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Abstract

Resolving the forward premium puzzle requires a volatile foreign exchange rate risk premium that covaries negatively with the expected depreciation rate. Earlier work has shown how models featuring consumption habits can generate such premia when either trade costs or 'deep habits' are assumed. We show that as long as consumption habits are slow-moving and shocks are highly persistent, a standard small open endowment economy — without any additional features — can address the puzzle. Moreover endogenising the labour supply decision in the small open economy can improve the model's ability to match risk premia observations so long as it makes business cycles less synchronised.

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Summary

Investors require compensation (or a 'premium') to hold risky financial asset. So if some currencies are perceived to be riskier than others, investors may demand a foreign exchange (FX) premium to invest in those currencies. This paper presents a small open economy model that can explain why FX premia arise in currency markets. We use this model to examine how well it resolves the so-called uncovered interest rate parity (UIP) puzzle. UIP is simply a condition that follows from financial market arbitrage. It ensures that the interest rate return on a domestic currency asset should equal the interest rate on each foreign currency assets, less the expected appreciation of the domestic currency. The puzzle stems from the empirical observation that high interest rate currencies tend to appreciate – contrary to what UIP would predict.

A key feature of our model is that households are assumed to have consumption habits, ie households get used to a 'habit' level of consumption, and only attain higher utility if actual consumption rises relative to that level.

We demonstrate that our model will only resolve the UIP puzzle if it produces significant precautionary savings effects, where savings rise in response to increased uncertainty. And these savings effects will only occur if we assume quite persistent productivity shocks combined with very slow-moving consumption habits.

In our model, changes in precautionary savings are a result of changes in households' attitude towards risk, and changes in economic prospects. In the face of bad shocks, for example, households increase their precautionary savings if they expect consumption to be low relative to their habits level. Thus, the slower is the adjustment of habits to the shock, the larger will be the revisions in precautionary savings. These revisions are also larger when the shocks are more persistent.

To understand the combined role of slow-moving consumption habits and persistent shocks in resolving the UIP puzzle, consider how a temporary fall in productivity in the rest of the world works its way through our model. The drop in foreign productivity causes an *ex ante* excess demand for foreign goods which is eliminated by a rise in the relative price of foreign goods, ie a

domestic currency depreciation. But since this is ultimately a temporary shock, the domestic currency is expected to appreciate back towards its initial steady state.

However, the same negative foreign shock also triggers a large increase in foreign precautionary savings, putting downward pressure on foreign interest rates and hence causing domestic interest rates to exceed foreign rates at the same time as the domestic currency is expected to appreciate, thus potentially resolving the puzzle. But at the same time the increase in foreign households' borrowing to smooth their consumption (known as 'intertemporal substitution') will tend to put upward pressure on foreign interest rates and hence cause domestic rates to lie below foreign rates at the same time as the domestic currency is expected to appreciate (ie in line with the predictions of UIP). So we can only account for the tendency for high interest rate currencies to appreciate if the precautionary savings effects outweigh the intertemporal substitution effects. This would be the case if the shock is very persistent and consumption habits are very slow-moving.

We initially show our result at work in a model with fixed labour supply. We then examine how our result changes when we allow domestic households in our small open economy to vary their hours worked. In our model, this extension makes domestic consumption less synchronised with foreign consumption. To ensure that risk is efficiently shared across countries, the real exchange rate would have to fluctuate more. We find that a more volatile real exchange rate combined with a stronger precautionary savings effect actually improves the model's ability to address the UIP puzzle. But when we allow both domestic and foreign households to vary their hours worked, consumption is both smooth and synchronised across countries. This dampens the FX premium volatility and impedes the model's ability to resolve the UIP puzzle.

1 Introduction

Would an investment strategy that borrows in a low interest rate currency and invests the proceeds in a high-yielding currency be profitable? The naive answer is 'yes' since the interest rate differential on the borrowing-lending spread represents a potential profit opportunity. The traditional answer is 'no' since any excess return from investing in a high interest rate currency would be offset by an associated depreciation in that particular currency. But, actually, the empirical evidence indicates that not only is this strategy profitable on average but it yields returns that exceed the interest rate differential. This is because, in practice, high interest rate currencies frequently appreciate over time. And therefore, rather than offsetting any interest rate differentials, the exchange rate movements actually increase the profit of this particular investment strategy (see Cavallo (2006)).[1] This finding is well known in the academic literature and is known as the forward premium anomaly or the uncovered interest rate parity (UIP) puzzle (see Fama (1984)).[2]

A traditional open economy model cannot replicate the forward premium anomaly as it typically assumes linear UIP holds. When investors are assumed to be risk-neutral, any cross-country differences in interest rates are associated with offsetting movements in expected depreciation. A large literature has tried previously to account for the forward premium anomaly. One strand of research, explored in Bekaert (1996), attributes the failure of UIP to the existence of time-varying foreign exchange (FX) risk premia. When risk is allowed for, risk-averse investors may require additional compensation to hold riskier assets. A risk-adjusted UIP condition breaks the tight link between expected changes in the exhange rate and interest rate differentials. But as demonstrated by Fama (1984), the FX risk premium embodied in this UIP condition needs to have certain dynamics properties in order to resolve the forward premium anomaly. The challenge for this strand of the literature has been to come up with an open economy model that generates an FX risk premium with the time series properties that resolves this long-lasting anomaly.[3]

[1]For further studies assessing the profitability of such strategies see Bilson (1981) and Della Corte and Tsiakas (2009).

[2]While the original forward premium anomaly is documented by Fama (1984), a number of papers have looked at the robustness of his result (see Baillie and Bollerslev (2000), Bansal and Dahlquist (2000), Bansal (1997) and Flood and Rose (1996) for recent contributions and Sarno (2005) for a comprehensive survey of the literature). Importantly, it has been shown that the puzzle is not robust to short-run analysis (see Lyons and Rose (1995) and Chaboud and Wright (2005) for an assessment of intraday data). For this reason, the current paper proposes a quarterly model of the exchange rate risk premium.

[3]There are other strands of literature that have tried to rationalise the forward premium anomaly using theoretical models. 'Peso problem'-type arguments and other explanations related to irrational market participant behaviour can be found in the early literature (see Engel (1996) for a survey). More recently, Bacchetta and van Wincoop (2005) and Gourinchas and Tornell (2004) examine the role of

In this paper, we re-examine the forward premium anomaly using a standard open economy macro framework as in Gali and Monacelli (2005) and De Paoli (2009). Having consumption habits in the model is crucial in order to resolve the UIP puzzle. These ensure that in periods where the domestic currency is expected to appreciate, domestic investors are less risk-averse than foreign households, and this translates into domestic interest rates being higher than foreign interest rates. We are not the first to explore the role of consumption habits in solving the UIP puzzle. Both Verdelhan (2006) and Moore and Roche (2007) have shown how models featuring consumption habits can help rationalise the UIP puzzle. These papers assume that investors are subject to permanent shocks and have Campbell and Cochrane (1999) type preferences. The former considers an endowment economy subject to trade costs. The latter presents a monetary model which adds so-called 'deep habits' to the Campbell and Cochrane (1999) setting. Unlike these authors, we do not rely on such additional model features and instead we use a linear, additive external habit specification which nests those in Uhlig (2004) or Smets and Wouters (2007).[4] This formulation enables us to demonstrate the precise role played by the persistence of habit formation in resolving the UIP puzzle.

Our analysis shows a standard small open endowment economy can go a long way towards addressing this puzzle while replicating key macroeconomic moments as long as consumption habits are slow-moving and shocks are highly persistent. Consumption habits alone are not sufficient to generate a constellation of movements in exchange rates and interest rates that resolve the UIP puzzle. It is crucial that investors expect changes to economic conditions – as captured by their excess consumption level – to persist.

We also show that moving towards a general equilibrium setting where the labour supply decision in the small open economy is endogenous, improves the model's ability to match risk premia observations so long as it makes business cycles less synchronised. Such a setting can generate larger real exchange rate fluctuations and would increase FX premium volatility.

Nevertheless, perhaps more realistically, when both the domestic and foreign labour supply

imperfect information. Alvarez, Atkeson and Kehoe (2005) look at the properties of the FX risk premium in a model with asset market segmentation. In addition, Lyons and Rose (1995) proposes a framework in which, for small Sharpe ratios, the forward bias does attract speculative capital and therefore persists until this ratio is sufficiently large (see Leon and Valente (2006) for an empirical test of this 'limits to speculation' hypothesis) .

[4]Campbell and Cochrane (1999) demonstrate that up to a first-order approximation the habit formation process they impose is equivalent to an autoregressive linear specification similar to the one we use.

decisions are endogenous, the model has more difficulty matching observed risk premia. This setting produces very smooth consumption dynamics, small fluctuations in the FX premium and behaves similarly to a model solved under certainty equivalence. Such a caveat is similar to the findings in the closed economy literature (see Jermann (1998) and Rudebusch and Swanson (2008)) which have stressed the pivotal role of real rigidities – such as adjustment costs in labour supply. Our results suggest that similar model features may be needed in an open economy setting.

The paper is structured as follows: in Section 2 we present the model. Section 3 discusses how our model could generate an FX premium consistent with the Fama puzzle. Section 4 presents simulation results illustrating how model features affect the model's ability to address this puzzle. Finally, the concluding remarks are presented in Section 5.

2 Model

As in Lucas (1982), we analyse the role of the foreign exchange rate risk premium in a two-country general equilibrium context. But our theoretical framework assumes home bias in consumption and, thus, incorporates deviations from purchasing power parity and fluctuations in the real exchange rate. The size of this consumption home bias depends on the degree of openness and the relative size of the economy. This specification allows us to characterise the small open economy by taking the limit of the home size to zero. Prior to applying the limit, we derive the optimal equilibrium conditions for the general two-country model. After the limit is taken, the two countries, Home and Foreign, represent the small open economy and the rest of the world, respectively. This setup follows closely that of Gali and Monacelli (2005) or De Paoli (2009). Moreover, apart from the case of endogenous labour supply, we also analyse the case of inelastic labour supply, which translates into an endowment economy model (as in Lucas (1982)).

The original Gali and Monacelli (2005) or De Paoli (2009) specifications feature a Calvo-type sticky price-setting in order to address monetary policy issues. However, in the present paper we abstract from such issues and consider a flexible price version of these models, with the added feature of a slow-moving external consumption habit formation.[5]

[5]The flexible price allocation of such models is equivalent to the case where the central bank targets output price inflation.

2.1 The goods market

The world economy is populated with a continuum of agents of unit mass, where the population in the segment $[0, n)$ belongs to country H (Home) and the population in the segment $(n, 1]$ belongs to country F (Foreign). The utility function of a consumer j in country H is given by:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j, X_s) - V(y_s(j), \varepsilon_t) \right]. \tag{1}$$

Households obtain utility from consumption $U(C_t, X_t) = \frac{(C_t^j - hX_t)^{1-\rho} - 1}{1-\rho}$ and contribute to the production of a differentiated good $y(j)$ attaining disutility $V(y_t, \varepsilon_{Y,t}) = \frac{\epsilon_t^{-\eta} y_t^{1+\eta}}{1+\eta}$.[6] Productivity shocks are denoted by ϵ_t. The parameter, ρ, is the inverse of the intertemporal elasticity of substitution while η is the inverse of the Frisch labour supply parameter.

We assume that agents have external consumption habits X_t which follows an ARMA process where ϕ controls its persistence, ie

$$X_t = (1-\phi)C_{t-1} + \phi X_{t-1}, \tag{2}$$

and, similarly, in the foreign economy

$$X_t^* = (1-\phi)C_{t-1}^* + \phi X_{t-1}^*. \tag{3}$$

The consumption basket C_t is a constant elasticity of substitution (CES) aggregate over domestic, $C_{H,t}$, and foreign produced tradables $C_{F,t}$:

$$C_t = \left[\upsilon^{\frac{1}{\theta}} \left(C_{H,t} \right)^{\frac{\theta-1}{\theta}} + (1-\upsilon)^{\frac{1}{\theta}} \left(C_{F,t} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

with the corresponding domestic price index, P_t, defined as

$$P_t = \left[\upsilon \left(P_{H,t} \right)^{1-\theta} + (1-\upsilon) \left(P_{F,t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{5}$$

[6]This specification would be equivalent to one in which the labour market is decentralised. These firms employ workers who have disutility of supplying labour and this disutility is separable from the consumption utility.

The sub-indices C_{Ht} and C_{Ft} are Home and Foreign consumption of the differentiated products produced in countries H and F. These are defined as follows:

$$C_{H,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_{F,t} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c_t(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad \textbf{(6)}$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated products. The consumption-based price indices that correspond to the above specifications of preferences are given by

$$P_{H,t} = \left[\left(\frac{1}{n} \right) \int_0^n p_t(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_{F,t} = \left[\left(\frac{1}{1-n} \right) \int_n^1 p_t(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad \textbf{(7)}$$

We consider the case in which the law of one price holds, and thus, $P_{H,t} = S_t P^*_{H,t}$ and $P_{F,t} = S_t P^*_{F,t}$ where S_t is the nominal exchange rate (the Home-currency price of Foreign currency). However, as in Sutherland (2005), we assume a particular specification of υ and υ^* which implies that consumers at home and abroad have a consumption home bias. Thus, we define the real exchange rate, Q_t as the value of the domestic consumption basket per unit of the foreign consumption basket, ie

$$Q_t = \frac{S_t P^*_t}{P_t}. \quad \textbf{(8)}$$

In order to characterise a small open economy setting, we assume that the parameter governing domestic consumers' preference for foreign goods, $(1 - \upsilon)$, is a function of the relative size of the foreign economy, $1 - n$, as well as the degree of openness, λ in the domestic economy:

$$(1 - \upsilon) = (1 - n)\lambda. \quad \textbf{(9)}$$

Hence, a more open domestic economy (higher λ) would – *ceteris paribus* – imply a larger share of foreign goods in the domestic consumption basket. Similarly, the greater the size of the foreign economy relative to the domestic economy (higher $1 - n$), the larger the share of foreign goods in the domestic consumption basket.

Agents in the Foreign economy have preferences analogous to **(1)**, **(4)** and **(6)**. Moreover, foreign consumers' preferences for home goods also depend on the relative size of the home economy, n, and the degree of openness of the domestic economy λ, that is

$$\upsilon^* = n\lambda. \quad \textbf{(10)}$$

From consumers' preferences, we can derive the total demand for a generic good h, produced in country H, and the demand for a good f, produced in country F:

$$y_t^d(h) = \left[\frac{p_t(h)}{P_{H,t}}\right]^{-\sigma} \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[\upsilon C_t + \frac{\upsilon^*(1-n)}{n}\left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right], \tag{11}$$

$$y_t^d(f) = \left[\frac{p_t(f)}{P_{F,t}}\right]^{-\sigma} \left[\frac{P_{F,t}}{P_t}\right]^{-\theta} \left[\frac{(1-\upsilon)\,n}{1-n} C_t + (1-\upsilon^*)\left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right]. \tag{12}$$

Following De Paoli (2009), to portray our small open economy, we use the definition of υ and υ^* and take the limit for $n \to 0$. Consequently, conditions **(11)** and **(12)** can be rewritten as

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[(1-\lambda)C_t + \lambda \left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right], \tag{13}$$

$$Y_t^* = C_t^*. \tag{14}$$

Moreover, combining equations **(5)**, **(8)**, **(13)** and **(14)** and log-linearising the resulting expression, we can summarise the demand-side equilibrium in the small open economy as follows

$$y_t = (1-\lambda)c_t + \lambda y_t^* + \gamma q_t. \tag{15}$$

Note that lower-case variables denote log deviations from steady state (ie $x = \log(X/\bar{X})$) and $\gamma = \theta\lambda(2-\lambda)/(1-\lambda)$. For the foreign economy the demand condition is simply

$$y_t^* = c_t^*. \tag{16}$$

Hence, total demand for foreign goods is independent of domestic economic conditions.

Turning now to the supply side of the model, we assume that each household, i, contributes to the production of a differentiated good $y(j)$. Since all prices are assumed to be flexible, the output price of the differentiated good, $y(j)$, is set at a constant mark-up over marginal cost. In aggregate terms, we can express the relative output price as

$$\frac{P_{H,t}}{P_t} = \frac{\sigma}{\sigma-1}\frac{V_y\left(Y_t, \varepsilon_{Y,t}\right)}{U_c(C_t, X_t)}. \tag{17}$$

Using the definition of aggregate consumer prices **(5)** and the real exchange rate **(8)**, we can write the economy-wide supply curve in log-linear term as

$$\rho\left(1-h\right)^{-1}\left(c_t-hx_t\right)+\eta y_t+\lambda(1-\lambda)_t^{-1}q_t-\eta\varepsilon_t=0. \quad \textbf{(18)}$$

Similarly, the supply condition for the foreign economy is

$$\rho\left(1-h\right)^{-1}\left(c_t^*-hx_t^*\right)+\eta y_t^*-\eta\varepsilon_t^*=0. \quad \textbf{(19)}$$

In the subsequent sections we will also consider the case in which domestic and foreign countries are endowment economies. This specification can be represented by assuming an infinitely elastic labour supply. In particular, we can characterise this case by taking the limit of $\eta\rightarrow\infty$. Under this specification, equations **(18)** and **(19)** become, respectively,

$$y_t = \varepsilon_t, \quad \textbf{(20)}$$

$$y_t^* = \varepsilon_t^*. \quad \textbf{(21)}$$

From equations **(16)** and **(19)** (or from **(21)**), we can see that the entire dynamics in the rest of the world are independent of those in the small open economy. Therefore we can treat foreign variables as exogenous from the point of view of the small open economy.

2.2 The asset market

Following Chari and McGrattan (2002), we assume that households have access to a complete set of contingent claims, ie financial markets are complete both domestically and internationally. In this environment, the intertemporal marginal rate of substitution (in nominal terms) is equalised across countries. In particular, we can write

$$\frac{Q_{t+1}}{Q_t}=\frac{M_{t+1}^*}{M_{t+1}}, \quad \textbf{(22)}$$

where the domestic and foreign stochastic discount factors, M_{t+1} and M_{t+1}^*, are given by

$$M_{t+1} = \frac{U_C\left(C_{t+1},X_{t+1}\right)}{U_C\left(C_t,X_t\right)}, \quad \textbf{(23)}$$

$$M_{t+1}^* = \frac{U_C\left(C_{t+1}^*,X_{t+1}^*\right)}{U_C\left(C_t^*,X_t^*\right)}. \quad \textbf{(24)}$$

In addition, we assume that households also have access to a domestic as well as a foreign bond. The domestic bond is a one-period real bond that pays out in units of the domestic consumption basket. The foreign bond is also a one-period real bond but its payout is in units of the foreign consumption basket. The one-period risk-free returns on the domestic and the foreign bonds are R_t and R_t^* respectively. Thus, the following Euler equations determine how the domestic household would price both types of bonds:

$$1 = E_t\left[\beta M_{t+1} R_t\right], \qquad \textbf{(25)}$$

$$1 = E_t\left[\beta M_{t+1} R_t^* \frac{Q_{t+1}}{Q_t}\right]. \qquad \textbf{(26)}$$

There is a corresponding set of Euler equations that determine how the foreign household would price the two bonds.

2.2.1 Time-varying risk aversion

If the utility function is defined as in **(1)**, and consumption habits are *external* to households, the coefficient of relative risk aversion (CRRA hereafter) can be written as

$$\eta_t = \frac{\rho}{S_t},$$

where

$$S_t = \frac{C_t - hX_t}{C_t},$$

represents the surplus consumption ratio. Thus, the presence of habits implies a countercyclical CRRA. That is, risk aversion falls in periods of high consumption (and high surplus ratio).

2.3 Equilibrium

The demand and supply equations, **(15)**, **(16)**, **(18)** and **(19)**, together with the asset pricing conditions **(23)**, **(24)**, **(22)**, **(25)**, **(26)**, and the definition of habits **(2)**, **(3)** determine the evolution of $(Y_t,\ Y_t^*, C_t, C_t^*, Q_t, M_t, M_t^*, R_t^*,\ R_t,\ X_t,\ X_t^*)$ given the exogenous process for $(\varepsilon_t, \varepsilon_t^*)$. These shocks are assumed to follow an AR(1) process where γ and γ^* are the domestic and foreign autoregressive coefficients.

3 FX premia and the Fama puzzle

This section discusses how a time-varying FX risk premium can potentially resolve the forward premium anomaly (the so-called Fama puzzle). We start off by briefly reviewing the so-called Fama regressions that document the failure of risk-neutral uncovered interest rate parity (UIP). We then go on to discuss which time series properties the FX risk premium should inherit in order to resolve the Fama puzzle. We end the section by describing how our model could generate an FX risk premium with such properties.

3.1 The failure of the risk-neutral UIP

We can derive the risk-neutral uncovered interest rate parity condition by taking a log-linear (first-order) approximation of equations **(25)** and **(26)**.

$$E_t[\Delta q_{t+1}] = r_{t+1} - r^*_{t+1} \quad \textbf{(27)}$$

And, given the definition of the real exchange rate (equation **(8)**), we can write the above equation in nominal terms

$$E_t[\Delta s_{t+1}] = i_{t+1} - i^*_{t+1} \quad \textbf{(28)}$$

where $i_t = r_t + \Delta p_t$ and $i^*_t = r^*_t + \Delta p^*_t$ are domestic and foreign nominal interest rates. In general, the literature has tested the risk-neutral (or linear) UIP condition by testing the relationship between the so-called 'forward premium' ($f_t - s_t$ – where f_t is the forward exchange rate) and changes in the nominal exchange rate. In particular, these studies substitute the interest rate differential in equation **(28)** with such a forward premium and regress the change in the exchange rate on this premium.[7] More specifically, the regression used to test the linear UIP condition is

$$\Delta s_{t+1} = \alpha_0 + \alpha_1(f_t - s_t) + \upsilon_{t+1} \quad \textbf{(29)}$$

where υ_{t+1} is the regression error term. The linear relationship implied by equation **(28)** suggests that α_1 should be equal to one. So when domestic interest rates exceed foreign rates (ie $f_t - s_t > 0$) , the domestic currency should depreciate by the same amount as the interest rate differential. Hence, when $\alpha_1 = 1$, any excess return from investing in a high interest currency would be exactly offset by the capital loss from an associated depreciation of that particular

[7] This procedure is justified by the no-arbitrage 'covered interest rate parity' (CIP) condition stating that the interest rate differential is equal to the forward premium, ie $f_t - s_t = i_t - i^*_t$. We should note that even though the linear UIP condition **(28)** is widely rejected by the data, there is extensive evidence that the CIP condition holds (see Sarno (2005) and Sarno and Taylor (2003) for some useful surveys).

currency.

But, as documented in Fama and French (1989) and many other studies (see, for example, Hodrick (1987) and Engel (1996)), α_1 is actually found to be negative. Using monthly data from 1990 to 2007, a regression of the three-month £/$ forward premium on the three-month £/$ depreciation rate yields a statistically significant estimate for α_1 of -0.1.[8] These findings present strong evidence against equation **(28)** as a negative α_1 implies that high interest rate currencies would tend to appreciate over time. So investing in a high interest rate currency not only yields a relatively higher interest rate. Any currency appreciation over the holding period would increase the investment return even further. So a negative α_1 suggest that there exist excess returns in currency markets. These results are often referred to as the 'forward premium anomaly puzzle' and equation **(29)** is often called the 'Fama regression' and α_1 the 'UIP coefficient'.

3.2 Risk premium explanation of UIP failure

Behind the linear UIP condition lies the assumption that investors are both rational and risk neutral. Therefore, relaxing these assumptions could help explain why equation **(27)** fails to match empirical observations. In the current work, we relax the second assumption and consider that investors are risk averse. In this setting, the presence of a foreign exchange rate risk premium breaks the equality between interest rate differentials and expected depreciation. That is, defining the FX risk premium as the excess return on domestic bonds (once returns are expressed in the same currency), we have

$$r_{t+1} - r^*_{t+1} = E_t[\Delta q_{t+1}] + fxp_t. \qquad \textbf{(30)}$$

By inspection of equation **(30)**, we can see that in order to have interest rate differentials moving in opposite direction to the expected depreciation rate, changes in the FX premium have to more than offset movements in expected depreciation rates. In other words, the necessary conditions to obtain a negative correlation between interest rate differentials and expected depreciation (consistent with the finding that $\alpha_1 < 0$) are: (1) an fxp_t that covaries negatively with $E_t[\Delta q_{t+1}]$ and (2) that is more volatile than $E_t[\Delta q_{t+1}]$. In the next section we demonstrate under which conditions our model can generate an FX risk premium with such characteristics.[9]

[8]The validity of the inferences from these regressions have been questioned in the literature (see Sarno (2005) for a review). Nevertheless, the failure of linear UIP at short horizon appear to be an accepted stylised fact in the literature.

[9]These conditions were first documented by Fama and French (1989).

3.3 Time-varying risk aversion and FX risk premium

The cyclical nature of the exchange rate in our model is standard and well understood. A positive (negative) domestic productivity shock depreciates (appreciates) the domestic currency today and implies that the currency is expected to appreciate (depreciate) back to its initial steady state. But what are the associated movements in the interest rate differential? In our model, this depends on the size of the so-called *intertemporal* substitution effect relative to the *precautionary savings* effect.

As shown in the appendix, a log-normal approximation of the pricing equations **(25)** and **(26)**, implies that

$$r_{t+1} - r^*_{t+1} = \underbrace{E_t[m^*_{t+1} - m_{t+1}]}_{\text{Relative intertemporal substitution effect}} + \frac{1}{2} \underbrace{\left(var_t(m^*_{t+1}) - var_t(m_{t+1})\right)}_{\text{Relative precautionary savings effect}} \quad \textbf{(31)}$$

or,

$$r_{t+1} - r^*_{t+1} = E_t[\Delta q_{t+1}] + \frac{1}{2} var_t(m^*_{t+1}) - \frac{1}{2} var_t(m_{t+1}). \quad \textbf{(32)}$$

So in a world that ignores uncertainty, the interest differential would be positively correlated with the expected depreciation rate and entirely determined by the intertemporal substitution effects at home and abroad. But in our setting, relative uncertainty at home and abroad affect the equilibrium interest rate differential through the relative precautionary savings effect. Using equation **(30)**, we can see this effect is the sole determinant of FX risk premium fxp_t

$$fxp_t = \frac{1}{2} var_t(m^*_{t+1}) - \frac{1}{2} var_t(m_{t+1}). \quad \textbf{(33)}$$

But what determines the cyclical properties of precautionary saving? In a closed economy setting, De Paoli and Zabczyk (2008) show that the precautionary saving effect is countercyclical provided that the model features enough habits and consumption persistence.[10] They show that investors' willingness to engage in precautionary savings (driven by the perceived degree of uncertainty $var_t(m_{t+1})$) increases after adverse shocks only when these are expected to persist. Thus, countercyclical risk aversion is not sufficient to generate countercyclical precautionary savings, as this also depends on investors expectations about future prospects.

[10] De Paoli and Zabczyk (2009) shows that similar conditions are necessary to generate a countercyclical risk premium in a closed economy model featuring habit formation.

In particular, their paper demonstrates that in an endowment economy the variance of the stochastic discount factor can be written as

$$var_t(m_{t+1}) = \kappa_0 + \kappa_1[(1 - \phi - \gamma)c_t + \phi x_t],$$

where $\kappa_0 = \frac{\rho^2\sigma_\varepsilon^2}{(1-h)^2}$, $\kappa_1 = \frac{2h\rho^2\sigma_\varepsilon^2}{(1-h)^3}$ and σ_ε^2 is the exogenous endowment shock volatility. By inspection of κ_0 and κ_1, we can see that the strength of precautionary savings is determined by the overall uncertainty (σ_ε^2) and that such savings are only time varying with habit persistence ($h \neq 0$). Given that x_t is a predetermined variable at time t, the above equation shows that $var_t(m_{t+1})$ is countercyclical (ie $\frac{\partial var_t(m_{t+1})}{\partial c_t} < 0$) as long as $\phi + \gamma > 1$.[11]

As our numerical simulations will illustrate below a result similar to De Paoli and Zabczyk (2008) holds in our open economy model. In particular when shocks and habits are persistent (ie both ϕ and γ are large), relative precautionary savings are countercyclical. Moreover, consistent with the results in Verdelhan (2006), if relative precautionary savings are countercyclical, the covariance between the FX premium and expected depreciation is negative.

To understand this point, consider the case of how a temporary fall in productivity in the rest of the world works its way through our model. The drop in foreign productivity causes an *ex ante* excess demand for foreign goods which is eliminated by a rise in the relative price of foreign goods, ie a domestic currency depreciation. But since this is ultimately a temporary shock, going forward the domestic currency is expected to appreciate back towards its initial steady state (hence $E_t[\Delta q_{t+1}] < 0$ in equation **(30)**).

At the same time, countercyclicality in relative precautionary savings would imply that an adverse foreign shock triggers a greater increase in foreign precautionary savings relative to domestic, and, thus an increase in the FX premium (see equation **(30)**). The fact that domestic interest rates now exceed foreign rates at the same time as the domestic currency is expected to appreciate would be consistent with a negative Fama regression coefficient, $\alpha_1 < 0$, and would resolve the forward premium puzzle.

[11] This expression was derived using a second-order approximation of the stochastic discount factor. The derivations can be found in the appendix .

4 Model simulations results

4.1 Model calibration

We now turn to our simulation results. Since our model is essentially the flexible price version of the Gali and Monacelli (2005) model, our model calibration follows closely their parameterisation. Table A summarises our calibration strategy.

Table A: Parameters used in the baseline calibration

Parameter	Description	Value
β	Discount factor	0.99
ρ	Household inverse IES	5
h	Habit parameter	[0, 0.85]
ϕ	Habit persistence parameter	[0, 0.99]
λ	Share of imports in domestic economy	0.08
θ	Import/domestic tradable ES	1
η	Inverse of Frisch elasticity of labour supply	$\left[3/(1-h)), \infty\right]$
γ	AR(1) coefficient for domestic productivity	[0.66, 0.9977]
γ^*	AR(1) coefficient for foreign output	[0.86, 0.9977]
σ_{ε}	Standard deviation of domestic productivity	0.71%
σ_{ε^*}	Standard deviation of foreign output	0.78%
$corr(\varepsilon_t, \varepsilon_t^*)$	Correlation between domestic and foreign shock	0.3

The discount factor is set at $\beta = 0.99$ which implies a steady-state interest rate of 4% p.a. in a quarterly model. The elasticity of substitution across traded goods, θ, equals 1. Our value for the import share, λ, follows the parameter estimates found in Lubik and Schorfheide (2007). Using Bayesian estimation techniques, they estimate the Gali-Monacelli model on UK data and find that λ equals 8%.[12] As in Chari and McGrattan (2002), we assume an intertemporal elasticity of substitution, ρ, equal to 5.[13]

Consider next the calibration of the labour supply elasticity. We will examine two cases: first, we simulate a model where hours worked is fixed. In other words, we consider the case where the domestic and foreign economies are endowment economies. This set-up can be represented by

[12]This implies a UK import share that is less than the 25% observed in UK data. Their low estimate of λ is probably caused by the fact that their model – as ours – assumes all goods are traded. Therefore the model cannot capture the additional relative non-tradable price volatility. We believe that assuming a lower import share is an appropriate modelling shortcut.

[13]Chari and McGrattan (2002) have pointed out that open economy models with complete markets imply a tight link between real exchange rate volatility and consumption volatility. And they claim that you need to have fairly high values for the intertemporal elasticity of substitution, ρ, to get any volatility in the real exchange rate.

assuming an infinitely elastic labour supply (ie $\eta \to \infty$). Then, we consider a model where hours worked can vary endogenously and set the inverse of the Frisch labour supply parameter, η, equal to $\frac{3}{1-h}$.[14]

As in Gali and Monacelli (2005) domestic productivity, ε_t, is assumed to have a standard deviation of 0.71 while the foreign productivity shock, ε_t^*, has a standard deviation of 0.78%.[15] The shocks are assumed to be positive correlated ie $corr\left(\varepsilon_t, \varepsilon_t^*\right) = 0.3$.

Since we are interested in the role that shock and habit persistence play in determining the cyclical properties of the FX premium, we carry out simulations for different values of γ and ϕ. We use two alternative specifications for the shock persistence parameters γ , γ^*: as in Gali and Monacelli (2005) we set the domestic AR(1) coefficient, γ , equal to 0.66 while the foreign AR(1) coefficient, γ^*, equals 0.86. Alternatively, we consider the shock persistence parameters of Smets and Wouters (2007) in which the shock persistence equals 0.9977. Heaton (1995) simulates his model using habit persistence parameters, ϕ, ranging from 0.05 to 0.95. We simulate our model for $\phi = 0$ and $\phi = 0.99$. Our calibration assumes that h equals 0.85 similar to the findings of Julliard, P. Karam, Laxton and Pesenti (2004) and Baanerjee and Batini (2003) using US and UK data respectively.

4.2 Results

We argued in Section 3.3 that in order to have an FX risk premium that covaries negatively with the expected depreciation, the model dynamics must exhibit sufficiently high shock and habit persistence. In this section, we illustrate this result numerically by varying the parameters γ and ϕ. The subsequent section examines the role of endogenous labour supply.

4.2.1 The role of shock and habit persistence

Column 3 in Table B contains the simulation results for the case of no consumption habits ($h = \phi = 0$) and low shock persistence parameters (ie $\gamma = 0.66$ and $\gamma^* = 0.86$). Overall, our

[14] There is a lot of uncertainty surrounding the correct value for η. It ranges from 0.47 (Rotemberg and Woodford (1997)) to 6 (Canzoneri, Cumby and Diba (2008)). We found in our model that introduction of consumption habits makes consumption too smooth relative to the data and thus requires us to choose a value of η equal to 20.

[15] These standard deviations are in line with those estimated by Lubik and Schorfheide (2007) who find the standard deviation of UK productivity shock to equal 0.72% while world real shocks have a volatility of 1.25%.

model generates theoretical moments that are close to what we observe in UK data (see column 2). Consumption growth is slighly less volatile than what is observed in the data (0.64% versus 0.75% in the data). Similarly the real exchange rate is less volatile than observed (2.69% versus 3.25% in the data). On the other hand, the risk-free rate is more volatile (1.13%) relative to the data (0.85%). Since the degree of risk aversion and hence the FX premium is constant – given that the model assumes no habits – the UIP coefficient, α_1, is 1.

Table B: Simulation results

Model variables	*Data*	*No habits*	*+Habits*	*+Habit & shock pers*
$\sigma_{\Delta y}$	0.76%	0.78%	0.78%	0.71%
$\sigma_{\Delta c}$	0.75%	0.64%	0.75%	0.71%
$\sigma_{\Delta q}$	3.25%	2.69%	5.05%	4.59%
σ_r	0.85%	1.13%	25.42%	1.29%
Fama risk premia requirements				
$\sigma_{\Delta q^e}$	$< \sigma_{fxp}$	0.96%	2.20%	0.19%
σ_{fxp}	$> \sigma_{\Delta q^e}$	0	0.01%	0.27%
$cov\left(fxp_t, \Delta q^e_{t+1}\right)$	< 0	0	0.00005%	-0.0003%
UIP coef α_1	-0.1^*	1	0.99	0.20

Note: Column 2 contains empirical moments (1976 Q1 to 2007 Q3) for data on the UK base interest rate (series RB.q from Bank of England Quarterly Model) as well as log first-differences for UK output (series GDPKP.q from Bank of England Quarterly Model), UK consumption of non-durables and services (series util.q+utip.q from UK Office for National Statistics) and the sterling effective real exchange rate (series Q.q from Bank of England Quarterly Model). Columns 3 to 5 are the unconditional moments produced in Dynare ++ freeware assuming 10,000 simulation periods. We use a third-order approximation of the model presented in Section 2 in order to capture the time variation in risk premium. Note that we use a log-linear version of the demand and supply conditions. This is done in order to facilitate the simulation of our model with highly persistent shocks. For consistency we use this log-linearisation throughout our numerical exercises.

* Estimate is from a regression of the three-month £/$ forward premium on the three-month £/$ depreciation rate based on monthly data from 1990 to 2007.

Column 4 of Table B presents the simulation results for the case where consumption habits are introduced. Adding consumption habits to the model causes the FX risk premium to be time varying. But as our simulation results illustrate, the Fama UIP coefficient, α_1, is very close to one. Even with consumption habits , $h = 0.85$, neither conditions (1) or (2) stated in Section 3.2 are satisfied. Since the volatility of the expected depreciation rate is 2.2%, the FX risk premium is not sufficiently volatile to satisfy the first Fama condition. In addition, the correlation between the FX premium and the expected depreciation is positive.

We explained in Section 3.3 how countercyclical relative precautionary savings were instrumental in terms of producing a negative correlation between fxp_t and $E_t[\Delta q_{t+1}]$. We have also argued that business cycle properties of precautionary savings are a function of the degree of habit persistence as well as the persistence of the exogenous stochastic process. The final column in Table B illustrates this point by presenting simulation results from our benchmark model with persistent productivity shocks ($\gamma = \gamma^* = 0.9977$) as well as a slow-moving consumption habits ($\phi = 0.99$).

Our model implies consumption growth that is slightly less volatile ($\sigma_{\Delta c} = 0.71\%$) than in the data (0.75%). But the standard deviation of real exchange rates in our model is somewhat higher than in the data (4.59% versus 3.25% in the data).

The combination of very persistent shocks and slow-moving consumption habits help us in three directions. It introduces the aforementioned countercyclicality in λ_t. It makes the household's attitude to risk more sensitive to business cycle fluctuations, increasing FX premium volatility ($\sigma_{fxp} = 0.27\%$). Finally, it also implies a less volatile expected depreciation rate ($\sigma_{\Delta q^e} = 0.19\%$).[16] As a result, our model generates a UIP coefficient, α_1, equal to 0.20. Although this specification leads to a smaller value of α_1, it still cannot replicate the negative UIP coefficients documented in the literature. Nevertheless, the obtained value of α_1 is not far from the estimate of -0.1 reported in Table B.

4.2.2 The role of endogenous labour supply

So far, we have assumed that hours worked is assumed to be constant, ie the domestic and foreign economies were both endowment economies. But how would our results change if we allowed households to vary their hours worked in response to an exogenous shock?

A number of papers (see Jermann (1998)) have illustrated how closed economy models with endogenous production have difficulty matching asset pricing moments. In particular, these models imply a lower premia relative to the premia generated by an endowment economy. An endowment economy prevents households from adjusting their hours worked and hence forces them to change their consumption patterns in response to any adverse shock. This increases

[16] The combination of slow-moving habits and persistent shocks generate a very persistent series for consumption. In an open economy model with complete markets, the dynamics of the real exchange rate is closely tied to consumption. Hence, the high persistence in consumption translates into high persistence in real exchange rate. This reduces the volatility in the expected depreciation rate.

consumption volatility, the volatility of the stochastic discount factor and ultimately the size of the risk premia.

In an open economy, we show that the implications of endogenising labour supply for risk premia depends on the symmetry assumptions between the small open economy and the rest of the world. Table C illustrates this point.

Column 3 in Table C contains the simulation results for the case where domestic households can vary their hours worked ($\eta = 3/(1-h)$) but foreign households cannot ($\eta^* \to \infty$). All other parameters remain similar to our previous setup with very persistent shocks and slow-moving consumption habits.

Table C: The role of endogenous labour supply

Model variables	$Data$	$Endog.\ labour\ (H)$	$Endog.\ labour\ (H+F)$
$\sigma_{\Delta y}$	0.76%	1.81%	0.44%
$\sigma_{\Delta c}$	0.75%	0.45%	0.11%
$\sigma_{\Delta n}$	0.67%	1.99%	0.49%
$\sigma_{\Delta q}$	3.25%	13.56%	2.66%
σ_r	0.85%	1.11%	0.25%
Fama risk premia requirements			
$\sigma_{\Delta q^e}$	$< \sigma_{fxp}$	0.87%	0.14%
σ_{fxp}	$> \sigma_{\Delta q^e}$	1.26%	0.007%
$cov\left(fxp_t, \Delta q^e_{t+1}\right)$	< 0	-0.009%	$-6.5x10^{-6}\%$
UIP coef α_1	-0.1^*	-0.24	1.03

The setup with endogenous domestic labour supply reduces the Fama coefficient further. In fact, in this case the model produces a negative coefficient ($\alpha_1 = -0.24$) as is typically found in the literature. Clearly adding endogenous labour supply reduces the volatility of domestic consumption. Indeed, we find that our model now generates a smoother consumption series (0.45%) relative to our previous setup with no labour supply (0.71%, see Table B). This has two important implications. First, it reduces the the size of the *intertemporal substitution effect* relative to the *precautionary savings effect.* Second, this also reduces the correlation between domestic and foreign consumption since it is only domestic agents who can smooth their consumption by altering their hours worked. Consequently, the real exchange rate would have to adjust by more to ensure efficient international risk-sharing (see equation **(22)**).

The first effect improves the model's ability to resolve Fama's puzzle since it reduces the covariance between the FX premium and the expected depreciation rate (-0.009%). The second effect also helps reduce the Fama coefficient since the higher real exchange rate volatility translates into a more volatile FX premium (1.26%). Thus, even though introducing endogenous labour supply leads to a fall in domestic consumption volatility, it also reduces the Fama coefficient in our calibration.[17]

Nevertheless, as shown in column 4 in Table C, these results change when labour is assumed to be endogenous in both economies. This setup assumes that the inverse of the labour supply elasticities, η and η^*, are equal to $3/(1-h)$. This model generates very smooth consumption series in both economies (domestic consumption volatility is 0.11%), since households at home and abroad can insure themselves against risk by varying their hours worked. The real exchange rate therefore needs to move less to ensure efficient international risk-sharing (2.66%). Consequently the volatility of the FX premium is only 1 basis point and the UIP coefficient equals 1.

So, when endogenous labour supply is introduced symmetrically across countries, changes in agents' attitude towards risk have smaller implications for economic dynamics. These results accord more with the closed economy findings of Jermann (1998) and Rudebusch and Swanson (2008), who suggest that DSGE models have difficulty matching asset pricing facts such as the size and variability of equity and bond term premia.

5 Conclusion

In this paper, we examine the properties of the foreign exchange rate risk premium in a canonical general equilibrium small open economy model. We show that the combination of persistent shocks and slow-moving consumption habits in an endowment setting could improve the model's ability to address the Fama puzzle. These features ensure that not only risk aversion but also precautionary savings are countercyclical. Consequently they help generate a FX risk premium that covaries negatively with the expected depreciation rate. In addition, the same features also increase FX premium volatility.

[17] The reverse result is true for lower values of η. In this case, the model exhibits too much consumption smoothing causing the volatility of the FX premium to fall relative to the volatility of the expected depreciation rate.

We also show that endogenising labour supply in the small open economy improves the model's ability to address the Fama puzzle. In particular, allowing domestic households to vary their hours worked decreases the cross-country correlation of consumption and increases real exchange rate volatility. It also increases the relative size of the precautionary savings effect which is a pivotal mechanism in our model to address Fama's puzzle.

Nevertheless, as in the closed economy, modelling the labour/leisure decision in both countries impedes the model's ability to match risk premia observations. When both domestic and foreign households can vary their hours worked, consumption is both smooth and synchronised across countries. This dampens the FX premium volatility. So even though our model has time-varying risk aversion and persistent model dynamics, it behaves very much like a model solved under certainty equivalence.

To examine our findings further, an interesting extension could be to introduce capital accumulation to our model. In a closed economy setting, the introduction of capital has a similar effect to endogenising labour supply. It would be interesting to see whether the same result holds in an open economy.

Finally, our results highlight the role of precautionary savings. Without these, our model would imply the traditional linear uncovered interest rate parity condition. It has been shown that models with credit constraints embody significant precautionary savings effects (see Aiyagari (1994)). Exploring how credit constraints in an open economy affect the FX risk premium may therefore also constitute an interesting avenue for future research.

Appendix A: Deriving the FX premium

From the Euler equation **(25)**, we can write that

$$(\beta R_{t+1})^{-1} = E_t\left[M_{t+1}\right]. \qquad \textbf{(A-1)}$$

Thus, assuming a log-normal stochastic discount factor, M_{t+1}:

$$r_{t+1} = -\log\beta - E_t\left[m_{t+1}\right] - \frac{1}{2}var_t(m_{t+1}). \qquad \textbf{(A-2)}$$

In the case of foreign bonds, we have:

$$\left(\beta R^*_{t+1}\right)^{-1} = E_t\left[M_{t+1}\frac{Q_{t+1}}{Q_t}\right]. \qquad \textbf{(A-3)}$$

And assuming that the stochastic discount factor, M_{t+1}, and the real exchange rate, Q_{t+1}, are jointly log-normal we can write the above pricing equation as follows:

$$r^*_{t+1} = -\log(\beta) - E_t\left[m_{t+1}\right] - E_t\left[\Delta q_{t+1}\right] - \frac{1}{2}var_t(m_{t+1}) - \frac{1}{2}var_t(q_{t+1}) - cov_t(m_{t+1}, q_{t+1}). \qquad \textbf{(A-4)}$$

Therefore, combining equations **(A-2)** and **(A-4)**, we have

$$r_{t+1} - r^*_{t+1} - E_t\left[\Delta q_{t+1}\right] = \frac{1}{2}var_t(q_{t+1}) + cov_t(m_{t+1}, q_{t+1}) \qquad \textbf{(A-5)}$$

or, defining the foreign exchange rate risk premium as the excess return on domestic bonds (that is $fxp_t = r_{t+1} - r^*_{t+1} - E_t\left[\Delta q_{t+1}\right]$)

$$fxp_t = \frac{1}{2}var_t(q_{t+1}) + cov_t(m_{t+1}, q_{t+1}). \qquad \textbf{(A-6)}$$

Moreover, using log-linear relationship given by the risk-sharing condition (**(22)**), that is,

$$\Delta q_{t+1} = m^*_{t+1} - m_{t+1} \qquad \textbf{(A-7)}$$

we can rewrite equation **(A-6)** as

$$fxp_t = \frac{1}{2} cov_t(m^*_{t+1} - m_{t+1}, \Delta q_{t+1}) + cov_t(m_{t+1}, \Delta q_{t+1}) \qquad \textbf{(A-8)}$$

or simplifying,

$$fxp_t = \frac{1}{2}[cov_t(m^*_{t+1}, q_{t+1}) + cov_t(m_{t+1}, q_{t+1})]. \qquad \textbf{(A-9)}$$

Alternatively, we could combine equations **(A-6)** and **(A-7)** as follows:

$$fxp_t = \frac{1}{2} var_t(m^*_{t+1} - m_{t+1}) + cov_t(m_{t+1}, m^*_{t+1} - m_{t+1}) \qquad \textbf{(A-10)}$$

or

$$fxp_t = \frac{1}{2} var_t(m^*_{t+1}) + \frac{1}{2} var_t(m_{t+1}) - cov_t(m_{t+1}, m^*_{t+1}) - var_t(m_{t+1}) + cov_t(m_{t+1}, m^*_{t+1}) \qquad \textbf{(A-11)}$$

and, thus

$$fxp_t = \frac{1}{2} var_t(m^*_{t+1}) - \frac{1}{2} var_t(m_{t+1}). \qquad \textbf{(A-12)}$$

Appendix B: Cyclical properties of precautionary savings

In order to obtain the cyclical properties of precautionary savings we need to derive the determinants of $var_t(m_{t+1})$. So, given that

$$
\begin{aligned}
m_{t+1} = \log(M_{t+1}) &= \log\left(\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}}\right) \\
&= \log\left(\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{C_{t+1}^{-\rho}} \frac{C_t^{-\rho}}{(C_t - hX_t)^{-\rho}} \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}\right) \\
&= \log\left(S_{t+1}^{-\rho} S_t^{\rho} C_{t+1}^{-\rho} C_t^{\rho}\right) \\
&= -\rho\left(c_{t+1} - c_t + s_{t+1} - s_t\right)
\end{aligned}
$$

we can write

$$
\begin{aligned}
var_t(m_{t+1}) &= \mathbf{E}_t\left(m_{t+1} - \mathbf{E}_t m_{t+1}\right)^2 \\
&= \rho^2 \mathbf{E}_t\Big((c_{t+1} - \mathbf{E}_t c_{t+1}) + (s_{t+1} - \mathbf{E}_t s_{t+1})\Big)^2 \\
&= \rho^2\Big(var_t(c_{t+1}) + 2cov_t(c_{t+1}, s_{t+1}) + var_t(s_{+1})\Big).
\end{aligned}
$$

Moreover, in a closed endowment economy, which assumes an AR(1) process for consumption, ie

$$c_{t+1} = \rho c_t + \varepsilon_{t+1} \text{ and } \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

we know that

$$var_t(c_{t+1}) = var_t\left(\rho c_t + \varepsilon_{t+1}\right) = var_t(\varepsilon_{t+1}) = \sigma_\varepsilon^2.$$

In addition, up to a second-order approximation, we can write

$$s_{t+1} = \Psi_1\left(c_{t+1} - \frac{1}{2}(1-h)^{-1}c_{t+1}^2 - \widetilde{x}_t + c_{t+1}\widetilde{x}_t(1-h)^{-1} - \frac{1}{2}(1-h)^{-1}\widetilde{x}_t^2\right) - \log(1-h)$$

where $\log(X_{t+1})$ is denoted by $\widetilde{x}_t$ and we define $\Psi_1 := h/(1-h)$.

Therefore, we can compute that

$$
\begin{aligned}
var_t(s_{t+1}) = \Psi_1^2\Big(& var_t\left(c_{t+1}\right) + \frac{1}{4}(1-h)^{-2}var_t(c_{t+1}^2) + \widetilde{x}_t^2(1-h)^{-2}var_t\left(c_{t+1}\right) \\
&- (1-h)^{-1}cov_t(c_{t+1}, c_{t+1}^2) + 2\widetilde{x}_t(1-h)^{-1}var_t c_{t+1} - (1-h)^{-2}\widetilde{x}_t cov_t(c_{t+1}, c_{t+1}^2)\Big)
\end{aligned}
$$

and limiting attention to terms of order not higher than four this leaves

$$
\begin{aligned}
var_t(s_{t+1}) &= \Psi_1^2\Big(var_t\big(c_{t+1}\big) - (1-h)^{-1}cov_t(c_{t+1}, c_{t+1}^2) + 2\widetilde{x}_t(1-h)^{-1}var_t c_{t+1}\Big) \\
&= \Psi_1^2\Big(\sigma_\varepsilon^2 - (1-h)^{-1}\big(2\gamma\, c_t\sigma_\varepsilon^2 + cov_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big)\big) + 2\widetilde{x}_t(1-h)^{-1}\sigma_\varepsilon^2\Big).
\end{aligned}
$$

Finally, we can write

$$
\begin{aligned}
cov_t(c_{t+1}, s_{t+1}) &= \Psi_1\big(var c_{t+1} - \frac{1}{2}(1-h)^{-1}cov_t\big(c_{t+1}, c_{t+1}^2\big) + \widetilde{x}_t(1-h)^{-1}var_t c_{t+1}\big) \\
&= \Psi_1\big(\sigma_\varepsilon^2 - \frac{1}{2}(1-h)^{-1}\big(2\gamma\, c_t\sigma_\varepsilon^2 + cov_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big)\big) + \widetilde{x}_t(1-h)^{-1}\sigma_{\varepsilon}^2\big).
\end{aligned}
$$

Collecting the non-time varying terms, we find that the constant in the expansion of $var_t(m_{t+1})$ is

$$
\begin{aligned}
&\rho^2\Big(\sigma_\varepsilon^2 + 2\Psi_1\big(\sigma_\varepsilon^2 - \frac{1}{2}(1-h)^{-1}cov_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big)\big) + \Psi_1^2\Big(\sigma_\varepsilon^2 - (1-h)^{-1}cov_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big)\Big)\Big) \\
&\qquad = \rho^2\Big(\sigma_\varepsilon^2\big(1+\Psi_1\big)^2 - (1-h)^{-1}\big(\Psi_1 + \Psi_1^2\big)cov_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big)\Big)
\end{aligned}
$$

and so

$$
var_t(m_{t+1}) = \rho^2\frac{1}{(1-h)^3}\Big(\sigma_\varepsilon^2(1-h) - h\, cov_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big)\Big) + 2\sigma_\varepsilon^2\rho^2\frac{h}{(1-h)^3}\Big(\widetilde{x}_t - c_t\gamma\Big).
$$

It is easy to show that for an $\mathcal{N}(0, \sigma_\varepsilon^2)$ variable $cov_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big) = 0$ as

$$
\mathrm{cov}_t\big(\varepsilon_{t+1}, \varepsilon_{t+1}^2\big) = \mathbf{E}_t(\varepsilon_{t+1})(\varepsilon_{t+1}^2 - \mathbf{E}_t\varepsilon_{t+1}^2) = \mathbf{E}_t\varepsilon_{t+1}^3 = \mathbf{E}\varepsilon_{t+1}^3 = \mu^3 + 3\mu\sigma_\varepsilon^2 = 0.
$$

Recalling the fact that $\widetilde{x}_t = \log X_{t+1}$ and so, $\widetilde{x}_t = c_t(1-\phi) + \phi x_t$

$$
var_t(m_{t+1}) = \frac{\rho^2\sigma_\varepsilon^2}{(1-h)^2} + \frac{2\sigma_\varepsilon^2\rho^2 h\big(1-\phi-\gamma\big)}{(1-h)^3}\; c_t + \frac{2\sigma_\varepsilon^2\rho^2 h\phi}{(1-h)^3}\; x_t,
$$

or

$$
var_t(m_{t+1}) = \kappa_0 + \kappa_1[(1-\phi-\gamma)c_t + \phi x_t],
$$

where $\kappa_0 = \frac{\rho^2\sigma_\varepsilon^2}{(1-h)^2}$ and $\kappa_1 = \frac{2h\rho^2\sigma_\varepsilon^2}{(1-h)^3}$.

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