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Endogenous choice of bank liquidity: the role of fire sales

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Viral V Acharya,[(1)] Hyun Song Shin[(2)] and Tanju Yorulmazer[(3)]

Abstract

Banks' liquidity is a crucial determinant of the adversity of banking crises. In this paper, we consider the effect of fire sales and entry during crises on banks' *ex-ante* choice of liquid asset holdings. We consider a setting with limited pledgeability of risky cash flows relative to safe ones and a differential expertise between banks and outsiders in employing banking assets. When a large number of banks fail, market for assets clears only at fire-sale prices and outsiders enter the market if prices fall sufficiently low. In such states, there is a private benefit of liquid holdings to banks from purchasing assets. There is also a social benefit since greater banking system liquidity reduces inefficiency from liquidation of assets to outsiders. When pledgeability of risky cash flows is high, for instance, in countries with well-developed capital markets, banks hold less liquidity than is socially optimal due to risk-shifting incentives; otherwise, banks may hold even more liquidity than is socially optimal to capitalise on fire sales. However, if there is a systemic cost associated with crises, for example, in the form of fiscal costs associated with provision of deposit insurance, then socially optimal liquidity may always be higher than the privately optimal one, and, in turn, regulation in the form of prudent liquidity requirements may be desirable. We provide some international evidence on banks' liquid holdings that is consistent with model's predictions.

Key words: Crises, systemic risk, distress, limited pledgeability, lender of last resort.

JEL classification: G21, G28, G32, E58, D61.

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Summary

A central difficulty during banking crises is one of finding ready buyers of distressed assets. If a bank needs to restructure its balance sheet during a crisis, the potential buyers of its assets are other banks that may have also been severely affected and thus may not have enough equity capital or debt capacity to purchase assets. Hence, during crisis periods, asset prices fall below their fundamental value, giving rise to 'cash-in-the-market' (or fire-sale) pricing. Surviving banks that do have enough liquidity during such states stand to make windfall profits from purchasing assets at fire-sale prices. Even if crises arrive infrequently, the potential gains from acquisitions at fire sales could be large. This gives banks incentives to hold liquid assets, not merely to increase the chances of surviving the crisis, but also so that in the event that they survive the crisis, they will have resources to take advantage of fire sales.

We present a model of banks' choice of *ex-ante* liquidity that is driven by such strategic considerations. We examine the portfolio choice of banks maximizing their profits in the presence of fire sales that are endogenously derived in an equilibrium setup of the banking industry. While risky assets are attractive to banks given their limited liability, cash flows of risky assets are illiquid and have limited pledgeability (that is, financing capacity) compared to cash flows of safe assets. This limited pledgeability of risky cash flows, coupled with the potential for future acquisitions at fire-sale prices, induces banks to hold liquid assets in their portfolios.

In this setting, we show that banks' equilibrium holding of liquid assets is decreasing in the pledgeability of risky cash flows. In turn, bank liquidity is also decreasing in the health of the economy. During economic upturns, expected profits from risky assets are high and so is their pledgeability. An important implication of this result is that adverse asset-side shocks that follow good times result in deeper fire-sale discounts since bank balance sheets feature low liquidity in such times, whereby conditional on adverse shocks, there is lower aggregate liquidity to clear the market for assets.

We also compare the privately optimal levels of bank liquidity with benchmark levels that maximise the overall banking sector output. The pledgeability of risky cash flows turns out to be the critical determinant of whether banks hold too little or too high liquidity relative to the socially optimal level. When pledgeability is high, banks hold less liquidity than is socially

optimal due to the preference for risk induced by limited liability; otherwise, banks may hold even more liquidity than is socially optimal in order to capitalise on fire sales. This latter result may seem surprising but is explained simply. Fire sales result in transfers of value among banks but do not lead to any aggregate welfare gains or costs, and thus, liquidity hoarded to capitalise on fire sales may in some cases be excessive from the standpoint of maximising banking sector output. In particular, inefficiently high levels of bank liquidity and by implication inefficiently low levels of intermediation arise when pledgeability of risky cash flows is sufficiently low, for example, during crises or in banking sectors of emerging markets.

We present descriptive cross-country evidence on the asset liquidity of banks across countries. This evidence suggests that banks' choice of liquidity seems to vary along dimensions that would be correlated with difficulty in raising external finance and the severity of financial distress. We show that banks hold more liquid assets in those countries that have (i) less developed accounting standards; (ii) lower total market capitalisation relative to GDP; and, (iii) lower liquidity in stock markets. We discuss how our model's implications on management of liquidity by banks over the business cycle square up with existing evidence and the recently documented facts concerning leverage targeting by banks.

We also analyse the effect of entry by outsiders (to the banking sector) for acquisition of assets during crises. Since outsiders may lack expertise relative to surviving banks, they may enter only when fire sales are sufficiently deep. Once they enter, they increase the aggregate pool of liquidity and stabilise prices. This reduces *ex-ante* returns to liquidity for banks and they hold lower levels of liquid assets in their portfolios. This implies that even when outsiders are second-best users of assets, their entry can potentially unlock liquid hoardings of banks in emerging markets and lead to greater intermediation by their banking sectors.

Finally, we consider the effect of various resolution policies on banks' choices. Bailouts in our model result in lower equilibrium bank liquidity holdings only if they are excessive. In contrast, liquidity grants to surviving banks that are not contingent on banks' liquidity holdings always lower equilibrium liquidity holdings. However, if the amount of liquidity provided is increasing in liquid holdings of surviving banks, then incentives for banks to hold liquid assets are strengthened. These results illustrate that the resolution policies can have subtle effects on bank liquidity depending on whether these policies are optimal or excessively forbearing, and whether they are unconditional or contingent on quality of bank balance sheets at the time of resolution.

1 Introduction

In the aftermath of a major financial crisis, one of the most pressing tasks for a country is to restructure its banking sector which emerges saddled with large non-performing claims against distressed borrowers — claims that are backed by collateral whose prices have fallen to a fraction of their levels before the crisis. After the fires have been put out, there follows a protracted period of banking sector resolution. The crises in Scandinavia (Sweden in particular) in 1992, Mexico in 1994-95, Thailand, Korea and Indonesia in 1997, Turkey in 2001, and the sub-prime crisis of 2007-08, are all instances that highlight the two-stage nature of financial crises, where the initial 'acute' stage is followed by the longer-term 'chronic' stage of bank restructuring.

Bank insolvency shares many of the principles for dealing with corporate insolvency in general. However, the public interest imperative in maintaining a sound banking sector means that resolution of insolvent banks has invariably been a matter where the government takes the lead. The favoured approach to bank resolution has been the setting up of a government-sponsored body that takes on the assets of the banking sector temporarily on its balance sheet for eventual sale to purchasers after restructuring of the liabilities. Indeed, the frequency and similarity of bank restructuring problems around the world has given rise to two official documents on the 'best practice' for the resolution of insolvent banks, issued by the International Monetary Fund (2003) and the Basel Committee on Banking Supervision (2002).[1] An earlier paper by Santomero and Hoffman (1998) describes the procedures used in Scandinavia and during the US savings and loans crisis.

The common theme that runs throughout the academic research and the official documents is the difficulty of finding ready buyers of distressed assets in the midst of crisis, or in the immediate aftermath of such a crisis. During times of distress, when a bank needs to restructure its balance sheet, the potential buyers of its assets are other banks that have also been severely affected by the crisis. Indeed, these potential buyers are also likely to be experiencing similar problems, and may not have enough liquid resources to purchase these assets. This theme is a familiar one from corporate finance (see Williamson (1988) and Shleifer and Vishny (1992)), but leads to especially acute problems in banking given the high sensitivity of banking assets to major macroeconomic shocks. Allen and Gale (1994, 1998) have noted, for example, how

[1] Lindgren, Garcia and Saal (1996) show that during the period 1980-96, of the 181 IMF member countries, 133 experienced significant banking problems. Such problems have affected developed, as well as developing and transitional countries.

‘cash-in-the-market’ (or fire-sale) pricing of assets during crises may cause asset prices to fall below their fundamental value.

On the one hand, surviving banks may not have the resources to purchase the failing banks’ assets in such systemic situations. As a result, assets may end up in the hands of buyers from outside the banking sector. Often, such outside buyers are short-term holders who repackage or securitise the assets for selling on to portfolio investors. However, outsiders may be unable to realise the full value of the assets for the familiar reason that bank assets (loans in particular) derive much of their value from the monitoring and collection efforts of loan officers who can influence the actions of the debtors. Hence, when distressed assets end up in the hands of outsiders, we may expect deadweight costs from inefficient allocation of assets.

On the other hand, surviving banks stand to make windfall profits if they can purchase distressed assets at low prices and take over the depositor base of a failed competitor bank, potentially increasing its market share of loans. Even if crises arrive infrequently, the potential gains from acquisitions at fire sales would be large, and we would expect banks to position themselves to take advantage of such opportunities. The most important ingredient of such pre-positioning is banks’ choice of the portfolio of assets. The objective would be to hold enough liquid assets so that in the event that the bank survives the crisis, it will have resources to take advantage of the low purchase price of distressed assets.

In this paper, we follow up on this theme theoretically and provide some empirical support for the theoretical predictions. First, we examine in a model the implications of endogenous price effects, namely fire-sales and entry, on the portfolio choice of banks with the goal of ascertaining both the equilibrium level of liquid asset holdings of banks, and the socially optimal level of liquidity. We show that banks’ equilibrium holding of liquid assets are decreasing in the pledgeability of risky cash flows. On the one hand, lower pledgeability increases the extent to which crises lead to large price discounts in fire sales, and, on the other hand, lower pledgeability reduces the liquidity banks can raise contingent on survival. The pledgeability of risky cash flows also turns out to be the critical determinant of whether banks hold too little liquidity relative to the socially optimal level. When pledgeability is high, as would be the case in countries with well-developed capital markets, banks hold less liquidity than is socially optimal due to the preference for risk induced by limited liability; otherwise, banks may hold even more liquidity than is socially optimal in order to capitalise on fire sales. This latter result may seem surprising but is explained simply. Fire sales result in transfers of value among banks but do not lead to any aggregate welfare gains or costs, and thus, liquidity hoarded to capitalise on fire sales may in some cases be excessive from the standpoint of maximising banking sector output. In particular, inefficiently high levels of bank liquidity and by implication inefficiently low levels of intermediation arise when pledgeability of risky cash flows is sufficiently low, for example, during crises or in banking sectors of emerging markets. A more detailed description of the model follows shortly.

Second, we provide descriptive cross-sectional evidence on the asset liquidity of banks across countries. This evidence suggests that banks' choice of liquidity does vary along dimensions that we would expect to be correlated with difficulty in raising external finance and severity of financial distress. We show that banks hold more liquid assets in those countries that have (i) less developed accounting standards; (ii) lower total market capitalisation relative to GDP; and, (iii) lower liquidity in stock markets. While it seems plausible that these findings are consistent with a precautionary motive for liquidity hoardings, our model shows that they are also consistent with a purely strategic one.

Finally, our analysis also touches on a few important themes in the regulation of financial institutions. We extend the theoretical framework to incorporate costly provision of deposit insurance. Note that our model abstracts from any systemic effects of bank failures such as contagion. This is natural in our framework given the presence of deposit insurance and the absence of any interbank linkages. Hence, costly provision of deposit insurance can be considered as a metaphor for systemic costs arising from bank failures. We show that the presence of such systemic costs makes it more likely that banks will hold less liquidity than is socially optimal. Indeed, if the systemic cost is sufficiently high, we show that the socially optimal liquidity may *always* be higher than privately optimal bank liquidity, and, in turn, regulation in the form of prudent liquidity requirements may become desirable.

We also consider the effect on bank liquidity of resolution policies such as government-sponsored bailouts and granting of liquidity to surviving banks. Bailouts in our model result in lower equilibrium bank liquidity holdings only if they are excessive in the sense of covering more banks than is necessary to avoid liquidations to outsiders. In contrast, liquidity grants to surviving banks that are not contingent on banks' liquidity holdings always lower equilibrium liquidity holdings. However, if the amount of liquidity provided is increasing in liquid holdings of surviving banks, then incentives for banks to hold liquid assets are strengthened. These results illustrate that public sector resolution practices can have subtle effects on bank liquidity depending on whether these policies are optimal or excessively forbearing, and whether they are unconditional or contingent on quality of bank balance sheets at the time of resolution.

Section 2 presents the related literature. Sections 3 and 4 set up the benchmark model without outsiders and characterise the effect of fire sales on bank liquidity. Section 5 considers liquidity-endowed outsiders and the effect of their entry on bank liquidity. Section 6 exhibits empirical evidence on the liquid asset holdings of banks. Section 7 reverts to the model examining the effect of costly provision of deposit insurance and closure policies of the regulator on liquidity choices of banks. Section 8 concludes. All proofs not in the main text are in the Appendix.

2 Related literature

Our paper is motivated in part by the policy-related literature on bank restructuring and financial crises. The official documents from the IMF (2003) and the Basel Committee on Banking Supervision (2002) arose from an extensive consultation process among the leading industrialised countries following the series of financial crises in the late 1990s. The prefaces to these official documents make it clear that the two reports were co-ordinated attempts to provide advice on 'best practice' on bank resolution, distilling insights from the experience gained from tackling banking crises in the 1980s and 1990s. The literature on banking sector resolution is vast (see Acharya and Yorulmazer (2005) for a summary), but shares the key themes examined in this paper.

More broadly, our paper has links to the recent literature on the role of foreign direct investment (FDI) flows in the aftermath of financial crises. Aguiar and Gopinath (2005) have recently documented evidence that the high FDI flows into the crisis-stricken countries of the 1997 Asian financial crisis had many of the features of fire sales: median offer price to book ratios were substantially lower for cash-strapped firms' purchase, especially in 1998 when national players had low liquidity, resulting in a boost in mergers and acquisitions involving foreign players. Their paper provides a systematic empirical counterpart to the hypothesis raised by Krugman (1998) that the investment flows into Asia following the crisis in 1997 and Mexico following the crisis in 1995 were suggestive of western firms taking advantage of low prices of real assets.[2] Although we do not address explicitly the role of 'foreign' outsiders in what follows, our model has important implications for their role following a widespread financial crisis. In particular, the welfare implications of our model on the issue of domestic outsider involvement in the resolution of banking problems are closely related to the issue of foreign entry, as we will detail below.

From a more narrow modelling perspective, the relationship between liquidity and asset prices has been used in the literature to examine a number of interesting issues such as financial market runs (Bernardo and Welch (2004) and Morris and Shin (2004), strategic lending and trading (Donaldson (1992) and Brunnermeier and Pedersen (2005), contagion through asset prices (Diamond and Rajan (2001), Gorton and Huang (2004), Schnabel and Shin (2004), Allen and Gale (2005) and Cifuentes, Ferrucci and Shin (2005), and optimal failure resolution (Acharya and Yorulmazer (2004, 2005). While liquidity can affect asset prices, most of the literature cited above treats the level of liquidity of banks as exogenous.

In this paper, we concentrate on the effect of liquidity on (endogenously derived) fire sales during systemic crises, and, in turn, their effect on equilibrium liquidity. On this score, our paper is more in the spirit of recent papers by Allen and Gale (2004) and Gorton and Huang (2004) who investigate how liquidity is endogenously determined. Allen and Gale (2004), for example, build a model where bank runs result in fire-sale liquidation of banking assets. Speculators endogenously choose the level of the liquid asset, which they use to purchase banking assets. Since on average the liquid asset has a lower return than the risky asset, speculators have to be compensated for holding liquid assets, which can only be possible if they can purchase the risky asset at a discount leading to cash-in-the-market pricing of the risky asset. The distinction between these papers and ours arises from the facts that (i) we model limited pledgeability of risky cash flows, à la Holmstrom and Tirole (1998), and show it to be a crucial determinant of bank liquidity; (ii) we consider a model with continuum of banks which provides a richer industry equilibrium setting; and, finally, (iii) we provide policy implications for the effect of regulatory closure policies on bank liquidity.

Perotti and Suarez (2002) consider a dynamic model where reducing competition in the banking

[2]Krugman's article provides some interesting headlines from newspapers that talk about foreign entry due to fire-sale prices in crisis-stricken countries: 'Korean companies are looking ripe to foreign buyers' (*New York Times*, 27 Dec), 'Some U.S. companies see fire sale in South Korea' (*Los Angeles Times*, 25 Jan), 'Some companies jump into Asia's fire sale with both feet' (*Chicago Tribune*, 18 Jan), 'While some count their losses in Asia, Coca-Cola's chairman sees opportunity' (*Wall Street Journal*, 6 Feb). In news related to the banking sector, Seoul Bank and Korea First Bank were under consideration for auction to foreign bidders.

industry by selling failed banks to surviving banks increases the charter value of surviving banks and gives banks *ex-ante* incentives to stay solvent. Such a strategic benefit is present in our model in a different guise as the fire-sale prices at which surviving banks purchase failed banks. However, in contrast to their paper, our model focuses on the on asset sales, fire-sale prices and banks' endogenous choice of liquidity.[3]

Since liquid assets usually have lower returns than illiquid assets, banks may rationally choose to rely on an interbank market or a lender of last resort (LOLR). Bhattacharya and Gale (1987) build a model of the interbank market where individual banks that are subject to liquidity shocks coinsure each other against these shocks through a borrowing-lending mechanism. However, in this model, the composition of liquid and illiquid assets in each bank's portfolio and the liquidity shocks are private information. Hence, banks have an incentive to underinvest in liquid assets and free-ride on the common pool of liquidity in the interbank market. Repullo (2005) shows that the existence of LOLR results in banks holding a lower level of the liquid asset as they factor in the LOLR as a potential source of liquidity.[4] While we do not consider interbank lending in this paper, we study in Section 7 the implications of bailouts and different variants of liquidity provision on the liquidity choice of banks.

3 Benchmark model

Before presenting the formal model, we first give an informal description of the building blocks, and the key assumptions. We consider a setting with a large number of banks. Banks solve a portfolio choice problem as to how much to invest in risky assets, which are assumed to have diminishing returns to scale, and how much to park in the safe asset as liquid reserves. This portfolio choice problem acquires an intertemporal dimension given the limited pledgeability of risky cash flows and the benefit from holding liquidity in states where banks can profit from asset purchases. Specifically, while banks have a preference for the risky asset due to its 'option' value in the traditional risk-shifting sense, there is a counteracting preference for the safe asset due to its greater liquidity relative to the risky asset. Banks' choice of liquidity trades off the expected returns from the two kinds of assets (adjusted for option value) taking account of this need for

[3] Also, see Wagner (2007) that analyse liquidity provision at banks when there are fire sales that are subject to deadweight loss. Wagner (2007) shows that banks do not internalise the deadweight loss, which results in underprovision of private liquidity.

[4] Gonzalez-Eiras (2003), using Argentinean data, tests this argument. He investigates the episode in December 1996 when the Central Bank of Argentina signed an agreement to have access to contingent credit lines with a group of international banks that enhanced its ability to act as a LOLR. He shows that banks have relied on the enhanced ability of the Central Bank of Argentina for liquidity and this has resulted in an approximately 6.7% reduction in banks' liquid asset holdings.

intertemporal transfers of liquidity. The socially optimal level of liquid asset holdings in banks' portfolio maximises the value of banks as a whole (that is, without any risk-shifting problem). Throughout our analysis, we assume that deposits are insured by the regulator. To start with, there is no cost of providing insurance to depositors, in which case the assumption of insured deposits does not play a key role in determination of liquidity choices of banks. We introduce outsiders in Section 5 and costly deposit insurance in Section 7.

The formal model is outlined in Figure 1. We consider an economy with two periods and four dates: $t = 0, \frac{1}{2}, 1, 2$, banks, bank owners, depositors and a regulator. There is a continuum of banks with measure 1 where each bank can borrow from a continuum of depositors of measure 1. Bank owners as well as depositors are risk-neutral, and obtain a time-additive utility u_t where u_t is the expected wealth at time t. Depositors receive a unit of endowment at $t = 0$ and $t = 1$, and have access to a reservation investment opportunity that gives them a utility of 1 per unit of investment. In each period, that is at date $t = 0$ and $t = 1$, depositors choose to invest their good in this reservation opportunity or in their bank.

Deposits take the form of a simple debt contract with maturity of one period. In particular, the promised deposit rate is not contingent on investment decisions of the bank or on realised returns.[5]

Banks collect one unit of deposits from depositors and make investments to maximise the expected profits at $t = 1$ and $t = 2$, where discounting has been ignored since it does not affect any of our results. In particular, banks choose a portfolio by investing l units in a safe asset and the remaining $(1 - l)$ units in a risky asset, which is to be thought of as a portfolio of loans to firms in the corporate sector. The performance of the corporate sector determines the random output at date $t + 1$ for an investment at date $t = 0, 1$.

Suppose R_t is the promised return on a unit of bank loan given at date t. We denote the random repayment on this loan as $\widetilde{R}_t$, $\widetilde{R}_t \in \{0, R_t\}$. The probability that the return from these loans is high in period t is α_t. We assume the high returns R_t, as well as their associated probabilities α_t to be different in the two periods. This helps isolate the effect of each return and probability on our results. We also assume that banks' return from their risky investments are independent.

[5] In this paper, we do not model why banks use debt finance. In Calomiris and Kahn (1991) and Diamond and Rajan (2001) debt financing can be desirable.

For simplicity, we assume the return for each bank is independent so that, by law of large numbers, α_t is also the proportion of banks that have the high return. In our model, there is aggregate uncertainty in the sense that α_t is a random variable with a continuous distribution f over the unit interval [0, 1].

We assume that the risky technology $\widetilde{R}_0$ has diminishing returns to scale, that is, the return R_0 is decreasing in $(1-l)$. In order to get a closed-form solution, we use a setup similar to Holmstrom and Tirole (2001) and let $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$. Hence, R_0 takes values between $\left(b - \frac{1}{2}\right)$ and b, and $\frac{dR_0}{dl} = \frac{1}{2} > 0$. For simplicity we assume that $\widetilde{R}_1$ is a constant returns to scale technology with $R_1 > 1$. This helps us concentrate on the effect of choice of liquid asset only in the first period and simplifies the analysis without affecting our results significantly.

At the intermediate date $t = 1/2$, the outcome of the first-period investments in the risky asset becomes public information, though banks can collect these returns fully only at $t = 1$.

The safe asset is completely liquid and pays one unit at any date for each unit invested. The risky asset is however not completely liquid due to a moral-hazard problem at the bank level. From date $t = 1/2$ to date $t = 1$, if the bank does not exert effort, then when the return is high, it cannot generate R_t but only $(R_t - \overline{\Delta})$ and its owners enjoy a non-pecuniary benefit of $B \in (0, \overline{\Delta})$. For the bank owners to exert effort, appropriate incentives have to be provided by giving bank owners a minimum share of the bank's profits. We denote this share as θ. If r_t is the cost of borrowing deposits, then the incentive-compatibility constraint is:

$$\alpha_t \theta (R_t - r_t) \geqslant \alpha_t \left[\theta((R_t - \overline{\Delta}) - r_t) + B\right]. \qquad (IC) \qquad \mathbf{(1)}$$

Using this constraint, we can show that bank owners need a minimum share of $\overline{\theta} = \frac{B}{\overline{\Delta}}$ to monitor these loans properly.[6] Therefore, the bank can generate at most a fraction $\tau = \left(1 - \overline{\theta}\right)$ of its future income in the capital market if it is required to exert effort to monitor loans.[7] We assume that at $t = 0$, the entire share of the bank profits belongs to the bank owners, and therefore, moral hazard is not a concern at the beginning. Hence, the expected profit for a surviving bank from the

[6] See Hart and Moore (1994) and Holmstrom and Tirole (1998) for models with similar incentive-compatibility constraints.

[7] The bank-level moral hazard in our model can be addressed by greater ownership of the bank by insiders. Caprio, Laeven and Levine (2007) study the ownership patterns of 244 banks across 44 countries, collecting data on the 10 largest publicly listed banks in those countries. They document that banks in general are not widely held (where a widely-held bank is one that has no legal entity owning 10% or more of the voting rights), a finding that is similar to that of La Porta, Lopez-de-Silanes and Shleifer (1999) for corporations in general. This observation is stronger in those countries which have weaker shareholder protection laws. Importantly, they also find that greater inside ownership of banks enhances bank valuation, especially in those countries where the shareholder protection laws are weaker. Overall, these findings are consistent with the key assumptions of our model since weaker shareholder protection laws should imply a greater risk of cash-flow appropriation by insiders, and, in turn, lead to greater inside ownership of banks in equilibrium.

risky asset in the second period when it chooses the good project is $\overline{p} = [\alpha_1(R_1 - r_1)] = E(\pi_2)$.

We assume that deposits are fully insured in the first period. Note that the second period is the last period in our model and there is no further investment opportunity. As a result, our analysis is not affected by whether deposits are insured for the second investment or not.

Finally, we make technical assumptions (A1)–(A4) which are contained in the Appendix. We refer to these at a few relevant points of our analysis.

If a bank's return from the first-period investment is high, then the bank operates one more period and makes the second-period investment. For a bank to continue operating for another period, it needs to pay its old depositors r_0. But, by our assumption (A2), a failed bank cannot generate the necessary funds to avoid default. Thus, if the return is low, then the bank is in default and the deposit insurance provider puts up the bank for sale at $t = 1/2$.

When banks with the high return from the first period investment want to acquire failed banks' assets, they use the liquid asset in their portfolio and/or try to raise funds from the capital market against their future return. However, because of moral hazard, banks cannot fully pledge their future income, but only a fraction τ of it. Formally, a surviving bank can generate $\tau\left[((1-l)R_0 - r_0) + \overline{p}\right]$ units from the capital market at $t = 1/2$.[8]

Depending on the first-period returns, some banks (say a proportion k out of 1) fail. Since banks are identical at $t = 0$, we denote the possible states at $t = 1$ with k, the proportion of bank failures.

4 Analysis

We analyse the model proceeding backwards from the second period to the first period.

The surviving banks operate for another period at $t = 1$. The probability of having the high return for each bank is equal to α_1 for all banks. As this is the last period, there is no further investment opportunity and no asset sales take place in this period. Since the risky asset has a

[8]We assume that banks can generate funds only against the future profits from their own investments but not against the future profits from the assets they plan to purchase. For the case where this assumption is relaxed without disturbing the key results on fire sales, see the unabridged version of Acharya and Yorulmazer (2005).

higher expected return than the safe asset and there is no asset purchase opportunity, banks invest all their funds in the risky asset at $t = 1$. The expected pay-off to the bank from its second-period investment, $E(\pi_2)$, is thus $\alpha_1[R_1 - r_1]$.

Next, we investigate the sale of failed banks' assets and the resulting asset prices.

4.1 Asset sales and liquidation values

In examining the purchase of failed banks' assets, several interesting issues arise. First, surviving banks may compete with each other if there are enough resources with them to acquire all failed banks' assets. Second, unless the game for asset acquisition is specified with reasonable restrictions, an abundance of equilibria arises. To keep the analysis tractable and at the same time reasonable, we make the following assumptions:

(i) The regulator pools all failed banks' assets and auctions these assets to the surviving banks. When only a part of the total failed banks' assets are sold and the remaining are bailed out, the assets to be sold are chosen randomly.

(ii) Denoting the surviving banks as $i \in [0, (1 - k)]$, each surviving bank submits a schedule $y_i(p)$ for the amount of assets they are willing to purchase as a function of the price p at which a unit of the banking asset (inclusive of associated deposits) is being auctioned.

(iii) The regulator cannot price-discriminate in the auction.

(iv) The regulator determines the auction price p so as to maximise the output of the banking sector, but subject to the natural constraint that portions allocated to surviving banks add up at most to the number of failed banks, that is,

$$y_2(p) + \int_0^{1-k} y_i(p) \leq k. \qquad \textbf{(2)}$$

(v) We focus on the symmetric outcome where all surviving banks submit the same schedule, that is, $y_i(p) = y(p)$ for all $i \in [0, 1]$.

First, we derive the demand schedule for surviving banks. Note that a surviving bank can generate a maximum return of $\overline{p}$ from the risky asset in the second period. Hence, the maximum price a surviving bank is willing to pay for a failed bank's asset is $\overline{p}$. Note that the resources available with a surviving bank for purchasing failed banking assets is equal to

$$L = l + \tau\left[((1-l)R_0 - r_0) + \alpha_1 (R_1 - r_1)\right], \tag{3}$$

when the return from the risky asset is enough to pay old depositors, that is, when $(1-l)R_0 \geqslant r_0$, which holds when $l \leqslant l_{max} = \left[\sqrt{b^2-2} + (1-b)\right]$ for $R_0 = \left(b - \frac{1-l}{2}\right)$. It can be shown that under assumptions (A1) and (A2), banks never hold a level of liquidity l greater than l_{max} in equilibrium (see Appendix), so that this condition is always satisfied.

Note that the expected profits of a surviving bank from the asset purchase can be calculated as: $y(p)[\overline{p} - p]$. The surviving bank wishes to maximise these profits subject to the resource constraint $y(p) \cdot p \leq L$. Hence, for $p < \overline{p}$, surviving banks are willing to purchase the maximum amount of failed banks' assets using their resources. Thus, demand schedule for surviving banks is

$$y(p) = \frac{L}{p}. \tag{4}$$

For $p > \overline{p}$, the demand is $y(p) = 0$, and for $p = \overline{p}$, $y(p)$ is indeterminate. In words, as long as purchasing bank assets is profitable, a surviving bank wishes to use up all its resources to purchase failed banks' assets.

Next, we analyse how the regulator allocates the failed banks' assets and the price function that results. The regulator cannot set $p > \overline{p}$ since in this case we have $y(p) = y_2(p) = 0$. If $p \leqslant \overline{p}$, and the proportion of failed banks is sufficiently small, then the surviving banks have enough funds to pay the full price for all the failed banks' assets. More specifically, for $k \leq \underline{k}$, where

$$\underline{k} = \left(\frac{L}{L + \overline{p}}\right), \tag{5}$$

the regulator sets the auction price at $p^*(k) = \overline{p}$. At this price, surviving banks are indifferent between any quantity of assets purchased. Hence, the regulator allocates a share $y(p^*) = \left(\frac{k}{1-k}\right)$ to each surviving bank.

For values of $k > \underline{k}$, surviving banks cannot pay the full price for all failed banks' assets. Formally, for $k > \underline{k}$, the regulator sets the price at $p^*(k) = \left(\frac{(1-k)L}{k}\right)$. Note that, in this region, surviving banks use all available funds and the price falls as the number of failures increase. This effect is basically the cash-in-the-market pricing as in Allen and Gale (1994, 1998) and is also akin to the industry-equilibrium hypothesis of Shleifer and Vishny (1992).

The resulting price function is formally stated in the following proposition and is illustrated in Figure 2.

Proposition 1 The price of failed banks' assets as a function of the proportion of failed banks is as follows:

$$p^*(k) = \begin{cases} \overline{p} & \text{for} \quad k \leqslant \underline{k} \\ \frac{(1-k)L}{k} & \text{for} \quad k > \underline{k} \end{cases} . \qquad \textbf{(6)}$$

From equation **(5)**, one can easily see that as banks hold less of the liquid asset, $\underline{k}$ decreases, that is, the region over which the price is equal to the fundamental price $\overline{p}$ shrinks. In turn, from Proposition 1 and Figure 2, one can easily see that for all values of k, when banks hold less of the liquid asset, prices deviate more from the fundamental price, that is, $\left[\overline{p} - p^*(k)\right]$ (weakly) increases. This gives us the following corollary.

Corollary 2 For all k, as aggregate liquidity l decreases, prices deviate more from the fundamental price $\overline{p}$, that is, $\left[\overline{p} - p^*(k)\right]$ (weakly) increases.

4.2 Banks' choice of liquidity

In the first period, all banks are identical. Hence, we consider a representative bank. Formally, the objective of each bank is to choose a portfolio of the safe and the risky asset, namely $(l, 1-l)$, at date 0 that maximises the sum of expected profits at $t = 1$ and $t = 2$, the expected profits from their own investments, from the asset purchases when they survive and losses from the opportunity cost of holding liquid assets in their portfolio.

Using the prices derived in Proposition 1, we can calculate profits for surviving banks from asset purchases. When only a small proportion of banks fail, $k \leq \underline{k}$, surviving banks pay the full price for the acquired assets and do not capture any surplus from the asset purchase. In these cases, from an *ex-post* standpoint, banks carry excess liquidity in their portfolio and incur losses from forgone investment in the risky asset.

When the proportion of failed banks is higher, $k > \underline{k}$, each surviving bank captures a surplus from asset purchase that equals

$$y(p^*) \cdot \left[\overline{p} - p^*\right] = \frac{k\overline{p}}{(1-k)} - L. \quad \textbf{(7)}$$

In all cases, bank owners of failed banks have no continuation pay-offs.

Given this analysis, we can formalise each bank's portfolio choice that gives rise to a competitive equilibrium as follows. Bank i's problem is to choose l_i that maximises

$$E(\pi(l_i)) = E\left(\alpha_0 \left[\left[(l_i + (1-l_i)R_0(l_i)) - r_0\right] + L\left(\frac{\left[\overline{p} - p^*(k)\right]}{p^*(k)}\right) + \overline{p}\right]\right), \quad \textbf{(8)}$$

where $p^*(k)$ is the market clearing price given in Proposition 1. Recall that the return for each bank is independent so that, by law of large numbers, α_0 is also the proportion of banks that have the high return. Hence, we have $k = (1 - \alpha_0)$.

The first-order condition (FOC) for the maximisation problem is given as:

$$E\left[\alpha_0\left(\left[1 - R_0 + (1-l_i)\frac{dR_0}{dl}\right] + \left[1 - \tau R_0 + \tau(1-l)\frac{dR_0}{dl}\right]\left[\frac{\left[\overline{p} - p^*(k)\right]}{p^*(k)}\right]\right)\right] = 0 \quad \textbf{(9)}$$

We define

$$\phi = \alpha_0 \left[\frac{\overline{p} - p^*(k)}{p^*(k)}\right], \quad \textbf{(10)}$$

as the expected benefit from asset purchase per unit of liquidity. See Figure 3 for an illustration of ϕ as a function of k. Note that ϕ is independent of l when viewed from a price-taking bank's perspective, but in equilibrium, $p^*(k)$ depends on the aggregate liquidity in state k. Hence, banks' equilibrium choice of liquid asset holdings is given by a fixed point that is formally stated below and is illustrated in Figure 4.

Proposition 3 Banks' choice of liquidity $\widehat{l}$ that satisfies the FOC in **(9)** is given by

$$\widehat{l} = \min\left\{1, \max\left\{0, 1 - b + \frac{E(\alpha_0) + E(\phi)}{E(\alpha_0) + \tau E(\phi)}\right\}\right\}. \quad \textbf{(11)}$$

The unique aggregate level of liquidity l^* is the fixed point of

$$\widehat{l}(E(\alpha_0), \tau, E(\phi(\alpha_0, \tau, l^*))) = l^*. \quad \textbf{(12)}$$

Note that $\widehat{l}$ is a (weakly) declining function of aggregate liquidity l. The intuition for this is that if aggregate liquidity is low, then the deviation of prices from the fundamental value is high, creating a motive to hold liquidity to acquire failed banks at lower prices. Conversely, if aggregate liquidity is high, then the expected gain from asset purchases is low and the incentives of a bank to carry liquid buffers is low as well.

Several aspects of the banks' private choice of liquidity l^* deserve mention and will play an important role in comparison to the socially optimal choice of liquidity. First, if $\tau = 1$, then $l^* = 0$, the portfolio choice that trades off simply the expected returns to the bank owners from the risky asset and the safe asset. In particular, in this case both assets are fully liquid so that portfolio choice is not affected by intertemporal liquidity considerations.

Note that the strategic benefit of holding liquid assets for an individual bank, given by ϕ, depends on the liquidity in the whole market, since the market liquidity l^* affects the price $p^*(k)$. The endogenous determination of prices, and, in turn, of the strategic benefit to banks from acquiring other banks, is an important distinguishing feature of our model.

Second, if $\tau < 1$, then liquidity cannot be generated against full expected value of uncertain cash flows. As a result, there is an intertemporal motive to hold liquidity. Specifically, liquid holdings exceed those from the portfolio choice problem as liquid assets dominate risky assets in states where there is a strategic benefit from acquiring failed banks at cash-in-the-market prices ($k > \underline{k}$).

4.3 Comparative statics

In this section, we analyse how banks' choice of liquidity is affected by model parameters. Since $R_0 = b - \left(\frac{1-l}{2}\right)$, as b increases, the return from the risky asset increases. This also increases the liquidity banks can generate against their profits in the first period. Hence, as b increases, the liquid asset becomes less attractive and banks choose a lower level of the liquid asset l^*. This relation is apparent from equation **(11)**.

Next, we investigate effects of the development of capital markets and the business cycle on banks' choice of liquidity, which form the primary testable implications of our model. In developed economies, we would expect highly developed capital markets where banks can generate funds freely against future profits. Hence, one can interpret τ in our model as a metaphor for the level of development in capital markets. Also, we know that the cost of issuing capital rises during economic downturns. Thus, in line with this empirical evidence, we can say that during economic downturns, the pledgeability of future returns, τ, decreases. We show below that for low values of τ, that is for less-developed economies and during economic downturns, banks hoard more liquidity since they cannot have easy or cheap access to capital markets for raising funds.

Also, during boom periods it is more likely that risky investments will pay-off well. To this end, we consider two different probability distributions, f and g, to represent recessions and boom periods, respectively, by assuming that g first order stochastically dominates (FOSD) f. We show that in equilibrium, banks invest less in the liquid asset during boom periods. Combining these two results, we get the following formal Proposition.

Proposition 4 Banks' choice of liquidity l^* has the following features:

(i) As the pledgeability of future returns, τ, increases, privately optimal levels of liquidity decrease.

(ii) Let f and g be two probability distributions for α_0, where g FOSD f. Let l^*_f and l^*_g be the liquid asset holdings of banks under probability distributions f and g, respectively. We have $l^*_f > l^*_g$.

Note that from expression **(10)**, ϕ is (weakly) decreasing in α_0 (see Figure 3). Increased probability of the high return has two effects on banks' choice of liquidity that work in the same direction. First, the expected return from the risky asset increases, which makes the risky asset more attractive. Also, the proportion of failed banks decreases, which limits the opportunity for making profits from asset purchases at cash-in-the-market prices. This, in turn, makes the liquid asset less attractive. Similarly, as τ increases, banks can generate more funds from the capital market. Hence, banks do not have to heavily rely on their liquid asset holdings which yield lower return than risky assets.

We can combine these two effects by modelling the business cycle in a simple way by assuming that if g FOSD f then $\tau_g > \tau_f$. This assumption amplifies the effect of the business cycle on banks' choice of liquidity. Also, from Corollary 2, we know that as liquidity decreases, we observe bigger deviations in the price of banking assets from its fundamental value of $\overline{p}$. Hence, crises preceded by boom periods result in lower asset prices and higher price volatility, giving us the following result.

Corollary 5 During economic upturns, banks' choice of liquidity l^* decreases. This, in turn, results in bigger deviations in the price of banking assets from their fundamental value of $\overline{p}$, that is, $\left[\overline{p} - p^*(k)\right]$ increases.

4.4 Socially optimal liquidity

In the following analysis, we derive the liquidity level of banks l^{**} that maximises the expected total output generated by the banking sector, given as:

$$E(\Pi) = E\left[l + \alpha_0(1-l)R_0(l)\right] + \alpha_1 R_1. \tag{13}$$

The first-order condition for the socially optimal level of l is thus given as:

$$1 - E(\alpha_0)\left[R_0(l) - (1-l)\frac{dR_0}{dl}\right] = 0. \tag{14}$$

Again, we use $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$ to get the following proposition, which formalises the socially optimal level of liquidity, denoted by l^{**}.

Proposition 6 The socially optimal level of liquidity satisfying the FOC in **(14)** is given as:

$$l^{**} = \min\left\{1, \max\left\{0, 1 - b + \frac{1}{E(\alpha_0)}\right\}\right\}. \tag{15}$$

Furthermore, we have $l^{**}_f > l^{**}_g$, when g FOSD f.

Note that the socially optimal level of liquidity is determined by only the portfolio choice. In contrast to the private choice of banks, asset sales do not play a role. When b increases, the return from the risky asset increases and the socially optimal level of liquidity l^{**} decreases, which can be seen from equation **(15)**. Furthermore, l^{**} is independent of τ. but is higher during recessions as was the case with privately optimal bank liquidity.

4.5 Comparing socially and privately optimal levels of liquidity

In this section, we compare the privately and socially optimal levels of liquidity. We show that a crucial determinant of the relationship between privately and socially optimal levels of liquidity is the extent of pledgeability of risky cash flows. When pledgeability is high, banks hold less liquidity than is socially optimal due to risk-shifting incentives, whereas when pledgeability is sufficiently low, (somewhat counterintuitively) banks may hold even more liquidity than is socially optimal. The intuition for this latter result in the context of our model is that banks stand to gain from acquiring failed banks in some states where there is no misallocation cost but only transfers within the banking system.

Similarly, we also show that the privately optimal level of liquidity is inefficiently low during economic downturns (even though in terms of absolute magnitude it is higher in downturns than in boom times).

Proposition 7 Comparing the privately and socially optimal liquidity levels, we obtain that:

(i) There exist critical values $\tau^*(E(\alpha_0))$, *such that, the privately optimal level of liquidity is higher than the socially optimal level if and only if* $\tau < \tau^*(E(\alpha_0))$.

(ii) *There exists a critical value* $\alpha_0^*(\tau)$, *such that, the privately optimal level of liquidity is higher than the socially optimal level if and only if* $E(\alpha_0) > \alpha_0^*(\tau)$.

Furthermore, $\tau^*(E(\alpha_0)) < E(\alpha_0)$ *and conversely* $\alpha_0^*(\tau) > \tau$.

5 Entry and inefficient liquidations

In the benchmark model, only banks were present in the market for banking assets. Hence, the sale of banking assets did not result in any misallocation of banking assets. In this section, we analyse the effect of entry by outsiders.

We introduce outside investors who are risk-neutral and competitive and have funds w to purchase banking assets were these assets to be liquidated.[9] These investors are from outside the banking sector so that although they have funds for asset purchases, they do not have the skills to generate the full value from banking assets. In particular, outsiders are inefficient users of banking assets relative to the bank owners, provided that bank owners exert effort. Often such

[9] In this model, the liquidity of outsiders is exogeneously given. See Allen and Gale (1998) for an endogenous choice of outsider liquidity.

outsiders are short-term holders who repackage or securitise the assets for selling on to portfolio investors. However, outsiders may be unable to realise the full value of the assets for the familiar reason that bank assets (loans in particular) derive much of their value from the monitoring and collection efforts of loan officers who can influence the actions of the debtors. Hence, when distressed assets end up in the hands of outsiders, we may expect deadweight costs from inefficient allocation of assets.

To capture this formally, we assume that outsiders cannot generate R_t in the high state but only $(R_t - \Delta)$. We also assume that $\overline{\Delta} > \Delta$ so that outsiders can generate more than what the banks can generate from bad projects.[10]

Next, we investigate the sale of failed banks' assets and the resulting asset prices in the presence of outsiders. The demand schedule for surviving banks does not change and we can derive the demand schedule for outsiders in a similar way. Let $\underline{p} = [\alpha_1 ((R_1 - \Delta) - r_1)] = [\overline{p} - \alpha_1 \Delta]$, the expected profit for the outsiders from the risky asset in the second period.

For $p < \underline{p}$, outsiders are willing to supply all their funds for the asset purchase. Thus, optimal demand schedule is $y_2(p) = k$. For $p > \underline{p}$, the demand is $y_2(p) = 0$, and for $p = \underline{p}$, $y_2(p)$ is indeterminate. Thus, for $p > \underline{p}$, there is limited participation in the market for banking assets.

Next, we analyse how the regulator optimally allocates the failed banks' assets and the price function that results. We know that in the absence of financial constraints, the efficient outcome is to sell all assets to surviving banks. However, surviving banks may not be able to pay the threshold price of $\underline{p}$ for all assets. If prices fall further, these assets become profitable for outsiders and they participate in the auction. Formally, as long as price is higher than $\underline{p}$, outsiders do not participate in the asset market. However, for $k > \overline{k}$, where

$$\overline{k} = \left(\frac{L}{L + \underline{p}} \right), \qquad \textbf{(16)}$$

surviving banks cannot pay the threshold price of $\underline{p}$ for all assets. At this point, outsiders have a

[10]The notion that outsiders may not be able to use the banking assets as efficiently as the existing bank owners is akin to the notion of *asset-specificity*, first introduced in the corporate-finance literature by Williamson (1988) and Shleifer and Vishny (1992). There is strong empirical support for this idea in the corporate-finance literature, as shown, for example, by Pulvino (1998) for the airline industry, and by Acharya, Bharath, and Srinivasan (2004) for the entire universe of defaulted firms in the United States over the period 1981 to 1999 (see also Berger, Ofek and Swary (1996) and Stromberg (2000)). In the evidence of such specificity for banks and financial institutions, James (1991) shows that the liquidation value of a bank is typically lower than its market value as an ongoing concern. In particular, his empirical analysis of the determinants of the losses from bank failures reveals a significant difference in the value of assets that are liquidated and similar assets that are assumed by acquiring banks.

positive demand and are willing to supply all their funds for the asset purchase. For the moment, we assume that outsiders have enough funds to purchase all assets, that is, $w \geqslant \underline{p}$. As a result, with the injection of outsider funds, prices can be sustained at $\underline{p}$. This price function is stated in the following proposition and is illustrated in Figure 6. We analyse the effect of outsider wealth w when $w < \underline{p}$, that is, when outsiders do not have enough funds to purchase all assets at $\underline{p}$ in the next section.

Proposition 8 The price of assets as a function of the proportion of failed banks is:

$$p^*(k) = \begin{cases} \overline{p} & \text{for} \quad k \leqslant \underline{k} \\ \frac{(1-k)L}{k} & \text{for} \quad k \in (\underline{k}, \overline{k}] \\ \underline{p} & \text{for} \quad k > \overline{k} \end{cases} . \quad \textbf{(17)}$$

5.1 Banks' choice of liquidity

The introduction of outsiders (weakly) increases the price for failed banks. In particular, for $k > \overline{k}$, with the injection of outsiders' funds, the price stays at $\underline{p}$. This decreases the benefit ϕ from holding the liquid asset in terms of profit from asset purchase. In this case, banks' problem, as well as the expected benefit from asset purchase ϕ per unit of liquidity, can be stated in the same way as in equations **(8)** and **(10)**, respectively. However, for asset price, we employ values in equation **(17)**, rather than the values in equation **(6)**. As a result, the value for ϕ changes for a high proportion of bank failures, that is, for $k > \overline{k}$. In particular, we have $\phi = \alpha_0 (\alpha_1 \Delta)$, for $k > \overline{k}$. See Figure 7 for an illustration of ϕ as a function of k.

Note that ϕ is not monotone increasing in k. The reason for this is that, for $k > \overline{k}$, with the participation of outsiders, price never falls below $\underline{p}$ and the profit for a surviving bank from purchasing a unit of failed banks' asset is bounded by $(\alpha_1 \Delta)$, whereas a bank survives only with probability α_0. Hence, as α_0 decreases, the marginal gain from holding the liquid asset goes down for $k > \overline{k}$. Since, ϕ is no longer monotone in α_0, the comparative statics result on $E(\alpha_0)$ in this case is not as clean as the result in the benchmark case. However, we can derive interesting results on the effect of expertise $(\alpha_1 \Delta)$ and the wealth of outsiders (w) on banks' choice of liquidity.

Comparative statics

So far, we assumed that the liquidity of outsiders w is greater than $\underline{p}$, so that there is always enough liquidity in the market to keep the price for assets at $\underline{p}$. Below, we relax this assumption and allow for lower levels of outsider funds.

In particular, when outsiders have limited funds w, that is, when $w < \underline{p}$, if the crisis is very severe (sufficiently large k), the total liquidity available within the surviving banks and outsiders may not be enough to sustain the price for assets at $\underline{p}$. Thus, we may observe a second region where the price is downward sloping as a function of the proportion of failed banks k. In other words, there is cash-in-the-market pricing in this region given the limited liquidity of the *entire* set of market players bidding for assets. In particular, for $k > \overline{\overline{k}}$, where

$$\overline{\overline{k}} = \left(\frac{L + w}{L + \underline{p}}\right), \tag{18}$$

the price is again strictly decreasing in k and is given by

$$p_w^*(k) = \left(\frac{(1-k)\,L + w}{k}\right), \tag{19}$$

and $y(p_w^*) = \left(\frac{L}{p_w^*}\right)$ and $y_2(p_w^*) = \left(\frac{w}{p_w^*}\right)$.

This price function is illustrated in Figure 8 and is given as follows:

$$p_w^*(k) = \begin{cases} \overline{p} & \text{for} \quad k \leqslant \underline{k} \\ \frac{(1-k)L}{k} & \text{for} \quad k \in (\underline{k}, \overline{k}] \\ \underline{p} & \text{for} \quad k \in (\overline{k}, \overline{\overline{k}}] \\ \frac{[(1-k)L]+w}{k} & \text{for} \quad k > \overline{\overline{k}} \end{cases}. \tag{20}$$

As in the benchmark case, using the price in equation **(20)**, we can calculate profits for surviving banks from asset purchases. In this case, the difference is that for $k > \overline{\overline{k}}$, surviving banks can acquire assets at prices lower than $\underline{p}$, which increases expected profit.

Bank i's problem can be stated in the same way as in the benchmark case (equation **(8)**), except for the fact that instead of ϕ, we have

$$\phi_w = \alpha_0 \left[\frac{\overline{p} - p_w^*(k)}{p_w^*(k)}\right], \tag{21}$$

as the expected benefit from asset purchase per unit of liquidity. Note that for $k \leqslant \overline{\overline{k}}$, $\phi_w = \phi$, whereas for $k > \overline{\overline{k}}$, we have $\phi_w > \phi$. Since $E(\phi_w) > E(\phi)$, the unique aggregate level of liquidity l_w^* is higher than l^* given in Proposition 3. Furthermore, as outsider wealth w increases, the threshold $\overline{\overline{k}}$ increases. Also, as w increases, the price p_w^* weakly increases for each k. This, in turn, decreases the private benefit ϕ_w and induces banks to hold less liquid asset: as w increases, l_w^* decreases.

We observe a similar effect of $(\alpha_1 \Delta)$ on banks' choice of liquidity. In particular, as the wedge between the expertise of banks and outsiders widens, that is, as $(\alpha_1 \Delta)$ increases, the price for assets (weakly) decreases for all values of k. Just like a decrease in outsider wealth w, this increases ϕ. As a result, banks hold more liquidity and l^* increases. We combine these results in the following proposition.

Proposition 9 As $(\alpha_1 \Delta)$ increases, banks' choice of liquidity l^* increases. With limited outsider funds, that is, for $w < \underline{p}$, banks' choice of liquidity $\widehat{l}_w$ is given by

$$\widehat{l}_w = \min\left\{0, \max\left\{0, 1 - b + \frac{E(\alpha_0) + E(\phi_w)}{E(\alpha_0) + \tau E(\phi_w)}\right\}\right\}, \qquad \textbf{(22)}$$

where ϕ_w is given by **(21)**. The unique aggregate level of liquidity l_w^* is the fixed-point of

$$\widehat{l}_w(E(\alpha_0), \tau, E(\phi(\alpha_0, \tau, l^*))) = l_w^*. \qquad \textbf{(23)}$$

Furthermore, as w increases l_w^* decreases.

Next, we analyse the socially optimal level of liquidity and compare it with the banks' choice derived in this section.

5.2 Socially optimal liquidity

To start with, we assume that outsider wealth w is greater than $\underline{p}$. The socially optimal liquidity level l of each bank maximises the objective function

$$E(\Pi) = E\left[l + \alpha_0(1 - l)R_0(l)\right] + \alpha_1 R_1 - (\alpha_1 \Delta) \int_{\overline{k}}^{1} f(k)\left[k - \frac{(1-k)L}{\underline{p}}\right] dk \qquad \textbf{(24)}$$

where $\left[k - \frac{(1-k)L}{\underline{p}}\right]$ represents the units of assets purchased by outsiders, which multiplied by $(\alpha_1 \Delta)$ represent the social welfare loss arising from their lack of expertise relative to banks.

On the one hand, as banks hold more liquid assets, the first expression decreases since in expected terms, risky asset has a higher return than the safe asset. On the other hand, as banks hold more liquid assets, they have more resources to acquire failed banking assets, which decreases the misallocation cost.

The first-order condition for the socially optimal level of l is thus given as:

$$1 - E(\alpha_0)\left[R_0(l) - (1-l)\frac{dR_0}{dl}\right] - (\alpha_1\Delta)\frac{d}{dl}\left[\int_{\bar{k}}^{1} f(k)\left[k - \frac{(1-k)L}{\underline{p}}\right] dk\right] = 0. \quad \textbf{(25)}$$

Again, to get closed-form solutions, we use $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$. Also, let

$$E(\gamma) = \left(\frac{\alpha_1\Delta}{\underline{p}}\right)\int_{\bar{k}}^{1} f(k)(1-k)\, dk, \quad \textbf{(26)}$$

which can be interpreted as the marginal reduction in expected misallocation cost for an additional unit of liquidity within the set of surviving banks (see Figure 7). We can characterise the socially optimal level of liquidity, denoted by l^{**} as follows.

Proposition 10 When outsider wealth w exceeds $\underline{p}$, the socially optimal level of liquidity satisfies the FOC **(25)** and is given as:

$$\hat{\hat{l}} = \min\left\{1, \max\left\{0, 1 - b + \frac{1 + E(\gamma)}{E(\alpha_0) + \tau E(\gamma)}\right\}\right\}. \quad \textbf{(27)}$$

The unique level of liquidity l^{**} is the fixed point of $\hat{\hat{l}}(\alpha_0, \tau, E(\gamma(\alpha_0, \tau, l^{**}))) = l^{**}$.

As a function of equilibrium liquidity l, $\hat{\hat{l}}$ behaves similar to $\hat{l}$ (Figure 4). When aggregate liquidity is high, misallocation costs are low and it becomes less desirable to carry additional liquidity. Similarly, if aggregate liquidity is low, the misallocation region is large and carrying additional liquidity is attractive from a social standpoint.

Note that when $\tau = 1$, the socially optimal liquidity l^{**} may exceed zero, the level of privately optimal liquidity for this value of τ. This is because bank owners are concerned only about their return when they survive ('risk-shifting') whereas from a social standpoint, the relevant trade-off is between the expected return of the two assets (and not between the 'option' value of risky asset against the return on the safe asset). It is important to point out that this deviation between the privately optimal and the socially optimal levels of liquidity arises purely due to agency conflict between bank owners and depositors (or the deposit-insurance provider) and not because of any considerations of intertemporal transfers of liquidity.

While this case ($\tau = 1$) is intuitive and well-known, the more interesting possibility arises when risky asset is illiquid ($\tau < 1$) so that intertemporal motive to hold liquidity arises. We show below that the privately optimal level of liquidity may exceed the socially optimal level in this case.

Comparative statics

In this section, we analyse the effect of model parameters on the socially optimal level of liquidity. As b increases, the return on risky asset improves and the socially optimal level of liquidity l^{**} decreases, which can be seen from equation **(27)**.

The limited liquidity of outsiders w also affects l^{**}. As in the benchmark case with $w \geqslant \underline{p}$, we can derive the socially optimal level of liquidity for $w < \underline{p}$. In this case, the socially optimal liquidity level l of each bank maximises the objective function

$$E(\Pi) = E\left[l + \alpha_0(1-l)R_0(l)\right] + \alpha_1 R_1 - (\alpha_1 \Delta) \int_{\bar{k}}^{1} f(k) \left[k - \frac{(1-k)L}{p_w^*(k)}\right] dk \qquad \textbf{(28)}$$

where $\left[k - \frac{(1-k)L}{p_w^*(k)}\right]$ represents the units of failed banks' assets purchased by outsiders at the price $p_w^*(k)$. We can write the first-order condition as:

$$1 + E(\alpha_0)\ \left[-b + (1-l)\right] + E(\gamma_w)\ \left[1 + \tau\left(-b + (1-l)\right)\right] = 0, \text{ where} \qquad \textbf{(29)}$$

$$E(\gamma_w) = E(\gamma) - \frac{(\alpha_1 \Delta)}{\underline{p}} \left[\int_{\bar{\bar{k}}}^{1} f(k)(1-k)\left[1 - \frac{kw\underline{p}}{\left[(1-k)L + w\right]^2}\right] dk\right] \qquad \textbf{(30)}$$

is the marginal reduction in expected misallocation cost for an additional unit of liquidity within the set of surviving banks. We thus obtain that:

Proposition 11 When outsiders have limited funds w, the socially optimal level of liquidity that satisfies the FOC in **(29)** is given as:

$$\widehat{\widehat{l}}_w = \min\left\{1, \max\left\{0, 1 - b + \frac{1 + E(\gamma_w)}{E(\alpha_0) + \tau E(\gamma_w)}\right\}\right\}, \qquad \textbf{(31)}$$

where $E(\gamma_w)$ is given in equation **(30)**. The unique level of liquidity l^{**} is the fixed point of $\widehat{\widehat{l}}_w(\alpha_0, \tau, E(\gamma(\alpha_0, \tau, l_w^{**}))) = l_w^{**}$. Furthermore, for $w > \left(\frac{L\underline{p}}{2L+\underline{p}}\right)$, the socially optimal level of liquidity l_w^{**} decreases as outsider wealth w increases.

Consider now the effect of $(\alpha_1 \Delta)$ on the socially optimal level of liquidity. Note that as $E(\gamma)$ increases, the socially optimal level of liquidity l^{**} increases. Hence, we need to examine the sign of $\frac{\partial E(\gamma)}{\partial(\alpha_1 \Delta)}$. We have

$$\frac{\partial E(\gamma)}{\partial(\alpha_1 \Delta)} = \left(\frac{1}{\underline{p}}\right)\left[\int_{\overline{k}}^{1} f(k)(1-k)\,dk - (\alpha_1 \Delta)\, f(\overline{k})(1-\overline{k})\,\frac{\partial \overline{k}}{\partial(\alpha_1 \Delta)}\right]. \quad \textbf{(32)}$$

Note that there are two effects at work here. On the one hand, as outsiders become less experienced, that is, as $(\alpha_1 \Delta)$ increases, the threshold $\overline{k}$ increases since $\frac{\partial \overline{k}}{\partial(\alpha_1 \Delta)} > 0$. Hence, as $(\alpha_1 \Delta)$ increases, the region over which we observe misallocation cost shrinks. This has a positive effect on social welfare and relaxes the burden of holding liquid assets to prevent misallocation cost. On the other hand, conditional on ending up in states where some of the banking assets have to be liquidated to outsiders, that is, for $k > \overline{k}$, the misallocation cost per unit of banking asset from sales to outsiders increases. Thus, the combined effect of an increase in $(\alpha_1 \Delta)$ on the socially optimal level of liquidity is not unambiguous.

5.3 Comparing socially and privately optimal levels of liquidity

In this section, we compare the privately and socially optimal levels of liquidity in the presence of outsider entry in the market for asset sales. As argued above, the conflict of interest between bank owners and senior claimants of the bank tends to push the privately optimal level of liquidity below the socially optimal level. In particular, this always holds with $\tau = 1$.

However, if $\tau < 1$, bank owners have an intertemporal motive to hold liquidity: surviving banks make profits from asset purchases when the proportion of failures is above $\underline{k}$, that is, $k > \underline{k}$, but since $\tau < 1$ they cannot pledge risky cash flows fully to capitalise on this benefit. Hence, there is a benefit from carrying liquidity into such states. In contrast, social welfare losses materialise only when the proportion of failures is above $\overline{k}$, that is, $k > \overline{k}$. For the intermediate region $[\underline{k}, \overline{k}]$, while banks gain by purchasing assets at cash-in-the-market prices, there is no social welfare loss. Thus, if τ is sufficiently small, then the private incentive to hold liquidity for intertemporal transfers can prevail over the risk-shifting incentive, and, in turn, privately optimal level of liquidity can exceed the social one. To summarise, if sufficient liquidity cannot be raised against risky cash flows in a contingent fashion in future, then banks may carry excess liquidity (inefficiently bypassing profitable lending opportunities) in order to stand ready for acquiring failed banks at attractive prices.

By the same token, given a value of τ, when the intermediate region $[\underline{k}, \overline{k}]$ is not very wide, that is, Δ is not very large, banks hold less than the socially optimal level of liquidity: the risk-shifting incentive dominates in this case. In other words, when the difference between the fundamental value of bank assets and the price outsiders are willing to pay for them is not very high, banks choose to hold less than socially optimal levels of liquidity.

We provide a formal presentation of the above discussion in the following proposition which is illustrated in Figure 9.

Proposition 12 With the possibility of outsider entry, the privately optimal and the socially optimal levels of liquidity are related as follows:

(i) *There exist critical values $\tau^*(\Delta)$ and $\tau^{**}(\Delta)$, such that, for $\tau > \tau^*(\Delta)$, the socially optimal level of liquidity is higher than the privately optimal level, and for $\tau < \tau^{**}(\Delta)$, the privately optimal level of liquidity is higher than the socially optimal level, where $\tau^{**}(\Delta) \leqslant \tau^*(\Delta)$.*

(ii) *There exists a critical value $\Delta^*(\tau)$, such that, for $\Delta < \Delta^*(\tau)$, the socially optimal level of liquidity is higher than the privately optimal level, and $\Delta^*(0) > 0$ and $\Delta^*(1) = \Delta_{\text{max}}$. Finally, $\tau^*(\Delta) > \tau^{**}(\Delta) = 0$, for all $\Delta < \Delta^*(0)$.*

6 Some evidence on bank liquidity

So far, our focus has mainly been to present a theoretical model for analysing the liquidity choice of banks in anticipation of financial crises. In this section, we provide some anecdotal and descriptive empirical evidence that is consistent with the model's implications for liquidity holdings of banks.

6.1 Hoarding of liquidity by banks for gains during crises

We focus below on one salient historical anecdote of a bank hoarding liquidity for strategic gains during crises – that of National City Bank from the United States banking system during the pre-Federal Reserve era.[11]

[11] Casual empiricism suggests however that such cases are not uncommon. In fact, our private communications with bankers suggest that during the most recent sub-prime crisis of 2007 too, one of the perceived reasons for drying up of interbank lending markets has been the

Cleveland and Thomas in their book *Citibank* provide a memorable account of how National City Bank, that eventually became Citibank, grew from a small treasury unit into one of the biggest commercial banks under its president Stillman, who anticipated the 1893 and 1907 crises and built up liquidity and capital before the crises to benefit from the difficulties of its competitors. In terms of actual levels of bank liquidity, the reserve ratio of National City Bank was 42.6% and 26.9% right before the 1893 and 1907 crises, respectively, while these ratios were lower at 25.2% and 24.9% for all other New York City banks. Also, for the 1907 crisis, the capital to net deposits stood at 35.2% for National City Bank, whereas it was 27.5% for all other New York City banks.

What was the impact of such positioning of the balance sheet by National City Bank in terms of cash and capital? Cleveland and Thomas report that during the 1893 (1907) crises, while National City Bank increased its deposits by 12.4% (23.5%), deposits in all other New York City banks decreased by 14.5% (increased by only 9.2%). Furthermore, during the 1893 (1907) crises, while National City Bank increased its loans and discounts by 14.7% (10.2%), loans and discounts in all other New York City banks decreased by 9.1% (increased by only 3.7%). In other words, evidence shows that National City Bank expanded its business operations while other banks were simultaneously experiencing a shrinkage. We document below that *hoarding liquidity to acquire business that belonged to distressed institutions* (in case of 1907 crisis, the New York-based trusts) was indeed the strategy followed by the bank. Below is the paragraph about the 1907 crisis from Cleveland and Thomas' book (page 52) which illustrates this point succinctly:

National City Bank again emerged from the panic a larger and stronger institution. At the start, National City had higher reserve and capital ratios than its competitors, and during the panic it gained in deposits and loans relative to its competitors. Stillman (President) had anticipated and planned for this result. In response to Vanderlip's (Vice President) complaint in early 1907 that National City's low leverage and high reserve ratio was depressing profitability, Stillman replied: "I have felt for sometime that the next panic and low interest rates following would straighten out good many things that have of late years crept into banking. What impresses me most important is to go into next Autumn (usually a time of financial stringency) ridiculously strong and liquid, and now is the time to begin and shape for it... If by able and judicious management we have money to help our dealers when trust companies have suspended, we will have all the business we want for many years."

hoarding of liquidity by banks for acquisitions of troubled institutions at fire-sale prices, the other two reasons being precautionary motive from the risk of being distressed oneself and adverse selection about borrowing institutions.

6.2 Bank liquidity and ease of external finance

To provide more systematic evidence on bank liquidity, we appeal first to the robust implication of our analysis that the greater is the difficulty banks face in raising external finance, the more would banks hold liquid assets. We explore this hypothesis by examining the liquid asset holdings of banks in a cross-section of countries.

In a recent paper, Freedman and Click (2006) show that banks in developing countries choose to channel only a modest portion of their funds to private sector borrowers, while keeping a sizable percentage of their deposits in liquid assets, such as cash, deposits with other banks, central bank debt, and short-term government securities. They construct a liquidity ratio for banks, defined as the ratio of liquid assets to total deposits, using the International Financial Statistics provided by the IMF.[12] They show that for developing countries the ratio ranges from 14% in South Africa to 126% in Argentina, with a mean value of 45%, with values of 2% for the United Kingdom, 6% for the United States, 21% for Japan, 31% for France and 34% for Germany, with an average of 19% for developed countries.

They attribute this difference among developed and developing countries to banks' reluctance to lend in developing countries. Such reluctance, they argue, could be a response to inefficiencies in credit markets resulting from factors such as higher reserve requirements, greater macroeconomic risk and volatility, and significant deficiencies in the legal and regulatory environment which make it difficult to enforce contracts and foreclose on collateral. Also, the risk-free rate is set so high in some emerging countries that there is little incentive for their banks to lend to private sector. In this paper, we argue that an alternative channel may also be at work. Banks in poor legal and regulatory environments may find it difficult to raise liquidity against future profits and thus end up hoarding greater liquidity. Such cash hoardings may be inefficiently high and result in low levels of intermediation by the banking sector.

We expand on the data set of Freedman and Click (2006) to cover about 70 countries with data on liquidity ratios dating back to September 2003. First, we link bank liquidity to a number of institutional variables that capture country's financial development in terms of quality of disclosures, and the extent of stock and credit intermediation (relative to country's size). These

[12] In particular, they calculate liquid assets as the sum of reserves (line 20) and claims on central government (usually line 22A), and total deposits as the sum of demand deposits (line 24), time and savings deposits (line 25), money market instruments (line 26A), and central government deposits (line 26D).

proxies should thus all measure the ease of raising external finance. Specifically, we employ five measures based on Rajan and Zingales (1998, 2003), which are:

1. **Accounting standards** is an index developed by the Center for International Financial Analysis and Research ranking the amount of disclosure in annual company reports in each country. Though this index from Rajan and Zingales dates back to 1990, they report that it does not change much over time.
2. **Total capitalisation to GDP** is the ratio of the sum of equity market capitalisation (as reported by the IFC) and domestic credit (IFS line 32a-32f but not 32e) to GDP. Stock market capitalisation is measured at the end of the earliest year in the 1980s for which it is available.
3. **Domestic credit to GDP** is the ratio of domestic credit to the private sector, which is from IFS line 32d, over GDP.
4. **Deposits to GDP** is the ratio of domestic deposits to the GDP, based on data for 1999.
5. **Stock market capitalisation to GDP** is the ratio of the aggregate market value of equity of domestic companies divided by GDP, based on data for 1999.

We find that in the cross-section of countries, the correlation of country-level average for the banking system's ratio of liquid assets to total deposits with these five measures is uniformly and significantly negative, the values being -0.55, -0.38, -0.36, -0.33, and -0.50, respectively. We also plot the best regression fit of the liquidity ratio to accounting standards (Figure 11) and to total capitalisation to GDP (Figure 12). The graphs illustrate that the negative relationship is quite robust to the exclusion of outliers such as Argentina, whose liquidity ratio has been inflated due to the recent economic and political turmoil.

While this evidence is striking, it is potentially also consistent with the explanation of Freedman and Click (2006) that these measures of financial development (especially domestic credit to GDP and to some extent accounting standards) also proxy for frictions in the market for lending. That is, the negative relationship may be due to lower attractiveness of risky loans in these countries rather than due to greater attractiveness of safe assets. To help at least partially address this issue, we examine data on international stock market liquidity measured over the period 1989 to 2000 from Levine and Schmukler (2005). In particular, we consider for a subset of countries three measures of stock market liquidity, namely **Turnover in Domestic Market**, and two inverse proxies, **Illiquidity Ratio** of Amihud (2002), and **Proportion of Zero Return Days** advocated by Bekaert, Harvey and Lundblad (2003).[13]

While the first two measures show little correlation with the banking system liquidity ratio, we find that the third measure of stock market illiquidity, the proportion of zero return days, is significantly positively correlated. The correlation is 0.25 (Figure 13 shows the best regression fit of banks' Liquidity Ratio versus the Proportion of Zero Return Days). When the Brazil outlier is excluded, the correlation is around 0.35, the corresponding correlations with accounting standards and total capitalisation to GDP being -0.25 and -0.60, respectively (for the limited sample where stock market liquidity proxies are available).

This suggests that the relationship between financial development and bank liquidity may not entirely be due to credit market frictions. A part of this relationship may also stem from the fact that financial development is associated with greater ease of external finance, which reduces the attractiveness of liquidity in banks' portfolio choice. Overall, this cross-country evidence is consistent with the view that the hoarding of liquidity buffers for profitable investments such as acquisitions may be a potentially important determinant of equilibrium levels of bank liquidity. It should be noted that the results do not however distinguish this strategic motive for liquidity hoarding from the precautionary one.

6.3 Bank liquidity and the business cycle

In order to provide further evidence in support of our model's implications, we next appeal to the second robust implication that bank liquidity is countercyclical, that is, lower during economic upturns and higher as recessions approach (or are anticipated). On this implication, we rely on extant empirical evidence.

Aspachs *et al* (2005) analyse the determinants of UK banks' liquidity holdings and find evidence supportive of this hypothesis. They use balance sheet and profit and loss data, for a panel of 57 UK-resident banks, on a quarterly basis, over the period 1985 Q1 to 2003 Q4. These data are obtained from the Bank of England Monetary and Financial Statistics and relate to the banks'

[13] In particular, the Illiquidity Ratio for a stock is measured as the average of daily ratio of absolute return to dollar volume traded, where the average is taken over the entire time period. The stock-level measure is then averaged across all stocks to construct the Illiquidity Ratio for the entire market. The Proportion of Zero Return Days is simply the fraction of trading days for a stock on which there is no change in its price, where the fraction is calculated over the entire time period and then averaged across all stocks in a given stock market.

resident (UK) activity, excluding activities abroad. They measure liquidity as the sum of cash, reverse repos, bills and commercial papers and comprise in addition all types of investments securities, such as equities and bonds. They use two alternative liquidity ratios. The first is the share of liquid assets in the bank's total assets. This measure captures the split between liquid and illiquid assets on the bank's balance sheet. And, to capture the liquidity mismatch inherent in the bank's balance sheet, they use a second measure, which is the ratio of liquid assets to total deposits. However, their results do not change materially whether they use ratio of liquidity over assets, or the ratio of liquidity over deposits.

In their regression analysis, they test among other effects the role of GDP growth in determining banks' liquid asset holdings. They find that banks in the UK appear to hold smaller (larger) amounts of liquidity, relative to both total assets and total deposits, in periods of stronger (weaker) economic growth. In particular, a 1% increase in GDP growth results in about a 2% decrease in liquidity, where the effect is significant at the 1% level. In other words, banks appear to build up their liquidity buffers during economic downturns and draw them down in economic upturns. Again, while business cycle fluctuations are certainly associated with fluctuations in demand for risky loans, their evidence, put together with the cross-country evidence, provides at least preliminary support for our model's business-cycle hypothesis. More research differentiating the alternative determinants of banks' liquid asset holdings and perhaps employing other empirical measures for the overall health of banking system is warranted.

Some recent literature (most notably, Adrian and Shin (2008), Figures 1, 2, 7 and 10) has focused on targeting of leverage ratios by banks and its implications for the business cycle. In particular, this literature has argued that individual bank risk management leads to unwinding of assets in response to negative asset-side shocks, which depresses prices and leads to more unwinding, causing significant price drops. It has also been documented that there is a negative relationship between equity cushion maintained by banks and their total assets. We elaborate below that these facts are potentially consistent with risk management at banks being primarily achieved by management of their liquidity.

Leverage ratios would be targeted by banks in a 'net' sense, that is, with leverage being net of cash reserves or liquid holdings of banks. Negative asset-side shocks increase the risk of a crisis giving banks incentives to build up their liquid buffers, for example, by liquidating risky assets and saving the proceeds. If such shocks are systematic, there may not be a sufficiently large pool of outsider buyers (such as pension funds, insurance companies, university endowments, hedge funds, etc, depending on the type of assets) to absorb liquidations by banks, resulting in fire-sale discounts in prices.[14] As asset liquidations increase, size of banking assets falls but due to liquidation proceeds and the anticipated gains on cash balances, the net equity cushions rise. These effects would be exaggerated if negative asset-side shocks are associated with a deterioration in market liquidity and cost of raising external finance (see, for example, Acharya and Pedersen (2005), Figure 1) since this would strengthen banks' strategic (and precautionary) motives to increase liquid buffers.

While this cross-country and business-cycle support for our model's implications is arguably preliminary and only suggestive, we find it intriguing and promising for detailed investigation in future research.

7 Policy analysis

In this section, we revert to theoretical analysis and examine the effect of regulatory policies on banks' choice of liquidity. In the benchmark model, we assumed that the provision of deposit insurance did not have any fiscal costs. We relax this assumption and show that such a cost for deposit insurance increases the socially optimal level of liquidity. We also analyse the effect of different resolution policies for bank failures on banks' choice of liquidity.

7.1 Costly deposit insurance

In practice, the provision of deposit insurance may result in significant fiscal costs, if its ordinary funding and accumulated resources prove inadequate (as is often the case, in a major crisis). These fiscal costs may be linked to a variety of sources, such as (i) distortionary effects of tax increases required to fund deposit insurance; (ii) the likely effect of huge government deficits on the country's exchange rate, manifested in the fact that banking crises and currency crises have often occurred as twins in many countries (especially in emerging market economies). Ultimately, the fiscal cost we have in mind is one of immediacy: government expenditures and inflows during the regular course of events are smooth, relative to the potentially rapid growth of off balance sheet contingent liabilities such as deposit-insurance funds, costs of bank bailouts

[14]While we have not yet discussed the role of outside capital in our model, we do so in the next section.

etc.[15] Furthermore, note that our model abstracts from any systemic effects of bank failures such as contagion. This is natural in our framework given the presence of deposit insurance and the absence of any interbank linkages. Hence, costly provision of deposit insurance can be considered as a metaphor for systemic costs arising from bank failures.

We introduce such a systemic cost in the form of a social cost from provision of deposit insurance and show that the socially optimal level of liquidity can be *always* higher than the privately optimal level once the cost is sufficiently high. Throughout, we concentrate on the case with $w \geqslant \underline{p}$, so that the price for failed banks' assets never falls below $\underline{p}$. However, our results hold for all other values of outsider wealth w.

When a bank fails, it owes r_0 units to its depositors. It has l units of cash and the remaining $(r_0 - l)$ units are paid through the deposit insurance fund. The regulator collects proceeds from the sale of assets to cover some of the costs of providing deposit insurance. Hence, the regulator needs to inject $(r_0 - l - p^*(k))$ units of funds. We assume that the regulator faces the following linear cost function, $c(x) = ax$, $a > 0$, when she provides x units of funds. While we could also consider a convex cost function, the linear cost assumption leads to more transparent results.

Thus, the objective function of the regulator, denoted by $E(\Pi)$, can be expressed as

$$E\left[l + \alpha_0(1-l)R_0(l)\right] + \alpha_1 R_1 - (\alpha_1 \Delta) \int_{\overline{k}}^{1} f(k) \left[k - \frac{(1-k)L}{\underline{p}}\right] dk - a \int_{0}^{1} f(k) k \left[r_0 - l - p^*(k)\right] dk \quad \textbf{(33)}$$

where $\left[k - \frac{(1-k)L}{\underline{p}}\right]$ represents the units of failed banks' assets purchased by outsiders.

For failures to have a cost for the regulator, we need $(r_0 - l - p^*(k)) > 0$. Note that this is satisfied when $(r_0 - l - \overline{p}) > 0$, which is equivalent to $l < (r_0 - \overline{p})$. This is a stronger assumption than (A2), which is, $l < (r_0 - \tau \overline{p})$.

[15]See, for example, the discussion on fiscal costs associated with banking collapses and bailouts in Calomiris (1998). Hoggarth, Reis and Saporta (2002) find that the cumulative output losses have amounted to a whopping 15%-20% of annual GDP in the banking crises of the past 25 years. Caprio and Klingebiel (1996) argue that the bailout of the thrift industry cost $180 billion (3.2% of GDP) in the United States in the late 1980s. They also document that the estimated cost of bailouts were 16.8% of GDP for Spain, 6.4% for Sweden and 8% for Finland. Honohan and Klingebiel (2000) find that countries spent 12.8% of their GDP to clean up their banking systems whereas Claessens, Djankov and Klingebiel (1999) set the cost at 15%-50% of GDP.

The first-order condition for the socially optimal level of l is thus given as:

$$1 - E\left[\alpha_0\left(R_0(l) - (1-l)\frac{dR_0}{dl}\right)\right] - (\alpha_1\Delta)\frac{d}{dl}\left[\int_{\overline{k}}^{1} f(k)\left[k - \frac{(1-k)L}{\underline{p}}\right]dk\right]$$
$$+ a\int_0^1 kf(k)dk + \frac{d}{dl}\left(a\int_0^1 kf(k)\left[p^*(k)\right]\,dk\right) = 0. \quad \textbf{(34)}$$

Again, to get closed-form solutions, we use $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$. Also, let

$$E(\widetilde{\gamma}) = \left(\frac{\alpha_1\Delta}{\underline{p}}\right)\int_{\overline{k}}^{1} f(k)(1-k)\,dk + a\int_{\underline{k}}^{\overline{k}} f(k)(1-k)\,dk, \quad \textbf{(35)}$$

which can be interpreted as the marginal reduction in expected misallocation cost for an additional unit of liquidity within the set of surviving banks. See Figure 10 for an illustration of $\widetilde{\gamma}$ as a function of k. We can now formally state the socially optimal level of liquidity under this case, denoted by $\widetilde{l}^{**}$.

Proposition 13 The socially optimal level of liquidity with a fiscal cost of deposit insurance satisfies the FOC in **(14)** and is given as:

$$\widetilde{l} = \min\left\{1, \max\left\{0, 1 - b + \frac{1 + a + E(\widetilde{\gamma})}{E(\alpha_0) + \tau E(\widetilde{\gamma})}\right\}\right\}. \quad \textbf{(36)}$$

The unique level of liquidity $\widetilde{l}^{**}$ is the fixed point $\widetilde{l}(\alpha_0, \tau, E(\gamma(\alpha_0, \tau, l^{**}))) = l^{**}$.

Furthermore, $\widetilde{l}^{**}$ is increasing in the fiscal cost parameter a so that $\widetilde{l}^{**} > l^{**}$ for all $a > 0$.

Finally, if the fiscal cost parameter is sufficiently high, $a > \left(\frac{\alpha_1\Delta}{\underline{p}}\right)$, then $\widetilde{l}^{**} > l^*$, that is, the socially optimal level of liquidity always exceeds the privately optimal level.

There are two properties of social welfare costs with costly deposit insurance that are important. First, with a fiscal cost of deposit insurance, a social cost is incurred whenever a bank fails regardless of whether the number of bank failures is high enough to result in sales to outsiders. Since funds to be provided by the deposit insurer decrease in the liquidity carried by failed banks, the socially optimal level of liquidity increases in the fiscal cost parameter. Second, the fiscal cost incurred is increasing in the number of bank failures for two reasons: first, simply because more banks have failed, and second, because with fewer surviving banks there is lower overall liquidity and hence lower proceeds for the regulator from sale of failed banks.

Given these properties, if the fiscal cost parameter a is sufficiently high, then it can be guaranteed that the socially optimal level of liquidity always exceeds the privately optimal level, in contrast to the case in the absence of costs for deposit insurance, where banks may in fact carry more liquidity than is socially optimal as in Proposition 12. This is the case because the private benefit (ϕ) is identical to the social cost (γ) for $k > \bar{k}$, but for $k \in [\underline{k}, \bar{k}]$, the private benefit is positive but there is no social cost (no misallocation). If liquidity cannot be raised easily against future cash flows, then banks carry liquidity buffers to capitalise on this private benefit.

With a fiscal cost of deposit insurance, that is, $a > 0$, the social cost $\widetilde{\gamma}$ exceeds ϕ for $k > \bar{k}$ and is positive in the range $[\underline{k}, \bar{k}]$. Thus, with a sufficiently large, it is in fact the case that the incentive of banks to carry liquidity buffers for intertemporal transfers is not high enough to internalise the fiscal costs of deposit insurance incurred by the regulator.

Thus, the fiscal cost of providing deposit insurance unambiguously raises the socially optimal level of liquidity, and if the cost function is sufficiently steep, then the socially optimal level of liquidity *always* exceeds the privately optimal level.

From a policy standpoint, the implication of the results so far can be summarised as follows. Regulation requiring banks to hold a minimum level of liquidity is justifiable in our model if (i) the pledgeability of risky assets is sufficiently high, the case where risk-shifting incentives imply that banks will hold too little liquidity or the safe asset, and/or (ii) the cost of providing deposit insurance is expected to be substantial, since this cost is not internalised by banks.[16]

7.2 Effect of regulatory closure policies

In this section, we consider three different policies the regulator may employ to prevent misallocation of banking assets resulting from entry of outsiders. We compare these policies in terms of the incentives they create for banks to hold liquid assets.

Note that for $k > \bar{k}$, to prevent the misallocation cost resulting from sales to outsiders, the regulator may choose to intervene. In particular, the regulator may choose to bail out failed

[16]It should be pointed out that with collection of deposit insurance premium, this result would depend on the current size of the deposit insurance fund. It is also important to point out that unless liquidity of banks can be contracted upon while determining the deposit insurance premium, our results would be qualitatively unaffected. That is, collecting a fixed deposit insurance premium would not overturn our results except for inducing a dependence on the current size of the deposit insurance fund.

banks, in which case, the bailed-out banks continue operation for the second period. The regulator may also provide liquidity to surviving banks so that these banks have enough funds to purchase all failed banks' assets. To keep the analysis simple, we assume that the misallocation cost from sales to outsiders is high enough so that the regulator would like to employ one of these policy options. For a detailed normative analysis of these closure options, see Acharya and Yorulmazer (2005).

Bailouts: The regulator would like to bail out all failed banks that have to be sold to outsiders, that is, a proportion

$$b(k) = k - \frac{\overline{L}(k)}{\underline{p}} \quad \textbf{(37)}$$

when $k > \overline{k}$, where $\overline{L}(k) = (1-k)L$ is the total liquidity within the set of banks. We assume that the regulator chooses which banks to bail out randomly and a bailed-out bank keeps the entire share of future profits.

For $w \geqslant \underline{p}$, bailouts do not affect the price, which will be identical to $p^*(k)$ in equation **(17)** (see Figure 6). Hence, the private benefit to banks from holding the liquid asset, ϕ, stays the same (see Figure 7). In other words, failed banks that would have been acquired by outsiders without the regulatory assistance are now being bailed out and this does not have an effect on the private gain from asset purchases for the surviving banks. As a result, banks' choice of liquidity is not affected from the bailout policy.[17]

However, if the regulator intervenes in the form of 'excessive' bailouts, that is, by bailing out more than $b(k)$ banks, then the private benefit to surviving banks from holding the liquid asset falls (as there are fewer banks for them to acquire). In turn, banks hold lower liquidity. Since, in practice, bailouts are likely to suffer from a too-many-to-fail problem and be driven by political economy considerations, it is reasonable to expect that in data, anticipated government support to failed banks lowers banks' liquidity *ex ante*. In addition to the theoretical work of Repullo (2005) and the empirical work of Gonzalez-Eiras (2003) on this topic (discussed in the Related literature section), Aspachs *et al* (2005) provide supportive evidence from the United Kingdom. In particular, they show that the greater the market expectation of support from the public authorities for a given bank (proxied by the Fitch support ratings for UK banks) in case of

[17]Note that bailouts decrease the proportion of banking assets that are potentially up for sale. Hence, for $w < \underline{p}$, bailouts can only increase the price for asset sales. This, in turn, decreases the benefit of holding liquidity to buy banking assets at fire-sale prices, that is, ϕ decreases. Hence, for $w < \underline{p}$, with bailouts, banks hold less liquidity.

liquidity crises, the lower the liquidity buffer bank holds.

Unconditional liquidity provision: Alternatively, the regulator can grant liquidity to surviving banks so that they have enough funds to purchase all failed banks' assets. In particular, for $k > \overline{k}$, the regulator can grant each surviving bank liquidity of $\widehat{l}(k)$ units, where

$$\widehat{l}(k) = \left(\left(\frac{k}{1-k}\right)\underline{p} - L\right), \tag{38}$$

so that each surviving bank can acquire $\left(\frac{k}{1-k}\right)$ units of assets at price $\underline{p}$. We denote the strategic benefit of the liquid asset that arises from asset purchases in this case as $\widehat{\phi}$. For $k \leqslant \overline{k}$, the regulator does not intervene, the price is $p^*(k)$ and we have $\widehat{\phi} = \phi$. For $k > \overline{k}$, surviving banks make a profit from the asset purchase that is given as

$$= \left(\left(\frac{k}{1-k}\right)\overline{p} - L\right).$$

This profit is decreasing in l since $\frac{\partial L}{\partial l} > 0$. Hence, the strategic benefit of the liquid asset from asset purchases is decreasing in the liquid asset holding of banks. The reason for this is that banks can get the needed liquidity from the regulator and by holding liquidity they increase the effective price they pay for acquiring assets. Hence, under unconditional liquidity provision policy, banks' liquidity is lower than the level without the liquidity provision policy.

Conditional liquidity provision: Another policy option is to grant liquidity to surviving banks but to make it conditional on the liquid assets banks hold in the first place. Note that, as in the previous case, the total liquidity that needs to be provided by the regulator to prevent sales to outsiders is $\widehat{L}(k) = k\underline{p} - \overline{L}(k)$. In this case, the regulator provides each bank a proportion z of the liquid asset the bank has in its portfolio, where $z = \frac{\widehat{L}(k)}{\overline{L}(k)}$. This way, the regulator provides a total liquidity of $\widehat{L}(k)$ units to surviving banks, enough to prevent sales to outsiders.

With this liquidity provision policy, for $k > \overline{k}$, the regulator subsidises sales for surviving banks and each surviving bank acquires $\left(\frac{(1+z)L}{\underline{p}}\right)$ units of assets by using their liquidity of L units. Hence, banks pay an effective price of $\widetilde{p}(k)$ for each unit, where

$$\widetilde{p}(k) = \begin{cases} \overline{p} & \text{for} \quad k \leqslant \underline{k} \\ \frac{(1-k)L}{k} & \text{for} \quad k \in (\underline{k}, \overline{k}] \\ \frac{\underline{p}}{1+z} & \text{for} \quad k > \overline{k} \end{cases}. \tag{39}$$

In this case, we can show that the strategic benefit of holding the liquid asset is given by

$$\widetilde{\phi} = \alpha_0 \left(\frac{\overline{p} - \widetilde{p}(k)}{\widetilde{p}(k)} \right).$$

Note that for $k \leqslant \overline{k}$, we have $\widetilde{\phi} = \phi$ and for $k > \overline{k}$, we have $\widetilde{\phi} = \phi + \left(\frac{z\overline{p}}{\underline{p}}\right) > \phi$. Hence, the strategic benefit of holding the liquid asset is higher in this case compared to the case without the liquidity provision policy (and, in turn, higher compared to unconditional liquidity provision policy). This implies that under the conditional liquidity provision policy, banks have incentives to hold more of the liquid asset.

8 Conclusions

Our objective in this paper has been to develop a theoretical framework for bank portfolio choice between liquid (safe) and illiquid (risky) assets that is set against the backdrop of potential crisis, entry and bank resolution. Given the potentially large rewards and costs that arise in the context of crisis and resolution, the endogeneity of bank portfolios is easily motivated in the positive theory of bank behaviour. Although our focus in the paper has been mainly theoretical, the cross-country empirical evidence on bank portfolios is suggestive of systematic variation in banks' choice of liquidity in the face of differing institutional quality variables. This evidence holds out some hope that a more comprehensive empirical study can unearth and provide further insights into the relationship between financial development and banking system liquidity.

Of greater consequence for policy, we have been able to conduct a normative welfare analysis comparing the equilibrium liquidity choice of banks to the socially optimal level. Welfare analysis is made possible in our model due to our assumption that outside investors (from outside the banking sector) do not have the skills to generate the full value from banking assets. This assumption is motivated by the familiar argument that bank assets (loans in particular) derive much of their value from the monitoring and collection efforts of loan officers who can influence the actions of the debtors. Hence, when distressed assets end up in the hands of outsiders, we may expect deadweight costs from inefficient allocation of assets. The empirical evidence reported by James (1991) on the substantial losses incurred due to liquidation of bank assets in the United States is one aspect of such inefficiency. We may expect the losses to be much larger in emerging market countries with less developed markets for distressed assets and the poor quality of legal and accounting infrastructure backing the insolvency process.

The policy implications that flow from our analysis are worthy of further study, both theoretically in more sophisticated set-ups, for example, allowing for dynamics, but also empirically in the arena of bank resolution, for example, in the form of specific case studies linking the regulatory choice of closure policies to the *ex-ante* choice of bank liquidity. In particular, the role of foreign investors in bank restructuring presents important trade-offs for a country in the aftermath of a major financial crisis. Foreign capital will be attracted by the very low prices of distressed assets, and fulfil the role of the 'purchaser of last resort' when domestic capital is exhausted. However, the ultimate welfare effects of such foreign entry will depend on the complex interplay between the cushioning of price in the event of a crisis, the *ex-ante* portfolio choices in anticipation of such entry, and the ability of the foreign entrant to manage the assets they acquire.

Foreign entry also presents important distributional questions. The perception that foreigners are able to buy up large swathes of the banking sector at fire-sale prices, and then sell them off at a large profit once the crisis has abated, presents important challenges, not least in political economy, as witnessed by the current fierce debates in Korea about the role of foreign private equity firms. Such issues concerning the role of foreign entry are being studied separately in a companion piece to the current paper.

Appendix

Technical model assumptions: We make the following parametric assumptions to analyse the model.

(A1) $b > 2$: In this case, the return from the bank's portfolio, $[l + (1-l)R_0]$, without the profits from the asset purchase, is decreasing as the liquid asset l in bank's portfolio increases. This creates the trade-off between the liquid and the illiquid asset *only once benefits from fire sales are introduced*. In other words, the pure portfolio choice problem would lead to liquidity choice of $l = 0$. Furthermore, this condition also guarantees that $R_0 > r_0 = 1$.[18]

(A2) $\tau < \min\{1/b, b/\overline{p}\}$: $\tau < 1/b$ guarantees that the liquidity banks have for asset purchases increases as they hold more liquid asset l in their portfolio and $\tau < b/\overline{p}$ guarantees that $r_0 \geqslant 1 > l + \tau\overline{p}$, so that a failed bank cannot generate the needed funds to avoid default.

(A3) $\overline{\Delta} < \left(b - \frac{3}{2}\right)$: Note that the maximum value $\overline{\Delta}$ can take, denoted by $\overline{\Delta}_{max}$, is equal to $(R_0 - r_0)$. This condition guarantees that $\overline{\Delta} < \overline{\Delta}_{max}$.

(A4) $B \leqslant \left(\frac{\overline{\Delta}^2}{b-1}\right)$: This condition guarantees that banks cannot generate a higher proportion of their future profits in the capital market when they invest in the bad project. In particular, when bank owners are left with a share of profits less than $\overline{\theta}$, they shirk, which results in a lower return from these investments. However, in that case, they can generate a higher proportion of their future profits in the capital market, that is, they can generate up to $\left(R_t - \overline{\Delta} - r_t\right)$. Banks can generate higher funds from the capital market when they choose the good project if

$$\left(1 - \overline{\theta}\right)(R_t - r_t) \geqslant R_t - \overline{\Delta} - r_t, \qquad \textbf{(40)}$$

which gives us $\overline{\theta} \leqslant \frac{\overline{\Delta}}{(R_t - r_t)}$. Thus, we have $\overline{\theta} = \frac{B}{\overline{\Delta}} \leqslant \frac{\overline{\Delta}}{(R_t - r_t)}$. In that case, it is optimal to leave a minimum share of $\overline{\theta}$ of future profits to bank managers, both for higher output as well as better liquidity generation through the capital market. This condition simplifies to $B \leqslant \left(\frac{\overline{\Delta}^2}{b-1}\right)$.

Proof that $l \leq l_{max}$ in equilibrium: In the first case when $(1-l)R_0 \geqslant r_0$, a proportion τ of the remaining return from the risky asset, that is, $\tau\left[((1-l)R_0 - r_0)\right]$ can be pledged in the capital

[18] We show below that when $b > 2$, there is a threshold level of liquidity, denoted by l_{max}, such that for $l > l_{max}$, liquidity that banks have for asset purchase decreases as l increases. In turn, this implies that in equilibrium, we always have $l \leqslant l_{max}$.

market. Thus, from equation **(3)**, we have

$$\frac{\partial L}{\partial l} = 1 - \tau R_0 + \tau(1-l)\left(\frac{\partial R_0}{\partial l}\right) = 1 - \tau b + \tau(1-l). \qquad \textbf{(41)}$$

Hence, for $\tau < \left(\frac{1}{b-1+l}\right)$, we have $\frac{\partial L}{\partial l} \geqslant 0$ and liquidity available for asset purchase increases as banks hold more of the liquid asset in their portfolio. A sufficient condition for this to hold is $\tau < 1/b$, which holds by our assumption (A2).

For the other case, $l > l_{max}$ and the return from the risky asset is not enough to pay old depositors. Hence, some of the liquid asset l has to be used to pay old depositors, which gives us

$$L = [l + ((1-l)R_0 - r_0)] + \tau\overline{p}. \qquad \textbf{(42)}$$

Thus, for $b > 2$, which holds by (A1), $\frac{\partial L}{\partial l} < 0$ and the liquid asset available for asset purchase decreases as banks hold more of the liquid asset. Furthermore, without the asset purchase, the expected return on bank's portfolio is $E\left[\alpha_0\left(l + (1-l)\left(b - \frac{1-l}{2}\right)\right)\right]$, is decreasing in l for $b > 2$. Hence, for $b > 2$, banks never hold a level of liquidity l greater than l_{max} in equilibrium. $\diamondsuit$

Proof of Proposition 3: We have $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$ and $\frac{dR_0}{dl} = \frac{1}{2}$. Plugging these expressions into the FOC in **(9)**, we get:

$$E(\alpha_0)\ [1 - b + (1-l)] + E(\phi)\ [1 + \tau[-b + (1-l)]] = 0, \qquad \textbf{(43)}$$

where ϕ is the expected benefit from assets purchase per unit of failed banks' assets. From here, we can find banks' choice of liquidity $\widehat{l}$ that satisfies the FOC as:

$$\widehat{l} = 1 - b + \frac{E(\alpha_0) + E(\phi)}{E(\alpha_0) + \tau E(\phi)}, \qquad \textbf{(44)}$$

which is given in Proposition 3.

We have the following:

$$\phi = \begin{cases} 0 & for \quad k \leqslant \underline{k} \\ \alpha_0\left(\frac{(1-\alpha_0)\overline{p}}{\alpha_0 L} - 1\right) & for \quad k > \underline{k} \end{cases}. \qquad \textbf{(45)}$$

Note that $\underline{k}$ is continuous in l. Thus, $E(\phi)$ is continuous in l. Hence, $\widehat{l}$ is continuous in l. Since, $\widehat{l}$ is a continuous function from the compact, convex set [0, 1] into itself, by Brouwer's fixed-point theorem, a fixed point of the mapping in equation **(44)** exists. Next, we show that the fixed point is unique.

Note that as l increases, the aggregate level of liquidity increases, the region over which the price of the failed banks' assets fall below their fundamental value shrink, that is, $\frac{\partial k}{\partial l} > 0$. Hence, we have $\frac{\partial E(\phi)}{\partial l} < 0$. Note that, we have $\frac{\partial \widehat{l}}{\partial E(\phi)} = (1-\tau)E(\phi) > 0$, that is, as the expected private benefit from holding the liquid asset decreases, banks hold less liquid asset in their portfolio. Thus, we have $\frac{\partial \widehat{l}}{\partial l} < 0$. As a result, the fixed point is unique. ♢

Proof of Proposition 4: First, we prove part (i). Note that if $\widehat{l}$ given in equation **(11)** increases, the privately optimal level of liquidity l^* increases. We have

$$\begin{aligned} sign\left(\frac{\partial \widehat{l}}{\partial \tau}\right) &= sign\left[\left(\frac{\partial E(\phi)}{\partial \tau}\right)[E(\alpha_0)+\tau E(\phi)] - [E(\alpha_0)+E(\phi)]\left(E(\phi)+\tau\left(\frac{\partial E(\phi)}{\partial \tau}\right)\right)\right] \\ &= sign\left[\left(\frac{\partial E(\phi)}{\partial \tau}\right)[(1-\tau)E(\alpha_0)] - E(\phi)\,[E(\alpha_0)+E(\phi)]\right]. \end{aligned}$$

We have $\frac{\partial E(\phi)}{\partial \tau} < 0$, since $\frac{\partial E(\phi)}{\partial p^*} < 0$ and $\frac{\partial p^*}{\partial \tau} > 0$. Hence, we have $\frac{\partial \widehat{l}}{\partial \tau} < 0$, that is, the privately optimal level of liquidity l^* decreases as τ increases.

Next, we prove part (ii). Note that ϕ is (weakly) increasing in k, therefore, is (weakly) decreasing in α_0. Hence, if g FOSD f, we have $E_g(\phi) < E_f(\phi)$. We have $\frac{\partial \widehat{l}}{\partial E(\phi)} > 0$. Hence, if g FOSD f, we have $l_g^* < l_f^*$. ♢

Proof of Proposition 7: From the expressions for these two values of liquidity, we have

$$\widehat{l} - \widehat{\widehat{l}} = \frac{E(\alpha_0)+E(\phi)}{E(\alpha_0)+\tau E(\phi)} - \frac{1}{E(\alpha_0)} = \frac{E(\phi)\,[E(\alpha_0)-\tau] - E(\alpha_0)\,[1-E(\alpha_0)]}{E(\alpha_0)\,[E(\alpha_0)+\tau E(\phi)]}. \quad \textbf{(46)}$$

Note that a sufficient condition for the socially optimal level of liquidity to be higher that the privately optimal level of liquidity is $E(\alpha_0) < \tau$. Hence, we analyse the case where $E(\alpha_0) \geqslant \tau$. As $E(\alpha_0)$ converges to 1, we have the privately optimal level of liquidity to be higher than the socially optimal level. Next, note that $\widehat{l} = \widehat{\widehat{l}}$ when

$$E(\phi)\,[E(\alpha_0)-\tau] = E(\alpha_0)\,[1-E(\alpha_0)]. \quad \textbf{(47)}$$

Since the left-hand side is decreasing in τ, but the right-hand side is not affected by τ, this equation implicitly defines a unique critical $\tau^*(E(\alpha_0))$ such that $\widehat{l} < \widehat{\widehat{l}}$ if and only if $\tau > \tau^*(E(\alpha_0))$.

Using the implicit function theorem, we get:

$$E(\phi)\left[\frac{d\tau^*}{dE(\alpha_0)}\right] = E(\phi) + [E(\alpha_0) - \tau]\left[\frac{dE(\phi)}{dE(\alpha_0)}\right] + [1 - 2E(\alpha_0)]\,. \quad \textbf{(48)}$$

Note that $\left(\frac{dE(\phi)}{dE(\alpha_0)}\right) < 0$ so that we obtain

$$\frac{d\tau^*}{dE(\alpha_0)} < 0 \text{ for } E(\alpha_0) < \left(\frac{1 - E(\phi)}{2}\right). \quad \textbf{(49)}$$

See Figure 5 for an illustration. ♢

Proof of Proposition 10: From the FOC in **(14)**, we have:

$$\underbrace{1 - E(\alpha_0)\left[R_0(l) - (1-l)\frac{dR_0}{dl}\right]}_{=T_1} \underbrace{- (\alpha_1\Delta)\frac{d}{dl}\left[\int_{\overline{k}}^{1} f(k)\left[k - \frac{(1-k)L}{\underline{p}}\right] dk\right]}_{=T_2} = 0, \quad \textbf{(50)}$$

where

$$T_1 = 1 + E(\alpha_0)\ [-b + (1-l)]\,. \quad \textbf{(51)}$$

Using Leibniz's rule, we get:

$$\begin{aligned} T_2 \quad = \quad & -(\alpha_1\Delta)\left[\int_{\overline{k}}^{1} f(k)\left[-\frac{(1-k)}{\underline{p}}\right]\left[1 - \tau R_0(l) + \tau(1-l)\left(\frac{dR_0}{dl}\right)\right] dk\right] \\ & + (\alpha_1\Delta)\underbrace{\left[\overline{k} - \frac{(1-\overline{k})L}{\underline{p}}\right]}_{=0}\left(\frac{d\overline{k}}{dl}\right). \end{aligned} \quad \textbf{(52)}$$

Note that at $k = \overline{k}$, all failed banks' assets are purchased by surviving banks and the second term in equation **(52)** is equal to 0. Using $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$ and $\frac{dR_0}{dl} = \frac{1}{2}$, we get:

$$T_2 = [1 + \tau\,(-b + (1-l))]\ \underbrace{\left[\left(\frac{(\alpha_1\Delta)}{\underline{p}}\right)\int_{\overline{k}}^{1} f(k)(1-k)\,dk\right]}_{=E(\gamma)}. \quad \textbf{(53)}$$

Thus, we have the FOC as

$$1 + E(\alpha_0)\ [-b + (1-l)] + E(\gamma)\ [1 + \tau\,(-b + (1-l))] = 0. \quad \textbf{(54)}$$

Note that equation **(54)** looks very much like the FOC for banks' choice of liquidity in equation **(43)**, except for the fact that instead of $E(\phi)$, the expected benefit from asset purchase per unit of failed banks' assets, we have $E(\gamma)$, which can be interpreted as the marginal reduction in misallocation cost from an increase in surviving banks' liquid asset holdings. Also, in the first expression, we have 1 instead of α_0, since banks can benefit from their liquid assets only when they survive, which happens with a probability of α_0, whereas the regulator always benefits from banks' liquid assets.

From here, we can find the socially optimal level of liquidity $\widehat{\widehat{l}}$ that satisfies the FOC as:

$$\widehat{\widehat{l}} = 1 - b + \frac{1 + E(\gamma)}{E(\alpha_0) + \tau E(\gamma)}, \quad \textbf{(55)}$$

which is given in Proposition 10.

We have the following:

$$\gamma = \begin{cases} 0 & for \quad (1 - \alpha_0) \leqslant \overline{k} \\ \alpha_0 \left(\frac{\alpha_1 \Delta}{\underline{p}} \right) & for \quad (1 - \alpha_0) > \overline{k} \end{cases} \quad \textbf{(56)}$$

Note that $\overline{k}$ is continuous in l. Thus, $E(\gamma)$ is continuous in l. Hence, $\widehat{\widehat{l}}$ is continuous in l. Since, $\widehat{\widehat{l}}$ is a continuous function from the compact, convex set [0, 1] into itself, by Brouwer's fixed-point theorem, a fixed point of the mapping in equation **(55)** exists. Next, we show that the fixed point is unique.

Note that as l increases, the aggregate level of liquidity increases, the region over which sales to outsiders take place shrinks, that is, $\frac{\partial \overline{k}}{\partial l} > 0$. Hence, we have $\frac{\partial E(\gamma)}{\partial l} < 0$. We have

$$\frac{\partial \widehat{\widehat{l}}}{\partial E(\gamma)} = \frac{E(\alpha_0) + \tau E(\gamma) - \tau \left[1 + E(\gamma)\right]}{\left[E(\alpha_0) + \tau E(\gamma)\right]^2} = \frac{E(\alpha_0) - \tau}{\left[E(\alpha_0) + \tau E(\gamma)\right]^2}.$$

Thus, for $E(\alpha_0) > \tau$, we have $\frac{\partial \widehat{\widehat{l}}}{\partial E(\gamma)} > 0$, which gives us $\frac{\partial \widehat{\widehat{l}}}{\partial l} < 0$. As a result, the fixed point is unique. $\diamondsuit$

Proof of Proposition 11: We have the FOC as:

$$T_1 + T_2 \underbrace{- (\alpha_1 \Delta) \frac{d}{dl} \left[+ \int_{\overline{\overline{k}}}^{1} f(k) \left[\left(\frac{kw}{(1-k)L + w} \right) - \left(k - \frac{(1-k)L}{\underline{p}} \right) \right] dk \right]}_{= T_3} \quad \textbf{(57)}$$

where T_1 and T_2 are the same as in the benchmark case and are given in equations **(51)** and **(53)**, respectively. Using Leibniz's rule, we get:

$$T_3 = -(\alpha_1 \Delta) \int_{\overline{\overline{k}}}^{1} f(k)(1-k)\left(\frac{dL}{dl}\right)\left[\frac{1}{\underline{p}} - \frac{kw}{[(1-k)L+w]^2}\right] dk, \quad \textbf{(58)}$$

since at $k = \overline{\overline{k}}$ we have $p^*_w(k) = \underline{p}$, which gives us $\left(\frac{L}{\underline{p}}\right) = \left(\frac{\overline{\overline{k}}L}{(1-\overline{\overline{k}})L+w}\right)$. We have $\left(\frac{dL}{dl}\right) = [1 + \tau(-b + (1-l))]$. Thus, the FOC can be written as

$$1 + E(\alpha_0)\ [-b + (1-l)] + E(\gamma_w)\ [1 + \tau\,(-b + (1-l))] = 0, \quad \textbf{(59)}$$

where $E(\gamma_w)$ is given in equation **(30)**. From here, we can find the socially optimal level of liquidity $\widehat{\widehat{l}}_w$ that satisfies the FOC as:

$$\widehat{\widehat{l}}_w = 1 - b + \frac{1 + E(\gamma_w)}{E(\alpha_0) + \tau E(\gamma_w)}, \text{ where} \quad \textbf{(60)}$$

$$\gamma_w = \begin{cases} 0 & for \quad k \leqslant \overline{k} \\ \alpha_0\left(\frac{\alpha_1\Delta}{\underline{p}}\right) & for \quad \overline{k} < k \leqslant \overline{\overline{k}} \\ \alpha_0\left(\frac{\alpha_1\Delta}{p^*_w(k)}\right)\left(\frac{w}{(1-k)L+w}\right) & for \quad k > \overline{\overline{k}} \end{cases} \quad \textbf{(61)}$$

As in the benchmark case, we can show that a fixed point of the mapping in equation **(55)** exists and is unique. Furthermore, we know that for $E(\alpha_0) > \tau$, we have $\frac{\partial \widehat{\widehat{l}}}{\partial\gamma} > 0$. Hence, if $\widehat{\widehat{l}}_w > \widehat{\widehat{l}}$ if and only if $E\left(\gamma_w\right) > E(\gamma)$.

Next we analyse how the socially optimal level of liquidity l_w changes with outsider wealth w. From equation **(61)**, we have $E\left(\gamma_w\right) > E(\gamma)$ when

$$\int_{\overline{\overline{k}}}^{1} f(k)(1-k)\left[1 - \frac{kw\underline{p}}{[(1-k)L+w]^2}\right] dk < 0 \quad \textbf{(62)}$$

A sufficient condition for the inequality in the expression **(62)** to hold is

$$kw\underline{p} \ > \ [(1-k)L+w]^2, \text{ that is,} \quad \textbf{(63)}$$

$$g \ = \ kw\underline{p} - [(1-k)L+w]^2 > 0 \text{ for all } k \in [\overline{\overline{k}}, 1]. \quad \textbf{(64)}$$

Note that as the function g increases, $E\left(\gamma_w\right)$ and the socially optimal level of liquidity l_w increases. We have

$$\frac{\partial g}{\partial k} = w\underline{p} + 2\,[(1-k)L+w]\,L > 0. \quad \textbf{(65)}$$

We also have $g = w\underline{p} - w^2 > 0$, for $k = 1$. And, at $k = \overline{\overline{k}}$, we have $\overline{\overline{k}}\underline{p} = \left[(1-\overline{\overline{k}})L + w\right]$.
Hence,

$$\begin{aligned} g(\overline{\overline{k}}) &= w\overline{\overline{k}}\underline{p} - \left[(1-\overline{\overline{k}})L + w\right]^2 = w\left[(1-\overline{\overline{k}})L + w\right] - \left[(1-\overline{\overline{k}})L + w\right]^2 \\ &= -\left[(1-\overline{\overline{k}})L\right]\left[(1-\overline{\overline{k}})L + w\right] < 0. \end{aligned} \quad \textbf{(66)}$$

Thus, we cannot get a sufficient condition for $E\left(\gamma_w\right) \lesseqgtr E(\gamma)$. Now, let

$$h = \frac{kw\underline{p}}{[(1-k)L + w]^2}. \quad \textbf{(67)}$$

Note that as the function h increases, $E\left(\gamma_w\right)$ and the socially optimal level of liquidity l_w increases. We have

$$\frac{\partial h}{\partial w} = \frac{k\underline{p}\,[(1-k)L + w]^2 - 2kw\underline{p}\,[(1-k)L + w]}{[(1-k)L + w]^4} = \frac{k\underline{p}\,[(1-k)L - w]}{[(1-k)L + w]^3}. \quad \textbf{(68)}$$

If $\frac{\partial h}{\partial w} < 0$ for $k \in [\overline{\overline{k}}, 1]$, then the socially optimal level of liquidity l_w decreases as outsider wealth w increases.

We have $\frac{\partial h}{\partial w} < 0$ when $w > (1-k)L$. For $k = 1$, this trivially holds.

For $k = \overline{\overline{k}}$, we have

$$w > \left(\frac{\underline{p} - w}{L + \underline{p}}\right)L \iff w\left(L + \underline{p}\right) > \left(\underline{p} - w\right)L \iff w > \left(\frac{L\underline{p}}{2L + \underline{p}}\right).$$

Hence, for $w > \left(\frac{L\underline{p}}{2L+\underline{p}}\right)$, we have $\frac{\partial h}{\partial w} < 0$ for $k \in [\overline{\overline{k}}, 1]$ and the socially optimal level of liquidity l_w decreases as outsider wealth w increases.

Proof of Proposition 12: We investigate how the difference between the privately and socially optimal levels of liquidity behaves as a function of τ and Δ.

From Proposition 3 and 10, we have

$$\widehat{l} = 1 - b + \frac{E(\alpha_0) + E(\phi)}{E(\alpha_0) + \tau E(\phi)} \quad \text{and} \quad \widehat{\widehat{l}} = 1 - b + \frac{1 + E(\gamma)}{E(\alpha_0) + \tau E(\gamma)}. \quad \textbf{(69)}$$

Note that for regions where $k \in [0, \underline{k}]$ and $k \in [\overline{k}, 1]$, ϕ and γ are identical. However, in the interim range of failures, $k \in [\underline{k}, \overline{k}]$, surviving banks gain from asset purchases through cash-in-the-market prices while there is no social welfare loss since all banking assets are operated by the most efficient users. Thus, in this region, we have $\gamma = 0$ and $\phi > 0$. This implies that $E(\phi) > E(\gamma)$ for a given level of aggregate liquidity. Given these facts, we first prove part (i).

In the extreme case where $\tau = 1$, $E(\phi) = 0$ and $E(\gamma) = 0$, so that for all Δ,

$$\widehat{l} = 2 - b \leqslant \widehat{\widehat{l}} = 1 - b + \frac{1}{E(\alpha_0)}. \quad \textbf{(70)}$$

Since $\left(\widehat{\widehat{l}} - \widehat{l}\right)$ is continuous in τ, there exists a critical level $\tau^*(\Delta) \leqslant 1$, such that, for all $\tau > \tau^*(\Delta)$, socially optimal level of liquidity is higher than the privately optimal level of liquidity.

Next, for $\tau = 0$, we obtain that $\widehat{l} > \widehat{\widehat{l}}$ if and only if

$$h(\Delta) = E(\alpha_0) + E(\phi) - \left[1 + E(\gamma)\right] > 0. \quad \textbf{(71)}$$

For $\Delta = 0$, we know that $E(\phi) = E(\gamma) = 0$, so that $h(0) = E(\alpha_0) - 1 < 0$. Next, we have

$$\frac{\partial h}{\partial \Delta} = \frac{\partial h}{\partial \Delta}\left[E(\phi) - E(\gamma)\right], \quad \textbf{(72)}$$

which is greater than 0 as shown below.

We know that except for the region $k \in [\underline{k}, \overline{k}]$, ϕ and γ are identical. Thus we have:

$$E(\phi) - E(\gamma) = \int_{1-\overline{k}}^{1-\underline{k}} \phi f(\alpha_0) d\alpha_0. \quad \textbf{(73)}$$

Note that in this region, we have

$$\phi = \alpha_0 \left(\frac{(1-\alpha_0)\overline{p}}{\alpha_0 L} - 1\right) = \frac{\overline{p} - \alpha_0(\overline{p} + L)}{L} = \frac{\overline{p}}{L} - \frac{\alpha_0}{\underline{k}}. \quad \textbf{(74)}$$

Note that as Δ increases, $\underline{p}(= \overline{p} - (\alpha_1 \Delta))$ decreases. Thus, $\overline{k}$ increases whereas $\underline{k}$ does not change. Hence, the interval $[\underline{k}, \overline{k}]$ widens and $(E(\phi) - E(\gamma))$ increases. Formally, using Leibniz's rule, we get

$$\frac{\partial\left(E(\phi) - E(\gamma)\right)}{\partial \Delta} = \phi(1-\underline{k})\left[\frac{\partial\left(1-\underline{k}\right)}{\partial \Delta}\right] - \phi(1-\overline{k})\left[\frac{\partial\left(1-\overline{k}\right)}{\partial \Delta}\right] + \int_{1-\overline{k}}^{1-\underline{k}} \frac{\partial(\phi)}{\partial \Delta} f(\alpha_0) d\alpha_0. \quad \textbf{(75)}$$

Note that $\phi(1-\underline{k}) = 0$. And since $\underline{k}$ does not change with $(\alpha_1 \Delta)$, from equation **(74)**, we have $\frac{\partial(\phi)}{\partial \Delta} = 0$. Thus, we have

$$\frac{\partial\left(E(\phi) - E(\gamma)\right)}{\partial \Delta} = -\phi(1-\overline{k})\left[\frac{\partial\left(1-\overline{k}\right)}{\partial \Delta}\right]. \quad \textbf{(76)}$$

Note that $\overline{k}$ increases with Δ so that $\left[\frac{\partial(E(\phi)-E(\gamma))}{\partial\Delta}\right] > 0$.

In other words, there exists a critical Δ^* such that $h(\Delta^*) = 0$ and $h(\Delta) > 0$ for all $\Delta > \Delta^*$, and $h(\Delta) < 0$ otherwise.

Since $\left(\widehat{\widehat{l}} - \widehat{l}\right)$ is continuous in τ, there exists a critical level $\tau^{**}(\Delta) \leqslant 1$, such that, for all $\tau \leqslant \tau^{**}(\Delta)$, the privately optimal level of liquidity is higher than the socially optimal level of liquidity.

Next, we prove part (ii). In fact, for any τ, a sufficient condition to obtain that $\widehat{\widehat{l}} > \widehat{l}$ is

$$h(\Delta) = E(\alpha_0) + E(\phi) - \left[1 + E(\gamma)\right] < 0.$$

As shown for $\tau = 0$, it is the case that $\frac{\partial h}{\partial \Delta}$ for all τ and $h(\tau, 0) < 0$.

It follows that there exists a $\Delta^*(\tau)$ such that for all $\Delta < \Delta^*(\tau)$, $h(\tau, \Delta) < 0$ and the socially optimal level of liquidity is higher than the privately optimal level of liquidity. Furthermore, $\Delta^*(0) > 0$ and $\Delta^*(1) = \Delta_{\max}$. $\diamondsuit$

Proof of Proposition 13: From the FOC in **(34)**, we have:

$$1 - E\left[\alpha_0\left(R_0(l) - (1-l)\frac{dR_0}{dl}\right)\right] - (\alpha_1\Delta)\frac{d}{dl}\left[\int_{\overline{k}}^{1} f(k)\left[k - \frac{(1-k)L}{\underline{p}}\right] dk\right] + a\int_0^1 kf(k)dk + \frac{d}{dl}\left(a\int_0^1 kf(k)\left[p^*(k)\right] dk\right) = 0.$$

Note that we have $k = (1-\alpha_0)$. Using the specification $R_0(l) = \left[b - \frac{1}{2}\right]$, we get

$$1+E(\alpha_0)\ [-b + (1-l)] + E(\gamma)\ [1 + \tau\,(-b + (1-l))] + aE(1-\alpha_0) + \frac{d}{dl}\left(a\int_0^1 kf(k)\left[p^*(k)\right] dk\right) = 0. \tag{77}$$

We can write the last expression as

$$= a\left[\frac{d}{dl}\left(\overline{p}\int_0^{\underline{k}} kf(k)\, dk + \int_{\underline{k}}^{\overline{k}} kf(k)\left[p^*(k)\right] dk + \underline{p}\int_{\overline{k}}^{1} kf(k)\, dk\right)\right]. \tag{78}$$

Using the Leibnitz's rule, we get:

$$= a \left[\underline{p}\left(\overline{k} f(\overline{k}) \frac{d\overline{k}}{dl}\right) - \overline{p}\left(\underline{k} f(\underline{k}) \left(\frac{d\underline{k}}{dl}\right)\right) + \int_{\underline{k}}^{\overline{k}} \frac{d}{dl}\left[f(k)\left[p^*(k)\right]\right] dk \right]$$

$$+a\overline{p}\left(\underline{k} f(\underline{k}) \left(\frac{d\underline{k}}{dl}\right)\right) - a\underline{p}\left(\overline{k} f(\overline{k}) \frac{d\overline{k}}{dl}\right) \quad \textbf{(79)}$$

$$= a \left[\int_{\underline{k}}^{\overline{k}} \left[f(k)(1-k)\frac{dL}{dl}\right] dk \right]. \quad \textbf{(80)}$$

We have $\frac{dL}{dl} = 1 + \tau\left[-b + (1-l)\right]$. Using this, we have the FOC as:

$$1+E(\alpha_0)\ [-b + (1-l)]+aE(1-\alpha_0)+[1+\tau\ (-b+(1-l))]+\underbrace{\left[E(\gamma) + a\int_{\underline{k}}^{\overline{k}} f(k)(1-k)\ dk\right]}_{=E(\widetilde{\gamma})} = 0, \quad \textbf{(81)}$$

where

$$\widetilde{\gamma} = \begin{cases} 0 & \text{for} \quad k \leqslant \underline{k} \\ a\alpha_0 & \text{for} \quad \underline{k} < k \leqslant \overline{k} \\ \alpha_0\left(\frac{\alpha_1\Delta}{\underline{p}}\right) & \text{for} \quad k > \overline{k} \end{cases}. \quad \textbf{(82)}$$

From here, we can find the socially optimal level of liquidity $\widetilde{l}$ that satisfies the FOC as:

$$\widetilde{l} = 1 - b + \frac{1 + E(\widetilde{\gamma}) + a(1 - E(\alpha_0))}{E(\alpha_0) + \tau E(\widetilde{\gamma})}, \quad \textbf{(83)}$$

which is given in Proposition 13.

Note that $\overline{k}$ is continuous in l. Thus, $E(\widetilde{\gamma})$ is continuous in l. Hence, $\widetilde{l}$ is continuous in l. Since, $\widetilde{l}$ is a continuous function from the compact, convex set [0, 1] into itself, by Brouwer's fixed-point theorem, a fixed point of the mapping in equation **(83)** exists. Next, we show that the fixed point is unique.

Note that as l increases, the aggregate level of liquidity increases, the region over which sales to outsiders take place shrinks, that is, $\frac{\partial \bar{k}}{\partial l} > 0$. Hence, we have $\frac{\partial E(\widetilde{\gamma})}{\partial l} < 0$. We have

$$\frac{\partial \widetilde{l}}{\partial E(\widetilde{\gamma})} = \frac{E(\alpha_0) + \tau E(\widetilde{\gamma}) - \tau \left[1 + E(\widetilde{\gamma}) + a(1 - E(\alpha_0))\right]}{\left[E(\alpha_0) + \tau E(\widetilde{\gamma})\right]^2} = \frac{E(\alpha_0) - \tau \left[1 + a(1 - E(\alpha_0))\right]}{\left[E(\alpha_0) + \tau E(\gamma)\right]^2}.$$

Thus, for $E(\alpha_0) > \tau \left[1 + a(1 - E(\alpha_0))\right]$, we have $\frac{\partial \widetilde{l}}{\partial E(\widetilde{\gamma})} > 0$, which gives us $\frac{\partial \widetilde{l}}{\partial l} < 0$. As a result, the fixed point is unique. $\diamondsuit$

Next, we show that with the cost of deposit insurance the socially optimal level of liquidity increases. We need to show $\widetilde{l} > \widehat{\widehat{l}}$.

Note that $E(\widetilde{\gamma}) > E(\gamma)$. We have

$$\frac{1 + E(\widetilde{\gamma}) + a(1 - E(\alpha_0))}{E(\alpha_0) + \tau E(\widetilde{\gamma})} > \frac{1 + E(\gamma) + a(1 - E(\alpha_0))}{E(\alpha_0) + \tau E(\gamma)} > \frac{1 + E(\gamma)}{E(\alpha_0) + \tau E(\gamma)}, \qquad \textbf{(84)}$$

which gives us $\widetilde{l} > \widehat{\widehat{l}}$. Hence, the socially optimal level of liquidity when there is a cost for the deposit insurance is higher. Thus, with the cost of deposit insurance, the socially optimal level of liquidity is higher than the privately optimal level for a larger set of parameter values. Furthermore, we have $\frac{\partial \widetilde{l}}{\partial a} > 0$. Hence, as the cost of providing deposit insurance increases, the socially optimal level of liquidity increases. In particular, for $a > \left(\frac{\alpha_1 \Delta}{\underline{p}}\right)$, we have $E(\widetilde{\gamma}) > E(\phi)$ and the socially optimal level of liquidity is always higher than the privately optimal level. $\diamondsuit$

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Table 1: Cross-country Data on Bank Liquidity, Financial Development and Stock-market Liquidity

Country	Liquidity ratio	Add. liquidity $	Add. liquidity/GDP	Account. stds	Total cap/GDP	Dom. Credit/GDP	Deposits/GDP	Stock market cap.	Turnover in Dom. Market	Amihud Illiquidity ratio	Prop. of Zero return days
United States	6.5	0	0				0.17	1.52			
Argentina	126.4	26.91	29.2				0.24	0.15	0.4	1.67	0.48
Armenia	34.7	0.07	2.9								
Australia	1.31			75	0.82	0.28	0.49	1.13			
Austria	16.43			54	1	0.77	0.7	0.17			
Bangladesh	28.31				0.2	0.07					
Belgium	50.42			61	0.65	0.29	0.85	0.82			
Bolivia	17.3	0.31	4.1								
Bosnia-Herzigova	16.8	0.21	3.7								
Brazil	98.6	107.21	28.1	54	0.33	0.23	0.17	0.45	0.33	0.61	0.4
Bulgaria	29.1	1.25	7.3								
Canada	14.01			74	0.98	0.45	0.61	1.22			
Chile	20.99			52	0.74	0.36	0.19	1.05	0.09	1.12	0.5
Colombia	33	4.85	6.8						0.1	1.31	0.63
Costa Rica	28.06				0.53	0.26					
Croatia	37.1	4.89	19.8						0.05		
Denmark	12.43			62	0.56	0.42	0.54	0.67			
Dominican Republic	33.1	1.7	9.1								
Egypt	41.4	26.66	31.4	24	0.74	0.21	0.51	0.29	0.3	1.18	0.2
El Salvador	31.3	0.19	1.4								
Finland	12.79			77	0.52	0.48					
France	31			69	0.7	0.54	0.47	1.17			
Germany	34			62	1.08	0.78	0.35	0.67			
Greece	27.28			55	0.47	0.44			0.6	1.02	0.23
Guatemala	32.5	1.42	6.1								
Honduras	25.8	0.54	8.5								
Hungary	46	12.05	16.2						0.65	1.21	0.28
India	44.9	103.93	20.2	57	0.5	0.24	0.09	0.46	0.34	2.07	0.23
Indonesia	69.8	57.33	31.8						0.96	1	0.55
Israel	18.34			64	1.18	0.67			0.26	0.27	0.15
Italy	21.46			62	0.98	0.42	0.28	0.68			
Jamaica	72.2	2.23	27.7								
Japan	21						0.53	0.95			
Jordan	53.2	4.85	54.9						0.13		
Kenya	39.7	1.44	11.5								
Korea	13.59			62	0.63	0.5			3.87	0.24	0.17
Lithuania	37.8	1.09	7.1						0.11	3.64	0.66
Malaysia	23.69			76	1.19	0.48			1.29	0.57	0.25
Mexico	53.5	69.14	11.4	60	0.39	0.16			0.32	0.26	0.27
Moldova	33.4	0.1	7								
Morocco	36.9	8.47	21.7								
Netherlands	19.17			64	0.91	0.6	0.69	2.03			
New Zealand	7.2			70	0.59	0.19					
Nicaragua	67.2	0.97	39.4								
Nigeria	65.2	5.62	12								
Norway	7.18			74	0.63	0.34	0.49	0.7			
Pakistan	50.85				0.53	0.25					
Paraguay	39	0.37	8.1								
Peru	39	5.05	8.9	38	0.28	0.28			0.75	1.33	0.51
Philippines	36.6	12.09	16	65	0.46	0.28			0.71	1.41	0.46
Poland	38.6	25.73	12.8						0.78	1.21	0.27
Portugal	8.42			36	0.82	0.52			0.27	1.19	0.43
Romania	54.5	4.91	10.9						0.24	1.26	0.75
Russia	45.9	31.42	9.2						0.28	1.67	0.69
Singapore	30.07			78	1.96	0.57					
South Africa	13.7	5.79	4.5					1.2	0.22	0.44	0.37
Spain	24.98			64	1.02	0.76	0.21	1.2			
Sri Lanka	29.08				0.44	0.21					
Sweden	0.49			83	0.79	0.42	0.71	0.69			
Switzerland	10.17						0.39	1.77			
Tanzania	35.4	0.47	5.6								
Turkey	71.13			51	0.35	0.14			2.09	0.3	0.42
Uganda	62.26	0.53	9.1								
UK	2			78	0.78	0.25	0.39	2.25			
Ukraine	18	0.85	2.1								
Venezuela	49.29			40	0.34	0.3			0.13	0.78	0.52
Zimbabwe	25.75				1.01	0.3			0.13	1.21	0.56

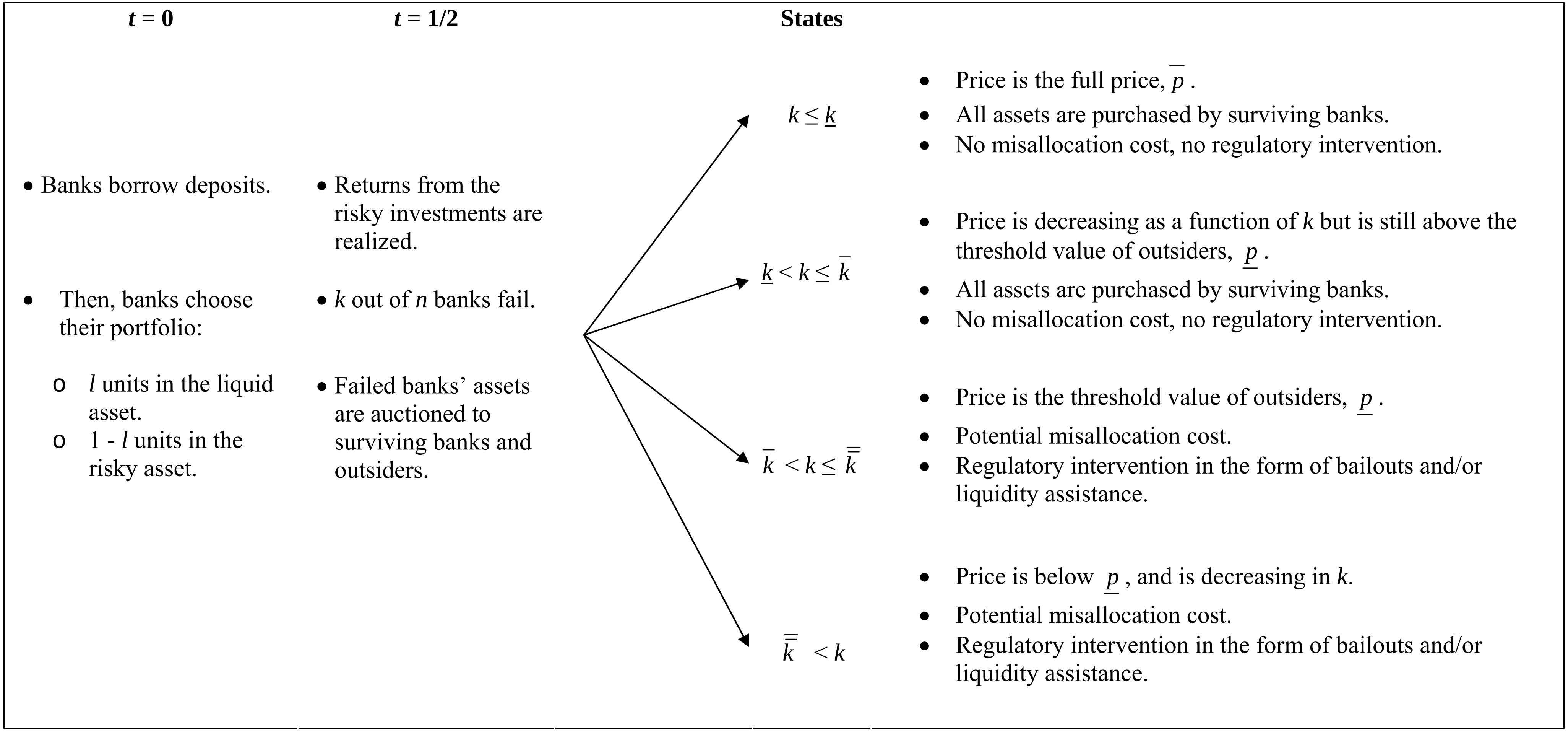

Figure 1: Timeline of the model.

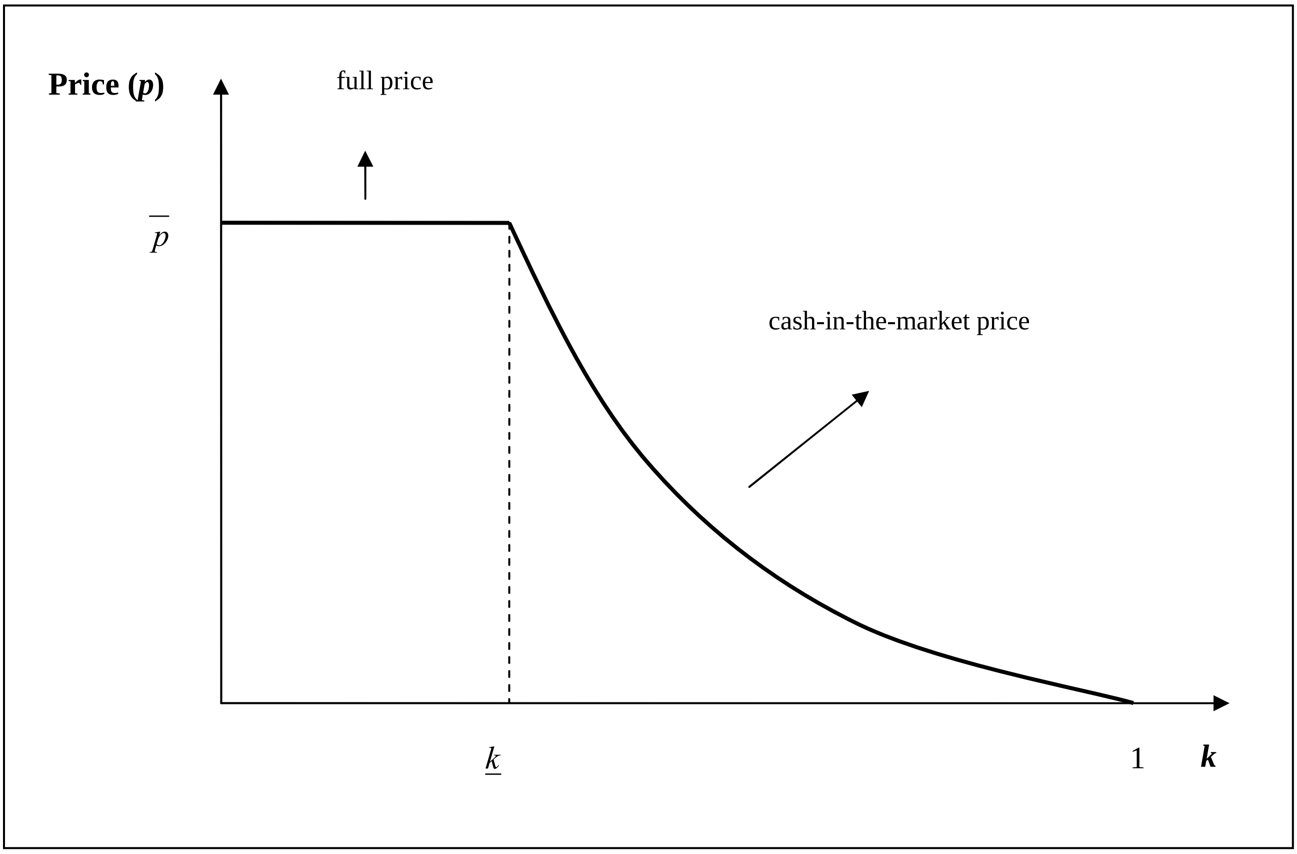

Figure 2: Price in Proposition 1.

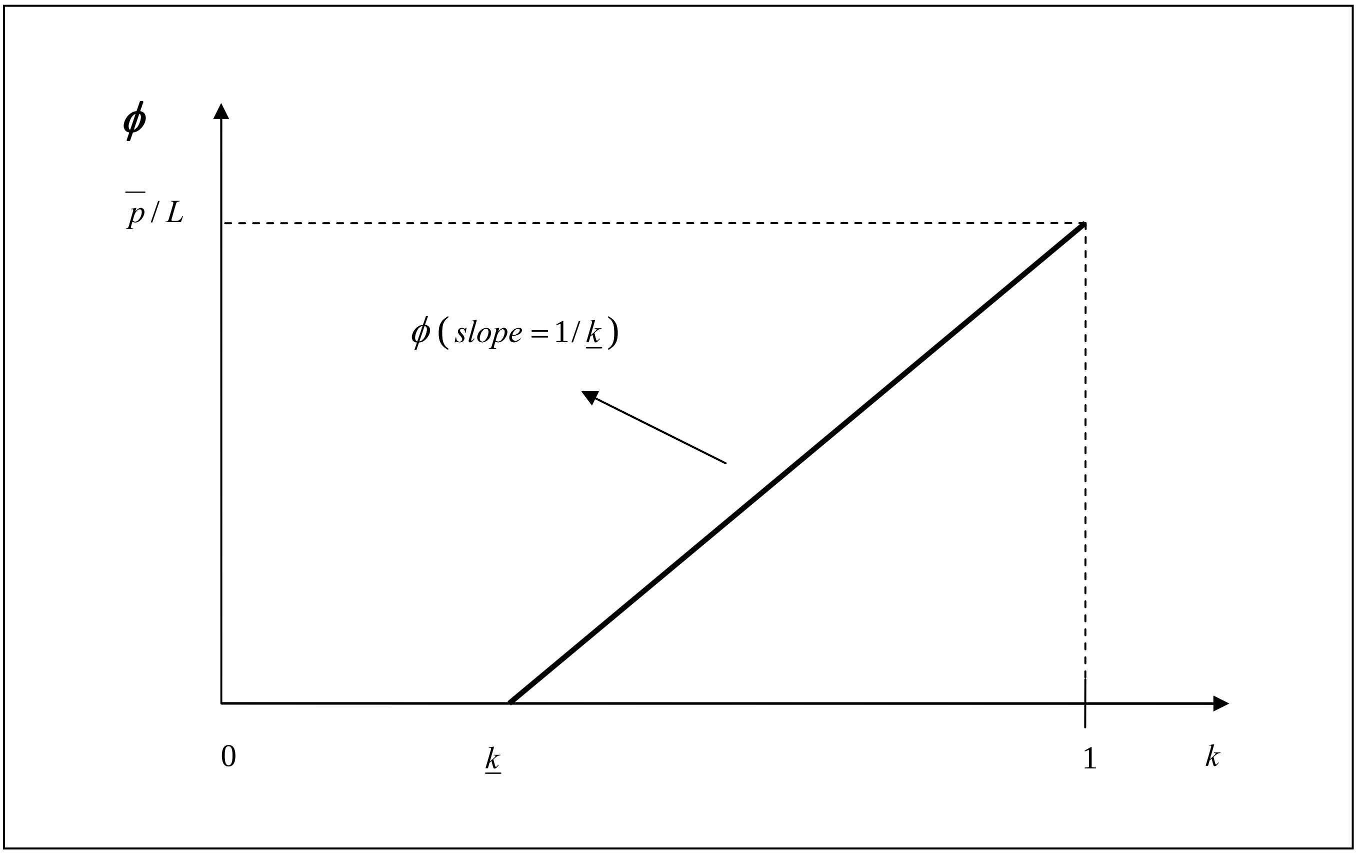

Figure 3: Marginal private (ϕ) benefit from the liquid asset (no outsiders).

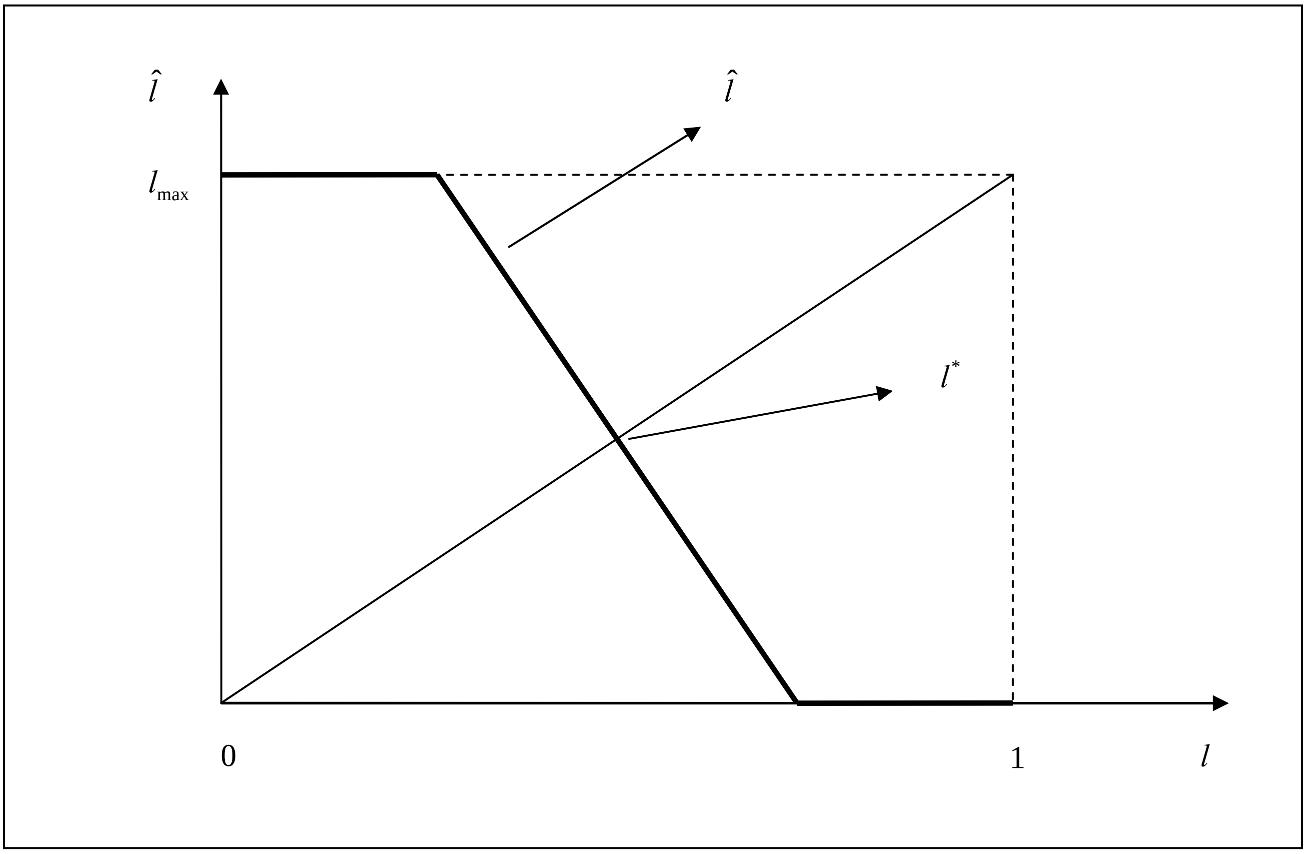

Figure 4: Privately optimal choice of liquidity and the equilibrium (Proposition 2).

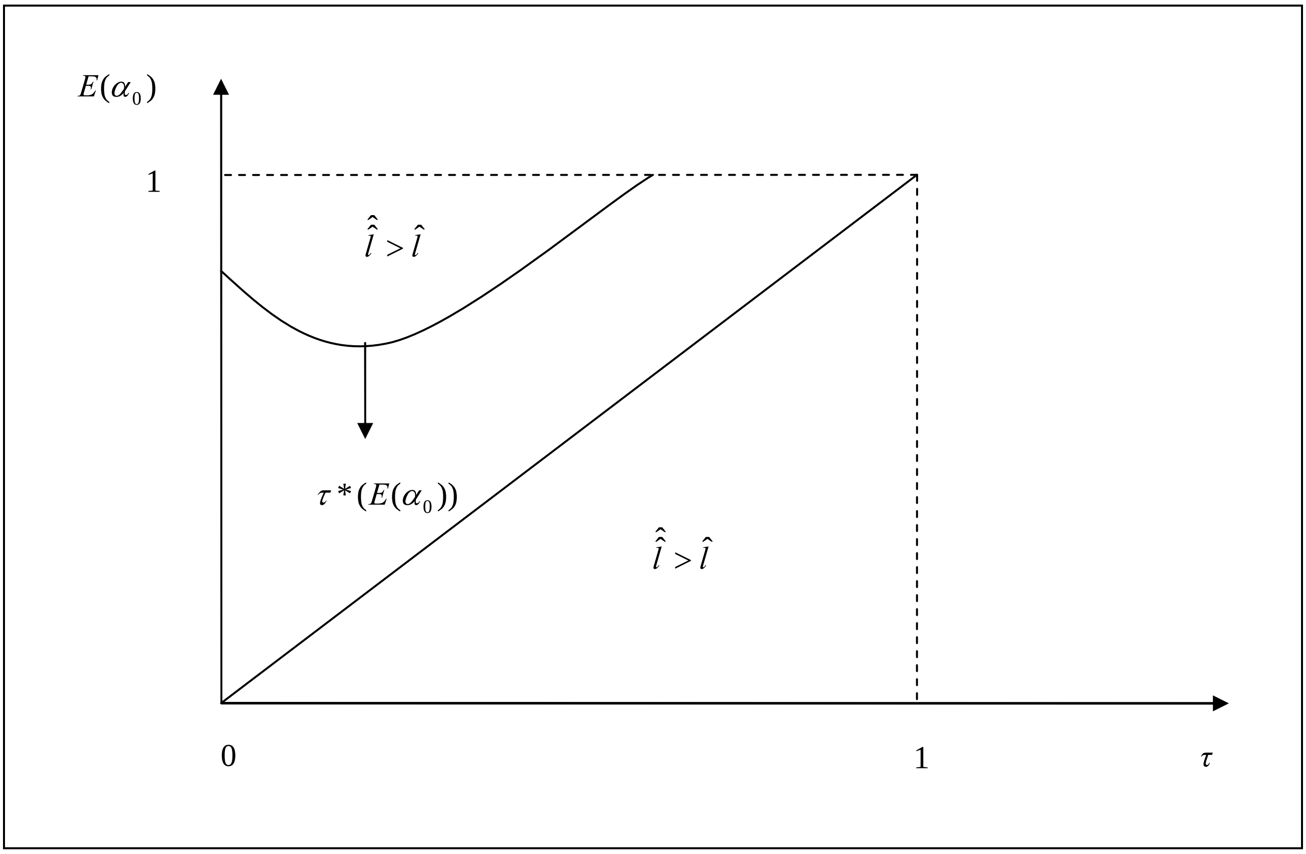

Figure 5: Comparison of privately and socially optimal levels of liquidity.

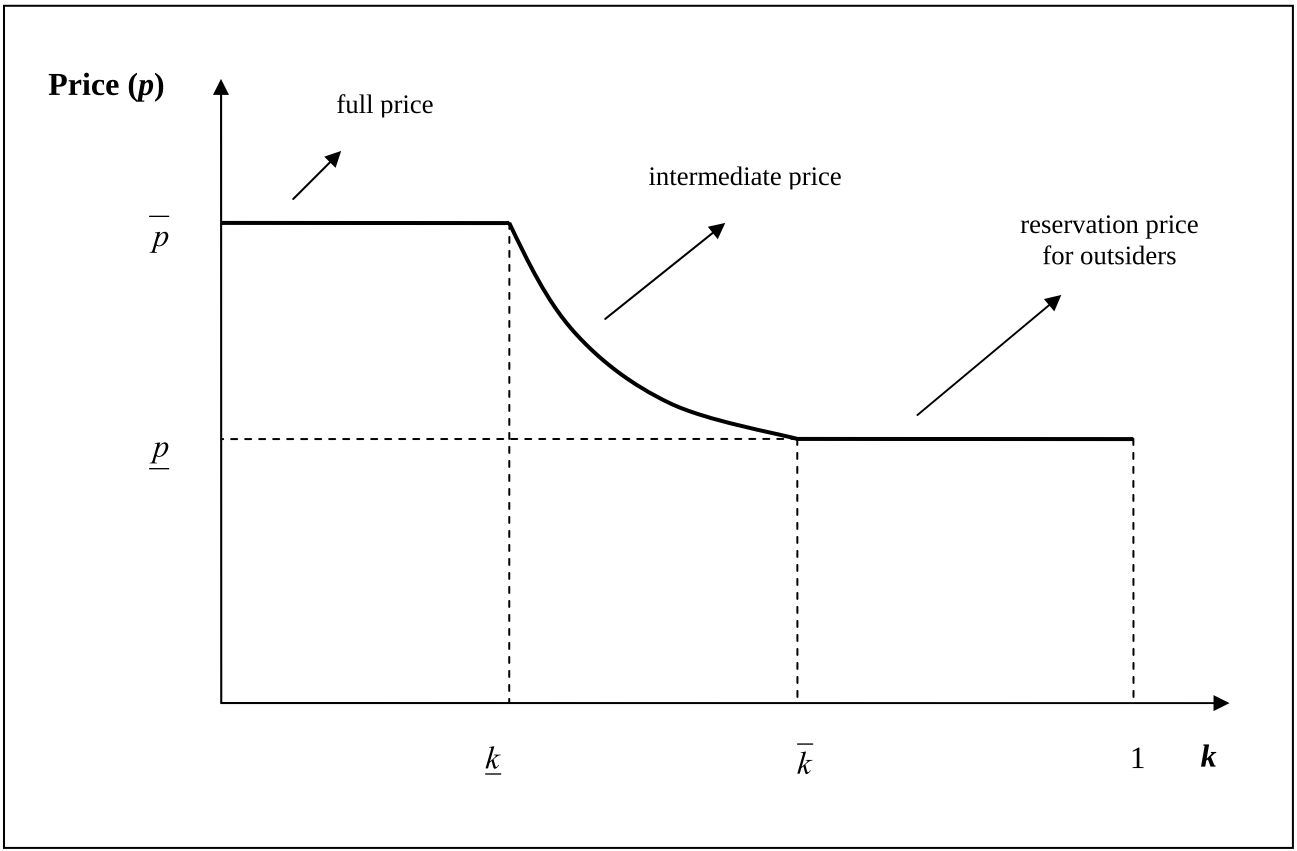

Figure 6: Price in Proposition 5.

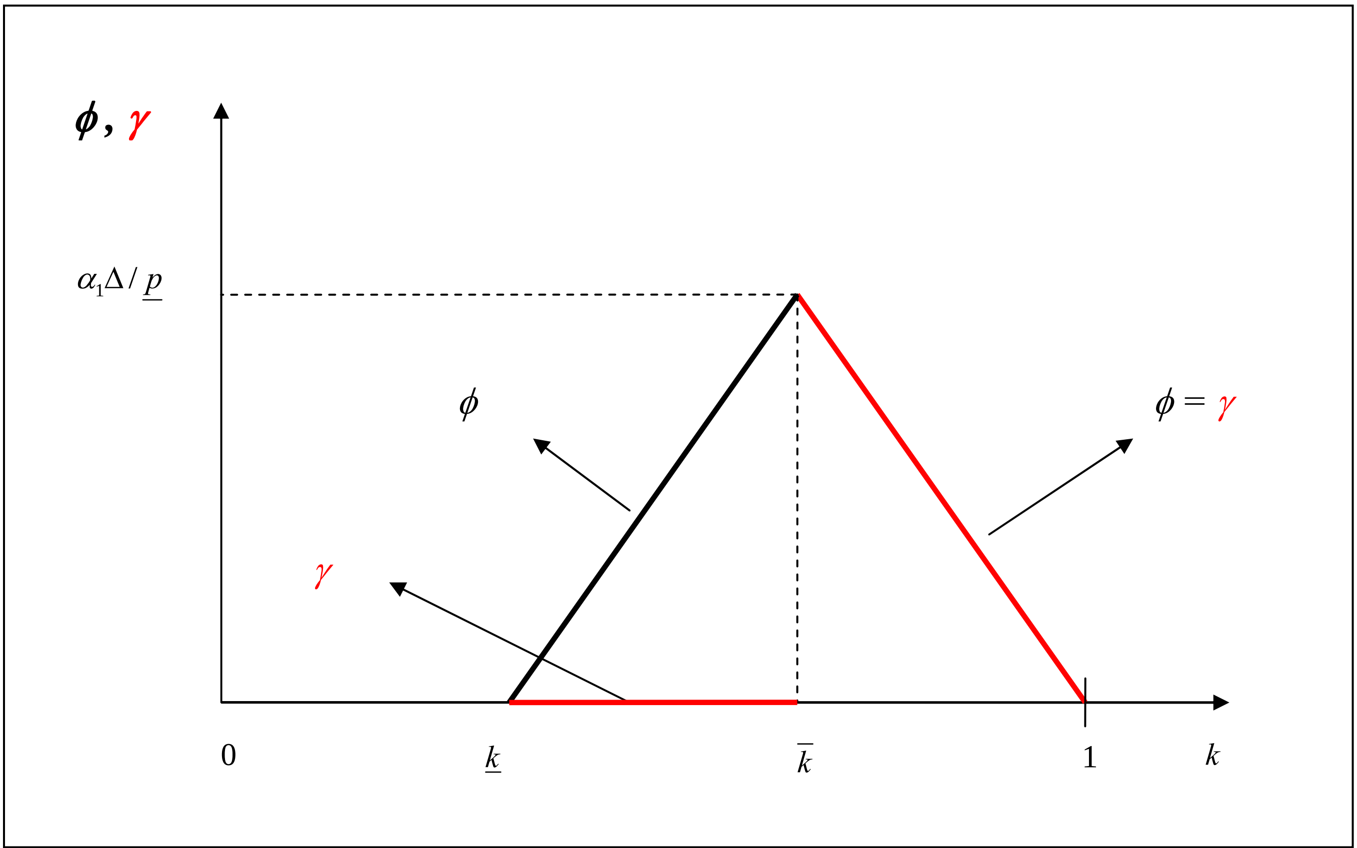

Figure 7: Marginal private (ϕ) and social (γ) benefit from the liquid asset for $w \geq \underline{p}$.

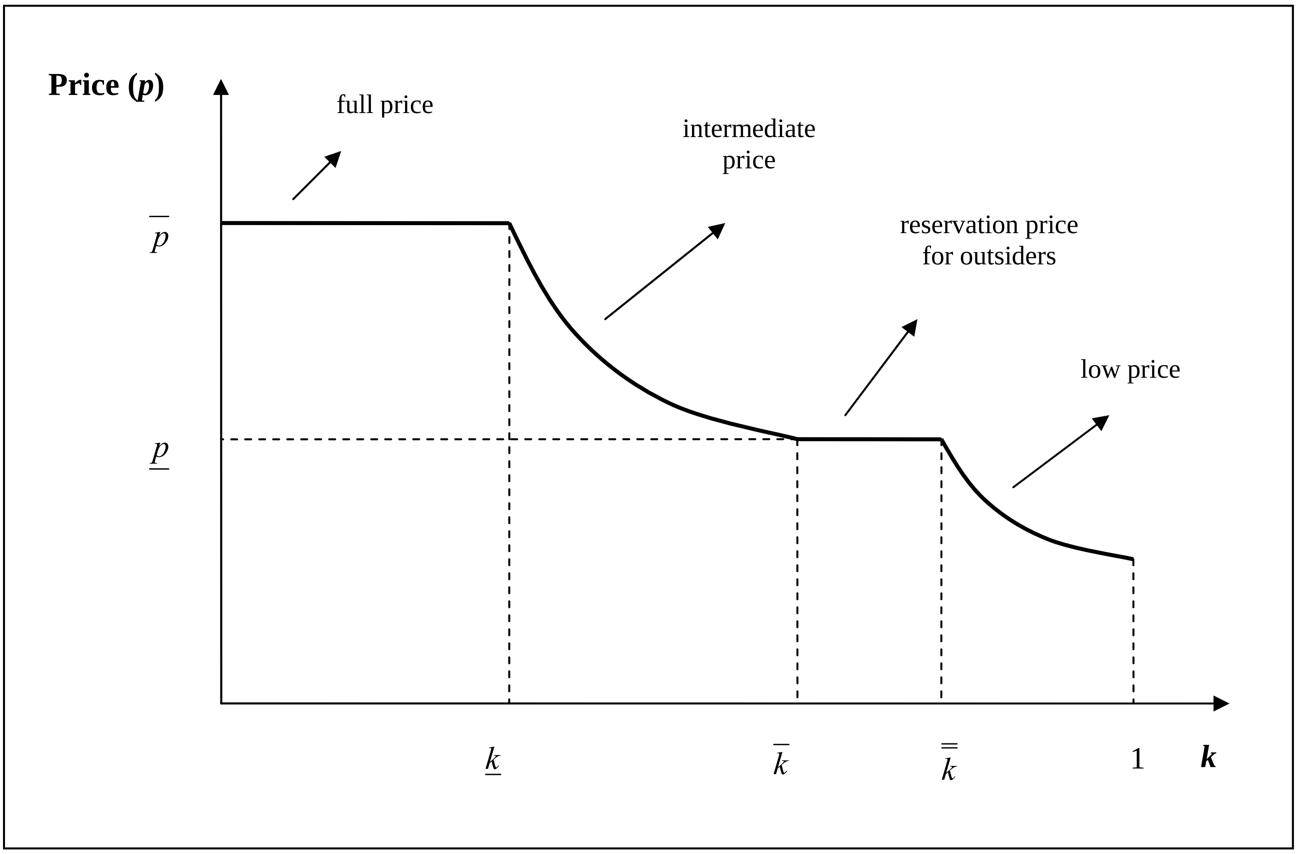

Figure 8: Price with $w < \underline{p}$.

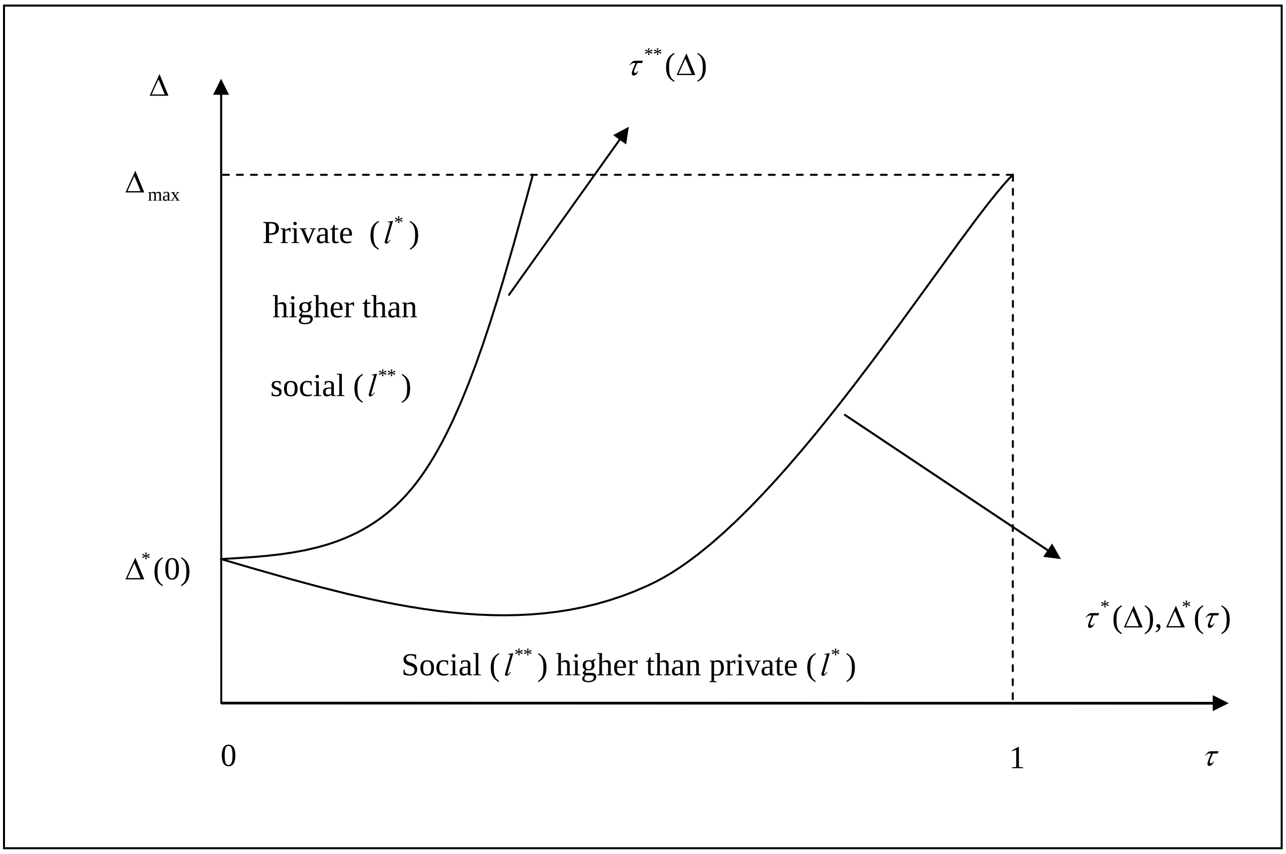

Figure 9: Comparing socially and privately optimal levels of liquidity (Proposition 4).

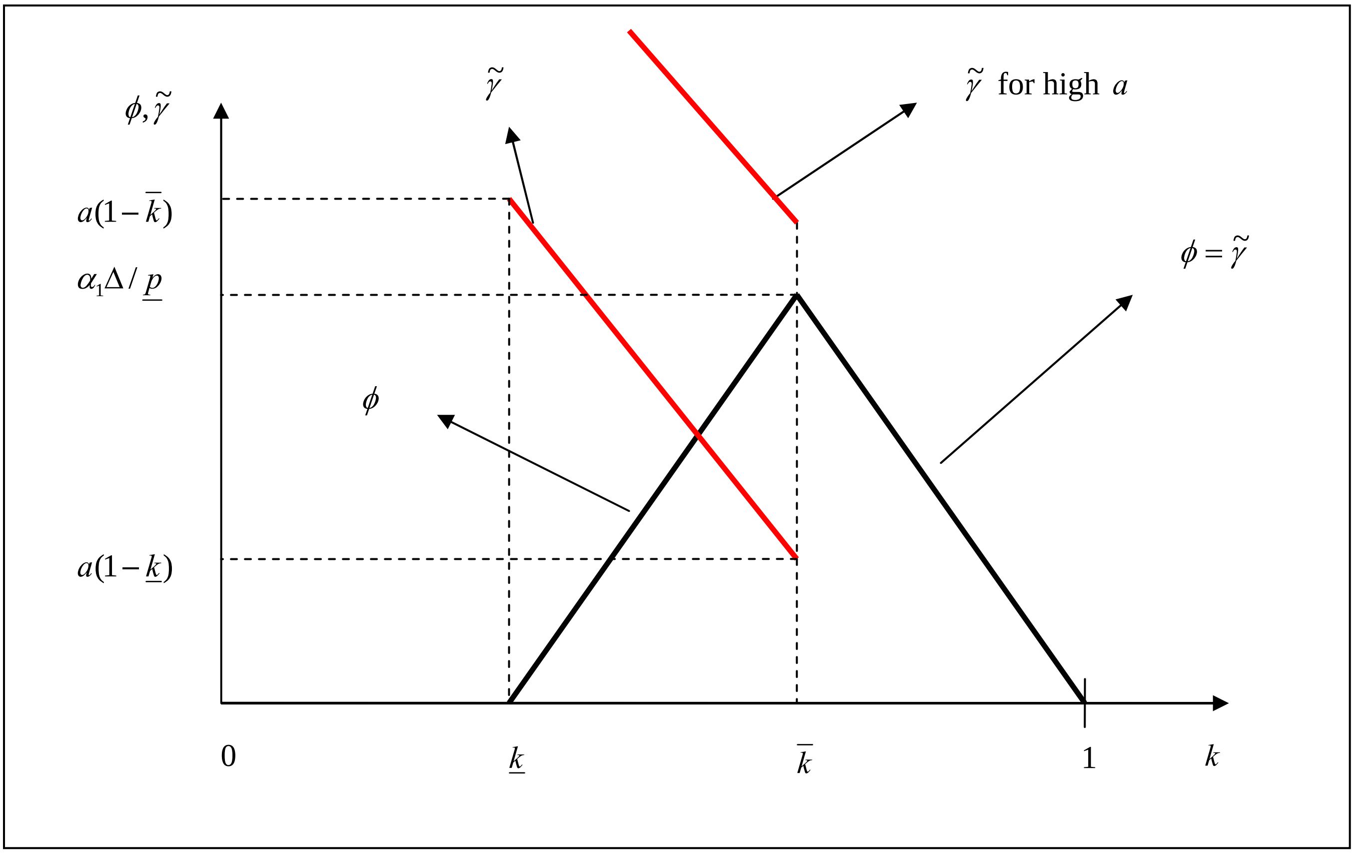

Figure 10: Expected marginal private and social benefit from the liquid asset.

Figure 11: Liquidity ratio and its Fitted value vs Accounting standards

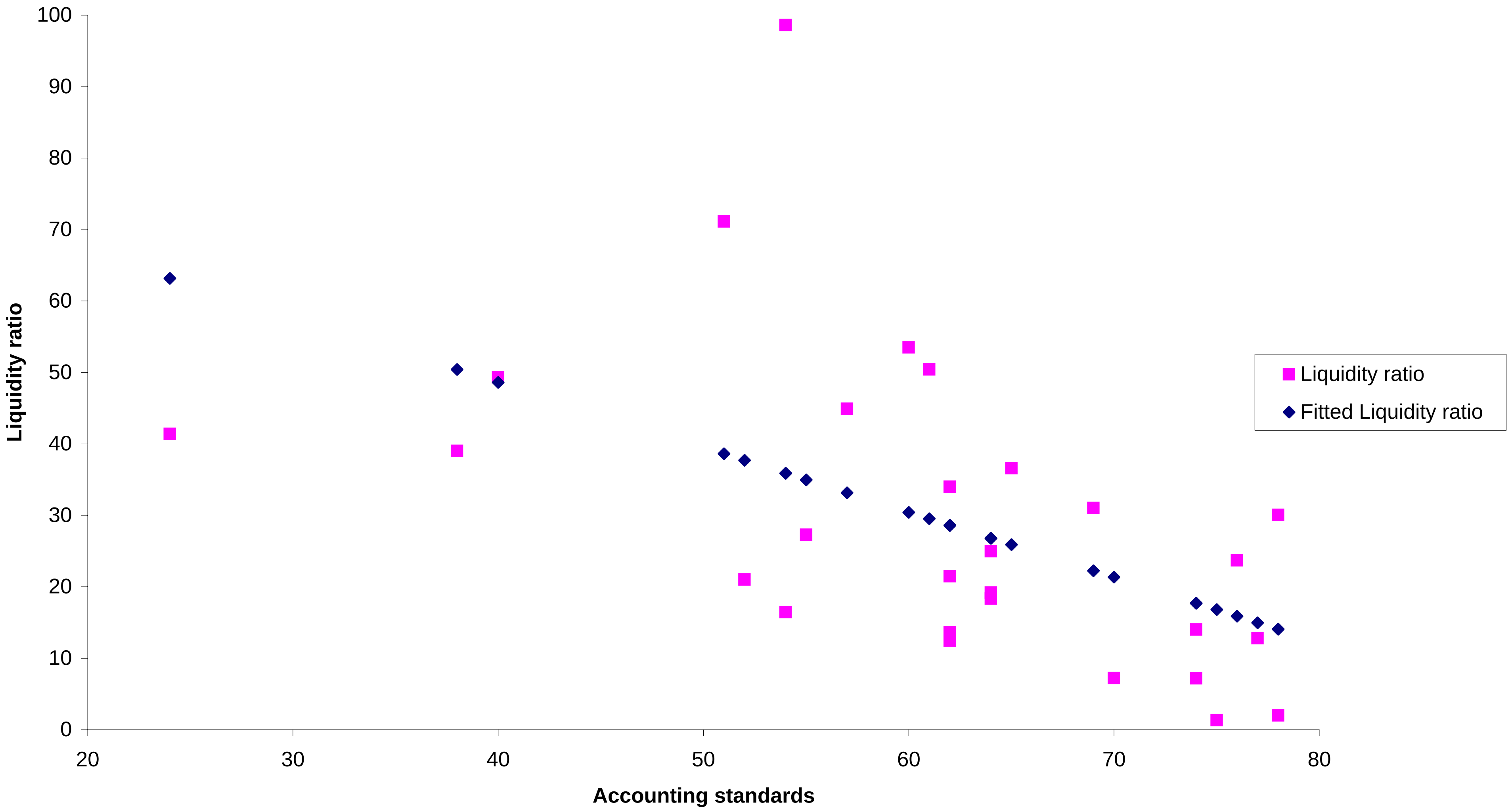

Figure 12: Liquidity ratio and its Fitted value vs Total Cap to GDP ratio

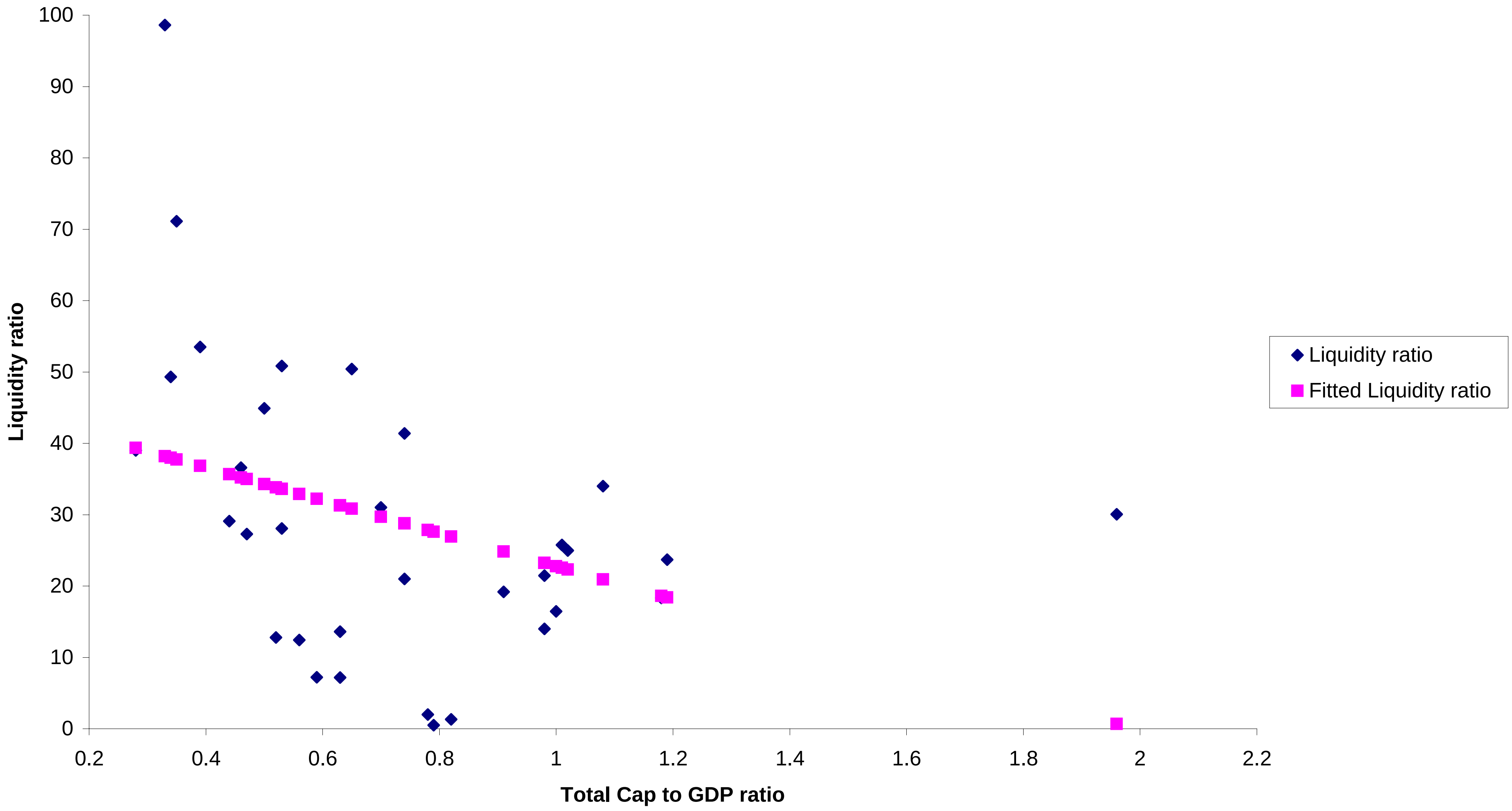

Figure 13: Liquidity ratio vs Stock market illiquidity (% Zero return days)