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Abstract

The identification of reduced-form VAR model had been the subject of numerous debates in the literature. Different sets of identifying assumptions can lead to very different conclusions in the policy debate. This paper proposes a theoretically consistent identification strategy using restrictions implied by a DSGE model. Monte Carlo simulations suggest the proposed identification strategy is successful in recovering the true structural shocks from the data. In the face of misspecified model restrictions, the data tend to push the identified VAR responses away from the misspecified model and closer to the true data generating process.

Key words: VAR identification, model misspecification, DSGE model.

JEL classification: F41, E52.

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Summary

Monetary policy making in central banks requires a profound understanding of the way the economy reacts to the shocks that continually bombard it. So banks call upon a wide range of economic models to help them in this undertaking. Since the pioneering work of Sims, vector autoregressive (VAR) models have been used extensively by applied researchers, forecasters and policymakers to address a range of economic issues. These models comprise equations explaining a small number of key macroeconomic variables where each equation includes the same set of explanatory variables, lagged values of all the variables in the system. The basic VAR is therefore unable to tell us about the detailed structure of the relationship or shocks, which is what the policymaker really wants to know, as it is a 'reduced-form' model. To unpack the shocks hitting the system and their effects on the economy, we need to 'identify' the model with extra assumptions.

Although VARs have been very successful in capturing the dynamic properties of macroeconomic time-series data, the decomposition of these statistical relationships back to coherent economic stories is still subject to a vigorous debate. However, the outcomes of the VAR analysis depend crucially on these assumptions and the various competing identification restrictions cannot be easily tested against the data. Even though several procedures have been proposed in the literature, shock identification remains a highly controversial issue.

A type of model that is not susceptible to this problem is the dynamic stochastic general equilibrium (DSGE) model. In this case, economic theory is used to define all the linkages between variables. The tight economic structure solves the identification problem, but at a cost. As theory is never able to fully explain the data, an agnostic VAR will almost certainly 'fit' the data better.

This paper proposes an identification strategy for VARs that extends an idea introduced by Harald Uhlig, a 'penalty function' that effectively weights various restrictions suggested by theory - in his case, the signs of various effects. So we construct a penalty function that is based on *quantitative* restrictions implied by a DSGE model. To assess the usefulness of the proposed identification strategy, we present a series of Monte Carlo experiments (where many experiments

are carried out on an artificial model, randomly differing in the shocks hitting the system). First, we investigate the ability of the method to recover the true set of structural shocks; second, we examine the source of bias in the identified VAR responses relative to the true data generating process; and third, we assess how the proposed identification strategy performs using restrictions from a misspecified model. We also present an application using a seven-variable VAR model estimated on US data. The structural shocks are identified using restrictions from a classic medium-scale DSGE model developed by Frank Smets and Raf Wouters.

A number of interesting results emerge from the analysis. First, by using the correct model restrictions, the identification procedure is successful in recovering the initial impact of the shocks from the data. Second, despite using restrictions from misspecified models, the data tends to push the VAR responses away from the misspecified model and closer to that of the true data generating process. Third, the proposed identification strategy systematically gives smaller bias compared with other popular identification schemes.

1 Introduction

Since the pioneer work of Sims (1980), vector autoregressive (VAR) models have been used extensively by applied researchers, forecasters and policymakers to address a range of economic issues. Although VAR models have been very successful in capturing the dynamic properties of the macroeconomic time-series data, the decomposition of these statistical relationships back to coherent economic stories is still under large debate. The key source of this disagreement arises from the difficulty in identifying structural disturbances from a set of reduced-form residuals. The sampling information in the data is not sufficient and several assumptions are needed in order to recover the mapping between the structural and the reduced-from errors.[1] However, the outcomes of the VAR analysis depend crucially on these assumptions and the various competing identification restrictions cannot be easily tested against the data.

The literature have proposed a number of different exact-identification strategies. First and the most popular, is the Choleski short-run restriction on the VAR's reduced-form covariance matrix. Under the Choleski scheme, the ordering of the variables is particularly important for the structural economic interpretation of the VAR (see, Lutkepohl (1993) and Hamilton (1994)). Furthermore, as Canova (2005) explains, the Choleski decomposition implies 'zero-type' restrictions that are rarely consistent with dynamic stochastic general equilibrium (DSGE) models. A similar procedure was introduced by Blanchard and Quah (1989) by imposing long-run relationships that are consistent with economic theory. However, a number of studies such as Chari, Kehoe and McGrattan (2005), Christiano, Eichenbaum and Vigfusson (2006), Erceg, Guerrieri and Gust (2005) and Ravenna (2007) have concluded that long-run restrictions are inadequate in recovering the true structural disturbances. The main reason is that it is often difficult to obtain an accurate estimate of the long-run impacts because of the truncation bias associated with VAR models. Therefore, imposing long-run restrictions based on these bias estimates can lead to misleading conclusions.

More recently, Faust (1998), Canova and De Nicolo (2002) and Uhlig (2005) propose an identification scheme that imposes 'sign' or 'qualitative' restrictions on the structural responses. The strategy recognises there are infinite number of observationally equivalent mappings between the structural and the reduced-form errors, and the idea of the sign restriction is to select

[1] The discussions here focus on exact VAR identification.

a subset of these mappings that are consistent with certain qualitative features. An attractive feature of this procedure is that it makes VAR and DSGE models more comparable than with other identification strategies. Researchers can then use qualitative information in the form of sign restrictions implied by DSGE models to help identify structural VAR shocks; for example Liu (2008) and Peersman and Straub (2009) use this approach. Although attractive to applied researchers, as highlighted by Uhlig (2005) and explicitly illustrated by Fry and Pagan (2007), this type of identification scheme fails to deliver a unique identification mapping. There can be a range of impulse responses that are consistent with the sign restrictions. This leads to large uncertainty around the model's estimates (see, Paustian (2007)) and makes policy inference less informative.

Uhlig (2005) also discussed an alternative procedure using the 'penalty-function' approach. The idea is to find a set of orthogonal shocks that minimise some specific penalty function. This is certainly a less agnostic approach relative to the pure sign restriction method. Nevertheless, the procedure produces a unique set of structural shocks and therefore reduces the degree of uncertainty related to the identification procedure. However, the choice of the penalty function remains arbitrary and difficult to motivate from an economic perspective. An alternative procedure is proposed by Del Negro and Schorfheide (2004), who developed a methodology to generate a prior distribution using a DSGE model for a time-series VAR. The DSGE-VAR approach relaxes the tight theoretical restrictions of the DSGE model by making use of the model and data, summarised by the likelihood of the model. While the DSGE-VAR has been a very useful tool for model comparison and forecasting, as Sims (2008) pointed out, it remains difficult to use the DSGE-VAR for policy analysis, for example impulse response analysis. The main reason is that the DSGE-VAR still faces the same identification problems as with standard VAR models. Del Negro and Schorfheide (2004) suggest using the identification matrix from the DSGE model as an approximation; however, the resultant variance-covariance matrix will no longer be the same as the estimated DSGE-VAR Del Negro and Schorfheide (see 2004, footnote 17).

This paper proposes an identification strategy that extends Uhlig's (2005) penalty-function approach to a more formal setting. In particular, we construct a penalty function that is based on quantitative restrictions implied by a DSGE model. To assess the usefulness of the proposed identification strategy, we present a series of Monte Carlo experiments. First, we investigate the ability of the algorithm to recover the true set of structural shocks; second, we examine the source of bias in the identified VAR responses relative to the true data generating process; and third, we assess how the proposed identification strategy performs using restrictions from a misspecified model. We also present an application using a seven-variable VAR model estimated on US data. The structural shocks are identified using restrictions from a medium-scale DSGE model by Smets and Wouters (2007).

There are several advantages in adopting this approach. First, as Pagan (2003) argued, there is an inherent trade-off between theoretical and empirical coherence for macroeconomic models.[2] One can think of the approach presented here as bringing the statistical VAR analysis closer to the structural DSGE models along the 'Pagan-curve' but without sacrificing empirical coherence. Second, despite applying more restrictions on the behaviour of the impulse response functions of the VAR, the proposed method does not change the empirical fit of the VAR model. Rather, it simply selects a unique identification mapping from the infinite number of observationally equivalent ones. This is a subtle but important difference with Del Negro and Schorfheide (2004) DSGE-VAR, where the approach here maintains the same variance covariance as the reduced-form representation of the VAR for structural identification. Third, the identified VAR can be used as a useful cross-check against the structural model's dynamic behaviour.

A number of interesting results emerge from the analysis. First, by using the correct model restrictions, the identification procedure is successful in recovering the initial impact of the shocks from the data. We identify two sources of bias relating to the difference between the VAR and the true response at longer horizons. One relates to the *small sample bias*, while the second part is due to the *truncation bias* that arises from using a finite-ordered VAR to approximate a vector autoregressive moving average (VARMA) process. Even in large samples, the truncation bias remains the dominant source of bias. Second, despite using restrictions from misspecified models, the data tends to push the VAR responses away from the misspecified model and closer to that of the true data generating process. Third, the proposed identification strategy systematically gives smaller bias compared with other identification schemes such as Choleski decomposition and pure sign restrictions.

[2] Pagan (2003) illustrates this by placing various types of models along a concave 'modelling frontier' with the degree of theoretical coherence and the degree of empirical coherence along each axis.

The paper is organised as follows. Section 2 outlines the methodology of the proposed identification strategy. Section 3 briefly describes the Monte Carlo experiments and outlines the medium-scale DSGE model used for the data generating process. Section 4 reports the results from the Monte Carlo experiments. Section 5 presents an application of the proposed identification strategy using a seven-variable VAR model estimated on US data. Section 6 contains concluding remarks and proposes direction for future research.

2 Methodology

2.1 A review of the identification problem

The ability of using VAR models to address key macroeconomic policy questions depends crucially on the identification of the reduced-form residuals. Even though several procedures have been proposed in the literature, shock identification remains a highly controversial issue. To illustrate the identification problem, consider the following stylised structural model:[3]

$$A_0 Y_t = A(L)^h Y_t + \eta_t \tag{1}$$

$$Y_t = A_0^{-1} A(L)^h Y_t + A_0^{-1} \eta_t \tag{2}$$

where Y_t is a $(n \times 1)$ vector of endogenous variables, A_0 is a $(n \times n)$ matrix of coefficients, $A(L)^h = A_1 L + \cdots + A_h L^h$ is a h^{th} order lag polynomial and $E(\eta_t \eta_t') = I$ gives the variance-covariance matrix of the structural innovations. Equation **(1)** is the structural model and equation **(2)** is the corresponding reduced-form representation. The key parameters of interest are A_0 and $A(L)$. However, the sampling information in the data is not sufficient to identify both A_0 and $A(L)$ separately without further identifying restrictions. There is an infinite combination of A_0 and $A(L)$ all imply exactly the same probability distribution for the observed data. To see this, premultiplying the model in **(1)** by a full rank matrix Q, which leads to the following new model:

$$Q A_0 Y_t = Q A(L) Y_t + Q \eta_t \tag{3}$$

$$Y_t = A_0^{-1} Q^{-1} Q A(L) Y_t + A_0^{-1} Q^{-1} Q \eta_t \tag{4}$$

The reduced-form representation of the two models in equations **(2)** and **(4)** are exactly the same. That implies both models in **(1)** and **(3)** are observationally equivalent. Without additional

[3]For the moment, we remain agnostic as to the form of the structural model or where it comes from. More details are provided in the next subsection.

assumptions – identifying restrictions – no conclusions regarding the structural behaviour of the 'true' model can be drawn from the data.

As explained by Canova (2005, Chapter 4), popular identification schemes such as the Choleski decomposition and long-run restrictions impose 'zero-type' restrictions that cannot be easily justified by a large class of DSGE models. In particular, DSGE models hardly display the type of recursive structures that are typically assumed by Choleski or long-run identification schemes. This raises the question whether these identification schemes are the appropriate choices in relation to the economic theory.

The identification scheme proposed by Faust (1998), Canova and De Nicolo (2002) and Uhlig (2005) seem to overcome some of the difficulties by imposing sign or/and shape (pure sign) restrictions on the structural responses. Although attractive to applied researchers, the procedure fails to deliver unique identification mapping as is highlighted by Uhlig (2005) and Fry and Pagan (2007). In other words, there can be a range of impulse responses that are consistent with the sign restrictions. This can lead to large uncertainty around the model's estimates and less reliable inference.

2.2 The mapping between DSGE and VAR model

To see the links between structural and reduced-form VAR models, it is useful to explore the mapping between the two. To be more specific, we are going to consider the class of structural models that are usually based on agent's optimisation behaviour and rational expectation formation - DSGE models. Generally, the solution of a linearised DSGE model can be summarised by the following state-space representation:[4]

$$X_t = B(\theta) X_{t-1} + \Gamma(\theta) \eta_t \tag{5}$$

$$Y_t = A(\theta) X_t \tag{6}$$

where X_t is an $n \times 1$ vector of state variables, Y_t is an $m \times 1$ vector of variables observed by an econometrician, and η_t represents an $k \times 1$ vector of economic shocks such that $E(\eta_t) = 0$ and $E(\eta_t \eta_t') = I$.[5] The matrices $A(\theta)$, $B(\theta)$ and $\Gamma(\theta)$ are functions of the underlying structural

[4]The solution of the model can be obtained by using either Blanchard and Kahn (1980) or Sims's (2002) type algorithms.

[5]In the notation here, x_t also includes the current values of exogenous shock processes.

parameters of the DSGE model. Equation **(6)** is usually referred to as the state equation (or policy function) that describes the evolution of the underlying economy, and equation **(5)** is the observation equation that relates the state of the economy with the set of observable variables. For notational convenience, we will drop the indication that the matrices A, B and Γ are functions of the structural parameters θ.

From the work of Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007), Christiano *et al* (2006) and Ravenna (2007), the state-space representation of the DSGE model described above has an infinite order VAR process representation, VAR(∞), if and only if the eigenvalues of the following matrix

$$M = \left[I_n - \Gamma\left(A\Gamma\right)^{-1}A\right]B \tag{7}$$

are less than one in absolute terms and the number of the shocks coincides with the number of observed variables, ie: $m = k$. This is known as 'Poor Man's invertibility condition' or simply the 'invertibility condition' as in Fernandez-Villaverde *et al* (2007). If this condition holds, the set of observable variables Y_t can be written as a VAR(∞) such that

$$Y_t = \sum_{j=1}^{\infty} \Delta_j Y_{t-j} + A\Gamma\eta_t \tag{8}$$

where

$$\Delta_j = ABM^{j-1}\Gamma\left(A\Gamma\right)^{-1}$$

On the other hand, a reduced-form VAR(h) model can be estimated on a set of stationary macroeconomic time series Y_t to provide a summary of its statistical properties

$$y_t = \sum_{j=1}^{h} A_j y_{t-j} + \upsilon_t \tag{9}$$

where υ_t is normally distributed with zero mean and variance-covariance Σ_υ matrix. Assuming the DSGE model in equation **(8)** is the true data generating process (DGP) for Y_t, the reduced-form VAR in equation **(9)** can provide a reasonably good approximation of the process Y_t as the number of lags (h) tends to infinity. In which case, the mapping between the reduced-form and structural shocks can be uniquely defined as (Christiano *et al* (2006), Proposition 1)

$$\upsilon_t = A\Gamma\eta_t \tag{10}$$

or

$$\Sigma_\upsilon \equiv E\left(\upsilon_t\upsilon_t'\right) = A\Gamma\Gamma'A'. \tag{11}$$

It is this unique mapping that this paper exploits to help identify reduced-form VAR shocks.

2.3 DSGE restrictions for structural VARs

In addition to the pure sign restriction approach, Uhlig (2005) also discusses an alternative procedure using the 'penalty-function'. The idea behind the procedure is to find a set of orthogonal shocks that minimises some specific penalty function. However, the choice of the penalty function remains arbitrary and difficult to motivate from an economic perspective. The identification strategy described here essentially extends the 'penalty-function' approach to a more formal setting. In particular, we exploit the mapping between the DSGE and the VAR model as shown earlier to construct the penalty function. This is attractive because it provides a theoretically consistent way of identifying structural VAR shocks, and the identifying assumptions are motivated from restrictions implied by DSGE models. Furthermore, the procedure can help bring together the two distinct approaches of macroeconomic modelling.

Assuming the DSGE model is the true DGP with variance-covariance matrix $A\Gamma\Gamma'A'$, Lutkepohl and Poskitt (1991) show that the estimated variance covariance of a VAR(h) model converges to the true variance covariance when the number of lags tend to infinity ($h \to \infty$) as the sample size tends to infinity ($T \to \infty$). The rate which the sample size tends to infinity must be faster than the rate which h^3 tends to infinity, that is

$$\widehat{\Sigma}_{\upsilon} \to A\Gamma\Gamma'A' \text{ as } \frac{h^3}{T} \to 0 \qquad \textbf{(12)}$$

where $\widehat{\Sigma}_{\upsilon}$ is the estimated VAR variance covariance or the reduced-form covariance. In practice, the two key assumptions underlying the above condition undoubtedly breaks down. First, mostly DSGE models are tools designed to explain certain subsets of stylised facts. Despite the recent success in improving its empirical performance, misspecification remains a concern (Del Negro, Schorfheide, Smets and Wouters (2007)). Second, samples of macroeconomic time-series data are limited, so the number of lags that can be included in the VAR is quite restrictive. Consequently, the estimated VAR variance covariance can be quite different to the one implied by the DSGE model.

In the pure-sign restriction case, $\widehat{\Sigma}_{\upsilon}$ is decomposed into $\widehat{\Sigma}_{\upsilon} = \hat{C}PP'\hat{C}'$, where $\hat{C}$ is the Choleski factor of $\widehat{\Sigma}_{\upsilon}$ and P is an orthonormal matrix such that $PP' = I$. The matrix P is selected in such a way to meet the researcher's belief regarding the qualitative properties of the impulse responses. As discussed earlier, the selection of P is non-unique. The proposed identification strategy here essentially selects a unique matrix P to minimise the 'distance' between the contemporaneous response of the VAR and the DSGE model. The procedure can be summarised as the following minimisation problem

$$P^* = \arg\min_P \left\{ \left\| vec\left(\widehat{C}P - A\Gamma\right)\right\|_2 + \sum_{j=1}^{k}\sum_{i=1}^{m}\delta_{ij} I\left(sign_{ij}\right)\right\} \tag{13}$$

subject to

$$PP' - I = 0 \tag{14}$$

where $\|\cdot\|_2$ stands for the Euclidian norm, $I\left(sign_{ij}\right)$ is an indicator function for variable i in response to shock j that takes values 0 if the sign restrictions are satisfied and 1 otherwise and δ_{ij} is a positive number. A few remarks are worth noting. The first part of equation **(13)** resembles Uhlig's (2005) penalty function although here the function is based on restrictions from an optimising DSGE model. The second part of equation **(13)** is analogous to the pure sign restrictions. The parameter δ_{ij} controls for the importance attached to the sign restrictions.[6] The key difference is that by imposing additional restrictions from a DSGE model it will ensure a unique identification matrix $\hat{C}P^*$ for the VAR. The difference between the identified VAR responses relative to the DSGE model will depend on how plausible the restrictions are in the face of the data summarised by $\hat{A}(L)^h$ and $\widehat{\Sigma}_v$. If these restrictions are deemed far away from the empirical evidence, then the difference can be quite large, and *vice versa*. There is no closed-form solution readily available for the above minimisation problem, so we resort to numerical methods for the simulation experiments.

3 Monte Carlo experiments

This section sets out a series of Monte Carlo experiments designed to evaluate the usefulness of the proposed identification procedure.

[6]The first part of equation **(13)** penalises positive and negative deviations from the DSGE responses in exactly the same manner. However, it is of greater economic interest for the VAR to deliver the same signs for the impulse responses compared to finding the smallest absolute deviation. The algorithm achieve this by attaching a relatively large weight to δ_{ij}.

3.1 *The model for the data generating process*

The model used for the Monte Carlo study is based on the model developed by Smets and Wouters (2007).[7] This is an estimated medium-scale DSGE model that incorporates various sources of nominal and real frictions to match US post-war business cycle fluctuations. In this model, the steady state of the economy follows a deterministic trend according to the rate of labour-augmenting technological progress. Households select consumption and labour efforts to maximise their non-separable utility preferences. Agents' consumption behaviour exhibits habit formation and households are assumed to supply differentiated labour services to firms. This gives households monopoly power over wage negotiations and therefore aggregate wages are sticky. In addition, households, who face capital adjustment costs, optimally decide how much capital to rent to firms and how much capital to accumulate.

On the production side, firms minimise the cost of production by optimally selecting the amount of labour and capital inputs subject to capital utilisation costs and the wage rate set by households. Given demands for its product, firms re-optimise prices infrequently in a Calvo-type fashion. Finally, wages and prices that are not re-optimised every period are partially indexed to the past inflation. Appendix A summarises the key linearised equation of the model. Readers who are interested in the agents' decision problems are recommended to consult the references mentioned above directly. The model's key parameters values are taken directly from Smets and Wouters'(2007) study and summarised in Table A.

In the original model, Smets and Wouters assume seven exogenous driving processes or shocks. These are required in order to match the seven observable variables used in the estimation. Here, we simplify the model to contain only four shocks, namely a government spending shock, a price mark-up shock, a wage mark-up shock and a monetary policy shock.[8]

[7]Smets and Wouters'(2007) model is based on the earlier work of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003).

[8]The original model also includes a net worth shock, technology shock and an investment specific shock. In principle, it is possible to include all seven shocks for the Monte Carlo experiments but this would increase the computation burden for the Monte Carlo experiment significantly.

3.2 Monte Carlo design

To investigate the properties of the identification strategy described in Section 2.3, we set up two Monte Carlo experiments. First, we investigate the properties of the identified structural VAR using restrictions from the true model specification (we refer to this as the benchmark model M_0). Second, we perform the same experiment but using restrictions from a series of misspecified models. In both cases, we simulate 500 samples of 200 observations of the observable vector (output growth, inflation, wage growth and the nominal interest rate), Y_t, using the model and parameters described in Section 3.1.[9] We also look at the properties of the VAR using a large sample of data, 100,000 observations.

3.2.1 Experiment one: benchmark model restrictions

For each simulated sample Y_t^i, $i = 1, \ldots, 500$:

1. Estimate the *benchmark* model using maximum likelihood estimation (MLE). The likelihood of the model is constructed via the Kalman filter and maximised using Sim's CSMINWEL algorithm. This gives the contemporaneous impact matrix of the DSGE model ($A^i \Gamma^i$) as functions of the model's structural parameters (θ^i).
2. Estimate a reduced-form VAR(2) model using ordinary least squares (OLS) and compute the variance-covariance matrix ($\hat{\Sigma}_v^i$) of the reduced-form errors.
3. Decompose $\hat{\Sigma}_v^i$ into $\widehat{C} P P' \widehat{C}'$ and find an orthonormal matrix P^* such that it minimises the loss function in equation **(13)** . We use Matlab's *fmin* function to find the minimum. To ensure the minimisation algorithm finds the unique global minimum, we repeat the minimisation procedure 1,000 times using different random starting values.
4. Construct impulse responses from the identified structural VAR (SVAR), $\widehat{C} P^* Y_t^i = \hat{A}(L)^2 Y_t^i + e_t$.

[9] We generate 10,000 observations for each sample and keep the last 200 for the Monte Carlo experiment.

3.2.2 *Experiment two: restrictions from misspecified models*

We consider five types of model misspecifications:

- M_1: Model with no habit formation ($h = 0$);
- M_2: Model with no price and wage indexation ($i_p = i_w = 0$);
- M_3: Model with no MA terms for price and wage mark-up shocks ($\mu_p = \mu_w = 0$);
- M_4: Model with no habit formation, price and wage indexation (M_1 and M_2);
- M_5: Model with no interest rate smoothing term in the Taylor rule ($\rho = 0$).

This is certainly not an exhaustive list of potential misspecifications that one can consider, but it does provide a way of evaluating the usefulness of the identification strategy using restrictions from a misspecified model. For each M_i, we repeat the same steps as the first experiment with the exception in step 1, the benchmark model is replaced with the misspecified models (M_i).

4 Results from the Monte Carlo study

4.1 Benchmark model

First we present the results of the Monte Carlo study for the benchmark model specification using 500 samples of 200 observations. Chart 1 in Appendix C plots the median impulse response functions (IRFs) for output growth, short-term interest rate, inflation and the wage growth with respect to a government spending, monetary policy, price mark-up and wage mark-up shock.[10] The black lines correspond to the responses of the true DGP described in Section 3.1 . The blue lines correspond to the median responses (across 500 samples) of the benchmark model (M_0) estimated using MLE. The red lines correspond to the median responses (also across 500 samples) of the SVAR(2) model using the identification strategy described in Section 2.3.

[10]The first four top left hand side panels correspond to the government spending shock; the top right four panels correspond to the monetary policy shock; the bottom left four panels correspond to the wage mark-up shock; and the bottom right four panels correspond to the price mark-up shock.

To compare the estimated responses with the DGP, we compute the bias of the impulse responses from the true DGP as

$$\text{bias}_T = 100 \sum_{t=1}^{T} \sum_{i=1}^{M} \sum_{j=1}^{K} \frac{|\Psi_{t,i,j} - \bar{\Psi}_{t,i,j}|}{\bar{\Psi}_{t,i,j}} \tag{15}$$

where $\Psi_{t,i,j}$ is the t'th period's impulse response of the estimated benchmark or VAR model for variable i to shock j, $\bar{\Psi}_{t,i,j}$ is the DGP equivalent and the bias is calculated as the sum across all the M variables, K shocks up to periods T. The first three columns in Table B provides a summary of the Monte Carlo simulations using small samples. The first column displays the average bias over 500 samples between the estimated DSGE model relative to the true DGP at different horizons. Similarly, the second column displays the same statistics for the SVAR model. The third column shows the ratio between the benchmark model and the SVAR. If the ratio is greater than one, this means the SVAR responses are closer to the true DGP compared with the benchmark model and *vice versa* for a ratio less than one.

Looking at the first two quarters following the shock, the responses of the benchmark model and the SVAR are very similar compared with the true DGP. This can also be seen graphically from Chart 1 where the SVAR (red line) and the benchmark model (blue line) overlap with the impulse responses from the true DGP. However, at longer horizons both the SVAR and the benchmark model's responses deviate away from the true responses. The bias of the SVAR is greater than the estimated benchmark model.

The discrepancy between the estimated benchmark model and its DGP is largely attributed to the small-sample *bias* in estimating DSGE models. Liu and Theodoridis (2009) investigates the small-sample properties of a similar medium-scale DSGE models in more detail. Essentially, the bias comes about because of the non-negativity constraints placed on the model's parameters and the non-linear mapping between the structural and reduced-form representation of the model. The resultant structural parameter estimates can be highly non-normal in small samples. However, the bias disappear when a sufficiently large sample of data is used for the estimation. In the Monte Carlo experiment, we demonstrate that with a sample size of 100,000 observations, it is sufficient to eliminate the bias completely. Graphically this can be seen from Chart 2 where the estimated benchmark model's responses and the DGP lie on top of each other exactly.

4.1.1 *Bias of the SVAR impulse response functions*

As discussed earlier, the bias from the VAR model is greater than the estimated benchmark model. To provide an economic interpretation for the bias, it is useful to consider a SVAR in a similar form as in equation **(1)**:

$$A(L)Y_t = e_t = A_0^{-1}\eta_t \quad \textbf{(16)}$$

$$Y_t = A(L)^{-1}A_0^{-1}\eta_t = R(L)A_0^{-1}\eta_t \quad \textbf{(17)}$$

where $A(L) = I - A_1L - \cdots - A_hL^h$ is the lag polynomial matrix, A_0 is the contemporaneous impact matrix, $R(L) = A(L)^{-1}$, e_t and η_t are $K \times 1$ vectors of reduced-form innovations and structural innovations. From equation **(17)**, one can see that the response of Y_t to the underlying structural innovation, η_t, is influenced both by the reduced-form moving average terms, $R(L)$, and by the identifying restrictions placed on A_0. Erceg *et al* (2005) usefully categorise the bias of a SVAR model relative to the true DGP (generated from a DSGE model) into three components:

$$\text{SVAR bias} = \text{R bias} + \text{A bias} + \text{Truncation bias} \quad \textbf{(18)}$$

The first part, the 'R bias', reflects the small-sample error in estimating the reduced-form moving average terms, the $R(L)$ coefficients in equation **(17)** . The second part, referred to, as the 'A bias' reflects the error associated with transforming the reduced form into its structural form by imposing certain identifying restrictions, the A_0 matrix. Lastly, the 'truncation bias' that arises because a finite-ordered VAR ($h < \infty$) is chosen to approximate the true dynamics implied by the model. King, Plosser and Rebelo (1988) were among the first to recognise that DSGE models imply a vector autoregressive moving average (VARMA) representation and Cooley and Dwyer (1998) emphasised that this is the case for most popular DSGE models. The solution of Smets and Wouters's (2007) model can be shown to have similar VARMA representation. More recently, Kapetanios, Pagan and Scott (2007) document that the truncation bias from medium to large-scale models can be very large.

It is important to recognise that the three types of biases are not necessary independent of each other, they can interact and exacerbate the overall bias of the SVAR responses. For example, using a fixed sample size, a larger truncation bias can increase the R bias related to the estimation of the reduced-form coefficients. Similarly, for a fixed set of identifying assumptions, the imprecision in estimating $R(L)$ can exacerbate the A bias associated with the identification of the structural shocks.

To investigate the relative importance of the biases for our SVAR model, we re-estimate the same VAR(2) model using 100,000 observations and the identifying restrictions come from estimating the benchmark model using the same data set. The results from this experiment are plotted along side the benchmark model's responses in Chart 2 (red line).[11] To aid comparison, we also reproduced the small-sample SVAR responses in the same chart (magenta line). The numerical calculations are summarised in the seventh column in Table B

By using the correct model restrictions (the estimated benchmark model now coincides with the DGP), the proposed identification strategy is able to recover the true impact matrix (A_0). This can be shown in Table B where the bias of the SVAR's first-period responses are very close to zero. Furthermore, the biases (at various horizons) are also smaller relative to the small-sample estimates. Although using a large data set helped eliminate the identification error (A bias) and reduced the small-sample error in estimating the reduced-form coefficients (R bias), large differences still exists at longer horizons.

In contrast to Erceg *et al*'s (2005) findings, we find the truncation bias plays the dominant role in explaining the difference between the VAR and the DSGE model's responses. Chart 3 plots the bias of the SVAR model in terms of the number of lags and the horizons for the impulse responses. To compute the bias, we re-estimate the VAR using different lag lengths based on 100,000 observations and the identification matrix is computed as before. As one might expect, for a fix number of lags the bias is larger at longer horizons (see explanations in Ravenna (2007)). On the other hand, the bias is a monotonic function decreasing with the number of lags. It is interesting to note that the bias decreases in a non-linear fashion. Moving from one to four lags, the bias decreases by 35%, whereas the simulation results indicate even after 50 lags, the bias remains around 15% of the VAR(1) model. This is in line with the evidences provided by Kapetanios *et al* (2007) that in order to approximate a medium to large-scale DSGE model, one would require a significant large number of lags for the VAR.

The speed in which the truncation bias decreases with the number of lags will depend on the model's specification and parameters. More specifically, Fernandez-Villaverde *et al* (2007) point out that the closer the largest (absolute value) eigenvalue of the matrix M in equation **(7)** is to one, the more lags are needed in order to approximate the true DGP. In our case, the largest

[11] Note, the estimated benchmark model responses using 100,000 observations are exactly the same as the DGP.

eigenvalue of the matrix M is indeed very close to one and therefore it is not surprising that a large-order VAR is needed to approximate the dynamics of the model.[12]

4.1.2 Proposed identification versus existing strategies

To compare the proposed identification with existing identification strategies, Chart 1 also includes the responses of the VAR model identified using the Choleski decomposition (green line) and sign restrictions (light blue line) implied by the model.[13] In all cases, the Choleski and sign restriction identification schemes produced impulse responses that are much further away from the DGP. The standard 'price puzzle', for example, is evident in the Choleski scheme and experiments with different recursive ordering structure produced similar results. It is interesting to note that even though the sign restrictions deliver the right signs by construction, it consistently overestimates the impact of the shock.

The summary statistics in Table B revealed similar conclusions.[14] The size of the bias using the Choleski scheme is between 8 and 11 times larger than the proposed identification strategy at shorter horizons and the sign restriction scheme is 31 to 48 times larger. At longer horizons, the truncation bias dominates and the differences are much smaller.

4.2 VAR identification using misspecified models

The Monte Carlo results so far assumes that the identifying restrictions come from the true estimated model. A natural question to ask is how the proposed identification scheme will perform in the face of model misspecification, this subsection investigates the issue in more detail. We repeat the same Monte Carlo experiments as before, the key difference being that the restrictions will come from a misspecified model as listed in Section 3.2. This is certainly not an exhaustive list of potential misspecifications one can consider, but it does provide a way of evaluating the usefulness of the identification scheme when restrictions are derived from

[12] We also experimented with a simple three-equation New Keynesian model where the eigenvalue of the matrix M is much smaller, in which case a VAR(2) together with the proposed identification strategy provides an excellent match with the DGP's impulse responses. These results are available on request.

[13] For the Choleski decomposition, the variables are ordered as output, inflation, wage growth and short-term interest rate.

[14] The fourth column in Table B shows the ratio of the total bias between Choleski VAR relative to the VAR using DSGE restrictions. Similarly, the fifth column reports the same ratio for the VAR using sign restrictions.

misspecified models. The aim here is to compare the SVAR results against the misspecified model, rather than exploring the importance of the misspecification on the model's properties.

Table C displays the bias of the impulse responses across different horizons for the estimated misspecified model and the SVAR.[15] The model with no interest rate smoothing term in the Taylor rule (M_5) appears to give the largest bias. Since the VAR identification depends on the restrictions implied by the misspecified model, the bias for the SVAR model is also the largest for M_5. At shorter horizons (one and two quarters), the bias for the SVAR models is smaller than that of the misspecified models.[16] This is an interesting result, even though the identification bias (A bias) might be larger because of restrictions from a misspecified model, the data tend to push the SVAR responses closer to the true DGP. Therefore, information from the data are useful in correcting some of the bias from using misspecified model restrictions.

At longer horizons, the truncation bias (as discussed earlier) dominates and the bias of the SVAR is around two times that of the misspecified model. It is interesting to note that once we take into account the truncation bias (proxied by the benchmark model), the bias of the SVAR model is comparable with that from the misspecified model.

The results from this experiment shows that despite misspecifications in the DSGE models, the restrictions implied by the model are still useful in identifying the SVAR shocks. The resultant dynamic properties of the SVAR is comparable with the misspecified model.

5 Application: seven-variable SVAR model for the United States

To demonstrate how the proposed identification scheme can be applied in practice, we estimate a seven-variable VAR using US data from 1966 Q1 to 2004 Q4. The data set is taken from the Smets and Wouters' (2007) paper which includes: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, log difference of GDP deflator and the Federal funds rate. Although Smets and Wouters (2007) compare the estimated DSGE model's forecast performance with reduced-form VARs, they did not present any comparisons between the DSGE model's impulse response functions with a VAR model. This

[15]The bias for the benchmark model is also included in Table C for relative comparison.

[16]With the exception of model M_5 where the bias between the misspecified model and the VAR are fairly similar.

partly reflects the difficulty in finding the appropriate set of identifying restrictions for the VAR. From that perspective, our identification scheme is a natural candidate for comparison analysis. One can view this as a diagnostic tool for analysing the dynamic behaviour of the estimated DSGE model.

We follow the same procedure set out earlier in Section 2.3 and the restrictions are based on Smets and Wouters' (2007) original model with all seven shocks.[17] First, we estimate the VAR using simple OLS regression to obtain the reduced-form variance-covariance matrix ($\hat{\Sigma}_{\upsilon}$). Next, we find an orthogonal matrix P^* that minimised the distance in the first-period response between the DSGE and the SVAR model. The DSGE model's response is based on the parameter estimates obtained by Smets and Wouters (2007) as listed in Table A. For diagnostic comparison, the log-likelihood of the VAR is calculated to be -1,064 *versus* -1,256 for the DSGE model.

5.1 Impulse response functions

In Charts 4 and 5, we present the impulse response functions for two of the most frequently analysed shocks: an unexpected productivity shock and a monetary policy shock.[18] The blue line corresponds to the Smets and Wouters (2007) estimated model and the red line correspond to the response of the SVAR model. It is worth highlighting that all the variables' responses have the same sign across the two models. However, there are some interesting differences in terms of the magnitudes and adjustment paths to the shocks.

For the productivity shock, the SVAR model tends to suggest a smaller impact on hours worked and the nominal interest rate. The impact on consumption, real wages and inflation is slightly higher but less persistent. For the monetary policy shock, the SVAR model gives a larger but more temporary response for inflation. On the other hand, the SVAR displays a much more persistent behaviour for hours worked, real wages and the interest rate compared with the DSGE model. Interestingly, the response of output, consumption and investment are very similar across the two models.

[17] The seven shocks include: a government spending shock, a price mark-up shock, a wage mark-up shock, a monetary policy shock, a net worth shock, a technology shock and an investment specific shock.

[18] The results of other shocks are available upon request to the authors.

5.2 *Forecast error variance decomposition*

In addition to the impulse response analysis, we also compute the forecast error variance (FEV) decomposition for both models. The results for the SVAR are summarised in Chart 6 and Chart 7 summarises Smets and Wouters'(2007) estimated DSGE model. For the SVAR, productivity and investment shocks are the main drivers of the FEV for output growth, whereas in the DSGE model, preference and government spending shocks play the dominant role. Similarly for consumption growth, investment shocks are more important than preference shocks in the SVAR model. For hours worked, productivity shocks explain over 50% of the FEV as opposed to the wage mark-up shock identified in the DSGE model. Investment shocks are the key factor in explaining investment growth across both models.

These observations highlight an important contrast across the two models, the SVAR tends to suggest real shocks, such as investment and productivity shocks, play a relatively more important role than nominal shocks (government spending and wage mark-up shocks) for real economic variables. The sum of real shocks account for 80% of output, 87% of consumption, 70% of hours worked and 65% of investment at the twelve-quarter horizon.[19] On the other hand, the DSGE model tends to suggest both real and nominal shocks play an equally important role.

For the nominal interest rate, both models suggest the contribution to the FEV is (roughly) equally divided among all seven shocks over the medium term. The DSGE model identifies both price and wage mark-up shocks to be the key drivers of the FEV for inflation, whereas the SVAR also attributes part of the FEV to investment shocks. For wage growth, while both models agree on the importance of wage mark-up shocks, the SVAR points to a much smaller role for price mark-up shocks over the medium term. Another interesting feature is that the DSGE model identifies wage mark-up shocks to be the dominant contributor of the unconditional variance for interest rate, inflation and wage growth, whereas the SVAR highlights the importance of productivity and price mark-up shocks.

[19]We classify real shocks to include productivity, preference and investment shocks. All other shocks are classified as nominal shocks.

6 Conclusion

Issues relating to the identification of VAR models have been subject to numerous debates in the literature. The key source of this disagreement arises from finding a set of appropriate identifying assumptions to disentangle the reduced-form residuals back into structural disturbances. The sampling information in the data is often insufficient to distinguish between these different sets of assumptions. This paper proposes an identification strategy that extends Uhlig's (2005) penalty-function approach to a more formal setting. In particular, we construct a penalty function that is based on quantitative restrictions implied by a DSGE model. We present a series of Monte Carlo experiments to assess the usefulness of the proposed identification strategy. We also present an application using a seven-variable VAR model estimated on US data and compare this with the results obtained from a medium-scale DSGE model by Smets and Wouters (2007).

A number of interesting results emerge from the analysis. First by using the correct model restrictions, the identification procedure is successful in recovering the initial impact of the shocks from the data. We identify two sources of bias relating to the difference between the SVAR and the true response at longer horizons. One is related to the *small-sample bias*, while the other is due to the *truncation bias* that arises from using a finite-order VAR to approximate a VARMA process. When a large sample is used, the truncation bias remains the dominant source of bias. Second, despite using restrictions implied by a misspecified model, the data tend to push the VAR responses away from the misspecified model and closer to the true DGP. Third, the proposed identification strategy systematically gives smaller bias compared with other identification schemes such as the Choleski decomposition and pure sign restrictions.

The identification procedure proposed here is mainly applied to VAR models with relatively small number of variables. But increasingly, the empirical literature emphasises the importance of estimating statistical models based on a large information set. Examples include the large Bayesian VAR model put forward by Bandbura, Giannone and Reichlin (2010) and the factor augmented VAR model of Bernanke, Boivin and Eliasz (2005). Future research could therefore be directed towards exploiting information contained in DSGE models to help identify VAR models with a large number of variables.

Appendix A: The linearised DSGE model

This appendix briefly discusses some of the key linearised equilibrium conditions of Smets and Wouters'(2007) model. Readers who are interested in the agents' decision problems are recommended to consult the references mentioned above directly. All the variables are expressed as log deviations from their steady-state values, $\mathbb{E}_t$ denotes expectation formed at time t, '$-$' denotes the steady-state values and all the shocks (η_t^i) are assumed to be normally distributed with zero mean and unit standard deviation.

The demand side of the economy consists of consumption (c_t), investment (i_t), capital utilisation (z_t) and government spending $\left(\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g\right)$ which is assumed to be exogenous. The market clearing condition is given by

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \qquad \textbf{(A-1)}$$

where y_t denotes the total output and Table A provides a full description of the model's parameters. The consumption Euler equation is given by

$$c_t = \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \left(1 - \frac{\lambda/\gamma}{1+\lambda/\gamma}\right) \mathbb{E}_t c_{t+1} + \frac{(\sigma_C - 1)\left(\bar{W}^h \bar{L}/\bar{C}\right)}{\sigma_C (1+\lambda/\gamma)} (l_t - \mathbb{E}_t l_{t+1}) - \frac{1-\lambda/\gamma}{\sigma_C (1+\lambda/\gamma)} (r_t - \mathbb{E}_t \pi_{t+1}) \qquad \textbf{(A-2)}$$

where l_t is the hours worked, r_t is the nominal interest rate and π_t is the rate of inflation. If the degree of habits is zero ($\lambda = 0$), equation **(A-2)** reduces to the standard forward-looking consumption Euler equation. The linearised investment equation is given by

$$i_t = \frac{1}{1+\beta\gamma^{1-\sigma_C}} i_{t-1} + \left(1 - \frac{1}{1+\beta\gamma^{1-\sigma_C}}\right) \mathbb{E}_t i_{t+1} + \frac{1}{\left(1+\beta\gamma^{1-\sigma_C}\right)\gamma^2\varphi} q_t \qquad \textbf{(A-3)}$$

where i_t denotes the investment and q_t is the real value of existing capital stock (Tobin's Q). The sensitivity of investment to real value of the existing capital stock depends on the parameter φ (see, Christiano *et al* (2005)). The corresponding arbitrage equation for the value of capital is given by

$$q_t = \beta\gamma^{-\sigma_C}(1-\delta)\mathbb{E}_t q_{t+1} + \left(1 - \beta\gamma^{-\sigma_C}(1-\delta)\right)\mathbb{E}_t r_{t+1}^k - (r_t - \mathbb{E}_t \pi_{t+1}) \qquad \textbf{(A-4)}$$

where $r_t^k = -(k_t - l_t) + w_t$ denotes the real rental rate of capital which is negatively related to the capital-labour ratio and positively to the real wage.

On the supply side of the economy, the aggregate production function is define as

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha) l_t\right) \tag{A-5}$$

where k_t^s represents capital services which is a linear function of lagged installed capital (k_{t-1}) and the degree of capital utilisation, $k_t^s = k_{t-1} + z_t$. Capital utilisation, on the other hand, is proportional to the real rental rate of capital, $z_t = \frac{1-\psi}{\psi} r_t^k$. The accumulation process of installed capital is simply described as

$$k_t = \frac{1-\delta}{\gamma} k_{t-1} + \frac{\gamma - 1 + \delta}{\gamma} i_t \tag{A-6}$$

Monopolistic competition within the production sector and Calvo-pricing constraints gives the following New Keynesian Phillips Curve for inflation

$$\begin{aligned}\pi_t = {} & \frac{i_p}{1+\beta\gamma^{1-\sigma_C} i_p}\pi_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1+\beta\gamma^{1-\sigma_C} i_p}\mathbb{E}_t \pi_{t+1} \\ & - \frac{1}{\left(1+\beta\gamma^{1-\sigma_C} i_p\right)} \frac{\left(1-\beta\gamma^{1-\sigma_C}\xi_p\right)\left(1-\xi_p\right)}{\left(\xi_p\left(\left(\phi_p - 1\right)\varepsilon_p + 1\right)\right)}\mu_t^p + \varepsilon_t^p \end{aligned} \tag{A-7}$$

where $\mu_t^p = \alpha\left(k_t^s - l_t\right) - w_t$ is the marginal cost of production and $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \sigma_p \eta_t^p - \mu_p \sigma_p \eta_{t-1}^p$ is the price mark-up price shock which is assumed to be an ARMA(1,1) process. Monopolistic competition in the labour market also gives rise to a similar wage New Keynesian Phillips Curve

$$\begin{aligned} w_t = {} & \frac{1}{1+\beta\gamma^{1-\sigma_C}} w_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1+\beta\gamma^{1-\sigma_C}}\left(\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}\right) - \frac{1+\beta\gamma^{1-\sigma_C} i_w}{1+\beta\gamma^{1-\sigma_C}}\pi_t \\ & + \frac{i_w}{1+\beta\gamma^{1-\sigma_C}}\pi_{t-1} - \frac{1}{1+\beta\gamma^{1-\sigma_C}} \frac{\left(1-\beta\gamma^{1-\sigma_C}\xi_w\right)(1-\xi_w)}{\left(\xi_w\left((\phi_w - 1)\varepsilon_w + 1\right)\right)}\mu_t^w + \varepsilon_t^w \end{aligned} \tag{A-8}$$

where $\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1-\lambda}\left(c_t - \lambda c_{t-1}\right)\right)$ is the households' marginal benefit of supplying an extra unit of labour service and the wage mark-up shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \sigma_w \eta_t^w - \mu_w \sigma_w \eta_{t-1}^w$ is also assumed to be an ARMA(1,1) process.

Finally, the monetary policy maker is assumed to set the nominal interest rate according to the following Taylor-type rule

$$r_t = \rho r_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right] + r_{\Delta y}\left[\left(y_t - y_t^p\right) + \left(y_{t-1} - y_{t-1}^p\right)\right] + \varepsilon_t^r \tag{A-9}$$

where y_t^p is the flexible prices-wages and zero mark-up shocks level of output and $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \sigma_r \eta_t^r$ is the monetary policy shock.

Appendix B: Tables

Table A: Parameter descriptions and estimated values from Smets and Wouters (2007)

Symbols	Description	M_0
γ	Steady-State Growth Rate	1.00
π	Steady-State Inflation	1.00
Φ	Fixed Cost	1.50
S"	Steady-State Capital Adjustment Cost Elasticity	5.74
α	Capital Production Share	0.19
σ	Intertemporal Substitution	1.38
h	Habit Persistence	0.71
ξ_w	Wages Calvo Parameter	0.70
σ_l	Labour Supply Elasticity	1.83
ξ_p	Prices Calvo Parameter	0.66
i_w	Wage Indexation	0.58
i_p	Price Indexation	0.24
z	Capital Utilisation Adjustment Cost	0.27
ϕ_π	Taylor Inflation Parameter	2.04
ϕ_r	Taylor Inertia Parameter	0.81
ϕ_y	Taylor Output Gap Parameter	0.08
ϕ_{dy}	Taylor Output Gap Change Parameter	0.22
ρ_g	Government Spending Shock Persistence	0.97
ρ_{ms}	Policy Shock Persistence	0.15
ρ_p	Price Mark-up Shock Persistence	0.89
ρ_w	Wage Mark-up Shock Persistence	0.96
ma_p	Price Mark-up MA Term	0.69
ma_w	Wage Mark-up MA Term	0.84
σ_g	Government Spending Shock Uncertainty	0.53
σ_{ms}	Policy Shock Uncertainty	0.24
σ_p	Price Mark-up Shock Uncertainty	0.14
σ_w	Wage Mark-up Shock Uncertainty	0.24

Table B: Accumulated bias for the benchmark model and SVAR model

	Small-sample estimates					Large-sample estimates			
Horizon	M_0	$SVAR_D$	$\frac{M_0}{SVAR_D}$	$\frac{SVAR_C}{SVAR_D}$	$\frac{SVAR_S}{SVAR_D}$	M_0	$SVAR_D$	$\frac{SVAR_C}{SVAR_D}$	$\frac{SVAR_S}{SVAR_D}$
1	1.5	1.6	0.9	11.0	48.5	0.0	0.3	66.9	279.3
2	2.5	3.7	0.7	8.3	31.9	0.0	2.3	13.6	50.2
3	4.9	28.3	0.2	3.0	10.2	0.0	5.6	8.7	29.3
4	8.4	28.3	0.3	3.0	10.2	0.0	27.3	4.4	8.8
8	39.2	98.4	0.4	2.8	5.5	0.0	95.4	3.7	5.0
12	68.4	190.2	0.4	2.2	3.3	0.0	184.3	2.8	3.3
16	86.1	253.7	0.3	1.9	2.7	0.0	236.5	2.5	2.9
20	104.5	311.9	0.3	1.8	2.4	0.0	281.7	2.4	2.8

1. M_0 corresponds to the bias for the estimated benchmark model, $SVAR_D$, $SVAR_C$ and $SVAR_S$ correspond to the SVAR(2) model using DSGE, Choleski and sign restrictions.

2.The small sample estimates are computed by taking the median of the 500 samples and the large sample estimates are computed using 100,000 observations.

Table C: Accumulated bias for misspecified models and the SVAR model

	Period 1			Periods 2			Periods 8			Periods 12		
	DSGE	SVAR	Ratio	DSGE	SVAR	Ratio	DSGE	SVAR	Ratio	DSGE	SVAR	Ratio
M_0	1.5	1.6	0.9	2.5	3.7	0.7	39.2	98.4	0.4	68.4	190.2	0.4
M_1	6.1	5.9	1.0	10.3	9.5	1.1	61.9	131.8	0.5	85.3	230.0	0.4
M_2	2.1	1.9	1.1	6.3	4.4	1.4	74.1	109.7	0.7	97.2	213.4	0.5
M_3	4.2	5.0	0.8	9.8	7.9	1.2	72.2	117.4	0.6	109.8	203.3	0.5
M_4	5.7	5.4	1.0	11.3	9.3	1.2	68.7	128.9	0.5	104.1	235.3	0.4
M_5	12.8	12.9	1.0	19.8	19.6	1.0	117.5	190.0	0.6	164.0	263.4	0.6

1. The accumulated bias is calculated as the sum (across different number of periods) of the absolute percentage difference between the estimated DSGE model or the VAR's impulse responses with the DGP.

2. The ratio measure is simply the bias of the estimated DSGE model relative to that of the SVAR model.

Appendix C: Charts

Chart 1: Median impulse response functions of the benchmark model: 500 samples of 200 observations

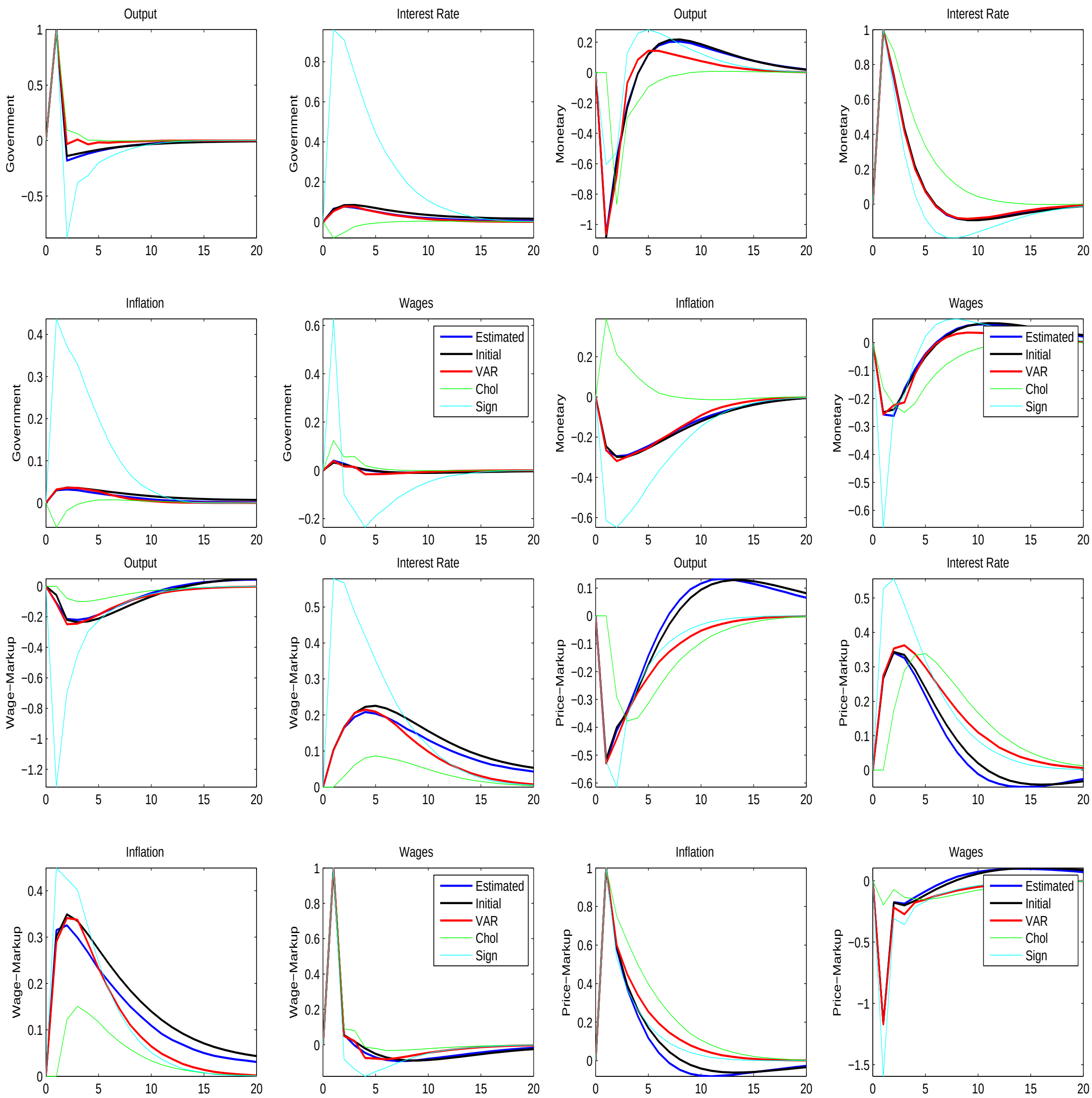


Chart 2: Impulse response functions of the benchmark model: 100,000 observations

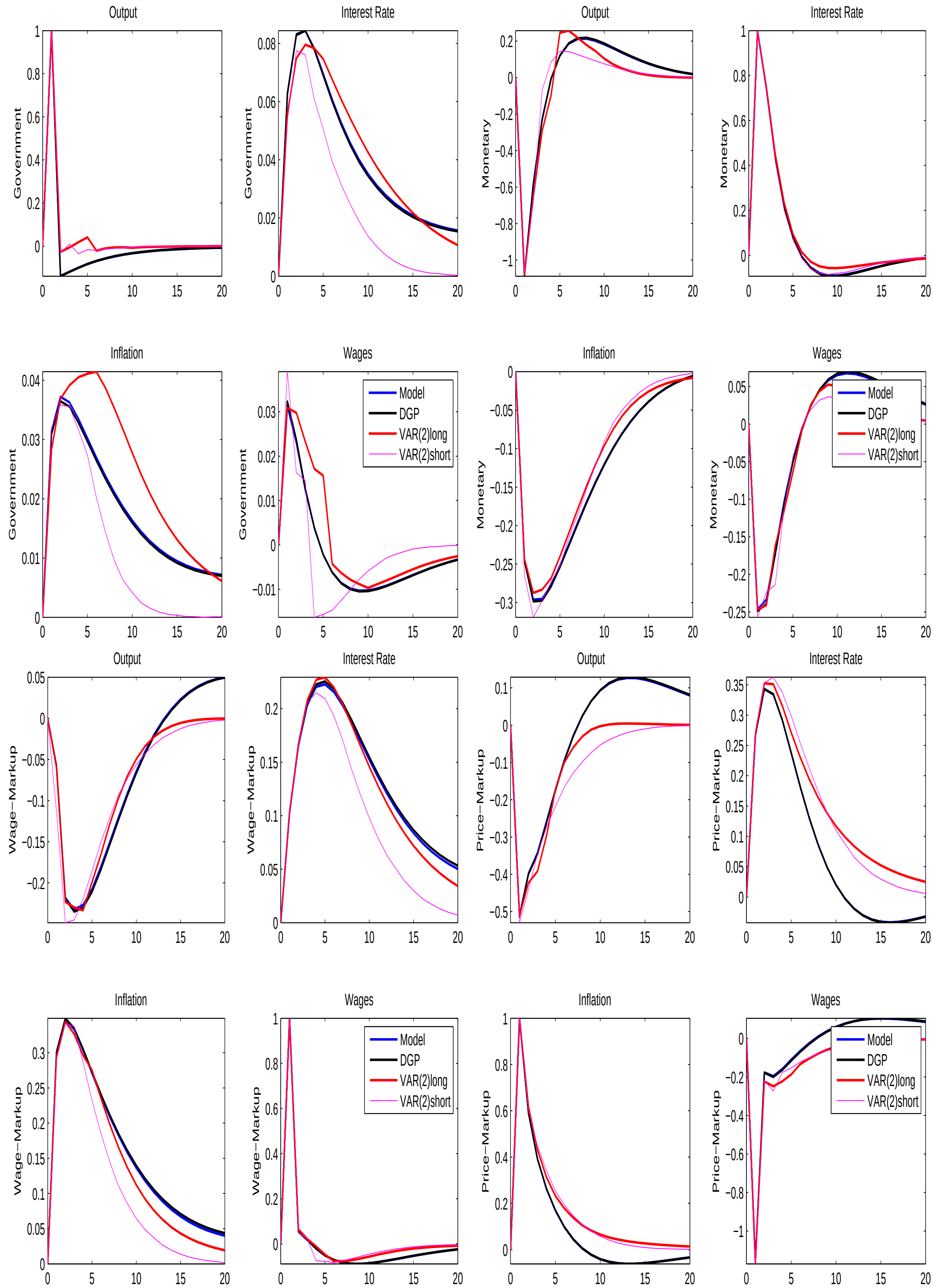


Chart 3: Truncation bias (accumulated) of the VAR model

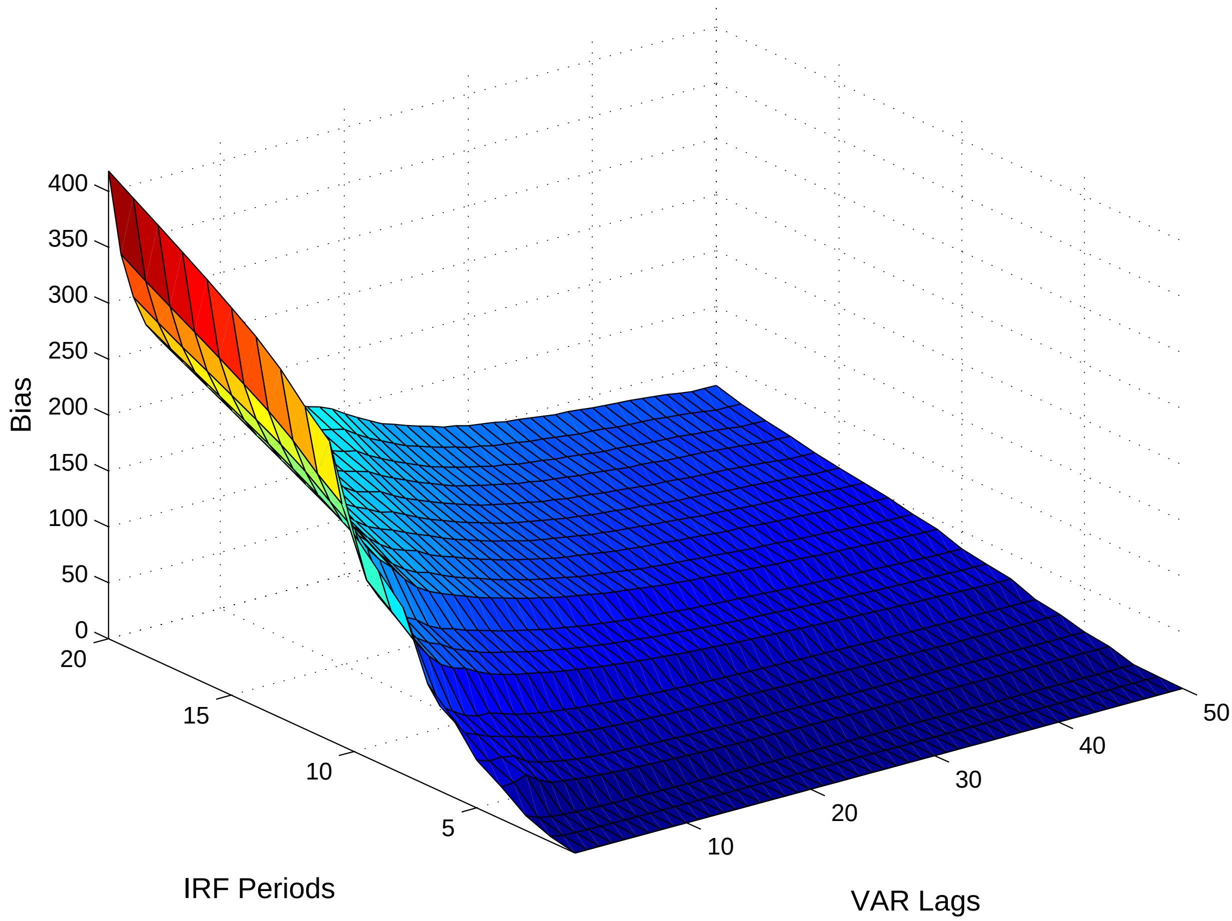


Chart 4: US structural VAR: productivity shock

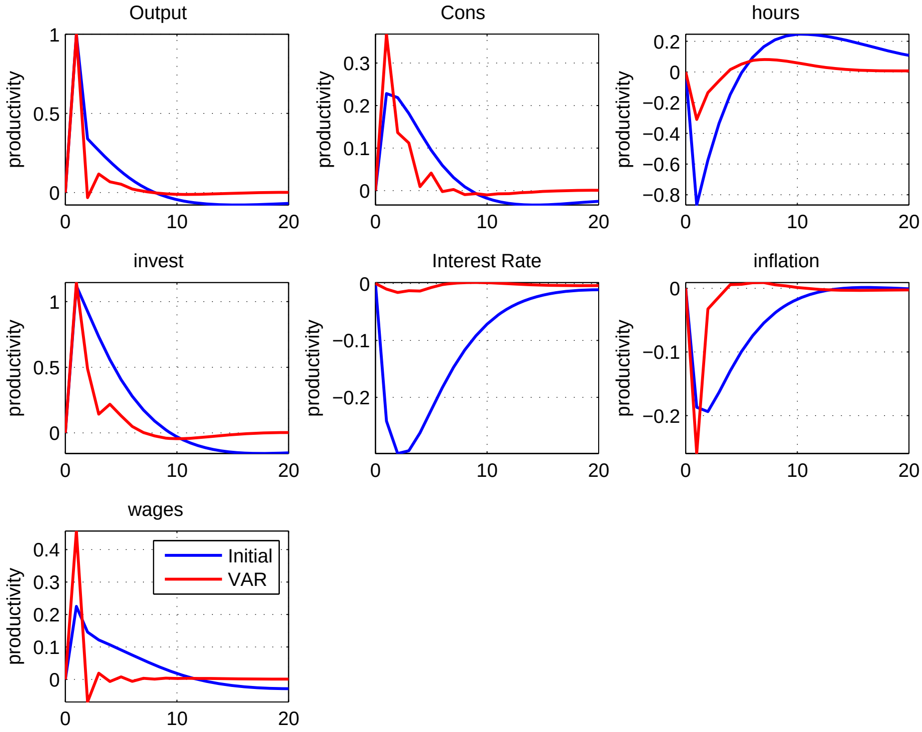


Chart 5: US structural VAR: monetary policy shock

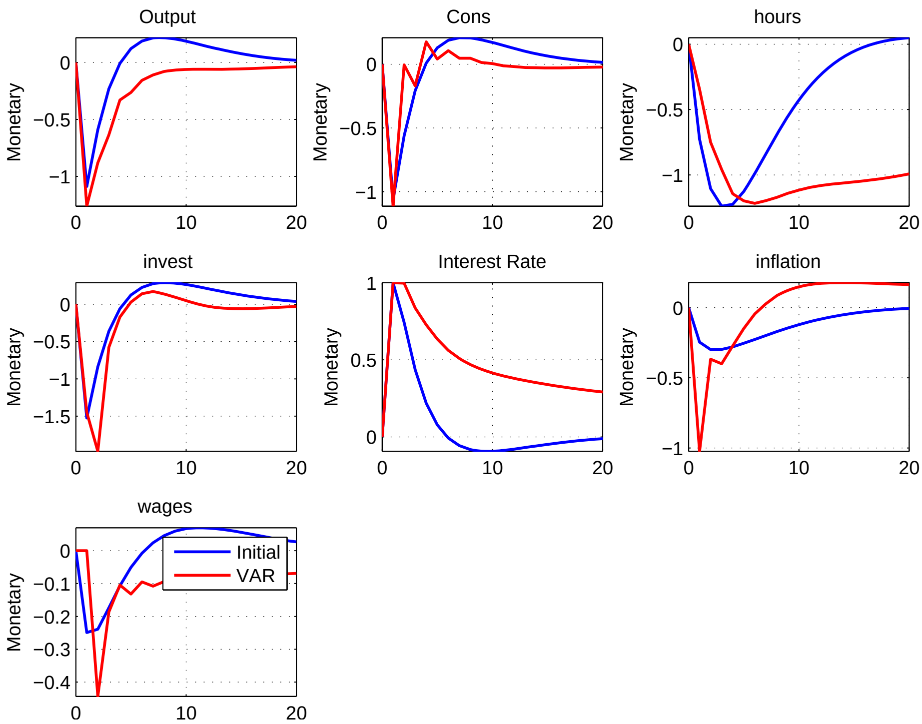


Chart 6: US structural VAR forecast error variance decomposition

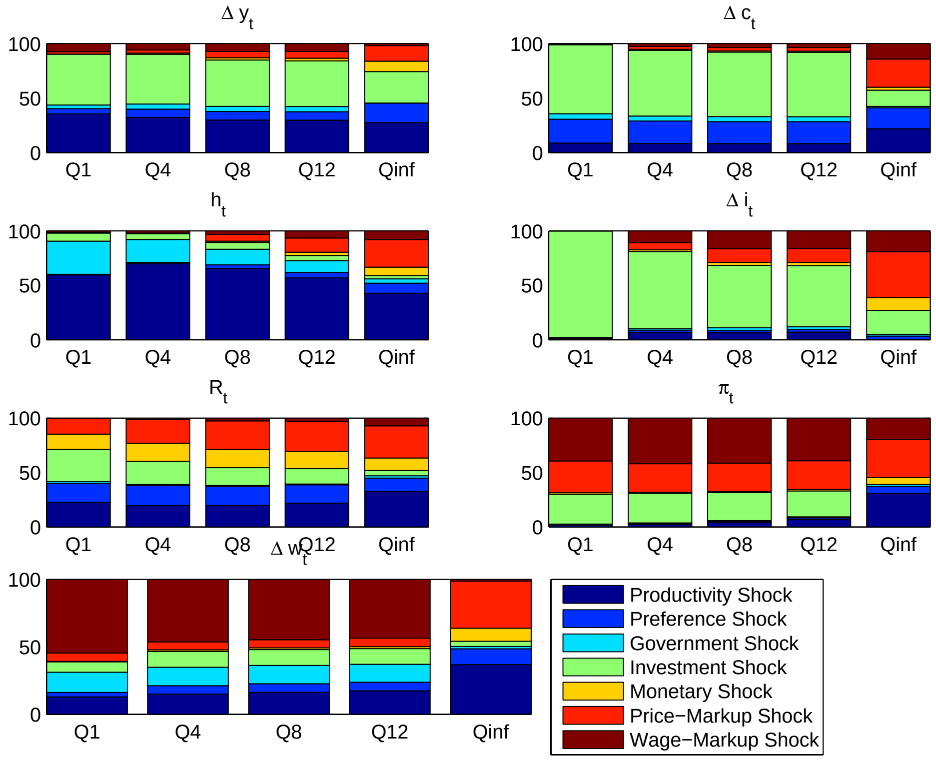
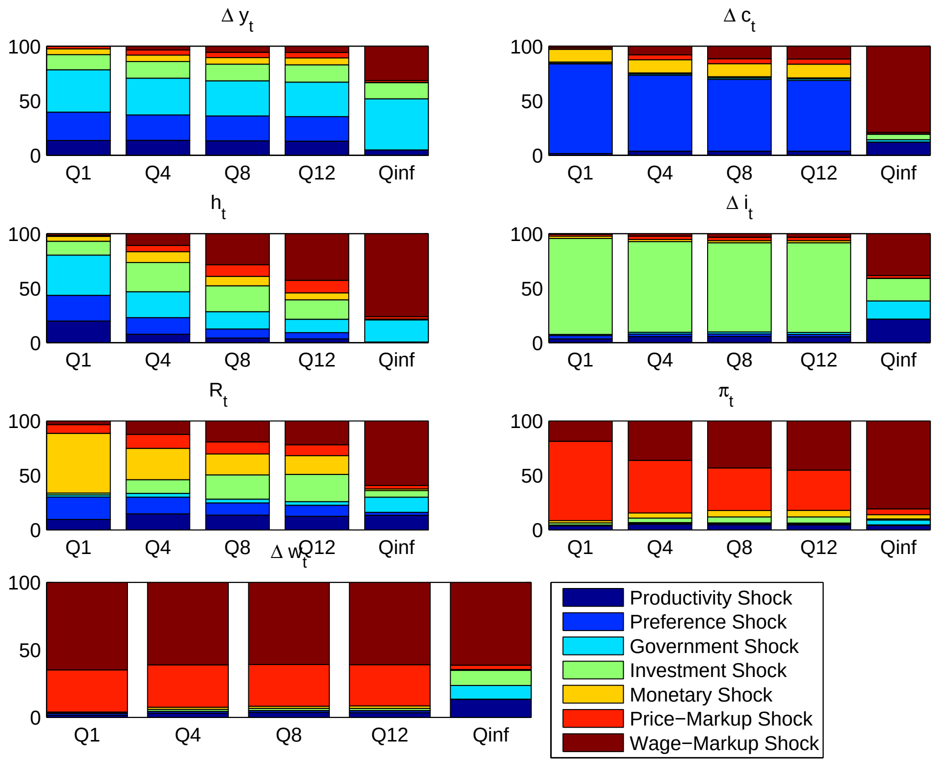


Chart 7: Smets and Wouters' model forecast error variance decomposition



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