.04)
Buildings								
DOLS (lags)	-0.14	-0.14	-0.17	-0.21	-0.16	-0.15	-0.15	-0.17
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)

Notes: 1970:1 - 2005:2; all regressions include a constant. Standard errors in parentheses. The Caballero measure is an aggregate of plant and machinery and ICT capital with a user-cost measure similar to that used in Caballero (1994). Autocorrelation consistent standard errors are presented for DOLS estimates.

differences cannot be explained by differences in the data or methods used. Second, the biases that we expect to see in these equations do not disappear at similar rates to Caballero (1994) and Schaller (2006). Indeed, the corrections for these biases seem to, at least initially, push our estimates towards zero. But these effects are not monotonic and our estimates can vary between -0.1 and -0.5 depending on the number of lags and leads we use to correct for the biases.

4.1.2 Panel results for aggregated capital

The estimates based on aggregate data clearly reject the hypothesis of a unit cost of capital elasticity. The possibility remains, however, and that is we simply do not have a long enough span of data to purge our estimates of the inherent biases. We tackle this possibility by exploiting panel data. In this section we extend our simple specification to panel setting. As discussed in Section 2.2, we use a class of single-equation panel estimators suitable for cointegrated data. In particular we start by estimating a simple pooled-panel model and then we demonstrate the biases inherent in these estimates via fixed-effects FM-OLS and DOLS models. Initially we simply estimate these models for all physical capital, but extend our specification to allow for the separate treatment of different capital assets.

Our starting point is to estimate the UCE, for all physical capital, for 31 market sector industries from 1970-2000. Our initial baseline estimates are shown in Table E. We start by estimating, for reference, a simple fixed-effects model,

$$k_{it} = \gamma_i - \sigma c_{it} + v_{it}, \quad t = 1, .., T; \quad i = 1, ...N \tag{30}$$

Note that, in this model, σ is assumed to be homogenous. As discussed above, the pooled estimator of σ is consistent but will have a second-order bias problem that may not be removed, even in large samples. Accordingly, our initial panel estimate is for an elasticity of -0.1. This estimate is (statistically) significantly lower from our estimates based on aggregate data. But, given the known problems with the estimator we put little weight on this result. We move onto estimating the pooled DOLS and FM-OLS estimators, proposed by Phillips and Moon (1999) and Kao and Chiang (2000).[12] The pooled FM-OLS estimator gives an estimate of -0.3; the DOLS of between -0.4 and -0.6, depending on lag-truncation chosen.[13] Monte Carlo evidence presented by Kao and Chiang (2000) suggests that the DOLS estimator does a better job of removing biases compared with the FM-OLS estimator in panels of similar size to ours. Given this, our initial panel estimates deliver very similar estimates to those based on aggregate data.

As discussed above these estimates will not, in general, be consistent estimates of the mean cointegrating vector if the actual UCE is heterogeneous. It seems likely that cointegrating vectors will vary across industries. But we can test this conjecture statistically: an F-test based on the restriction that all the cross-sectional UCE equal rejects the homogeneity restriction at any level of significance. This implies cross-sectional heterogeneity is an important feature of our data. Given this we use the mean-group FM-OLS and DOLS estimators of Pedroni (2001).

Results from these estimators suggest our pooled estimators are overestimating the UCE. One important assumption that is maintained in the derivation of the limiting distribution of the various estimators discussed above is that the errors are independent across cross-sectional units. As the price of investment goods are affected by aggregate shocks, it is not clear *a priori* that this assumption will be satisfied. To address this we re-estimate these models with common time effects. As shown in Table B, our data appear integrated of order one, even with the inclusion of time effects. While the pooled results do not change with the inclusion of time effects the mean-group estimators suggest that the UCE is very close to -0.4, and could be slightly higher

[12] Thanks is given to Min-Hsien Chiang and Chihwa Kao for making their programs for estimating pooled panel-models available. See Chiang and Kao (2002) for details.

[13] Unreported results using both lags and leads give near-identical results.

Table E: Panel estimates of the target cost of capital elasticity

Lags	1	2	3	4	Lags	1	2	3	4
All assets					*Buildings*				
POLS	−0.13				POLS	−0.30			
	(−6.0)					(−15.2)			
PFMOLS	−0.30				PFMOLS	0.11			
	(−11.3)					(6.0)			
PDOLS	−0.40	−0.45	−0.49	−0.60	PDOLS	−0.61	−0.60	−0.59	−0.64
	(−21.5)	(−15.8)	(−16.7)	(−19.8)		(−39.4)	(−33.1)	(−31.3)	(−32.3)
BFMOLS	−0.19				BFMOLS	−0.16			
	(−10.9)					(−8.5)			
BDOLS	−0.17	−0.17	−0.09	−0.09	BDOLS	−0.13	−0.12	−0.07	−0.05
	(−9.8)	(−9.5)	(−5.2)	(−4.9)		(−12.6)	(−13.6)	(−16.0)	(−15.7)
Plant					*ICT assets*				
POLS	−0.34				POLS	−1.24			
	(−9.8)					(−45.3)			
PFMOLS	0.40				PFMOLS	−0.58			
	(8.1)					(−14.2)			
PDOLS	−0.85	−0.85	−0.84	−0.88	PDOLS	−0.92	−0.94	−0.95	−0.97
	(−27.1)	(−17.4)	(−16.6)	(−16.8)		(−14.1)	(−13.7)	(−13.4)	(−13.2)
BFMOLS	−0.41				BFMOLS	−1.39			
	(13.4)					(−92.7)			
BDOLS	−0.49	−0.56	−0.60	−0.62	BDOLS	−1.40	−1.44	−1.47	−1.51
	(−16.9)	(−17.9)	(−20.1)	(−28.5)		(−84.7)	(−86.0)	(−88.0)	(−92.1)

Notes:
POLS: Pooled OLS estimator (least-squares dummy variable estimator);
PFMOLS: Pooled FMOLS proposed by Phillips and Moon (1999);
PDOLS: Pooled DOLS, Mark and Sul (2003);
BFMOLS: Mean-group FMOLS estimator, Pedroni (2000);
BDOLS: Mean-group DOLS estimator, Pedroni (2001).
Asymptotic t-statistics in parentheses. For DOLS and FM-OLS estimators these are derived using a Bartlett kernel estimate of the long-run variance matrix with lag window of 5. Estimates use data from 31 annual observations (1970-2001) and 31 market sector industries.

depending on the lag-length chosen in the DOLS regressions.[14] Taken together these results suggest that there is no reason to think that our initial aggregate time-series estimates of the UCE are misleading. Indeed, our results provide very similar results but with a higher degree of precision: based on the estimates using the mean-group DOLS estimator we can reject the hypotheses that the UCE is -0.3 or -0.5 at the 1% significance level.

[14] For a discussion of the issues related to choosing the number of lags and/or leads to use in a DOLS regression, see Westerlund (2005). In general, one wants to choose as large a number of lags as possible. But choosing too many lags can result in a deterioration of the small-sample properties of the estimators. Information criterion can help but these suggest relatively few lags should be added (1 or 2) in the present application.

4.1.3 *Panel results for individual capital assets*

What about our estimates for other assets? We undertake the same estimation for plant, buildings and ICT capital. In all cases, the pooled OLS estimator appears significantly biased. The pooled FM-OLS – without common time effects – is also inconsistent with the other estimates presented in Table E and Table F. For buildings investment, aggregate estimates of the UCE are very low – close to -0.15. The pooled estimates – particularly when common time effects are included – show a much larger estimate. For example, the pooled DOLS estimate is close to –0.6 without common time effects and close to -0.8 with them included. The between dimension estimators are somewhat smaller, however: with time effects the DOLS estimator is around -0.3. As pointed out by Caballero (1994), the degree of bias in estimates of the UCE will depend on the degree of serial correlation (adjustment costs) and endogeneity of the user cost. With a fixed supply of land and high transaction costs for buildings, we put weight on the result from our panel estimates that the UCE may be larger for buildings. However, given apparent heterogeneity in the data set, the pooled estimators seem misleading.

Table F: Panel estimates of the target cost of capital elasticity, including common time effects

Lags	1	2	3	4	Lags	1	2	3	4
All assets					*Buildings*				
POLS	−0.11 (−4.8)				POLS	−0.43 (−18.1)			
PFMOLS	−0.36 (−12.5)				PFMOLS	−0.76 (−29.3)			
PDOLS	−0.42 (−13.7)	−0.48 (−15.3)	−0.54 (−16.6)	−0.67 (−19.8)	PDOLS	−0.78 (−28.8)	−0.77 (−27.7)	−0.75 (−26.0)	−0.78 (−26.0)
BFMOLS	−0.40 (−14.3)				BFMOLS	−0.44 (−17.0)			
BDOLS	−0.41 (−12.3)	−0.47 (−11.4)	−0.44 (−15.4)	−0.58 (−11.6)	BDOLS	−0.34 (−18.7)	−0.33 (−15.5)	−0.26 (−12.8)	−0.23 (−12.4)
Plant					*ICT assets*				
POLS	−0.34 (−9.5)				POLS	−0.67 (−23.9)			
PFMOLS	−0.85 (−17.5)				PFMOLS	−0.14 (−3.2)			
PDOLS	−0.85 (−17.1)	−0.85 (−16.6)	−0.85 (−15.8)	−0.88 (−15.8)	PDOLS	−0.12 (−2.9)	−0.15 (−3.2)	−0.16 (−3.5)	−0.20 (−4.1)
BFMOLS	−0.26 (−18.3)				BFMOLS	−0.71 (−23.9)			
BDOLS	−0.26 (−13.5)	−0.29 (−14.2)	−0.26 (−21.1)	−0.26 (−20.1)	BDOLS	−0.68 (−32.9)	−0.63 (−32.3)	−0.54 (−32.4)	−0.46 (−31.6)

Notes: Asymptotic *t*-statistics in parentheses. For DOLS and FM-OLS estimators these are derived using a Bartlett kernel estimate of the long-run variance matrix with lag window of 5. Estimates use data from 31 annual observations (1970-2001) and 31 market sector industries.

For plant and machinery capital, our baseline estimates from aggregate data seem broadly consistent with the results from our panel. The pooled estimates show a wide dispersion of estimates: the pooled FM-OLS estimator, without time effects, is significantly *positive* at 0.4 whereas the pooled DOLS estimate is close to -0.9. But, the between estimators seem to give estimates in a much tighter range, centred around the time-series estimate. Given this the mean-group DOLS estimator suggests that the UCE might be somewhat smaller for plant and machinery capital.

Elasticities for ICT capital have typically been found to be large. For example, Tevlin and Whelan (2003) found an elasticity close to -1.6 and McMahon *et al* (2003) found an elasticity close to -1.3. The latter, based on UK data, is in-line with our aggregate estimates. Do our panel estimates coincide with these? Again, there is a range of elasticities. But the estimates with time effects are much smaller and, in particular, the mean-grouping estimators are significantly smaller than -1. How do we interpret this? Clearly, time-dependence between the cross-sectional units has an influence on our results; common shocks appear to be particularly important.

The panel estimates are broadly consistent with those from aggregate data. In terms of the parameter of interest – the private sector UCE – our regressions on panel data add weight to the hypothesis that the values are close to -0.4. Importantly there is no evidence of a unit elasticity for the United Kingdom. Asset-level panel regressions suggest that the individual results for each asset are closer to our benchmark elasticity estimate. Our favoured estimate for buildings is larger than in the aggregate data whereas those for plant and ICT appear somewhat smaller. While these estimates are still statistically distinct, they are closer than those typically found using evidence based on aggregate data.

4.1.4 Sensitivity analysis: the influence of cross-sectional dependence

As pointed out above, one important assumption that is maintained in the derivation of the limiting distribution of the various estimators discussed above is that, conditional on common time effects, the errors are independent across cross-sectional units: that is $\{v_{it}\}_{t=0}^{T}$ in equation **(8)** are independent across i. The evidence on the empirical implications of violating this assumption is mixed. Pesaran (2006) finds that the performance of naïve estimators that ignore common effects are substantially biased when such effects are present. On the other hand, Coakley *et al* (2001) and Coakley *et al* (2005) present Monte Carlo evidence that suggests such effects may be less severe. A critical difference between these studies is that Pesaran (2006) allows for a multiple common factors; if cross-sectional dependence is the result of common shocks then common time effects may be sufficient to ameliorate the problem of cross-sectional dependence. On the face of it this possibility seems plausible in the current setting: the common element in the capital-output ratio and the user cost can be thought of as deriving from a common shock.

If this is not the case, however, our results may be affected. To address this possibility we estimate the UCE using Pesaran's (2006) common correlated effect mean group (CCEMG) estimator. Given the dimensions of our panel, Pesaran (2006) shows that the CCEMG estimator exhibits substantially less cross-sectional bias than the standard mean-group estimator when cross-sectional dependence is the result of multiple factors. Furthermore, this result holds even if the errors are difference stationary (see Kapetanios *et al* (2006)). Against this we are now estimating the long-run association between the capital-output ratio and user cost and our regression no longer has the interpretation of a cointegrating vector. Indeed, if our estimates conform to the assumptions of the panel DOLS estimator above – namely that there is cointegration at the level of the cross-sectional unit and that the innovations are independent across industries – the CCEMG estimator itself may well be biased.

The CCEMG estimator is calculated by estimating the following regression for each cross-section,

$$k_{it} = \gamma_i - \sigma_i c_{it} + \beta_1 \overline{k}_t + \beta_1 \overline{c}_t + v_{it} \quad \textbf{(31)}$$

where

$$\overline{k}_t = N^{-1} \sum_{i=1}^{N} k_{it}, \quad \text{and} \quad \overline{c}_t = N^{-1} \sum_{i=1}^{N} c_{it} \quad \textbf{(32)}$$

The CCEMG is then given by,

$$\widehat{\sigma}_{MG} = N^{-1} \sum_{i=1}^{N} \widehat{\sigma}_i \quad \textbf{(33)}$$

where $\widehat{\sigma}_i$ is simply the OLS estimator of σ_i. As such the CCEMG estimator amounts to the standard linear mean group estimate of σ on data filtered using cross-sectional means in order to purge the data of the influence of common effects. Pesaran (2006) shows that a consistent (non-parametric) estimate of the variance of the estimator is given by,

$$\widehat{\Sigma}_{MG} = \frac{1}{N-1} \sum_{i=1}^{N} (\widehat{\sigma}_i - \widehat{\sigma}_{MG})^2 \quad \textbf{(34)}$$

Table G shows the results from the CCEMG estimator for each asset. One conclusion that emerges is that the inclusion of the common time effects does not change the results significantly. The only exception to this, is the case of ICT assets, here the CCEMG estimator reduces the size of the estimate dramatically: from -1.4 to -0.2. However all the estimates for the CCEMG estimator presented in the second column of Table G are statistically indistinguishable (at the 5% level) from the estimates based on the group-mean DOLS estimates – with common time effects – for the corresponding assets. As such it is evident that accounting for possible cross-sectional dependence, at least on the basis suggested by Pesaran (2006), does not change the inference made above with respect to the UCE.

Table G: Common correlated effects estimates of the target cost of capital elasticity

Asset	*MG estimate*	*CCEMG estimate*
All	−0.17	−0.21
	(0.10)	(0.08)
Buildings	−0.16	−0.34
	(0.10)	(0.10)
Plant	−0.37	−0.43
	(0.19)	(0.20)
ICT assets	−1.38	−0.23
	(0.09)	(0.13)

Notes: Table compares estimates using Pesaran's (2006) CCEMG estimator with those from a standard MG estimate of **(31)**. Standard errors (given by equation **(33)** are shown in parentheses.

5 Concluding remarks

A great deal of effort has been employed in estimating the effect on investment of its price – not least because, under standard assumptions, we are also estimating the elasticity of substitution between capital and labour. Despite this, the elasticity has proved elusive; estimates vary greatly. Recent time-series work for the United States and Canada has suggested that, once the biases inherent in estimating cointegrating vectors are properly accounted for, the user cost elasticity could, indeed, be close to unity. The contribution of this paper is to provide a robust investigation of whether these results carry over to the United Kingdom. Our aggregate regressions suggest that the UCE is close to -0.4. This implies that our estimate of the elasticity of substitution between capital and labour is economically and statistically different from the neoclassical benchmark of a unit elasticity. Furthermore, estimates based on different capital assets suggest that the UCE differs greatly by asset.

It is possible that either biases inherent in estimating cointegrating vectors are very large in the United Kingdom or that our data is affected by aggregation bias. Either way, we can do better by exploiting panel data. We use unique industry panel data set for the United Kingdom to investigate these possibilities. This greatly improves the power of our test statistics. But we must be careful to adequately account for the possible biases introduced by the related problems of cross-sectional heterogeneity and cross-sectional dependence. The results from using our panel data for aggregate capital are clear: our aggregate estimates cannot be rejected but the unit elasticity can. However, the data from our panel does suggest that the dispersion of UCE estimates between different capital assets could be narrower than previously thought.

Appendix A: The neoclassical theory of capital accumulation

In this section we consider a simple version of the firm's problem. We assume all markets are competitive, information is symmetric and the firm's financial policy does not affect investment (ie the Modigliani-Miller theorem holds): the firm is able to issue as much debt as it chooses at an exogenous risk-free interest rate, R. We distinguish between capital assets, labour inputs and intermediate inputs. We will start with a version of the firm's problem in which there are no internal costs of adjustment. The firm maximises the value of future revenues, according to the following Bellman equation:

$$V_t(K_{t-1}) = \left\{ \max_{I_t, L_t, M_t} X_t\left(K_t, L_t, M_t, I_t\right) + \beta_{t+1}\mathbb{E}_t\left[V_{t+1}\left(K_t\right)\right] \right\} \quad \textbf{(A-1)}$$

where V_t is the value of the firm in period t, $X_t\left(K_t, L_t, M_t, I_t\right)$ is the firm's net revenue function, depending on inputs of capital K_t, labour L_t, intermediate goods M_t and the quantity of investment I_t. We can allow for multiple inputs by assuming that K, L and M are vectors. β_{t+1} is the discount factor and is given by $\beta_{t+1} \equiv (1 + R_{t+1})^{-1}$, where R is the risk-free interest rate. The relevant constraint for this problem is the capital accumulation identity,

$$K_{it} = (1 - \delta_i)K_{i,t-1} + I_{it} \quad \text{for } i = 1, ..., N \quad \textbf{(A-2)}$$

Here δ_i is the depreciation rate for each type of capital asset. The revenue function has the form,

$$X_t = p_t F(K_t, L_t, M_t) - p_t^K I_t - w_t L_t - p_t^M M_t \quad \textbf{(A-3)}$$

where $F(\cdot)$ is the production function, p_t is the price of output, p_t^K is an N dimensional vector of prices of capital assets, w_t is the wage rate and p_t^M is a vector of intermediate goods prices.

This problem gives rise to the following first-order necessary conditions:

$$\frac{\partial V_t}{\partial I_{it}} = 0 \Rightarrow \frac{\partial X_t}{\partial I_{it}} = -\lambda_{it} \quad \text{for } i = 1, ..., N \quad \textbf{(A-4)}$$

$$\frac{\partial V_t}{\partial K_{it}} = 0 \Rightarrow \frac{\partial X_t}{\partial K_{it}} + \beta_{t+1}\mathbb{E}_t\frac{\partial V_{t+1}}{\partial K_{it}} = \lambda_{it} \quad \text{for } i = 1, ..., N$$

$$\textbf{(A-5)}$$

$$\Rightarrow \frac{\partial X_t}{\partial K_{it}} + (1 - \delta_i)\,\beta_{t+1}\mathbb{E}_t\left[\lambda_{i,t+1}\right] = \lambda_{it}$$

$$\frac{\partial V_t}{\partial L_t} = 0 \Rightarrow \frac{\partial X_t}{\partial L_t} = 0 \quad \textbf{(A-6)}$$

$$\frac{\partial V_t}{\partial M_t} = 0 \Rightarrow \frac{\partial X_t}{\partial M_t} = 0 \quad \textbf{(A-7)}$$

where λ_{it} is the shadow value of receiving an extra unit of capital asset i in time t. If we assume perfect competition,[15] then $\frac{\partial X_t}{\partial I_{it}} = -p_{it}^K$ and $\frac{\partial X_t}{\partial K_{it}} = p_t \frac{\partial F}{\partial K_{it}}$. Using these conditions to eliminate λ_{it} and $\mathbb{E}_t \left[\lambda_{i,t+1}\right]$ from equations **(A-4)** and **(A-5)** we have,

$$\frac{\partial F}{\partial K_{it}} = \frac{p_{it}^K}{p_t}\left[1 - \left(\frac{1-\delta_i}{1+R_{t+1}}\right)\mathbb{E}_t\left(\frac{p_{i,t+1}^K}{p_{it}^K}\right)\right] \equiv C_{it} \qquad \textbf{(A-8)}$$

This equation shows that, setting aside the treatment of taxes, in the absence of adjustment costs and if prices are set in competitive markets, firms will adjust capital to equate its marginal product with the real user cost.

How do we turn this relationship into one we can test? We start by assuming that capital is only comprised of one good and output is produced using a constant elasticity of substitution (CES) production function: ie $F(\cdot)$ is given by,

$$Y_t = \left[\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \qquad \textbf{(A-9)}$$

where σ is the elasticity of substitution. This implies,

$$\frac{\partial F}{\partial K_t} = \alpha\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} = C_t \qquad \textbf{(A-10)}$$

which leads to the following investment equation (lower-case variables are in logs),

$$k_t = \mu + y_t - \sigma c_t \qquad \textbf{(A-11)}$$

where $\mu = \sigma \ln \alpha$.

Given the discussion thus far, we can estimate equation **(A-11)** using the techniques described in Section 2. However, we would like to extend this logic to the multi-asset environment. A multi-asset version of equation **(A-9)** takes the following form

$$Y_t = \left[\sum_{i=1}^N \alpha_i K_{it}^{\frac{\sigma-1}{\sigma}} + \left(1 - \sum_{i=1}^N \alpha_i\right) L_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \qquad \textbf{(A-12)}$$

this implies that we have an N equation system:

$$\begin{aligned} k_{1t} &= \mu_1 + y_t - \sigma c_{1t} \\ &\vdots \\ k_{Nt} &= \mu_N + y_t - \sigma c_{1t} \end{aligned} \qquad \textbf{(A-13)}$$

[15] Or a constant global mark-up.

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