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Deep habits and the cyclical behaviour of equilibrium unemployment and vacancies

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Abstract

We extend the standard textbook search and matching model by introducing deep habits in consumption. The cyclical fluctuations of vacancies and unemployment in our model can replicate those observed in the US data, with labour market tightness being 20 times more volatile than consumption. Vacancies display a hump-shaped response to technology shocks as well as autocorrelation coefficients that are in line with the empirical evidence. Our model preserves the assumption of fully flexible wages for the new hires and the calibration is consistent with the estimated elasticity of unemployment to unemployment benefits. The numerical simulations generate an artificial Beveridge curve which is in line with the data.

Key words: Consumption, business cycles, labour market fluctuations, search and matching, wage bargaining.

JEL classification: E21, E24, E32, J41, J64.

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Contents

Summary	3
1 Introduction	5
2 The model	7
2.1 The labour market	7
2.2 The household's problem	8
2.3 The firm's problem	10
2.4 Wage bargaining	12
2.5 Aggregation and the competitive equilibrium	13
3 The source of amplification	13
4 Calibration	14
5 Model evaluation	16
5.1 US data	17
5.2 Simulation results	17
5.3 Impulse responses	19
6 Conclusions	22
References	23



Summary

Understanding the determinants of unemployment fluctuations along the business cycle is an important topic for policymakers, since the degree of slack in the labour market affects both wage and price inflation. However, there is no agreement as yet on the sources of fluctuations in unemployment and vacancies. The standard model which attempts to explain these quantities allows for ‘matching’ of vacant jobs to unemployed workers. In the US data, a standard test bed for labour market models, employment and vacancies are about ten times more volatile than productivity, and the standard textbook matching model of the labour market fails to replicate this fact.

The most successful extension of the standard model that manages to replicate the high degree of volatility in labour market variables is based on the assumption that wages of all workers are sticky. However, recent studies show that what matters for the decision of job creation is only the volatility of the wages of newly hired workers. Intuitively, the decision on whether to create a marginal job only depends on the profitability of the marginal worker, which is only a function of his or her productivity and wage. Empirical evidence shows that the wages of newly hired workers do not exhibit sticky behaviour. Hence, the assumption of sticky wages cannot explain the volatility of unemployment and vacancies over the business cycle.

This paper provides a new mechanism of fluctuation in labour market variables, which does not rely on the assumption that wages for the newly hired workers are sticky. It is based on the notion of ‘habits’ in consumption, where households utility from consumption depends partly on past levels of aggregate (‘external’) consumption, sometimes described as ‘catching up with the Joneses’ behaviour. This has proved to be very helpful in explaining many features of the economy. The new variant that we apply to the labour market is that workers form habits in consumption on particular varieties of goods, rather than on the average consumption basket in the economy. So some households will form habits on the consumption of cars, others on the consumption of clothes, food, or various amenities, and so on. If this is the case, each firm should internalise the impact of their pricing policy on habit formation. In other words, when setting prices firms anticipate that higher consumption in the current period implies higher habits and higher future consumption. In a model with deep habits, firms exploit the upturns of the business cycle to increase the stock of habits. In order to do so, they need to increase employment. The



assumption of deep habits therefore helps making the behaviour of vacancies and employment more strongly procyclical.

We show that a model with deep habits is able to replicate successfully the qualitative and quantitative behaviour of labour market variables along several dimensions. Our paper therefore reinforces the idea that deep habits have a wide range of macroeconomic implications. Previous work in the literature has shown that deep habits can account for the countercyclicality of mark-ups, the positive response of consumption to a government expenditure shock, the price puzzle and the incomplete pass-through. Our work uncovers an important implication of deep habits for the labour market.



1 Introduction

In a seminal contribution, Shimer (2005) shows that the standard textbook matching model is unable to account for the high volatility of unemployment and vacancies as observed in the US data. Following his work, many studies have attempted to generate new sources of amplification in labour market variables. On the one hand, Gertler and Trigari (2009), Hall (2005a), Hertweck (2006), Kennan (2010), Menzio (2005), Moen and Rosen (2006) and Rudanko (2009) have shown that introducing wage rigidities for new hires into standard matching models can generate fluctuations in labour market variables that are in line with the data. But Pissarides (2009) and Haefke, Sonntag and van Rens (2009) argue that this assumption does not appear to be supported by microeconomic evidence. These findings suggest that explanations of the unemployment volatility puzzle should preserve the cyclical volatility of wages. On the other hand, Hagedorn and Manovskii (2008) have shown that an alternative calibration of the standard search and matching model can generate sufficiently large fluctuations in unemployment and vacancies. Their results are driven by the value of the non-market activity being set close to the value of search to the worker. Costain and Reiter (2008) have however shown that this calibration implies an implausible elasticity of labour supply to unemployment insurance.

In the light of the above critiques, we develop a model where wages are fully flexible and analyse the implied business cycle properties using a non-controversial calibration. Following the work by Ravn, Schmitt-Grohe and Uribe (2006), we introduce deep habits into a search and matching model to explain the cyclical behaviour of labour market variables. In their model habits are formed at the good-specific level rather than at the aggregate level. To facilitate the comparison with Shimer (2005), we assume that technology shocks are the only source of fluctuations, and employment is the only factor of production. Ravn *et al* (2006) show that deep habits are key in matching the countercyclical behaviour of mark-ups in the United States. In this study, we show that deep habits are also key in generating volatility in labour market variables.

In our model, artificial series for vacancies and unemployment match the empirical moments calculated using US data. As a result, the volatility of labour market tightness is about 20 times the volatility of aggregate consumption. The model overcomes the critique posed by Fujita and Ramey (2007), who show that the standard matching framework fails to generate hump-shaped vacancy responses to technology shocks. Our model is also successful at matching the persistence of vacancies with the data.

This work is also related to Faccini and Ortigueira (2010), where it is shown that investment-specific technology shocks can generate cyclical fluctuations in labour market variables that match the data. Our paper improves on search and matching models with time-varying job destruction rates, such as Faccini and Ortigueira (2010), Ramey (2008) and Shimer (2005), to the extent that we are able to generate a downward-sloping Beveridge curve. The correlation between vacancies and unemployment in our model is close to the corresponding measure in the data.

Rotemberg (2006) shows that a search and matching model with exogenous shocks to mark-ups can match the volatilities in labour market variables. In his model, a mark-up shock leads to an expansion in employment and to a fall in the marginal product of labour which dampens the increase in the real wage paid to new workers. This mechanism of amplification stands in sharp contrast with the mechanism of the standard model, where technology shocks are the only source of fluctuations. As pointed out by Shimer (2005), Nash bargaining implies that wages absorb most of the incentives for job creation, which are generated by changes in productivity.

Our main contribution is that we endogenise the dynamics of mark-ups into a search and matching model by introducing deep habits. In our economy, deep habits magnify the impact of technology shocks on vacancies by generating fluctuations in the real marginal cost, which can be interpreted as the shadow value of output. In the standard matching model, job creation depends on the current return of an additional worker, that is, on the difference between the productivity and the wage. In a model with monopolistic competition, this current period return is evaluated at the shadow value of output. As a result, fluctuations in the real marginal cost affect the incentive to post vacancies.

As in Ravn *et al* (2006), fluctuations in the real marginal cost are driven by the *intertemporal effect* and the *price-elasticity effect*. A technology shock increases the present value of per unit profits and generates an incentive for firms to invest in consumer base. Firms do so by building up the stock of habits. In turn, this increase in habits is achieved by higher current sales, which entails a decrease in the mark-up and an increase in the real marginal cost. This effect is known as the intertemporal effect. In addition, an expansion in aggregate demand increases the price elasticity, which in turn decreases the mark-up and raises the marginal cost. Ravn *et al* (2006) refer to this effect as the price-elasticity effect. The main difference between our mechanism of amplification and Rotemberg (2006) is the following: in Rotemberg (2006) there is no

intertemporal effect, but only a price-elasticity effect, which is generated by exogenous shocks to the elasticity of demand.

The paper is organised as follows. The first section presents the model. The second section discusses the main mechanism of amplification and propagation. The third section compares the results of our simulations with the data. The last section concludes.

2 The model

2.1 The labour market

The labour market is frictional in that firms fill jobs by posting vacancies. The technology that matches jobs with workers is given by

$$M_t = M v_t^\zeta u_t^{1-\zeta}, \quad (1)$$

where M_t denotes the aggregate flow of hires at time t , u_t denotes aggregate unemployment, v_t aggregate vacancies. The parameter $\zeta \in (0, 1)$ is the elasticity of the matching function with respect to vacancies and the parameter M is the efficiency of matching. At time t , vacancies are filled with probability $q(\theta_t) \equiv M_t/v_t = \theta_t^{\zeta-1}$, where $\theta_t = v_t/u_t$ denotes labour market tightness. The assumption of constant returns to scale in the matching function implies that workers find jobs with probability $M_t/u_t = \theta_t q(\theta_t)$.

Following Blanchard and Gali (2008), we assume that workers matched at the beginning of time t become immediately productive. The law of motion for aggregate employment, denoted by n_t , can be written as

$$n_t = M_t + (1 - \rho) n_{t-1}, \quad (2)$$

where ρ is the exogenous job destruction rate. Unemployment evolves according to the following law of motion

$$u_t = 1 - (1 - \rho) n_{t-1}. \quad (3)$$

This condition states that the stock of workers searching for a job at time t is given by the measure of workers who did not have a job at $t - 1$, $1 - n_{t-1}$, plus the measure of workers who lost their job at the end of $t - 1$, ρn_{t-1} .

2.2 The household's problem

The economy is populated by a unit measure of identical households, indexed by $i \in [0, 1]$. Households have preferences over different consumption varieties, indexed by $j \in [0, 1]$. Workers can be either employed or unemployed. Members of the household employed at firm j earn a wage rate w_{jt} and suffer disutility from working, while the unemployed members receive unemployment subsidies b . We assume that the workers can perfectly insure against idiosyncratic shocks. This assumption implies that all household members enjoy the same level of consumption. We denote by $r_{t,t+1}$ the price of the state contingent asset at time t that pays one unit of consumption at time $t + 1$. Following Ravn *et al* (2006), we assume that household preferences exhibit external habit formation in consumption at the good-specific level rather than at the aggregate level. This consumption externality has been coined as deep habits or, alternatively, as catching up with the Joneses good by good.

Household i solves two problems: an intratemporal and an intertemporal problem. The former problem is to minimise total consumption expenditure, $\int_0^1 p_{jt} c_{jt}^i dj$, subject to the following consumption object

$$x_t^i = \left[\int_0^1 (c_{jt}^i - \zeta s_{jt-1})^{1-1/\epsilon} dj \right]^{1/(1-1/\epsilon)}, \quad (4)$$

where s_{jt} denotes the stock of external habit in the consumption of good j at time $t - 1$, $\zeta \in [0, 1]$ the degree of external habit formation of each variety and ϵ the intratemporal elasticity of substitution of the habit adjusted consumption across varieties. The stock of habits is assumed to evolve over time according to the following law of motion

$$s_{jt} = \vartheta s_{jt-1} + (1 - \vartheta) c_{jt}, \quad (5)$$

where the parameter $\vartheta \in (0, 1)$ measures the speed of adjustment of the stock of external habits to changes in the average level of consumption of variety j . A value of ϑ equal to 0 implies that the stock of habits exhibits no persistence.

By minimising expenditure with respect to c_{jt}^i , we derive the individual consumption demands of variety j by household i

$$c_{jt}^i = \left(\frac{p_{jt}}{p_t} \right)^{-\epsilon} x_t^i + \zeta s_{jt-1}, \quad (6)$$

where $p_t \equiv \left[\int_0^1 p_{jt}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$ is the nominal price index and p_{jt} is the price of good j . The consumption demand for each variety j is decreasing in the relative price of good j , p_{jt}/p_t , and increasing in both the level of habit adjusted consumption, x_t^i , and, for positive values of ζ , in the

aggregate stock of habits s_{jt-1} . Aggregating across households implies that

$$P_t x_t^i = \int_0^1 P_{jt} (c_{jt}^i - \zeta s_{jt-1}) dj.$$

The second problem of the household i is to maximise their lifetime utility by choosing the consumption object, x_t^i , and state contingent bonds, d_{t+1}^i . The period utility is defined as

$$\mathcal{U}(x_t^i, n_{it}) = \frac{(x_t^i)^{1-\sigma} - 1}{1-\sigma} - \chi \frac{(n_{it})^{1+\varphi}}{1+\varphi},$$

where the first term of this expression denotes utility from habit adjusted consumption and the second term disutility of work. The term $n_{it}^i = \int_0^1 n_{jt}^i dj$ represents aggregate employment at the household i and σ denotes the intertemporal elasticity of substitution between the consumption object at time t and $t + 1$. The parameter φ is the inverse of the Frisch elasticity of labour supply and χ is a constant.

The lifetime utility of household i is given by

$$\max_{x_t^i} E_t \sum_{s=0}^{\infty} \beta^s \mathcal{U}(x_{t+s}^i, n_{t+s}^i), \quad (7)$$

where E_t is the mathematical expectation operator conditional on the information available at time t and $\beta \in (0, 1)$ is the subjective discount factor. Household i 's budget constraint expressed in terms of the aggregate consumption good is given by

$$x_t^i + \omega_t + E_t r_{t,t+1} d_{t+1}^i = d_t^i + \int_0^1 w_{jt} n_{jt}^i dj + (1 - n_t^i) b + \pi_t^i, \quad (8)$$

where ω_t is equal to $\zeta \int_0^1 \left(\frac{p_{jt}}{p_t}\right) s_{jt-1} dj$, $(1 - n_t^i) b$ are the unemployment benefits received by the unemployed members of household i and π_t^i are the real profits of the firms distributed to each household i at time t . We assume that households face an additional constraint that prevents them from engaging in Ponzi games. Finally, the law of motion of employment is given by the following equation

$$n_t^i = (1 - \rho) n_{t-1}^i + \theta_t q(\theta_t) u_t^i. \quad (9)$$

Household i maximises (7) by choosing the processes x_t^i and d_{t+1}^i subject to condition (8) and a no Ponzi-game constraint. The household takes π_t , ω_t , w_t and d_0 as given. The first-order conditions are

$$\lambda_t^i = \mathcal{U}_x(x_t^i, n_t^i), \quad (10)$$

and

$$\lambda_t^i E_t r_{t,t+1} = \beta E_t \lambda_{t+1}^i. \quad (11)$$

The first equation equalises the Lagrange multiplier λ_t^i to the marginal utility of the consumption object. The second equation is the standard Euler condition that sets the marginal cost of

consumption at time t equal to the marginal benefit of consumption at time $t + 1$. In addition, the non-arbitrage condition with respect to employment is given by

$$\mathcal{W}_{jt}^i = w_{jt}^i - \left[b - \frac{\mathcal{U}_n(x_t^i, n_t^i)}{\lambda_t^i} \right] + E_t \beta \frac{\lambda_{t+1}^i}{\lambda_t^i} (1 - \rho) \mathcal{W}_{jt+1}^i [1 - \theta_{t+1} q(\theta_{t+1})], \quad (12)$$

where \mathcal{W}_{jt}^i denotes the value to the household of having an additional worker employed at firm j .

2.3 The firm's problem

There is a unit mass of monopolistically competitive firms, each of which produces a particular variety of the final good. Each variety j is produced using labour as the sole factor of production. The production process exhibits decreasing returns to scale, and it is given by $y_{jt} = A_t n_{jt}^\alpha$. The variable y_{jt} denotes the output of firm j and A_t is an aggregate technology shock that follows a stochastic process of the form

$$\ln A_t = \varrho \ln A_{t-1} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, \varsigma), \quad (13)$$

where ϱ is the persistence of the technology shock and ς is the standard deviation of the innovation ε_t . Firms open vacancies at the beginning of each period in order to control the level of employment. When posting vacancies, firm j takes the matching probabilities as given.

The assumptions of decreasing returns in production and Nash bargaining imply that both the marginal product of labour and wages depend on the firm's size. Following Stole and Zwiebel (1996), Cahuc, Marque and Wasmer (2008) argue that if vacancies are opened *before* wages are bargained, firms should anticipate how vacancies affect the negotiated wage. This bargaining problem is known as *intrafirm bargaining*. In contrast, we assume that the opening of vacancies, the hiring of workers and the bargaining of wages occur *simultaneously*. In principle, *intrafirm bargaining* has the potential to drive a wedge between large firm models and the standard one-worker-one-firm characterisation as in Shimer (2005). We abstract from *intrafirm bargaining* to isolate the effect of deep habits on labour market variables.¹

Opening vacancies is costly in that the resources that could be otherwise devoted to producing the consumption good are diverted to hiring. We therefore assume that each firm faces a resource constraint of the form:

$$A_t n_{jt}^\alpha = c_{jt} + C(v_{jt}), \quad (14)$$

¹Faccini and Ortigueira (2010) show that, when neutral technology shocks are the only source of fluctuation, *intrafirm bargaining* has negligible implications for the cyclical behaviour of labour market variables. Another study by Krause and Lubik (2007) finds similar results.

where $C(v_{jt}) = \kappa v_{jt}$ is the vacancy cost function.

By adding up the cross-sectional individual demands for good j , we can recover aggregate demand for good j . Analytically, we simply integrate expression (6) over i to obtain

$$c_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{-\epsilon} x_t + \zeta s_{jt-1}, \quad (15)$$

where $x_t = \int_0^1 x_t^i di$ is a habit adjusted measure of aggregate consumption across households. The demand for good j depends on the sum of a price-elastic term, $(p_{jt}/p_t)^{-\epsilon} x_t$, and a price inelastic term ζs_{jt-1} . An expansion in aggregate demand increases the weight of the price-elastic term in the demand function, which implies that the price elasticity of demand for good j is positively related to aggregate demand. Since mark-ups are inversely related to the price elasticity of demand, the deep habits model predicts that mark-ups are time-varying and countercyclical.

Monopolistic competition implies that each firm sets their price by taking the prices of all other firms as given. At the announced price, each firm stands ready to meet the demand for its own variety. We define the profit of firm j at time t by $\phi_{jt} = p_{jt}c_{jt}/p_t - w_t n_{jt}$. The problem of the representative firm is to choose the processes c_{jt} , p_{jt} , s_{jt} and v_{jt} so as to maximise the present discounted value of expected profits,

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} \left[\frac{p_{jt+s}}{p_{t+s}} c_{jt+s} - w_{jt+s} n_{jt+s} \right],$$

subject to the resource constraint, (14), the law of motion of employment,

$$n_{jt} = (1 - \rho) n_{jt-1} + v_{jt} q(\theta_t), \quad (16)$$

the law of motion of the habit stock, (5), and demand for good j , (15). The first-order conditions with respect to vacancies, consumption, the stock of habits and prices are:

$$C'(v_{jt}) m c_{jt} = \mathcal{J}_{jt} q(\theta_t), \quad (17)$$

$$m c_{jt} = \frac{p_{jt}}{p_t} - v_{jt} + (1 - \vartheta) \psi_{jt}, \quad (18)$$

$$\psi_{jt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [\vartheta \psi_{jt+1} + \zeta v_{jt+1}], \quad (19)$$

and

$$c_{jt} = \epsilon \left(\frac{p_{jt}}{p_t} \right)^{-\epsilon-1} x_t v_{jt}. \quad (20)$$

where $m c_{jt}$, \mathcal{J}_{jt} , ψ_{jt} and v_t are the Lagrange multipliers associated with the constraints in equations (5), (14), (15) and (16), respectively. The shadow value of output, which is denoted in our model by $m c_{jt}$, is the contribution of an additional unit of output to the profits of the firm,

being also equal to the firm's real marginal costs. In addition, the non-arbitrage condition with respect to employment is

$$\mathcal{J}_{jt} = \alpha A_t mc_{jt} n_{jt}^{\alpha-1} - w_{jt} + E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) \mathcal{J}_{jt+1}, \quad (21)$$

where \mathcal{J}_{jt} denotes the Lagrange multiplier associated with the employment constraint, which is equal to the current period marginal value of employment to the firm. By combining (17) with (21), we can find an expression for the job creation condition

$$\frac{C'(v_{jt})}{q(\theta_t)} mc_{jt} = \alpha A_t mc_{jt} n_{jt}^{\alpha-1} - w_{jt} + \beta (1 - \rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{C'(v_{jt+1})}{q(\theta_{t+1})} mc_{jt+1}. \quad (22)$$

This condition states that firms will expand employment up to the point where the marginal cost equals the marginal benefit of employing an additional worker. The LHS of expression (22) measures the expected cost of increasing employment at the margin. Since the adjustment costs in our set-up are interpreted as forgone output, the expected cost of employment must be equal to the additional cost of posting a vacancy, denoted by $C'(v_{jt})$, times the average duration of a vacancy, $1/q(\theta_t)$, evaluated at the shadow value of output.

The first-order condition with respect to consumption, equation (18), expresses the shadow value of output as the sum of three components. The first two terms represent the current period revenues associated with a marginal increase in output. This is equal to the revenue p_{jt}/p_t , obtained on the marginal unit of production net of the forgone revenue on inframarginal quantities, v_{jt} . The third component of equation (18) is the shadow value of the habit stock and is equal to the present value of the expected revenues associated with an additional sale of good j .

2.4 Wage bargaining

Wages are renegotiated every period and are the solution to a standard Nash bargaining problem. This bargaining protocol implies that wages maximise the weighted product of the worker's and the firm's value of employment. Formally, the wage solves the following condition

$$w_{jt} = \arg \max \mathcal{W}_{jt}^\gamma \mathcal{J}_{jt}^{1-\gamma},$$

where γ denotes the bargaining power of the worker. The first-order condition yields the standard sharing rule:

$$\gamma \mathcal{J}_{jt} = (1 - \gamma) \mathcal{W}_{jt}. \quad (23)$$

By plugging (12) and (21) into (23), and using $(1 - \gamma) E_t \mathcal{W}_{jt+1} = \gamma E_t \mathcal{J}_{jt+1}$, it follows that:

$$w_{jt} = \gamma \alpha A_t mc_{jt} n_{jt}^{\alpha-1} + (1 - \gamma) \left[b - \frac{U_n(x_t, n_t)}{\lambda_t} \right] + \gamma \beta (1 - \rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \mathcal{J}_{jt+1} \theta_{t+1} q(\theta_{t+1}).$$

Replacing the first-order condition for vacancies, (17), into the above equation we obtain

$$w_{jt} = \gamma \alpha A_t m c_{jt} n_{jt}^{\alpha-1} + (1 - \gamma) \left[b - \frac{U_n(x_t, n_t)}{\lambda_t} \right] + \gamma \beta (1 - \rho) E_t \frac{\lambda_{t+1}}{\lambda_t} C'(v_j) m c_{jt+1} \theta_{t+1}. \quad (24)$$

This condition states that the real wage is a weighted average of the marginal product of the worker plus the cost of replacing the worker and the opportunity cost of working.

2.5 Aggregation and the competitive equilibrium

Using the assumption of symmetry, aggregation in our model economy is simple and can be obtained by integrating over households and firms. We simply remove the i and j subscripts from the equations in the main text to obtain the aggregate economic behaviour in our model economy. A stationary competitive equilibrium is a set of stationary processes $v_t, \lambda_t, \mu_t, \phi_t, n_t, w_t, v_t, u_t, M_t, c_t, x_{ct}, m c_t, \theta_t, r_{t,t+1}$ satisfying equations (1)-(3), (5), (10)-(11), (13), (15), (17)-(21) and (24), given the exogenous stochastic process $\{A_t\}_{t=0}^{\infty}$ and the initial conditions s_{t-1} and n_{t-1} .

3 The source of amplification

In this section we discuss the mechanism of amplification. To gain intuition, we take a closer look at the job creation condition

$$\frac{C'(v_{jt})}{q(\theta_t)} m c_{jt} = \alpha A_t m c_{jt} n_{jt}^{\alpha-1} - w_{jt} + (1 - \rho) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{C'(v_{jt+1})}{q(\theta_{t+1})} m c_{jt+1}.$$

Employment fluctuations in our model are driven by the shadow value of output, which is strongly procyclical and volatile. The real marginal cost enters the job creation condition mainly by affecting the marginal revenue product and the real wage. The sum of these two terms is the current period net return of a marginal worker, which is increasing in $m c_{jt}$. As a result, an increase in the real marginal cost will create a higher incentive to post vacancies.

To understand the dynamics of the real marginal cost, we copy the first-order condition with respect to consumption:

$$m c_{jt} = \frac{p_{jt}}{p_t} - v_{jt} + (1 - \vartheta) \psi_{jt}. \quad (25)$$

As we have previously discussed, the first two terms on the RHS of equation (25) are the current period revenues associated with a marginal increase in output. The third component is the present value of the expected revenues associated with an additional sale of the good. We start by discussing the role of this last term.

By iterating forward equation (19), we can derive an expression for ψ_{jt} as a sum of the discounted shadow values of expected sales

$$\psi_{jt} = \zeta E_t \sum_{s=0}^{\infty} \vartheta^s \beta^{s+1} \frac{\lambda_{t+s+1}}{\lambda_t} v_{jt+s+1}.$$

Therefore, the introduction of deep habits magnifies the impact of a positive technology shock on the present value of per unit profits via higher expected future sales. This effect is what Ravn *et al* (2006) refer to as the *intertemporal effect* of deep habits. Higher future sales in our model translate into higher vacancies through the job creation condition. It is worth noticing that this expression is increasing in the degree of habit formation ζ .

The *intertemporal effect* of deep habits is re-enforced by a *price-elasticity effect*. By combining (15) and (20), we can derive an expression for v_{jt} that depends on the relative price of good j , $v_{jt} = -c_j \partial (p_{jt}/p_t) / \partial c_j$. The variable v_{jt} is positive because the partial derivative $\partial (p_{jt}/p_t) / \partial c_j$ is negative. Factoring p_{jt}/p_t on the first two terms on the RHS of (25), we can write $p_{jt}/p_t - v_{jt} = p_{jt}/p_t (1 + \varepsilon_{p,c})$, where $\varepsilon_{p,c}$ denotes the price elasticity of consumption demand for good j . Since the price-elasticity is procyclical with deep habits, it follows that a positive technology shock increases the real marginal cost. An alternative way to see this effect is to substitute for v_{jt} using equation (20). In the symmetric equilibrium, where $p_{jt} = p_t$, it follows that $v_t = c_t / (\varepsilon x_t)$. Substituting for $x_t = c_t - \zeta s_{t-1}$, we can rewrite v_t as

$$v_t = \frac{c_t}{\varepsilon (c_t - \zeta s_{t-1})}.$$

This expression is decreasing with c_t , and hence the price elasticity is procyclical. In the following section we disentangle the impact of the *price-elasticity effect* and the *intertemporal effect* on the marginal cost by analysing the impulse responses.

4 Calibration

In this section we assign numerical values to the parameters of the model following a standard calibration exercise. We rely on two sources of information. Some parameters are set using *a priori* information. The remaining ones are set such that the stationary equilibrium of the model

matches a number of stylised facts observed in the post-WWII US economy. One period of time in our model equals one quarter in the data.

We discuss first the parameters that are set using *a priori* information. The subjective discount factor, β , is set to 0.99, implying a quarterly real interest rate of about 1%. The degree of external habit formation ζ , the persistence of the habit stock, ϑ , and the intertemporal elasticity of substitution, σ , are set to 0.86, 0.85 and 2, respectively, as estimated by Ravn, Schmitt-Grohe and Uribe (2004). The rate of job destruction ρ is set to 0.08 as estimated by Davis, Haltiwanger and Schuh (1996). The elasticity of the matching function ξ is set to 0.5, the standard value used in the literature. This choice lies within the range of plausible values, between 0.5 and 0.7, as reported by Petrongolo and Pissarides (2001) in a survey of the literature. The inverse of the Frisch elasticity of labour supply ϕ is set to 1. The value of this parameter is in line with the business cycle literature. The coefficient χ in the utility function is normalised to 1 and only the bargaining power parameter γ is set arbitrarily. We choose the standard value of 0.5 in order to facilitate comparability with the existing literature, which typically focuses on the case of symmetric Nash bargaining.

We set the remaining five parameters, namely the elasticity of substitution ϵ , the elasticity of the production function with respect to employment, α , the constant of the vacancy cost function, κ , the constant of the matching function, M , the bargaining power of the worker, b , to match: *i*) a 20% equilibrium mark-up as in Schmitt-Grohe and Uribe (2007); *ii*) a labour share of income of 66%; *iii*) an unemployment rate of 10% as in Hall (2005b). This rate of unemployment, which is somewhat higher than in the data, can be justified by interpreting workers who are unmatched in the model as being both actively and passively searching for a job. These passive job seekers are sometimes defined as workers marginally attached to the labour force, meaning that they would be willing to work if they received an offer. As such, they are not captured by the standard ILO definition of unemployment. As pointed out by Trigari (2006), this interpretation is consistent with the abstraction in the model from participation decisions; *iv*) a vacancy filling rate of 70% as in Trigari (2006)²; *v*) a replacement ratio - computed as the sum of unemployment benefits and the disutility of work over the wage - equal to 70%, as in Gertler, Sala and Trigari (2008).

The choices of γ and b are very important for the analysis of cyclical fluctuations in labour market variables. Hagedorn and Manovskii (2008) argue that, in the standard textbook matching

²The same value is used by Cooley and Quadrini (1999) and den Haan, Ramey and Watson (2000).

model with small firms, the combination of a replacement ratio of 95% and a bargaining power of 0.05 generates labour market fluctuations that are in line with the empirical evidence. In their study, the replacement ratio is equal to the value of non-market activity that includes both unemployment subsidies and the value of leisure. Their calibration has been criticised on the grounds that it generates an excessive sensitivity of unemployment to unemployment benefits. In our calibration, the replacement ratio generates a semi-elasticity of unemployment to unemployment benefits around 2, in line with that estimated by Costain and Reiter (2008). Our choice for the bargaining power $\gamma = 0.5$ is standard, and together with our choice of b , it ensures that our results are not driven by the Hagedorn and Manovskii effect.

Our calibration implies a quarterly job finding rate of about 80%. This result is not directly implied by our calibration exercise and it corresponds to the empirical estimates calculated by Shimer (2005).³ Finally, we set the persistence of the technology shock ϱ to 0.95. Table A provides a summary of the parameters used in the baseline calibration of our hypothetical model economy.

Table A: Calibrated parameter values

Description	Parameter	Value
Intertemporal Elasticity of Substitution	σ	2
Discount Factor	β	0.99
Frisch Elasticity	φ	1
Degree of External Habit Formation	ζ	0.86
Persistence of the Habit Stock	ψ	0.85
Efficiency of the Matching Technology	M	0.75
Elasticity of the Matching Function	ξ	0.5
Cost of posting a vacancy	κ	0.19
Separation Rate	ρ	0.08
Bargaining Power	γ	0.5
Unemployment Benefit	b	0.41
Labour Share	α	2/3
Elasticity of Substitution between varieties	ϵ	8.6
Persistence of Productivity Shock	ϱ	0.95

5 Model evaluation

We start this section by documenting the stylised facts of the US labour market, and we then assess the ability of our model to replicate these facts. We close the section by discussing the

³Shimer estimates a monthly job finding rate of 0.45. This would imply a quarterly finding rate of 0.83 following the formula $f_q = 1 - (1 - f_m)^3$, where f_q and f_m denote the quarterly and the monthly finding rate, respectively.

responses of the labour market variables to a productivity shock.

5.1 US data

Table B reports the standard deviations, the autocorrelations and the cross-correlations of both consumption and labour market variables using US data. All data series are quarterly and cover the period ranging from 1951 Q1 to 2006 Q2. The summary statistics refer to the cyclical component of the logged variables, which are detrended using a Hodrick-Prescott filter with a smoothing parameter of 1600.

Table B: Summary statistics, quarterly US data 1951-2006

	c	n	v	u	θ
σ_x/σ_c	1	1.04	11.03	9.77	20.37
Autocorrelation	0.82	0.92	0.91	0.87	0.90
Correlation Matrix	1	0.6	0.77	-0.65	0.73
	-	1	0.87	-0.91	0.91
	-	-	1	-0.92	0.98
	-	-	-	1	-0.98
	-	-	-	-	1

Notes. Standard deviations and correlations in this table correspond to quarterly series, detrended using a Hodrick-Prescott filter with smoothing parameter of 1600.

A key stylised fact is that vacancies (v) and unemployment (u) are about 11 times and 10 times more volatile than consumption. The combination of these two facts implies that labour market tightness, $\theta = v/u$, is about 20 times more volatile than consumption. The inability of the standard search and matching model to replicate the data in this dimension has triggered a significant amount of research. Another important stylised fact is that vacancies are highly autocorrelated, with a first-order autocorrelation coefficient of 0.91. The data also displays a strongly negative contemporaneous correlation between vacancies and unemployment. This correlation coefficient, which is equal to -0.92 , is also known as the slope of the Beveridge curve.

5.2 Simulation results

The model is solved using first-order perturbation methods following the study by Schmitt-Grohe and Uribe (2004). We then simulate the unconditional moments of the key variables c , n , v , u ,

and θ . An advantage of the unconditional approach is that, up to first order, the moments are independent of the volatility of the productivity shock, ζ . The relative standard deviations and correlations are reported in Table C below.

Table C: Labour productivity shocks - deep habits model

	c	n	v	u	θ	
σ_x/σ_c	1	1.03	12.51	8.84	20.14	
Autocorrelation	0.96	0.89	0.95	0.94	0.95	
Correlation Matrix	c	1	0.87	0.79	-0.87	0.88
	n	-	1	0.77	-1.00	0.91
	v	-	-	1	-0.77	0.96
	u	-	-	-	1	-0.92
	θ	-	-	-	-	1

The simulated statistics reveal that the model is able to replicate the data surprisingly well. Vacancies and unemployment are about 12.5 and 8.84 times more volatile than aggregate consumption. This result implies that labour market tightness is 20 times more volatile than consumption, which corresponds to the value that we observe in the data. In our model deep habits generate a mechanism of amplification that is sufficiently strong to solve the Shimer's puzzle.

Table D: Autocorrelations of the vacancy series

Lags	1	2	3	4
US Data	0.91	0.69	0.43	0.15
Model with Deep Habits	0.89	0.73	0.55	0.38
Model with Superficial Habits	0.85	0.75	0.68	0.63

Notes. Correlations in this table correspond to quarterly series, detrended using a Hodrick-Prescott filter with smoothing parameter of 1600.

The model performs well also on other dimensions. Table D shows that the dynamic autocorrelation of the artificial vacancy series is in line with its empirical counterpart. Moreover, as we show in the following section, vacancy series are hump-shaped in response to a technology shock. This is another important result since Fujita and Ramey (2007) have emphasised the inability of the standard model to generate persistent vacancy series and hump-shaped responses. All artificial series of the labour market variables exhibit first-order autocorrelation coefficients that are in line with the values observed in the data. Similarly, the simulated cross-correlations have about the same order of magnitude as the actual correlations.

Table E: Labour productivity shocks - superficial habits model

	c	n	v	u	θ	
σ_x/σ_c	1	0.20	2.35	1.72	3.90	
Autocorrelation	0.96	0.85	0.96	0.95	0.97	
Correlation Matrix	c	1	0.95	0.95	-0.95	0.99
	n	-	1	0.83	-1.00	0.94
	v	-	-	1	-0.83	0.97
	u	-	-	-	1	-0.94
	θ	-	-	-	-	1

The contemporaneous correlation between vacancies and unemployment in our model is -0.77, and this result compares with a value of -0.92 in the data. Our model improves upon models with time-varying stochastic job destruction rates, which exhibit a counterfactual upward-sloping Beveridge curve.

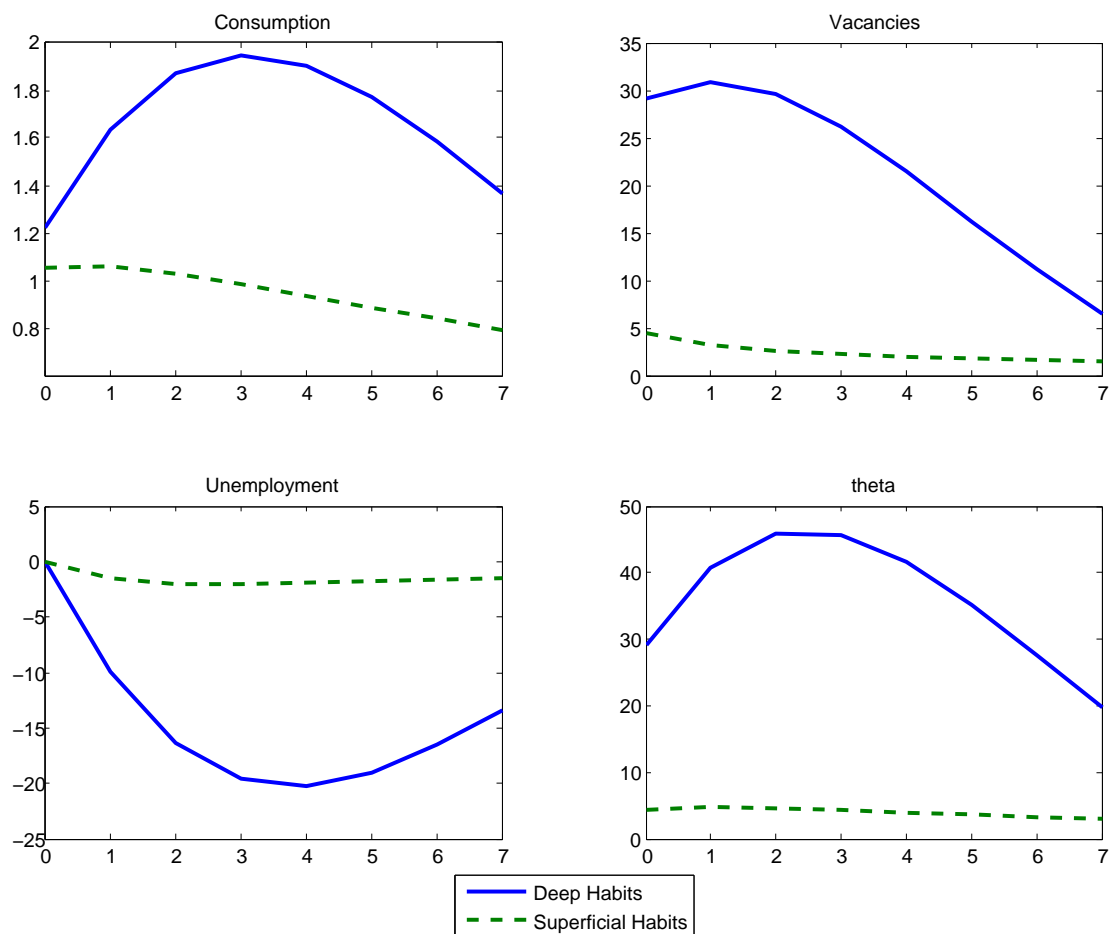
In order to disentangle the contribution of deep habits to the cyclical behaviour of unemployment and vacancies, we simulate a version of the same model, where deep habits are substituted with superficial habits. The results, which are reported in Table E, show that the model with superficial habits exhibits no amplification. Vacancies and unemployment are now only two times more volatile than consumption, and tightness nearly four times. The procyclical behaviour of the marginal cost induced by the assumption of deep habits is therefore key in explaining the volatility of labour market variables.

5.3 *Impulse responses*

In this section we study how the model responds to a 1% increase in the technology shock. In order to disentangle the effect of deep habits, we compare the dynamic properties of our model with an otherwise identical model featuring superficial habits. The real marginal cost is constant with superficial habits and procyclical with deep habits. As discussed in Section 3, the real marginal cost plays a key role in generating amplification of a technology shock to labour market variables.

Chart 1 displays the impulse responses for consumption, unemployment, vacancies and tightness in both models with deep habits and superficial habits. In addition, in Chart 2 we show the responses of the marginal cost, the shadow value of the habit stock, ψ_t , and the shadow value of consumption, v_t in the deep habits model. As explained in Section 3, these two variables explain

Chart 1: Impulse responses to a technology shock under deep and superficial habits

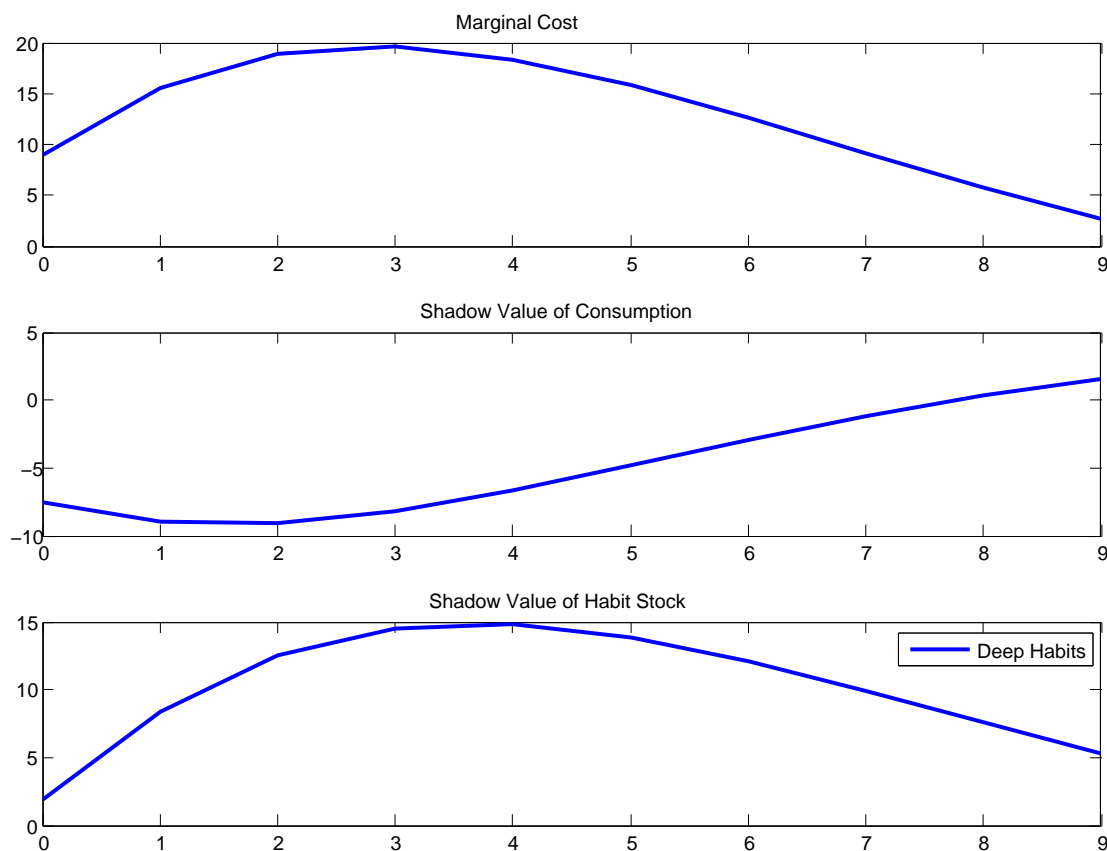


Notes. Impulse responses are measured in percentage deviations (%) from the steady state. Horizontal axes display the number of quarters after the shock.

the fluctuations in the real marginal cost. Equation (25) shows that the real marginal cost is increasing in ψ_t and decreasing in ν_t . The shadow value of the habit stock captures the *intertemporal effect* of deep habits and the shadow value of consumption the *price-elasticity effect*.

Chart 2 shows that, following a technology shock, both the intertemporal effect and the price-elasticity effect increase the real marginal costs. Both sources are quantitatively important. The chart shows that in the deep habit model an increase in the real marginal cost generates strong responses in vacancies and unemployment. When we shut down the real marginal cost channel by replacing deep habits with superficial habits, the model exhibits no amplification to a technology shock.

Chart 2: Impulse responses to a technology shock under deep habits



Notes. Impulse responses are measured in percentage deviations (%) from the steady state. Horizontal axes display the number of quarters after the shock.

In the standard search and matching model a technology shock renders employment more productive, and firms respond by increasing vacancies. However, with Nash bargaining, wages soak up most of the incentives for job creation, and the model exhibits little amplification properties. Under deep habits a technology shock generates an additional incentive for firms to post vacancies. Firms exploit the higher productivity of labour to build up the stock of habits. In addition, the price elasticity of demand increases with aggregate consumption, reinforcing the expansionary effect on vacancies and employment.

As a result, a technology shock increases consumption, the shadow value of the habit stock, the real marginal cost and vacancies on impact. While consumption and the real marginal cost peak after three quarters, the shadow value of the habit stock and vacancies peak after the fourth quarter and the first quarter respectively. Unemployment and the shadow value of consumption fall in the wake of a productivity shock. The trough in unemployment occurs after four periods

and that of the shadow value of consumption after two periods. All series are hump-shaped.

6 Conclusions

Search and matching models featuring nominal wage rigidities for new hires have been successful at generating amplification and propagation of technology shocks to labour market variables. These models provide a successful alternative to Hagedorn and Manovskii (2008), who show that a high opportunity cost of work generates sufficient volatility in unemployment and vacancies. However, recent evidence has cast doubts on these assumptions. Pissarides (2009) calls for alternative explanations of the unemployment volatility puzzle that are consistent with the estimated elasticity of labour supply to unemployment insurance and preserve the flexibility of wages.

This paper takes up on this challenge. To this end, we maintain the assumption that wages are renegotiated at every period through Nash bargaining, and we extend the standard search and matching model by introducing deep habits in consumption. To facilitate the comparison with Shimer (2005), we assume that technology shocks are the only source of fluctuations and employment is the only factor of production. We show that the introduction of deep habits into a search and matching model creates a powerful mechanism of amplification whereby technology shocks are transmitted onto labour market variables. Deep habits generate a procyclical real marginal cost, which is in line with the data. This procyclicality in the real marginal cost is at the source of the amplification. Our model is capable of matching the data in several dimensions: it solves the unemployment volatility puzzle and generates persistent, hump-shaped vacancy series as well as plausible correlations of labour market variables.

This paper reinforces the conclusions of Ravn *et al* (2006) that deep habits have a wide range of macroeconomic implications. Previous work by Ravn *et al* (2006), Ravn, Schmitt-Grohe, Uribe and Uuskula (2008) and Ravn, Schmitt-Grohe and Uribe (2008) shows that deep habits can account for the countercyclicality of mark-ups, the positive response of consumption to a government expenditure shock, the price puzzle and the incomplete pass-through. Our work uncovers an important implication of deep habits for the labour market.



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