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Abstract

Empirical evidence suggests that risk premia are higher at business cycle troughs than they are at peaks. Existing asset pricing theories ascribe moves in risk premia to changes in volatility or risk aversion. Nevertheless, in a simple general equilibrium model, risk premia can be *procyclical* even though the volatility of consumption is *constant* and despite a *countercyclically* varying risk aversion coefficient. We show that agents' expectations about future prospects also influence premium dynamics. In order to generate countercyclically varying premia, as found in the data, one requires a combination of hump-shaped consumption dynamics or highly persistent shocks and habits. Our results, thus, suggest that factors which help match activity data may also help along the asset pricing dimension.

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Contents

Summary

Risky assets, such as stocks, tend to yield higher returns than safer assets, such as bonds. This difference in returns reflects the fact that investors require extra compensation (or a ’premium’) for bearing risk. Evidence suggests that the size of this risk premium depends on whether the economy is in a period of stagnation or prosperity. In particular, investors require higher premia during economic slowdowns than during booms. This empirical regularity has been termed ‘premium countercyclicality’, and accounting for it in a theoretical framework is the focus of this paper.

We assume that investors form ‘consumption habits’. That is, they get used to a certain reference level of consumption, which, much like real-life habits, is allowed to change over time. Allowing for habits has two main implications. First, it means that good times correspond to periods when actual consumption is high *relative* to this reference level. Second, it implies that in those good times, agents tend to be less averse to bearing risk (ie risk aversion is countercyclical). Our first, somewhat surprising, finding is that it is possible for more risk-averse agents to demand lower compensation for bearing risk. The remainder of the paper then analyses why this is the case and highlights conditions which guarantee that risk premia fall in good times and increase in bad times — as found in the data.

We first demonstrate that investors’ assessment of future prospects is crucial in determining the behaviour of premia. We then show how the interplay of different model parameters, such as the speed with which investors change their habits or the persistence of shocks affecting the economy, jointly influence investors’ assessment of the future. We prove that, in our simple model, to generate countercyclical risk premia, shocks to economic conditions have to be long-lasting, and consumption habits have to adjust slowly to these shocks.

To understand the intuition behind this result, consider a bad shock which pushes down the level of consumption. If the shock is temporary and households very quickly change their habits, then next period they will be used to a lower level of consumption, while actual consumption will tend to revert back to its previous (higher) level. Hence, households hit by the negative shock have every reason to expect consumption next period to be high relative to the benchmark.

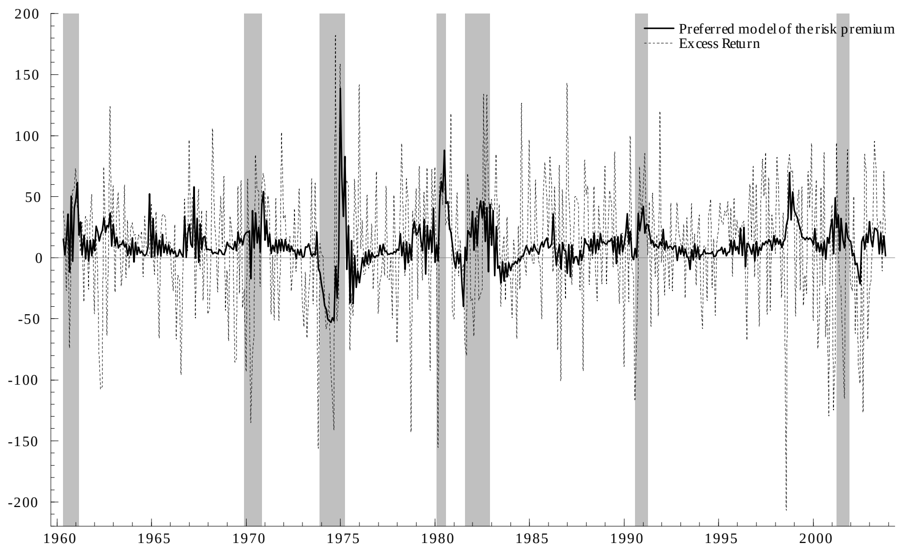
Accordingly, even though risk aversion increases as a result of the bad shock, prospects of *good times* ahead make agents take on more risk and actually lead to a compression of premia. This is why temporary shocks and quickly adjusting habits translate into procyclical risk premia.

We then extend our analysis and investigate the likely behaviour of risk premia given more complicated dynamics of consumption, similar to those that might arise in modern macroeconomic models (and, arguably, in the data which they attempt to fit). A typical feature of these models is that they produce a 'hump-shaped' response of consumption to shocks. That is, following a bad shock, consumption will initially be expected to fall before recovering. As a result, bad shocks can lead to a reduction in risk-taking and an increase in risk premia, even if habits adjust quickly. Thus, under this specification, the conditions for countercyclical premia become less stringent. This result suggests that features which help generate hump-shaped consumption responses are likely to generate more realistic risk premium behaviour.

1 Introduction

Available empirical estimates suggest that risk premia vary over time (see Chart 1 for an example).[1] Harvey (1989) showed that US equity risk premia are higher at business cycle troughs than they are at peaks. Subsequent results of Bekaert and Harvey (1995), He, Kan, Ng and Zhang (1996) and Li (2001) confirmed these findings. Cochrane and Piazessi (2005) find that the term premium is countercyclical in the United States while Lustig and Verdelhan (2007) document strong countercyclicality in the exchange rate risk premium. The two most popular asset pricing models attribute this variation either to countercyclical changes in risk aversion (Campbell and Cochrane (1999)) or to changes in the volatility of the consumption process (Bansal and Yaron (2004)). This paper uses a simple, standard, general equilibrium setup to demonstrate analytically that risk premia can be *procyclical* even though the volatility of consumption is *constant* and despite a *countercyclically* varying risk aversion coefficient. This seems puzzling and raises the question of what other factors can cause risk premia to vary over time? Identifying them and explaining why they matter are the focus of this paper.

Chart 1: An estimate of the US equity risk premium — taken from Smith, Sorenssen and Wickens (2007)



[1]The model underlying Chart 1 can be interpreted as a variant of Merton's intertemporal CAPM. The estimate of the risk premium is obtained assuming that the equity risk premium changes over time with the conditional covariance between returns on a broad stock market index and macroeconomic variables such as real industrial production, money and inflation.

We believe our study is important for several reasons. First, observed changes in asset prices appear to be associated with fluctuations in premia, and so having a good understanding of factors driving premia is crucial for modelling asset prices.[2] Moreover, once the factors affecting risk premia are well understood, asset price data can be meaningfully used to back out shocks affecting the economy. Finally, given the increasing frequency with which dynamic stochastic general equilibrium (DSGE) models are being used to address asset pricing puzzles, it is important to clarify how and why changes in standard modelling assumptions translate into different dynamics of premia.[3]

Our analysis proceeds in the simplest possible setup — that of Lucas' (1978) endowment economy. We deliberately leave capital and the entire production side of the economy out of the picture because, as pointed out by Mehra and Prescott (1985), this imposes no constraints on the set of joint equilibrium processes on consumption and asset prices.[4] The parsimony of our model allows us to derive tractable results and enables us to analyse the impact of expected consumption dynamics on premia — as all we need to do is specify the shock process appropriately. Nevertheless, since some of our results rely on approximations, we subsequently use numerical simulations conducted on the De Paoli, Weeken and Scott (2007) model to show that the intuition we develop as well as our conclusions continue to hold in a fully fledged DSGE framework.

In our model we allow for consumption habits in the utility function.[5] These have been helpful in accounting for asset price properties — eg in Campbell and Cochrane (1999) and have also proved useful in many areas in macro — see for instance Carroll, Overland and Weil (2000), Fuhrer (2000) or Christiano, Eichenbaum and Evans (2005). Unlike Campbell and Cochrane (1999), however, we use a linear, additive external habit specification which nests those used in

[2] Campbell and Cochrane (1999, page 207), for instance, argue that *a slowly time-varying, countercyclical risk premium* is key for matching asset pricing data.

[3] For examples of such papers, see for instance Jermann (1998), Boldrin, Christiano and Fisher (2001), Uhlig (2007) or Rudebusch and Swanson (2008).

[4] While introducing capital can change the dynamics of consumption, the results presented in this paper remain valid *conditional* on a given path for consumption. Arguably, this argument only holds true in a setup in which marginal utility is only a function of consumption — which we shall assume throughout. See for instance Uhlig (2007) for an interesting analysis of asset pricing implications of consumption-leisure non-separabilities.

[5] Abel (1990) distinguishes between 'habits' and 'catching up with the Joneses'. The former depends on an agent's own past level of consumption, while the latter depends on some external benchmark. Abel's 'habit' variable is treated as endogenous in the utility maximisation problem while those capturing 'catching up with the Joneses' are taken as exogenous. This distinction got slightly blurred in Campbell and Cochrane's (1999) paper, where they referred to 'keeping up with the Joneses' as 'habits'. To avoid confusion we refer to 'Abel's habits' as 'internal habits', while 'keeping up with the Joneses', which is what we focus on in this paper, shall be termed 'external habits' (or just 'habits' when no confusion can arise).

Uhlig (2004) or Smets and Wouters (2007).[6] Other than implying a time-varying risk aversion coefficient, this setup has the added appeal that there is a single parameter directly controlling habit persistence.

So what determines the risk premium's pattern of cyclical variation? In the remainder of the paper we demonstrate how premium cyclicality depends on agents' assessment of future prospects. We investigate how the interplay of habit and shock persistence affects premia in the presence of exponentially decaying and hump-shaped consumption responses. We show analytically that the standard external habit specification implies counterfactually *procyclical* premium variation unless shocks or habits are sufficiently persistent or unless the impulse response of consumption is hump-shaped.

Several existing contributions explore issues related to those we analyse. Li (2007) documents that premia in the framework of Campbell and Cochrane (1999) are not robustly countercyclical, a point similar to the one we make in a different setup. Den Haan (1995) shows that the slope of the yield curve changes with the endowment specification used — which is closely related to our findings on the role of shock persistence. Equally, there are many papers showing how habits in the utility function help match empirical properties of anything from exchange rate risk premia (Verdelhan (2006)) to yield curve responses (Wachter (2006)), though we are unaware of any which explicitly investigate the impact of structural model characteristics on premium dynamics.

In the following sections we describe the model, state our analytical results, provide the intuition and present simulation results before summarising and concluding.

[6]Campbell and Cochrane (1999) assume a non-linear habit. However, they argue that up to a first-order approximation the process for surplus consumption they impose is equivalent to an autoregressive habit specification — similar to the one we use.

2 Model and notation

Agents, indexed by $i \in [0, 1]$ choose consumption C_t^i, investment in riskless bonds B_t^i and investment in risky assets S_t^i to maximise the expected discounted value of lifetime utility:

$$\max_{C_t^i,\, B_t^i,\, S_t^i} \mathbf{E}\left(\sum_{t=0}^{+\infty} \beta^t \frac{\left(C_t^i - hX_t\right)^{1-\rho} - 1}{1-\rho}\right) \quad \textbf{(1)}$$

$$\text{s.t. } X_t := (1-\phi)C_{t-1} + \phi X_{t-1} \quad \textbf{(2)}$$

$$C_t^i + V_t^f B_t^i + V_t^r S_t^i = B_{t-1}^i + S_{t-1}^i\left(V_t^r + D_t\right) \quad \textbf{(3)}$$

where X_t denotes the external habit, C_t corresponds to aggregate consumption, V_t^f is the time t price of a one-period bond paying a unit of the consumption good next period and V_t^r is the price of a perfectly divisible risky asset entitling its owner to the stream of dividends $D_{t+1}, D_{t+2}, \ldots$.[7] The parameter ϕ (where $0 \leq \phi < 1$) determines the persistence of habits and $C_{-1}, C_{-2}, \ldots$ are assumed given. Note that for $\phi = 0$ our specification simplifies to one in which habits are purely a function of last period's aggregate consumption while for $h = 0$ we are back in the standard setup of Lucas (1978) and Mehra and Prescott (1985).

The standard first-order conditions with respect to asset holdings are

$$R_{t+1}^f \cdot \mathbf{E}_t\, \mathcal{M}_{t+1}^i = 1 \qquad\qquad \mathbf{E}_t\, \mathcal{M}_{t+1}^i\, R_{t+1}^r = 1$$

where the stochastic discount factor $\mathcal{M}_t^i$, marginal utility of consumption Λ_t^i and gross returns on bonds R_t^f and risky assets R_t^r are defined as

$$\mathcal{M}_{t+1}^i := \beta \cdot \frac{\Lambda_{t+1}^i}{\Lambda_t^i} \Lambda_{t+1}^i := \left(C_{t+1}^i - hX_{t+1}\right)^{-\rho} \qquad R_{t+1}^f := \frac{1}{V_t^f} R_{t+1}^r := \frac{V_{t+1}^r + D_{t+1}}{V_t^r}.$$

We use a textbook definition of the equity risk premium rp_t

$$rp_t := \mathbf{E}_t\left(\log\left(R_{t+1}^r\right)\right) - \log\left(R_{t+1}^f\right) := \mathbf{E}_t r_{t+1}^r - r_{t+1}^f \quad \textbf{(4)}$$

where lower-case letters denote logs — ie $r_{t+1}^i, i \in \{f, r\}$ corresponds to log returns. For analytical tractability, we restrict attention to a representative agent model, ie we assume that individual consumption C_t^i as well as the marginal utility of consumption Λ_t^i and the stochastic discount factor $\mathcal{M}_t^i$ equal their respective aggregate equivalents C_t, Λ_t and $\mathcal{M}_t$.

We define the coefficient of relative risk aversion as

$$\eta(C_t, X_t) := -C_t \cdot \frac{U_{cc}(C_t, X_t)}{U_c(C_t, X_t)}$$

[7] It could, for example, correspond to dividends on an equity share or to a call option on next period's bond with strike price K (in which case $D_1 = (V_{t+1}^f - K)^+$ and $D_i = 0$ for all $i > 1$).

where $U_y(\cdot,\cdot)$ denotes the partial derivative of utility function $U(\cdot,\cdot)$ with respect to y.[8] Since this coefficient measures agents' willingness to enter pure consumption gambles, given habits fixed at reference level X_t, we shall frequently refer to it as *consumption* risk aversion.[9] Finally, let excess consumption C_t^e and the surplus ratio S_t be defined as

$$C_t^e := C_t - hX_t \qquad\qquad S_t := \frac{C_t - hX_t}{C_t} = \frac{C_t^e}{C_t}.$$

It is easy to show that if the utility function and external habits are as in equations **(1)**–**(2)** then the coefficient of consumption risk aversion is countercyclical. To see this, note that $\eta_t = \rho\, C_t/(C_t - hX_t)$ and so:

$$\frac{\partial \eta_t}{\partial C_t} = -\rho \frac{hX_t}{(C_t - hX_t)^2} \leq 0. \qquad \textbf{(5)}$$

To analyse the determinants of risk premia in the model, we can derive a second-order approximation to the first-order conditions. This approximation implies

$$rp_t + \frac{1}{2} var_t\, r_{t+1}^r \approx \rho\, cov_t(c_{t+1}^e, r_{t+1}^r). \qquad \textbf{(6)}$$

Jensen's inequality term aside, the risk premium is proportional to the *excess consumption* relative risk aversion coefficient ρ and the conditional covariance of returns r_{t+1}^r with excess consumption c_{t+1}^e. As shown in the appendix, and under the assumptions therein, we can express $cov_t(c_{t+1}^e, r_{t+1}^r)$ as

$$cov_t(c_{t+1}^e, r_{t+1}^r) = cov_t(c_{t+1}, r_{t+1}^r)\, \mathbf{E}_t \frac{1}{S_{t+1}}. \qquad \textbf{(7)}$$

Equation **(7)** demonstrates that agents' expectations about the surplus ratio matter because they affect the covariance of excess consumption and returns. Combined with equation **(6)** this shows that if $cov_t(c_{t+1}, r_{t+1}^r)$ is time invariant, then only changes in these expectations are going to affect risk premium cyclicality.[10] Equation **(7)** can further be rewritten as

$$rp_t + \frac{1}{2} var_t\, r_{t+1}^r \approx \eta_t\, cov_t(c_{t+1}, r_{t+1}^r)\, \mathbf{E}_t \frac{S_t}{S_{t+1}}. \qquad \textbf{(8)}$$

Equations **(7)** and **(8)** clearly summarise the forces driving risk premia in our model. They demonstrate that the risk premium is determined by the coefficient of risk aversion η_t, the covariance of consumption and returns as well as expectations about the growth of the surplus

[8]For notational parsimony, we shall typically stress the dependence of η on C_t and X_t by referring to it as η_t (with the point at which the derivatives are evaluated to be inferred from the context).

[9]Note that in our setup excess consumption $C_t^e := C_t - hX_t$ plays the role of consumption in a no-habit model. It is important not to confuse η_t — the state-dependent measure of risk preferences towards pure consumption, with the coefficient of *excess consumption* relative risk aversion which, given CRRA utility in C_t^e, is a constant equal to ρ, and which measures aversion to 'excess consumption' lotteries.

[10]Many partial-equilibrium finance papers assume that the covariance of consumption and returns is constant. Numerical simulations we conducted on a number of DSGE models also suggest that fluctuations in these covariances are small.

ratio. Importantly, they show that if agents' expectations of the future improve following a bad shock (or deteriorate following a good shock) then the risk premium can be procyclical even though the risk aversion coefficient is countercyclical.

In the next sections we will discuss how different specifications for habits and endowments affect agents' expectations and thus premium cyclicality.

3 Results

3.1 Case of auto-regressive consumption dynamics

In this section we investigate factors driving risk premium cyclicality under the assumption that consumption following a shock converges back to steady state exponentially or follows a unit root process. The following proposition formalises our results.

Proposition 1 *If the conditional variance of returns* $var_t(r^r_{t+1})$ *and their conditional covariance with consumption* $cov_t(r^r_{t+1}, c_{t+1})$ *are constant and log consumption follows*

$$c_t = \gamma c_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim i.i.d.(0, \sigma^2), \qquad \gamma \in [0, 1] \tag{9}$$

then the derivative of the risk premium rp_t *with respect to the current shock realisation can be expressed as*[11]

$$\frac{\partial rp_t}{\partial \varepsilon_t} \approx E_t S^{-2}_{t+1} C^{-1}_{t+1} h(1-\phi)\left[C_t - \gamma \sum_{s=0}^{t} \phi^s C_{t-s}\right]. \tag{10}$$

The proof of this and all subsequent propositions can be found in the appendix. Since, in general, the sign of the risk premium is ambiguous, in order to build some intuition we now focus on two popular specifications — one in which habits fully adjust in a single period and the other in which consumption is a random walk, as in Hall (1978).

Corollary 2 *If habits only depend on past periods' consumption* ($\phi = 0$) *then the risk premium*

[11] Equation **(10)** holds exactly under the additional assumption that excess consumption and risky returns are jointly conditionally log normal and that the conditions of Stein's lemma are satisfied. Clearly, under the additive linear habit preference specification these distributional assumptions generate ill-defined maximisation problems. Resolving this issue is beyond the scope of this paper, but a discussion of conditions guaranteeing that utility is well defined as well as analytical formulae for asset prices in this model can be found in Zabczyk (2008).

is procyclical

$$\frac{\partial rp_t}{\partial \varepsilon_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\gamma) \geq 0. \tag{11}$$

Under the assumptions of Corollary 2, following an adverse shock, agents expect consumption to improve (given the AR(1) nature of the consumption process) while habits will unambiguously fall. Since habits adjust fully in a single period, excess consumption, which is all agents care about, is expected to *increase* following the negative shock. Thus, even despite higher risk aversion, agents will require lower compensation for bearing risk — ie premia will fall.[12] The fact that the risk premium is procyclical if $\phi = 0$ is important as that assumption is frequently used in macro models.[13]

Corollary 3 *Under the assumptions of Proposition 1 if log consumption follows a random walk ($\gamma = 1$) and habits are persistent ($\phi > 0$) then the risk premium is countercyclical as*

$$\frac{\partial rp_t}{\partial \varepsilon_t} = -E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi) \sum_{s=1}^{t} \phi^s C_{t-s} < 0. \tag{12}$$

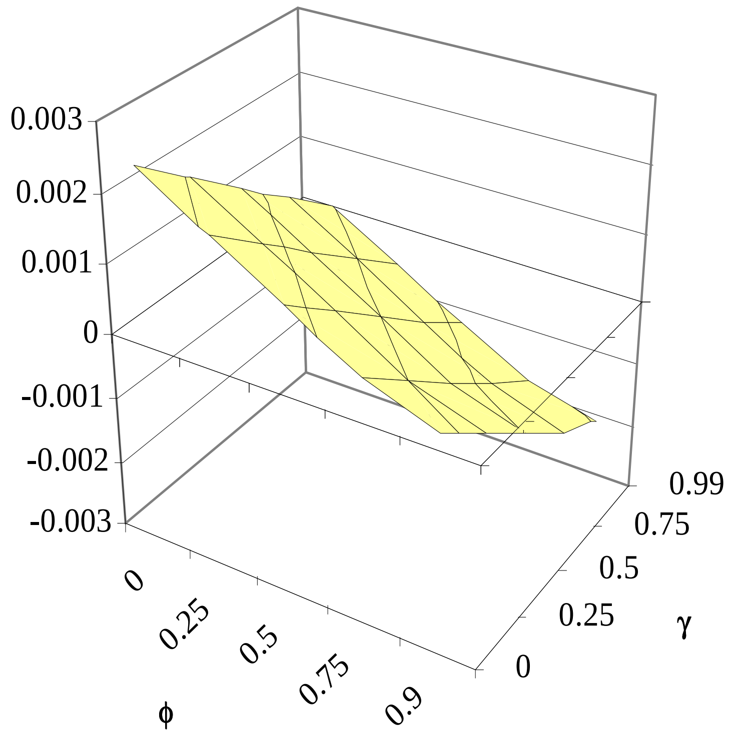
Under the assumptions of Corollary 3, shocks to consumption are permanent and habits adjust gradually. In this setting, after adverse shocks, expected excess consumption falls leading to an increase in the risk premium. This shows that a combination of permanent shocks and persistent habits generates countercyclically varying risk premia. Equation **(10)** generalises this point and shows that a sufficiently persistent shock yields countercyclical premium variation. While the effect of increasing habit persistence ϕ is less clear-cut, Chart 2 suggests that raising ϕ has a similar effect.

So, these results demonstrate that despite the countercyclicality of the risk aversion coefficient, premia need not necessarily be countercyclical. To further develop intuition and clarify the economics we can refer back to the specification of utility. Inspecting expression **(1)** it should be clear that when consumption is close to the habit level even small fluctuations in C_t result in large swings in marginal utility. Because risk-averse agents dislike these large swings, they would like to keep consumption far away from the reference level hX_t.

[12] Alternatively, one can think that agents' 'excess consumption' risk has fallen.

[13] Models that assume $\phi = 0$ include Smets and Wouters (2007) or Uhlig (2007). This result does not necessarily establish that premia in those models are procyclical as the implied consumption dynamics might differ from the one we assume here — see also subsequent sections for a discussion.

Chart 2: Cyclicality of the equity risk premium in a Lucas tree economy with an external habit (chart shows $\partial rp_t/\partial \varepsilon_t$ as a function of habit persistence ϕ and shock persistence γ)



So let us now consider a shock which forces C_t closer to hX_t. By virtue of the argument above, agents become more risk-averse as new gambles could push their consumption into regions of even more volatile marginal utility. This is the logic behind formula **(5)**. However even though risk aversion increases, the behaviour of premia will depend on the persistence of shocks and habits and, more generally, on expectations about the future path of consumption. This is clearly demonstrated in equation **(8)**.

To see why, let us assume that habits adjust very quickly (eg $\phi = 0$, as in Corollary 2) — which means that they will fall significantly next period — and that the shock was temporary (ie $\gamma < 1$) — implying that consumption is expected to unwind. Both of these factors would tend to push consumption C_{t+1} away from hX_{t+1} into regions in which agents feel more comfortable. So, even though their current situation might have deteriorated, the expectation that things are going to improve significantly would tend to make them take on more risk. The increase in risk-taking would, in turn, translate into a compression of premia. So, in this situation a bad shock would be associated by a fall in premia — ie premia would evolve procyclically.

However, if habits took very long to adjust (corresponding to high values of ϕ) then, even if consumption would be expected to partially recover (corresponding to low values of γ), consumers could still be worse off relative to the steady state. Accordingly, they would be unwilling to take on risk and, as a result, premia would have to be higher, ie they would be countercyclical. Equally, if habits adjusted very quickly but shocks were permanent (Corollary 3), then following a bad shock consumers would expect to be back at their benchmark utility level and so their willingness to take on risk would be unchanged and so would be risk premia.

Of course it is possible that even if some deep, 'fundamental' shocks follow AR dynamics, the consumption process might not. In the data and in most large-scale dynamic general equilibrium models, consumption does not necessarily behave as posited in equation **(9)**. Many authors have postulated that the response of consumption to shocks displays a 'hump'. If this is the case then, following a bad shock, consumption will initially be expected to fall before recovering. This means that even if habits adjust quickly, expected consumption next period might still fall relative to habits resulting in a fall in risk-taking and pushing premia up. In the next section we investigate whether this mechanism might generate countercyclical premia even in the absence of persistent habits.

3.2 Case of hump-shaped consumption dynamics

We now investigate how the cyclicality of the risk premium is determined when consumption displays a hump-shaped adjustment profile. To capture this idea in the simplest possible way, we now model log consumption as an ARMA(1,1) process, ie[14]

$$c_t = \gamma c_{t-1} + \varepsilon_t + \theta \varepsilon_{t-i}, \qquad \varepsilon_t \sim \mathcal{N}.i.d.(0, \sigma^2), \qquad \gamma \in [0, 1]. \qquad \textbf{(13)}$$

Proposition 4 *If the conditional variance of returns $var_t(r^r_{t+1})$ and their conditional covariance with consumption $cov_t(r^r_{t+1}, c_{t+1})$ are constant and log consumption follows an ARMA process as in equation **(13)** then*

$$\frac{\partial rp_t}{\partial \varepsilon_t} \approx E_t S^{-2}_{t+1} C^{-1}_{t+1} h(1-\phi) \left[C_t - (\gamma + \theta) \sum_{s=0}^{t} \phi^s C_{t-s} \right]. \qquad \textbf{(14)}$$

[14]The results discussed in this section naturally generalise to arbitrary ARMA(1,K) processes.

In particular, if habits only depend on past periods' consumption ($\phi = 0$) and $\gamma + \theta \geq 1$ then

$$\frac{\partial rp_t}{\partial \varepsilon_t} \approx E_t S_{t+1}^{-2} C_{t+1}^{-1} h C_t \left[1 - (\gamma + \theta) \right] \leq 0. \qquad \textbf{(15)}$$

Equation **(14)** demonstrates that θ can play a similar role to γ. It suggests that models with hump-shaped consumption responses may be able to generate countercyclical premia without persistent habits or shocks. To understand the intuition we focus on the simpler case described in equation **(15)**. This shows that if $\gamma + \theta > 1$ then the risk premium is unambiguously countercyclical. Notably this condition implies that log consumption increases (decreases) further in the period after a positive (negative) shock, following which it converges back to its steady state. In this case, after a bad shock, agents expect the future to get worse (ie excess consumption to decrease) and therefore require higher compensation for bearing risk.

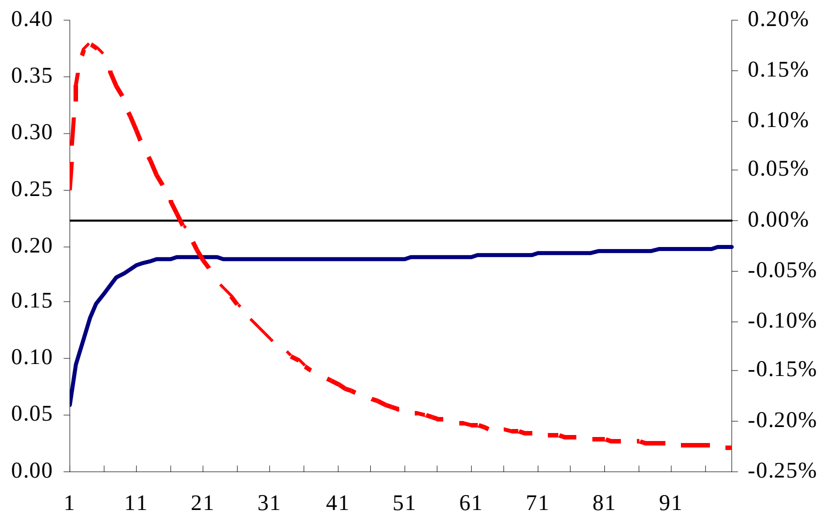
3.2.1 Premium dynamics in a production economy

In order to illustrate how our results can be helpful in making sense of the properties of premia in more general settings, we scrutinised the dynamics of the equity risk premium in a production economy similar to Christiano *et al* (2005). In particular, we used the DSGE model documented in De Paoli *et al* (2007). This framework features habit formation, but sets ϕ equal to 0, explicitly models capital, and allows for many rigidities. While the structural shocks are AR(1), ie they lack the moving average component of an ARMA(1,1) process, the model's internal propagation mechanism generates a hump-shaped response of consumption to productivity shocks — see Chart 3.

To compute the response of the equity risk premium to a productivity shock, we solved the model to third order using perturbation methods — as implemented in *Dynare++* and Perturbation AIM.[15] The chart confirms that the equity risk premium is indeed countercyclical. This is a consequence of the fact that De Paoli *et al* (2007) assume highly persistent shocks — which is mirrored in the profile of consumption — and is 'helped' by the hump in the impulse response of consumption. The latter confirms the findings of Proposition 4, which states that even with $\phi = 0$ the hump-shaped response can yield countercyclically varying premia. Equally, Proposition 1

[15] Third order is the lowest which allows for time variation in risk premia. As is well known — and as discussed in Schmitt-Grohe and Uribe (2004) — a first-order approximation would imply that premia are zero at all times, while a second-order approximation would only allow for constant premia.

Chart 3: Response of consumption (solid line, LHS) and the equity risk premium (dashed line, RHS) to a productivity shock in the model of De Paoli, Scott and Weeken (2007)



suggests that if it were not for this endogenous hump, the equity risk premium in this model would be procyclical.

4 Summary and conclusions

We have used a simple, general equilibrium model to analyse the determinants of risk premium dynamics. We have demonstrated that risk premia can be *procyclical* even though the volatility of consumption is *constant* and despite a *countercyclically* varying risk aversion coefficient. We have documented, however, that persistent habits, shocks or a hump-shaped consumption process are all likely to make the premium countercyclical. Our results suggest that the countercyclicality of the premium rests on agents' belief that changes in economic conditions, as summarised by the surplus ratio, are persistent.

Implications of different model features for the equilibrium properties of macroeconomic variables are well understood. But while the asset pricing implications of such models have recently started to receive wider attention, the understanding of the role of various structural characteristics is still limited. We hope that by focusing on factors affecting risk premium cyclicality, a crucial determinant of asset price dynamics, this paper has made some progress in this direction. Our results suggest, in particular, that factors which help match activity data — such as persistent shocks or consumption habits — may also help along the asset pricing dimension.

Appendix

Derivation of equation (7)

Stein's lemma postulates that, if X and Y are jointly normally distributed, then

$$cov(g(X), Y) = E[g'(X)]cov(X, Y).$$

Therefore, under the assumption that c_{t+1} and r^r_{t+1} are jointly normally distributed, we can express $cov_t(c^e_{t+1}, r^r_{t+1})$ as

$$cov_t(c^e_{t+1}, r^r_{t+1}) = \lambda_t \, cov_t \left(c_{t+1}, r^r_{t+1}\right) \quad \textbf{(A-1)}$$

where

$$\lambda_t := E_t(\partial c^e_{t+1}/\partial c_{t+1}). \quad \textbf{(A-2)}$$

This derivation can also be found in Li (2001).

Proof of Proposition 1

Equation **(A-1)** combined with the assumptions of Proposition 1 implies that the derivative of the risk premium with respect to the current shock realisation, which determines the cyclicality of the premium, equals

$$\frac{\partial rp_t}{\partial \varepsilon_t} \approx \rho \, cov_t \left(c_{t+1}, r^r_{t+1}\right) \frac{\partial \lambda_t}{\partial \varepsilon_t}.$$

Assuming the asset in question is risky, and so covaries positively with consumption, the cyclicality of the premium will be entirely determined by the sign of the derivative of λ_t, which we shall now establish.

From the definition of the surplus ratio, it follows that

$$\lambda_t = E_t(1/S_{t+1}). \quad \textbf{(A-3)}$$

Recursive use of the definition of habits — equation **(2)** — makes it possible to express this in terms of consumption and model parameters as

$$\lambda_t = E_t\left[1 - \left(C^\gamma_t \mathrm{e}^{\varepsilon_{t+1}}\right)^{-1} h(1-\phi) \sum_{s=0}^{+\infty} \phi^s C_{t-s}\right]^{-1} \quad \textbf{(A-4)}$$

where we have substituted out for C_t using the definition of the endowment process **(9)**. The chain rule implies that

$$\frac{\partial \lambda_t}{\partial \varepsilon_t} = \frac{\partial \lambda_t}{\partial C_t} \cdot \frac{\partial C_t}{\partial c_t} \cdot \frac{\partial c_t}{\partial \varepsilon_t}. \quad \textbf{(A-5)}$$

It follows directly from the respective definitions that

$$\frac{\partial C_t}{\partial c_t} = C_t \frac{\partial c_t}{\partial \varepsilon_t} = 1$$

and so to determine the cyclicality of the risk premium it suffices to determine the sign of $\partial \lambda_t / \partial C_t$. From **(A-3)** we obtain that

$$\frac{\partial \lambda_t}{\partial C_t} = E_t - S_{t+1}^{-2} \frac{\partial S_{t+1}}{\partial C_t} = -E_t S_{t+1}^{-2} \frac{\partial}{\partial C_t} \left[1 - \left(C_t^{\gamma} \, \mathrm{e}^{\varepsilon_{t+1}} \right)^{-1} h(1-\phi) \sum_{s=0}^{+\infty} \phi^s C_{t-s} \right]$$

where, in the first equality, we have used the fact that C_t is measurable with respect to the natural filtration and swapped the order of integration and differentiation. Computing the derivative of the term in the square brackets we finally arrive at

$$\frac{\partial \lambda_t}{\partial C_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi) \left[1 - \gamma \; C_t^{-1} \sum_{s=0}^{+\infty} \phi^s C_{t-s} \right]$$

which, can be plugged into equation **(A-5)** to yield

$$\frac{\partial \lambda_t}{\partial \varepsilon_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi) \left[C_t - \gamma \; \sum_{s=0}^{+\infty} \phi^s C_{t-s} \right]. \; Q.E.D. \qquad \textbf{(A-6)}$$

Proof of Proposition 4

From the proof of Proposition 1 we know that

$$\frac{\partial rp_t}{\partial \varepsilon_t} = \rho \, cov_t \left(c_{t+1}, r_{t+1}^r \right) \frac{\partial \lambda_t}{\partial \varepsilon_t}$$

where

$$\lambda_t = E_t \left[1 - C_{t+1}^{-1} h(1-\phi) \sum_{s=0}^{+\infty} \phi^s C_{t-s} \right]^{-1}. \qquad \textbf{(A-7)}$$

The chain rule implies

$$\frac{\partial \lambda_t}{\partial \varepsilon_t} = \frac{\partial \lambda_t}{\partial C_t} \cdot \frac{\partial C_t}{\partial c_t} \cdot \frac{\partial c_t}{\partial \varepsilon_t} + \frac{\partial \lambda_t}{\partial C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial c_{t+1}} \cdot \left(\frac{\partial c_{t+1}}{\partial \varepsilon_t} + \frac{\partial c_{t+1}}{\partial c_t} \frac{\partial c_t}{\partial \varepsilon_t} \right).$$

It follows directly from the respective definitions that

$$\frac{\partial C_t}{\partial c_t} = C_t, \qquad \frac{\partial C_{t+1}}{\partial c_{t+1}} = C_{t+1}, \qquad \frac{\partial c_{t+1}}{\partial c_t} = \gamma\,, \qquad \frac{\partial c_t}{\partial \varepsilon_t} = 1, \qquad \frac{\partial c_{t+1}}{\partial \varepsilon_t} = \theta_1$$

and so, since

$$\frac{\partial \lambda_t}{\partial C_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi), \; \frac{\partial \lambda_t}{\partial C_{t+1}} = -E_t S_{t+1}^{-2} C_{t+1}^{-2} h(1-\phi) \sum_{s=0}^{+\infty} \phi^s C_{t-s}$$

therefore

$$\frac{\partial \lambda_t}{\partial \varepsilon_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi) \left[C_t - \left(\theta + \gamma \right) \sum_{s=0}^{+\infty} \phi^s C_{t-s} \right] . Q.E.D. \qquad \textbf{(A-8)}$$

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