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Multivariate methods for monitoring structural change

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Abstract

Detection of structural change is a critical empirical activity, but continuous 'monitoring' of time series for structural changes in real time raises well-known econometric issues. These have been explored in a univariate context. If multiple series co-break, as may be plausible, then it is possible that simultaneous examination of a multivariate set of data would help identify changes with higher probability or more rapidly than when series are examined on a case-by-case basis. Some asymptotic theory is developed for a maximum CUSUM detection test. Monte Carlo experiments suggest that there is an improvement in detection relative to a univariate detector over a wide range of experimental parameters, given a sufficiently large number of co-breaking series. The method is applied to UK RPI inflation in the period after 2001. A break is detected which would not have been picked up by univariate methods.

Key words: Monitoring, structural change, panel, CUSUM, fluctuation test.

JEL classification: C100, C590.

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Summary

Monetary policy makers need to know what is happening now in the economy, and also to have some idea what will happen in the future. To do the latter, they need to forecast. But a major practical problem is that one of the main causes of forecast failure is structural change. Often, as David Hendry and Mike Clements have emphasised, this manifests itself as a 'mean shift'; for example, a step change in a growth rate. In some cases, policymakers might have a good idea when such changes take place. For example, the shift to inflation targeting in the United Kingdom in 1992 and the move to Bank independence in 1997 were clear structural changes, with likely consequences for inflation. But in other cases, such as the period after a large rise in energy prices, the case may not be so obvious. It would be helpful to have statistical techniques that help us look for evidence of such changes in the data.

However, such a continuous 'monitoring' of series for structural changes, period after period, raises well-known econometric issues. Statistical tests are designed so that accepting a false hypothesis happens only a small proportion of times, often set at 5%. The idea is that if we do such a test only once, then there is only one chance in 20 of making this type of mistake. In this way we can be quite confident that results are unlikely to have been generated by chance; it is a cautious approach. But it is easy to see that if such a test is repeated many times then eventually it will accept a false hypothesis (in this case, that a break has happened) purely by chance. This has led to the development of techniques looking at single time series accounting for this problem. But in practice such tests are not very successful in detecting breaks, as they must be inherently conservative.

The simple insight explored in this paper is that if several time series of data have a structural break at roughly the same time ('co-break'), as may often be plausible, then it is possible that simultaneous examination of a set of such variables helps identify changes with higher probability or more rapidly than when each is examined on a case-by-case basis. Naturally, this need not necessarily imply that there is a break in some aggregate series of interest, although it may do so. Some statistical theory is developed for such a method, which cumulates forecast errors from many series and picks the maximum at each point to construct a 'CUSUM' (cumulated sum) detection test, or 'detector'. Breaks leading to forecast failure often manifest themselves by mean shifts, even if the shock to the variable is temporary. We therefore focus on

this type. Monte Carlo experiments (simulating data generating processes thousands of times) suggest that there is an improvement in detection relative to a single variable test over a wide range of experimental parameters, given a sufficiently large number of co-breaking series. It should be clear, however, that this method only has the potential to detect the existence of a general break; it is not informative about the precise nature of that object.

One very natural application is UK retail prices index (RPI) inflation in the period after 2001. This is partly because many subcomponent series are published (about 80 on a consistent basis over the relevant period). But there are also several reasons to suppose that at least some of these may have experienced breaks. Although inflation is determined by monetary policy in the long run, in the short run large fluctuations in important prices may lead to the breakdown of the type that we consider in empirical relationships. From 1992 to 2001 there was a high degree of stability in aggregate RPI inflation. But thereafter house price inflation fluctuated fairly widely (peaking at over 25% per year) and energy prices rose dramatically after 2004. As policymakers, we are mainly interested in the aggregate, and not all of the subcomponents need to have co-broken for there to be an impact on total RPI, so this method could be useful. On the other hand, we should recognise that breaks may be offsetting, so that there may be no effect on the aggregate series.

It turns out that univariate methods would not have detected any breaks in the aggregate series, but the multivariate method would have indicated a potential break in 2005, which would then suggest further examination using other methods. And it appears from an examination of the data over the whole period, and therefore with the benefit of hindsight, that such a break may have occurred in the aggregate RPI series. It should be noted, however, that the evidence is not overwhelming, and there is no sign of a break in the inflation series that was targeted for much of this period (RPI excluding mortgage interest payments). Nevertheless, this new method may be a useful addition to the toolkits of policymakers and other forecasters.

1 Introduction

Detection of structural change is a critical empirical activity, for the obvious reason that if such changes are ignored then econometric relations are misspecified, from which numerous problems may flow. An area where it may be particularly important is forecasting. Clements and Hendry argue forcefully (in, eg, 1998a, b) that the main source of forecast error is structural change; Hendry (2000) argues that the dominant cause of these failures is the presence of deterministic shifts. Moreover, even where there are not permanent shifts in the true data-generation process, in misspecified forecasting models a wide variety of events leading to forecasting failure may manifest themselves as mean shifts. One pertinent example may be a large movement in a variable or variables excluded from the forecasting model. Stock and Watson (1996) looked at many forecasting models of a large number of US time series, and found evidence for parameter instability in a significant proportion of the relations. Groen, Kapetanios and Price (2009) examine the Bank of England record for output growth and inflation forecasts, assessed against some statistical benchmarks. They find the inflation forecast record is very good, and suggest that this flows partly from the Bank's ability to detect mean shifts in the series. So this is an important issue for macroeconomic policymaking.

Break detection has a long history. The seminal paper testing for a break at a known point was Chow (1960). Andrews (1993) introduced a methodology that allowed for unknown break-points: one influential paper is Bai and Perron (1998). All these tests are for breaks against specific alternatives. While effective in that case, they are ineffective when the break is not covered by the particular alternative. In addition, by their nature they require some end-of-sample observations (typically, 10% to 15%) to perform the test. So real-time detection is impossible.

An alternative to this general methodology was the CUSUM[1] approach of Brown, Durbin and Evans (1975).[2] The advantage of the CUSUM test flows from the fact that there are many ways to reject the hypothesis of no structural change. While Wald, Lagrange multiplier and likelihood ratio tests are efficient against specific alternatives, the CUSUM test's usefulness lies partly in the fact that it offers a graphical view of deviations from constancy. But formal significance tests based on boundary conditions can be constructed for hypotheses likely to be observed in

[1]Based on a cumulative sum of squared errors, and an example of a fluctuation test. While some authors (eg Chu, Stinchcombe and White (1996)) reserve the term for tests based on variation in parameters, others (eg Zeileis, Leisch, Kleiber and Hornik (2005)) apply it more generally.

[2]Extended to dynamic models by Krämer, Ploberger and Alt (1988).

practice. Thus the method is more likely to be robust under different break scenarios. Moreover, there is no sample-trimming problem. On the negative side, however, after detection it is hard to establish the cause of the break. Furthermore, Monte Carlo studies reveal that in practice they have low power of break detection: there is a price to be paid for generality.

But all these tests are 'retrospective', in the sense that they are designed to test for change in particular data sets; a one-off experiment. The problem that is usually faced in practice is the continual monitoring of a series via repeated tests, whereby tests are applied in successive periods or at intervals. It is not hard to see that as the monitoring period increases, the probability of rejecting a true null hypothesis of no break will eventually approach unity, following the law of iterated logarithm. It is possible this will occur quite rapidly, as (eg) Chu *et al* (1996) show with some simulations. The challenge, then, is to find suitable boundary conditions to obviate this. Consequently Chu *et al* (1996) introduced a sequential testing procedure using fluctuation tests with asymptotically correct size. Zeileis *et al* (2005) subsequently explored some extensions in dynamic models, and Leisch, Hornik and Kuan (2000) generalised the class, extending Kuan and Hornik (1995).

But there may be a way to extend the tests to obtain more effective monitoring procedures. The idea is that there are common breaks, also known as 'co-breaks',[3] in multiple time series. This has a natural appeal. For example, changes in monetary regime may affect steady-state inflation, and we would expect this to be reflected in disaggregate inflation measures. Or shifts in total factor productivity growth may occur simultaneously in different industries or countries. If the series under consideration are components of an aggregate, such as inflation, it follows that if a component breaks, or if components co-break, then a break also occurs in the aggregate. This can be important for policymakers; for example, detecting changed steady-state inflation after a new monetary regime. An obvious question to ask, therefore, is whether detection of such a common break is easier with multiple series.

Bai, Lumsdaine and Stock (1998) develop some asymptotic distribution theory for maximum likelihood detection of a break in a multivariate model, but theirs is the multivariate equivalent of the fixed-sample tests. Our approach is instead to extend the monitoring framework of Chu *et al* (1996) to a multivariate setting. Chu *et al* (1996) establish that a limiting distribution applies to

[3]Clements and Hendry (1999) are mainly concerned with co-breaks in non-stationary systems, analagous to cointegrating relations. In our paper co-breaking signifies that some fraction of the series considered experience a break at approximately the same point in time.

the univariate CUSUM test statistic such that the critical boundary condition is an increasing function of time.

We assume that there is a set of variables which are generated by structurally stable processes over some initial period. There is then a subset of these variables that co-break at some point. The task of the econometrician is to detect that co-breaking point by a monitoring process that starts after the initial sample. To do these we propose a multivariate detector that takes the residuals from a set of equations recursively estimated over a monitoring period. The null is that there are no breaks in any series: the alternative is that at least one series breaks, and if more than one series breaks, their respective break points are temporally close (co-breaks). We construct CUSUM statistics from the normalised residuals purged of cross-equation correlation, and examine the asymptotic behaviour of the maximum absolute cumulative sum. A version of the Chu *et al* (1996) result is shown to apply asymptotically to that supremum statistic. Using Monte Carlo methods, we explore the small sample properties of the detector under different configurations of the proportion of series co-breaking at different dates, under differing monitoring periods, sample lengths and numbers of series. To anticipate, the result is that provided the proportion of series is sufficiently large, the multivariate detector increases the probability of detection relative to a single series with a break in the majority of cases examined. Similarly, the speed with which a break is detected is also improved. The broad pattern of the results is preserved whether or not there is cross-sectional correlation in the data. It is worth emphasising that gains from considering the multivariate detector are apparent even for a relatively small proportion of co-breaking series.

In Section 2 we set out the theory underlying our proposed test. In Section 3 we perform some Monte Carlo experiments. Section 4 applies the multivariate detector to UK RPI annual inflation data, monitoring for a break after 2001. The method does indeed pick up potential breaks that univariate methods fail to capture. Section 5 concludes. All proofs are relegated to an Appendix.

2 Theoretical considerations

Our interest focuses on a seemingly unrelated set of regressions given by

$$Y_{j,t} = X'_{j,t}\beta_{j,t} + \epsilon_{j,t}, \quad t = 1, ... \quad j = 1, ..., p \qquad \textbf{(1)}$$

where $X_{j,i}$ is a $k_j \times 1$ random vector and $\beta_{j,t}$, is a $k_j \times 1$ non-stochastic vector. Throughout the analysis the following non-contamination assumption is made:

Assumption 1 $\beta_{j,t} = \beta_j, \quad \text{for } t = 1, 2, ..., m, \quad j = 1, ..., p.$

The entertained null hypothesis is $H_0 : \beta_{j,t} = \beta_j, \quad \text{for } t = m + 1, ...$ The regressions are potentially related in two ways. The first is standard, in that we assume that $\epsilon_t = (\epsilon_{1,t}, ..., \epsilon_{p,t})' \sim (0, \Sigma)$ for some positive definite matrix Σ where $\Sigma[j, j] \equiv \sigma_j^2$. The second way in which these regressions are related provides the motive for considering multivariate monitoring schemes. We consider alternative hypotheses of the form $H_1 : \beta_{j,t}$ changes at some $T_{0,j} \geq m + 1$ for some $j \in J_{p_1} = \{j_1, .., j_{p_1}\}$ where $p_1 > 1$ and $T_{0,j_k}/T_{0,j_l} = 1 + o(1)$ for all $1 \leq k, l \leq p_1$. In words, a subset of the processes under consideration roughly co-breaks, in the sense that these processes exhibit breaks in close temporal proximity.

The univariate CUSUM detector is specified as follows. Let

$$\hat{\beta}_{j,T} = \left(\sum_{i=i}^{T} X_{j,i} X'_{j,i} \right)^{-1} \left(\sum_{i=1}^{T} X_{j,i} Y_{j,i} \right)$$

be the OLS estimator for the j-th equation at time T. Define recursive residuals as $\omega_{j,k} = 0$ and

$$\omega_{j,T} = \hat{\epsilon}_{j,T}/\nu_{j,T}^{1/2},$$

$$\nu_{j,T} = 1 + X'_{j,T} \left(\sum_{i=1}^{T-1} X_{j,i} X'_{j,i} \right)^{-1} X_{j,T},$$

$$\hat{\epsilon}_{j,T} = Y_{j,T} - X'_{j,T}\hat{\beta}_{j,T-1}, \quad T = k + 1, ..., m, ...$$

The T-th cumulated sum of recursive residuals is

$$Q_{j,t}^m = \sigma_j^{-1} \sum_{i=k}^{T} \omega_{j,i},$$

where $\hat{\sigma}_j^2$ is some consistent estimate of σ_j^2. An obvious choice for this is the estimate of σ_j^2 based on the OLS estimate of β_j obtained in the non-contamination period $1, ..., m$. It is well known (see, eg, Krämer *et al* (1988)) that under the null hypothesis, H_0,

$$\left\{t \to m^{-1/2} Q^m_{j,t},\ \ t \in [0,\infty)\right\} \Rightarrow \{t \to W(t),\ \ t \in (0,\infty)\} \qquad \textbf{(2)}$$

where $\Rightarrow$ denotes the weak convergence of the associated probability measures. This result can be used to motivate the following monitoring scheme

$$\lim_{m\to\infty} \Pr\left\{|Q^m_{j,t}| \geq \sqrt{m} g(n/m), \quad \text{for some } n \geq m\right\} = \Pr\left(|W_j(t)| \geq g(t), \quad \text{for some } t \geq 1\right) \qquad \textbf{(3)}$$

where $W_j(t)$ is a standard Brownian motion. The probability on the right hand side of **(3)** does not have a closed-form solution for any arbitrary $g(t)$. In specific instances though it does. So as to keep the discussion general we will parameterise $g(t)$ as follows: $g(t) = g(t,a)$ such that

$$\Pr\left(|W_j(t)| \geq g(t), \quad \text{for some } t \geq 1\right) = f_g(a) = \alpha$$

where there is a unique mapping $f_g(a) = \alpha$ for all $\alpha \in (0,1)$. Admissible functions $g(t)$ are discussed in detail in Chu *et al* (1996). In particular, in what follows and in common with Chu *et al* (1996), we assume that g belongs to the class of regular functions as defined in (5)-(6) of Chu *et al* (1996).

As a natural extension of the above univariate detector we suggest the following multivariate detector. Let $\hat{\Sigma}$ denote a consistent estimator of Σ. We suggest estimating this from the residuals of univariate OLS estimations of the regressions **(1)**, over the non-contamination period $1, ..., m$, since we would like to consider relatively large values of p. Let $\omega_t = (\omega_{1,t}, ..., \omega_{p,t})'$ and $\tilde{\omega}_t = (\tilde{\omega}_{1,t}, ..., \tilde{\omega}_{p,t})'$ where $\tilde{\omega}_t = \hat{\Sigma}^{-1/2}\omega_t$. The t-th cumulated sum of recursive residuals is $\tilde{Q}^m_{j,t} = \sum_{i=k}^{t} \tilde{\omega}_{j,i}$. Define the maximum absolute cumulative sum as

$$\tilde{Q}^m_{\max,t} = \max_{j=1,\dots,p} \left|\tilde{Q}^m_{j,t}\right|.$$

We make the following assumption:

Assumption 2 (i) For all j, $m^{-1}\sum_{t=1}^{m} X_{j,t} \xrightarrow{p} b_j$ and $m^{-1}\sum_{t=1}^{m} X_{j,t}X'_{j,t} \xrightarrow{p} M_j$ where b_j and $M_j = E(X_{j,t}X'_{j,t})$ are non-stochastic $k_j \times 1$ vectors and non-stochastic full-rank $k_j \times k_j$ matrices respectively. (ii) $E(\epsilon_t\epsilon'_t) = \Sigma < \infty$ where Σ is a positive definite symmetric matrix. (iii) For all j, $\epsilon_{j,t}$ satisfies a functional central limit theorem, ie,

$$m^{-1/2}(\sigma_{0j}M_j)^{-1/2}\sum_{t=1}^{[m\ell]} X_{j,t}\epsilon_{j,t} \Rightarrow W(\ell), \quad \ell \in (0,1),$$

where $E(\epsilon^2_{j,t}) = \sigma_{0j}$.

This assumption is quite mild. In particular, only the validity of a functional central limit theorem and existence of second moments is assumed about the error terms, $\epsilon_{j,t}$, allowing a wide variety of data-generating processes to be accommodated. Then we have the following theorem stating the asymptotic behaviour of the multivariate detector under the null hypothesis.

Theorem 1 Let Assumptions 1 and 2 hold. Let g be regular and such that $t^{1/2}g(t)$ is eventually non-decreasing. Further, assume that the null hypothesis holds, ie, that, for all j, $Y_{j,t} = X'_{j,t}\beta_{j,t} + \epsilon_{j,t}, \ \ t = 1, ..., m, ...$. Then,

$$\lim_{m\to\infty} \Pr\left\{\tilde{Q}^m_{\max,t} \geq \sqrt{m}g(n/m, a), \quad \text{for some } n \geq m\right\} = 1 - (1 - f_g(a))^p \tag{4}$$

The next theorem examines the case where $p \to \infty$.

Theorem 2 Let Assumptions 1 and 2 hold. Let $p = o(T^{1/2})$. Let g be regular and such that $t^{1/2}g(t)$ is eventually non-decreasing. Further, assume that the null hypothesis holds, ie, that, for all j, $Y_{j,t} = X'_{j,t}\beta_{j,t} + \epsilon_{j,t}, \ \ t = 1, ..., m, ...$. Then

$$\lim_{m,p\to\infty} \Pr\left\{\tilde{Q}^m_{\max,t} \geq \sqrt{m}g(n/m, a_p(\alpha)), \quad \text{for some } n \geq m\right\} = \alpha \tag{5}$$

where $a_p(\alpha)$ is chosen so that $\lim_{p\to\infty} 1 - (1 - f_g(a_p(\alpha)))^p = \alpha$

Theorem 3 provides an example of a function g and a sequence $a(p)$ that satisfies Theorem 2.

Theorem 3 Let Assumptions 1 and 2 hold. Let $p = o(T^{1/2})$. Further, assume that the null hypothesis holds, ie, that, for all j, $Y_{j,t} = X'_{j,t}\beta_{j,t} + \epsilon_{j,t}, \ \ t = 1, ..., m, ...$. For $g(t) = \left((t+1)\left[a^2 + \ln(t+1)\right]\right)^{1/2}$, the sequence $a_p = C\ln(p)^{1/2}$ is admissible for Theorem 2.

Next, we provide some local power results for the simple location model. We focus on the simple location model for simplicity and because the results we will obtain for this case provide clear implications for more general models. Assuming the simple model

$$Y_{j,t} = \beta_{j,t} + \epsilon_{j,t}, \ \ t = 1, ..., \ \ j = 1, ..., p \tag{6}$$

we consider the following local alternative

$$H_{1,T} : \beta_{j,t} = \beta_j, t \leq T_{T,j,0}, \quad \beta_{j,t} = \beta_j + \beta_1/\sqrt{T}, t \geq T_{T,j,0} + 1, \quad \text{for all } j \tag{7}$$

where $T_{T,j,0}/T_{T,i,0} = 1 + o(1)$. We have the following theorem on the local power of the univariate and multivariate CUSUM detector procedures in this simple case.

Theorem 4 Let Assumptions 1 and 2 hold. Let g be regular and such that $t^{1/2}g(t)$ is eventually non-decreasing. Then, under the local alternative $H_{1,T}$, we have that for the univariate detector procedure with significance level α,

$$\lim_{m\to\infty} \Pr\left\{|Q_{j,t}^m| \geq \sqrt{m}g(n/m, a^*), \quad \text{for some } n \geq m\right\} = f_1(a^*, \tau)$$

where $f(a^*) = \alpha$, $\tau = \lim_{T\to\infty} \frac{T_{T,0}}{T}$, and f_1 is defined in **(A-4)**. For the multivariate detector procedure with significance level α, we have

$$\lim_{m\to\infty} \Pr\left\{\tilde{Q}_{\max,t}^m \geq \sqrt{m}g(n/m, a^{**}), \quad \text{for some } n \geq m\right\} = 1 - (1 - f_1(a^{**}, \tau))^p$$

where $1 - (1 - f(a^{**}))^p = \alpha$.

The preceding discussion has been focused on a particular monitoring scheme based on the work of Chu *et al* (1996) and a particular multivariate extension. Both these choices are mainly made for expositional clarity and to illustrate the potential of exploiting a multivariate data set in the context of monitoring structural breaks. Our work can be generalised in both these dimensions. In particular, rather than focusing on the recursive residual approach of Chu *et al* (1996), we can consider other fluctuation processes which can be generalised in a similar fashion to the multivariate setting. These include processes based on estimates as in Leisch *et al* (2000) or Zeileis *et al* (2005), or on OLS residuals as in Zeileis *et al* (2005), or on scores as in Zeileis (2005).

There are also alternatives to the use of the maximum for the multivariate detector. One such can be based on the average of the absolute individual cumulative sums given by

$$\tilde{Q}_{ave,t}^m = \frac{1}{p}\sum_{j=1}^{p}\left|\tilde{Q}_{j,t}^m\right|.$$

Whereas the maximum should be superior when only a subset of the series under investigation undergo structural change, the average is likely to work better when most of the series are affected. To investigate this possibility we briefly consider this in our Monte Carlo study.

3 Monte Carlo study

In this section we report results of an extensive Monte Carlo study on the properties of the multivariate break detector. We consider the simple location model

$$Y_{j,t} = \beta_{j,t} + \epsilon_{j,t}, t = 1, ..., T$$

Following the notation of the previous sections we set $T = 100, 200, 400$, $m = [p_m T]$, $p_m = 0.25, 0.5, 0.75$, $T_{T,j,0} = T_{T,0} = m + (T - m)b$, $b = 0.25, 0.5, 0.75$. $\beta_{j,t} = 1, t \leq T_{T,0}$ and $\beta_{j,t} = 1 + \beta_j, t \geq T_{T,0} + 1$ for $j = 1, ..., [p_b p]$ where $\beta_j \sim N(1, 1)$, $p_b = 0.1, 0.2, 0.4, 0.6, 0.8, 1$ and [.] denotes integer part. For reference: m/T is the proportion of the entire sample at which monitoring starts; p_b is the proportion of the series which exhibit a break; b is the proportion of the monitoring period at which the break occurs; and p is the number of series.

The presence of cross-sectionally correlated errors may have an effect on the performance of the multivariate break detector. Therefore, we also carry out a Monte Carlo experiment where we allow for cross-sectional error correlation. In particular, we specify that the first five auxiliary diagonals of the error-covariance matrix around the main diagonal be non-zero. The elements of these diagonals are specified to be uniformly distributed random variables in [0.03, 0.33]. Covariance matrix draws that were not positive definite were discarded. Symmetry of the covariance matrix is respected by restricting the elements of the auxiliary diagonals below the main diagonal to be the same with those of the auxiliary diagonals above the main diagonal.

Results on the probability of break detection and the relative mean delay of break detection for both the multivariate break detector and the univariate break detector based on an analysis of $Y_{1,t}$ are reported in Appendix B both for cross-sectionally uncorrelated and correlated errors. All these results relate to the multivariate detector based on the maximum absolute cumulative sum of residuals. We also report a selection of results for the maximum absolute cumulative sum of residuals. These are also given in Appendix B. In particular, and in order to conserve space, we report results on the probability of break detection for the case of cross-sectionally uncorrelated errors.

We begin by examining the uncorrelated case. Section B.1 reports the probability of detecting a break for the univariate and multivariate cases.[4] The messages are straightforward and accord with our intuition. First, it is clear that as b (measuring the lateness of the break) increases, performance falls in both the univariate and multivariate case. Similarly, as the proportionate length of the monitoring period rises, performance falls. When only a small number of series co-break, the multivariate method is inferior to the univariate detector. But the multivariate

[4]For ease of comparisons between the univariate and multivariate cases, the univariate panel reports for all p_b, but the variation in each column is simply due to differing draws.

performance increases rapidly with that proportion, so that in most cases examined performance is superior at $p_b = 0.2$. The increase in detection probability is very marked in many cases. For example, for the realistic case where $T = 100$ and $p = 20$, and where there is a relatively long sample before monitoring starts ($m/T = 0.75$), the probability roughly trebles for $b = 0.50$ and 0.75, and rises from 0.4 to 1.0 for $b = 0.25$. The results are all the more striking, given they are compared to detection in a series known to exhibit a break.

Section B.2 reports the relative mean delay in detection, where there is a qualitatively similar pattern to the results; higher detection probabilities are associated with swifter detection.

The results for the correlated case are reported in Tables B10-B18 of Appendix B, Sections B.3 and B.4. Once again, the probability of detection tends to rise with p_b and p. Similarly, the delay declines in the same pattern, although the gain is smaller relative to the uncorrelated case. In general, all patterns reported for the case of uncorrelated errors, remain for the case of correlated errors.

Finally, we consider the relative performance of the maximum versus the average of the absolute cumulative sums of residuals. We contrast Tables B2, B4 and B6 with B19, B20 and B21. The results closely mirror our expectations. The detector based on the maximum is superior when only a small subset of the series under investigation undergo structural change. On the other hand, the average works better when a larger proportion of the series undergo structural change. As a result we view these two summaries of the individual cumulative sums as complementary to each other, having the upper hand in naturally interpretable circumstances. We also note that the reduction in detection ability for small numbers of co-breaking series for the average measure is typically greater than the increase in detection for large numbers. Thus the maximum detector statistic may be considered to be more robust, especially if there is an *a priori* belief that only a minority of series are co-breaking.

4 Empirical application

In this section we present an empirical application that illustrates the potential of the multivariate break detector. As we have noted earlier, one possible way in which the multivariate break detector may be of use is to consider whether disaggregated data may be used to provide information on the presence of breaks in an aggregate series. A natural test bed for this is price inflation data, where there is a large number of easily obtainable and interpretable component series, which aggregate in a relatively straightforward way. We use UK RPI data, for which we have 77 component indices available over our sample. For this empirical application we focus on the multivariate detector based on the maximum of the absolute cumulative sums of residuals.

Monthly data for the UK RPI series and its components are available on a consistent basis at this level of aggregation from January 1987. We drop the first 50 periods so that our series for annual inflation rates begin in March 1992 (so forecast errors commence in April 1992) and end in August 2007 (the last period available at the time of writing). We choose this starting point as 1992 marks the transition to a formal inflation targeting regime in the United Kingdom. In addition to making a natural historical point to choose, the previous period exhibited more inflation volatility than the post-1992 period (King (2002)), so that arguably a potential structural break is excluded from the sample. This is important as in the test framework the series under investigation is assumed not to have undergone structural change prior to the start of monitoring.[5]

Some further discussion of the monetary regime and history may be relevant. The first UK inflation target was introduced on 8 October 1992. It constituted a rate of annual RPIX[6] inflation of lying between 1% and 4%, with the objective for inflation to be in the lower half of that range by the end of that Parliament. In June 1995 the then Chancellor of the Exchequer announced an inflation target of the mid-point, 2.5%, or less. In May 1997, on Bank independence, this was changed to a symmetric target of 2.5%. Thus although it could be argued there had been changes in regime, in practice the post-1992 period is often considered to be a single monetary regime.[7] In April 2003 it was announced that the target would switch at some unspecified point to consumer prices index (CPI)[8] inflation, and this was implemented at 2.0% in December of that year. However, it is likely that this had only a small change on the regime. There is a wedge between the two measures of inflation, and although the average varies over time, CPI inflation had largely tended to lie below RPIX inflation. Nevertheless, as CPI inflation is constructed differently from RPIX, an interesting policy question is whether the changed target would manifest itself in a change in the average level of RPI or RPIX inflation.[9]

[5] Or, if they did, then that structural change was properly modelled.

[6] Total RPI excluding mortgage interest payments.

[7] Characterised as inflation targeting; see King (2002).

[8] The UK CPI is defined equivalently to the euro-area HCPI.

[9] It should also be noted that the detector is sensitive to other forms of break, such as changing variance. It may be that after RPIX ceased to be targeted in 2003 its variance may have increased, even if the mean were constant.

But other events occurred in the monitoring period. These include a period of volatile house price inflation, which affects both RPIX and, to a greater extent, RPI inflation.[10] There was also a large and continued rise in the price of energy and some other commodities, which appears to have started around 2004.[11]

Thus there are several reasons to think that there may have been a structural break in this period. Having said this, it should be clear that in the long run the level of inflation is determined by monetary policy and not by temporary changes in relative prices. The point is that, as observed above, such temporary changes in the environment may manifest themselves as intercept shifts; this does not imply a permanently changed level of long-run inflation.

Consequently, we consider a monitoring period that starts in September 2001, roughly two years before the events including the change in targeting regime associated with the switch to a CPI inflation target, the beginning of the energy price rises and the decline in house price inflation from its peak. Charts 1 and 2 show the data for RPI and RPIX inflation over the period since 1992.[12]We monitor the RPI index and its components individually with the univariate break detector, as well as the whole panel of components using the maximum multivariate detector.[13] We consider critical values that correspond to a 95% significance level. We pre-whiten each inflation series by fitting an AR(1) model. In the first case we examine, we use all data available. After extracting the residuals they are examined for a mean shift using the simple location model. We consider such a shift following our prior belief that in this particular instance focus should be placed on shifts in the mean of the inflation process, as opposed to the dynamics of the process, which we keep fixed *a priori*.

Our results make interesting reading. Out of 77 RPI components 65 reveal no breaks up to and including August 2007. Of the twelve components where we do find breaks, nine reveal a break at the very beginning of the monitoring period. The univariate detector cannot find a break in the aggregate RPI series. By contrast, the multivariate detector detects a break at December 2003.

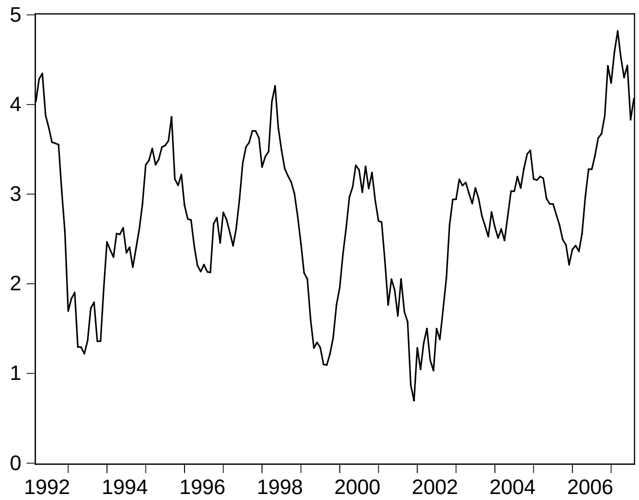
[10]This alone means that the wedge between RPIX and CPI inflation is itself moderately large and variable. Over our monitoring period annual house price inflation (the average of the Halifax and Nationwide series) peaked at 25.0% in 2003 Q1, with a low of 2.9% in 2005 Q3.

[11]The dollar price of the Brent marker rose from about $28 in January 2004 to about $55 in January 2006.

[12]Chart 2 suggests that 1992 may predate the period of RPIX inflation stability. Consequently we repeated the exercises reported below for a start date twelve months later, maintaining the same monitoring period. The break dates we identify and report below turn out to be unchanged.

[13]We know that the avergae detector is more effective when many series break, but is less powerful when only a few break. As our candidate breaks may not affect all series equally, we consider the maximum detector to be more appropriate.

Chart 1: Annual RPI inflation



This is a remarkable result, coinciding precisely with the shift to a specific CPI target, which was not pre-announced (other than the intention to change the measure at some point). We consider the timing too precise for it to be plausibly capturing the regime shift, rather than some other structural change.

Chart 2: Annual RPIX inflation

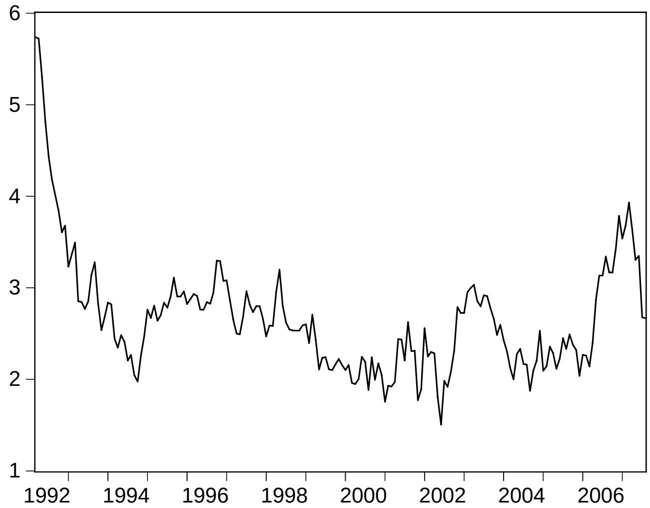


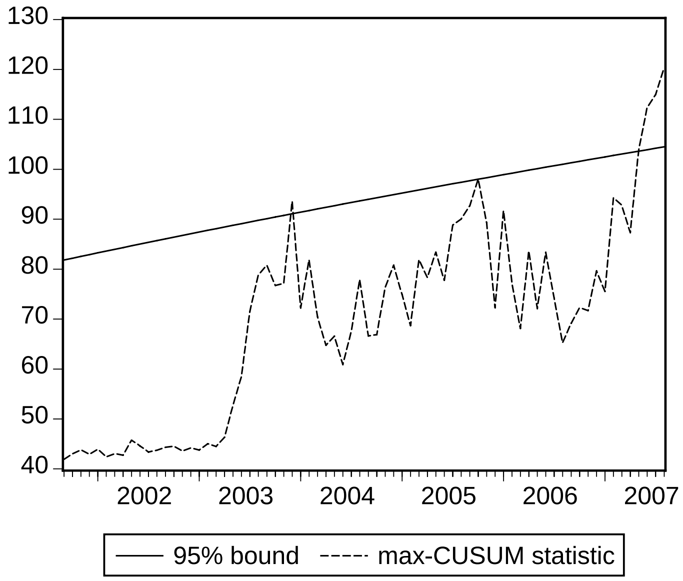
Chart 3 illustrates the detector. There is a marked rise in the detector statistic in 2003, just prior to the identified break. The statistic then remains close to the bound, falling below but just exceeding it again in October 2005. By the end of the period the statistic is well above the bound.

The pre-whitening has been conducted using whole-sample information. It is important that we maintain a constant dynamic structure, interested as we are in mean shifts. In a real exercise, the pre-whitening would need to be conducted only over the monitoring period. Thus we also consider that case. In terms of the variation between the univariate and multivariate procedures, the results are similar. For the univariate detector the vast majority of series report no break; only seven series report a break, and these do not include the aggregate RPI series. As previously, the multivariate detector again detects a break, although at a later date, October 2005.[14] The detector is shown in Chart 4. One feature is that the detector exceeds the critical bound to a larger extent at the end of the period.

Not all of the subcomponents need to have co-broken for there to be an impact on aggregate RPI. On the other hand, it is important to remember that breaks may be offsetting, so that there could be no effect on the aggregate series. A rise in one price may be balanced by a fall in another.

[14]This is the date at which the detector statistic crosses the boundary for the second time in Chart 3.

Chart 3: Maximum CUSUM detector: pre-whitened using whole sample



The detector, based as it is on the CUSUM, does not give a magnitude for the shift in mean, if any. It would be natural to test for structural breaks using standard techniques such as those in Bai and Perron (1998), but the identified dates allow insufficient observations to do so.[15] Table A therefore reports the results of estimating AR(1) processes with shift dummies in either December 2003 or October 2005, for both RPI and RPIX, which amount to simple Wald tests for the hypotheses in question (mean shifts). In all cases, before the break the estimated unconditional mean of the inflation rate was close to 2.5%. For RPIX there are insignificant long-run shifts of 0.2 and 0.6 percentage points after the two candidate break points. For RPI, which includes housing costs and was not targeted, the estimated shifts are larger and significant at the 10% level.[16] These amount to weak evidence for a structural break in the unconditional mean of RPIX using the full sample, that could not have been detected using standard methods in real time or by univariate monitoring techniques, but which were indicated by the multivariate monitoring technique we have developed in this paper.[17]

[15] Standard practice is to trim the start and end of the sample by 15% to perform the Bai and Perron (1998) test. This would lose approximately 28 observations from the end of the sample, which brings us before the later break identified by the monitoring method.

[16] As mentioned in footnote 12, we repeated the exercise starting estimation twelve months later, and found the break dates remained the same. Neither does the shorter sample make any real difference to the estimated means or shifts, which for RPIX are unchanged to 1 decimal point, and for RPI differ by no more than 0.1.

[17] As Hendry and his co-authors have emphasised, mean shifts may often be seen as symptons of structural breaks, rather than to be taken literally. In this case the obvious cause is the very rate of energy price inflation after 2004.

Chart 4: Maximum CUSUM detector: pre-whitened using pre-monitoring sample

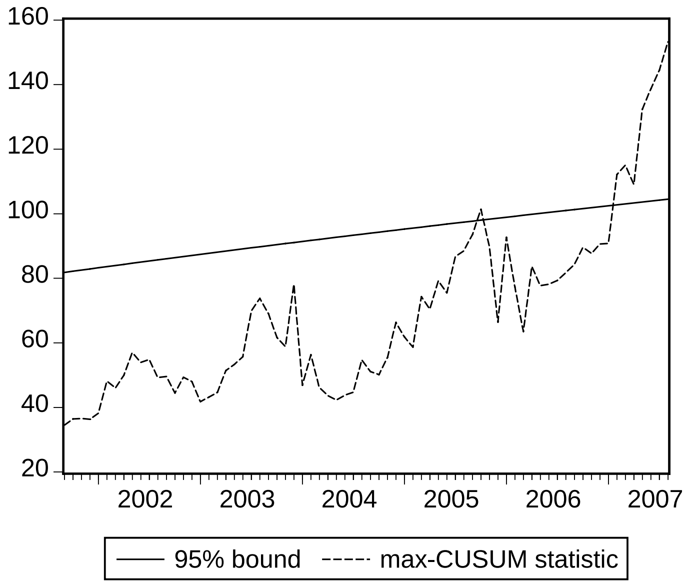


Table A: Shift dummies in AR(1) processes; long-run impact

Series	Break	Mean	Shift (se)
RPIX	2003m12	2.50	0.16 (0.30)
	2005m10	2.48	0.56 (0.37)
RPI	2003m12	2.39	1.36 (0.78)*
	2005m10	2.48	1.91 (1.01)*

* indicates significance at the 10% level

Sample April 1992 - August 2007.

5 Conclusions

'Monitoring' of series for structural breaks raises special econometric problems, primarily because classical methods are invalid in repeated experiments. Fortunately, a methodology exists for testing individual relationships that can cope with this, by defining appropriate boundary conditions for various types of test, including the well-known CUSUM. This has the additional advantage that no end-of-sample period is required. But an unexplored avenue that may lead to earlier and more reliable detection is to extend the approach to include multiple series. The idea is motivated by the recognition that in many instances (for example, with inflation measures) it is plausible that several series co-break simultaneously. Asymptotic distribution theory is developed for a panel CUSUM detection test, based on the supremum from individual CUSUMs constructed from normalised recursive residuals purged of cross-equation correlation. A Monte Carlo exercise strongly suggests that, given a sufficient number of co-breaking series, the method does increase both the speed with which breaks are detected and the probability of detection in a wide variety of situations.

The theoretical and simulated results are supported by tests using subcomponents of UK RPI inflation, where the multivariate method suggests the existence of breaks after 2001, in 2003 or 2005, for which univariate methods provide no evidence at all. Moreover, using all the data now available, there is some weak evidence for mean shifts in total RPI inflation at the identified break dates. These may well have been temporary breaks induced by the large fluctuations in house and energy prices over this period, which still continue at the time of writing. Naturally, we cannot be sure how large or prolonged any such breaks will be.

So we have demonstrated a technique that appears to improve the power of monitoring tests. This leaves open the question of what to do after a break is detected, for example when forecasting, as standard methods of modeling breaks suffer from a lack of observations for estimation. This is the subject of our further research.

Appendix A: Proofs

Proof of Theorem 1

Using Corollary 3.5 of Chu *et al* (1996), we have that, for finite p,

$$\lim_{m\to\infty} \Pr\left\{\tilde{Q}^m_{\max,t} \geq \sqrt{m}g(n/m,a), \quad \text{for some } n \geq m\right\} = \tag{A-1}$$
$$\Pr\left(\max_{j=1,\dots,p} |W_j(t)| \geq g(t,a), \quad \text{for some } t \geq 1\right)$$

where $W(t) = (W_1(t), ..., W_p(t))'$ is a multivariate standard Brownian motion. But

$$\Pr\left(\max_{j=1,\dots,p} |W_j(t)| \geq g(t,a), \quad \text{for some } t \geq 1\right) = \tag{A-2}$$
$$1 - \Pr\left\{\max_{j=1,\dots,p} |W_j(t)| \leq g(t,a), \quad \text{for all } n \geq m\right\}$$

Then,

$$\Pr\left\{\max_{j=1,\dots,p} |W_j(t)| \leq g(t,a), \quad \text{for all } n \geq m\right\} = \tag{A-3}$$
$$\Pr\left(\{|W_1(t)| \leq g(t,a), \quad \text{for all } n \geq m\} \cap ... \cap \left\{|W_p(t)| \leq g(t,a), \quad \text{for all } n \geq m\right\}\right)$$

But, by the independence between $W_1(t), ..., W_p(t)$, we get

$$\Pr\left(\{|W_1(t)| \leq g(t,a), \quad \text{for all } n \geq m\} \cup ... \cup \left\{|W_p(t)| \leq g(t,a), \quad \text{for all } n \geq m\right\}\right) =$$
$$\prod_{i=1}^{p} \Pr\{|W_1(t)| \leq g(t,a), \quad \text{for all } n \geq m\} = (1 - f_g(a))^p$$

proving the theorem.

Proof of Theorem 2

We first give a Lemma that is of use in the main proof.

Lemma 1 Let $Y_{i,T}$ be random scalars. Then, $\sup_i Y_{i,T} \xrightarrow{d} Y$ as $T, n \to \infty$ sequentially implies that $\sup_i Y_{i,T} \xrightarrow{d} Y$ as $N, T \to \infty$ jointly, if $E\left|\sup_i Y_{i,T}\right|^\theta < \infty$, for some $\theta > 1$.

Given the assumptions of Theorem 2, Theorem 1 and the above Lemma it is sufficient to use sequential asymptotics. To see this simply note that $E\left|\sup_i \tilde{Q}^m_{i,t}\right|^\theta < \infty$, for some $\theta > 1$, follows from $E\left|\epsilon_{i,t}\right|^\theta < \infty$ and $E\left\|X_{i,t}\right\|^\theta < \infty$ which, in turn follow from the fact that both $\epsilon_{i,t}$ and $X_{i,t}$ are assumed to have finite second moments.

Then, it is sufficient to prove that every element of $\hat{\Sigma}^{-1/2}\omega_t - \Sigma^{-1/2}\omega_t$ is $o_p(1)$. But, by $T^{1/2}$-consistency of every element of $\hat{\Sigma}$, and continuity of the inverse, it follows that every element of $\hat{\Sigma}^{-1/2}$ is $T^{1/2}$-consistent for $\Sigma^{-1/2}$. Since every element of $\Sigma^{-1/2}\omega_t$ is a function of p elements of $\hat{\Sigma}^{-1/2}$, it follows that every element of $\hat{\Sigma}^{-1/2}\omega_t - \Sigma^{-1/2}\omega_t$ is $O_p(pT^{-1/2})$-consistent. Hence the result follows by the assumption that $p = o(T^{1/2})$.

Proof of Theorem 3

By (8) of Chu *et al* (1996), we have that $f_g(a) = e^{-a^2/2}$. It is sufficient to show that the limit of $1-(1-f_g(a_p))^p$ where $a_p = \ln(p)^{1/2}$ is bounded. Then, an appropriate choice for C gives $\lim_{p\to\infty} 1-(1-f_g(a_p(\alpha)))^p = \alpha$. Setting $1-(1-f_g(a))^p = \alpha$ gives $a = \left[-2\ln\left[1-(1-\alpha)^{1/p}\right]\right]^{1/2}$. It is sufficient to show that

$$\left|\lim_{p\to\infty} \frac{\ln\left(1-C_1^{1/p}\right)}{\ln p}\right| = C_2 < \infty$$

for some $0 < C_1 < 1$. By L'Hopital's rule we get that

$$\lim_{p\to\infty} \frac{\ln\left(1-C_1^{1/p}\right)}{\ln p} \approx \lim_{p\to\infty} \frac{\left(1-C_1^{1/p}\right)^k}{p^k}$$

for all integer $k > 0$ where $\approx$ is defined so that $a \approx b$ is equivalent to $a/b = O(1)$. This immediately implies that $\lim_{p\to\infty} \frac{\left(1-C_1^{1/p}\right)^k}{p^k} \approx 1$ proving the result.

Proof of Theorem 4

We simplify our analysis by disregarding the normalisation by $1 + X'_{j,T}\left(\sum_{i=1}^{T-1} X_{j,i}X'_{j,i}\right)^{-1} X_{j,T}$ to obtain the recursive residuals since this normalisation term converges to 1 almost surely asymptotically. Further, since $T_{T,j,0}/T_{T,i,0} = 1 + o(1)$, it is asymptotically appropriate to simplify the analysis by setting $T_{T,j,0} = T_{T,0}$ for all j. We have that, under the local alternative,

$$\beta_{T-1} = \beta + \frac{1}{T-1}\sum_{i=1}^{T-1}\epsilon_i,$$

if $T-1 < T_{T,0}+1$ and

$$\beta_{T-1} = \beta + \frac{1}{T-1}\sum_{i=1}^{T-1}\epsilon_i + \frac{1}{T-1}\sum_{i=T_{T,0}+1}^{T-1}\frac{\beta_1}{\sqrt{T}} = \beta + \frac{1}{T-1}\sum_{i=1}^{T-1}\epsilon_i + \frac{(T-1-T_{T,0})\beta_1}{(T-1)\sqrt{T}}$$

if $T-1 > T_{T,0}$. Similarly,

$$\hat{\epsilon}_T = \epsilon_T - \sum_{i=1}^{T-1} \epsilon_i,$$

if $T-1 < T_{T,0}+1$ and

$$\hat{\epsilon}_T = \epsilon_T - \sum_{i=1}^{T-1} \epsilon_i - \frac{(T-1-T_{T,0})\beta_1}{(T-1)\sqrt{T}} + \frac{\beta_1}{\sqrt{T}} = \epsilon_T - \sum_{i=1}^{T-1} \epsilon_i + \frac{T_{T,0}\beta_1}{(T-1)\sqrt{T}}$$

if $T-1 > T_{T,0}$. Then, under the local alternative hypothesis, and for $t > T_{T,0}$,

$$Q_{j,t}^{m} = Q_{j,t}^{*m} + \sum_{s=T_{T,0}+1}^{t} \frac{T_{T,0}\beta_1}{(s-1)\sqrt{T}}$$

where $Q_{j,t}^{*m}$ is the cumulative sum under the null hypothesis. $\sum_{s=T_{T,0}+1}^{t} \frac{T_{T,0}\beta_1}{(s-1)\sqrt{T}}$ can be written as a function of t (and $T_{T,0}$) and is asymptotically of the order $\sqrt{t}\ln t$. We write

$$g_1(t, T_{T,0}) = \sum_{s=T_{T,0}+1}^{t} \frac{T_{T,0}\beta_1}{(s-1)\sqrt{T}} \rightarrow g_1(t,\tau),$$

as $T \rightarrow \infty$ where $\tau = \lim_{T\rightarrow\infty} \frac{T_{T,0}}{T}$. Without loss of generality let us assume that $\beta_1 > 0$. Then,

$$\lim_{m\rightarrow\infty} \Pr\left\{|Q_{j,t}^{m}| \geq \sqrt{m}g(n/m, a^*), \quad \text{for some } n \geq m\right\} = \qquad \textbf{(A-4)}$$

$$\Pr\left(|W_j(t) + g_1(t,\tau)| \geq g(t,a^*), \quad \text{for some } t \geq 1\right) \equiv f_1(a^*,\tau)$$

Unfortunately, the nature of $g(t,a^*)$ and $g_1(t,\tau)$ implies that a closed form solution for $f_1(a^*,\tau)$ is not readily available (see also Theorem A of Chu *et al* (1996)). Similarly and using arguments from the proof of Theorem 1, it is straightforward to show that for the multivariate detector

$$\lim_{m\rightarrow\infty} \Pr\left\{\tilde{Q}_{\max,t}^{m} \geq \sqrt{m}g(n/m, a^{**}), \quad \text{for some } n \geq m\right\} = 1 - (1 - f_1(a^{**},\tau))^p$$

Proof of Lemma 1

Sequential convergence implies that, for all i, there exists Y_i such that $Y_{i,T} \stackrel{d}{\rightarrow} Y_i$ as $T \rightarrow \infty$. Then, from Lemma 6 of PM the result of the Lemma follows if we show that

$$\limsup_{N,T} \left| E\left(f\left(\sup_i Y_{i,T}\right)\right) - E\left(f\left(\sup_i Y_i\right)\right)\right| = 0, \ \forall f \in \mathcal{C} \qquad \textbf{(A-5)}$$

where $\mathcal{C}$ is the space of all bounded continuous real functions on $\mathbb{R}$. Without loss of generality let the functions f be such that $\left|f^{(k)}(x)\right| \leq 1$ where $f^{(k)}(x)$ denotes the k-th derivative function of $f(x)$. Fix f. Let

$$g(h) = \sup_x \left|f(x+h) - f(x) - f'(x)h\right|$$

Set $x = \sup_i Y_{i,T}$ and $h = \sup_i Y_{i,T} - \sup_i Y_i$. It follows by the triangle inequality that

$$\limsup_{N,T} \left| E\left(f\left(\sup_i Y_{i,T}\right)\right) - E\left(f\left(\sup_i Y_i\right)\right)\right| \leq \qquad \textbf{(A-6)}$$

$$\limsup_{N,T} \left| E\left(f'\left(\sup_i Y_{i,T}\right)\left(\sup_i Y_{i,T} - \sup_i Y_i\right)\right)\right| + \limsup_{N,T} \left| E\left(g\left(\sup_i Y_{i,T} - \sup_i Y_i\right)\right)\right|$$

But since $\left| f^{(k)}(x)\right| \leq 1$

$$\limsup_{N,T} \left| E\left(f'\left(\sup_i Y_{i,T}\right)\left(\sup_i Y_{i,T} - \sup_i Y_i\right)\right)\right| \leq \limsup_{N,T} \left| E\left(\sup_i Y_{i,T}\right) - E\left(\sup_i Y_i\right)\right| \quad \textbf{(A-7)}$$

Also by the mean value theorem and for some finite M

$$g(h) \leq M \min\{|h|, h^2\}$$

Thus,

$$\limsup_{N,T} \left| E\left(g\left(\sup_i Y_{i,T} - \sup_i Y_i\right)\right)\right| \leq M \limsup_{N,T} E\left|\sup_i Y_{i,T} - \sup_i Y_i\right| \quad \textbf{(A-8)}$$

From **(A-7)** and **(A-8)**, it follows that the result of the Lemma is true if

$$\limsup_{N,T} E\left|\sup_i Y_{i,T} - \sup_i Y_i\right| = 0 \quad \textbf{(A-9)}$$

However, uniform integrability of $|\sup_i Y_{i,T}|$ implies **(A-9)**. By Theorem 12.10 of Davidson (1994) $E\left|\sup_i Y_{i,T}\right|^{\theta} < \infty$ implies uniform integrability of $|\sup_i Y_{i,T}|$. Hence, the result of the Lemma follows.

Appendix B: Tables

B.1 Results on probability of break detection for uncorrelated processes

Table B1: Results for univariate detector (single series with break considered), $T = 100$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.650	0.585	0.320	0.667	0.595	0.384	-	-	-
	0.2	0.664	0.579	0.350	0.679	0.576	0.400	-	-	-
	0.4	0.653	0.541	0.310	0.695	0.612	0.355	-	-	-
	0.6	0.674	0.577	0.319	0.672	0.615	0.364	-	-	-
	0.8	0.642	0.579	0.338	0.712	0.622	0.384	-	-	-
	1	0.670	0.562	0.331	0.716	0.615	0.371	-	-	-
0.50	0.1	0.594	0.474	0.184	0.609	0.473	0.239	0.635	0.511	0.232
	0.2	0.617	0.488	0.185	0.594	0.512	0.209	0.612	0.506	0.221
	0.4	0.594	0.504	0.197	0.599	0.484	0.204	0.630	0.482	0.228
	0.6	0.595	0.462	0.196	0.617	0.487	0.209	0.639	0.496	0.236
	0.8	0.595	0.445	0.182	0.629	0.481	0.204	0.635	0.479	0.240
	1	0.618	0.459	0.172	0.605	0.490	0.180	0.589	0.498	0.237
0.75	0.1	0.461	0.265	0.112	0.434	0.229	0.117	0.443	0.246	0.092
	0.2	0.438	0.305	0.104	0.430	0.280	0.096	0.404	0.210	0.086
	0.4	0.417	0.269	0.113	0.432	0.265	0.116	0.407	0.208	0.088
	0.6	0.449	0.275	0.122	0.431	0.278	0.114	0.412	0.225	0.100
	0.8	0.450	0.274	0.127	0.424	0.243	0.116	0.398	0.257	0.096
	1	0.439	0.264	0.133	0.419	0.271	0.099	0.394	0.252	0.106

Table B2: Results for multivariate detector, $T = 100$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.473	0.362	0.142	0.226	0.182	0.113	-	-	-
	0.2	0.706	0.585	0.266	0.353	0.270	0.120	-	-	-
	0.4	0.923	0.830	0.439	0.531	0.409	0.182	-	-	-
	0.6	0.977	0.939	0.606	0.664	0.516	0.247	-	-	-
	0.8	0.993	0.977	0.734	0.800	0.614	0.327	-	-	-
	1	0.999	0.991	0.821	0.875	0.757	0.401	-	-	-
0.50	0.1	0.495	0.348	0.098	0.627	0.425	0.119	0.313	0.183	0.077
	0.2	0.720	0.552	0.184	0.856	0.705	0.172	0.520	0.290	0.096
	0.4	0.921	0.809	0.291	0.985	0.902	0.370	0.789	0.586	0.177
	0.6	0.975	0.910	0.424	0.998	0.979	0.504	0.919	0.710	0.258
	0.8	0.996	0.956	0.553	1.000	0.994	0.628	0.975	0.857	0.359
	1	0.999	0.976	0.626	1.000	0.999	0.781	0.997	0.950	0.487
0.75	0.1	0.271	0.133	0.058	0.378	0.178	0.067	0.492	0.175	0.065
	0.2	0.454	0.197	0.070	0.623	0.273	0.076	0.723	0.351	0.075
	0.4	0.694	0.394	0.084	0.860	0.514	0.107	0.965	0.611	0.145
	0.6	0.809	0.484	0.140	0.960	0.699	0.162	0.993	0.849	0.219
	0.8	0.925	0.608	0.152	0.986	0.821	0.207	1.000	0.928	0.354
	1	0.956	0.698	0.201	0.997	0.893	0.281	1.000	0.979	0.461

Table B3: Results for univariate detector (single series with break considered), $T = 200$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.783	0.724	0.511	0.763	0.702	0.489	0.762	0.690	0.496
	0.2	0.775	0.721	0.484	0.758	0.706	0.505	0.744	0.704	0.493
	0.4	0.794	0.719	0.500	0.796	0.723	0.494	0.769	0.701	0.462
	0.6	0.755	0.697	0.516	0.757	0.704	0.462	0.785	0.706	0.485
	0.8	0.777	0.709	0.474	0.770	0.709	0.497	0.758	0.674	0.494
	1	0.763	0.730	0.512	0.781	0.703	0.475	0.747	0.686	0.469
0.50	0.1	0.737	0.636	0.363	0.718	0.653	0.376	0.754	0.618	0.383
	0.2	0.693	0.608	0.361	0.761	0.626	0.345	0.715	0.606	0.392
	0.4	0.724	0.589	0.342	0.721	0.626	0.369	0.731	0.618	0.382
	0.6	0.710	0.648	0.338	0.714	0.646	0.354	0.722	0.617	0.403
	0.8	0.729	0.630	0.362	0.762	0.645	0.345	0.730	0.653	0.368
	1	0.729	0.640	0.364	0.725	0.634	0.350	0.720	0.603	0.369
0.75	0.1	0.556	0.419	0.179	0.558	0.424	0.152	0.595	0.418	0.190
	0.2	0.558	0.427	0.179	0.569	0.425	0.149	0.563	0.450	0.171
	0.4	0.577	0.406	0.165	0.559	0.400	0.143	0.565	0.397	0.167
	0.6	0.568	0.441	0.171	0.537	0.401	0.153	0.589	0.408	0.170
	0.8	0.582	0.436	0.161	0.553	0.423	0.159	0.587	0.421	0.156
	1	0.591	0.414	0.168	0.559	0.411	0.140	0.552	0.422	0.166

Table B4: Results for multivariate detector, $T = 200$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.661	0.598	0.319	0.807	0.734	0.325	0.469	0.330	0.104
	0.2	0.890	0.825	0.527	0.971	0.936	0.591	0.762	0.555	0.238
	0.4	0.983	0.968	0.768	0.999	0.995	0.849	0.955	0.845	0.441
	0.6	0.999	0.995	0.894	1.000	0.999	0.950	0.990	0.945	0.578
	0.8	1.000	1.000	0.950	1.000	1.000	0.986	1.000	0.987	0.710
	1	1.000	1.000	0.974	1.000	1.000	0.998	1.000	0.996	0.857
0.50	0.1	0.660	0.510	0.238	0.822	0.719	0.341	0.954	0.839	0.385
	0.2	0.841	0.739	0.367	0.981	0.912	0.539	0.998	0.973	0.633
	0.4	0.986	0.921	0.593	1.000	0.995	0.785	1.000	1.000	0.875
	0.6	0.996	0.982	0.747	1.000	1.000	0.904	1.000	1.000	0.968
	0.8	1.000	0.999	0.841	1.000	1.000	0.968	1.000	1.000	0.992
	1	1.000	0.999	0.912	1.000	1.000	0.989	1.000	1.000	0.999
0.75	0.1	0.418	0.282	0.081	0.591	0.372	0.090	0.814	0.513	0.114
	0.2	0.671	0.467	0.139	0.822	0.579	0.133	0.969	0.797	0.201
	0.4	0.870	0.678	0.207	0.967	0.839	0.208	0.998	0.958	0.339
	0.6	0.958	0.815	0.265	0.997	0.939	0.319	1.000	0.995	0.504
	0.8	0.978	0.904	0.336	1.000	0.985	0.407	1.000	1.000	0.640
	1	0.995	0.951	0.407	1.000	0.993	0.493	1.000	1.000	0.730

Table B5: Results for univariate detector (single series with break considered), $T = 400$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.836	0.770	0.625	0.813	0.805	0.655	0.826	0.780	0.601
	0.2	0.823	0.791	0.603	0.850	0.791	0.623	0.828	0.751	0.603
	0.4	0.829	0.796	0.614	0.835	0.788	0.628	0.839	0.777	0.604
	0.6	0.828	0.761	0.623	0.841	0.768	0.620	0.804	0.764	0.601
	0.8	0.837	0.795	0.590	0.827	0.803	0.633	0.811	0.767	0.604
	1	0.826	0.753	0.622	0.834	0.785	0.622	0.822	0.767	0.600
0.50	0.1	0.805	0.734	0.520	0.798	0.725	0.469	0.787	0.757	0.567
	0.2	0.806	0.750	0.559	0.800	0.701	0.484	0.836	0.749	0.517
	0.4	0.798	0.730	0.525	0.807	0.709	0.498	0.784	0.724	0.558
	0.6	0.830	0.720	0.515	0.803	0.705	0.467	0.821	0.767	0.514
	0.8	0.801	0.755	0.534	0.788	0.727	0.496	0.816	0.756	0.528
	1	0.808	0.745	0.510	0.818	0.729	0.488	0.819	0.750	0.579
0.75	0.1	0.719	0.594	0.333	0.644	0.537	0.255	0.677	0.553	0.259
	0.2	0.720	0.595	0.312	0.676	0.555	0.251	0.690	0.574	0.265
	0.4	0.718	0.582	0.299	0.666	0.542	0.263	0.695	0.556	0.281
	0.6	0.726	0.568	0.307	0.705	0.508	0.256	0.697	0.565	0.299
	0.8	0.693	0.591	0.284	0.642	0.550	0.267	0.670	0.569	0.288
	1	0.724	0.585	0.296	0.687	0.547	0.250	0.717	0.559	0.305

Table B6: Results for multivariate detector, $T = 400$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.779	0.707	0.507	0.938	0.907	0.743	0.994	0.978	0.837
	0.2	0.944	0.920	0.740	0.996	0.995	0.934	1.000	1.000	0.975
	0.4	0.997	0.995	0.933	1.000	1.000	0.991	1.000	1.000	1.000
	0.6	0.999	0.999	0.984	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.709	0.649	0.390	0.916	0.861	0.548	0.990	0.971	0.752
	0.2	0.930	0.864	0.603	0.995	0.977	0.822	1.000	1.000	0.936
	0.4	0.997	0.977	0.827	1.000	1.000	0.963	1.000	1.000	0.996
	0.6	0.998	0.998	0.932	1.000	1.000	0.991	1.000	1.000	1.000
	0.8	1.000	0.999	0.973	1.000	1.000	0.998	1.000	1.000	1.000
	1	1.000	1.000	0.987	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.624	0.432	0.161	0.788	0.625	0.244	0.939	0.823	0.271
	0.2	0.816	0.665	0.260	0.956	0.850	0.343	0.998	0.945	0.430
	0.4	0.959	0.877	0.404	0.999	0.976	0.584	1.000	0.997	0.696
	0.6	0.992	0.967	0.554	1.000	0.999	0.727	1.000	1.000	0.853
	0.8	1.000	0.989	0.644	1.000	1.000	0.827	1.000	1.000	0.939
	1	1.000	0.995	0.735	1.000	1.000	0.890	1.000	1.000	0.967

B.2 Results on relative mean delay of break detection for uncorrelated processes

Results presented are the ratio of the absolute delay of break detection of the multivariate detector to the absolute delay of break detection of the univariate detector

Table B7: Results for $T = 100$

		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	1.372	1.246	1.119	1.802	1.432	1.187	-	-	-
	0.2	1.067	1.086	1.064	1.697	1.339	1.250	-	-	-
	0.4	0.669	0.829	0.977	1.455	1.245	1.141	-	-	-
	0.6	0.436	0.686	0.923	1.183	1.210	1.186	-	-	-
	0.8	0.310	0.598	0.856	1.005	1.087	1.145	-	-	-
	1	0.225	0.519	0.742	0.819	0.984	1.038	-	-	-
0.50	0.1	1.140	1.110	1.075	1.109	1.134	1.160	1.383	1.326	1.201
	0.2	0.963	0.989	1.052	0.879	1.027	1.123	1.226	1.200	1.162
	0.4	0.693	0.827	0.960	0.639	0.796	0.999	1.002	1.045	1.109
	0.6	0.567	0.690	0.900	0.540	0.659	1.008	0.851	0.935	1.111
	0.8	0.511	0.612	0.839	0.474	0.574	0.933	0.698	0.819	1.099
	1	0.466	0.554	0.809	0.412	0.492	0.776	0.561	0.707	1.022
0.75	0.1	1.141	1.095	1.130	1.069	1.038	1.080	1.067	1.088	1.074
	0.2	1.040	1.090	1.053	0.940	1.038	1.038	0.931	1.012	1.090
	0.4	0.898	1.018	1.094	0.800	0.963	1.078	0.733	0.929	1.058
	0.6	0.816	0.985	1.078	0.682	0.891	1.070	0.597	0.824	1.064
	0.8	0.741	0.931	1.082	0.612	0.807	1.006	0.520	0.750	1.013
	1	0.680	0.886	1.073	0.546	0.761	0.949	0.450	0.668	0.998

Table B8: Results for $T = 200$

		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	1.492	1.318	1.252	1.195	1.197	1.262	2.009	1.598	1.307
	0.2	0.895	0.983	1.129	0.618	0.830	1.164	1.416	1.378	1.248
	0.4	0.365	0.698	1.024	0.295	0.599	0.966	0.890	1.058	1.031
	0.6	0.202	0.504	0.824	0.156	0.442	0.761	0.544	0.870	1.112
	0.8	0.120	0.423	0.706	0.080	0.403	0.668	0.378	0.673	0.966
	1	0.070	0.411	0.643	0.036	0.337	0.603	0.225	0.542	0.865
0.50	0.1	1.209	1.186	1.080	0.944	1.039	1.091	0.822	0.960	1.193
	0.2	0.880	0.951	1.019	0.664	0.787	0.944	0.556	0.712	1.051
	0.4	0.592	0.720	0.886	0.465	0.568	0.872	0.441	0.545	0.901
	0.6	0.468	0.583	0.822	0.377	0.450	0.703	0.387	0.453	0.777
	0.8	0.415	0.506	0.754	0.366	0.440	0.623	0.328	0.405	0.613
	1	0.382	0.462	0.634	0.323	0.393	0.580	0.301	0.352	0.563
0.75	0.1	1.123	1.080	1.069	1.055	1.079	1.026	0.939	1.028	1.156
	0.2	0.993	1.028	1.009	0.882	1.008	1.049	0.729	0.889	1.045
	0.4	0.787	0.885	0.960	0.674	0.808	1.017	0.575	0.701	0.959
	0.6	0.684	0.807	1.001	0.568	0.740	0.939	0.502	0.622	0.951
	0.8	0.632	0.761	0.928	0.514	0.680	0.994	0.453	0.571	0.884
	1	0.566	0.694	0.864	0.497	0.600	0.952	0.408	0.510	0.883

Table B9: Results for $T = 400$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	1.408	1.193	1.203	0.592	0.869	1.005	0.324	0.671	0.964
	0.2	0.613	0.799	0.957	0.127	0.511	0.744	0.070	0.455	0.665
	0.4	0.100	0.508	0.700	0.058	0.310	0.519	0.056	0.322	0.490
	0.6	0.014	0.377	0.621	0.134	0.267	0.445	0.115	0.255	0.438
	0.8	0.086	0.326	0.497	0.194	0.237	0.383	0.160	0.212	0.349
	1	0.127	0.241	0.484	0.206	0.190	0.324	0.198	0.186	0.318
0.50	0.1	1.263	1.256	1.130	0.826	0.842	0.930	0.592	0.733	1.076
	0.2	0.779	0.924	1.025	0.511	0.590	0.774	0.441	0.519	0.734
	0.4	0.471	0.616	0.892	0.373	0.416	0.507	0.327	0.389	0.536
	0.6	0.426	0.490	0.704	0.316	0.370	0.423	0.297	0.335	0.490
	0.8	0.367	0.433	0.619	0.282	0.328	0.411	0.277	0.302	0.362
	1	0.342	0.376	0.555	0.271	0.284	0.370	0.261	0.249	0.354
0.75	0.1	1.214	1.169	1.148	0.918	0.975	0.998	0.786	0.915	1.052
	0.2	0.970	1.025	1.049	0.683	0.818	0.909	0.593	0.764	1.024
	0.4	0.662	0.838	1.007	0.508	0.644	0.918	0.482	0.590	0.913
	0.6	0.599	0.713	0.977	0.434	0.548	0.788	0.418	0.521	0.780
	0.8	0.507	0.650	0.826	0.386	0.490	0.712	0.378	0.490	0.680
	1	0.487	0.601	0.816	0.369	0.468	0.586	0.361	0.443	0.700

B.3 Results on probability of break detection for correlated processes

Table B10: Results for univariate detector (single series with break considered), $T = 100$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.679	0.567	0.343	0.638	0.578	0.366	-	-	-
	0.2	0.663	0.608	0.332	0.648	0.558	0.337	-	-	-
	0.4	0.675	0.586	0.337	0.704	0.603	0.340	-	-	-
	0.6	0.694	0.587	0.346	0.672	0.577	0.349	-	-	-
	0.8	0.662	0.587	0.353	0.657	0.598	0.345	-	-	-
	1	0.655	0.596	0.339	0.700	0.620	0.365	-	-	-
0.50	0.1	0.626	0.478	0.255	0.590	0.451	0.204	0.619	0.451	0.200
	0.2	0.612	0.495	0.223	0.591	0.488	0.191	0.601	0.491	0.189
	0.4	0.655	0.528	0.207	0.587	0.437	0.199	0.579	0.477	0.184
	0.6	0.615	0.494	0.235	0.605	0.451	0.205	0.577	0.495	0.207
	0.8	0.639	0.517	0.226	0.597	0.483	0.229	0.577	0.459	0.179
	1	0.596	0.497	0.239	0.580	0.472	0.212	0.619	0.499	0.173
0.75	0.1	0.392	0.230	0.104	0.470	0.278	0.117	0.437	0.264	0.107
	0.2	0.404	0.237	0.092	0.461	0.252	0.113	0.419	0.255	0.104
	0.4	0.416	0.235	0.093	0.448	0.276	0.107	0.415	0.270	0.083
	0.6	0.385	0.240	0.090	0.450	0.258	0.102	0.388	0.240	0.106
	0.8	0.403	0.237	0.087	0.462	0.280	0.119	0.410	0.249	0.097
	1	0.425	0.231	0.105	0.417	0.263	0.131	0.427	0.267	0.109

Table B11: Results for multivariate detector, $T = 100$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.435	0.327	0.127	0.253	0.168	0.109	-	-	-
	0.2	0.672	0.529	0.188	0.344	0.268	0.125	-	-	-
	0.4	0.892	0.768	0.359	0.529	0.398	0.178	-	-	-
	0.6	0.969	0.902	0.448	0.643	0.503	0.227	-	-	-
	0.8	0.987	0.948	0.602	0.750	0.639	0.328	-	-	-
	1	0.995	0.979	0.728	0.840	0.716	0.369	-	-	-
0.50	0.1	0.522	0.359	0.140	0.651	0.391	0.095	0.340	0.198	0.087
	0.2	0.757	0.555	0.204	0.857	0.686	0.169	0.526	0.358	0.130
	0.4	0.911	0.803	0.328	0.971	0.881	0.330	0.824	0.585	0.164
	0.6	0.977	0.885	0.432	0.998	0.965	0.469	0.933	0.751	0.293
	0.8	0.997	0.950	0.500	1.000	0.996	0.637	0.971	0.862	0.394
	1	0.997	0.981	0.604	1.000	1.000	0.762	0.995	0.942	0.514
0.75	0.1	0.299	0.181	0.088	0.447	0.202	0.089	0.569	0.261	0.104
	0.2	0.504	0.271	0.098	0.656	0.329	0.098	0.787	0.399	0.104
	0.4	0.726	0.413	0.126	0.876	0.515	0.124	0.961	0.678	0.175
	0.6	0.807	0.520	0.176	0.955	0.666	0.161	0.998	0.844	0.268
	0.8	0.909	0.613	0.168	0.981	0.810	0.263	1.000	0.952	0.374
	1	0.947	0.680	0.243	0.997	0.867	0.277	1.000	0.983	0.523

Table B12: Results for univariate detector (single series with break considered), $T = 200$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.787	0.689	0.475	0.755	0.725	0.481	0.773	0.700	0.455
	0.2	0.763	0.700	0.500	0.798	0.699	0.478	0.747	0.700	0.465
	0.4	0.762	0.697	0.485	0.754	0.687	0.536	0.768	0.701	0.487
	0.6	0.767	0.697	0.465	0.780	0.685	0.510	0.762	0.714	0.512
	0.8	0.781	0.699	0.494	0.789	0.696	0.511	0.753	0.707	0.458
	1	0.763	0.704	0.507	0.784	0.709	0.510	0.757	0.714	0.512
0.50	0.1	0.749	0.617	0.363	0.710	0.621	0.358	0.734	0.627	0.370
	0.2	0.716	0.625	0.356	0.721	0.619	0.349	0.711	0.620	0.364
	0.4	0.707	0.641	0.355	0.713	0.627	0.413	0.734	0.616	0.372
	0.6	0.738	0.613	0.379	0.720	0.639	0.338	0.712	0.630	0.357
	0.8	0.707	0.627	0.368	0.713	0.612	0.341	0.725	0.642	0.362
	1	0.699	0.622	0.380	0.717	0.610	0.345	0.717	0.630	0.362
0.75	0.1	0.560	0.423	0.173	0.613	0.440	0.189	0.566	0.409	0.155
	0.2	0.537	0.416	0.172	0.614	0.487	0.186	0.564	0.429	0.159
	0.4	0.536	0.391	0.139	0.589	0.459	0.192	0.566	0.449	0.186
	0.6	0.544	0.418	0.152	0.603	0.464	0.210	0.533	0.432	0.174
	0.8	0.532	0.391	0.149	0.609	0.472	0.211	0.599	0.454	0.157
	1	0.563	0.390	0.165	0.589	0.478	0.189	0.573	0.434	0.157

Table B13: Results for multivariate detector, $T = 200$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.709	0.581	0.330	0.832	0.757	0.411	0.517	0.395	0.142
	0.2	0.884	0.826	0.547	0.974	0.928	0.612	0.748	0.578	0.221
	0.4	0.992	0.966	0.762	1.000	0.998	0.856	0.939	0.860	0.428
	0.6	1.000	0.994	0.894	1.000	1.000	0.953	0.984	0.950	0.583
	0.8	1.000	0.999	0.939	1.000	1.000	0.977	1.000	0.986	0.728
	1	1.000	1.000	0.978	1.000	1.000	0.998	1.000	0.994	0.853
0.50	0.1	0.663	0.526	0.249	0.829	0.735	0.296	0.958	0.851	0.353
	0.2	0.845	0.756	0.379	0.977	0.908	0.496	0.994	0.967	0.584
	0.4	0.983	0.930	0.601	1.000	0.988	0.748	1.000	1.000	0.867
	0.6	0.997	0.983	0.708	1.000	1.000	0.894	1.000	1.000	0.974
	0.8	0.999	0.994	0.808	1.000	1.000	0.940	1.000	1.000	0.993
	1	1.000	0.997	0.893	1.000	1.000	0.962	1.000	1.000	0.998
0.75	0.1	0.476	0.312	0.104	0.636	0.386	0.120	0.795	0.494	0.110
	0.2	0.661	0.492	0.149	0.829	0.593	0.156	0.954	0.717	0.162
	0.4	0.867	0.667	0.197	0.966	0.802	0.220	0.995	0.943	0.303
	0.6	0.959	0.774	0.256	1.000	0.926	0.318	1.000	0.991	0.453
	0.8	0.986	0.881	0.301	0.999	0.973	0.434	1.000	0.999	0.566
	1	0.990	0.911	0.381	1.000	0.986	0.470	1.000	1.000	0.701

Table B14: Results for univariate detector (single series with break considered), $T = 400$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.839	0.795	0.611	0.849	0.808	0.648	0.830	0.782	0.644
	0.2	0.839	0.770	0.616	0.837	0.817	0.616	0.833	0.794	0.633
	0.4	0.817	0.783	0.615	0.834	0.780	0.629	0.851	0.784	0.631
	0.6	0.828	0.831	0.625	0.824	0.808	0.627	0.833	0.788	0.641
	0.8	0.834	0.790	0.650	0.824	0.811	0.645	0.829	0.791	0.602
	1	0.821	0.775	0.637	0.843	0.790	0.643	0.827	0.816	0.639
0.50	0.1	0.789	0.733	0.513	0.801	0.754	0.548	0.790	0.726	0.481
	0.2	0.805	0.720	0.513	0.822	0.725	0.567	0.794	0.705	0.478
	0.4	0.807	0.718	0.510	0.800	0.751	0.534	0.793	0.719	0.461
	0.6	0.790	0.764	0.513	0.803	0.719	0.503	0.778	0.713	0.455
	0.8	0.799	0.706	0.506	0.798	0.743	0.534	0.785	0.726	0.489
	1	0.809	0.720	0.499	0.825	0.735	0.552	0.779	0.704	0.530
0.75	0.1	0.684	0.572	0.256	0.682	0.559	0.282	0.680	0.552	0.292
	0.2	0.673	0.560	0.272	0.711	0.556	0.285	0.695	0.568	0.289
	0.4	0.689	0.527	0.262	0.691	0.567	0.316	0.700	0.556	0.300
	0.6	0.681	0.549	0.256	0.700	0.578	0.274	0.684	0.560	0.294
	0.8	0.676	0.535	0.272	0.688	0.562	0.305	0.714	0.578	0.270
	1	0.685	0.557	0.297	0.709	0.581	0.295	0.706	0.575	0.301

Table B15: Results for multivariate detector, $T = 400$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.782	0.734	0.504	0.930	0.911	0.727	0.993	0.971	0.790
	0.2	0.946	0.922	0.744	0.996	0.993	0.896	1.000	1.000	0.950
	0.4	0.999	0.997	0.938	1.000	1.000	0.992	1.000	1.000	0.998
	0.6	1.000	1.000	0.988	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.992	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.752	0.661	0.382	0.916	0.849	0.588	0.987	0.965	0.680
	0.2	0.935	0.859	0.618	0.996	0.972	0.814	1.000	0.998	0.905
	0.4	0.993	0.980	0.813	1.000	1.000	0.942	1.000	1.000	0.990
	0.6	1.000	0.998	0.925	1.000	1.000	0.989	1.000	1.000	1.000
	0.8	1.000	1.000	0.959	1.000	1.000	0.996	1.000	1.000	1.000
	1	1.000	1.000	0.986	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.592	0.438	0.152	0.792	0.611	0.214	0.923	0.771	0.234
	0.2	0.811	0.645	0.236	0.949	0.831	0.321	0.997	0.940	0.390
	0.4	0.960	0.856	0.380	0.999	0.969	0.504	1.000	0.998	0.634
	0.6	0.990	0.928	0.447	1.000	0.992	0.653	1.000	0.999	0.789
	0.8	0.999	0.969	0.536	1.000	0.999	0.751	1.000	1.000	0.884
	1	1.000	0.992	0.655	1.000	1.000	0.831	1.000	1.000	0.931

B.4 Results on relative mean delay of break detection for correlated processes

Results presented are the ratio of the absolute delay of break detection of the multivariate detector to the absolute delay of break detection of the univariate detector

Table B16: Results for $T = 100$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	1.253	1.114	1.050	1.317	1.154	1.025	-	-	-
	0.2	1.089	1.072	1.044	1.274	1.106	1.021	-	-	-
	0.4	0.899	0.988	1.031	1.212	1.093	1.016	-	-	-
	0.6	0.780	0.927	1.014	1.094	1.047	1.017	-	-	-
	0.8	0.701	0.903	1.000	1.005	1.014	1.002	-	-	-
	1	0.654	0.847	0.983	0.932	0.991	0.999	-	-	-
0.50	0.1	1.114	1.056	1.028	1.048	1.042	1.023	1.224	1.088	1.024
	0.2	0.939	1.000	1.009	0.921	0.984	1.017	1.125	1.060	1.010
	0.4	0.830	0.932	0.997	0.758	0.902	1.005	0.938	1.002	1.005
	0.6	0.716	0.892	0.974	0.675	0.840	0.988	0.829	0.954	1.010
	0.8	0.678	0.850	0.979	0.632	0.806	0.988	0.752	0.893	0.986
	1	0.635	0.806	0.973	0.594	0.790	0.953	0.709	0.866	0.964
0.75	0.1	1.050	0.999	0.997	1.066	1.028	1.007	1.015	1.034	1.022
	0.2	0.974	0.994	0.998	0.966	1.003	1.011	0.912	0.997	1.021
	0.4	0.873	0.956	0.990	0.856	0.977	1.003	0.780	0.950	1.007
	0.6	0.824	0.944	0.972	0.773	0.946	1.001	0.695	0.890	1.009
	0.8	0.773	0.914	0.989	0.731	0.905	0.995	0.627	0.833	0.987
	1	0.729	0.906	0.986	0.667	0.881	1.012	0.592	0.809	0.974

Table B17: Results for $T = 200$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	1.169	1.070	1.030	1.020	1.070	1.039	1.394	1.193	1.048
	0.2	0.907	0.994	1.025	0.852	0.949	1.001	1.193	1.136	1.050
	0.4	0.707	0.882	0.982	0.678	0.855	0.992	0.932	1.018	1.018
	0.6	0.660	0.834	0.951	0.651	0.816	0.961	0.807	0.939	1.014
	0.8	0.638	0.809	0.956	0.624	0.779	0.944	0.728	0.875	0.981
	1	0.598	0.787	0.947	0.606	0.779	0.919	0.676	0.860	0.959
0.50	0.1	1.135	1.059	1.023	0.958	1.008	1.021	0.873	0.979	1.038
	0.2	0.932	0.965	1.011	0.777	0.919	0.992	0.735	0.885	1.000
	0.4	0.719	0.886	0.983	0.663	0.840	0.972	0.649	0.814	0.963
	0.6	0.679	0.838	0.967	0.613	0.797	0.943	0.587	0.778	0.933
	0.8	0.626	0.814	0.958	0.593	0.778	0.926	0.572	0.767	0.917
	1	0.603	0.784	0.946	0.577	0.753	0.923	0.540	0.748	0.902
0.75	0.1	1.058	1.048	1.017	1.074	1.056	1.025	0.966	1.012	1.018
	0.2	0.968	0.977	0.993	0.950	1.013	1.018	0.847	0.981	1.023
	0.4	0.842	0.941	0.980	0.800	0.944	1.013	0.727	0.902	1.012
	0.6	0.770	0.924	0.993	0.719	0.897	1.011	0.632	0.842	1.008
	0.8	0.703	0.871	0.975	0.701	0.874	1.014	0.626	0.807	0.976
	1	0.695	0.847	0.972	0.665	0.847	0.989	0.598	0.785	0.972

Table B18: Results for $T = 400$										
		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	1.105	1.064	1.028	0.921	0.992	1.024	0.822	0.960	1.025
	0.2	0.838	0.934	0.984	0.733	0.885	0.990	0.736	0.879	0.996
	0.4	0.665	0.848	0.945	0.671	0.821	0.947	0.679	0.833	0.961
	0.6	0.637	0.822	0.919	0.627	0.825	0.911	0.637	0.802	0.933
	0.8	0.623	0.797	0.913	0.610	0.812	0.923	0.610	0.799	0.910
	1	0.605	0.778	0.904	0.618	0.799	0.916	0.600	0.809	0.906
0.50	0.1	1.106	1.063	1.027	0.915	1.001	1.016	0.777	0.920	0.999
	0.2	0.867	0.945	0.996	0.777	0.885	0.987	0.691	0.837	0.966
	0.4	0.712	0.846	0.966	0.666	0.841	0.946	0.624	0.799	0.917
	0.6	0.648	0.829	0.946	0.627	0.788	0.934	0.577	0.772	0.900
	0.8	0.630	0.789	0.930	0.604	0.793	0.920	0.568	0.765	0.896
	1	0.611	0.794	0.919	0.608	0.775	0.919	0.553	0.742	0.889
0.75	0.1	1.112	1.050	1.012	1.001	1.034	1.021	0.915	1.009	1.024
	0.2	0.943	1.007	1.008	0.861	0.950	1.016	0.779	0.928	1.026
	0.4	0.803	0.910	1.002	0.711	0.894	1.006	0.678	0.849	1.002
	0.6	0.739	0.894	0.984	0.660	0.844	0.987	0.622	0.808	0.978
	0.8	0.691	0.850	0.983	0.635	0.803	0.967	0.605	0.786	0.959
	1	0.655	0.834	0.978	0.630	0.803	0.955	0.602	0.782	0.946

B.5 Results for the multivariate detector based on average absolute cumulative sums

Table B19: Results for multivariate detector (average absolute cumulative sum), $T = 100$

		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.392	0.318	0.146	0.410	0.269	0.095	-	-	-
	0.2	0.693	0.539	0.217	0.672	0.536	0.184	-	-	-
	0.4	0.930	0.863	0.471	0.922	0.791	0.387	-	-	-
	0.6	0.985	0.947	0.683	0.982	0.933	0.556	-	-	-
	0.8	0.997	0.986	0.810	0.999	0.971	0.651	-	-	-
	1	0.999	1.000	0.906	0.998	0.991	0.715	-	-	-
0.50	0.1	0.319	0.204	0.065	0.552	0.385	0.108	0.820	0.586	0.176
	0.2	0.591	0.376	0.112	0.881	0.664	0.198	0.982	0.864	0.354
	0.4	0.900	0.732	0.217	0.992	0.951	0.479	1.000	0.996	0.661
	0.6	0.977	0.909	0.388	1.000	0.993	0.716	1.000	1.000	0.851
	0.8	0.997	0.969	0.576	1.000	0.999	0.844	1.000	1.000	0.944
	1	1.000	0.989	0.706	1.000	1.000	0.929	1.000	1.000	0.980
0.75	0.1	0.162	0.079	0.045	0.279	0.123	0.065	0.627	0.343	0.107
	0.2	0.361	0.150	0.060	0.555	0.260	0.062	0.940	0.667	0.169
	0.4	0.643	0.327	0.084	0.902	0.570	0.128	0.999	0.929	0.379
	0.6	0.821	0.494	0.135	0.980	0.794	0.216	1.000	0.992	0.585
	0.8	0.946	0.699	0.149	0.999	0.923	0.286	1.000	1.000	0.735
	1	0.976	0.788	0.232	1.000	0.969	0.391	1.000	1.000	0.863

Table B20: Results for multivariate detector (average absolute cumulative sum), $T = 200$

		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.510	0.351	0.150	0.758	0.603	0.286	0.935	0.855	0.451
	0.2	0.766	0.691	0.307	0.952	0.909	0.569	0.996	0.987	0.781
	0.4	0.974	0.940	0.649	1.000	0.998	0.909	1.000	1.000	0.985
	0.6	0.997	0.994	0.870	1.000	1.000	0.984	1.000	1.000	0.999
	0.8	0.999	0.998	0.956	1.000	1.000	0.999	1.000	1.000	1.000
	1	1.000	1.000	0.985	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.424	0.263	0.080	0.626	0.425	0.111	0.915	0.762	0.252
	0.2	0.723	0.534	0.169	0.932	0.799	0.271	1.000	0.979	0.610
	0.4	0.957	0.874	0.422	1.000	0.987	0.685	1.000	1.000	0.966
	0.6	0.995	0.976	0.674	1.000	0.999	0.901	1.000	1.000	0.998
	0.8	0.999	0.996	0.830	1.000	1.000	0.978	1.000	1.000	0.999
	1	1.000	1.000	0.922	1.000	1.000	0.996	1.000	1.000	1.000
0.75	0.1	0.243	0.118	0.062	0.399	0.182	0.070	0.609	0.289	0.056
	0.2	0.480	0.249	0.093	0.725	0.418	0.089	0.948	0.717	0.121
	0.4	0.851	0.558	0.151	0.963	0.830	0.215	1.000	0.984	0.388
	0.6	0.959	0.774	0.219	0.998	0.966	0.368	1.000	0.997	0.636
	0.8	0.990	0.917	0.347	1.000	0.991	0.584	1.000	1.000	0.839
	1	0.998	0.969	0.454	1.000	1.000	0.714	1.000	1.000	0.949

Table B21: Results for multivariate detector (average absolute cumulative sum), $T = 400$

		p = 10			p = 20			p = 40		
m/T	p_b	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 0.25$	$b = 0.50$	$b = 0.75$
0.25	0.1	0.582	0.463	0.200	0.787	0.697	0.308	0.971	0.943	0.649
	0.2	0.855	0.797	0.437	0.979	0.946	0.672	1.000	0.999	0.947
	0.4	0.988	0.982	0.840	1.000	1.000	0.975	1.000	1.000	1.000
	0.6	0.999	0.999	0.964	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.1	0.514	0.360	0.130	0.725	0.517	0.137	0.936	0.787	0.266
	0.2	0.831	0.695	0.306	0.965	0.868	0.425	0.997	0.989	0.724
	0.4	0.993	0.957	0.703	1.000	0.999	0.904	1.000	1.000	0.993
	0.6	0.999	0.995	0.899	1.000	1.000	0.993	1.000	1.000	1.000
	0.8	0.999	0.999	0.966	1.000	1.000	0.999	1.000	1.000	1.000
	1	1.000	1.000	0.994	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.1	0.363	0.196	0.067	0.500	0.302	0.091	0.694	0.410	0.086
	0.2	0.667	0.400	0.126	0.859	0.632	0.161	0.980	0.847	0.214
	0.4	0.931	0.784	0.312	0.998	0.951	0.471	1.000	0.998	0.687
	0.6	0.995	0.946	0.517	1.000	0.998	0.754	1.000	1.000	0.939
	0.8	0.998	0.991	0.696	1.000	0.999	0.908	1.000	1.000	0.992
	1	1.000	0.997	0.824	1.000	1.000	0.961	1.000	1.000	1.000

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