BANK OF ENGLAND

Working Paper No. 374

How do different models of foreign exchange settlement influence the risks and benefits of global liquidity management?

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August 2009

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Abstract

Large, international banking groups have sought to centralise their cross-currency liquidity management: liquidity shortages in one currency are financed using liquidity surpluses in another currency. The nature of risks to financial stability emerging from global liquidity management depends on how these foreign exchange transactions settle. I analyse these risks in a game of asymmetric information. The main result is that the transition from local to global liquidity management, and better co-ordination in settlement of foreign exchange transactions, have two effects. On the one hand, the likelihood rises that payments are delayed beyond their due date. On the other hand, solvency shocks are less likely to be passed on to other banks. The main assumption is that lending between subsidiaries of the same banking group takes place under symmetric information, while external interbank market loans are extended under asymmetric information. More co-ordinated settlement increases the exposure of the intragroup lender relative to the interbank lender and leads to more informed lending.

Key words: Liquidity risk, foreign exchange settlement.

JEL classification: G2, G32, F36, D82.

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The views expressed in this paper are those of the author, and not necessarily those of the Bank of England. I would like to thank Mark Manning, Erlend Nier, Julian Oliver, William Speller and Gabriel Sterne for repeated thorough reviews of earlier drafts. I would also like to thank James Chapman and participants in a workshop on Payment Systems and Liquidity Management at the Bank of England and participants in a joint ECB/Bank of England conference on Payments and Monetary and Financial Stability for helpful suggestions. All remaining errors are mine. This paper was finalised on 30 April 2009.

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ISSN 1749-9135 (on-line)

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Summary

In response to greater internationalisation, financial groups have adopted a wide range of approaches to liquidity risk management, a defining characteristic of which is the degree of centralisation. Under local liquidity management, each subsidiary of a financial group maintains a separate pool of liquidity in its local currency and funds its obligations domestically in each market. Under global liquidity management, financial groups also fund liquidity shortfalls (or recycle liquidity surpluses) via intragroup, cross-currency and/or cross-border transfers of liquidity or collateral: there is a global flow of liquidity within the group.

In practice, there are many barriers to managing liquidity globally. When banks are concerned about their counterparties' credit risk, one of these barriers can be the design of the settlement infrastructure for the cross-currency transfer of liquidity. A key design feature is whether the settlements of the two currencies involved in the foreign exchange (FX) transaction occur simultaneously, or at least closely co-ordinated in time. At the moment, facilities are available for simultaneous next-day settlement, but not for simultaneous same-day settlement. This paper shows that while there are benefits to increased co-ordination for same-day settlement of foreign exchange transactions, there may also be costs for financial stability.

In order to understand the argument, consider the case of a global bank, A, with two legally independent subsidiaries in the United Kingdom and the United States, referred to as A(UK) and A(US). A(UK), which may be subject to severe credit risk, is faced with requests to make an unusually large number of payments. Incoming payments are only expected for the following day. In response to these payment requests, A(UK) could either delay the payments, or attempt to raise sufficient funds on the interbank market (for example, via an overnight loan, or via an FX swap) to be able to execute them. Suppose A(UK) decided to take out an FX swap. Each foreign exchange transaction requires two settlements, one in the payment system of each currency. When A(UK) buys sterling against dollars for same-day settlement, it effectively borrows sterling from a UK counterparty, B, and promises that its US subsidiary, A(US), will pay dollars to B's US correspondent on the same day. If the settlement of the dollar payment occurs later than the settlement of the sterling payment, then B is exposed to the risk that A(UK) might default in-between the two settlements. Once the dollar transfer has taken place, A(US) is exposed to the risk that A(UK) might default. As a result, A(UK) may be left short of liquidity

for two reasons: if B is concerned about A(UK)'s credit risk, it may refuse to enter into the foreign exchange transaction with A(UK). Or, A(UK) may also be unable to raise funds because A(US) refuses to execute the dollar transfer on A(UK)'s behalf.

The likelihood that the foreign exchange transaction will take place in the presence of counterparty credit risk depends, therefore, on the information that A(US) and A(UK)'s UK counterparty have about A(UK)'s insolvency risk. The main assumption of this paper is that information flows freely between the two subsidiaries but not between different banks. Thus, A(UK)'s domestic counterparty charges an interest rate appropriate to the *expected* risk of A(UK), whereas A(US) charges an interest rate appropriate to A(UK)'s *actual* risk. In both cases, the interest rate is proportional to the time the lender carries this exposure. The better co-ordinated the settlement, the less time can expire between the settlement of the sterling and dollar payments, the longer A(US)'s exposure, and the shorter the exposure of A(UK)'s UK counterparty. If A(UK)'s actual risk is higher than its UK counterparty expects, A(UK)'s cost of an FX swap increases. Conversely, the cost of an FX swap falls when A(UK)'s risk is below average. As a result, with better co-ordination, only the less risky banks find funding, while riskier banks delay payments.

Delaying payments thereby becomes a signal for high solvency risk, and this signal becomes more precise when the co-ordination in FX settlement increases. In practice, a bank's failure to execute payment requests that are contractually due might therefore trigger further liquidity outflows. Other creditors of A(UK) might refuse to roll over funds and eventually drive A(UK) into insolvency. To keep the model tractable, I do not model these further consequences of A(UK)'s inability to make payments in detail but simply assume that A(UK) incurs a fixed cost if it delays payments beyond their due date.

The main result of the paper is that better co-ordination of FX settlements has two, potentially offsetting, effects on risk. On the one hand, it reduces the likelihood that solvency shocks are transmitted from one institution to another. If a bank was close to insolvency, it would not be able to refinance itself at all in response to liquidity outflows, neither domestically nor via FX transactions. Should such a bank eventually default, this default shock remains more contained because it had not entered (additional) loan agreements as part of an FX swap, or an overnight loan. But on the other hand, that bank would have to delay the payment of its obligations beyond their due date.

1 Introduction

In 2006, a report by the Joint Forum[1] found that financial groups have adopted a wide range of approaches to liquidity risk management in response to greater internationalisation, a defining characteristic of which is the degree of centralisation. Under local liquidity management, each subsidiary of a financial group maintains a separate pool of liquidity in its local currency and funds its obligations domestically in each market. Under global liquidity management, financial groups also fund liquidity shortfalls (or recycle liquidity surpluses) via intragroup, cross-currency and/or cross-border transfers of liquidity or collateral: there is a global flow of liquidity within the group.

The Joint Forum report found that most financial groups expect to rely upon intragroup, cross-border and cross-currency transfers in stress situations. There are, however, significant barriers to the free transferability between jurisdictions and affiliates within a banking group. Such barriers may be due to legal, operational, or time-zone restrictions. Moreover, diversification benefits in global liquidity management may be precluded when liquidity needs are correlated across different parts of a group because of system-wide liquidity shocks affecting more than one jurisdiction simultaneously, or by the presence of 'reputational contagion' between entities within a group.

This paper focuses on the operational dimension in a stressed situation in which counterparty credit risk is important even on short horizons. How does the type of settlement of FX transactions affect the nature of risks to financial stability that emerge from global liquidity management in the context of this model? A key design feature is the degree of co-ordination of the settlements of the two currencies which are exchanged in an FX transaction. This paper shows that while there are benefits to increased co-ordination for same-day settlement of foreign exchange transactions, there may also be costs for financial stability.[2]

The model investigates the reaction of a single multinational bank, A, to a variety of (domestic and global) liquidity shocks. The multinational bank has two subsidiaries, referred to as A(UK) and A(US), in the two countries in which it operates. When hit by a liquidity outflow, it can

[1] Joint Forum (2006).

[2] Since 2002, simultaneous (payment versus payment, PvP) next-day settlement is available via the payment service provided by CLS. PvP settlement is, however, not yet available on the same day on which transactions are concluded.

choose between borrowing the missing funds on the domestic market; entering an FX swap; or delaying the payment to the following day. Delay is less innocuous than it may seem at first sight: a bank that delays payments beyond their due date enters default, even when it expects to be able to make the payment on a future date. In practice, such a situation might trigger further liquidity outflows, and its creditors to refuse to roll over funds, eventually driving the bank into insolvency. The paper does not model these further consequences of illiquidity but, for the sake of tractability, simply assumes that the liquidity-short subsidiary incurs a fixed cost if it delays payments beyond the due date. Instead, we take particular care to model the choice between obtaining funds domestically, or via an FX swap.

Recall that when a bank exchanges foreign against domestic currency, this exchange involves two transactions: a payment of the domestic currency from a domestic bank to the domestic subsidiary, and a payment of the foreign currency from the foreign subsidiary to the domestic bank's foreign correspondent. If the payment in the domestic system settles before the payment in the foreign system, the domestic bank is exposed to the domestic subsidiary's failure. This exposure ends only when its foreign correspondent receives the payment. From then onwards, the foreign subsidiary is exposed to the domestic subsidiary's failure.

When modelling the credit relationships that arise during the settlement of FX transactions, we focus on a stress scenario in which counterparty risk (not differences in money market rates) determines the price of the FX transaction. Thus, the key determinant of the price is the degree of asymmetric information between liquidity-rich and liquidity-short banks.

We assume that only external but not internal (within-group) credit relationships suffer from asymmetric information between the borrower and the lender. When liquidity is managed locally, a liquidity-short subsidiary of a global bank – A(UK) – has to obtain funds domestically. In contrast, when liquidity is managed globally, A(UK) can additionally access foreign-denominated liquidity at a foreign subsidiary – A(US) – of the same group by exchanging it into the home currency. Because it is part of the same banking group, A(US) is considerably better informed about the domestic subsidiary's insolvency risk. It will only enter the FX transaction if it judges A(UK) to be reasonably safe.[3]

[3] Subsidiaries of a global banking group are independent legal entities and can, in principle, fail independently. I assume here that one subsidiary only suffers from another subsidiary's failure if it was exposed because of outstanding intragroup loans; in particular, I abstract from 'reputational contagion' between the subsidiaries.

Consequently, A(US)'s willingness to support A(UK) with an FX swap provides A(UK)'s domestic counterparties with a signal of A(UK)'s creditworthiness. In addition, the duration of A(UK)'s domestic counterparty's exposure to A(UK) is shorter in an FX transaction than in a (domestic) overnight loan. Both factors mean that A(UK)'s counterparties are willing to enter an FX transaction with A(UK), but unwilling to grant A(UK) an overnight loan. Conversely, if A manages its liquidity globally, and A(US) decides against granting liquidity support to A(UK), A(UK) will find that its domestic funding market is closed as well. This is the first key result of the paper. In contrast, under local liquidity management, A(US) is unable to support A(UK), so A(UK)'s domestic counterparties would not interpret A(US)'s failure to support A(UK) as a judgement on A(UK)'s credit risk. In this case, A(UK) might still be able to raise funds domestically via an overnight loan.

The second result links the type of FX settlement to risks to financial stability. In the model, an FX transaction involves a sequence of an uninformed credit relationship (between A(UK) and its domestic bank counterparty) and an informed credit relationship (within the global group). How long each stage lasts depends on the degree of co-ordination in FX settlement. In an extreme case, it is simultaneous, in which case A(UK)'s domestic counterparty bears no credit risk. But the longer the first, uninformed stage lasts, the larger the share of external, uninformed finance in the transaction.

Under the (admittedly stark) assumption that there are no other impediments to global liquidity management than the design of the FX settlement infrastructure, the transition from local to global liquidity management, and a better co-ordination of settlement of FX transactions, would both lead to more informed lending relationships. Improved information would have two consequences for financial stability. First, the *transmission* of solvency shocks from one institution to another would be less likely because banks with high solvency risks would not be able to refinance themselves at all in response to liquidity outflows, neither domestically nor via FX transactions. But this implies that these banks would have to delay the payment of their obligations beyond their due date. Hence the second result: liquidity-short institutions may find it more difficult to obtain funds, and the likelihood rises that payments are delayed beyond their due date. These results continue to hold when we endogenise banks' *ex-ante* liquidity holdings.

The following section reviews related literature. Sections 3-5 present the model's set-up and guide the reader through the derivation of the main results. Section 6 shows how these results extend to a comparison of local with global liquidity management. A discussion of the model's main assumptions can be found in Section 7. Proofs are presented in the appendix.

2 Related literature

Net redemptions from the banking system are an exception in economies with well-developed financial markets: if liquidity leaves one bank, it generally flows via a payment system directly into the accounts of another bank. If there was no market failure in the domestic interbank market, no bank would ever experience a liquidity crisis, because it could always re-borrow the liquidity it lost. Global liquidity management would not have any advantages.

The key assumption in this paper is that there is a market failure in the domestic interbank market that prevents liquidity from being lent out by a liquidity-rich bank to a liquidity-short bank. This market failure is due to asymmetric information and was described as a screening problem by Stiglitz and Weiss (1981). Stiglitz and Weiss argued that borrower/lender relationships are characterised by asymmetric information. Credit rationing occurs in equilibrium because of two effects: first, a bank attracts only riskier borrowers when it increases its interest rate (adverse selection). Second, the borrower might be inclined to increase the risk of his project if the bank cannot perfectly monitor his choice (moral hazard). Gorton and Huang (2004), Mallick (2004) and Skeie (2004) suggest other reasons for market failures; but these are more likely to hold for longer-term exposures. In our model, exposures do not last more than 24 hours; hence our choice to make adverse selection and not moral hazard the cornerstone of the market imperfection in our model. Screening problems appear to be very important in reality: banks generally refuse to grant intraday credit to counterparties which do not have excellent credit status (instead of simply charging them a higher interest rate); and anecdotal evidence suggests that even high-quality credit institutions do not rely on uncollateralised borrowing in their contingency plans.

There are several other strands of models that dealt with related questions. Manning and Willison (2006) analyse the benefits of the cross-border use of collateral in payment systems. In contrast, we focus on a (more extreme) situation in which banks have no collateralisable assets: thus, banks have to resort to unsecured borrowing and/or an exchange of foreign currency to transfer their foreign liquidity holdings into domestic currency. (See Section 7 for a discussion.) The literature on (the limits of) internal capital markets discusses the degree to which internal borrowing takes place under symmetric information and identifies a variety of factors that inhibit the information flow within a multinational company (eg, Stein (1997), Scharfstein and Stein (2000)). We abstract from these frictions to keep the analysis tractable. Kahn and Roberds (2001) describe settlement banks' incentives in CLS Bank, which started to offer payment versus payment (PvP) settlement for some foreign exchange transactions (but not those requiring same-day settlement) in 2002. They argue that PvP increases the certainty that the counterparty will settle its part of the FX transaction. This improves banks' incentives to have good liquidity management procedures, but may also reduce their incentive to monitor counterparties. The authors do not discuss the relative merits of global and local liquidity management, nor do they show the trade-off between the likelihoods of transmission of shocks and of delay. Freixas and Holthausen (2004) study the cross-country integration of interbank markets. They argue that cross-country interbank market integration may not be perfect when banks have less knowledge about the solvency of foreign banks. We show to what extent this friction can be overcome by a global bank which has subsidiaries in both countries.

Fujiki (2006) shows that PvP settlement, together with free daylight overdrafts from the central bank, is one possibility to yield efficiency gains in an extension of Freeman's (1996) island model. Finally, there is a distinct strand of models investigating banks' reserve management: Tapking (2006) and Ewerhart *et al* (2004) show that interbank interest rates do not necessarily have to increase when liquidity becomes scarce. Ho and Saunders (1985) derive optimal interbank lending to meet reserve requirements.

3 Set-up

This section describes the main assumptions (in particular regarding the distribution of information) and the timing of the game. Figure 1 contains a stylised game tree with the global bank's most important decisions. Variables are also listed on page 55 for ease of reference. There are three days and three players: one global bank with two subsidiaries, G_E and G_W, and two local banks D_E and D_W, one in each country (East or West).[4] Banks are owned by their depositors (equity is zero) and maximise undiscounted end of day two pay-offs.

Day zero. The global bank invests its deposits into a risky, illiquid and a risk-free liquid asset.

[4]Under certain restrictions, a domestic bank could be interpreted as a continuum of small banks. The distribution of profits between lender an borrower is exogenous in our model and would not materially affect the results (see Section 7); hence, it should be robust to the assumptions regarding the market structure. But for the equilibrium selection argument to go through, we would have to assume that the amount the global bank borrows from the domestic banking sector is public information.

The risk-free asset pays off one for each unit invested, ie, it has a return of zero. With probability p_i, one unit of subsidiary i's risky asset has a pay-off of $(1+\rho)/p_i$. With probability $(1-p_i)$, it becomes worthless. Thus, one unit of the risky asset is expected to pay off $(1+\rho)$; after investment outlays, the expected profit is simply ρ per unit invested. (Notice that the pay-off assumptions imply that ρ does not depend on p_i: this simplifies the following analysis.) We assume that over the time period considered in this model – 24 hours – the only entity subject to solvency risk is one of the global bank's subsidiaries. This appears to be a reasonable simplification – usually, overnight credit risk is negligible – and also enables us to focus on the credit risk of only one counterparty, instead of having to trace default risk for up to four counterparties in an FX swap. The identity of the subsidiary that is subject to solvency risk is known (for example, rumours about its imminent default might already circulate); but the precise likelihood of its insolvency is only known within the global banking group. Formally, there are two randomisations which together determine the value of p_E and p_W. The first determines whether $p_E = 1$ or $p_W = 1$; both events have probability $1/2$ and are publicly observed. The second shock determines the value of p_W if $p_E = 1$, drawing p_W from a uniform distribution over [0, 1], and the value of p_E if $p_W = 1$, drawing p_E from a uniform distribution over [0, 1]. The realisation of this second shock is known within the global banking group, but not to the domestic banks. (See Section 7 for a discussion.)

Table 1: Types of shocks

		D_E	G_E	G_W	D_W
s_1	Countrywide liquidity shortage in E	$-\lambda$	$-\lambda$	λ	λ
s_2	Countrywide liquidity shortage in W	λ	λ	$-\lambda$	$-\lambda$
s_3	Eastern subsidiary experiences outflow, Western subsidiary an inflow	λ	$-\lambda$	λ	$-\lambda$
s_4	Eastern subsidiary experiences inflow, Western subsidiary an outflow	$-\lambda$	λ	$-\lambda$	λ
s_5	Global bank suffers liquidity outflow	λ	$-\lambda$	$-\lambda$	λ
s_6	Global bank experiences liquidity inflow	$-\lambda$	λ	λ	$-\lambda$

After having learnt the risk of its illiquid assets, the global bank decides how to allocate the deposit base of size 1 in each country into a risk-free liquid asset (shares L_E and L_W) and the risky illiquid asset (shares $1-L_E$ and $1-L_W$ of its deposit base). The exchange rate between currencies is 1:1.

Day one. Banks are hit by liquidity shocks (that is, requests to make payments) of size λ. These liquidity shocks are publicly observed, independent of p_i. Their structure is given in Table 1. Each of these shocks occurs with equal probability, and only one of them hits at a time. (Shocks s_3 and s_5 will be of most interest to us.) The assumption that these shocks are publicly observed is strong; in reality, the global bank's counterparties will only suspect that the global bank's liquidity buffers are running low. We nevertheless adopt this assumption because it provides a useful benchmark, and because it makes the inferences that counterparties draw from the global bank's behaviour more tractable. For simplicity, we have assumed that there is no global liquidity shock: globally, outflows equal inflows. Clearly, under this assumption the benefits of global liquidity management are maximal.[5]

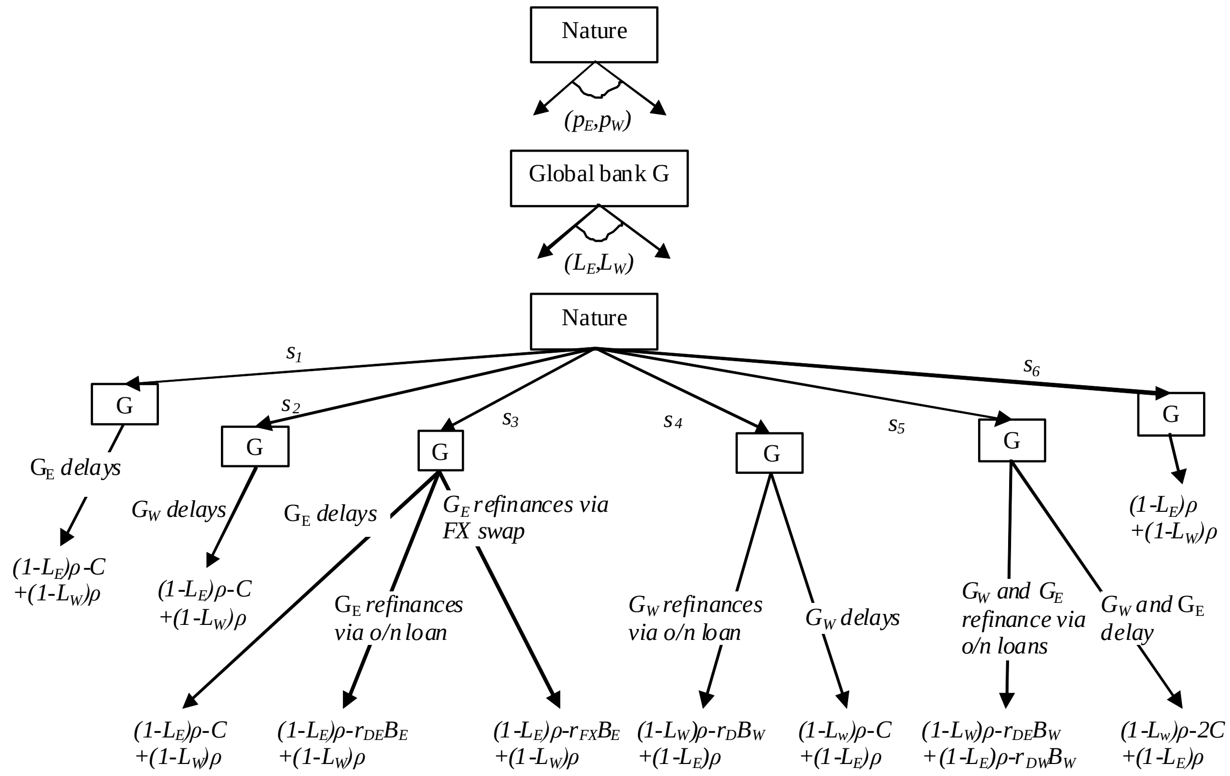
How the liquidity shock affects the global bank depends, of course, on its initial investment in liquid assets. It may have sufficient liquidity to absorb the shock. If, in contrast, it suffers from a shortfall, it may be able to raise B_E East-\$ (or B_W West-\$, respectively). For its Eastern subsidiary, G_E, it can choose between refinancing via a domestic interbank loan, transferring liquidity between subsidiaries via an FX swap (both with D_E as counterparty), and delaying payment until day two.[6] We assume that the per-dollar cost of the domestic interbank loan, r_D, and the per-dollar cost of the FX swap, $r_{FX} = s_D + r_G$, are such that all parties with an exposure to G_E expect to break even. (Notice that there are two components to the charge of an FX swap: s_D is paid to the interbank lender of domestic liquidity as a compensation for counterparty risk, while r_G is paid to the intragroup lender. The details are explained below.) Payment delay entails a fixed cost of C, independently of the amount of the payments that did not settle on day one: a proxy for the associated costs of having failed to fulfil a contract (payments are delayed beyond their due date), the risk of being declared in default (which may trigger additional payment requests, even if this 'technical' default might only last for a day), and the associated reputational costs. The model abstracts from any contagious effect delay may have on other banks.

For its Western subsidiary, G_W, the options are more restricted. This is because of date conventions used in the settlement of foreign exchange transactions explained in more detail in Section 5. The result is that G_W only has two options when it finds itself short of liquidity: payment delay and a domestic interbank loan with D_W as counterparty.

[5] But notice from Lemma 1 (below) that this assumption is not necessary to justify that G_E's and G_W's optimisation problems can be dealt with separately.

[6] The latter is also referred to as 'technical default'.

Figure 1: The global bank's investment and refinancing decisions and its expected pay-offs



Note: 'Nature' (= fate) determines the realisation of probabilities p_E and p_W and of the liquidity and real shocks.

Day two. During the subsequent 24 hours, each subsidiary's illiquid asset may be hit by the aforementioned real shock. If the shock hits a subsidiary, it is forced to default on its obligations, such that its creditor (if there is one) loses the entire principal amount of its loan.[7] This is referred to as 'transmission of losses' from the debtor to the creditor. Subsidiaries, in contrast to branches, are separately liable for their loans; creditors of one subsidiary cannot take recourse to the other subsidiary.

If the real shock does not hit G_i's illiquid asset, the illiquid investment pays off sufficient liquid assets such that obligations incurred on day one can be fulfilled. The assumption that the asset's pay-off is 'sufficient' is made so as to abstract from repercussions that refinancing decisions on day one may have for the availability of liquidity on day two.[8] Outstanding interbank loans are paid back and FX swaps reversed. Each bank then distributes its assets to its (local) depositors.[9]

[7]Notice that this implies that the illiquid asset cannot be used to collateralise the liquidity-short bank's loan.

[8]This is clearly a strong simplification but we would have faced this problem in all games with a finite number of periods.

[9]The need to enter an FX swap rather than just a simple FX transaction is motivated by the latter assumption.

The following section derives the subsidiaries' optimal refinancing decisions in the case of local liquidity management. They are also informative for global liquidity management: as we will argue in Section 5, the decisions of the Western subsidiary are identical in both cases.

4 Local liquidity management

When liquidity is managed locally, each subsidiary of the global bank seeks recourse to the domestic interbank market if it experiences a liquidity outflow and does not want to delay. In market i, it borrows B_i at rate r_{Di}. Its expected pay-off, π_G, is the sum of the profits of its subsidiaries, π_{Gi}, where

$$\pi_{Gi}(L_i, p_i) = (1 - L_i)\rho - \frac{1}{6}p_i C - 2 \cdot \frac{1}{6}p_i \min\{C, r_{Di} B_i\}$$

$(1 - L_i)\rho$ is the expected pay-off from the illiquid asset, given that a share $(1 - L_i)$ of G_i's balance sheet is invested in it. Each liquidity shock has a probability of $1/6$. If shock s_1 (s_2) occurs, G_E (G_W) has no choice but to delay and incurs a cost C.[10] (The reputational cost C only arises if G_i survives until day two; hence the multiplication by p_i.) If s_3 (s_4) occurs, G_E (G_W) chooses between delay (at cost C) and domestic refinancing (at cost $r_{Di} B_i$; again, the subsidiary only has to pay it if it does not go bankrupt). If s_5 occurs, both subsidiaries face the same decision. After the other shocks, the subsidiary has excess liquidity (which, by assumption, it lends out at zero expected profit).

We can solve each subsidiary's problem independently because liquidity management is local. Consider G_E. As borrowing is costly, G_E never borrows more than necessary to cover its liquidity shortfall: $B_E \leq \lambda - L_E$ in all equilibria. Because the cost of delay is independent of the amount outstanding at the end of day one, $B_E \in \{0, \lambda - L_E\}$ in equilibrium: G_E either borrows sufficiently to avoid delay, or nothing at all. If $B_E = \lambda - L_E$, its expected profit is

$$\pi_{GE}(L_E, p_E) = (1 - L_E)\rho - \frac{1}{6}p_E \begin{cases} 0 & \text{if} \quad L_E \geq \lambda \\ C + 2r_{DE}(\lambda - L_E) & \text{if} \quad \lambda - C/r_{DE} \leq L_E < \lambda \\ 3C & \text{if} \quad L_E < \lambda - C/r_{DE} \end{cases}$$

This is convex in L_E; hence $L_E \in \{0, \lambda\}$ in equilibrium. We refer to the choice $L_E = \lambda$ as 'hoarding liquidity'. Whether hoarding liquidity is profitable depends, among other factors, on the interest rate r_{DE} which the domestic lender D_E charges. By assumption, liquidity is provided

[10]The situation would be different if collateral could be transferred without frictions from one country to the other. Here, however, we focus on cross-country liquidity transfers via the FX market.

at an interest rate at which D_E expects to break even. Break-even is achieved if

$$r_{DE} = \frac{1 - E\left[p_E | p_E \in P_{DE}\right]}{E\left[p_E | p_E \in P_{DE}\right]} \tag{1}$$

where P_{DE} is the set of risk types that opt for the interbank loan in equilibrium.

There are three types of equilibria, one of which is not stable (in a sense made precise in the proof of Lemma 1) and has thus limited predictive value. I focus on the other two equilibria. In one, no lending takes place: either because both the opportunity costs of holding liquidity and the cost of delay are low, or because the interbank market breaks down because the lender is very suspicious: if, off equilibrium, he was approached for an interbank loan, he would believe that he faces a very risky borrower. In this case, $P_{DE} = \varnothing$. In the other equilibrium, the liquidity-short subsidiary is able to obtain funds on the interbank market independently of its risk. Here, $P_{DE} = [0, 1]$. Lemma 1 provides details.

Lemma 1 In all stable equilibria, G_E's equilibrium strategy fulfils one of the following:

D1: $P_{DE} = \varnothing$. Then

$$L_E = \begin{cases} 0 & \text{if} \quad p_E < 2\lambda\rho/C \\ \lambda & \text{if} \quad p_E \geq 2\lambda\rho/C \end{cases}$$

and high-risk banks delay if hit by a liquidity outflow. This equilibrium exists for all λ, C, and ρ.

D2: $P_{DE} = [0, 1]$. Then $L_E = 0$ independently of the borrowing subsidiary's risk, and the subsidiary takes out an overnight loan if hit by a liquidity outflow. This equilibrium exists if, and only if, $\lambda \leq C$ and $\rho \geq \frac{1}{6}(2 + C/\lambda)$.

The following lemma collects the results regarding the *ex-ante* likelihood of transmission of losses and delay under local liquidity management. Transmission of losses (from G_E to its domestic creditor D_E) occurs in circumstances in which G_E's illiquid assets fail (probability $(1 - p_E)$) given that G_E opted for refinancing after having been hit by a liquidity outflow (probability $2/6$). Delay occurs in circumstances in which G_E did not refinance after having been hit by a liquidity outflow: after s_1 hits; or if $p_E < 2\lambda\rho/C$ and s_3 or s_5 hit. (Recall that $p_E = 1$ with probability $1/2$.)

Lemma 2 Likelihood of delay and transmission of losses under local liquidity management.

1. In equilibrium D1, the likelihood of transmission of losses is zero. The likelihood of delay is

$$\begin{aligned}\frac{1}{6}+\left(2\cdot\frac{1}{6}\right)\Pr\left(p_E<2\lambda\rho/C|p\neq 1\right)\Pr\left(p\neq 1\right) &= \frac{1}{6}+\frac{1}{3}\left(2\lambda\rho/C\right)\frac{1}{2}\\ &= \frac{1}{6}+\frac{1}{3}\lambda\rho/C\end{aligned}$$

2. In equilibrium D2, the likelihood of delay is $1/6$. The likelihood of transmission of losses is

$$\begin{aligned}& \frac{1}{2}\cdot\frac{2}{6}E\left[1-p_E|p_E\in[0,1]\right]\Pr\left(p_E\in[0,1]\,|p\neq 1\right)\Pr\left(p\neq 1\right)\\ = \;& \frac{1}{3}\left(\int_0^1\left(1-p_E\right)dp_E\right)\frac{1}{2}=\frac{1}{12}\end{aligned}$$

Unsurprisingly, the likelihood of delay is increasing in ρ (because this increases the opportunity costs of holding liquidity *ex ante*) and λ (because this makes refinancing more costly), and decreasing in C (because for lower C, delay is associated with a lower penalty).

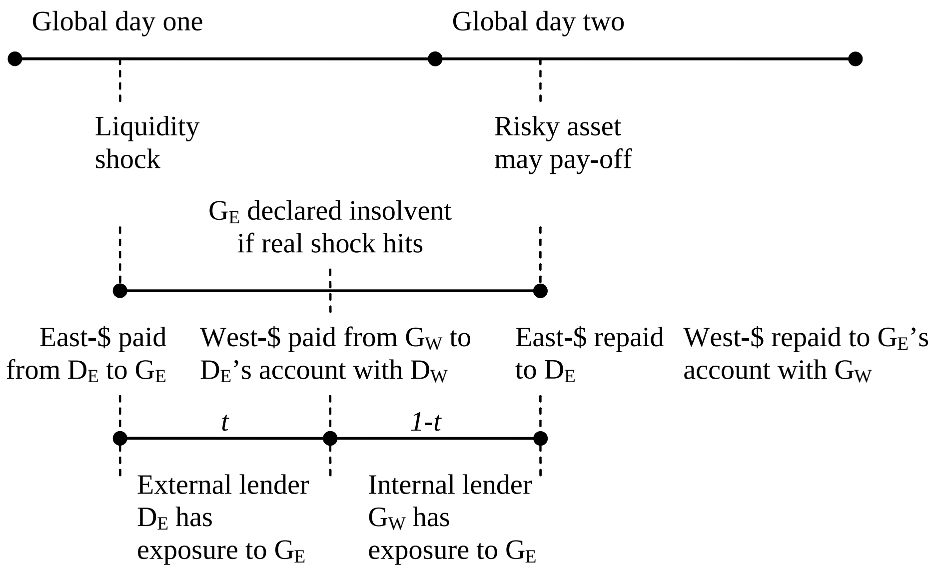
5 Globally centralised liquidity management

We assume that each subsidiary holds liquidity in its own currency but not in other currencies. A single unit within the global bank decides on how much liquidity each subsidiary holds. We impose that intragroup loans of liquidity are charged at an interest rate at which the lender just expects to break even; as in the case of local liquidity management, a more explicit modelling of the bargaining between lender and borrower is left for future research. We abstract from regulatory barriers to the transfer of liquidity across jurisdictions and focus instead on the influence the model of settlement of foreign exchange transactions has on the availability of funding for liquidity-short banks, and thereby on the likelihoods of delay and transmission of losses. Importantly, we assume that each subsidiary can default independently, and that there are no obligations by the surviving part of the global bank to support an insolvent subsidiary. See Section 7 for a discussion.

Intragroup liquidity transfers via an FX swap are only possible if two conditions are met: first, the local market in which the liquidity-short subsidiary is based must not be short of liquidity; and second, the banking group as a whole must be liquid.

To understand the first condition, consider the mechanics of an FX swap. To fix ideas, suppose that shock s_3 occurred (Figure 2).

Figure 2: Timeline for the settlement of an FX swap



G_E experienced a liquidity outflow, and G_W a corresponding liquidity inflow. Because G_W operates in a different currency area, only D_E can provide G_E with East-$. In an FX transaction, G_E borrows liquidity from D_E, promising that G_W will pay D_E's correspondent bank in the West (D_W) the corresponding amount in West-$ (in Figure 2 after a fraction t of the global day expired).[11] By paying the West-$ to D_W, G_W grants G_E an intragroup loan over this amount. On the following day, the transactions are reversed.

The second condition – that the entire banking group must be liquid – should be intuitive. One might argue that G_W would be able to raise funds on behalf of G_E. However, this is not possible because by assumption, G_W's equity is too small to absorb a write-off of the intragroup loan to G_E. Thus, G_W's domestic lenders would be exposed to G_E's default, just as G_E's domestic lenders. If the latter refuse to provide credit, the former will as well.

In addition to these constraints, an important impediment to foreign exchange trades arises from

[11] If G_E is declared insolvent before G_W paid D_W, D_E may lose the full face value of the swap. This form of principal risk is also referred to as FX settlement risk, or Herstatt risk. The duration of this exposure can be lengthy: for example, if a domestic European bank pays euros at 9 am in the European day, it may only receive the dollar via its correspondent at 5 pm in the US day, some 14 hours later.

calendar-day conventions. When two banks trade dollars for euros, both the sale and the purchase (the transfer of dollars and the transfer of euros) have to settle on the same calendar day. Otherwise, the bank that receives its payment only on the next calendar day effectively grants the other bank an overnight loan which is repaid in another currency. This implies that if a global bank finds itself short of West-$, and the payment system in the East has already closed, it will not be able to sell a surplus of East-$ against West-$ without incurring charges for an overnight loan from D_W. We capture this asymmetry by assuming that only the Eastern subsidiary G_E, but not the Western subsidiary G_W, has the option to raise liquidity via an FX transaction.

In summary, the liquidity-short subsidiary can only raise funds via an FX transaction after shock s_3. We can now prove our first lemma:

Lemma 3 G_W's optimal liquidity holdings are the same as under domestic liquidity management.

The proof is straightforward. Because of the above-mentioned calendar-day conventions, G_E cannot support a liquidity-short G_W. But could G_W still insure G_E? It would have an incentive to do so if G_E compensated it for the opportunity cost incurred. However, no such compensation exists. The reason is that the investment opportunities are the same in both countries, and technologies linear. Thus, both subsidiaries' opportunity costs for holding liquidity are the same. If these costs are too high to encourage G_E to invest in liquidity in the East, any compensation that G_W would require to invest in liquidity to protect G_E from liquidity shortages would also be too high for G_E.

The remainder of this section deals with G_E's optimal liquidity holdings. We analyse G_E's choice between delaying, raising liquidity via a domestic interbank loan from D_E, and raising liquidity via an FX swap when G_E is hit by a liquidity shortage on day one and its counterparties are unsure about its solvency risk. We solve the model backwards. Section 5.1 derives the lenders' and the borrower's participation constraints; Section 5.2 provides the results for the global bank's optimal refinancing choice, and 5.3 derives the link between the degree of co-ordination in settlement and the likelihood of delay and transmission of losses.

5.1 Participation constraints: determination of the cost of refinancing

This section derives the lenders' and the borrower's participation constraints. Assume that G_E, the borrower, might declare bankruptcy within the next 24 hours, while G_W is considered to be safe.[12] The precise probability of G_E's bankruptcy is the global banking group's private information. Lemma 4 describes the decision of G_W, the internal lender:

Lemma 4 G_W charges the borrower a per-dollar fee of

$$r_G = (1-t)\,\frac{1-p_E}{p_E}$$

for the intragroup loan.

Notice that this per-dollar fee is the full-information interest rate appropriate to the borrower's risk, $(1-p_E)/p_E$, times a factor for the duration $1-t$ of G_W's exposure to G_E. Of course, the shorter the duration of the exposure, and the lower G_E's risk of default, the lower the fee. The proof is in Section 4.

The Eastern domestic bank, D_E, now offers two products: an interbank loan at interest rate r_D, and an FX swap at a fee s_D. For the interbank loan, the participation constraint is, as before,

$$r_D \geq \frac{1-E\left[p|p \in P_I\right]}{E\left[p|p \in P_I\right]}$$

where all $p_E \in P_I$ opt for the interbank loan. Lemma 5 provides the corresponding per-dollar fee for the FX swap. Recall that we are considering a crisis scenario, in which counterparty risk is the main determinant of the price of the FX swap. The differential of (official or interbank) overnight interest rates is of secondary importance in such a situation; indeed, we neglected it completely in our model.

Lemma 5 As compensation for Herstatt risk, D_E charges the borrower a fee of

$$s_{DE} = \frac{t\left(1-E\left[p|p \in P_{FX}\right]\right)}{1-t\left(1-E\left[p|p \in P_{FX}\right]\right)}$$

for each unit borrowed, where all $p_E \in P_{FX}$ opt for the FX swap.

This fee is, of course, decreasing in the time t that D_E has exposure to G_E. It is also decreasing in D_E's expectation of G_E's risk $E\left[p|p \in P_{FX}\right]$. The proof is in the appendix.

[12] Compare Section 3: $p_W = 1$, $p_E \in [0,1]$.

The total per-dollar cost of the FX swap is the sum of both lenders' fees:

$$r_{FX}\left(p_E, t, P_{FX}\right) = (1-t)\,\frac{1-p_E}{p_E} + t\left(\frac{1-E\left[p|p\in P_{FX}\right]}{1-t\left(1-E\left[p|p\in P_{FX}\right]\right)}\right) \qquad \textbf{(2)}$$

If t is small, we speak of (relatively) co-ordinated settlement; if t is large, of (relatively) uncoordinated settlement of the two currency transactions in the first (spot) leg of the FX swap. The case $t = 0$ corresponds to PvP settlement. For $t \to 1$, the FX swap approaches a domestic overnight loan taken out by types P_{FX}. Lemma 6 in the appendix shows that r_{FX} is strictly declining in p_E: a part of the FX transaction is financed by an informed lender, who charges risky borrowers a higher fee.

5.2 The liquidity-short subsidiary's refinancing decision

This section starts out with a presentation of the formal results. A reader primarily interested in the intuition is invited to jump directly to Section 5.2.2.

5.2.1 Equilibrium refinancing decisions – formal results

Proposition 1 characterises the types of pure-strategy equilibria that can occur under global liquidity management.[13] We focus our attention on equilibria in which the global bank makes use of FX swaps where possible, and, among these equilibria, on those which are stable in the sense that they meet Cho and Kreps' Intuitive Criterion. In all these equilibria, the most risky banks delay payment when they find themselves short of liquidity. This is because any FX swap involves at least some portion of informed lending ($t < 1$), making refinancing costly, and because the expected costs of delay, $p_E C$, of the reputational penalty, is very small for very risky banks. In addition, in all these equilibria the least risky banks rely on refinancing in the case of a liquidity shortage. Because of their low risk, refinancing is cheap for them.

Proposition 1 shows that if some refinancing is done via an FX swap ($P_{FX} \neq \varnothing$), there is no recourse to overnight loans ($P_I = \varnothing$).[14] Put differently, the model predicts that once the tools for global liquidity management are in place, a liquidity manager will, whenever possible, refinance himself using internal funds rather than borrow externally. If the expected return, ρ, on the illiquid, risky asset is sufficiently high, the liquidity manager will opt to hold no liquid assets *ex*

[13] There may be additional mixed equilibria for specific parameter constellations, eg, for $t = t'$; these are ignored here.

[14] One can show that this result holds in all Bayesian Nash equilibria of this game.

ante (equilibria of type G1 and G2 in Proposition 1 below). Here, relatively safe borrowers refinance themselves, whereas risky borrowers delay payment when faced with a liquidity shock. If, in contrast, ρ is lower, some types of borrowers – those with an intermediate level of risk – opt for hoarding liquidity to avoid borrowing altogether. (Equilibria of type G3 and G4; the least risky borrowers continue to rely on their ability to obtain funds in case of a liquidity shortage. The most risky borrowers choose to delay payments: the expected costs of delay, $p_E C$, is small for them.)

To understand the difference between equilibria of types G1 and G2, recall that under domestic refinancing, we found two (stable) equilibria: D1 and D2. In the first, the domestic interbank market is closed – the domestic bank fears that if approached for a loan, it would face a very risky borrower. Hence it asks for a very high interest rate, discouraging the less risky borrower types. In equilibrium, there is no lending. In contrast, in D2, the domestic bank lends freely at a comparatively low rate, and attracts a mix of borrower types, so that it expects to break even on average.

The same reasoning continues to hold under global liquidity management. Even when liquidity is managed globally, all funds must be raised from domestic sources after shocks s_1, s_2, s_4 and s_5. This domestic market might again be closed (leading to equilibrium G1), or open (leading to equilibrium G2). The same distinction applies to G3 (closed domestic interbank market) and G4 (open domestic interbank market).

Proposition 1 In (Bayesian Nash) equilibria in which some types of G_E obtain funds via an FX swap ($P_{FX} \neq \varnothing$), and which pass Cho and Kreps' Intuitive Criterion, G_E's equilibrium strategy is given in Figure 3.

When the existence conditions are not met, the equilibrium is either not stable, or does not involve the use of FX swaps. (In particular, if the opportunity cost for liquidity, ρ, is low, G_E prefers to hoard liquidity rather than refinance via an FX swap.) Domestic banks provide funds at zero expected profit. A complete description of all equilibria which pass the Intuitive Criterion, and in which $P_{FX} \neq \varnothing$, is given by this behaviour of domestic banks, and all combinations of D1 and D2[15] as descriptions of G_W's equilibrium decisions with G1-G4.

[15]Replacing G_E by G_W in Lemma 1.

The proof of Proposition 1 is in the appendix. But the following two subsections provide some intuition. The first illustrates why G_E will find that the domestic refinancing market is closed if G_W is able to, but refuses to lend funds to G_E (ie, after shock s_3 occurred). The second shows how the equilibrium of type G3 is derived.

Figure 3: G_E's equilibrium strategy under global liquidity management

Existence for...	**No.**	**Hoard liquidity if p_E is in...**	**After s_5, refinance via domestic loan if p_E is in...**
Any t, and high ρ	G1	$\varnothing$	$\varnothing$
	G2	$\varnothing$	$[0, 1]$
Low t, and intermediate ρ	G3	$\left[2\lambda\rho/C, p'''\right]$	$\varnothing$
	G4	$\left[p_1''', p_2'''\right]$	$\left[0, p_1'''\right] \cup \left[p_2''', 1\right]$

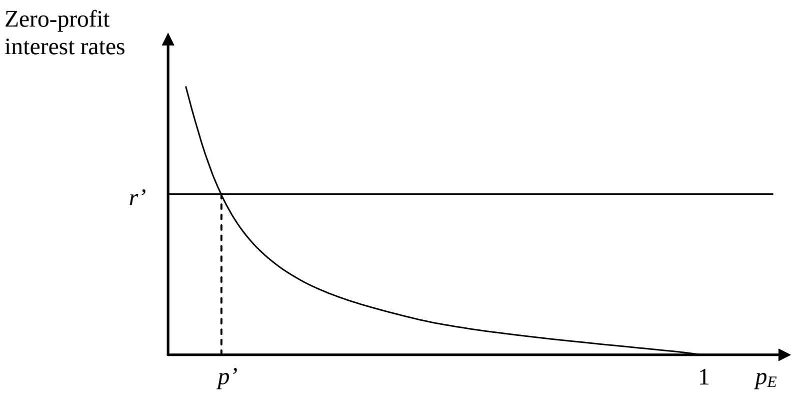
Existence for...	**No.**	**After s_3, refinance via FX swap if p_E is in...**
Any t, and high ρ	G1	$\left[p'', 1\right]$
	G2	$\left[p'', 1\right]$
Low t, and intermediate ρ	G3	$\left[p''', 1\right]$
	G4	$\left[p_2''', 1\right]$

Note: p'', p''', p_1''' and p_2''' are the success probabilities p_E of G_E's illiquid investment at which G_E's behaviour changes. (They are defined in the appendix.) If G_E neither hoards liquidity nor refinances, it delays. $L_E = 0$ if $L_E \neq \lambda$. A sufficient condition for G4 to meet the Intuitive Criterion is $p_2''' < E\left[p : p \in P_I\right]$.

5.2.2 *The availability of an FX swap crowds out refinancing via domestic overnight loans*

To see why external refinancing is crowded out after shock s_3 occurred, consider Figure 4. It is drawn for a special case, in which C is high (such that delay is not an option for any type); ρ is high (such that G_E would opt to hold no liquidity *ex ante* independently of its risk), and in which settlement is PvP for the FX swap (such that G_E faces a fully informed counterparty when it enters an FX swap, and an uninformed counterparty when it refinances domestically). The x-axis shows the likelihood p_E that G_E's illiquid assets pay off on day two. Thus, G_E's default risk is decreasing from the left to the right. The y-axis shows the interest rates at which G_W is willing to grant the intragroup loan (the downwards-sloping line), and the interest rate at which D_E is willing to grant the overnight loan (the horizontal line).

Figure 4: Crowding out of domestic interbank loans when global refinancing is available (after shock s_3)



The external lender, D_E, does not know p_E and offers the same contract independently of p_E. By contrast, the internal lender, G_W, knows the borrower's risk and offers less risky borrowers (higher p_E) a lower interest rate.[16] The proof that the borrower prefers G_W's offer independently of its risk now proceeds by contradiction. Suppose instead that there was a type $p_E = p' > 0$ which found itself confronted with the same offer r' by the external and the internal lender. By construction, both the informed and the uninformed lender expect to break even by offering $r_D = r_G(p')$. Then all riskier types $p_E < p'$ would strictly prefer to take the uninformed lender's offer. All less risky types, in contrast, would strictly prefer the informed lender's offer.

But this contradicts the assumption that both lenders expect to break even on their contracts. Thus, there is no $p' > 0$ at which both curves intersect. In all equilibria,[17] a liquidity-short subsidiary prefers the FX swap independently of its risk.

5.2.3 Derivation of Proposition 1 – intuition

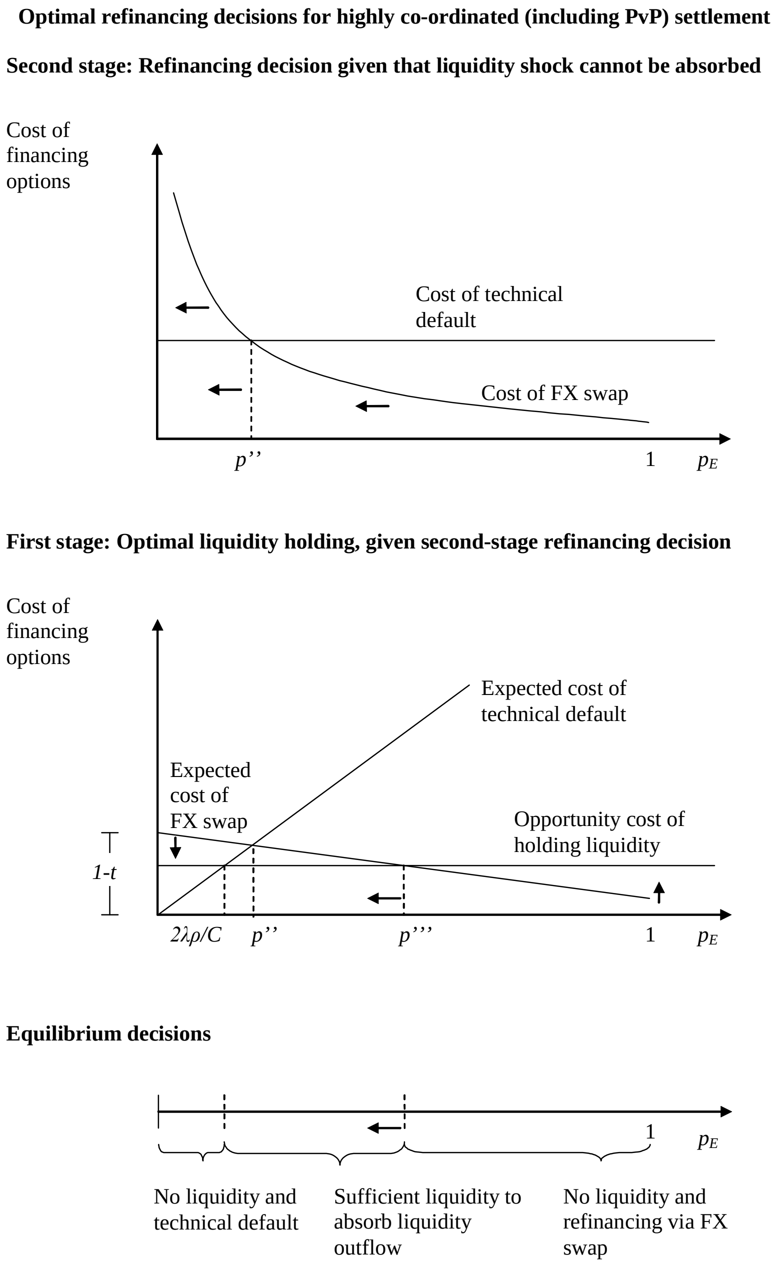
This section provides some graphical intuition for the derivation of equilibria of type G3. Figure 5 shows how the model is solved backwards in this case. It takes the result that the availability of

[16]Notice that this line is constructed assuming that G_W offers contracts such that he expects to break even on *each* contract.

[17]This result applies to all Bayesian Nash equilibria of the game, not only to those listed in Proposition 1.

an FX swap crowds out refinancing via domestic overnight loans as given.

Figure 5: Graphical derivation of an equilibrium of type G3



Note: The arrows indicate reactions to a decline in the degree of co-ordination in FX settlement (an increase in t).

The first panel shows the (second-stage) decision between refinancing via an FX swap and delay. Only borrowers with low risk ($p_E \geq p''$) prefer to take out the FX swap. The second panel shows the (first-stage) decision about how much liquidity to hold *ex ante*. The opportunity cost of holding liquidity is constant in p_E. This reflects the assumption that the expected return on the risky asset, ρ, is independent of p_E. Refinancing via an FX swap is preferred by the least risky banks ($p_E \geq p'''$). The most risky banks ($p_E \leq 2\lambda\rho/C$) do not refinance in response to a liquidity outflow and instead delay their payment. Borrowers with intermediate solvency risk prefer hoarding sufficient liquidity to avoid any borrowing. The third panel summarises the results.

The arrows indicate reactions to a decline in the degree of co-ordination in FX settlement (an increase in t). Notice that the equilibrium only exists for sufficiently small $t < t'$; only for these t is the expected cost of refinancing a declining function of p_E. These arrows ultimately determine the reaction of the likelihood of delay and of transmission of losses. Their direction is proven in the following section.

5.3 *Likelihood of delay and of transmission of losses*

Ex ante, the likelihood of delay is equal to the likelihood that a subsidiary is short of liquidity, subject to solvency risk, holds no liquidity, and delays rather than refinances. This likelihood depends, of course, on the equilibrium under consideration. Consider for example an equilibrium in which G_E's behaviour is described by G2, while G_W's behaviour is described by D2. Then G_W delays whenever $L_W = 0$ and G_W is hit by a liquidity outflow, ie, with probability $\frac{1}{6}$ (derived in Lemma 2). G_E delays with probability

$$\Pr(s_1) + \Pr(s_3)\Pr\left(p_E < p''\right) = \frac{1}{6}\left(1 + p''\right)$$

Corresponding expressions can be easily computed for the other equilibria. Here, I investigate how these likelihoods of delay change when the degree of co-ordination of settlement changes. G_W's behaviour, described by D1 and D2, does not depend on t. Thus, it is sufficient to compute how G_E's behaviour changes, that is, how a change in t influences $\Pr(p_E < p'')$ whenever G1 or G2 describe G_E's equilibrium behaviour, $\Pr(p_E < 2\lambda\rho/C)$ when G3 applies, and $\Pr\left(p_E < p_1'''\right)$ when G4 applies. Proposition 2 has the results; the proof is in the appendix.

Proposition 2 In equilibria of type G1-G3, the likelihood of delay falls the less co-ordinated the settlement of the two currency transactions in the spot leg of the FX swap.

The situation is more complicated in equilibria of type G4. Here, all types in $\left[0, p_1'''\right]$ delay, and the sign of $\partial p_1'''/\partial t$ depends on the equilibrium under consideration. The appendix explores the conditions under which more precise statements can be made. For example, if a comparatively large share of safer banks refinances via FX swaps (here: $p_2''' < E\left[p : p \in P_I\right]$), Proposition 2 also applies to G4.[18]

The likelihood of transmission of losses is equal to the likelihood that a subsidiary is short of liquidity, subject to solvency risk, holds no liquidity, and prefers refinancing over delay. Consider again an equilibrium in which G_E's behaviour is described by G2, while G_W's behaviour is described by D2. From Lemma 2, the likelihood of transmission of losses originating from G_W is $1/12$. For losses originating from G_E, the corresponding expression is (recall that G_E is only subject to refinancing risk with probability $1/2$):

$$\begin{aligned} & \Pr(s_3)\, E\left[1 - p_E | p_E \in P_{FX}\right] \Pr(p_E \in P_{FX}) + \Pr(s_5)\, E\left[1 - p_E | p_E \in P_I\right] \Pr(p_E \in P_I) \\ = \; & \frac{1}{6} \cdot \frac{1}{2} \cdot \left(\int_{p''}^{1} (1 - p)\, dp \right) + \frac{1}{6} \cdot \frac{1}{2} \cdot 1 = \frac{1}{24} \left(1 - p''\right)^2 + \frac{1}{12} \end{aligned}$$

For G_W, the likelihood of transmission of losses is independent of t, so that we can again focus on G_E. In particular, the reaction is proportional to the change induced on p'' whenever G1 or G2 describe G_E's equilibrium behaviour, on p''' when G3 applies, and on p_1''' when G4 applies.

When the domestic interbank market can be accessed by all banks independently of their risk (equilibria of type G2), or when banks completely rely on global refinancing (equilibria of types G1 and G3), there is a straightforward relationship between the co-ordination in the settlement for FX transactions and the likelihood of delay, and the transmission of losses.

The more time elapses between the settlement of the first (East-\$) and the second (West-\$) transactions of the spot leg of the FX swap (the larger t), the larger the share of exposure borne by the uninformed lender, the lower the fee the worst risk in P_{FX} is charged for the FX swap and hence the more attractive it is to take out the FX swap. As a result, P_{FX} contains more risky borrowers in equilibrium. Essentially, highly uncoordinated settlement allows relatively bad risks to hide among the good risks when taking out an FX swap. When PvP settlement is introduced, the informed lender carries all the risk, so he increases the charge for bad risks, which in

[18] Incidentally, $p_2''' < E\left[p : p \in P_D\right]$ is also a sufficient (but not necessary) condition for G4 to meet the Intuitive Criterion.

response opt for other alternatives (delay in G1 and G2; hoarding liquidity in G3). Proposition 3 states the result formally (the proof is in the appendix):

Proposition 3 In equilibria of type G1-G3, losses are more likely to be transmitted the less co-ordinated the FX settlement.

No unambiguous statement can be made for equilibria of type G4 . While it can be shown that better co-ordination of settlement (smaller t) causes the more risky banks to abandon global refinancing after s_3, the impact on the average quality of domestic borrowers is uncertain. Hence, lending after s_3 becomes less risky, but we cannot exclude that lending after s_5 becomes riskier. The net effect is uncertain. The appendix explores the conditions under which more precise statements can be made.

The distribution between domestic and global transmission of losses is also of interest. If G_E's default occurs while D_E has exposure, losses are contained domestically. If, in contrast, G_E's default occurs while G_W has exposure, losses are transmitted only within the same banking group, but across countries. Clearly, the likelihood of domestic transmission of losses is increasing in t in equilibria of types G1-G3. In contrast, the likelihood of global transmission of losses may be declining in t, given that G_W has, on average, exposure to worse risks, but only for a shorter duration.

Finally, it is interesting to see how the likelihood of delay and transmission of losses react to a change in the size of the liquidity shock, and the cost of delay. Proposition 4 shows that the likelihood of delay is decreasing the higher the cost of delay, and the smaller the liquidity shock in equilibria G1-G3. This is intuitive: the smaller the liquidity shock, the lower the refinancing costs via an FX swap, so the more likely a liquidity-short bank is to either hoard liquidity or to refinance when hit by a liquidity outflow. In G1 and G2, a greater likelihood of refinancing implies directly that transmission of losses has become more likely. (Here, the subsidiary holds no liquidity independently of its risk.) In contrast, the likelihood of transmission of losses falls in C/λ for G3. To understand why, compare the second panel in Figure 5: at p''', the marginal benefit of hoarding liquidity has increased because after s_1, delay is the only option when liquidity is insufficient. This causes the horizontal line (the opportunity cost of liquidity) to shift downwards. Hence, p''' (and borrowers with slightly lower risks) hoard liquidity: in the figure, p''' shifts to the right. Thus, the average quality of banks taking out an FX swap (those whose risk lies in $[p''', 1]$) increases.

Proposition 4 In equilibria of type G1-G4, the likelihood of delay falls when C/λ rises. The likelihood of transmission of losses rises in G1 and G2. In contrast, the likelihood of transmission of losses falls in C/λ for G3.

Again, the situation is more complicated in equilibria of type G4. If primarily the least risky banks refinance via FX swaps ($p_2''' < E[p : p \in P_I]$), then the likelihood of transmission of losses falls in C/λ also for G4.

Equally intuitively, both the likelihoods of transmission of losses and of delay tend to rise the opportunity cost of hoarding liquidity ρ. Proposition 5 has the details. The proof is in the appendix.

Proposition 5 In equilibria of types G1 and G2, the probability of delay is independent of ρ. In equilibria of type G3, the likelihood of delay and the likelihood of transmission of losses both rise in ρ. In equilibria of type G4, the likelihood of delay rises. If $p_2''' < E[p : p \in P_I]$, the likelihood of transmission of losses also rises.

6 Comparison between global and local liquidity management

A comparison between behaviour under local and global liquidity management is made difficult by the large number of equilibria under both scenarios. I have not been able to establish results that hold, for a given set of parameter values, across all possible equilibria under global and under local liquidity management. The following paragraphs present a number of more limited results that hold when comparing specific types of equilibria.

First, compare an equilibrium in which G_E's behaviour is described by G2, and G_W's behaviour by D2 – referred to as (G2,D2) – with an equilibrium in which both subsidiaries' behaviour is described by D2 – referred to as (D2,D2). We establish that (G2,D2) converges towards (D2,D2) as $t \to 1$. (The statement is made more precise and proven in the appendix. There is no corresponding property for the other equilibria.) Thus, the results in Propositions 2 and 3 for equilibria in which G_E behaves according to G2 are equally applicable for a comparison between local and global liquidity management: delay becomes more likely, and transmission of losses less likely when we move from local to global liquidity management.

The other two results refer to the level of precautionary liquidity holdings when the settlement of FX swaps is PvP ($t = 0$). Both suggest that optimal liquidity holdings tend to be lower under global liquidity management.

Comparing again (G2,D2) to (D2,D2), notice that subsidiaries do not hold any precautionary liquidity in either equilibrium. However, a prediction can be made for an interesting special case. If settlement is PvP, then there are situations in which it is optimal for the global bank to hold no precautionary liquidity under global liquidity management, whereas it would hold liquidity under local management. The converse is not true. (This statement is made precise in the appendix.) In this sense, optimal liquidity holdings may fall (but not rise) when a bank moves from local to global liquidity management.

Now compare (G3,D1) and (D1,D1). We first establish that whenever parameters are such that an equilibrium exists in which G_E behaves according to G3, no equilibrium exists in which he behaves according to G2 or D2. But D1 still exists. Thus, the natural comparison is between (G3,D1) and (D1,D1). Here, it is straightforward to show that liquidity holdings are smaller in (G3,D1) than in (D1,D1).

Finally, notice that the results rely on a comparison of liquidity holdings under G2 (G3) with D2 (D1). If G_W was also able to raise liquidity via an FX swap after shock s_4, he could also play according to G2 (G3), and the global bank's optimal liquidity holdings would fall further. However, as argued in Section 5, calendar-day conventions imply that an FX swap and a domestic overnight loan are equivalent refinancing instruments for G_W. Thus, the global bank's subsidiaries can only partially co-insure each other against local liquidity shocks. Western markets can insure Eastern markets, but not *vice versa*. Thus, one might interpret G_W's higher precautionary liquidity holdings as an explanation for the comparatively greater depth of the US market for short-term funds.

A considerable caveat to these statements is, of course, that it is not certain whether banks would indeed move from equilibrium D2 to G2, and D1 to G3 once they switch to global liquidity management. A study of the transition dynamics is left for future research.

7 Discussion

This section discusses the implications of some of the key assumptions made in the model.

Absence of reputational contagion. We assume that the Eastern subsidiary's default only impacts the Western subsidiary to the extent that the Western subsidiary loses the principal amount of the intragroup loan. However, in practice, the Western subsidiary would probably suffer some reputational damage as well.[19] For example, the Western subsidiary's depositors might withdraw their funds when the Eastern subsidiary delays payments. (In our set-up, this would correspond to an increase in the likelihood of shock s_5 relative to s_3.) If, in response, the Western subsidiary required a higher compensation for the intragroup loan, the cost of an FX swap would increase, and domestic interbank lending might not be crowded out. However, one might also argue that the Western subsidiary would be willing to lend at a lower rate if the Eastern subsidiary otherwise suffered refinancing problems on the following day.

In any case, the resulting greater correlation of local liquidity shocks would imply that the potential benefit of co-insurance would be reduced further. Equilibrium liquidity holdings would have to rise. (An extreme case of contagion is given when G_E and G_W are not subsidiaries, but branches, which could not fail separately. This is left for future research.)[20]

Abstraction from refinancing via cross-border collateral movements. Another option that banks might have available when managing their liquidity globally is the movement of collateral across borders. Continuing the example in which the Eastern subsidiary suffers a liquidity outflow, the Western subsidiary could sell (or repo) West-$ denominated collateral in the domestic market. The Eastern subsidiary could access this liquidity via an FX swap, exactly as discussed above. Thus, this case is covered by the model.

The situation would change if the Eastern subsidiary could use West-$ denominated collateral in its own domestic market. In this case, the Western subsidiary could lend the Eastern subsidiary

[19] Notice that for simplicity, we also abstract from any contagion as a consequence of technical default. This would have required more detailed modelling of the payment flows on each day and is left for future research.

[20] Relatedly, notice that we restricted attention to the case that only one subsidiary's loan portfolio was risky at a time. This appeared to be justifiable given our focus on risk of imminent failure (ie, within the next 24 hours). If we allowed both portfolios to default and G_W defaults before it takes over the exposure to G_E in the FX swap, but after G_E received East-$ from D_E, G_E would not be able to raise West-$ in time to pay D_E. Then D_E would effectively grant G_E an overnight loan and require additional payments. This would raise the costs of the FX swap for good risks, and reduce it for bad risks of G_E. In expectation, more risks might opt to take out the FX swap, but the comparative static results should remain qualitatively the same.

the West-$ denominated collateral. Asymmetric information would not play a role any more as the Eastern subsidiary could now collateralise the loan it raises domestically. This corresponds exactly to the situation of PvP settlement of FX swaps in the model in this paper, where only G_W has exposure to G_E. Thus, widening collateral requirements, and simplifying the process for cross-border transfer of collateral, could be another policy option to improve the availability of funds for liquidity-short banks in global liquidity management.

Opportunity costs of liquidity-rich banks and their bargaining power. We assume that both the internal lender and the external lender are willing to grant a loan at an interest rate at which they just break even. It might be desirable to endogenise the bargaining between the liquidity-short subsidiary and its potential lenders. At the moment, we effectively assume that the liquidity-short bank has all the bargaining power. Consider for the moment the opposite case in which the liquidity-short bank has no bargaining power, and the lender(s) make take it or leave it offers. Proposition 1 would then not hold any more. To see this, consider for simplicity the case of PvP settlement. The informed lender would raise the interest rate high enough until the liquidity-short bank is only just willing to take out the loan instead of delaying payments. This holds when the expected cost of borrowing $p_E r_G B_E$ equals the expected cost of delaying, $p_E C$. Equating these two expressions leads to $r_G = C/B_E$. Then the borrower's cost of the FX swap is constant in his risk p_E. It is also identical to what the outside lender would charge for a domestic overnight loan: r_{DE} would be set such that

$$E\left[p_E | p_E \in P_I\right] r_D B_E \leq E\left[p_E | p_E \in P_I\right] C$$

holds as an equality, leading to $r_{DE} = C/B_E$. In practice, the borrower is likely to have some bargaining power, such that better risks pay less for an FX swap than worse risks, and Proposition 1 would presumably go through as in the extreme case we consider.

The **specification of the outside option**, that is, of delay, also has important consequences for the results we derived. First, we assumed that a bank maximises its pay-off at the end of day two; the cost of delay incurred on day one does not matter for a bank that is declared bankrupt on day two. In equilibrium, a bank whose illiquid assets do not pay off has no assets left (it either had zero liquid assets when the liquidity shock hit, or borrowed only just enough to make all necessary payments). There is no additional penalty depositors could suffer from delay. This assumption implies that if funds are insufficient to absorb the liquidity shock, the liquidity-short bank either delays or takes out the overnight loan independently of its risk, because the *relative* size of the expected borrowing cost $p_E r_D B_E$ and the expected cost of delay $p_E C$ is independent of p_E. Second, the assumption that the costs of failing to raise sufficient funds are independent of the amount that a bank falls short appears reasonable when the alternative is delay.

8 Conclusion and future research

The main result of the paper is that better co-ordination of FX settlements reduces the likelihood that solvency shocks are transmitted from one institution to another: if a bank was close to insolvency, it would not be able to refinance itself at all in response to liquidity outflows, neither domestically nor via FX transactions. But this implies that such a bank would have to delay the payment of its obligations beyond their due date. (In practice, a bank's failure to execute payment requests that are contractually due might therefore trigger further liquidity outflows. Other creditors of A(UK) might refuse to roll over funds and eventually drive A(UK) into insolvency. In the paper, these effects are simply approximated by a fixed cost.) Thus, better co-ordinated settlement of FX transactions has benefits as well as risks for financial stability: it reduces the transmission of solvency shocks, but increases the chances of payment delay.

The paper could be extended in a number of ways. Most importantly, the second-round effects of delay and insolvency could be modelled in more detail. In the current version of the model, only the bank that delays suffers a loss. In practice, other banks might have expected to receive liquidity from this bank and now find themselves short of liquidity as well. Similarly, there might be second-round effects for the solvency shock as well: the equity of the defaulting bank's creditor is impaired. It might then be more likely to default on its own liabilities and could find it difficult to fund itself, if necessary, on the following day.

Appendix 1: Local liquidity management

We assume that equilibrium behaviour is identical after all shocks that permit domestic refinancing (s_3 and s_5 for G_E; s_4 and s_5 for G_W).

First recall that the (second-stage) decision over borrowing vs delay is independent of p_E. Thus, abstracting from the special case that all risks are indifferent between these options, either all $p_E : L_E(p_E) = 0$ delay (such that $P_{DE} = \varnothing$), or all $p_E : L_E(p_E) = 0$ take out an interbank loan (such that $P_{DE} = \{p_E \in [0,1] : L_E(p_E) = 0\}$). In the following, we distinguish equilibria by which types take out a loan (ie, by whether P_{DE} is empty).

1. Suppose $P_{DE} = \varnothing$. This equilibrium always exists given the off-equilibrium belief $\Pr(p_E = 0 | P_{DE} = \varnothing) = 1$. Then $L_E = 0$ is preferred over $L_E = \lambda$ if
$$(1-0)\rho - \frac{1}{6} 3 p_E C \geq (1-\lambda)\rho$$
equivalently, $p_E \leq 2\lambda\rho/C$. In this 'autarky' equilibrium (type D1), we have
$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \\ \lambda & \text{if} \quad p_E \geq 2\lambda\rho/C \end{cases}$$
D1 passes the Intuitive Criterion. Suppose any type who in equilibrium sets $L_E = 0$ successfully signalled his type. If he had an incentive to do so, then all riskier types would imitate him. But this means that pessimistic borrowers would set the interest rate at a level appropriate to the lowest type, which would discourage the deviation. Similarly, consider any type who in equilibrium sets $L_E = \lambda$. The type with the largest incentive to deviate is $p_E = 1$. Expected refinancing cost under external borrowing are continuously and strictly falling in p. Let p^* be the lowest type who is just indifferent between his equilibrium pay-off and the deviation pay-off. If $L_E(p^*) = \lambda$, then the equilibrium pay-offs of $\pi_{GE}(\lambda, p^*)$ and $\pi_{GE}(\lambda, 1)$ coincide (neither refinances in equilibrium).
Because expected refinancing cost under external borrowing are continuously and strictly falling in p, the deviation pay-off of $p_E = 1$ would be strictly higher than his equilibrium pay-off. If, in contrast, $L_E(p^*) = 0$, then $\pi_{GE}(\lambda, p^*) > \pi_{GE}(\lambda, 1)$, and the result (that the deviation pay-off of $p_E = 1$ would be strictly higher than his equilibrium pay-off) holds *a fortiori*.

2. Suppose instead $P_{DE} \neq \varnothing$. Then all $p_E \in P_{DE}$ prefer holding $L_E = 0$ over holding $L_E = \lambda$ if

$$\begin{aligned}
(1-0)\rho - \frac{1}{6}p_E\left(C + 2r_{DE}\left(\lambda - L_E\right)\right) &\geq (1-\lambda)\rho \\
\frac{1}{6}p_E\left(C + 2r_{DE}\left(\lambda - L_E\right)\right) &\leq \lambda\rho \\
p_E &< \frac{6\rho}{C/\lambda + 2r_D}
\end{aligned}$$

This is an upper bound to p_E, implying that P_{DE} must have the form $\left[0, p'\right]$ where $p' \leq 1$. However, such an equilibrium is unstable in the sense that it does not pass Cho and Kreps' Intuitive Criterion. To see this, consider an interval of deviating types $P^* = \left[p_1, p_2\right]$ where $p_1 < p' < p_2$, and where all types set $L^* \in (0, \lambda)$ and hence ask to borrow $B^* = \lambda - L^*$. Then pessimistic lenders will believe they face p_1 and ask for an interest rate $r^* = (1-p_1)/p_1$. Their deviation pay-off is

$$\pi_{GE}^{D}\left(L^*, p_E\right) = \left(1 - L^*\right)\rho - \frac{1}{6}p_E\left(C + 2\left(\lambda - L^*\right)\left(\frac{1-p_1}{p_1}\right)\right)$$

In contrast, their equilibrium pay-off is

$$\pi_{GE}^{E}\left(L_E, p_E\right) = \begin{cases} (1-\lambda)\rho & \text{if} \quad p_E \geq \frac{6\rho}{C/\lambda + 2r_D} \\ \rho - \frac{1}{6}p_E\left(C + 2\lambda r_{DE}\right) & \text{if} \quad p_E < \frac{6\rho}{C/\lambda + 2r_D} \end{cases}$$

It remains to show that $p_1 \in [0, 1]$. We clearly cannot have $p_1 = 0$. We show that $p_1 \in (0, p')$. All types $p_E \in \left[p_1, p'\right]$ prefer to deviate if

$$\left(1 - L^*\right)\rho - \frac{1}{6}p_E\left(C + 2\left(\lambda - L^*\right)\left(\frac{1-p_1}{p_1}\right)\right) \geq \rho - \frac{1}{6}p_E\left(C + 2\lambda r_{DE}\right)$$

Indifference point:

$$\begin{aligned}
\left(1 - L^*\right)\rho - \frac{1}{6}\left(p_1 C + 2\left(\lambda - L^*\right)\left(1 - p_1\right)\right) &= \rho - \frac{1}{6}p_1\left(C + 2\lambda r_{DE}\right) \\
-L^*\rho - \frac{1}{3}\left(\lambda - L^*\right)\left(1 - p_1\right) &= -\frac{1}{3}p_1 \lambda r_{DE} \\
3L^*\rho + \left(\lambda - L^*\right)\left(1 - p_1\right) &= p_1 \lambda r_{DE} \\
3L^*\rho + \left(\lambda - L^*\right) &= p_1\left(\lambda r_{DE} + \left(\lambda - L^*\right)\right) \\
\frac{3L^*\rho + \left(\lambda - L^*\right)}{\lambda r_{DE} + \left(\lambda - L^*\right)} &= p_1
\end{aligned}$$

This yields a lower bound to p_1; this lower bound is increasing in L^*: For $p_1 < 1$, need $3L^*\rho < \lambda r_{DE}$, and there are sufficiently small $L^* > 0$ for which this holds. Hence $\left(\lambda - L^*\right)\left(\frac{1-p_1}{p_1}\right) < \lambda r_{DE}$ and all $p_E < p_1$ strictly prefer their equilibrium pay-off, while some $p_E > p_1$ strictly prefer to deviate.

3. Finally, consider an equilibrium in which $P_{DE} = [0, 1]$ and there is no p' which is indifferent between $L_E = 0$ and $L_E = \lambda$. Then $r_D = 1$. This equilibrium exists under two conditions: first, in the second stage of the game, that borrowing is preferred over delay $(\lambda - C/r_{DE} \leq L_E)$ given $L_E = 0$, ie, that $\lambda \leq C$, and second, in the first stage of the game, that $L_E = 0$ is strictly preferred over $L_E = \lambda$ for all $p_E \in [0, 1]$, ie, that

$$\begin{aligned} (1-0)\,\rho - \frac{1}{6}p_E\,(C + 2r_{DE}\,(\lambda - 0)) &\geq (1-\lambda)\,\rho \\ \frac{1}{6}p_E\,(C + 2 \cdot 1 \cdot \lambda) &\leq \lambda\rho \end{aligned}$$

holds for all risks p_E, so also for $p_E = 1$. Hence, need

$$\begin{aligned} C + 2\lambda &\leq 6\lambda\rho \\ \rho &\geq \frac{1}{6}\,(2 + C/\lambda) \end{aligned}$$

is true. This equilibrium passes the Intuitive Criterion. The least risky type, $p_E = 1$, has the highest incentive to signal his type. But if he did so, then all riskier types would imitate him, down to $p_E = 1/2$. But this means that pessimistic borrowers would set the interest rate at the rate appropriate to $p_E = 1/2$, which is equal to the equilibrium interest rate. Hence, there would be no incentive to deviate.

Appendix 2: Global liquidity management

Proof of Lemma 4

This is straightforward. If G_W does not grant the intragroup loan, his pay-off is $\pi_{GW} = (1 - L_W)\,\rho$. If he grants the intragroup loan, and G_E defaults, his pay-off is (notice that we assume that the occurrence of the real shock is uniformly distributed during the day)

$$\begin{aligned}
\pi_{GW} &= \Pr\left(G_E \text{ defaults while } D_E \text{ has exposure}\right)(1 - L_W)\,\rho \\
&\quad + \Pr\left(G_E \text{ defaults while } G_W \text{ has exposure}\right)\left((1 - L_W)\,\rho - B_E\right) \\
&= t\,(1 - L_W)\,\rho + (1 - t)\left((1 - L_W)\,\rho - (1 - t)\,B_E\right) \\
&= (1 - L_W)\,\rho - (1 - t)\,B_E
\end{aligned}$$

If G_E does not default, the pay-off is $\pi_{GW} = (1 - L_W)\,\rho + r_G B_E$. Then G_W is willing to grant the intragroup loan if

$$(1 - p_E)\left((1 - L_W)\,\rho - (1 - t)\,B_E\right) + p_E\left((1 - L_W)\,\rho + r_G B_E\right) \geq (1 - L_W)\,\rho$$

equivalently, if $-(1 - p_E)(1 - t) + p_E r_G \geq 0$. G_W's break-even per-unit fee $r_G = (1 - t)(1 - p_E)/p_E$ solves this equation as an equality.

Proof of Lemma 5

Assuming that all $p_E \in P_{FX}$ choose the swap, D_E's pay-offs are $\pi_{DE} = s_D B_E$ if G_E does not default. If, in contrast, G_E defaults,

$$\begin{aligned}
\pi_{DE} &= \Pr\left(G_E \text{ defaults while } D_E \text{ has exposure}\right)(-B_E) \\
&\quad + \Pr\left(G_E \text{ defaults while } G_W \text{ has exposure}\right)(s_D B_E) \\
&= t\,(-B_E) + (1 - t)\,(s_D B_E)
\end{aligned}$$

D_E is willing to offer the FX swap if his expected pay-off from doing is (at least) equal to zero, ie, if

$$E\left[(1-p_E)\left(t\left(-B_E\right)+(1-t)\,s_D B_E\right)+p_E s_D B_E | p_E \in P_{FX}\right] \geq 0$$

D_E's break-even per-unit fee $s_D(t) = t\left(1 - E\left[p|p \in P_{FX}\right]\right) / \left(1 - t\left(1 - E\left[p|p \in P_{FX}\right]\right)\right)$ solves this equation as an equality.[21]

Comparative static properties of r_{FX}

Lemma 6 For a given $P_{FX} \neq \varnothing$, the following holds:

1. r_{FX} is strictly declining in p_E:

$$\frac{\partial r_{FX}}{\partial p_E} = \frac{\partial\left((1-t)\frac{1-p_E}{p_E}+t\left(\frac{1-x}{1-t(1-x)}\right)\right)}{\partial p_E} = -\frac{1-t}{p_E^2} < 0$$

2. G_E's expected payment for the FX swap, $p_E r_{FX}(p_E, t)\,B_E$, is linear in p_E. There is a unique $t'(P_{FX}) \in (0,1]$ such that

$$\frac{\partial\left(p_E r_{FX}\right)}{\partial p_E}\begin{cases} > 0 & \text{if and only if} \quad t > t' \\ = 0 & \text{if and only if} \quad t = t' \\ < 0 & \text{if and only if} \quad t < t' \end{cases}$$

Proof. The change of the total expected cost of the FX swap with respect to p_E is, for a given P_{FX}, independent of p_E:

$$\frac{\partial p_E r_{FX}}{\partial p_E} = -(1-t) + t\left(\frac{1-E\left[p|p \in P_{FX}\right]}{1-t\left(1-E\left[p|p \in P_{FX}\right]\right)}\right)$$

This derivative is positive if

$$t^2 - \frac{3-2E\left[p|p \in P_{FX}\right]}{1-E\left[p|p \in P_{FX}\right]}t + \frac{1}{1-E\left[p|p \in P_{FX}\right]} \leq 0$$

The solutions to the corresponding quadratic equation are

$$t_{1,2} = \frac{3-2E\left[p|p \in P_{FX}\right]}{2\left(1-E\left[p|p \in P_{FX}\right]\right)} \pm \frac{\sqrt{4\left(1-E\left[p|p \in P_{FX}\right]\right)^2+1}}{2\left(1-E\left[p|p \in P_{FX}\right]\right)}$$

[21] Notice that the forms of the expressions for r_G and s_D differ. This is because G_W does not enter a contract with G_E if G_E defaults while G_E has exposure to D_E. In contrast, D_E has received the principal and interest rate (via D_W) if G_E defaults while G_W has exposure.

It is straightforward to show that the larger root is always larger than one. Thus,

$$\frac{\partial\left(p_E r_{FX}\right)}{\partial p_E}\begin{cases} > 0 & \text{if} \quad t \geq t_2 \\ < 0 & \text{if} \quad t < t_2 \end{cases}$$

One can show that the smaller root t_2 is strictly increasing in $E\left[p|p \in P_{FX}\right]$. Also, $t_2 \in [0.38, 1]$ because

$$\begin{aligned}
\lim_{E\left[p|p\in P_{FX}\right]\to 0} t_2 &= \frac{3-2(0)}{2(1-0)} - \frac{\sqrt{4(1-0)^2+1}}{2(1-0)} = \frac{3}{2} - \frac{1}{2}\sqrt{5} = 0.38 \\
\lim_{E\left[p|p\in P_{FX}\right]\to 1} t_2 &= \lim_{x\to 1}\left(\frac{3-2x}{2(1-x)} - \frac{\sqrt{4(1-x)^2+1}}{2(1-x)}\right) = 1
\end{aligned}$$

Proof of Proposition 1

Overview

We first establish which types of Bayesian Nash equilibria exist in which there is some refinancing via foreign exchange transactions. (We neglect equilibria in which no global refinancing takes place: either because the opportunity costs of liquidity are so low that independently of their risk, banks hold sufficient liquidity to withstand the liquidity shock; or because FX markets are 'closed' because of uninformed lenders' off-equilibrium belief that they, if approached, would face the borrower with the worst risk. For the latter reason, all equilibria of the local liquidity management game are also equilibria of the global liquidity management game.)

Because the cost of refinancing depends on which types opt for the respective refinancing option, which in turn depends on the cost of refinancing, there may be multiple equilibria for a given value of our exogenous parameters. I only determine the structure that the equilibria can take – for example, that the most risky types opt for delay; intermediate types hoard liquidity; while the least risky types rely on refinancing via an FX swap. I do not, however, solve explicitly for all boundaries between these intervals. Explicit solutions are not necessary for our main conclusions regarding the likelihood of delay and transmission of shocks.

Equilibria differ also with respect to behaviour after a shock after which local refinancing is possible, but not global refinancing (s_5 for G_E). The domestic interbank market may be closed in this case (see Lemma 1: in this case, $P_I = \varnothing$). This influences pay-offs and the desirability of holding liquidity *ex ante*. We first consider open domestic interbank markets after s_5; proofs for the closed case are analogous.

For each type of equilibrium that we identify, we investigate whether it passes the Intuitive Criterion. In our case, this criterion eliminates those equilibria in which only the least risky types would hoard liquidity, whereas more risky types refinance. In such a situation, the least risky types can successfully signal their types (and obtain cheaper funds). Equilibria that do not pass the Intuitive Criterion are not listed in Proposition 1.

Claim

In (Bayesian Nash) equilibria in which some types of G_E obtain funds via an FX swaps ($P_{FX} \neq \varnothing$), G_E's equilibrium strategy fulfils one of the following:

Existence for..	**No.**	$\mathbf{p}_E$: $\mathbf{L}_E(p_E) = \boldsymbol{\lambda}$
Any t, high ρ	G1	$\varnothing$
	G2	$\varnothing$
Low t, intermediate ρ	G3	$\left[2\lambda\rho/C, p'''\right]$
	G4	$\left[p_1''', p_2'''\right]$
Intermediate t and ρ;	G5	$\left[p''', 1\right]$
or high t and (low or intermediate) ρ	G6	$\left[p''', 1\right]$

where G1-G3 pass the Intuitive Criterion, G4 only if $p_2''' < E\left[p : p \in P_I\right]$, while G5 and G6 do not pass this criterion. Also $P_I = \varnothing$, and $L_E = 0$ if $L_E \neq \lambda$. The definition of the indifferent types is as follows:

No.	P_I	$\mathbf{P}_{FX}$	**Definition of** p'''
G1	$\varnothing$	$\left[p'', 1\right]$	p''' does not exist
G2	$[0, 1]$	$\left[p'', 1\right]$	p''' does not exist
G3	$\varnothing$	$\left[p''', 1\right]$	$6\lambda\rho = p'''\left(2C + \lambda r_{FX}\left(p''', t, P_{FX}\right)\right)$

No.	P_I	$\mathbf{P}_{FX}$	**Definition of** p'''
G4	$[0, p_1'''] \cup [p_2''', 1]$	$[p_2''', 1]$	$6\lambda\rho = p_1''' (2C + \lambda r_{DE}(P_I))$ $6\lambda\rho = p_2''' \left(C + \lambda r_{DE}(P_I) + r_{FX}\left(p_2''', t, P_{FX}\right)\lambda\right)$
G5	$[0, p''']$	$[p'', p''']$	$6\lambda\rho = p''' (C + \lambda r_{DE}(P_I) + r_{FX}(p''', t, P_{FX})\lambda)$
G6	$\varnothing$	$[p'', p''']$	$6\lambda\rho = p''' (2C + \lambda r_{FX}(p''', t, P_{FX}))$

Proof

Recall that liquidity shocks are publicly observable – there is only asymmetric information regarding the solvency risk. We first write G_E's pay-off as an expectation over the liquidity shocks. We then compute G_E's expected cost of refinancing: it depends, naturally, on G_E's risk, and its financing choice. In the process, we determine relationships between P_{FX} (the set of types that opt for an FX swap) and P_I (the set of types that opt for an interbank loan). These sets also yield the answer to our final question: how, given these expected refinancing costs, G_E allocates his funds between liquid and illiquid assets.

Step 1: Computation of expected pay-offs

G_E's pay-off is, analogously to the case of local liquidity management, as follows:

- if s_1 occurs, and G_E is short of liquidity, it has no other option but to delay payment. G_E's pay-off is
$$(1 - L_E)\rho - \begin{cases} 0 & \text{if} \quad L_E \geq \lambda \\ p_E C & \text{if} \quad L_E < \lambda \end{cases}$$
- if s_2, s_4 or s_6 occurs, G_E has surplus liquidity (which, by assumption, it cannot lend out profitably), hence the pay-off is $(1 - L_E)\rho$.
- if s_3 occurs, and G_E is short of liquidity, it has the option to delay, to refinance via an overnight loan, or to refinance via an FX transaction. Its pay-off is
$$(1 - L_E)\rho - \begin{cases} 0 & \text{if} \quad L_E \geq \lambda \\ p_E \min\{r_I, r_{FX}\} B_E & \text{if} \quad L_E < \lambda \text{ and } \min\{r_I, r_{FX}\} B_E \leq C \\ p_E C & \text{if} \quad L_E < \lambda \text{ and } \min\{r_I, r_{FX}\} B_E > C \end{cases}$$
- if s_5 occurs, both G_E and G_W are short of liquidity, such that G_E's only option is to refinance domestically. If the domestic market remains open after s_5,

$r_{DE} = \left(1 - E\left[p_E | p_E \in P_I\right]\right) / E\left[p_E | p_E \in P_I\right]$ and G_E's pay-off is

$$(1 - L_E)\,\rho - \begin{cases} 0 & \text{if} \quad L_E \geq \lambda \\ p_E \min\{C, r_{DE} B_E\} & \text{if} \quad L_E < \lambda \end{cases} \qquad \textbf{(B-1)}$$

If the domestic interbank market is closed, then the pay-offs are as in **(B-1)**, with the exception that $\min\{C, r_{DE} B_E\}$ is replaced by C (the only remaining option is to delay).

Given that all shocks occur with probability $1/6$, the expected pay-off is, if the domestic market remains open,

$$\begin{aligned} & \pi_{GE}(L_E, p_E) \\ = \;& (1 - L_E)\,\rho - \begin{cases} \frac{3}{6} \cdot 0 & \text{if} \quad L_E \geq \lambda \\ \frac{1}{6} p_E C + \frac{1}{6} p_E \min\{C, \min\{r_I, r_{FX}\} B_E\} + \frac{1}{6} p_E \min\{C, r_{DE} B_E\} & \text{if} \quad L_E < \lambda \end{cases} \end{aligned}$$

This expression is declining in B_E. As borrowing is costly, $B_E \leq \lambda - L_E$ in all equilibria. Because the cost of delay is independent of the amount outstanding at the end of day one, $B_E \in \{0, \lambda - L_E\}$ in equilibrium: G_E either borrows sufficiently to avoid delay, or it delays (and borrows nothing). Also, because $\rho > 0$, we must have $L_E \leq \lambda$.

The expected pay-off can be rewritten as

$$\begin{aligned} & \pi_{GE}(L_E, p_E) \\ = \;& (1 - L_E)\,\rho - \frac{1}{6} p_E \begin{cases} 0 & \text{if} \quad L_E = \lambda \\ C + \min\{C, \min\{r_I, r_{FX}\}(\lambda - L_E)\} + \min\{C, r_{DE}(\lambda - L_E)\} & \text{if} \quad L_E < \lambda \end{cases} \end{aligned}$$

if the domestic interbank market remains open after s_5, and

$$\pi_{GE}(L_E, p_E) = (1 - L_E)\,\rho - \frac{1}{6} p_E \begin{cases} 0 & \text{if} \quad L_E = \lambda \\ 2C + \min\{C, \min\{r_I, r_{FX}\}(\lambda - L_E)\} & \text{if} \quad L_E < \lambda \end{cases}$$

if it is closed.

The following paragraphs establish the existence conditions for equilibria in which $P_{FX} \neq \varnothing$. The equilibria are grouped by whether or not some risks choose to refinance via an FX swap or an interbank loan: ie, whether in a certain class of equilibria, P_{FX} and P_I are non-empty. For each of the possible cases, conditions for the existence of such equilibria are derived.

Step 2: Proof that global refinancing crowds out less informed local refinancing after s_3

We first prove that $P_{FX} \neq \varnothing$ implies $P_I = \varnothing$. Suppose in contrast that in equilibrium, $P_I \neq \varnothing$. Pick any $p'_E \in P_I$.

1. Because p'_E prefers domestic over global refinancing, we must have $r_I \leq r_{FX}\left(p'_E\right)$. Now r_{FX} is strictly decreasing in p_E, so all $p_E < p'_E$ strictly prefer domestic to global refinancing. Thus, for all $p''_E \in P_{FX}$, $p''_E > p'_E$ (in words: if $P_I \neq \varnothing$, in equilibrium the less risky banks go for the FX swap).
2. Because p'_E prefers domestic refinancing over delay, no $p_E < p'_E$ delays: in both cases, the associated costs only have to be borne by the surviving bank; so the preference relation between domestic refinancing and delay is independent of p_E.

Hence, $\left[0, p'_E\right] \in P_I$. This implies that first, $r_I \geq \left(1 - p'_E\right)/p'_E$, and second, that $s_D < \left(1 - p'_E\right)/p'_E$ because only lower-risk banks choose the FX swap. But then p'_E would strictly prefer global refinancing. This contradicts $p'_E \in P_I$.

Step 3: Convexity of pay-offs

We can thus write the expected pay-offs as

$$\pi_{GE}\left(L_E, p_E\right) = \left(1 - L_E\right)\rho - \frac{1}{6}p_E \begin{cases} 0 & \text{if} \quad L_E = \lambda \\ C + \min\left\{C, r_{FX}\left(\lambda - L_E\right)\right\} + \min\left\{C, r_I\left(\lambda - L_E\right)\right\} & \text{if} \quad L_E < \lambda \end{cases}$$

if the domestic interbank market remains open after s_5, and

$$\pi_{GE}\left(L_E, p_E\right) = \left(1 - L_E\right)\rho - \frac{1}{6}p_E \begin{cases} 0 & \text{if} \quad L_E = \lambda \\ 2C + \min\left\{C, r_{FX}\left(\lambda - L_E\right)\right\} & \text{if} \quad L_E < \lambda \end{cases}$$

if it is closed.

It is easy to see that $\pi_{GE}(L_E, p_E)$ is piecewise linear and convex in L_E: for progressively higher L_E, $\partial \pi_{GE}(L_E, p_E)/\partial L_E$ rises from $-\rho$ to $-\rho + \frac{1}{6} p_E (r_{FX} + r_I)$ if the domestic interbank market remains open after s_5, and to $-\rho + \frac{1}{6} p_E r_{FX}$ if it remains closed. In addition, $\pi_{GE}(0, p_E) < \lim_{L_E \to \lambda} (L_E, p_E)$ because of the assumption that the cost of delay is independent of the amount delayed. Hence, the value of L_E that maximises π_{GE} must lie in a corner, ie, $L_E \in \{0, \lambda\}$.

Step 4: Types of equilibria when the domestic interbank market is open after s_5

Consider first the second stage (where L_E is given), and suppose that s_5 occurred. Among those types who have $L_E = 0$, if some types choose the interbank loan, then all types do so (because both C and r_I are independent of p_E). Hence, either $P_I = \varnothing$, or $P_I = \{p : L_E(p) = 0\}$. Here, we assume that the domestic interbank market is open after s_5; hence, $P_I = \{p : L_E(p) = 0\}$. Then G_E prefers $L_E = \lambda$ over $L_E = 0$ if

$$
\begin{aligned}
(1-\lambda)\rho &\geq (1-0)\rho - \frac{1}{6} p_E \left(C + \min\{C, r_{FX}(\lambda - 0)\} + \min\{C, r_I(\lambda - 0)\}\right) \\
-\lambda\rho &\geq -\frac{1}{6} p_E \left(C + (\min\{C, \lambda r_{FX}\} + r_I)\right) \\
6\lambda\rho &\leq p_E \left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p_E, t, P_{FX})\}\right)
\end{aligned}
$$

Let $P'' = \{p \in [0,1] : r_{FX}(p) = C/\lambda\}$ and $P''' = \{p \in [0,1] : 6\lambda\rho = p(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\})\}$. P'' and P''' may be empty. Or they may contain more than one point because r_{FX} depends on which risk types take out FX swaps, hence on the solution under consideration. Notice that:

- If $P_{FX} \neq \varnothing$, we must have $P'' \neq \varnothing$. Suppose in contrast that $P'' = \varnothing$. This may occur for two reasons. First, for all $p \in [0,1]$, delay is strictly preferred over refinancing via an FX swap when s_3 occurs. But then $P_{FX} = \varnothing$. Second, for all $p \in [0,1]$, refinancing via an FX swap is strictly preferred when s_3 occurs. But this cannot be true because for the most risky types, the costs of refinancing via an FX swap grows above all bounds, whereas the cost of technical default are bounded. Thus, we only consider $P'' \neq \varnothing$.

- If $P''' = \varnothing$, then for $P_{FX} \neq \varnothing$ we must have that for all $p \in [0, 1]$, $L_E = 0$ is strictly preferred over $L_E = \lambda$.

The following paragraphs simply characterise the equilibria depending on whether (or not) P''' is empty. This is done first for the case that $p_E r_{FX}(p_E)$ is strictly increasing in p_E (large t, settlement is not well co-ordinated), and then for the case that $p_E r_{FX}(p_E)$ is strictly decreasing in p_E (small t).

1. Suppose $p_E r_{FX}(p_E)$ is strictly increasing in p_E. Then the expected cost of not holding any liquid assets, $p\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\}\right)$ is strictly increasing in p_E. Thus, in equilibrium, at most the least risky types choose $L_E = \lambda$.

   (i) Suppose first that $P''' = \varnothing$. Then $L_E = 0$ for all p, $P_I = [0, 1]$, and for each $p'' \in P''$, the equilibrium takes the form G2: $P_{FX} = \left[p'', 1\right]$: given P_{FX}, $r_{FX}\lambda$ is strictly declining in p_E, whereas C is constant in p_E, so given P_{FX}, all $p_E < p''$ strictly prefer delay over refinancing, and *vice versa* for all $p_E > p''$. This equilibrium exists if

   $$p\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\}\right) < 6\lambda\rho$$

   for all p, hence also for $p = 1$, ie, if

   $$C/\lambda + r_I + r_{FX}(1, t, P_{FX}) < 6\lambda\rho$$

   (ii) Now suppose that $P''' \neq \varnothing$. Take any $p'' \in P''$, $p''' \in P'''$. Assume first that $p''' < p''$. But then $P_{FX} = \varnothing$: increasingness of $p\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\}\right)$ implies that all $p_E > p'''$ prefer $L_E = \lambda$ over $L_E = 0$; and all $p_E < p''$ (hence all $p_E < p'''$) strictly prefer delay over refinancing after s_3. Assume instead that $p''' \geq p''$. Then p''' is determined by indifference between $L_E = 0$ with refinancing and $L_E = \lambda$; that is, $p''' : 6\lambda\rho = p'''\left(C + \lambda r_I(P_I) + \lambda r_{FX}(p''', t, P_{FX})\right)$. Then the equilibrium takes the form G5:

   $$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq p''' \\ \lambda & \text{if} \quad p_E > p''' \end{cases}$$

   and $P_I = \left[0, p'''\right]$, $P_{FX} = \left[p'', p'''\right]$. This equilibrium does not fulfil the Intuitive Criterion as types around p''' would benefit from signalling their type, and would not be imitated by worse types in P_{FX} if they held a sufficiently large amount of liquidity. (See above for the precise argument in the case of local liquidity management.)

2. Suppose that for all p, $pr_{FX}(p)$ is strictly decreasing in p. (Ie, we need $t < t_2$, compare Lemma 6.)

   (i) If $P''' = \varnothing$, then for each $p'' \in P''$, the equilibrium takes the form G2.

   (ii) If $P''' \neq \varnothing$. Again, take any $p'' \in P''$, $p''' \in P'''$ such that $p''' > p''$. Then for each $p'' \in P''$, linearity of $pr_{FX}(p)$ implies that $p\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\}\right)$ reaches its maximum either at p'' (where $C = \lambda r_{FX}(p, t, P_{FX})$), or at $p = 1$. The former is true when $pr_{FX}(p, t, P_{FX})$ decreases faster in p than $p(C + \lambda r_I(P_I))$ increases. The slope of $pr_{FX}(p)$,

$$\frac{\partial p_E r_{FX}}{\partial p_E} = -(1-t) + t\left(\frac{1 - E\left[p|p \in P_{FX}\right]}{1 - t\left(1 - E\left[p|p \in P_{FX}\right]\right)}\right)$$

   is increasing in t and reaches a minimum of -1 at $t = 0$, when settlement is PvP; compare Lemma 6). If $C/\lambda + r_I(P_I) - 1 > 0$, then $p\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\}\right)$ reaches a maximum at $p = 1$. If, in contrast, $C/\lambda + r_I(P_I) - 1 < 0$, there is a $t_3 : 0 \leq t_3 < t_2$ such that for all $t \leq t_3$, $p\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\}\right)$ reaches a maximum at $p = p''$, whereas for all $t \in (t_3, t_2)$, $p\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p, t, P_{FX})\}\right)$ reaches a maximum at $p = 1$. Consider these cases in turn.

      a. If $t \leq t_3$ (recall that t_3 only exists if $C/\lambda + r_I(P_I) - 1 < 0$) and the opportunity cost of liquidity ρ is small enough such that P''' is not empty, and large enough to ensure that the lowest risk ($p = 1$) prefers $L_E = 0$ over hoarding liquidity ($6\lambda\rho > C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(1, t, P_{FX})\}$), then low types choose technical default after s_3, intermediate types hoard liquidity, and high types refinance via an FX swap. The equilibrium G4 is characterised by the indifferent types p_1''' and p_2''',

$$\begin{aligned}
6\lambda\rho &= p_1'''\left(2C + \lambda r_I(P_I)\right) \\
6\lambda\rho &= p_2'''\left(C + \lambda r_I(P_I) + r_{FX}\left(p_2''', t, P_{FX}\right)\lambda\right)
\end{aligned}$$

      and $P_{FX} = \left[p_2''', 1\right]$, $P_I = \left[0, p_1'''\right] \cup \left[p_2''', 1\right]$. Liquidity holdings are

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq p_1''' \text{ or } p_E \geq p_2''' \\ \lambda & \text{if} \quad p_1''' < p_E < p_2''' \end{cases}$$

      (Notice that if ρ is larger, we have the above case $P''' = \varnothing$. If ρ is smaller, p_2''' does not exist in [0, 1], and $P_{FX} = \varnothing$. If p_2''' exists, we must have $p_1''' < p'' < p_2'''$ because $C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(1, t, P_{FX})\}$ has a maximum at p''.)

To check whether this equilibrium passes the Intuitive Criterion, consider an interval of types including p_1'''.

- After s_3, they have no incentive to signal their type. For $p_E < p_1'''$, technical default is more attractive than taking out the FX swap. The FX swap is taken out only by better risks, hence if types risks around p_1''' managed to signal their type, the interest rate that they would be charged would exceed the interest rate charged by those taking out the FX swap.
- After s_5, they have no incentive to signal their type if they are riskier that the average type in P_I, ie, if $p_1''' < E\left[p : p \in P_I\right]$. (Recall $P_I = \left[0, p_1'''\right] \cup \left[p_2''', 1\right]$.)

Now consider an interval of types including p_2'''.

- After s_3, they form the lower bound of types taking out the FX swap, so signalling their types would make them worse off.
- After s_5, they have no incentive to signal their type if they are riskier that the average type in P_I, ie, if $p_2''' < E\left[p : p \in P_I\right]$.

Hence, a sufficient condition for G4 to pass the Intuitive Criterion is $p_2''' < E\left[p : p \in P_I\right]$. But it is not necessary, given that both s_3 and s_5 are equally likely, and that an incentive to signal their type never exists after s_5. A necessary and sufficient condition would be for the increase in the cost of FX swap (after signalling) after s_3 to (just) more than offset the decline in the cost of domestic borrowing after s_5. Equally well, the condition could be phrased in terms of likelihoods of s_3 and s_5. If a situation in which only one subsidiary of the global bank experiences a liquidity outflow (s_3) is sufficiently less likely than a situation in which both subsidiaries experience such an outflow (s_5), then $p_2''' < E\left[p : p \in P_I\right]$ is necessary and sufficient.

b. If, in contrast, $t \in (t_3, t_2)$, then expected refinancing costs are increasing despite the fact that the expected refinancing costs via an FX swap are decreasing. Thus, it is unsurprising that we get the same equilibrium as in the case where the expected refinancing costs via an FX swap are increasing (equilibrium G5).

Step 5: Types of equilibria when the domestic interbank market is closed after s_5

The proof is very similar – only $\lambda r_I(P_I)$ is replaced by C because after s_5, all types with $L_E = 0$ now delay payment. Hence G_E prefers $L_E = \lambda$ over $L_E = 0$ if $6\lambda\rho \leq p_E(\min\{r_{FX}\lambda, C\} + 2C)$. P''' can be redefined accordingly as $P''' = \{p \in [0,1] : 6\lambda\rho = p(\min\{r_{FX}(p)\lambda, C\} + 2C)\}$. The following equilibria pass the Intuitive Criterion:

- If $P''' = \varnothing$, then $L_E = 0$ for all types, $P_I = \varnothing$, and for each $p'' \in P''$, the equilibrium takes the form $P_{FX} = \left[p'', 1\right]$: given P_{FX}, $r_{FX}\lambda$ is strictly declining in p_E, whereas C is constant in p_E, so given P_{FX}, all $p_E < p''$ strictly prefer delay over refinancing, and *vice versa* for all $p_E > p''$.
- If $P''' \neq \varnothing$ and $pr_{FX}(p)$ is strictly decreasing in p, the equilibrium takes the form
$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \text{ or } p_E \geq p''' \\ \lambda & \text{if} \quad 2\lambda\rho/C < p_E < p''' \end{cases}$$
where here $p''' : 6\lambda\rho = p(\min\{r_{FX}(p)\lambda, C\} + 2C)$, and $P_{FX} = \left[p''', 1\right]$, $P_I = \varnothing$.

Comparative statics of P_{FX} and P_I with respect to t

Lemma 7 In all equilibria, the lower bound to P_{FX} strictly falls in t.

Proof. p'' is defined by $r_{FX}(p'') = C/\lambda$. Taking total derivatives yields
$$\left(\frac{\partial r_{FX}}{\partial p} - \frac{\partial(C/\lambda)}{\partial p}\right)dp + \left(\frac{\partial r_{FX}}{\partial t} - \frac{\partial(C/\lambda)}{\partial t}\right)dt = \frac{\partial r_{FX}}{\partial p}dp + \frac{\partial r_{FX}}{\partial t}dt = 0$$
such that
$$\frac{dp''}{dt} = -\frac{\partial r_{FX}}{\partial t} / \frac{\partial r_{FX}}{\partial p}$$

In the equilibria under consideration, $P_{FX} = \left[p, 1\right]$ for $p \in \{p'', p'''\}$. Then $E\left[p|p \in P_{FX}\right] = (p+1)/2$. Hence, an increase in p increases both the informed and the uninformed lending in the FX swap: the informed (intragroup) lender knows that he faces a better type, and the uninformed (external) lender knows that the mix of types he faces has better average quality. Thus $\partial r_{FX}/\partial p < 0$.

Regarding the derivative with respect to t,

$$\frac{\partial r_{FX}}{\partial t} = \frac{\partial\left((1-t)\frac{1-p}{p} + t\left(\frac{1-(p+1)/2}{1-t(1-(p+1)/2)}\right)\right)}{\partial t} = -(1-p)\frac{(-t+pt+2)^2-2p}{p(-t+pt+2)^2} < 0$$

It is straightforward to show that the right-hand side is negative as intuition suggests: a decrease in the share of informed lending reduces the costs for the marginal (ie, worst) borrower. Thus,

$$\frac{dp''}{dt} = -\frac{\partial r_{FX}}{\partial t}/\frac{\partial r_{FX}}{\partial p} < 0$$

The definition of p''' depends on the equilibrium under consideration. p''' is not defined in equilibria of type G2 and G1.

In G3,

$$6\lambda\rho = p'''\left(2C + \lambda r_{FX}\left(p''', t, P_{FX}\right)\right)$$

Taking total derivatives yields

$$\begin{aligned}
&\left(\frac{\partial\left(p\left(2C+\lambda r_{FX}\left(p,t,P_{FX}\right)\right)\right)}{\partial p} - \frac{\partial(6\lambda\rho)}{\partial p}\right)dp + \left(\frac{\partial\left(p\left(2C+\lambda r_{FX}\left(p,t,P_{FX}\right)\right)\right)}{\partial t} - \frac{\partial(6\lambda\rho)}{\partial t}\right)dt \\
= &\ \frac{\partial\left(p\left(2C+\lambda r_{FX}\right)\right)}{\partial p}dp + p\lambda\frac{\partial r_{FX}}{\partial t}dt = 0 \\
= &\ \left(2C + \lambda\frac{\partial\left(pr_{FX}\right)}{\partial p}\right)dp + p\lambda\frac{\partial r_{FX}}{\partial t}dt = 0
\end{aligned}$$

such that

$$\frac{dp'''}{dt} = -p\lambda\frac{\partial r_{FX}}{\partial t}/\left(2C + \lambda\frac{\partial\left(pr_{FX}\right)}{\partial p}\right)$$

From above, we know that $\partial r_{FX}/\partial t < 0$. Also, we assumed that

$$\frac{\partial\left(p\left(2C+\lambda r_{FX}\left(p,t,P_{FX}\right)\right)\right)}{\partial p} < 0$$

in the derivation of G3 for constant P_{FX}. Hence, if the average risk in P_{FX} improves as p''' increases, this property holds *a fortiori* for variable P_{FX}. Thus,

$$\frac{dp'''}{dt} < 0$$

In G4, the direct effect of an increase in t is a decrease in refinancing costs via an FX swap at p_2'''. The costs of the alternative, hoarding liquidity, are constant. At p_2''', expected refinancing costs are declining in p. Hence, a decline in p compensates for the increase in t, implying that $dp_2'''/dt < 0$.

Proof of Propositions 2 and 3 (delay and transmission of losses)

In G1 and G2, the likelihood of delay is proportional to $\Pr(p_E < p'')$; in G3, it is proportional to $\Pr(p_E < 2\lambda\rho/C)$, and in G4, to $\Pr\left(p_E < p_1'''\right)$. The likelihood of transmission of losses is proportional to

$$E\left[1 - p_E | p_E \geq p''\right] \Pr\left(p_E \geq p''\right) = \frac{1}{4}\left(1 - p''\right)^2$$

in G1 and G2; to

$$E\left[1 - p_E | p_E \geq p'''\right] \Pr\left(p_E > p'''\right) = \frac{1}{4}\left(1 - p'''\right)^2$$

in G3; and to

$$\begin{aligned}
& E\left[1 - p_E | p_E \in \left[0, p_1'''\right] \cup \left[p_2''', 1\right]\right] \Pr\left(p_E \in \left[0, p_1'''\right] \cup \left[p_2''', 1\right]\right) \\
& + E\left[1 - p_E | p_E \in \left[p_2''', 1\right]\right] \Pr\left(p_E \in \left[p_2''', 1\right]\right) \\
= & \int_0^x (1 - p)\, dp + 2\int_y^1 (1 - p)\, dp = \frac{1}{2} x (2 - x) + (1 - y)^2
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial\left(\frac{1}{2}x(2 - x) + (1 - y)^2\right)}{\partial x} &= 1 - x > 0 \\
\frac{\partial\left(\frac{1}{2}x(2 - x) + (1 - y)^2\right)}{\partial y} &= -2(1 - y) < 0
\end{aligned}$$

in G4. The following sections compute how these probabilities react to changes in t.

From Lemma 7, $\partial p'/\partial t < 0$ for $p' \in \{p'', p'''\}$; thus, the likelihood of delay is decreasing in t in G1 and G2, and independent of t in G3. Regarding the likelihood of transmission of losses, we have that

$$\frac{\partial}{\partial t}\left(\frac{1}{4}\left(1 - p'\right)^2\right) = \frac{1}{2}\left(1 - p'\right)\left(-\frac{\partial p'}{\partial t}\right) > 0$$

Thus, the likelihood of transmission of losses is decreasing in t for G1-G3.

For G4, the situation is more complicated. An increase in t has a direct effect only on p_2''' – refinancing costs via an FX swap fall for the riskiest bank in P_{FX} because

$p_2''' < E\left[p : p \in P_{FX}\right]$, hence p_2''' falls. For $6\lambda\rho = p_1'''\left(2C + \lambda r_I\left(P_I\right)\right)$ to continue to hold, we need (replacing $x = p_1'''$, $y = p_2'''$) the total derivative of this equation to equal zero, ie, that

$$\begin{aligned}
&\left(\frac{\partial\left(x\left(2C/\lambda + r_I\left(P_I\right)\right)\right)}{\partial x} - \frac{\partial\left(6\rho\right)}{\partial x}\right)dx + \left(\frac{\partial\left(x\left(2C/\lambda + r_I\left(P_I\right)\right)\right)}{\partial y} - \frac{\partial\left(6\rho\right)}{\partial y}\right)dy \\
&+\left(\frac{\partial\left(x\left(2C/\lambda + r_I\left(P_I\right)\right)\right)}{\partial t} - \frac{\partial\left(6\rho\right)}{\partial t}\right)dt \\
= &\left(2C/\lambda + \frac{\partial\left(xr_I\left(P_I\right)\right)}{\partial x}\right)dx + x\frac{\partial r_I\left(P_I\right)}{\partial y}dy = 0
\end{aligned}$$

The sign of the partial derivatives of r_I depends on the position of $E\left[p : p \in P_I\right]$:

- If $y < E\left[p : p \in P_I\right]$, then $\partial r_I\left(P_I\right)/\partial y < 0$, and $\partial r_I\left(P_I\right)/\partial x > 0$, hence $\partial\left(xr_I\left(P_I\right)\right)/\partial x > 0$. Hence, dx/dt has the same sign as dy/dt, and we have
$$\frac{\partial p_1'''}{dt}, \frac{\partial p_2'''}{dt} < 0$$
Then the likelihood of delay is falling in t. But a decline in x and y have opposite effects, so that the reaction of the likelihood of transmission of losses remains ambiguous.
- If $x < E\left[p : p \in P_I\right] < y$, then $\partial r_I\left(P_I\right)/\partial y > 0$, and $\partial r_I\left(P_I\right)/\partial x > 0$, hence $\partial\left(xr_I\left(P_I\right)\right)/\partial x > 0$. Then
$$\frac{\partial p_2'''}{dt} < 0 < \frac{\partial p_1'''}{dt}$$
Then the likelihood of delay and the likelihood of transmission of losses both rise in t.
- If $E\left[p : p \in P_I\right] < x$, then $\partial r_I\left(P_I\right)/\partial y > 0$, and $\partial r_I\left(P_I\right)/\partial x < 0$, hence the sign of $\partial\left(xr_I\left(P_I\right)\right)/\partial x$ is ambiguous, and we cannot determine the sign of $\partial p_1'''/dt$ unambiguously.

Proof of Proposition 4 (comparative statics)

The following sections compute how these probabilities react to changes in C/λ and ρ, respectively.

Comparative static properties with respect to C/λ

p'' is defined by $r_{FX}(p'') = C/\lambda$. Taking total derivatives yields

$$\left(\frac{\partial r_{FX}}{\partial p} - \frac{\partial (C/\lambda)}{\partial p}\right) dp + \left(\frac{\partial r_{FX}}{\partial (C/\lambda)} - \frac{\partial (C/\lambda)}{\partial (C/\lambda)}\right) d(C/\lambda) = \frac{\partial r_{FX}}{\partial p} dp - d(C/\lambda) = 0$$

such that

$$\frac{dp''}{d(C/\lambda)} = 1/\frac{\partial r_{FX}}{\partial p} < 0$$

Thus, for G1-G3, the likelihood of delay is decreasing in the cost of delay, and increasing in the size of the liquidity shock. Correspondingly, the likelihood of transmission of losses is increasing in C/λ in G1 and G2. In contrast, it is decreasing in G3 because $dp'''/d(C/\lambda) > 0$:

$$\begin{aligned} & \left(\frac{\partial (p''' (2C/\lambda + r_{FX}(p''', t, P_{FX})))}{\partial p'''} - \frac{\partial (6\rho)}{\partial p'''}\right) dp''' \\ & + \left(\frac{\partial (p''' (2C/\lambda + r_{FX}(p''', t, P_{FX})))}{\partial (C/\lambda)} - \frac{\partial (6\rho)}{\partial (C/\lambda)}\right) d(C/\lambda) \\ = & \left(\frac{\partial (p''' (2C/\lambda + r_{FX}(p''', t, P_{FX})))}{\partial p'''}\right) dp''' + p''' d(C/\lambda) = 0 \end{aligned}$$

By construction of G3, we know that $\partial (p''' (2C/\lambda + r_{FX}(p''', t, P_{FX}))) / \partial p''' < 0$ for constant P_{FX}, hence also for endogenous P_{FX}. Hence $dp'''/d(C/\lambda) > 0$. The intuition is that at p''', the cost of failing to hoard liquidity has increased because after s_1, delay is the only option when liquidity is scarce. Hence, p''' (and slightly better risks) hoard liquidity, and the average quality of banks taking out an FX swap increases.

In G4, the probability of delay is proportional to $\Pr\left(p_E < p_1'''\right)$. At p_1''', expected refinancing costs are upwards sloping in p; at p_2''', they are downwards sloping. Hence, the direct effect of an increase in C/λ is an increase in the slope of expected refinancing costs. Both p_1''' and p_2''' are defined by the intersection of expected refinancing costs with a constant (the cost of hoarding liquidity). This implies that the direct effect of the increase in C/λ reduces p_1''' and increases p_2'''. Thus, the probability of delay falls as C/λ rises. The reaction of the likelihood of transmission of losses again depends on the average risk in P_I. If $p_2''' < E\left[p : p \in P_I\right]$, then $E\left[p : p \in P_I\right]$ rises as p_1''' falls and p_2''' increases. Thus, an increase in C/λ means that less risky banks refinance, and the likelihood of transmission of losses falls. (These direct effects may be amplified or reduced by an indirect effect that an increase in C/λ has on P_I and P_{FX}, hence r_D and r_{FX}.)

Comparative static properties with respect to ρ

In G1 and G2, the subsidiary does not hold any liquidity because ρ is too high; consequently, a marginal change in ρ does not change either the likelihood of delay nor the likelihood of transmission of losses. In G3, it is immediate that the likelihood of delay, $\Pr\left(p_E < 2\lambda\rho/C\right)$, increases as ρ rises. Regarding the likelihood of transmission of losses, taking total derivatives of the definition of p''' yields

$$\begin{aligned}
& \left(\frac{\partial\left(p'''\left(2C/\lambda + r_{FX}\left(p''', t, P_{FX}\right)\right)\right)}{\partial p'''} - \frac{\partial\left(6\rho\right)}{\partial p'''}\right) dp''' \\
& + \left(\frac{\partial\left(p'''\left(2C/\lambda + r_{FX}\left(p''', t, P_{FX}\right)\right)\right)}{\partial \rho} - \frac{\partial\left(6\rho\right)}{\partial \rho}\right) d\rho \\
= \; & \frac{\partial\left(p'''\left(2C/\lambda + r_{FX}\left(p''', t, P_{FX}\right)\right)\right)}{\partial p'''} dp''' - 6d\rho = 0
\end{aligned}$$

By construction of G3, we know that $\partial\left(p'''\left(2C/\lambda + r_{FX}\left(p''', t, P_{FX}\right)\right)\right)/\partial p''' < 0$. Hence $dp'''/d\rho < 0$, and the likelihood of transmission of losses rises.

For G4, the same argument applies as above. At p_1''', expected refinancing costs are upwards sloping in p; at p_2''', they are downwards sloping. Both p_1''' and p_2''' are defined by the intersection of expected refinancing costs with a constant (the cost of hoarding liquidity). If this constant increases, the direct effect is an increase in p_1''' and a decrease in p_2'''. Thus, the probability of delay increases as ρ rises. If $p_2''' < E\left[p : p \in P_I\right]$, then $E\left[p : p \in P_I\right]$ falls as p_1''' increases and p_2''' falls. In this case, the likelihood of transmission of losses increases.

Comparison between global and local liquidity management

Convergence of (G2, D2) to (D2, D2)

Lemma 8 For $t \to 1$, equilibria of type (G2,D2) converge to (D2,D2).

Using $P_{FX} = \left[p'', 1\right]$ and $p'' : r_{FX}(p'') = C/\lambda$, one can solve explicitly for p'':

$$p'' = -\frac{\left(2C + 2\lambda - 5t\lambda + 2t^2\lambda - Ct - \sqrt{-(t-2)\left(4C\lambda + 2\lambda^2 - C^2 t - t\lambda^2 + 2C^2 - 6Ct\lambda\right)}\right)}{2t\left(C + \lambda(2-t)\right)}$$

Taking limits yields

$$\lim_{t\to 1} p'' = \frac{\lambda - C + \sqrt{(C-\lambda)^2}}{2(C+\lambda)} = \begin{cases} 0 & \text{if} \quad C > \lambda \\ (\lambda - C)/(\lambda + C) & \text{if} \quad C \leq \lambda \end{cases}$$

$t' < 1$ (see the proof of Lemma 6), so we have $\partial p_E r_{FX}(p_E)/\partial p_E > 0$. Then G2 exists if and only if $\rho > \frac{1}{6}\left(C/\lambda + r_I + r_{FX}(1, t, P_{FX})\right)$. In addition, in G2, $P_I = [0, 1]$, hence $r_I = 1$. Taking limits with respect to t yields

$$\begin{aligned} \rho &> \frac{1}{6}\left(C/\lambda + r_I\right) + \frac{1}{6}\lim_{t\to 1}\left((1-t)\frac{1-1}{1} + t\left(\frac{1 - E\left[p|p \in P_{FX}\right]}{1 - t\left(1 - E\left[p|p \in P_{FX}\right]\right)}\right)\right) \\ &= \frac{1}{6}\left(C/\lambda + 1\right) + \frac{1}{6}\left(\frac{1 - \frac{1}{2}\left(1 + \max\left\{0, \frac{\lambda - C}{C+\lambda}\right\}\right)}{1 - \left(1 - \frac{1}{2}\left(1 + \max\left\{0, \frac{\lambda - C}{C+\lambda}\right\}\right)\right)}\right) = \frac{1}{6}\left(C/\lambda + 1\right) + \frac{1}{6}\left(\frac{1 - \max\left\{0, \frac{\lambda - C}{C+\lambda}\right\}}{1 + \max\left\{0, \frac{\lambda - C}{C+\lambda}\right\}}\right) \end{aligned}$$

Now $C > \lambda$ implies that this condition is equal to $\rho > \frac{1}{6}(C/\lambda + 1) + \frac{1}{6} = \frac{1}{6}(2 + C/\lambda)$, which is the condition under which D2 exists. Also, for $C > \lambda$, $\lim_{t\to 1} p'' = 0$, such that $P_{FX} = [0, 1] = P_I$ and $L_E = 0$ in both equilibria. That is, for $t \to 1$, the global bank's subsidiaries do not hold liquidity in either equilibrium, and if hit by a liquidity outflow, they both refinance independently of their risk using an overnight loan (or, in G2, via an FX swap that is indistinguishable from an overnight loan).

Comparison of optimal liquidity holdings under local and global liquidity management

We would like to show that there are situations (combinations of parameter values) under which the global bank holds no precautionary liquidity when it manages liquidity globally, but would hold liquidity under local management. We focus on a comparison of an equilibrium (G2,D2) in which G_E's behaviour follows G2, and G_W's behaviour follows D2, with an equilibrium (D2,D2) in which both subsidiaries follow D2. (G2,D2) is an equilibrium under global liquidity management; (D2,D2) an equilibrium under local liquidity management. We need to show that (G2,D2) exists if (D2,D2) exists, but not *vice versa*.

Lemma 9 If $t = 0$, then (G2,D2) exists if (D2,D2) exists. The converse is not true.

To determine the condition under which (G2,D2) exists for $t = 1$, we reconsider G_E's choice for $t = 0$. p'' is defined by $r_{FX}(p, t, P_{FX}) = C/\lambda$. At $t = 0$, $r_{FX}(p) = (1 - p)/p$, see equation **(2)**, hence $p'' = \lambda/(\lambda + C)$. In G2, G_E's expected costs of refinancing reach their peak at p''. Hence, the liquidity-short subsidiary prefers to hold no precautionary liquidity reserves independently of its risk if

$$\begin{aligned} (1-\lambda)\rho &< (1-0)\rho - \frac{1}{6}p_E\left(C + \min\{C, r_{FX}(\lambda - 0)\} + \min\{C, r_I(\lambda - 0)\}\right) \\ 6\lambda\rho &> p_E\left(C + \lambda r_I(P_I) + \min\{C, \lambda r_{FX}(p_E, 0, P_{FX})\}\right) \end{aligned}$$

holds for all p_E, hence also at p''. Also, in G2, $P_I = [0, 1]$, hence $r_I(P_I) = 1$. Equivalently,

$$\begin{aligned} 6\lambda\rho &> \frac{\lambda}{\lambda + C}(C + \lambda + C) \\ 0 &> \frac{\lambda(C + \lambda + C)}{\lambda + C} - 6\lambda\rho = \lambda\frac{2C + \lambda - 6C\rho - 6\lambda\rho}{C + \lambda} \end{aligned}$$

which holds if

$$\begin{aligned} 2C + \lambda - 6C\rho - 6\lambda\rho &< 0 \\ \rho &> \frac{2C + \lambda}{6C + 6\lambda} \end{aligned}$$

In contrast, D2 exists if $\rho \geq \frac{1}{6}(2 + C/\lambda)$. Because

$$\frac{2C + \lambda}{6C + 6\lambda} - \frac{1}{6}(2 + C/\lambda) = -\frac{1}{6}\frac{C\lambda + \lambda^2 + C^2}{\lambda(C + \lambda)} < 0$$

the condition on D2 is stronger, meaning that (G2,D2) exists if (D2,D2) exists, but not *vice versa*.

Lemma 10 If $t = 0$, then there are no parameter values for which both (G3,D2) and (G2,D2) exist.

In G3, p''' is defined by

$$6\lambda\rho = p'''\left(2C + \lambda r_{FX}\left(p''', t, P_{FX}\right)\right)$$

equivalently $6\lambda\rho = p\left(2C + \lambda\frac{1-p}{p}\right)$, or

$$p = \lambda\frac{1-6\rho}{\lambda - 2C}$$

and we assumed that in G3, expected refinancing costs are decreasing in p at p''', ie, that

$$\frac{\partial p\left(2C + \lambda r_{FX}\left(p, t, P_{FX}\right)\right)}{\partial p} = \frac{\partial\left(2Cp + \lambda\left(1-p\right)\right)}{\partial p} = 2C - \lambda < 0$$

Then G3 exists if $\left[2\lambda\rho/C, p'''\right]$ is non-empty, ie, if

$$\lambda\frac{6\rho - 1}{2C - \lambda} - 2\lambda\rho/C > 0$$

equivalently, $\rho < \frac{1}{2}\frac{C}{\lambda + C}$. Now D2 exists if $\rho \geq \frac{1}{6}\left(2 + C/\lambda\right)$ and we have

$$\frac{1}{6}\left(2 + C/\lambda\right) - \frac{1}{2}\frac{C}{\lambda + C} = \frac{1}{6}\frac{2\lambda^2 + C^2}{\lambda\left(C + \lambda\right)} > 0$$

hence D2 does not exist when G3 exists. Hence G3 need to be compared with D1. In D1, liquidity holdings are

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \\ \lambda & \text{if} \quad p_E \geq 2\lambda\rho/C \end{cases}$$

whereas in G3, they are

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \text{ or } p_E > \lambda\frac{1-6\rho}{\lambda-2C} \\ \lambda & \text{if} \quad p_E \in \left[2\lambda\rho/C, \lambda\frac{1-6\rho}{\lambda-2C}\right] \end{cases}$$

Hence clearly, liquidity holdings in G3 are smaller than in D1.

Lemma 11 If $t = 0$, then there are no parameter values for which both G2 and G3 exist.

G3 exists if $\rho > \frac{2C+\lambda}{6C+6\lambda}$ and $2C - \lambda < 0$. G2 exists when $\rho > \frac{2C+\lambda}{6C+6\lambda}$. Now

$$\frac{2C + \lambda}{6C + 6\lambda} - \frac{1}{2}\frac{C}{\lambda + C} = \frac{1}{6}\frac{\lambda - C}{C + \lambda} > 0$$

Hence, G2 does not exist when G3 exists.

List of symbols

- D_E and D_W for the domestic banks in countries E and W; G_E and G_W for the subsidiaries of the global bank.
- λ is the size of the liquidity shock.
- p_E and p_W are the likelihoods that the subsidiaries' illiquid assets pay off R/p_E (R/p_W).
- P_{DE} (and P_{DW}) is the set of types of G_E (and G_W) that opt for an interbank loan when liquidity management is local.
- P_I is the set of types that opt for an interbank loan when liquidity management is global.
- P_{FX} is the set of types that opt for an FX swap.
- ρ is the expected net pay-off of each subsidiary's illiquid assets (ie, expected gross return minus one).
- C is the cost of delay.
- L_E and L_W: liquidity holdings of the global bank's subsidiaries. Total balance sheet size is one in period zero.
- B_E and B_W is the amount the liquidity-short branch borrows.
- r_{Di} is the per-dollar fee the domestic bank charges G_i for its loan when liquidity management is local.
- r_I is the per-dollar fee the domestic bank charges G_i for its loan when liquidity management is global.
- s_D is the per-dollar fee the domestic bank requires as compensation for Herstatt risk in an FX swap.
- r_G is the intragroup per-dollar fee the liquidity-rich subsidiary charges the liquidity-short subsidiary as part of the FX swap.

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