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Dynamics of the term structure of UK interest rates
Francesco Bianchi, Haroon Mumtaz and Paolo Surico

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Abstract

This paper models the evolution of monetary policy, the term structure of interest rates and the UK economy across policy regimes. We model the interaction between the macroeconomy and the term structure using a time-varying VAR model augmented with the factors from the yield curve. Our results suggest that the level, slope and curvature factors display substantial time variation, with the level factor moving closely with measures of inflation expectations. Our estimates indicate a large decline in the volatility of both yield curve and macroeconomic variables around 1992, when the United Kingdom first adopted an inflation-targeting regime. During the inflation-targeting regime, monetary policy shocks have been more muted and inflation expectations have been lower than in the pre-1992 era. The link between the macroeconomy and the yield curve has also changed over time, with fluctuations in the level factor becoming less important for inflation after the Bank of England independence in 1997. Policy rates appear to have responded more systematically to inflation and unemployment in the current regime. We use our time-varying macro-finance model to revisit the evidence on the expectations hypothesis.

Key words: Term structure, time-varying VAR, Bayesian estimation.

JEL classification: E50, E58.
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Summary

A number of recent papers have analysed the evolving dynamics of output and inflation using systems of equations known as vector autoregressions (VARs): a set of equations where the explanatory variables in each equation are the complete set of lagged variables in the system. GDP growth, inflation and the nominal interest rate are the typical variables included in VARs that describe the transmission mechanism of monetary policy. These empirical models are subject to the criticism that they include a limited amount of information. If, in reality, the central bank examines a wider set of variables when setting policy, estimates of the monetary policy shock derived from these small empirical models may be biased – ie not completely disentangled from non-policy shocks. As a consequence an accurate assessment of structural shifts may be hampered.

The aim of this paper is to investigate the evolution of UK macroeconomic dynamics using a VAR model that is less susceptible to this criticism. In particular, we augment the standard three-variable VAR with variables that describe the level, slope and curvature of the yield curve, which shows the pattern of interest rates at different maturities. These yield curve variables contain information about private sector expectations. This additional information may alleviate the biases referred to above by ensuring that the forward-looking aspect of monetary policy is accounted for in our empirical model. In addition, we allow the relationship between the yield curve and the macroeconomy (embodied in our VAR) to change over time. We use this model to investigate how the dynamics of UK macroeconomic variables have changed over time and how these changes are related to changing properties of the yield curve.

The main results can be summarised as follows. First, the level, slope and curvature factors display substantial time variation, with the level factor moving closely with measures of inflation expectations. Second, our estimates indicate a large decline in the volatility of both yield curve and macroeconomic variables around 1992, when the United Kingdom adopted inflation targeting. Third, and more important, during the inflation-targeting regime, monetary policy shocks have been more muted and inflation expectations have been lower than in the pre-1992 era. Fourth, the link between the macroeconomy and the yield curve has also changed over time, with fluctuations in the level factor becoming less important for inflation after Bank of England independence in 1997. In particular, policy rates appear to have responded more systematically...
to inflation and unemployment in the current regime. Finally, we use our time-varying macro-finance model to revisit the evidence on the expectations hypothesis (ie the hypothesis that in any given period the yield on a long-maturity bond is equal to the discounted sum of the expected yields on short-maturity bonds over the lifetime of the long-maturity bond). Our results suggest that time-varying dynamics in both the yield curve and the structure of the economy may explain part of the deviations from the expectation hypothesis found in fixed-coefficient models.
1 Introduction

After the introduction of the inflation-targeting regime in 1992, the United Kingdom experienced a significant change in the dynamics of main macroeconomic variables. Benati (2004) provides a comprehensive description of these changes.

The possible role played by monetary policy in bringing about this change in inflation dynamics and volatility has been analysed in a series of papers, both for the United States and the United Kingdom. For example, Cogley and Sargent (2002) report a significant change in the degree of ‘activism’ of US monetary policy. As in Clarida, Gali and Gertler (2000), the authors argue that the fall in the level and persistence of US inflation in the 1980s and the 1990s coincided with an increase in the degree of activism. Some of the subsequent literature has been less favourable to this ‘good policy’ hypothesis. For example, the evidence on US policy activism reported in Cogley and Sargent (2005) and based on an extended model is less clear cut than the authors’ earlier work. Primiceri (2005) suggests that ‘planting Greenspan in the 1970s’ would have had little impact on inflation during that period. Similarly Sims and Zha (2006) show that a model that allows for variation in the volatility of shocks fits US data better than a model that allows for a change in the monetary policy rule.¹

Although research on this issue is still at an early stage for the United Kingdom, initial results are equally contentious. For example, using a time-varying structural VAR, Benati (2008) shows that a fall in the volatility of demand and supply shocks can explain most of the recent stability in the United Kingdom’s output and inflation. However, the arguments in Castelnuovo and Surico (2006) and Benati and Surico (2007) suggest that these results may be the outcome of model misspecification. In particular, Castelnuovo and Surico (2006) and Benati and Surico (2007) argue that the amount of information incorporated in these VAR models is relatively limited. Typically, the VAR models used in these studies (eg Benati (2008)) consist of three or four variables – usually a short-term interest rate, output growth and inflation. This feature has two potential consequences. Firstly, missing variables could lead to biases in the reduced form VAR coefficients. Secondly, the omission of some variables could hinder the correct identification of structural shocks. For example, Lubik and Schorfheide (2004), show that when the Taylor principle is not satisfied (ie the monetary authority accommodates inflationary pressure), the

¹Note that this evidence is mostly based on time-varying VAR models. Based on a New Keynesian DSGE model Lubik and Schorfheide (2004) provide evidence in favour of a policy shift in the United States.
dynamics of the economy in a DSGE model are characterised by a latent variable. Lubik and Schorfheide (2004), Castelnuovo and Surico (2006) and Benati and Surico (2007) show that this latent variable is a function of inflation expectations and that the interpretation of structural VAR estimates may be misleading if expectations are not taken into account directly.

The aim of this paper is to use a time-varying VAR model that is less susceptible to this problem. In particular, this paper examines the changing dynamics of the UK economy using a time-varying VAR model that incorporates information about inflation expectations extracted from the term structure of interest rates. We augment a standard time-varying VAR model with factors extracted from the term structure to form a factor augmented VAR (FAVAR). These factors summarise information about the level and shape of the yield curve and, as our results show, the level of the yield curve is strongly correlated with measures of inflation expectations. By using this augmented VAR model, our aim is to minimise the possible omitted variable bias referred to above.

The basic premise of our paper is in line with a number of recent studies that have used similar models to highlight the link between the yield curve and the macroeconomy. Recent examples include Diebold, Rudebusch and Aruoba (2006b) and Diebold and Li (2006), who use a generalised version of the Nelson-Siegel model to show this link for the United States. Joyce, Kaminska and Lildholdt (2008) estimate a variety of affine term structure models for the United Kingdom over the period 1992-2006 and find that lower term premia account for the fall in long real rates during 2004 and 2005, the so-called Greenspan’s ‘conundrum’. Rudebusch and Wu (2007), Diebold, Li and Yue (2006a) and Cogley (2004) show that the dynamics of the US yield curve have changed over time. For the United Kingdom, Lildholdt, Panigirtzoglou and Peacock (2007) investigate historical fluctuations in the yield curve and try to determine if these are due to changes in the inflation target or monetary policy shocks.

The contribution of this paper is twofold. First, the analysis in this paper brings together the latest developments in the macro-finance literature on the bi-directional feedback between the yield curve and the economy, and the observation that both sides of this relationship have been historically characterised by substantial instabilities. We specify the link between macro and finance as in the Nelson-Siegel generalisation by Diebold et al (2006b), and model both the interactions and the evolution of the factors using time-varying coefficients and stochastic volatilities. Second, to our knowledge, this is the first paper that provides systematic
investigation into shifts in the link between the economy and the yield curve for the United Kingdom. In addition, this paper is one of the first to use information from the yield curve in an analysis of the ‘great stability’.

The main results can be summarised as follows. First, the estimated yield curve ‘factors’ display substantial time variation with the level factor moving closely with the one year ahead inflation forecasts produced by the National Institute of Economic and Social Research (NIESR). Second, the volatility associated with macroeconomic variables and the yield curve has declined over time with the inflation-targeting regime experiencing the most stability. Third, variance decomposition and impulse response analysis points to important changes in the practice of monetary policy in the United Kingdom. Finally, the addition of time variation in our FAVAR model leads to estimates of theoretical yields that are very close to actual data with deviations from the expectations hypothesis rare over our sample.

As for the UK great stability, our results point towards a remarkable improvement in the conduct of monetary policy, exemplified by a large and significant decline in the volatility of the policy shocks. Furthermore, the variance decomposition analysis reveals that the monetary policy shock accounted for about 70% of fluctuations in our measure of inflation expectation during the inflation peak of the early 1980s. After the introduction of the inflation-targeting framework in 1992, in contrast, the volatility of the policy shocks was smaller and associated with significantly lower inflation expectations.

The paper has four sections. Section 2 describes a generalisation of the Nelson-Siegel model using a FAVAR with time-varying coefficients and stochastic volatilities. Section 3 describes the data set used in the paper. The empirical results are presented in Section 4 while Section 5 concludes. Details on the estimation procedure are provided in the appendix.

2 Modelling yield curve and macro dynamics

Earlier empirical contributions based on US data have shown that the dynamics of the yield curve and key macroeconomic variables have evolved significantly over time. This is particularly true if the selected sample covers most of the post second world war period. While the recent

[2] Note, however, that a large number of recent papers have applied a variety of macro-finance models to UK data within a fixed coefficients framework. See for example, Lildholdt et al (2007).
macro-finance literature has convincingly advocated the case for the existence of a bi-directional link between the term structure and the rest of the economy, to the best of our knowledge no studies have yet tried to model time variations in the yield curve and the economy simultaneously. To this end, we design a generalisation of Nelson-Siegel interpolation in the context of a FAVAR model with time-varying coefficients and stochastic volatilities. It is worth emphasising that we also allow for time variation in the cross-correlations between macro and financial factors.

2.1 A generalisation of Nelson-Siegel model

Our model is a generalisation of the latent dynamic factor model used in Diebold et al (2006b). Following Nelson and Siegel (1987), Diebold et al (2006b) assume that information about the term structure of interest rates can be summarised by three factors that represent the ‘level’, ‘slope’ and ‘curvature’ of the yield curve. They include these yield curve factors and measures of real activity, inflation and the central bank rate in a VAR model which is used to model the interaction between these variables. We generalise this approach by allowing the parameters of the VAR model to be time varying.

An intuitive way to represent our model is to cast it into state-space form. The observation equation of the state-space system is based on the yield curve model developed by Nelson and Siegel (1987):

\[ y_t(\tau) = L_t + \frac{1}{\tau\lambda}S_t + \left( \frac{1}{\tau\lambda} - e^{-\tau\lambda} \right) C_t + e_t(\tau) \]  

where \( y_t(\tau) \) denotes yields at maturity \( \tau \) and \( L_t, S_t \) and \( C_t \) denote the (unobserved) level, slope and curvature factors. Equation (1) relates the yield data to the unobserved factors.

The dynamics of these factors are described by the following time-varying VAR

\[ Z_t = \alpha_t + \sum_{p=1}^{P} \beta_{t,p} Z_{t-p} + v_t \]  

where \( Z_t = \{ L_t, S_t, C_t, U_t, \pi_t, R_t \} \) denotes the data matrix and \( v_t = \omega_t \Omega_t^{1/2} \) with \( \omega_t \sim N(0, I) \). Note that along with the unobserved factors, \( Z_t \) contains three macroeconomic variables: the unemployment rate (\( U_t \)), annualised monthly inflation (\( \pi_t \)) and the policy interest rate (\( R_t \)).

Following Cogley and Sargent (2005) and Primiceri (2005) among others, we postulate a random

\[ \Omega_t = \sigma_t^2 \Omega \]

\[ \Omega \] is positive semi-definite and \( \sigma_t \) is a positive time varying parameter.

We choose the unemployment rate as our monthly ‘real activity’ indicator mainly because the volatility in variables such as industrial production resulted in estimation difficulties.
walk for the evolution of the VAR coefficients:

\[ \Phi_t = \Phi_{t-1} + \eta_t \]  

(3)

where \( \Phi_t = [\alpha_t, \beta_{t,p}] \).

The covariance matrix of the VAR innovations \( \nu_t \) is factored as

\[ VAR(\nu_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})' \]  

(4)

The time-varying matrices \( H_t \) and \( A_t \) are defined as:

\[
H_t \equiv \begin{bmatrix}
  h_{1,t} & 0 & 0 & 0 & 0 & 0 \\
  0 & h_{2,t} & 0 & 0 & 0 & 0 \\
  0 & 0 & h_{3,t} & 0 & 0 & 0 \\
  0 & 0 & 0 & h_{4,t} & 0 & 0 \\
  0 & 0 & 0 & 0 & h_{5,t} & 0 \\
  0 & 0 & 0 & 0 & 0 & h_{6,t}
\end{bmatrix}
\]

(5)

\[
A_t \equiv \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  \alpha_{21,t} & 1 & 0 & 0 & 0 & 0 \\
  \alpha_{31,t} & \alpha_{32,t} & 1 & 0 & 0 & 0 \\
  \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 & 0 & 0 \\
  \alpha_{51,t} & \alpha_{52,t} & \alpha_{53,t} & \alpha_{54,t} & 1 & 0 \\
  \alpha_{61,t} & \alpha_{62,t} & \alpha_{63,t} & \alpha_{64,t} & \alpha_{65,t} & 1
\end{bmatrix}
\]

(6)

with the \( h_{i,t} \) evolving as geometric random walks,

\[ \ln h_{i,t} = \ln h_{i,t-1} + u_t \]

Following Primiceri (2005), we postulate that the non-zero and non-one elements of the matrix \( A_t \) evolve as driftless random walks,

\[ a_t = a_{t-1} + \varepsilon_t \]  

(7)

Note that by ordering the policy rate last and imposing the normalisation (6) we are also identifying the monetary policy shock as the only shock that does not have a contemporaneous effect on the other variables in the system.\(^4\) We assume that the vector \([e (\tau)'_t, \nu'_t, \eta'_t, \tau'_t, \nu'_t]'\) is

---

\(^4\)As noted by Diebold et al (2006b), such ordering is also consistent with the fact that the yields are dated at the beginning of each month.
distributed as
\[
\begin{bmatrix}
e(\tau)_t \\
v_t \\
\eta_t \\
\epsilon_t \\
u_t
\end{bmatrix}
\sim N(0, V),
\]
where
\[
V = \begin{bmatrix}
R & 0 & 0 & 0 & 0 \\
0 & \Omega & 0 & 0 & 0 \\
0 & 0 & Q & 0 & 0 \\
0 & 0 & 0 & S & 0 \\
0 & 0 & 0 & 0 & G
\end{bmatrix}
\quad \text{and} \quad
G = \begin{bmatrix}
\sigma_1^2 & 0 & 0 & 0 & 0 \\
0 & \sigma_2^2 & 0 & 0 & 0 \\
0 & 0 & \sigma_3^2 & 0 & 0 \\
0 & 0 & 0 & \sigma_4^2 & 0 \\
0 & 0 & 0 & 0 & \sigma_5^2
\end{bmatrix}
\]  
(8)

The model in equations (1) to (9) provides a flexible framework for analysing the interaction between the yield curve and macroeconomy. In particular, the model allows us to investigate how this interaction has evolved over time while simultaneously accounting for changes in the volatility of the shocks. In addition, the Nelson-Siegel framework imposes some restrictions on the yield curve that may help to improve the fit of the model – it guarantees positive forward rates at all horizons and a discount factor that approaches zero as maturity increases. Note, however, that our model does not incorporate some of the additional structure seen in recent macro-finance models (e.g., Ang and Piazzesi (2003)). In particular, our model does not incorporate no-arbitrage restrictions. This is primarily because of technical constraints – imposing these restrictions in a time-varying framework is still a task in progress. A drawback of this simplification is that we cannot estimate the term premium directly from our model. To the extent that our yield-macro model with time-varying parameters and stochastic volatility is correctly specified, however, the residuals of the observation equations can be interpreted as estimates of the term premia.\(^6\)

2.2 Estimation

The model in equations (1) to (9) is estimated using the Bayesian methods described by Kim and Nelson (1999). In particular, we employ a Gibbs sampling algorithm that approximates the posterior distribution. The algorithm exploits the fact that given observations on \(Z_t\), the model is

\(^5\)Relative to a model which includes unrestricted factors from the yield curve.

\(^6\)Note also that the model is silent about the role of the real term structure, an aspect that is potentially important in terms of the great stability.
a standard time-varying parameter model.

A detailed description of the prior distributions and the sampling method is given in the appendix. Here we summarise the basic algorithm which involves the following steps:

1. Given initial values for the factors, simulate the VAR parameters and hyperparameters.
   - The VAR coefficients $\phi_t$ and the off-diagonal elements of the covariance matrix $\alpha_t$ are simulated using the methods described by Carter and Kohn (2004).
   - The volatilities of the reduced form shocks $H_t$ are drawn using the date-by-date blocking scheme introduced by Jacquier, Polson and Rossi (2004).
   - The hyperparameters $Q$ and $S$ are drawn from an inverse Wishart distribution while the elements of $G$ are simulated from an inverse gamma distribution.

2. Given initial values for the factors, draw the covariance matrix $R$.
   - Note that as in Diebold and Li (2006) we fix the value of $\lambda$ at 0.0609. Given data on $Z_t$ and $y(\tau)$ and a value for $\lambda$, the variances are simulated from an inverse gamma distribution.

3. Simulate the factors conditional on all the other parameters
   - This is done by employing the methods described by Bernanke, Boivin and Eliasz (2005) and Kim and Nelson (1999).

4. Go to step 1.

We use 60,000 Gibbs sampling replications and discard the first 58,000 as burn-in. The posterior moments vary little over the retained draws providing evidence of convergence.\(^7\)

3 Data set

We consider UK government bond yields with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months. The yields are derived from bid/ask average price quotes,

\(^7\)Estimates of the posterior moments across subsets of the retained draws is available on request.
from January 1970 through March 2006, using the methods developed by Svensson (1995). To initialise the factors and calibrate priors for the VAR, we use a pre-sample of three years starting in January 1970. Therefore the results presented in the following section refer to the period January 1973-March 2006. Inflation is measured as monthly changes in the consumer prices index, the policy instrument is the Bank Rate and real activity is measured by the unemployment rate.

4 Results

This section describes the empirical results of the generalised Nelson-Siegel model developed in Section 2. We report estimates of the factors and their stochastic volatilities, decompose the variance of the variables in our FAVAR and revisit the evidence on the expectations hypothesis.

4.1 Main features of the posterior

4.1.1 Factors

Chart 1 presents the estimates of the factors together with the central 68% posterior bands. In addition, we also show ‘empirical counterparts’ of the factors. These ‘empirical counterparts’ of the factors can be thought of as proxies for the level, slope and curvature of the yield curve and are calculated as simple functions of the yields at different maturities:

Level: \[ \frac{y_t (3) + y_t (24) + y_t (120)}{3} \]
Slope: \[ y_t (3) - y_t (120) \]
Curvature: \[ 2y_t (24) - y_t (3) - y_t (120) \]

These proxies or counterparts are regularly used by finance practitioners and provide a good cross-check on the Bayesian estimates of the yield curve factors.

The top left panel shows the level factor (black line), the bands (red lines) and the counterpart (blue line). The correlation between the level factor and its counterpart is 0.91, which is 14% higher than the number obtained by Diebold et al (2006b) using US data and a time-invariant

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8 Anderson and Sleath (2001) show that the variable roughness penalty (VRP) model performs better than the Svensson (1995) method in that small changes in the data at one maturity (such as at the long end) do not have a disproportionate effect on forward rates at other maturities. The use of the Svensson (1995) approach, however, allows us to extend the data on short-maturity yields back to the 1970s. Furthermore, Svensson (1995) is a generalisation of Nelson-Siegel. As this paper is primarily concerned with changes in macroeconomic and yield curve dynamics, having a long time span of data is crucial.

9 See Diebold et al (2006b).
Chart 1: Factors and their empirical counterparts

The bottom left panel reports the NIESR forecasts for inflation. The correlation between our estimated level factor and the forecasts of the NIESR, which is available at quarterly frequency over the period March 1973-December 2003, is remarkably high: 0.84.\textsuperscript{10} Finally, the slope and the curvature factors track well their empirical counterparts.

\textsuperscript{10}Note that the estimated correlation is significant at the 1\% level. The correlation between the first difference of the two series is 0.44 which is, again, significant at 1\%. 
4.1.2 Volatilities

Homoskedasticity is a recurrent assumption in the macro-finance literature. In this section, we show that, in fact, significant time variation also characterises the evolution of volatilities of the observed and unobserved factors.

Chart 2 plots the volatility of the orthogonalised shocks \((A_t v_t)\) to each equation in the time-varying VAR model (with \(v_t\) and \(A_t\) defined in equations (2) and (6) respectively). As stated in Section 2.1 this transformation of the reduced form shocks \(A_t v_t\) allows us to identify the shock to the interest rate equation as the monetary policy shock. Although shocks to the other equations do not have an economic interpretation, estimates of their volatilities still provide useful information about the movement of the variable described by these equations.

In Chart 2, we report median estimates and 68% central posterior bands of the square roots of the stochastic volatilities. The first row shows that the volatility of the innovations to the level, slope and curvature factors displays a stable declining path, with sporadic and short-lived increases, the last of which occurred around the ERM crisis of 1990. The volatility of shocks to inflation has also declined over time from its peak in the mid-1970s and the early 1980s. By contrast, the volatility of shocks to unemployment has been broadly stable over the sample period. Finally, note the volatility of the monetary policy shock has effectively disappeared in the post-1992 period.

Chart 3 plots a measure of the volatility of the endogenous variables in the VAR. This is calculated as

\[
\left( s_i \left[ \sum_{j=0}^{\infty} \tilde{A}_t^i \Omega_t \tilde{A}_t^{ij} \right] s_i' \right)^{1/2}
\]

where \(\tilde{A}\) denotes the VAR coefficients in companion form while \(s_i, i = 1..6\) represents selection vectors for the six variables in the time-varying VAR. This measure is an approximation to the standard deviation of infinite-horizon prediction errors.

The top row of the chart shows the volatility of the level factor. Note that the level factor can be
thought of as representing the level to which interest rates return in the long run. If one assumes that the policy rule of the central bank eventually pushes interest rates to this trend level, then the level factor can be thought of as a proxy for the neutral level of the interest rate. As argued in Cogley (2004), the volatility associated with the ‘target rate’ should be low if agents are more certain about the level at which interest rates will settle in the long run and therefore (the estimated volatility) provides a measure of the credibility of monetary policy. The chart shows that the volatility was high in the mid-1970s and reached its peak at the start of the Thatcher disinflation in 1980. Subsequently, the volatility declined until the ERM crisis of the early 1990s. This measure of volatility has been low (relative to previous peaks) in the current inflation-targeting regime. Note another possible implication of these results: low volatility of the level factor in the current regime may also reflect lower volatility of inflation expectations.
(which are highly correlated with the level factor – see Chart 1) providing further evidence that agents perceive the current regime to be credible. It is also worth noting that the volatilities of the slope and the curvature factor have also declined over time. This indicates a general decline in the variance of interest rates in the United Kingdom and matches the evidence for the United States presented in Cogley (2004).

The second row of Chart 3 points to a similar decline in the volatility of inflation and unemployment. The first peak in inflation volatility corresponds to the breakdown of income policies over the years 1975-77. The second peak in 1979-80 possibly reflected various events: another breakdown of income policies, high pay awards in the public sector including those coming from the Clegg Commission, and the one-off effect of the increase in VAT from 8% to
15%. Subsequently, inflation volatility declined with the last significant peak occurring in 1990. Since the introduction of inflation targeting at the end of 1992, inflation volatility has been low, on average. The variance of the unemployment rate was at its highest in the mid-1970s and during the start of the Thatcher deflation in the early 1980s, but has remained low since the mid-1980s.

4.2 Variance decomposition

In this section we decompose the unconditional variance of each endogenous variable in the FAVAR into contributions from the monetary policy shock and shocks to the level of the yield curve at each point in time and at different frequencies. A number of interesting questions can be assessed with this exercise. First, as an extension to Diebold et al (2006b) we can trace how the link between the macroeconomy and the yield curve has evolved over time. Second, it allows us to assess the role of economic events and changes in policy regimes.

As noted above, our identification scheme is based on the Cholesky decomposition of the covariance matrix with the variables ordered as: \( L_t, S_t, C_t, U_t, \pi_t, R_t \). As in Diebold et al (2006b), the term structure factors are ordered first as the yield data are dated at the beginning of each month.

Charts 4 and 5 display the proportion of the normalised spectra of each variable accounted for by the innovations in the monetary policy shock and the shock to the level factor equation (at different frequencies). These are calculated as the ratio of the spectral density due to the shock in question (for example the monetary policy shock) and the ‘total’ spectral density.\(^\text{11}\) That is, the charts depict the following ratio:

\[
\frac{f_{i|T}^* (\omega)}{f_{i|T} (\omega)}
\]

where

\[
f_{i|T} (\omega) = s (I - \tilde{\beta}_{i|T} e^{-i\omega})^{-1} \frac{\Omega_{i|T}}{2\pi} (I - \tilde{\beta}_{i|T} e^{-i\omega})^{-1} s'
\]

with \( \tilde{\beta}_{i|T} \) representing the VAR coefficients in companion form, \( I \) is a conformable identity matrix, \( s' \) is a selection vector that picks out the \( i^{th} \) variable and \( \omega = 0..\pi \) denotes the frequency. Values of \( \omega \) closer to zero represent variation in the long run.

\(^{11}\)These figures should be interpreted as follows: the Z axis in each figure represents the percentage contribution of each shock to the variation in each variable of the VAR at different frequencies at each point in the sample. The frequencies are depicted on the Y axis, while the X axis represents time.
The numerator of equation (10) \( f_{\omega T}^*(\omega) \) is calculated as:

\[
f_{\omega T}^*(\omega)_j = s \left( I - \tilde{\beta}_{\omega T} e^{-i\omega} \right)^{-1} \frac{\Omega_{\omega T}^*}{2\pi} \left( I - \tilde{\beta}_{\omega T} e^{-i\omega} \right)^{-1} \beta_j^*\]

where the covariance matrix \( \Omega_{\omega T}^* \) is built (using equation (4)) assuming that the only non-zero element in \( H \) is the variance of the shock that is under consideration. For example, when considering the contribution of the monetary policy shock, all elements of \( H \) except the last are set to zero. This implies that \( f_{\omega T}^*(\omega)_j \) represents the spectral density of variable \( i \) due to the monetary policy shock.

**Chart 4: Variance decomposition: contribution of the monetary policy shock**

Chart 4 plots on the vertical axis the fraction of variance explained by the monetary policy shock, while the horizontal axes report respectively the years and the frequency \( \omega \), with zero being the lowest frequency (ie a cycle of infinite duration) which is a commonly used measure of

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persistence. The chart shows that the monetary policy shock explains a large amount of variation in the yield curve factors in the mid-1970s (around the time of the first oil crisis in 1974) and in the mid-1980s. A similar pattern emerges in its contribution to the variation in inflation and the unemployment rate, with the policy shock important in the mid-1970s, the early and late 1980s.

The contribution of the monetary policy shock to the policy interest rate shows the most striking variation. In the pre-1992 period, the policy shock accounts for most of the variance of the policy rate, both in the long-run and at higher frequencies. This suggests that the policy rate before 1992 was mainly driven by concerns other than inflation and unemployment (and the information contained in the yield curve). After 1992, however, the policy shock explains only a very small fraction of short-term interest rate variability. This is suggestive of a systematic change in monetary policy, with policy rates moving in response to inflation and unemployment and deviations from this ‘rule’ substantially smaller in magnitude and importance.

Chart 5 shows how the shock to the level factor equation contributes to the movement in each of the endogenous variables. There are a number of reasons why we focus on this aspect of the yield curve. First, as noted above, the level of the yield curve may reflect long-run or trend nominal rates. Second, as chart 1 shows, the level factor moves closely with inflation expectations. One interpretation of this comovement is that the shocks to this variable possibly capture shifts in inflation expectations. This is the interpretation of the level of the yield curve and is a common theme in recent papers such as Kozicki and Tinsley (2001), Diebold et al (2006b) and Joyce et al (2008).

Chart 5 shows that the shock to the level factor equation explains (at most) close to 40% of the variation in inflation over most of the sample period. The ‘peaks’ in the total contribution occur at the start of the Thatcher era; around the time of the ‘sterling crisis’ in the mid-1980s; during the ERM crisis in 1992 and around the time of Bank independence. It is likely that these periods were characterised by considerable uncertainty about the level at which rates would settle eventually. This pattern also suggests that shocks to the central bank’s target were more important in driving inflation in the period before the Bank of England was granted independence. In other words, agents’ beliefs about the actions of the central bank have become more certain since 1997.

The contribution of the level factor to unemployment is more modest: fluctuating at most around

\[\text{Note that this percentage represents the contribution over the short and the long run.}\]
20% (in total across frequencies). The contribution to the policy rate rises dramatically after the early 1990s, with this increase coinciding with the fall in the importance of the monetary policy shock (see Chart 4).

The top row of Chart 5 indicates large changes in the importance of the level shock to the slope and curvature of the yield curve. In particular, during the post-1992 policy regime, the variations in the slope and curvature factors have been mainly driven by shocks to the level factor.

It is interesting to compare the results in Charts 4 and 5 with a similar exercise conducted by Diebold et al (2006b) for the United States using a fixed coefficient version of our model. Diebold et al (2006b) report that:
‘the variance decompositions suggest that the effects of the yield curve on the macro variables are less important than the effects of the macro variables on the yield curve.’ (page 326)

The results presented in this section point to different conclusions. For our data set and our model, we find that the level of yield curve has been quite important for the macroeconomic variables included in our system.

4.3 Impulse responses

Following Diebold et al (2006b) we consider the dynamic relationships between the macro and the yield curve variables through impulse response analysis. As in the previous section, we focus on the monetary policy shock and the level factor shock.

The time-varying nature of our model implies that unlike Diebold et al (2006b) we can explore how these dynamic relationships have changed over time. Note, however, that as the coefficients change over time this feature has to be taken into account when estimating the impulse response functions. Following Koop, Pesaran and Potter (1996) we define the impulse response functions as:

\[
IRF = E(\Psi_{t+k}^t, \mu) - E(\Psi_{t+k}^t)
\]

where \( \Psi \) denotes all the parameters and hyperparameters of the VAR and \( k \) is the horizon under consideration. Equation (13) states that the impulse response functions are calculated as the difference between two conditional expectations. The first term in equation (13) denotes a forecast of the endogenous variables conditioned on a shock \( \mu \). The second term is the baseline forecast, ie conditioned on the scenario where the shock equals zero.\(^{13}\)

Chart 6 shows the impulse responses to a monetary policy shock at four selected dates in the sample. The shock is normalised so that it increases the central bank interest rate by 100 basis points at all dates.

Consider the impact of the policy shock on the two macroeconomic variables. The response of inflation suggests some evidence of a price puzzle in 1978 and 1987. However, the confidence

\(^{13}\)The impulse responses are computed via Monte Carlo integration for 500 replications of the Gibbs sampler.
intervals indicate that the initial positive response is not significantly different from zero. The negative response of unemployment in 1978 is at odds with conventional theory. But, this negative effect disappears in the 1980s and the response is essentially zero in the inflation-targeting period.

The response of the level factor to the monetary policy shock suggests some interesting conclusions. As discussed in Diebold et al (2006b), the direction of the response of the level of the yield curve to a monetary contraction may depend on the credibility of monetary policy. For example, if monetary policy is credible, then the level factor may fall in response to higher policy rates because expectations of future inflation decline. The estimated response of the level factor in 1978 suggests that this was not the case. The level factor is unchanged in response to a policy contraction. This suggests that the central bank was unable to influence inflation expectations in the 1970s. In contrast, the response in the late 1980s and during the inflation-targeting regime is significantly negative suggesting a marked improvement in the credibility of monetary policy.

The response of the slope factor to the monetary policy shock is in line with the results for the
United States reported in Diebold et al (2006b). The slope factor rises immediately in response to monetary tightening, indicating an increase (decrease) in the negative (positive) slope of the yield curve. The response of the curvature factor is positive in the 1970s and becomes negative in the later part of the sample. This suggests that, in the 1970s, a monetary contraction did not necessarily have a large impact on longer-term rates leading to an increase in the curvature of the yield curve.

Chart 7: Impulse response to a level factor shock

Chart 7 displays the impulse response to a shock to the level factor. In the pre-inflation targeting period, inflation displayed a strong positive response to the level factor. This response has been insignificantly different from zero in the current monetary regime. Following Diebold et al (2006b), one interpretation of these estimates is that they represent the impact of an increase in inflation expectations. Under this interpretation of the shock, these results are consistent with the idea that during the pre-inflation targeting period, the response of the monetary authority to an increase in inflation expectations was less active leading to a significant increase in actual inflation. The initial fall in unemployment (in response to the level factor shock) possibly indicates that the real interest rate may not have risen significantly during this period.\(^\text{14}\)

\(^{14}\)In other words, the monetary authority did raise nominal interest rates but possibly not by the degree needed to obtain positive real rates.
contrast, during the inflation-targeting regime, the central bank countered the shock to inflation expectations actively and actual inflation remained relatively unchanged.

4.4 The evidence on the expectations hypothesis revisited

The expectations theory of the term structure predicts that movements in long rates are due to movements in expected future short rates. Any differences between actual long rates and expected short rates reflect a term premium, which is typically assumed to vary across maturities but remain constant over time. A substantial body of work has concentrated on testing the expectations hypothesis, with evidence in favour of the theory hard to find. Our framework allows us to revisit this problem using a time-varying generalisation of Nelson-Siegel model. In particular, our framework allows us to assess whether (the lack of) time variation in the dynamics of both yield curve and macroeconomic variables can account for the failure of the expectations hypothesis documented in earlier contributions: apparent deviations from the expectations theory may reflect neglected parameter instability.

The expectations hypothesis (EH) consistent (pure discount) bond yield is:

$$y_t^{EH} = \left( \frac{1}{\tau} \right) \sum_{i=0}^{\tau-1} E_t y_{t+i} (1) + c_t$$

(14)

where $\tau$ and $c_t$ represent the maturity and the term premium.

The right-hand side of (14), when computed using a fixed-coefficients model, involves the implicit assumption that agents form their expectations using a model of the economy that is fixed over time. This assumption is a strong one, especially in the light of policy changes that have taken place in the United Kingdom. In contrast, our time-varying VAR proxies changing monetary policy (and the response of agents) through the drifting VAR coefficients. Therefore our model implies that agents update their beliefs about the economy and monetary policy at each point in time. As in Cogley (2004) their forecast of the long-term interest rate is based on these updated beliefs.

Note that the law of motion for the VAR coefficients in equation (3) implies that future evolution in agents’ beliefs is a random variable. When computing the forecast of the long-term interest rate, we take this uncertainty into account through Monte Carlo integration (see for instance
Koop et al (1996)). This approach is different from the ‘anticipated utility’ version of the expectations hypothesis used in Cogley (2004) where agents update their beliefs each period but then keep them fixed over the forecast horizon. Note also that the Bayesian approach taken in this paper provides us with a very natural way of accounting for parameter uncertainty when constructing bands around the central predictions of the expectations hypothesis.\(^{15}\)

**Chart 8: Theoretical yields from the time-varying VAR**

![Chart 8: Theoretical yields from the time-varying VAR](image)

In Chart 8, we compare the theoretical yields constructed using (14) with actual yields (blue lines). The gray area represents central 90% posterior bands while the black lines are median values. Note that although this exercise does not amount to a formal test of the expectations hypothesis, it does allow us to assess if the results from our time-varying FAVAR are consistent with the predictions of the hypothesis. In addition, we can carry out the same exercise for a time-invariant model to infer the relative performance of our extended model. At each point in time, the forecasts of the one-month yield are based on the time-varying model (1)-(3), (8)-(9) conditional on the information available at time \(t - 1\). The theoretical yields track actual yields

\(^{15}\)In a classical framework, a time-varying parameter model imposes such a heavy computational burden that considering parameter uncertainty becomes unfeasible (see Carriero, Favero and Kaminska (2006) for an alternative procedure based on recursive estimations).
extremely well and the predictions of the model are accurate, especially at short maturities where the actual yields rarely fall outside the 90% confidence interval. At the five and ten-year maturities, the estimated theoretical yields still fit the data well. However, there are isolated but noticeable deviations of actual data from theoretical yields, especially during the mid-1970s, early 1980s and the early 1990s.

**Chart 9: Theoretical yields from the fixed-coefficients model**

Chart 9 presents the theoretical yields computed using a fixed-coefficients version of our FAVAR model. At horizons of up to two years, the fixed-coefficients model performs well, with results similar to the time-varying model. In stark contrast to the time-varying model, at longer horizons (five and ten years), the theoretical yields derived from the fixed-coefficients model deviate substantially from actual data, with the theoretical yields displaying much less variation than actual data. Note also that the uncertainty associated with these estimates is larger than in the time-varying model. These results again highlight the advantages of the time-varying specification. The time-varying parameter model includes more sources of variation than the fixed-coefficients model. The only source of variation in the latter is the shock to the VAR

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16Note also that the performance matches the results for the United States reported in Diebold *et al* (2006b).
disturbances. In the time-varying model, there is also a shock attached to each VAR coefficient in addition to the disturbance covariance matrix. This additional variation implies that the time-varying model is more suited to tracking shifts in agents’ beliefs.

### 4.4.1 The term premium

Chart 10 provides a closer inspection of the ten-year term premium obtained as the difference between actual yields and theoretical yields (at the ten-year horizon) obtained from the time-varying VAR model. Note that the grey-shaded area represents the 90% confidence interval.

**Chart 10: Term premium**

The term premium fluctuates around zero over most of the sample. There are two notable exceptions: the term premium is high during the oil crisis of the 1970s, being consistent with the view that investors were asking a positive premium to hold long-term bonds and the ERM crisis in 1992 which saw a sharp increase in this variable.

The vertical line marks the introduction of the inflation-targeting framework in December 1992. The positive premium in the following couple of years can be interpreted as evidence of the
credibility building of the new regime. Since independence was granted to the Bank of England in 1997, deviations from the expectations theory have been modest.

Chart 11: Standard deviation of the term premium

Finally, Chart 11 plots the evolution of the uncertainty associated with the estimated term premium.\textsuperscript{17} It is interesting to note that the decline in uncertainty towards the end of the sample coincides with the introduction of the inflation-targeting regime.

5 Conclusions

This paper has studied the evolution of the link between the yield curve and the UK economy. We extend the FAVAR model in Diebold \textit{et al} (2006b) to include time variation in the parameters and volatilities of the shocks. The shape of the yield curve changes over time with the level closely correlated with measures of inflation expectations. Estimates of volatilities associated with both the yield curve and the macroeconomy have declined over time.

The contribution of monetary policy to the change in inflation dynamics is reflected by the fact

\textsuperscript{17}This is calculated as the standard deviation across the draws from the Gibbs sampler.
that in the post-1992 period the monetary policy shock made a minimal contribution to the movement in policy interest rates – a large change from the pre-1992 period where the policy shock was the main driver of both the policy rate and the peak of inflation expectations during the early 1980s. The level of the yield curve did not respond to policy shocks in the 1970s, consistent with the notion that policy was not sufficiently credible over those years.
Appendix: Priors and estimation

Consider the time-varying VAR model given by equations (1) and (2).

Prior distributions and starting values

Factors

We center our prior on the factors (and obtain starting values) by using the least squares estimator employed by Diebold and Li (2006). The prior covariance of the states \( P_{0/0} \) is set equal to an identity matrix.

The prior on the diagonal elements of \( R \) is assumed to be inverse gamma:

\[
R_{ii} \sim IG(R_{i0}, 1)
\]

where \( R_{i0} = 1 \).

VAR coefficients

The prior for the VAR coefficients is obtained via a fixed-coefficients VAR model estimated over the sample January 1973 to December 1973. \( \Phi_0 \) is therefore set equal to

\[
\Phi_0 \sim N(\hat{\Phi}^{OLS}, V^{OLS})
\]

Elements of \( H_t \)

Let \( \hat{\Phi}^{ols} \) denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data described above. The prior for the diagonal elements of the VAR covariance matrix ((5)) is as follows:

\[
\ln h_0 \sim N(\ln \mu_0, I_6)
\]

where \( \mu_0 \) are the diagonal elements of \( \hat{\Phi}^{ols} \).
Elements of $A_i$

The prior for the off-diagonal elements $A_i$ is

$$A_0 \sim N(\hat{a}^{ols}, V(\hat{a}^{ols}))$$

where $\hat{a}^{ols}$ are the off-diagonal elements of $\hat{o}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V(\hat{a}^{ols})$ is assumed to be diagonal with the diagonal elements set equal to ten times the absolute value of the corresponding element of $\hat{a}^{ols}$.

Hyperparameters

The prior on $Q$ is assumed to be inverse Wishart

$$Q_0 \sim IW(\hat{Q}_0, T_0)$$

where $\hat{Q}_0$ is assumed to be $\text{var}(\hat{\phi}^{OLS}) \times 10^{-4}$ and $T_0$ is the length of the sample used for calibration.

The prior distribution for the blocks of $S$ is inverse Wishart:

$$S_{i,0} \sim IW(\tilde{S}_i, K_i)$$

where $i = 1..6$ indexes the blocks of $S$. $\tilde{S}_i$ is calibrated using $\hat{a}^{ols}$. Specifically, $\tilde{S}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by $10^{-3}$.

Following Cogley and Sargent (2005), we postulate an inverse Gamma distribution for the elements of $G$,

$$\sigma_i^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right)$$

Simulating the posterior distributions

Factors and factor loadings

This closely follows Bernanke et al (2005). Details can also be found in Kim and Nelson (1999).
Factors

Conditional on a value for \( \lambda \) and draws for the remaining parameters, the factors are drawn using the methods of Carter and Kohn (2004). For details see Kim and Nelson (1999).

Elements of \( R \)

As in Bernanke et al (2005) \( R \) is a diagonal matrix. The diagonal elements \( R_{ii} \) are drawn from the following inverse gamma distribution:

\[
R_{ii} \sim IG \left( \tilde{R}_{ii}, T + 1 \right)
\]

where

\[
\tilde{R}_{ii} = \hat{\epsilon}(\tau)' \hat{\epsilon}(\tau) + R_{i0}
\]

and \( \hat{\epsilon}(\tau) = y(\tau) - \left( \hat{L}_i + \frac{1-e^{-\tau \lambda}}{\tau \lambda} \hat{S}_i + \left( \frac{1-e^{-\tau \lambda}}{\tau \lambda} - e^{-\tau \lambda} \right) \hat{C}_i \right) \) with \( \hat{L}_i, \hat{S}_i, \hat{C}_i \) denoting a draw of the three factors. \( \lambda = 0.0609 \).

Time-varying VAR

Given an estimate for the factors, the model becomes a VAR model with drifting coefficients and covariances. This model has become fairly standard in the literature and details on the posterior distributions can be found in a number of papers such as Primiceri (2005). Here, we describe the algorithm briefly.

VAR coefficients \( \Phi_i \)

As in the case of the unobserved factors, the time-varying VAR coefficients are drawn using the methods described by Kim and Nelson (1999).

Elements of \( H_i \)

The diagonal elements of the VAR covariance matrix are sampled using the methods described by Jacquier et al (2004).
Element of $A_t$

Given a draw for $\Phi_t$ the VAR model can be written as

$$A_t^t(\tilde{Z}_t) = u_t$$

where $\tilde{Z}_t = Z_t - \alpha_t - \sum_{p=1}^{p} \beta_{t-p} Z_{t-p} = v_t$ and $\text{VAR}(u_t) = H_t$. This is a system of equations with time-varying coefficients and given a block diagonal form for $\text{VAR}(\tau_t)$ the standard methods for state space models described by Kim and Nelson (1999) can be applied.

VAR hyperparameters

Conditional on $Z_t, \Phi_t, H_t, \text{and } A_t$, the innovations to $\Phi_t, H_t, \text{and } A_t$ are observable, which allows us to draw the hyperparameters – the elements of $Q, S$, and the $\sigma_i^2$ – from their respective distributions.
References


