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## Evaluating and estimating a DSGE model for the United Kingdom

Richard Harrison and Özlem Oomen

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Richard Harrison<sup>(1)</sup> and Özlem Oomen<sup>(2)</sup>

### Abstract

We build a small open economy dynamic stochastic general equilibrium model, featuring many types of nominal and real frictions that have become standard in the literature. In recent years it has become possible to estimate such models using Bayesian methods. These exercises typically involve augmenting a stochastically singular model with a number of shocks to structural equations to make estimation feasible, even though the motivation for the choice of these shocks is often unspecified. In an attempt to put this approach on a more formal basis, we estimate the model in two stages. First, we evaluate a calibrated version of the stochastically singular model. Then, we augment the model with structural shocks motivated by the results of the evaluation stage and estimate the resulting model using UK data using a Bayesian approach. Finally, we reassess the adequacy of this *augmented and estimated* model in reconciling the dynamics of the model with the data. Our findings suggest that the shock processes play a crucial role in helping to match the data.

**Key words:** DSGE models, model evaluation, Bayesian estimation, monetary policy.

**JEL classification:** E4, E5.

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## Summary

It is impossible to conduct monetary policy without some understanding of how the economy works, and consequently economic models are vital in this process. The Bank of England uses many such models, some very abstract and others largely data driven. In this paper, we examine one that is both rich in theory and consistent with the data. We estimate it using UK macroeconomic data from 1955-2007.

Our approach has two stages. First, we derive predictions about the relationships between key economic variables in both the short run and long run, using judgement to select sensible values for the parameters so that we can deliver specific results. We then compare these with the actual behaviour of UK data. This comparison helps us to identify those relationships that fail to match data closely and hence where additional features may be required. In the second step, we incorporate these features, called ‘structural shocks’, and estimate the parameters that best fit the data. The shocks that we add are in the form of movements in the demand and supply curves that determine prices and quantities. We find that they are crucial in helping the model match reality.

We work with four key sectors: households, businesses, the monetary policy maker and the rest of the world. Households receive income from working and interest from past saving. They choose how much of their total income to spend on goods and services and how much of it to save, depending on the real rate of interest earned on saving. A higher real interest rate will, other things equal, encourage households to save more. Businesses produce the goods and services that households buy. They set the prices for their products and decide how much labour and capital to employ in order to maximise their profits. Importantly, businesses face costs of adjusting their prices which means that they find it best to change them gradually. The monetary policy maker sets the nominal interest rate by adjusting it in response to changes in inflation and output. The rest of the world, modelled using a set of estimated equations, affects the domestic economy through the demand for the goods and services that it produces.

Together, these features allow us to describe how households, businesses, the monetary policy maker and the rest of the world interact. The values of the parameters are an important determinant of the consequent behaviour of macroeconomic variables. For example, there is a



parameter that determines the willingness of households to substitute consumption spending today for consumption spending in the future. If households are less willing to substitute consumption today for future consumption, then their saving and consumption decisions will be less affected by changes in the real interest rate. Similarly, there are parameters that determine the costliness to businesses of changing prices (an example of a ‘friction’). Other things equal, if prices are more costly to adjust, a business prefers to adjust the amount of labour and capital it employs in response to a change in the demand for its products, rather than changing the price that it charges. To evaluate the model, we use data on consumption, gross domestic product, investment, total hours worked, real wages, the nominal interest rate and inflation. We choose the longest available data set in order to gain as much information as possible about the parameters, while recognising that there will be a trade-off against accuracy if, as is likely, their values change over time.

When we compare the model’s predictions about the relationships between key variables to the behaviour of UK data, we find some important differences. In many cases, the model predicts a much stronger relationship between variables in the short run than we observe in the data. And it predicts a weaker long-run correlation between the movements in consumption and output. It also predicts that real wages are less variable than we observe in the data.

So before we estimate the parameters, we incorporate additional shocks in the form of random movements in the demand and supply curves. For example, we assume that a household’s preferences for spending versus saving may vary somewhat over time. This means that, in some periods, households will be inclined to save less, even when the real interest rate is high (and *vice versa*). When we estimate the model, we find that these structural shocks are very important in helping it to better match the behaviour of the data. Our estimation results also suggest that the parameters that determine the costliness of adjusting prices are more in line with similar work using US data, rather than in studies using data from the euro area. But we do not have the whole story. For example, the estimated model does not explain nominal interest rates well. Ways to explore this could include extending the approach to allow for the fact that monetary policy may change over time.

## 1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are now a standard tool of macroeconomic research. Typically, such models are built from microeconomic foundations, along with a set of simple monetary and fiscal rules. To capture key properties of the data, these models often also incorporate numerous nominal and real frictions such as sticky prices and wages, habit formation in consumption and labour choices, and adjustment costs in investment and capital utilisation. DSGE models have been used to address a variety of macroeconomic questions including the analysis of business cycle fluctuations and effects of monetary and fiscal policies.<sup>1</sup>

Since Kydland and Prescott (1982), researchers have considered how to take DSGE models to data.<sup>2</sup> While previously DSGE models were typically calibrated, in recent years likelihood-based techniques have been used to estimate their parameters. Such estimation exercises typically involve augmenting a (stochastically singular) model with a number of shocks to structural equations to make estimation feasible, even though the motivation for the choice of these shocks is often unspecified. However, the choice of which shock to add is important since each will have a distinct and possibly profound impact on the model properties, affecting both the fit of the model and the story it tells about the data. An important contribution of this paper is that we take a more formal approach to the selection of such shocks. Specifically, following Smets and Wouters (2003), we estimate the DSGE model here using a Bayesian estimation methodology, but in contrast, we estimate the model in two steps. First, we evaluate a calibrated version of the singular model. Here, we adopt the approach developed by Watson (1993), which is based on comparison of model and data spectra, to assess the performance of the model. Examining the ability of the model to capture the properties of the data at different frequencies helps us discover which relationships might benefit most from the addition of structural shocks. Second, we augment the model with structural shocks identified in the first stage and estimate the parameters of the resulting model on UK data using Bayesian maximum likelihood. Finally, we reassess the adequacy of this augmented and estimated model in reconciling the dynamics of the model with the data.

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<sup>1</sup> Among many others, see Kydland and Prescott (1982), Cooley and Hansen (1992), Christiano and Eichenbaum (1995), Smets and Wouters (2003) and Adolfson *et al* (2007).

<sup>2</sup> Among many others, see Altug (1989), Sargent (1989), Watson (1993), Cogley and Nason (1995), King and Watson (1996), Wen (1998), Ireland (2004), Smets and Wouters (2003) and Adolfson *et al* (2007).



We consider a small open economy DSGE model with nominal and real rigidities. The model has four economic agents: households, firms, the government, and a monetary policy maker. Households decide how much to consume (of home and imported goods), how much to invest in the domestic capital stock, and how much to work, and on their holdings of money balances and net foreign assets. Firms employ capital services and labour and choose how much to produce and the price they charge for their products sold at home and abroad. The fiscal and monetary authorities follow simple policy rules. In order to induce a persistent response of real and nominal variables to shocks, the model features a number of real and nominal frictions that have become standard in the literature: sticky prices and wages (with partial indexation of prices and wages that cannot be re-optimised), costs of adjusting the stock and utilisation rate of capital, and external habit formation in consumption.

The model we use can be thought of as a smaller and simpler version of the ‘core’ part of the Bank of England Quarterly Model (BEQM).<sup>3</sup> The smaller scale of the model means that it is incapable of addressing the breadth of issues that can be analysed using BEQM. However, it is sufficiently compact to be estimated on the data using Bayesian methods. Estimation exercises of this type provide useful information about the likely values of the parameters that govern economic behaviour and the causes of business cycle fluctuations.

Turning to the empirical results, the main findings from the evaluation stage are as follows. We find that the dynamics of the UK data are poorly matched by the calibrated baseline model. While some of the deficiencies may be resolved by different choice of parameters, we perform a number of simple experiments suggesting that such changes alone are unlikely to produce a good match between the model and the data. Guided by Watson’s measure of fit and inspection of the coherence functions for model and data, we choose to add four shocks (i) to consumption in the form of a shock to preferences in the utility function, (ii) to investment in the form of a shock to capital adjustment costs, (iii) to inflation in the form of a mark-up shock, and (iv) to hours worked, again in the form of a shock to preferences in the utility function. Next, we estimate the resulting model with Bayesian likelihood over the period 1955 Q1-2007 Q1 using data on seven UK macroeconomic series: output, consumption, investment, total hours worked, real wages, the short-term nominal interest rate, and inflation. Our estimation results suggest that, in

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<sup>3</sup>See Harrison, Nikolov, Quinn, Ramsay, Scott and Thomas (2005) - chapter 3 describes the ‘core’ model. The Bank of England uses many such models, some very abstract and others largely data driven.

combination with moderate real rigidities (habit formation and capital adjustment costs), both price and nominal wage rigidities are important for matching the UK data. While nominal wage and price stickiness are favoured by the data, there is little evidence for strong effects from lagged inflation rates on price and wage-setting (often termed ‘indexation’ in this literature). In common with most papers that estimate similar models using Bayesian likelihood, we find that several shocks – total factor productivity (TFP), government expenditure, the consumption preference shock and the shock to capital adjustment costs – are estimated to be extremely persistent. Our reassessment of the estimated model suggest that the shock processes play a key role in helping the model match the data.

The remainder of this paper is organised as follows. In Section 2, we discuss the linearised DSGE model. In Section 3, we confront the model with UK data. In Section 4 we use the estimated DSGE model to analyse the impulse responses to various shocks. Finally, in Section 5 we conclude.

## **2 The linearised DSGE model**

In this section, we present and discuss the linearised DSGE model that we subsequently evaluate and estimate using UK data. The model is very similar to the DSGE models of Christiano *et al* (2005) and Smets and Wouters (2003), though we extend their closed economy setting to an open economy in a similar way to Adolfson *et al* (2007). As in these models, in order to induce a persistent response of real and nominal variables to shocks, we incorporate a number of real and nominal frictions: sticky prices and wages (with partial indexation of prices and wages that cannot be re-optimised), cost of adjusting stock and utilisation rate of capital, and external habit formation in consumption.

In the model, infinitely lived households have identical preferences over consumption, hours worked, and money balances. However, in the open economy setting here, households consume both domestically produced and imported goods. As in the closed economy, household preferences exhibit external habit formation in consumption. In other words, households gain utility from keeping consumption close to previous levels, as well as higher levels of lifetime consumption. Households own the capital stock in the economy. They rent capital services to the production sector and decide how much capital to accumulate. The total supply of capital



services on the rental market is determined by the capital accumulation and utilisation decision of households.<sup>4</sup> Adjusting the capital stock and the rate at which it is utilised is costly.<sup>5</sup> Households also supply labour services to firms. Following Erceg, Henderson and Levin (2000), we assume that each household supplies a differentiated labour service to the production sector, which implies that they can set their own wage. Monopolistically competitive households set wages in staggered contracts with timing like that in Calvo (1983). Additionally, we assume that wages that are not re-optimised in the Calvo model are partially indexed to past inflation rates. Wage decisions are therefore influenced by past inflation rates as well as current and expected future marginal rates of substitution. In the open economy setting here, households have access to a complete set of state-contingent claims, which insure them at the domestic level against wage income risk, as well as risk-free nominal bonds issued by foreign governments. Their portfolio decision leads to a UIP (uncovered interest rate parity) type relationship which relates the expected exchange rate change to the interest rate differential. We assume that households pay fees to hold foreign bonds.<sup>6</sup>

Firms are composed of producers and bundlers. Each monopolistically competitive producer supplies a single differentiated intermediate good to perfectly competitive bundlers using capital and labour only. In the open economy here, bundlers combine these intermediate goods with imported intermediate goods to produce final consumption goods which then they sell domestically and abroad. Firms engage in local currency pricing (LCP) and invoice exports in foreign currency. We assume that adjusting nominal prices is costly. We introduce Rotemberg (1982) price adjustment costs, so that nominal prices as well as wages are sticky.<sup>7</sup> We augment the Rotemberg price-setting by linking the cost of price changes to past inflation rates: price changes relative to past inflation are also costly to the firm. As a result, price decisions are influenced by past inflation rates as well as current and expected future marginal costs. Nominal price rigidities in both the import and export sectors, coupled with the assumption that firms engage in LCP, implies that exchange rate pass-through in the model is incomplete.

<sup>4</sup>In contrast to Adolfson, Laseen, Linde and Villani (2007) we do not allow for imported and exported final investment goods.

<sup>5</sup>As in Christiano *et al* (2005), we assume that the cost of capital utilisation is zero when capital utilisation rate is one. We model the cost of adjusting capital as a function of the lagged change in capital stock. This specification is similar to that in Christiano *et al* (2005).

<sup>6</sup>This assumption pins down the steady-state level of net foreign assets.

<sup>7</sup>We model price adjustment costs following Rotemberg for its simplicity and tractability, particularly given our assumption that firms set different prices for goods sold domestically and overseas.

The fiscal and monetary authorities follow simple rules. The government runs a balanced budget each period adjusting lump-sum taxes/transfers to ensure that its budget is balanced, and the monetary policy maker sets the short-term nominal interest rate according to a simple (Taylor-type) reaction function. Finally, we assume that the foreign variables in the model follow exogenous stochastic processes.

In the rest of this section, we describe the log-linearised version of the DSGE model with a focus on the key equations. The full set of linearised model equations are shown in Table D in Appendix C.<sup>8</sup> We use standard log-linearisation techniques in order to analyse the dynamics of the model's key equations. Namely, we take a first-order Taylor approximation of the model's equations around the non-stochastic steady state. The hat above a variable denotes its percentage deviation from this steady state. The *ss* in subscript denotes the steady-state level of a variable.

We start with the demand side of the model. The dynamics of consumption follows from the Euler equation and is given by:

$$\hat{c}_t = \frac{1}{1 + \psi^{hab} - \psi^{hab}\sigma^c} E_t \hat{c}_{t+1} + \frac{\psi^{hab} - \psi^{hab}\sigma^c}{1 + \psi^{hab} - \psi^{hab}\sigma^c} \hat{c}_{t-1} - \frac{\sigma^c}{1 + \psi^{hab} - \psi^{hab}\sigma^c} r \hat{r} g_t, \quad (1)$$

where  $E_t$  is the rational expectations operator. With external habit formation ( $0 < \psi^{hab} < 1$ ) today's consumption depends on past as well as expected future consumption, because households derive utility from keeping consumption close to its previous aggregate levels. Additionally, the elasticity of consumption with respect to the real interest rate ( $r \hat{r} g_t$ ) depends on the habit parameter as well as the intertemporal elasticity of substitution ( $\sigma^c > 0$ ). *Ceteris paribus*, a higher (lower) degree of habit persistence will reduce (increase) the interest rate elasticity of consumption for a given elasticity of substitution.

In the open economy setting here, households consume both domestically produced and imported goods. The optimal allocation of expenditure between domestic and imported goods imply:

$$\hat{c}h_t - \hat{c}m_t = -\sigma^m (\hat{p}h_t - \hat{p}m_t). \quad (2)$$

As the equation shows, consumption of domestic goods ( $\hat{c}h_t$ ) relative to imported goods ( $\hat{c}m_t$ ) depends on their relative prices ( $\hat{p}h_t - \hat{p}m_t$ ) as well as the substitution elasticity between them ( $\sigma^m$ ).

<sup>8</sup> An annex with full details of the model and derivation of its equations and steady-state is available at [www.bankofengland.co.uk/publications/workingpapers/wp380tech.pdf](http://www.bankofengland.co.uk/publications/workingpapers/wp380tech.pdf).

The equation that governs the accumulation of capital stock, with adjustment costs, is given by:

$$\begin{aligned}\hat{k}_t - \hat{k}_{t-1} = & \frac{\beta}{1 + \beta\epsilon^k}(E_t\hat{k}_{t+1} - \hat{k}_t) + \frac{\epsilon^k}{1 + \beta\epsilon^k}(\hat{k}_{t-1} - \hat{k}_{t-2}) + \frac{\beta\chi^z}{\chi^k(1 + \beta\epsilon^k)}E_t\hat{r}_{t+1} \\ & + \frac{\beta(1 - \delta)}{\chi^k(1 + \beta\epsilon^k)}E_t\hat{p}h_{t+1} - \frac{1}{\chi^k(1 + \beta\epsilon^k)}\hat{p}h_t - \frac{1}{\chi^k(1 + \beta\epsilon^k)}r\hat{r}g_t,\end{aligned}\quad (3)$$

where  $\beta$  is households' discount factor and  $\delta$  is capital's depreciation rate. Modelling capital adjustment costs as a function of the lagged change in capital stock leads to persistent deviations of the capital stock from its level at the steady state (when  $\epsilon^k \neq 0$ ) as shown above. The equation also shows that capital adjustment costs provide incentives for households to change the capital stock slowly. That is, *ceteris paribus*, a higher cost of capital adjustment parameter,  $\chi^k$ , will reduce the elasticity of the change in capital stock with respect to real interest rate, shadow price of capital ( $\hat{p}h_t$ ), and rental rate of capital ( $\hat{r}_t$ ).

The dynamics of investment are given by:

$$\hat{I}_t = \frac{1}{\delta}(\hat{k}_t - \hat{k}_{t-1}) + \hat{k}_{t-1} + \frac{\chi^z}{\delta}\hat{z}_t. \quad (4)$$

*Ceteris paribus*, a higher utilisation ( $\hat{z}_t$ ) rate depletes the capital stock carried over to the next period and as a result investment must increase to keep capital stock at its equilibrium level. The sensitivity of investment to changes in the utilisation rate is governed by the cost parameter,  $\chi^z$ .

Equilibrium in the goods market means that total domestic output ( $\hat{y}_t^v$ ) should equal total demand, which in the open economy here, consists of domestic consumption ( $\hat{c}h_t$ ), investment ( $\hat{I}_t$ ), exogenous government spending ( $\hat{g}_t$ ), and total export demand ( $\hat{x}_t$ ):

$$\hat{y}_t^v = \frac{\kappa^{hv}ch^{ss}}{y^{v,ss}}\hat{c}h_t + \frac{\kappa^{hv}I^{ss}}{y^{v,ss}}\hat{I}_t + \frac{\kappa^{hv}g^{ss}}{y^{v,ss}}\hat{g}_t + \frac{\kappa^{xv}x^{ss}}{y^{v,ss}}\hat{x}_t, \quad (5)$$

where  $\frac{\kappa^{hv}ch^{ss}}{y^{v,ss}}$  is the steady-state share of domestic consumption in output and equals

$1 - (\frac{\kappa^{hv}I^{ss}}{y^{v,ss}}\hat{I}_t + \frac{\kappa^{hv}g^{ss}}{y^{v,ss}}\hat{g}_t + \frac{\kappa^{xv}x^{ss}}{y^{v,ss}}\hat{x}_t)$ , where the terms in parenthesis are the steady-state shares of investment, government spending, and exports.  $\kappa^{hv}$  and  $\kappa^{xv}$  are parameters and they represent the share of domestic value added in final domestic and export output, respectively. We assume that the exogenous government spending follows a first-order autoregressive process:

$\hat{g}_t = \rho_g\hat{g}_{t-1} + \sigma_g\eta_t^g$ , where  $\eta_t^g$  is an i.i.d. normal error term with zero mean and unit variance.

Total export demand is given by:

$$\hat{x}_t = \hat{c}f_t - \sigma^x(\hat{q}_t - p\hat{x}f_t) + p\hat{x}_t = \hat{y}_t^x. \quad (6)$$

Domestic exports increase with an increase world demand ( $\hat{c}f_t$ ) but they decrease with an appreciation in the real exchange rate (a fall in  $\hat{q}_t$ , which is measured as the foreign price of domestic output), with relatively less expensive world exports ( $p\hat{x}f_t$ ), and with relatively more expensive domestic exports ( $p\hat{x}_t$ ). The elasticity of demand for domestic exports is given by  $\sigma^x > 1$ . And the equilibrium in the goods market requires that the demand for exports equals their supply ( $\hat{y}_t^x$ ).

The dynamics of the real exchange rate are described by:

$$\hat{q}_{t+1} - \hat{q}_t + \chi^{bf} n\hat{f}a_t = r\hat{r}f_t - r\hat{r}g_t = (r\hat{f}_t - r\hat{g}_t) - \frac{1}{1-\beta}[(\dot{p}_{t+1}^f - \dot{p}^{f,ss}) - (\dot{p}_{t+1} - \dot{p}^{ss})], \quad (7)$$

where  $r\hat{r}g_t$  and  $r\hat{r}f_t$  are the real interest rates at home and the rest of the world. The modified version of the real UIP condition says that, for agents to be indifferent between domestic and foreign bonds, the real interest rate differentials must be equal to the real depreciation allowing for an adjustment cost of adjusting (net) foreign asset holdings ( $n\hat{f}a_t$ ).<sup>9</sup> The second equation in the above expression merely restates real UIP condition in terms of nominal interest rates and inflation rates at home and abroad using the well-known Fisher parity condition.

We specify a VAR (vector autoregressive) process for the foreign variables in the model.

Specifically, we estimate a VAR in world demand ( $\hat{c}f_t$ ), world nominal interest rates ( $r\hat{f}_t$ ), world inflation ( $\dot{p}_t^f$ ) and world relative export prices ( $p\hat{x}f_t$ ). The sample period here is 1955 Q1–2007 Q1 and we select a lag length of four using the Akaike information criterion. These data are described in Appendix B.

We now turn to the supply side. As noted above, firms are composed of producers and bundlers. Each monopolistically competitive producer supplies a single differentiated intermediate good to perfectly competitive bundlers for the production of final domestic and export goods. The aggregate production function of the producer is given by:

$$\hat{y}_t^v = t\hat{f}p_t + \varphi^h \hat{h}_t + (1 - \varphi^h) \hat{k}_t^s. \quad (8)$$

Output is produced using labour services ( $\hat{h}_t$ ) and capital services ( $\hat{k}_t^s$ ), where the parameter  $\varphi^h$  is the elasticity of output with respect to labour and depends negatively on the substitution elasticity

<sup>9</sup>When the cost parameter is zero, this equation collapses to the standard real UIP condition.

between labour and capital and ( $\sigma^y$ ) and positively on the share of labour in production ( $1 - \alpha$ ).<sup>10</sup>

We assume that total factor productivity follows a first-order autoregressive process:

$t\hat{f}p_t = \rho_{tfp}t\hat{f}p_{t-1} + \sigma_{tfp}\eta_t^{tfp}$ , where  $\eta_t^{tfp}$  is an i.i.d. normal error term with zero mean and unit variance.

In the open economy here, perfectly competitive bundlers combine these domestic intermediate goods with imported intermediate goods, using a Leontief technology, to produce final consumption goods, which then they sell domestically and abroad. The equilibrium in the goods market is given by:

$$\hat{y}_t^v = \hat{y}_t^{hv} + \hat{y}_t^{xv} = \hat{m}_t^h + \hat{m}_t^x = \hat{y}_t^h + \hat{y}_t^x. \quad (9)$$

where  $\hat{y}_t^{hv}$  ( $\hat{y}_t^{xv}$ ) and  $\hat{m}_t^h$  ( $\hat{m}_t^x$ ) denote the domestic and imported intermediate good used in the production of domestic (exported) final consumption goods,  $\hat{y}_t^h$  ( $\hat{y}_t^x$ ).

As the capital used in the current period must be installed in the previous period, the current capital services depend on the rate of utilisation and accumulated capital:

$$\hat{k}_t^s = \hat{z}_t + \hat{k}_{t-1}. \quad (10)$$

The optimal utilisation rate of capital is governed by:

$$\hat{z}_t = \frac{1}{\sigma^z}(\hat{r}_t - \hat{p}_t^h). \quad (11)$$

The utilisation rate increases with the rental rate of capital services as it becomes more profitable to use the existing capital stock more intensively but decreases with the price of investment goods as it becomes more costly to replace depleted capital. And the sensitivity of utilisation is governed by the cost parameter,  $0 < \sigma^z < 1$ .

Turning to the monopolistically competitive producers, the dynamics of the producer price inflation in the domestic intermediate goods sector is given by the following New Keynesian Phillips Curve:<sup>11</sup>

$$\dot{p}_t^{hv} - \dot{p}^{ss} = \frac{\sigma^{hb} - 1}{\chi^{hv}(1 + \beta\epsilon^{hv})}r\hat{m}c_t^{hv} + \frac{\beta}{1 + \beta\epsilon^{hv}}(E_t\dot{p}_{t+1}^{hv} - \dot{p}_t^{ss}) + \frac{\epsilon^{hv}}{1 + \beta\epsilon^{hv}}(\dot{p}_{t-1}^{hv} - \dot{p}_t^{ss}), \quad (12)$$

where  $r\hat{m}c_t^{hv} = \hat{m}c_t - \hat{p}_t^{hv}$  and  $\dot{p}_t^{hv} - \dot{p}^{ss}$  are the real marginal cost (or the negative of the price mark-up,  $\hat{\psi}_t^{hv}$  shown in Table D) and the domestic producer price inflation, respectively. Current

<sup>10</sup>A full expression for the parameter  $\varphi^h$  appears in Table D.

<sup>11</sup>A similar equation for the producer price inflation in the exported intermediate goods sector is given in Table D.

inflation tends to rise when real marginal costs rise (as firms pass on higher costs in the form of price increases) or when expected inflation rises (as firms raise their price today anticipating future), or when past inflation rises (since producers use past inflation as a reference when setting current prices). The sensitivity of today's inflation to these variables will depend on the parameters that govern rigidities we have introduced into price-setting as well as the elasticity of substitution among domestic goods ( $\sigma^{hb} > 1$ ). For instance, when  $0 < \epsilon^{hv} < 1$ , the parameter that governs the degree of indexation to past inflation, is zero this equation reduces to a purely forward-looking Phillips curve, where past inflation does not influence current price-setting. And firms will tend to increase (decrease) prices less in face of higher (lower) costs if  $\chi^{hv}$  is large.

The dynamics of the price inflation in the imported intermediate goods sector is given by the following New Keynesian Phillips Curve:

$$\dot{p}_t^m - \dot{p}_t^{ss} = \frac{[1 - \beta(1 - \psi^{pm})]\psi^{pm}}{(1 - \psi^{pm})(1 + \beta\epsilon^m)} r\hat{m}c_t^f + \frac{\beta}{1 + \beta\epsilon^m} (E_t \dot{p}_{t+1}^m - \dot{p}_t^{ss}) + \frac{\epsilon^m}{1 + \beta\epsilon^m} (\dot{p}_{t-1}^m - \dot{p}_t^{ss}), \quad (13)$$

where  $r\hat{m}c_t^f = p\hat{x}f_t - \hat{q}_t - p\hat{m}_t$  is the real marginal cost of production in the rest of the world. Notice that the first term on the right-hand side is different to that appears in equation (12). This is due to the way we introduce rigidities into price-setting in these markets. In particular, we assume that producer prices are subject to Rotemberg-type price adjustment costs whereas import prices are set in Calvo-type staggered contracts. But, despite the specification of price rigidity, the relationship between import price inflation and real marginal costs remains. That is, import price inflation tends to rise when real marginal costs rise and the sensitivity of inflation is governed by the parameter,  $0 < \psi^{pm} < 1$ , which governs import price rigidity. That is, import prices will tend to rise by less (more) in face of higher marginal costs if the parameter  $\psi^{pm}$  is lower (higher), ie if a smaller (higher) fraction of producers are able to optimally set their prices in any given period.

The dynamics of inflation in the final domestic goods sector will be a weighted average of producer price and import price inflation as shown below:<sup>12</sup>

$$\hat{p}h_t = \kappa^{hv} \frac{ph^{vss}}{ph^{ss}} \hat{p}h^v_t + (1 - \kappa^{hv}) \frac{pm^{ss}}{ph^{ss}} \hat{p}m_t. \quad (14)$$

Turning to the labour market, the rigidities in wage-setting, as in Calvo (1983), combined with partial indexation of those wages that cannot be re-optimised to lagged wage inflation lead to the

<sup>12</sup>A similar equation for inflation in the exported final goods sector is given in Table D.

following dynamics of real wage inflation:

$$\begin{aligned}\dot{w}_t - \dot{p}^{ss} = & -\frac{\psi^w[1 - \beta(1 - \psi^w)]}{(1 - \psi^w)(1 + \beta\epsilon^w)}\left(1 + \frac{\sigma^w}{\sigma^h}\right)^{-1}(\hat{w}_t - \hat{m}\hat{r}s_t) \\ & + \frac{\beta}{(1 + \beta\epsilon^w)}(E_t\dot{w}_{t+1} - \dot{p}^{ss}) + \frac{\epsilon^w}{(1 + \beta\epsilon^w)}(\dot{w}_{t-1} - \dot{p}^{ss})\end{aligned}\quad (15)$$

Analogous to price inflation, the above expression describes a relationship between current nominal wage inflation ( $\dot{w}_t$ ), past and expected future wage inflation, and the gap between the real wage rate and the level that would prevail if wages were flexible. According to this expression, current wage inflation will tend to rise when the real wage falls relative to the marginal rate of substitution of consumption for leisure ( $\hat{m}\hat{r}s_t$ ). The lower the fraction of wages being reset, or the smaller the parameter  $0 < \psi^w < 1$ , the smaller the increase in current wage inflation. Moreover, the easier it is for firms to substitute between labour services (or the higher  $\sigma^w > 1$ ), and the more willing households to smooth labour hours (or the smaller  $\sigma^h > 0$ ) the smaller the increase in today's wage inflation.

Finally, the monetary policy reaction function is given by:

$$\hat{r}g_t = \theta^{rg}\hat{r}g_{t-1} + (1 - \theta^{rg})[\theta^p(\dot{p}_t - \dot{p}^{ss}) + \theta^y(\hat{y}_t^v - t\hat{f}p_t)] + \hat{m}p_t. \quad (16)$$

We assume that monetary policy is conducted through changes in the nominal interest rate. The monetary policy reaction function is a ‘Taylor rule’ with smoothing. The monetary authority responds to the deviation of inflation from its level at the steady state ( $\dot{p}_t - \dot{p}^{ss}$ ) and the output gap ( $\hat{y}_t^v - t\hat{f}p_t$ ). The parameter  $\theta^{rg}$  captures the degree of interest rate smoothing. We assume that the monetary policy shock is white noise:  $\hat{m}p_t = \sigma_{mp}\eta_t^{mp}$ , where  $\eta^{mp}$  is an i.i.d. normal error term with zero mean and unit variance.

All in all, 39 log-linearised equations shown in Table D determine the model's 39 endogenous variables. The system of linear equations is driven by seven exogenous shocks: total factor productivity ( $t\hat{f}p_t$ ), government spending ( $\hat{g}_t$ ), monetary policy ( $\hat{m}p_t$ ), world demand ( $\hat{c}f_t$ ), world nominal interest rates ( $\hat{r}f_t$ ), world inflation ( $\dot{p}_t^f$ ), and world relative export prices ( $p\hat{x}f_t$ ).

### 3 Taking the model to UK data

In this section, we confront the model described in Section 2 with UK data. After discussing the calibration of model parameters in Section 3.1 and the data in Section 3.2, we proceed in three steps. First, we evaluate a calibrated version of the model in Section 3.3. Here we adopt the method developed by Watson (1993) to assess the model's performance in replicating the key features of UK data. This analysis guides the choice of several key 'structural shocks' that are incorporated into the model in Section 3.3.2. Then the augmented model is estimated by Bayesian maximum likelihood in Section 3.4 and, finally, re-evaluated in Section 3.5.

#### 3.1 *Model parameters*

Before evaluating the log-linearised model, we need to set values for the model parameters. Our strategy for doing so is to split the parameters into two groups. The first group are parameters that are most important in determining the steady state of the model with little or no influence over its dynamic properties. In contrast, the second group are parameters that predominantly influence the dynamic behaviour of the model with little or no effect on its steady state.

We will hold the parameters in the first group fixed in the evaluation and estimation phases.<sup>13</sup> This choice is based on the observation that these parameters are more important in matching the first moments of the data rather than the dynamics, which is the focus of this paper. Thus, for this group of parameters, in most cases, we choose values that match the first moments of the model to those of the data. Where suitable data is not readily available, we use results from previous studies of UK data. Some parameters in this group are used to normalise prices and quantities in the model (which have no natural units). Some others reflect our assumptions and judgement. In Appendix A, we document the values chosen for the parameters in this group and the motivation for our choice. As for the parameters in the second group, we will experiment with different values for some of them when we evaluate the model in Section 3.3 before estimating them in Section 3.4 using Bayesian methods, where we will use the values documented in Appendix A to inform our prior distributions. For this reason, we prefer not to use estimates from UK studies to inform the parameter values in this group. Instead, we make use of a range of estimates for the

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<sup>13</sup> Many previous studies have fixed a subset of model parameters in a similar way, for instance see Smets and Wouters (2003), though the precise set that is fixed varies across the studies.



United States and the euro area using models with very similar structures to our own. We weight these estimates together to form the baseline parameter setting for our model. Our weighting scheme is mechanical and reflects a ‘meta prior’ that has two components. First, we attach more weight to parameter estimates from studies of the US economy since we believe that the UK economy is more similar to the US economy. And second, we attach more weight to parameter estimates from studies that use Bayesian maximum likelihood than on studies that match the model’s impulse responses to those from an estimated VAR. This reflects our view that Bayesian maximum likelihood makes use of the additional information in the data reflecting the economy’s response to a wider set of shocks. Of course, both aspects of our ‘meta prior’ are open to debate and the weighting scheme we use is to some extent arbitrary. The parameter values generated by this approach are documented in Appendix A.

### **3.2 Data**

We use data over the period 1955 Q1-2007 Q1 on seven UK macroeconomic variables: consumption, total hours worked, investment, inflation, nominal interest rates, real wages, and output. We convert all real variables to a per capita basis by dividing by an estimate of the working-age population prior to detrending. To detrend, we estimate a log-linear time trend (including a constant) for all real variables. For interest rates and inflation we divide the sample into three ‘regimes’ based on the analysis of Benati (2006). These regimes are 1955 Q1-1972 Q2, 1972 Q3-1992 Q4, and 1993 Q1 onwards. For each subsample, we take the mean of the series as a measure of ‘trend’. Our data is detrended so that the series we use are comparable with the variables that appear in the log-linearised equations of the model. The construction of these seven variables is discussed in more detail in Appendix B. The choice of the data set is driven by several considerations. First, we will estimate a VAR model on our data when evaluating the model in Sections 3.3 and 3.5, and hence, it is desirable to limit the total number of variables to avoid losing too many degrees of freedom. Second, the limited set of variables chosen should reflect the key macroeconomic series that the model seeks to match. And third, the set of variables should be rich enough to allow the identification of the important parameters in the estimation phase. Data for consumption, hours, investment and output are obvious candidates for models that are based on a real business cycle structure such as ours. Indeed, these variables are used by Watson (1993) and Wen (1998) in their studies of real business cycle (RBC) models using US data. The fact that the model considered here also includes nominal variables (and

nominal rigidities) motivates the inclusion of inflation and the nominal interest rate in our data set. Finally, the real wage is included because we also model nominal wage stickiness. The variables in our UK data set are therefore very similar to those used by Smets and Wouters (2003) in their estimation exercise using euro area data.

Additionally, we use data over the same period on four foreign processes: world demand, world nominal interest rates, world inflation, and world relative export prices. We construct world variables as averages of individual country data in G7 countries (excluding the United Kingdom). These data are detrended in the same way as the UK data. The construction of these foreign variables is discussed in detail in Appendix B.

### 3.3 *Evaluating the model*

In this section, we evaluate the fit of the calibrated version of the log-linearised model to UK data. The log-linearised model is driven by seven exogenous shocks: total factor productivity, government spending, monetary policy, world demand, world nominal interest rates, world inflation, and world relative export prices. When we estimate the model in Section 3.4, we will use a total of eleven data series: seven UK data series and four world data series. But unless we confine attention to at most seven data series, the model is ‘stochastically singular’.<sup>14</sup>

A variety of approaches have been proposed to address the singularity problem inherent in DSGE models. We choose to follow recently popular practice and augment the model with ‘structural shocks’.<sup>15</sup> This approach adds sufficient number of shocks to structural equations of the model in order to make maximum likelihood estimation possible. But adding such shocks potentially changes the behaviour of the model. Further, estimation exercises that follow this approach often find that the structural shocks explain a large proportion of the variation in key variables. This implies that the choice of exactly which shocks to add to the model could have a significant effect on the way it explains movements in the data, though to our knowledge this issue has not been

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<sup>14</sup>This is because there are only seven stochastic shocks that drive all of the variation in the model variables. This means that for any group of eight or more variables, there will exist linear combinations that are deterministic. And unless these exact relationships are replicated in the data, any attempt to estimate the model using maximum likelihood techniques will fail. Ingram *et al* (1994) provides a nice account of the stochastic singularity problem.

<sup>15</sup>Other approaches include estimating the model using as many observable variables as there are structural shocks as in Bouakez *et al* (2001) and Ireland (2001) or appending measurement errors to the observation equation of the state-space representation as in Sargent (1989) and Ireland (2004). Alvarez-Lois *et al* (2008) compares some of the approaches.

extensively investigated in previous studies.<sup>16</sup>

Since we will estimate the model using data on a total of eleven data series we need to add at least four structural shocks. One way to choose the shocks would be to include a large number of them and allow the estimation procedure to select the most important shocks. But doing so is likely to increase the identification problems inherent in the estimation of DSGE models. Our approach is to choose the structural shocks that help to address the most serious deficiencies in the model's ability to fit the data. To do so requires a technique that permits comparison of a stochastically singular model with the data. We use the approach developed by Watson (1993) and outlined below. There are other approaches that work with stochastically singular models,<sup>17</sup> but we prefer Watson's approach for two reasons. First, it encourages close inspection of the information contained in the spectral density matrices of the model and the data which helps to guide the process of model evaluation. Indeed Watson (1993, page 1,038) notes that '...one of the most informative diagnostics...is the plot of the model and data spectra... Some practical advice, therefore, is to present both model and data spectra as a convenient way of comparing their complete set of second moments.' Second, this approach generates a number of statistics that can be used to provide useful summaries of this information, aiding the communication of results.

### 3.3.1 Methodology

Watson (1993) develops a measure of fit that is based on the size of the stochastic error required to reconcile the autocovariances of the model,  $x_t$ , with those of the data,  $y_t$ .<sup>18</sup> The approach therefore focuses on the properties of the error process,  $u_t$ , defined such that the autocovariances of the model plus the error,  $(x_t + u_t)$ , match exactly the autocovariances of the data,  $y_t$ . Typically, researchers place some restrictions on the correlations between the model, data, and the error process. For example, assuming that the error process is uncorrelated with the model corresponds to the assumption that  $u_t$  represents measurement error. Or, assuming that the error process is uncorrelated with the data corresponds to the assumption that  $u_t$  represents forecast or signal extraction error. Watson, on the other hand, derives a *lower bound* on the variance of  $u_t$  without imposing any such restrictions. As noted by Watson (1993),  $u_t$  represents the

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<sup>16</sup>Alvarez-Lois *et al* (2008) argues that this issue may impede the usefulness of DSGE models in policy institutions.

<sup>17</sup>See, for example, Cogley and Nason (1995) and Bierens (2007).

<sup>18</sup>Our discussion of the approach draws on the excellent summary by Wen (1998).

‘approximation or abstraction error’ in the model compared with the data. And the lower bound on the variance of  $u_t$  is found by letting the covariance (or the correlation) between the model and the data to be maximised.<sup>19</sup> Watson uses the *lower bound* for the variance of  $u_t$  to construct an upper bound for the statistic that summarises the fit of the economic model.

To fix ideas, consider the simplest case, where  $y_t$  and  $x_t$  are univariate and uncorrelated. Then, Watson’s fit statistic  $\mathcal{W}$  is given by

$$\mathcal{W} = 1 - \frac{\sigma_{u*}^2}{\sigma_y^2},$$

where  $\sigma_{u*}^2$  is the minimum error variance. If the lower bound for the error variance,  $\sigma_{u*}^2$ , is found to be large, or in other words, if the upper bound for the fit statistic,  $\mathcal{W}$ , is found to be small, then this provides evidence that the model fits the data poorly. But, as stressed by Watson (1993), the converse is not necessarily true because the procedure identifies a lower bound on the variance of  $u_t$ . Thus, finding that the minimum error variance is small, merely implies that there are possible assumptions about the correlations between the model and the data for which the model fits the data well. As the equation above shows, the fit statistic  $\mathcal{W}$  is similar to  $R^2$  in a standard regression in that it is maximised at 1 (when the minimum error variance is zero). But it can also be negative (when the error variance exceeds that of data).

For the most general case, where  $y_t$ ,  $x_t$ , and hence  $u_t$  are multivariate and serially correlated, the method operates in the frequency domain, where spectral density matrices replace covariance matrices. Then, the maximisation problem to compute the lower bound, and hence the fit measure, can be solved independently for each frequency. This means that the fit measure can be computed by treating the spectral density matrices in the same way as covariance matrices of serially uncorrelated variables. The overall fit measure is then calculated by integrating across frequencies of interest.

One critique of Watson’s approach is that the fit measure is not invariant to the filter applied to the data and model outputs.<sup>20</sup> In our case this is of little importance since we use the same detrending method for the data in both the evaluation and estimation phases. To implement Watson’s approach we need to characterise the spectral density matrices of the UK data and the

<sup>19</sup>Watson (1993) demonstrates that, to solve the maximisation problem, a singular value decomposition can be used to deal with the fact that the covariance matrix of a stochastically singular model will not be of full rank. When all variables are not equally important, a weighting matrix can be used to focus attention on specific variables of interest.

<sup>20</sup>For example, Cogley and Nason (1995) note that Watson’s overall goodness of fit statistic is not invariant to linear filtering.

model described in Section 2. We discuss each in turn.

### 3.3.2 *Model and data spectra*

In this section, we study the behaviour of the model and data spectra across frequencies and variables.

*Estimating model and data spectra.* To estimate the data spectrum, we first estimate a VAR model on our data set. We choose the lag length of the VAR to be two using the Akaike information criterion. We then use the estimated VAR parameters (coefficients and the variance) to compute our estimate of the data spectra analytically. To assess the sampling uncertainty, the VAR coefficients are bootstrapped 10,000 times to construct confidence intervals for the estimated data spectra and the fit statistics based on them. The spectral density matrices for the DSGE model are computed in the same way, where we first solve for the state-space representation of the rational expectations equilibrium of the model. We compute the model spectra exactly since it has no sampling uncertainty.

*Studying model and data spectra.* Chart 2 in Appendix C plots the (base 10) logarithms of the baseline DSGE model and the estimated data spectra. The grey bands denote the 95% confidence interval of the data spectra and the solid green line is the model spectra. The X-axis denotes the periodicity of the cycle (ie the time period it takes for a cycle to repeat itself), which is equal to  $2\pi/\omega$ , where  $\omega$  is the frequency of the cycle (ie how many times a cycle repeats itself in a given period of time). Cycles that last 4-7 quarters coincide with the typical range for monetary policy. We label the cyclical movements within the 2-8 quarter periodicity range as *high frequency* and those within the 8-256 quarter range as *low frequency*. The dominant structure of the data spectrum for the variables in our analysis is that they generally decrease noticeably as frequency increases, with most of the power concentrated at low frequencies. This shape is known in the literature as the ‘typical spectral shape’ of an economic variable after Granger (1966). The downward-sloping spectrum is indicative of a correlation structure that is dominated by high, positive, and low-order serial correlation (as in the case of a first-order autoregressive process). However, the spectrum of inflation does not seem to display the typical spectral shape in that it is somewhat flatter. When compared to model spectra a number of points emerge. First, the variability in the model is generally low as the model spectra mostly lie below the data spectra.

This is especially true for investment, inflation, and output. Second, the slope of model spectra differs from the data spectra implying that the model cannot capture the persistence in the data. This is mostly true for hours worked and real wage. One possible explanation for the poor fit of the model is that the values of the parameters governing rigidities are not correctly calibrated. To investigate this possibility we consider two alternative model specifications alongside the baseline model: a *flexible price* specification (here onwards the FP model), where we relax nominal (price and wage) rigidities<sup>21</sup> and a *RBC* specification (here onwards the RBC model), where we additionally relax real rigidities<sup>22</sup> in the baseline DSGE model.<sup>23</sup> In Chart 2 we also show the spectra for these alternative models: the dotted blue line is the FP model and the dashed red line is the RBC model. As the chart shows, relaxing rigidities indeed increase the variability for some variables, bringing model and data spectra closer. This is especially true for output and consumption. But, for investment, the RBC model implies too much and the FP model implies too little variation relative to data. The alternative model specifications do not lead to an overall improvement in the slope of the spectra either. For hours worked alternative model specifications make virtually no difference and for inflation they perform even more poorly.

*Results from Watson's fit statistic.* Table E in Appendix C provides statistics that summarise the fit of the model to data. These statistics are derived based on Watson's methodology discussed in Section 3.3.1 and integrated across the whole frequency range to give an overall fit statistic ( $\mathcal{W}$ ). In Panel A, we present results for the baseline model and in Panels B and C we present results for the FP and RBC models. In each panel the first row presents results for  $\mathcal{W}$  under the equal weight scheme, where all variables receive equal weight. Rows 2-8, on the other hand, show results for  $\mathcal{W}$  when each individual variable receives all the weight. Watson's fit statistic are broadly in line with our conclusions above. For consumption and output we see that  $\mathcal{W}$  is among the highest where also the spectra of the data and the model come closest.  $\mathcal{W}$  is particularly low for hours worked, inflation, investment, and the nominal interest rate in line with the evidence in Chart 2. When we consider the alternative model specifications,  $\mathcal{W}$  improves for consumption, investment, and output. Improvements in  $\mathcal{W}$  are more dramatic when these variables are weighted individually. The inflation and real wage series, however, do worse under the alternative

<sup>21</sup>That is we set  $\psi^w = \psi^{pm} = 1$  and  $\chi^{hv} = \chi^{xv} = 0$ .

<sup>22</sup>That is we set  $z = 1$ ,  $\chi^k = 0$ , and  $\psi^{hab} = 0$ .

<sup>23</sup>We should note that, as part of sensitivity analysis, we have considered alternative values for many of the calibrated parameters in the model including those that govern the autocorrelations and standard deviations of the exogenous shock processes. However, we found that none of these experiments materially improved the overall fit of the model.

model specifications under the equal weighing scheme. Improvements in the nominal interest rate and hours worked are modest under the alternative models even when we consider the case where they receive all the weight. Overall, our results indicate that some model deficiencies can be resolved by different choice of parameters but that significant discrepancies remain.

*Coherence measures.* The analysis above has concentrated on the ability of the models to explain the variability in consumption, output, investment, real wage, hours worked, nominal interest rate, and inflation across the whole frequency range. Examining the coherence function can provide additional useful information.<sup>24</sup> Chart 3 in Appendix C shows the estimated coherence functions for output for data and the models. Grey bands denote the 95% confidence interval of the output coherence in the data. The output coherence for the baseline model is shown by the solid green line, for the FP model by the dashed blue line, and for the RBC model by the dotted red line. Several points emerge. First, in the data, pairwise coherence is highest at low frequency, where most of the variables discussed above have their spectral peak. Second, in the baseline model, coherence among variables is rather low at low frequencies but does broadly mimic pattern in data at high frequencies with the exception of nominal interest rate. The coherence between output and the nominal interest rate in the baseline model is highest at high frequencies. This is because fluctuations in the nominal interest rate and output over this frequency band are predominantly driven by the serially uncorrelated monetary policy shock reflecting the specification of the monetary policy rule in the model. Third, in contrast to findings from previous discussion, the alternative models generally perform worse than the baseline case and the deterioration compared to baseline case is most pronounced at high frequencies. When we relax rigidities the coherence measures generally become flatter and stronger perhaps suggesting a weakening in the propagation mechanisms of the model. For instance, for hours worked and investment, the coherence measure increases dramatically and comes close to unity (much higher than suggested by the data) under the FP and RBC model specifications. This suggests that output and hours worked as well as output and investment are predominantly driven by a common shock that seems to govern their dynamics once we remove rigidities. Hence, an independent shock to hours worked and investment might prove useful. Likewise for consumption and inflation.

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<sup>24</sup>The coherence between the two series lies in the range  $(0, 1)$  and gives a measure of the degree of correlation between the series at frequency  $\omega$ . A strong (weak) correlation is indicated by a coherence measure close to unity (zero). For instance, if two processes are driven by independent (perfectly correlated) shocks, then the coherence measure between them will be zero (one) at any given frequency indicating that the processes move independently (together).



*Structural shocks.* The evaluation stage reveals important information on how well the model captures the key properties of the data: the dynamics of the UK data are poorly represented by the calibrated baseline model. And while some of the model deficiencies may be resolved by different choice of parameters, discrepancies remain. Enriching the model with additional dynamics (in the form of structural shocks) to better capture data properties might prove useful. But which structural shocks to add? Our analysis above demonstrates that the rigidities in the model (eg habit formation, capital adjustment costs, price and wage stickiness) help to reduce the coherence of many variables with output at high frequencies by preventing all variables from moving together in response to the small number of common shocks. But the rigidities also tend to reduce the coherence with output at low frequencies, which is at odds with the data. In contrast, calibrations of the model with the rigidities switched off tend to produce more variation in the endogenous variables (which is more in line with the data) at the expense of stronger coherence with output at high frequencies.

Ideally, therefore, we wish to incorporate more variability in the model and increase low frequency coherence with output, while retaining the low coherence at high frequencies generated by the model's rigidities. To do so, we choose shocks that affect a relatively small number of equilibrium conditions in the model and that are likely to affect the spectra and coherence of a small number of variables in a predictable manner. Before providing details of how the shocks are added to the model, we first summarise the motivation for including them.

To influence the behaviour of consumption, we add a *preference shock*, in the form of a shock to the utility function. This shock will then appear in the Euler equation that relates consumption decisions to the real interest rate.<sup>25</sup> This shock has the potential to increase the coherence of consumption and output at low frequencies because consumption is a large fraction of output. So if the preference shock can explain a large fraction of low frequency consumption movements, the coherence with output is likely to increase. Precisely the same logic applies to the addition of a *shock to capital adjustment costs*. This shock appears in the first-order condition for capital and therefore has a direct effect on investment decisions.

To influence the behaviour of inflation, we insert a *mark-up shock* to the demand elasticity of goods sold domestically. This shock allows the behaviour of nominal interest rates and inflation

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<sup>25</sup>This shock also appears in the wage Phillips curve.



to become less correlated at lower frequencies. While the nominal rigidities in the model allow inflation and nominal interest rates to move in opposite directions in the short run (for example in response to a monetary policy shock), these nominal rigidities are relatively short-lived. So in the long run, the model behaves rather like a flexible price model. Including an additional source of inflation variation helps to reduce this tight link between nominal variables at lower frequencies. Finally, we add a *labour supply shock* in the form of a shock to the disutility from working in the utility function. This shock enters the wage Phillips curve and creates another source of variation in the labour market (which could manifest itself in terms of higher variability in either prices or quantities). Since both real wages and hours worked are insufficiently variable at low frequencies under the baseline calibration, this shock could create a source of variation that reduces this discrepancy with the data.<sup>26</sup>

Thus, guided by the Watson's measure of fit and the inspection of coherence functions for model and data, we choose to incorporate four shocks (i) to *consumption* in the form of a shock to preferences in the utility function, (ii) to investment in the form of a *shock to capital adjustment costs*, (iii) to prices in the form of a *mark-up shock*, and (iv) to *hours worked*, again in the form of a shock to preferences in the utility function.<sup>27</sup> We will assess the impact of these shocks on the model's fit in Section 3.5.

*Augmenting the model with structural shocks.* Incorporating the structural shocks discussed above will modify the equations of the model described in Section 2. We show how these shocks alter the model's equations in Table F in Appendix C. We assume that all shocks follow a first-order autoregressive process with an i.i.d. normal error term with zero mean and unit variance, that is  $\hat{\zeta}_t^i = \rho_i \hat{\zeta}_{t-1}^i + \sigma_i (1 - \rho_i^2)^{\frac{1}{2}} \eta_t^i$ , where  $i = con, hw, inf, inv$ , denotes shocks to consumption, hours worked (labour supply), inflation (price mark-up), and investment (capital adjustment costs), respectively. A positive *consumption shock* will initially increase today's consumption since households obtain more utility from a unit consumption. The impact depends on the intertemporal elasticity of substitution and the degree of habit formation (see first equation in Panel B of Table F). Further, a positive consumption shock will decrease the marginal rate of

<sup>26</sup>The choice of shocks, although guided by the inspection of spectral densities and coherence functions, still remains a subjective exercise. An alternative, suggested by the anonymous referee, would be to first estimate the model with shocks to all variables we would like to fit to data (ie consumption, hours worked, investment, inflation, nominal interest rate, and output) and then analyse how the spectra and coherence functions of the variables change as we eliminate each shock.

<sup>27</sup>These shocks have also been considered in Smets and Wouters (2003).

substitution (second equation in Panel B of Table F), so it will lead to a fall in current wage inflation (see equation (15)). Likewise, a positive *labour supply* shock will decrease the marginal rate of substitution and current wage inflation since households incur less disutility from supplying labour. A negative price *markup-shock* tends to increase the producer price inflation in the intermediate goods sector (see third equation in Panel B). In this sense, it can be thought of as a ‘cost-push’ shock, the effects of which will work through to consumer price inflation. Finally, a negative shock to *the cost of adjusting capital*, will temporarily increase investment (see fourth equation in Panel B and equation (4)).

### 3.4 Estimating the augmented model

In this section, we estimate the log-linearised model described in Section 2 using Bayesian maximum likelihood. We outline the estimation approach in Section 3.4.1 before discussing our priors in Section 3.4.2. Section 3.4.3 presents the results of the estimation.

#### 3.4.1 Estimation approach

We estimate the parameters listed in Table B in Appendix A together with the parameters governing the structural shocks added in Section 3.3.2 by Bayesian maximum likelihood. Since this approach has become commonplace in the estimation of medium-scale DSGE models, our discussion of the methodology is brief. Interested readers are referred to Smets and Wouters (2003) and An and Schorfheide (2007) for relevant discussions.

We denote the vector of parameters to be estimated as  $\theta$ . Estimation of these parameters by Bayesian maximum likelihood proceeds in two steps. First, we specify prior distributions for the parameters. Then we combine this prior information with the likelihood of the model and characterise the posterior distribution. From Bayes’s rule the posterior distribution satisfies:

$$p(\theta|Y_T) \propto p(\theta) p(Y_T|\theta), \quad (17)$$

where  $p(\theta)$  represents the prior distribution for the parameters and  $p(Y_T|\theta)$  is the likelihood of the model. That is, we use  $Y_T$  to denote the set of eleven macroeconomic time series described in Appendix B. The modal parameter vector  $\hat{\theta}$  can be found by numerically maximising the right-hand side of (17). However, to approximate the posterior distribution, we use Markov Chain Monte Carlo methods. Specifically, we used the Metropolis-Hastings algorithm to

generate 250,000 parameter observations on which to base inference.<sup>28</sup>

### 3.4.2 Priors

Our prior distributions are summarised in Table G in Appendix C. For the parameters listed in Table B, we use the average of the estimates reported in that table as the means of our prior distributions. As discussed in Section 3.1, these estimates are obtained from a range of studies for the United States and the euro area using models with very similar structures to our own. The tightness of the priors is determined by considering the range of estimates reported in these studies.<sup>29</sup> The cross-sectional variation in estimates of similar parameters in similar models using data from similar economies is clearly not the same as a prior about the distribution of a particular parameter in the model. Nevertheless, we do find this information useful as it allows us to discern the extent to which previous estimation exercises provide strong evidence about the likely values of parameters in the model. This approach leads us to set relatively tight priors for the parameters governing habit formation, the interest rate smoothing parameter in the monetary policy rule and the persistence of TFP and government spending shocks.<sup>30</sup> We set relatively loose priors for the other parameters, particularly those governing the monetary policy response inflation and price rigidities, where there is little consensus about both the absolute size of these parameters and their relative importance. For the parameters governing the structural shocks added to the model in Section 3.3.2, we set very loose priors as we have little information about the likely properties of these shocks. We use uniform distributions to describe our priors about the persistence parameters of the shock processes and inverted Gamma distributions with two degrees of freedom for the standard deviations. To set the means of the distributions for the shock standard deviation we simply assume that the variance of the structural shock is equal to one third of the variance of the endogenous variable that it helps most to fit. For example, the shock to the marginal utility of consumption is assumed to have a variance equal to one third the variance of consumption. These priors are designed to attribute relatively little importance to the contribution of these shocks to the variation in endogenous variables.

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<sup>28</sup>We generated four chains of length 125,000 and the last 62,500 of each chain were used for inference. Acceptance rates for each chain were 28%–29%.

<sup>29</sup>An alternative approach, suggested by the anonymous referee, would be to use the information contained in the posterior confidence intervals of the studies we cite.

<sup>30</sup>Note that setting relatively tight priors also ensures reasonably shaped Beta distributions for these parameters.

### 3.4.3 Estimation results

The results of the estimation exercise are presented in Table G in Appendix C. In discussing the results, it is useful to compare them to those of DiCecio and Nelson (2007), who estimate a similar model on similar UK data series.<sup>31</sup> In discussing the parameter estimates, we will note that the estimated posterior distributions for most parameters are not particularly skewed. Therefore, in most cases, when we refer to our ‘estimate’ of the parameter we will be referring to both the mean and the mode of the estimated distribution. In cases where there is a significant skew, we will discuss mean and mode estimates separately.

Considering first the household preference parameters, we observe that the posterior estimates for both the elasticity of intertemporal substitution ( $\sigma_c$ ) and habit formation parameter ( $\psi^{hab}$ ) are somewhat lower than the mean of the prior distributions assumed for them. DiCecio and Nelson (2007) find a much higher estimate for the habit formation parameter and they impose a higher value for the elasticity of intertemporal substitution (they assume log utility). So our estimation procedure finds that consumption is relatively insensitive to movements in the real interest rate not because of significant habit formation, but because consumers dislike substituting consumption across time periods. Our results are therefore broadly consistent with Nelson and Nikolov (2003) and Bergin (2003), who both estimate  $\sigma_c$  to be much lower than unity using UK data. We also find that the posterior estimate of the Frisch elasticity of labour supply is somewhat lower than our prior mean. Labour supply is therefore relatively inelastic with respect to changes in the real wage. This finding is again consistent with the estimates of Bergin (2003), who estimates a very low Frisch elasticity for the UK data.

Turning to the parameters governing production and investment, the posterior estimate of the capital adjustment cost parameter ( $\chi^k$ ) is slightly lower than the prior mean, but the confidence interval indicates that this estimate is extremely imprecise. DiCecio and Nelson (2007) estimate a much higher value for the adjustment cost parameter (equivalent to  $\chi^k \approx 650$ ) and the estimate of Bergin (2003) is even higher. The posterior estimate of  $\epsilon^k$  (the extent to which capital adjustment costs depend on lagged capital growth) is close to zero. The tight interval around the

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<sup>31</sup> Though similar, there are several differences between our analysis and that of DiCecio and Nelson (2007). First, DiCecio and Nelson (2007) use the closed economy model of Christiano *et al* (2005) that imposes a number of parameters (for example the intertemporal elasticity of substitution) that we have chosen to estimate. Second, their estimation methodology follows Christiano *et al* (2005) and chooses the parameters of the model to best match the impulse responses to a monetary policy shock in an identified VAR. Finally, there are some differences in the data series used to estimate the models.

mean and mode estimates suggests that the data strongly prefers a more conventional capital adjustment cost function to the ‘investment adjustment costs’ assumed in many other papers. Our estimate for the elasticity of capital utilisation ( $\sigma^z$ ) is somewhat lower than the prior mean and substantially lower than the estimate of DiCecio and Nelson (2007), who find that their estimation approach implies an infinite elasticity (that is, fixed capital utilisation).<sup>32</sup>

Our estimates of nominal rigidities show that, compared to the prior mean, the posterior estimates of price stickiness are lower for domestically produced goods ( $\chi^{hv}$ ), higher for exports ( $\chi^{xv}$ ), and substantially higher for import prices ( $\psi^{pm}$ ). The respective Calvo probabilities of optimally resetting prices (using the posterior mode estimates) are: 0.17, 0.33 and 0.08. These compare to the estimate of 0.06 found by DiCecio and Nelson (2007), who do not distinguish between prices for domestic absorption, exports and imports. Our estimate for the probability of optimally resetting nominal wages suggests slightly more wage stickiness than our prior assumption: the Calvo readjustment probability is around 0.16, compared with a value of almost unity (implying flexible nominal wages) estimated by DiCecio and Nelson (2007). Our estimation results do not provide clear evidence for the extent to which ‘indexation’ of nominal rigidities to past inflation rates is important: the posterior estimates of  $\epsilon^{hv}$ ,  $\epsilon^{xv}$ ,  $\epsilon^m$  and  $\epsilon^w$  are all close to the prior means. Overall, our results suggest a greater role for nominal wage stickiness than found by DiCecio and Nelson (2007) and somewhat less price stickiness.<sup>33</sup>

Our estimates of the monetary policy reaction function suggest a rather weaker long-run response of interest rates to inflation than the prior mean for this parameter, whereas the estimates of the coefficient on the output gap and the lagged interest rate are very close to our priors. The lower responsiveness of nominal interest rates to inflation is unsurprising, given that our sample period includes periods of highly volatile inflation during which many authors have argued that monetary policy was not appropriately directed to inflation control. See, for example, Batini and Nelson (2005). Given the regime shifts evident in the UK monetary policy, as documented by

<sup>32</sup>One possible explanation for these results may lie in the fact that the response of output to a monetary policy shock can be damped either by high capital adjustment costs or a low elasticity of capital utilisation, which may create an identification issue when parameters are estimated by fitting impulse response functions as in DiCecio and Nelson (2007) and Christiano, Eichenbaum and Evans (2005). This seems consistent with the sensitivity analysis presented in Christiano *et al* (2005), which suggests that imposing a very low (high) elasticity of utilisation results in relatively low (high) estimates of capital adjustment cost parameters.

<sup>33</sup>One reason for this difference may be the fact that DiCecio and Nelson (2007) estimate their model on data from 1979 onwards. When they extend their sample back further they find more evidence for nominal wage stickiness – particularly for the sample 1962 Q3–1979 Q1. Another issue that may be relevant for our results is the Del Negro and Schorfheide (2008) finding that posterior estimates about nominal rigidities can be heavily influenced by the choice of priors.

Nelson (2003) among others, the estimates of policy rule parameters may be somewhat unreliable as an approximation to recent policy behaviour. Of course, changes in the monetary policy regime may affect the estimated of other parameters too. A useful extension to the current analysis would be to examine the stability parameter estimates across subsamples or perhaps to allow for a discrete change in the monetary policy rule as in Adolfson *et al* (2007).

Finally, we discuss the estimated parameters of the stochastic shock processes. The posterior estimates suggest that four shocks are extremely persistent: total factor productivity, government expenditure, the consumption shock and the shock to capital adjustment costs. However, we find that the shocks to the mark-up and labour supply are not highly autocorrelated, particularly in the case of the labour supply shock. The estimated standard deviations of the shocks to these stochastic processes are in most cases significantly higher than the prior means. This is not surprising since our priors were intended to imply that no single shock dominated the variability in the endogenous variables, which led us to calibrate relatively low variances for each shock.<sup>34</sup>

### 3.5 *Evaluating the estimated model*

In Section 3.3, we evaluated the fit of the calibrated baseline model (the baseline model) to data by analysing the spectra and the coherence functions implied by the model and the data together with the Watson's statistics for fit. In this section, we evaluate the fit of the baseline model after it has been augmented with structural shocks and estimated (the estimated model) as discussed in Sections 3.3.2 and Section 3.4. Spectra and coherence functions for both of these models and data are presented in Charts 4 and 5 in Appendix C while Table H in the appendix shows fit statistics. Table H has three panels. Panel A presents Watson's fit measure for the estimated model. Panel B presents results for the baseline model. In Panel C we report Watson's fit measure for a version of the calibrated model, where we replace the calibrated values for parameters governing the shock processes with their estimated values.

The estimated model improves over the baseline model on the variables hours worked, investment, inflation, and real wage. As shown by Table H (Panels A and B), when all variables are weighted equally, the overall fit measure for investment increases from 0.186 to 0.867, for hours worked it increases from 0.124 to 0.658, for inflation it increases from -0.041 to 0.214, and

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<sup>34</sup>However, the standard deviation of the labour supply shock is estimated to be several orders of magnitude larger than the prior mean.

for real wage it increases from 0.398 to 0.544. The improvement on investment, hours worked, and real wage is striking when the individual series receives all the weight: the fit measures for investment, hours worked, and real wage increase to 0.907, 0.921, and 0.952, respectively, implying that the model's potential to explain the variation increases to more than 90%. The results for *investment* and *inflation* reflect the estimated model's improvement across the whole frequency range. This can be seen from Chart 4. As before, the grey band denotes the 95% confidence interval for the data spectra and the solid green line is the spectra for the calibrated model. The spectra for the estimated model is shown by the solid blue line. As the chart shows, for all frequencies, the investment and inflation spectra in the estimated model are much closer to the data spectra lying above the spectra of the baseline model.

For the investment series, the shock processes play an important role in reconciling its dynamics with those of the data. As Table H (Panel C) shows, increasing the persistence and standard deviation of the shocks in the baseline model (namely, TFP, government spending, and monetary policy) to their estimated values alone improves the fit measure to 0.577. The addition of structural shocks and estimation improves it further to 0.867 (Panel A). However, for the inflation series, the biggest improvement in the fit measure occurs when the model is augmented with additional structural shocks and estimated (Panel A). For the hours worked (real wage) series, results reflect the estimated model's improved performance over the low (high) frequency band. Indeed, for high frequencies, the fit of hours worked is worse (the solid blue line lies further away from the grey shaded area above the solid green line) indicating that the variability in the estimated model is too high relative to data and baseline model. This is consistent with the fact that the estimated elasticity of labour supply ( $\sigma^h$ ) is lower than our prior and the estimated standard deviation of the shock to labour supply is higher. So it seems that the data favours these results because they aid the fit of other variables. Interestingly, the estimation process seems to have little effect on the fit of the real wage series. As Table H (Panel C) shows, after including the estimated values for shock processes the fit measure for this variable improves to 0.426 just short of its level under the estimated model (Panel A). This result also holds for the model's fit over the high and low frequency ranges.

The estimated model has only modest deterioration in the fit of *output* series due to its slightly worse fit over high frequencies. The *consumption* series, on the other hand, performs considerably worse under the estimated model. When all variables are weighted equally, the



overall fit measure for consumption decreases from 0.339 (Panel B) to -0.452 (Panel A). When all the weight is placed on consumption, Watson's fit statistic falls from 0.817 to 0.477 suggesting a decline in the estimated model's potential to explain its movements. This decline reflects poorer performance over the low frequency range. As Chart 4 suggests the estimated model implies too much variability relative to data over this frequency band. This result seems inconsistent with the lower estimated value of the habit parameter ( $\psi^{hab}$ ) but is consistent with the higher estimated value for the standard deviation of shocks suggesting the relative importance of these shocks for consumption dynamics in the estimated model.<sup>35</sup> Another series that fits the data less well is the nominal interest rate. As Table H shows the fit measure for this variable falls from -0.224 (Panel B) to -1.149 (Panel A), again reflecting the model's poorer fit over the low frequency range. As Chart 4 shows the persistence of the nominal interest rate series in the estimated model is more than suggested by the data and predicted by the baseline model. This result might reflect the fact that persistence parameters are higher for most shocks in the estimated model.

Additionally, Chart 5 shows that, in contrast to the baseline model, the estimated model has a tendency to mimic the coherence pattern in the data better, since it exhibits higher coherence at low frequencies and lower coherence for high frequencies. One exception is the investment series. The coherence function for investment in the estimated model is low and essentially flat. This suggests that the investment shock in the estimated model is more important for investment dynamics than it is for output.

Overall, the estimated model seems to capture the properties of the UK data better for most variables. Improvements are most visible at low frequencies (though there are exceptions) reflecting the model's increased capability to capture the persistence in the data. However, the estimated model's performance over the high frequency range deteriorates significantly for consumption and hours worked suggesting that they are more volatile in the estimated model relative to data.

## 4 Impulse responses analysis

In this section, we use the estimated DSGE model to analyse the impulse responses to various shocks on the supply side (productivity, labour supply, and producer price mark-up shocks) and

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<sup>35</sup>Consumption performs considerably worse even when only the parameters of the shock processes are changed: -0.292 (Panel C).



the demand side (monetary policy, consumption and investment shocks) as well as shocks to the rest of the world variables.

Chart 6 in Appendix C shows the response of model variables to a positive TFP shock. Following the productivity shock, the domestic economy can (in the short term) produce more output for given amounts of labour and capital. This leads to an increase in output supplied. But because prices adjust slowly, aggregate demand does not immediately match this increase so employment and capital utilisation fall. This reduces the marginal cost of production for all firms. Firms respond by lowering prices to stimulate aggregate demand and reducing their demand for labour and capital services. The increased productivity leads to higher real wages as households set higher wages in light of the rise in their productivity. With higher real wages, demand rises steadily bringing utilisation and employment back to normal levels. Investment falls immediately after the shock but increases quickly afterwards and remains high to build up the capital stock. The investment response reflects the role of capital adjustment costs. The level of aggregate imports rise with the now higher level of consumption and output. To restore external balance the real exchange rate depreciates leading to an increase in exports. The depreciation in the real exchange rate also leads to a gradual increase in import prices acting to reduce the demand for imports. Inflation falls because the fall in aggregate prices outweighs the increase in import prices. Monetary policy responds to low inflation and output below potential (the simple ‘output gap’ measure in the monetary policy rule is adjusted for TFP movements) by cutting nominal interest rates.<sup>36</sup>

Chart 7 shows the effects of a positive labour supply shock. After the labour supply shock, households are willing to supply more labour services for a given wage rate. The effects of this shock on aggregate demand, inflation, and interest rates are qualitatively similar to those of a positive productivity shock. The main qualitative difference is that employment rises and real wage falls since households incur less disutility from supplying a unit of labour. Another difference is the fall in imports in face of depreciating real exchange rate (this is because in this case the substitution effect from higher import prices dominates the income effect from higher demand). Monetary policy responds by cutting nominal interest rates.

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<sup>36</sup>The above responses are qualitatively consistent with the results in the empirical literature on monetary policy shocks. See, for instance, Smets and Wouters (2003). That is, following a (unanticipated) positive productivity shock, consumption, output, and investment increase.

The effects of a price mark-up shock are shown in Chart 8. A positive mark-up shock tends to lower the producer price inflation in the domestic intermediate goods sector. The fall in consumer price inflation reflects this. Stimulated by lower prices, demand for domestic goods increases relative to imported goods. Firms respond by reducing export production as well as increasing employment and capital utilisation. Both the wage rate and rental rate rises. As the real exchange rate depreciates, exports rise steadily towards their pre-shock levels. Monetary policy responds to low inflation by cutting nominal interest rates.

Turning to the demand-side shocks, Chart 9 shows that, following a contractionary monetary policy shock, the short-term nominal and real interest rates rises. The rise in the real interest rate, reflecting the role of nominal rigidities, generates contraction in consumption, output, and investment. The impact on investment is about twice as large as that on output. Firms also reduce employment and the rate of capital utilisation in face of lower aggregate demand. The real wage rises in the very first period, but falls quickly afterwards as a result of reduced labour demand. The initial increase in the real wage reflects the balance of rigidities firms face in adjusting their factors of production and nominal prices. The nominal exchange rate appreciates with the increase in the nominal interest rate. Since domestic prices are sticky, this leads to an appreciation of the real exchange rate, which in turn reduces the demand for exports. The appreciation of the real exchange rate also acts to lower import prices but only gradually due to price stickiness. The fall in import prices generates an increase in import demand. However, the overall demand for imports falls reflecting the fall in aggregate demand. Inflation also falls, returning to steady state after about three years.<sup>37</sup>

Chart 10 shows that, in response to a government spending shock, consumption falls reflecting the crowding-out effect (ie higher government spending is financed by higher lump-sum taxes on households). The net effect on aggregate demand is positive. In face of higher domestic demand, firms increase capital utilisation and employment. While the rental rate rises, real wages are little changed (this is because the fall in their consumption makes households more willing to work, which largely offsets the effects of increased labour demand on wages). The level of aggregate exports initially fall as firms reduce export production to meet the higher domestic demand. The

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<sup>37</sup>The above responses are qualitatively consistent with the results in the empirical literature on monetary policy shocks. See, for instance, Christiano *et al* (2005), Smets and Wouters (2003) and Søndergaard (2004). That is, following a (unanticipated) contractionary monetary policy shock, interest rates rise, consumption, output, and investment fall in a hump-shaped fashion, real profits and real wages fall, and finally inflation falls.

appreciation in the real exchange rate leads to a gradual fall in import prices. However, the level of aggregate imports falls reflecting the lower level of consumption. Inflation falls a little since the fall in import prices outweigh domestic price pressures. Monetary policy responds to higher output by raising nominal interest rates.

The effects of a positive consumption shock is shown in Chart 11. The consumption shock tends to increase the utility households derive from a unit of consumption. Thus, following the shock, consumption, investment, and output all increase. To meet the higher demand, firms increase capital utilisation and employment and reduce export production. Although the rental rate of capital increases, real wages fall reflecting greater willingness of households to work to finance their increased demand for consumption. The increase in the overall level of imports reflects the higher level of aggregate demand and lower import prices. Inflation rises as a result of the increase in factor costs and demand pressures. Monetary policy responds by raising nominal interest rates.

A temporary fall in capital adjustment costs drives the surge in investment in Chart 12 and leads to the increase in output and employment but has little impact on consumption. Different to the consumption shock discussed above, in this case, the real wage rate rises in line with output and the utilisation and rental rate of capital falls in the medium term to bring investment back to steady state. Inflation increases as a result of the rise in factor costs and demand pressures and monetary policy responds by raising nominal interest rates.

Charts 13-15 show the effects of a shock to the world variables and are presented in Appendix C for reference.

#### ***4.1 A comparison of impulse responses: calibrated versus baseline model***

We assess the differences in the response of the model to various shocks before and after estimation by means of a stylised exercise. In discussing our results, we take as example the response of interest rates to a contractionary monetary policy shock and real wage to a positive TFP shock, but our results can be generalised to other variables and shocks in the model. These impulse responses are shown in the first and second rows of Chart 17, respectively. In the first column of the chart, we compare impulse responses from the estimated model (solid lines) to

those from the calibrated model (dashed lines). In the second column of the chart, impulse responses from the estimated model are compared to those from a version of the calibrated model where we replace the calibrated values for parameters governing the persistence and standard deviation of shock processes with the estimated values (dotted lines). A number of points emerge. First, responses of variables to shocks are qualitatively similar across different models. Second, interest rates respond by less to a monetary policy shock under the calibrated model mainly reflecting the smaller value for the standard deviation of this shock in this model (about three times): when we increase the calibrated value for the standard deviation to its estimated value impulse responses come much closer (where the remaining distance can be attributed to the differences in parameters other than those governing the shock processes). Third, the response of real wage to a productivity shock is shorter-lived under the calibrated model mainly reflecting the smaller value for the persistence of the shock in this model: when we increase the calibrated persistence to its estimated value impulse responses come much closer. Overall, the impulse responses from the estimated model is qualitatively similar to those from the calibrated model, but quantitatively they are deeper and longer-lived mainly due to the differences in the values for the parameters governing the shock processes. Again, this reveals the importance of the estimated shock processes in determining the dynamic behaviour of the estimated model.

## 5 Conclusion

In this paper, we have developed and estimated a medium-scale DSGE model for the United Kingdom. The model incorporates many of the features that are now standard in the literature that builds on Christiano *et al* (2005) and Smets and Wouters (2003). But it also includes features that are designed to make it relevant for the analysis of a small open economy such as the United Kingdom. Estimation proceeds in two stages. First, we evaluate a calibrated version of the stochastically singular model. Here, we adopt the approach developed by Watson (1993), which is based on comparison of model and data spectra. Second, we augment the model with four types of structural shocks, chosen on the basis of the results of the evaluation stage and estimate its parameters on UK data using Bayesian maximum likelihood. Finally, we reassess the adequacy of this augmented and estimated model.

Our approach to first assessing the fit of the calibrated model before moving to Bayesian estimation generates a greater understanding of the way in which the parameters can and cannot

help it to fit the data. This assists in the choice of structural shocks to add. The choice of structural shocks may be crucial both to the fit of the model and to the story that it tells about the economic forces governing the dynamics of the data. When we evaluate the performance of the calibrated model in matching UK data, we find that it performs poorly: it fails to match both the patterns of variability across frequencies for individual variables (as summarised by Watson's measure of fit) and the comovement of variables across frequencies (as summarised by coherence functions). While some changes to parameter values may improve the fit, we find that such changes alone are unlikely to produce a good match between the data and model.

Our assessment of the ways that the dynamics of the model fail to match those of the data leads us to augment it with four additional shocks – to consumption and hours worked in the form of shocks to preferences in the household utility function, to investment in the form of a shock to capital adjustment costs, and to inflation in the form of a mark-up shock – before estimating it by Bayesian likelihood. We find that in combination with moderate real rigidities (habit formation and capital adjustment costs), both price and nominal wage rigidities are important for matching the UK data. While nominal wage and price stickiness are favoured by the data, there is little evidence for strong effects from lagged inflation rates on price and wage-setting (often termed 'indexation' in this literature). The shock processes play a key role in matching the UK data and four of them – total factor productivity, government expenditure, the consumption preference shock and the shock to capital adjustment costs – are estimated to be extremely persistent.

This study suggests several promising avenues for future research. The analysis above has concentrated on assessing the fit of the model over the period 1955 Q1 and 2007 Q1. Evaluating the calibrated model, we find that it poorly matches the dynamics of the UK data and that the fit of the nominal interest rate and inflation rate are among the worst fitting variables, even when we experiment with different parameterisations. Moreover, while the fit of the inflation series improves after augmentation and estimation, the fit of the nominal interest rate worsens. Taken together, these results might reflect the misspecification present in the monetary policy rule. We assume that the policymaker sets the short-term nominal interest rate according to a simple Taylor-type reaction function. Given the regime shifts evident in the UK monetary policy, a useful extension would be to evaluate the model's fit once we account for the different policy regimes. Moreover, the model features a wide range of nominal and real frictions. Our evaluation and estimation results suggest that the data favours a parametrisation with moderate degrees of

real and nominal rigidities. But which of these frictions are really necessary to capture the dynamics of the data? To answer this question, one can re-evaluate the model's fit when these frictions are switched off one at a time. Such an analysis should give an idea about the model's performance with respect to various frictions.



## Appendix A: Model parameters

The first group of parameters are most important in determining the steady state of the model. These are shown in Table A. The fourth column of the table summarises the motivation for our choice. When an expression involving model variables appears without a comment, the parameter has been chosen to match the steady state of our model to our data set. For example, the substitution elasticity among the varieties of domestic goods ( $\sigma^{hb}$ ) is set to match the labour share with the average calculated from our data set. When a reference appears, we have either used parameter values directly from that study of UK data or used the parameter value that matches the steady-state of our model to the model in the reference. For example, we take the value of the elasticity of substitution between domestic and imported goods ( $\sigma^m$ ) directly from Harrison *et al* (2005), but we chose the utility weight of money balances ( $\kappa^{mon}$ ) to match the ratio of real money balances to output in Batini *et al* (2003). A group of parameters are used to normalise prices and quantities in the model (which have no natural units). Finally, a small group of parameters reflect our assumptions and judgements and therefore require further discussion.

The assumptions  $\beta = 0.99$  and  $\delta = 0.025$  are standard in the DSGE literature implying annual steady-state real interest and depreciation rates of around 4% and 10%, respectively. Our assumption that the elasticities of substitution among imported and exported goods are equal to  $\sigma^{hb}$  reflects an assumption of symmetry across the markets in the absence of reliable data. Similarly, we assume that steady-state world nominal and real interest rates equal those in the domestic economy. More controversial is our assumption about the elasticity of substitution between capital and labour in the production of domestic goods ( $\sigma^y = 0.5$ ). Ellis (2006) notes that estimates for the United Kingdom are typically below unity and his own estimates suggest a very low estimate of around 0.2. Our assumption reflects a desire to use this information but also to retain comparability with a number of DSGE estimation exercises that impose a unit substitution elasticity. We assume that the production function parameter  $\alpha = 0.3$  so that the labour share in the model approaches the value observed in the data in the limiting case of perfect competition and Cobb-Douglas technology. Another assumption worthy of some discussion is our judgement about the elasticity of substitution among varieties of labour ( $\sigma^w$ ). Again, this parameter not only affects the steady state of the model, but is also important in determining its

dynamic behaviour. As is apparent from equation (15), the parameter  $\sigma^w$  affects the slope of the wage Phillips curve. However, other researchers have noted the difficulty in identifying  $\sigma^w$  given that it is the ratio of this parameter with  $\sigma^h$  that affects the slope coefficient. Faced with this identification problem, these researchers impose a fixed value for  $\sigma^w$ . Our choice of  $\sigma^w$  is motivated by a simple and partial equilibrium calculation that associates unemployment with the monopolistic distortion caused by imperfect competition in the labour market. Specifically, we set the parameter so that the elimination of the wage mark-up ( $\frac{\sigma^w}{\sigma^w-1} \rightarrow 1$ ) increases employment by 5% if the labour supply curve is approximated as  $h_t = D(\frac{\sigma^w}{\sigma^w-1})^{-\sigma^h} w_t^{\sigma^h}$ , for some fixed  $D$ . That is we find  $\sigma^w$  such that  $(\frac{\sigma^w}{\sigma^w-1})^{-\sigma^h} = 1.05^{-1}$ . Despite the simplicity of this procedure, the resulting value of  $\sigma^w = 8.3$  sits comfortably in the range of values used in recent studies: comparable values for  $\sigma^w$  from Adolfson *et al* (2007), Batini *et al* (2003), Christiano *et al* (2005), Edge *et al* (2003), Juillard *et al* (2006), Smets and Wouters (2003) and Smets and Wouters (2007) are 21, 7.06, 21, 4, 7.25, 3 and 3, respectively. Finally, we should note that the value of the parameter  $\chi^{bf}$  has no effect on the steady state of the model (though a positive value is required to ensure that the model is dynamically stable around the assumed steady-state net foreign asset position). In principle, the value of  $\chi^{bf}$  will affect the dynamic properties of the model through its effect on the UIP equation (see equation (7)). In practice, the dynamics of the model seems relatively unaffected by a wide range of values for  $\chi^{bf}$  so we chose an arbitrarily small value that is still sufficient to ensure dynamic stability.

The second group of parameters govern the dynamics of the model. These are shown in Table B below. The table shows that there is a general consensus for estimates of some parameters. For example, the ranges of estimates for the degree of habit formation and the coefficients of the monetary policy rule are reasonably tight. For other parameters, the range of estimates is rather wider. One particular difference is in the relative importance of price and wage stickiness: studies based on euro-area data typically find stickier prices compared with those based on US data (which tend to find slightly more wage stickiness). Finally, we note that in the absence of any estimates for  $\epsilon^k$ , we impose the parameter at 0.5. This is midway between the assumption that adjustment costs apply to the capital stock ( $\epsilon^k = 0$ ) and the assumption that they apply to the flow of investment ( $\epsilon^k = 1$ ). The latter assumption is imposed in most of the studies cited in Table B. When calibrating the variance of the shocks to TFP, government spending, and the monetary policy rule we use estimates of the contribution of these shocks to output and nominal interest rate variability in previous studies of US and euro data as shown in Table B. Based on the



weighted average across different studies, we set the variance of TFP and government spending to be respectively 14% and 3% of the variance of UK output (measured as log-deviations from trend). And the variance of the monetary policy shock is set to be 11% of the variance of nominal interest rate. Specifically, we set the standard deviation of the TFP shock to be 0.006, the standard deviation of the government spending shock to be 0.008, and the standard deviation of monetary policy shock to be 0.0007. The calculations are as follows. For the TFP,  $\sigma_{tfp}^2 = (1 - \rho_{tfp}^2) \times 0.14 \times 0.0011 = 0.00003$ , where 0.0011 is the variance of log output deviations from trend and where  $\rho_{tfp} = 0.89$  as shown in Table B. For the government spending, we also adjust for the share of government spending in output, which is 0.19 in our sample, to get  $\sigma_g^2 = (1 - \rho_g^2) \times 0.03 \times 0.0011 \times (0.19)^{-2} = 0.00007$ , where  $\rho_g^2 = 0.96$  as shown in Table B. For the monetary policy shock,  $\sigma_{mp}^2 = [1 - (\theta^{rg})^2] \times 0.11 \times 0.00002 = 0.0007$ , where 0.00002 is the variance of the nominal interest rate in our sample. Note that, for simplicity, we assume that the nominal interest rate is given by a first-order autoregressive process with coefficient  $\theta^{rg} = 0.87$  as shown in Table B.

**Table A: First group parameter values**

Parameter	Value	Description	Motivation
$\beta$	0.99	discount factor	assumption
$\delta$	0.025	depreciation rate	assumption
$\alpha$	0.30	share of capital	$\frac{h^{ss}w^{ss}}{y^{v,ss}}=0.7$
$\sigma^m$	1.77	e.o.s: domestic and imported goods	Harrison <i>et al</i>
$\sigma^{hb}$	9.67	e.o.s: domestic goods	$\frac{w^{ss}h^{ss}}{y^{v,ss}}=0.7$
$\sigma^{xb}$	9.67	e.o.s: export goods	assumption: $\sigma^{xb} = \sigma^{hb}$
$\sigma^{mb}$	9.67	e.o.s: imported goods	assumption: $\sigma^{mb} = \sigma^{hb}$
$\sigma^w$	8.3	e.o.s: labour services	assumption
$\sigma^y$	0.50	e.o.s: capital and labour	assumption
$\sigma^x$	1.50	e.o.s: domestic and foreign exports	Harrison <i>et al</i>
$\psi^m$	0.25	weight on import consumption	Batini <i>et al</i> : $\frac{pm^{ss}cm^{ss}}{ph^{ss}ch^{ss}}=0.14$
$\chi^{bf}$	0.01	cost parameter: foreign denominated bonds	normalisation
$\chi^z$	0.035	cost parameter: utilisation rate of capital	normalisation: $z^{ss}=1$
$\kappa^{hv}$	0.94	share of domestic inputs in domestic production	Batini <i>et al</i> : $\frac{\kappa^{xv}}{\kappa^{hv}}=0.8$
$\kappa^{xv}$	0.75	share of domestic inputs in export production	$\frac{y^{x,ss}}{y^{v,ss}}=0.2$
$\kappa^c$	1.62	scale parameter in consumption index	normalisation: $ph^{ss}=1$
$\kappa^x$	0.24	scale parameter in export demand	normalisation: $q^{ss}=1$
$\kappa^{mon}$	0.03	utility weight of money balances	Batini <i>et al</i> : $\frac{mon^{ss}}{y^{v,ss}}=0.3$
$\kappa^h$	6.43	utility weight of labour supply	normalisation: $y^{v,ss}=1$ (normalisation)
$nf^{a^{ss}}$	-0.99	steady-state net foreign assets	$\frac{y^{x,ss} - (cm^{ss} + mi^{h,ss} + mi^{x,ss})}{y^{v,ss}}=0.01$
$tfp^{ss}$	0.24	steady-state total factor productivity	$\frac{I^{ss}}{y^{v,ss}}=0.14$
$g^{ss}$	0.19	steady-state government spending	$\frac{I^{ss}}{y^{v,ss}}=0.14$
$pxf^{ss}$	0.90	steady-state world export prices	normalisation: $pm^{ss}=1$
$cf^{ss}$	1	steady-state world consumption	normalisation
$RRF^{ss}$	1.01	steady-state world nominal interest rate	assumption
$\dot{p}^{ss}$	0	steady-state domestic inflation	assumption
$\dot{p}^{f,ss}$	0	steady-state world inflation	assumption

**Table B: Second group parameter values**

Parameter	Value	Description	US				Euro area	
			A	B	C	D	E	F
$\psi^{hab}$	0.69	habit formation in consumption	0.71	0.83	0.65	0.64	0.57	0.69
$\psi^w$	0.21	Calvo parameter: wage contracts	0.27	0.02	0.36	0.07	0.26	0.30
$\psi^{pm}$	0.40	Calvo parameter: import prices contracts						0.40
$\sigma^c$	0.66	inverse of risk aversion coefficient	0.72			0.36	0.74	
$\sigma^h$	0.43	labour supply elasticity	0.52			0.25	0.42	
$\epsilon^{hv}$	0.26	weight on lagged domestic producer inflation	0.22			0.00	0.47	0.21
$\epsilon^{xv}$	0.14	weight on lagged export producer inflation						0.14
$\epsilon^w$	0.58	weight on lagged wage inflation	0.59			0.32	0.76	0.52
$\epsilon^m$	0.17	weight on lagged import price inflation						0.17
$\epsilon^k$	0.50	cost parameter: capital stock						
$\chi^{hv}$	326	cost parameter: domestic producer prices	47	137	33	18	879	540
$\chi^{xv}$	43	cost parameter: export producer prices	43					43
$\chi^k$	201	cost parameter: capital stock	219	79	99	70	271	349
$\sigma^z$	0.56	cost parameter: utilisation rate of capital	0.85			0.77	0.17	
$\theta^{rg}$	0.87	parameter in monetary policy rule	0.81			0.78	0.96	0.87
$\theta^p$	1.87	parameter in monetary policy rule	2.03			2.23	1.68	1.71
$\theta^y$	0.11	parameter in monetary policy rule	0.08			0.23	0.10	0.08
$\rho_{tfp}$	0.89	persistence: productivity shock	0.95	0.89			0.82	0.91
$\rho_g$	0.96	persistence: spending shock	0.97	0.95			0.95	
			Contribution to $\sigma_y^2$ (%)					
$\sigma_{tfp}$	0.006	standard deviation: productivity shock	30	5			8	
$\sigma_g$	0.008	standard deviation: spending shock	5	0.5			4	
			Contribution to $\sigma_y^2$ (%)					
$\sigma_{mp}$	0.0007	standard deviation: policy shock	15	8			10	
Weights			0.2	0.2	0.1	0.1	0.2	0.2

A: Smets and Wouters (2007), B: Juillard *et al* (2006), C: Christiano *et al* (2005), D: Edge *et al* (2003), E: Smets and Wouters (2003), F: Adolfson *et al* (2007). Note that the values of parameters reported from other studies correspond to the 'baseline', 'benchmark' or 'preferred' model specifications. The estimates reported in these studies are often for slightly different parameters than ours: for example, many studies assume Calvo contracts in price-setting whereas we assume Rotemberg-style price adjustment costs. To convert the reported estimates into values for our parameters, we employ the transformations documented in Harrison *et al* (2005) (see pages 110–11, footnotes 47, 48 and 52).

## Appendix B: Data sources and construction

Our data for *endogenous variables* and *exogenous forcing processes* cover the period 1955 Q1–2007 Q1.

*Endogenous variables:* The endogenous variables used for estimation are output, consumption, investment, total hours worked, real wage, the short-term nominal interest rate, and inflation. We convert all real variables to a per capita basis by dividing by an estimate of the working-age population prior to detrending. We use the ONS quarterly series (MGSL.Q) for UK population aged 16 and over from 1971 Q1. For the period 1955 Q1 to 1970 Q4, we construct a quarterly population measure by interpolating annual ONS data (DYAY.A) for the total UK resident population (mid-year estimates) which is then spliced to the quarterly series for population aged 16 and over. Nominal variables are expressed as quarterly rates. To detrend we estimate a log-linear time trend (including a constant) for all real variables. For interest rates and inflation we divide the sample into three ‘regimes’ based on the analysis of Benati (2006). These regimes are 1955 Q1–1972 Q2, 1972 Q3–1992 Q4 and 1993 Q1 onwards. For each subsample, we take the mean of the series as a measure of ‘trend’. To arrive at our final data, we subtract the estimated trend components. Chart 1 below plots these implied ‘gaps’. Data on *output*, *consumption*, and *investment* are taken directly from the ONS quarterly series for the whole sample. They are seasonally adjusted and chain-volume measures. We use GDP at basic prices (ABMM.Q) for output, final consumption expenditure of households and NPISHs (ABJR.Q + HAYO.Q) for consumption, and total gross fixed capital formation (NPQT.Q) for investment in our data set. We construct a measure of *inflation* by first constructing the deflator associated with our measure of consumption and then taking the log first difference of this deflator. This consumption deflator is given by  $(ABJQ.Q + HAYE.Q) / (ABJR.Q + HAYO.Q)$ , where the term in the numerator is the *nominal* final consumption expenditure of households and NPISHs. For the *nominal interest rate*, we use the three-month Treasury bill rate series from the IMF’s International Financial Statistics (IFS) database (IFS 11260 C ZF) from 1957 Q1 onwards. For the period to 1956 Q4, we use data from Mills (1999), taking the quarterly averages of the monthly series. For *total hours worked*, we use the ONS quarterly series (YBUS.Q) for total actual weekly hours worked from 1971 Q1 onwards. Prior to 1970 Q4, we construct annual



measures of total hours by multiplying the series for ‘average annual hours per person engaged, whole economy’ and ‘number of persons engaged, whole economy’ provided in O’Mahony (1999). These data were interpolated to form a quarterly series and then spliced onto the quarterly ONS series YBUS.Q. To construct our measure of *real wages*, we first construct a measure of nominal wages by summing the quarterly ONS series for wages and salaries (ROYJ.Q) and employers’ social contributions (ROYK.Q), and income from self-employment (ROYH.Q). Because the series for ROYH.Q starts in 1987 Q1, before that date we proxy the sum by ROYJ.Q + ROYK.Q + g(ROYJ.Q), where g is the average of the ROYH.Q/ROYJ.Q ratio since 1987 Q1. The nominal wage series is then divided by total hours, consumption deflator and total population to create an hourly real wage series.

Forcing processes: We specify forcing processes for the following variables: world nominal interest rate, world consumer price inflation, world demand, and world price of exports relative to price of consumption. In each case, we construct world variables as a geometric average of individual country data using their respective trade weights. That is:

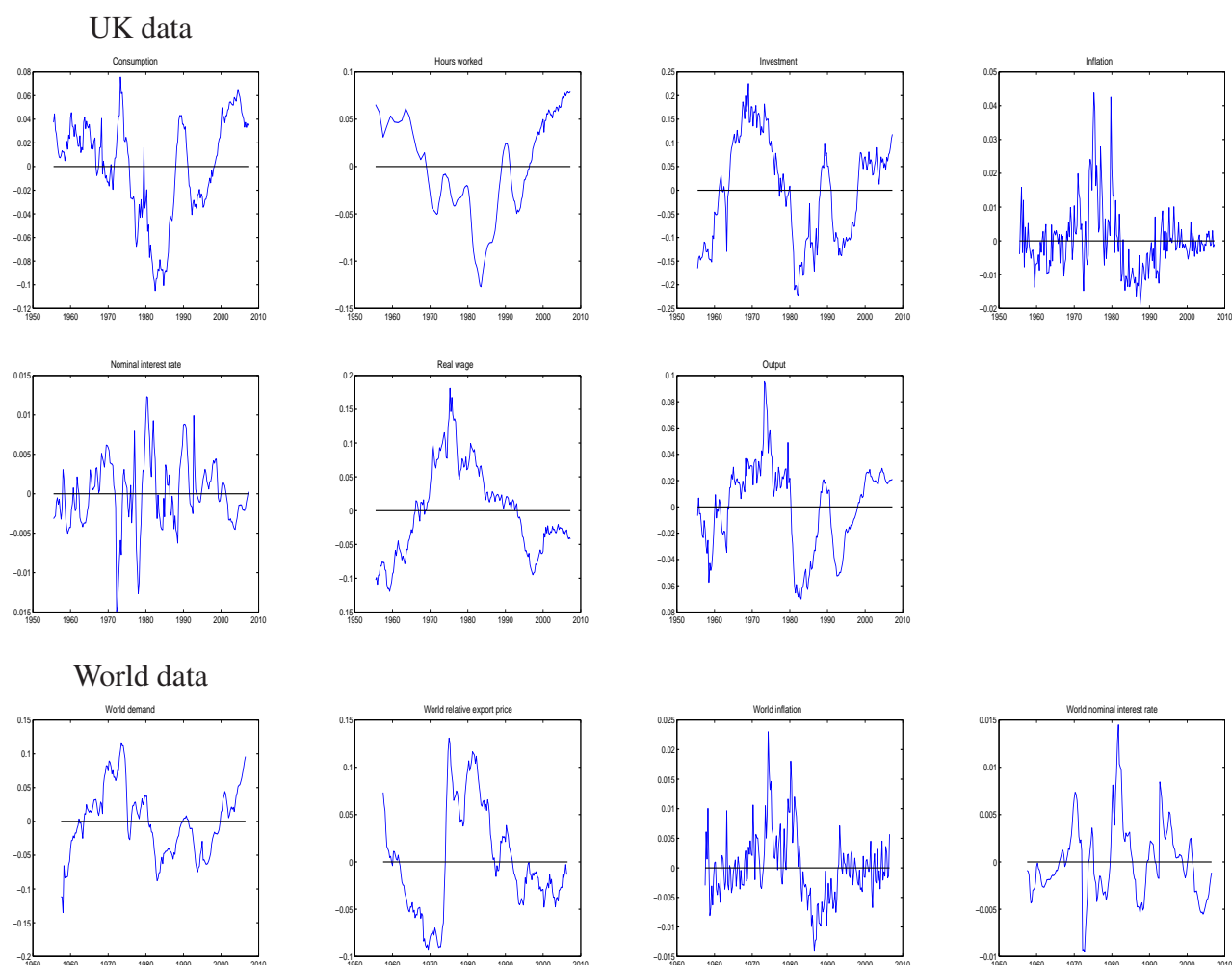
$$X_t = \left( \prod_{j=1}^N X_{j,t}^{w_j} \right)^{1/\sum_j w_j},$$

where  $X_{j,t}$  is the data for country  $j$  at time  $t$  and  $w_j$  is the weight for country  $j = 1, \dots, N$  and  $\sum_j w_j = 1$  when data for all countries are available. We choose  $N = 6$  by selecting the G7 countries (excluding the United Kingdom): Canada, France, Germany, Italy, Japan, and the United States. Over the period 1973–2004, trade with these countries accounted for an average of 57% of total UK trade. The trade weights for these countries are the average trade weights computed over this period and they are 0.037 for Canada, 0.173 for France, 0.254 for Germany, 0.095 for Italy, 0.112 for Japan, and 0.328 for the United States. The data source for each variable are summarised in the table below. The key source is the IMF’s IFS database. Where data for a subset of countries are not available we simply compute the weighted world average for the data series that are available (adjusting  $\sum_j w_j$  accordingly). We detrend the foreign variables in the same way as the UK data (described above) except that we allow for a quadratic time trend in (log) world demand. Chart 1 below plots the implied ‘gaps’.

**Table C: Data source for world variables**

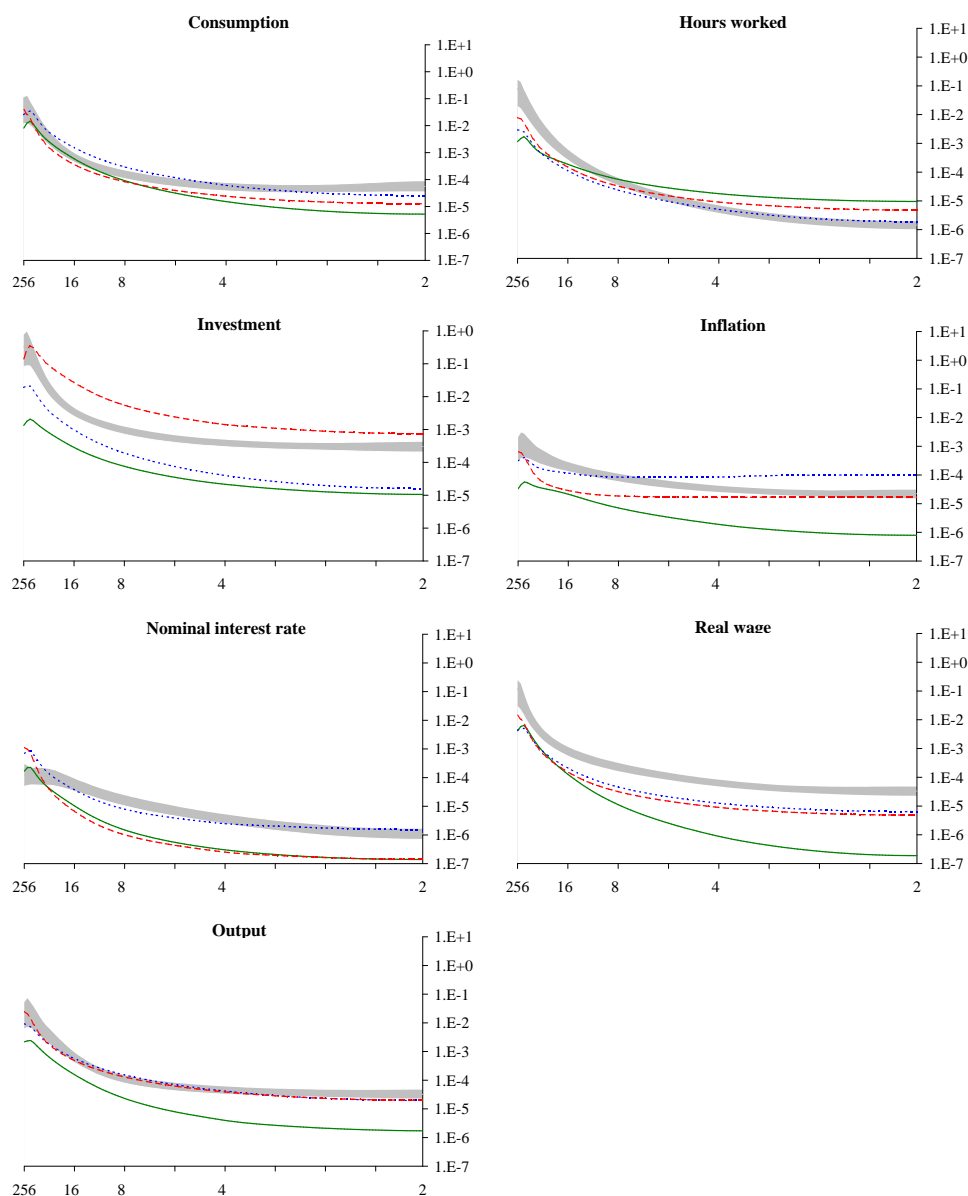
	Data	Source	Exceptions
Interest rate	Three-month T-bill rate (period average)	Global Financial Data	JAP & FR from 1960 Q1
Inflation	CPI (2000=100)	IFS, Thomson Datastream (GER)	
Demand	Industrial production (FR, GER, JAP, IT)	IFS	GER from 1958 Q1
	GDP (US, CAN)	IFS	
Export price	Export prices (US, FR, JAP)	IFS	FR from 1990 Q1
	Unit value of exports (CAN, GER, IT)	IFS	

**Chart 1: Detrended data series**



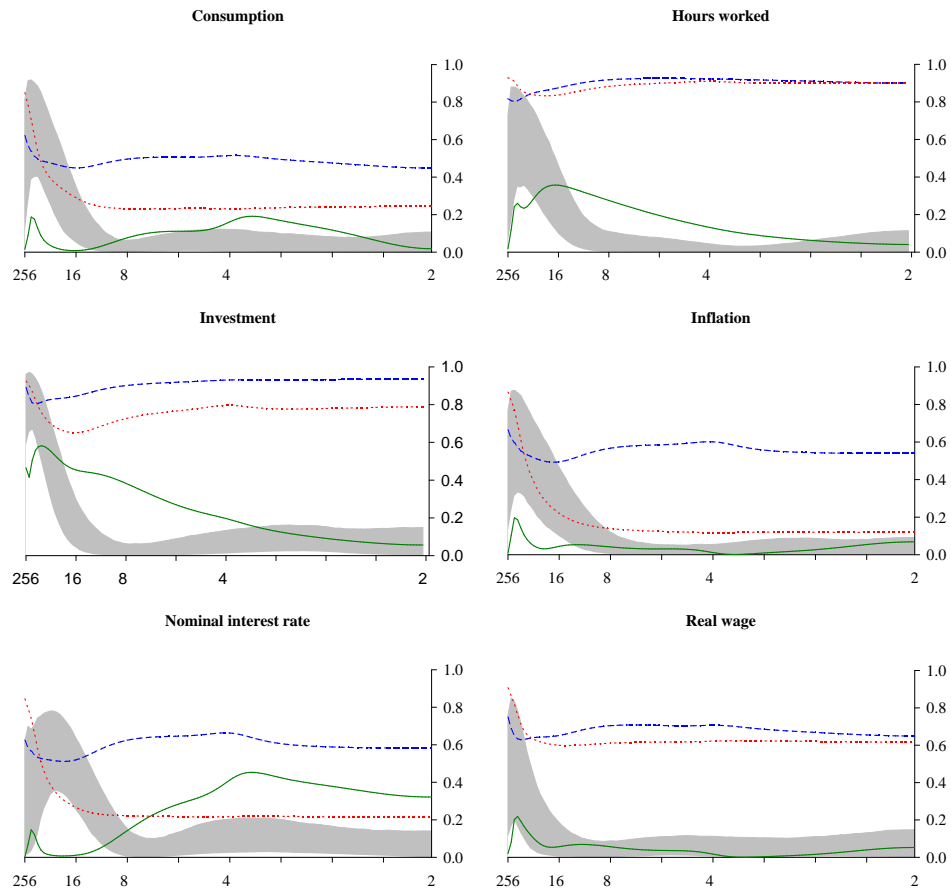
## Appendix C: Charts and tables

Chart 2: Spectra of data and models



Note. Grey bands denote the 95% confidence interval of data spectra. The solid green line is the spectra for the baseline model, the dotted blue line is the spectra for the FP model, and the dashed red line is the spectra for the RBC model. For details refer to Section 3.3.2.

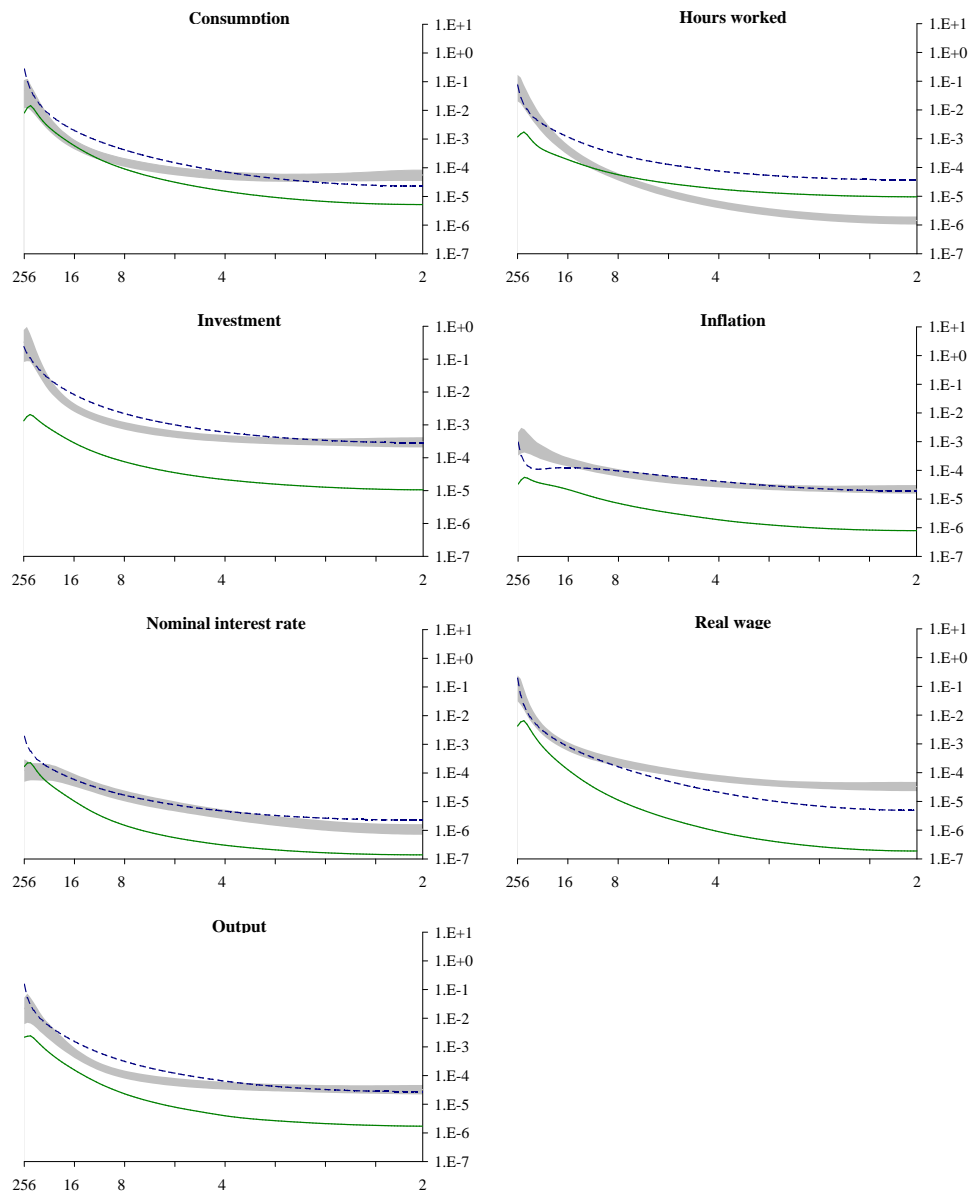
**Chart 3: Coherence function of data and models**



Note. The chart shows the coherence function between output and consumption, hours worked, investment, inflation, nominal interest rate, and real wage respectively. Grey bands denote the 95% confidence interval of the coherence function of the data. The solid green line is the coherence function for the baseline model, the dashed blue line is the coherence measure for the FP model and the dotted red line is the coherence function for the RBC model. For details refer to Section 3.3.2.

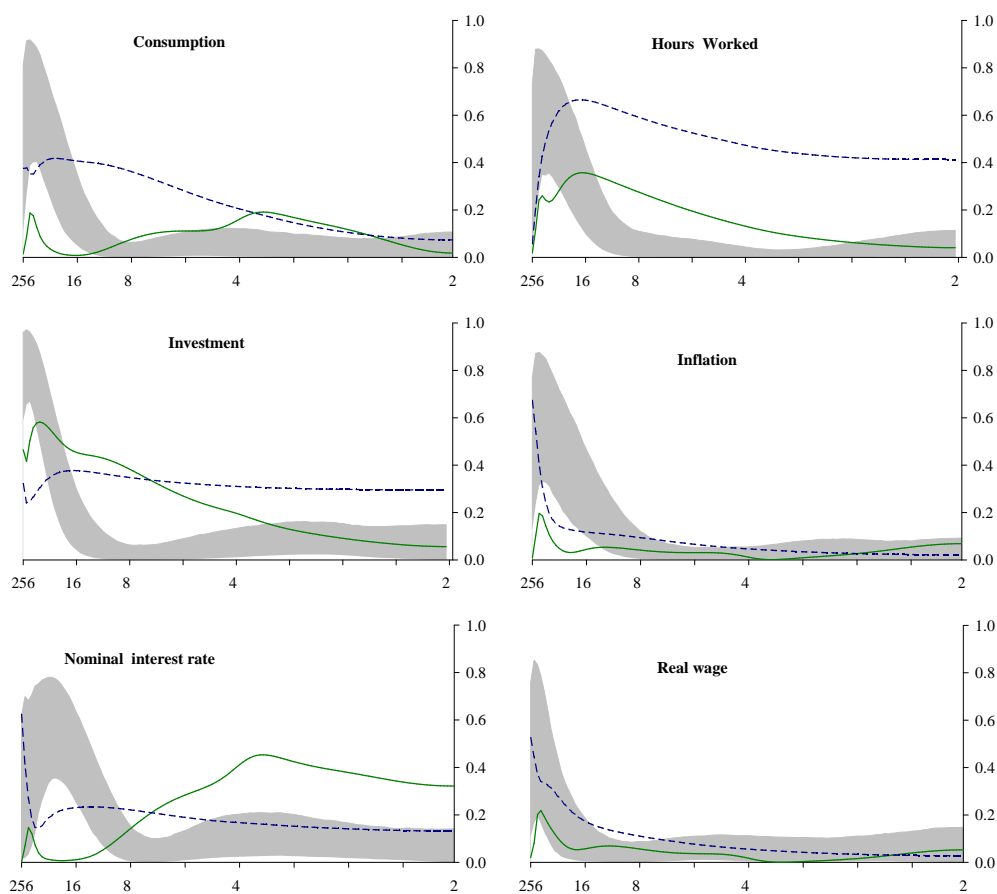


**Chart 4: Spectra of data and estimated model**



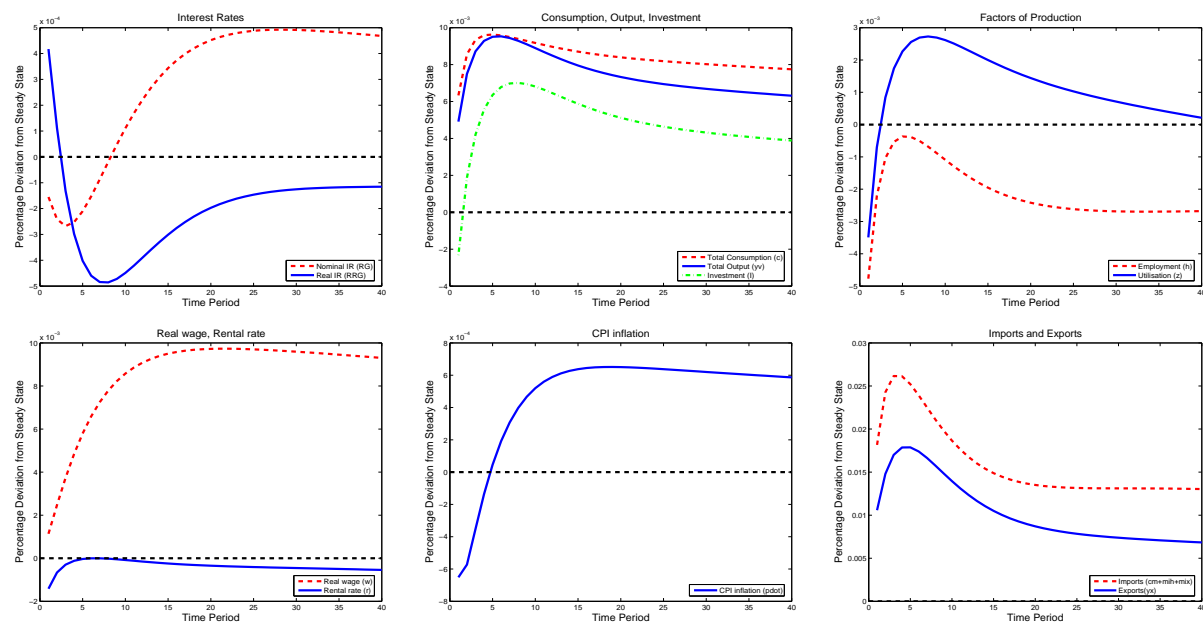
Note. Grey bands denote the 95% confidence interval of data spectra. The solid green line is the spectra for the calibrated baseline model and the dashed blue line is the spectra for the baseline model after it has been augmented with additional structural shocks and estimated. For details refer to Sections 3.3.2 and 3.4.

**Chart 5: Coherence function of data and estimated model**

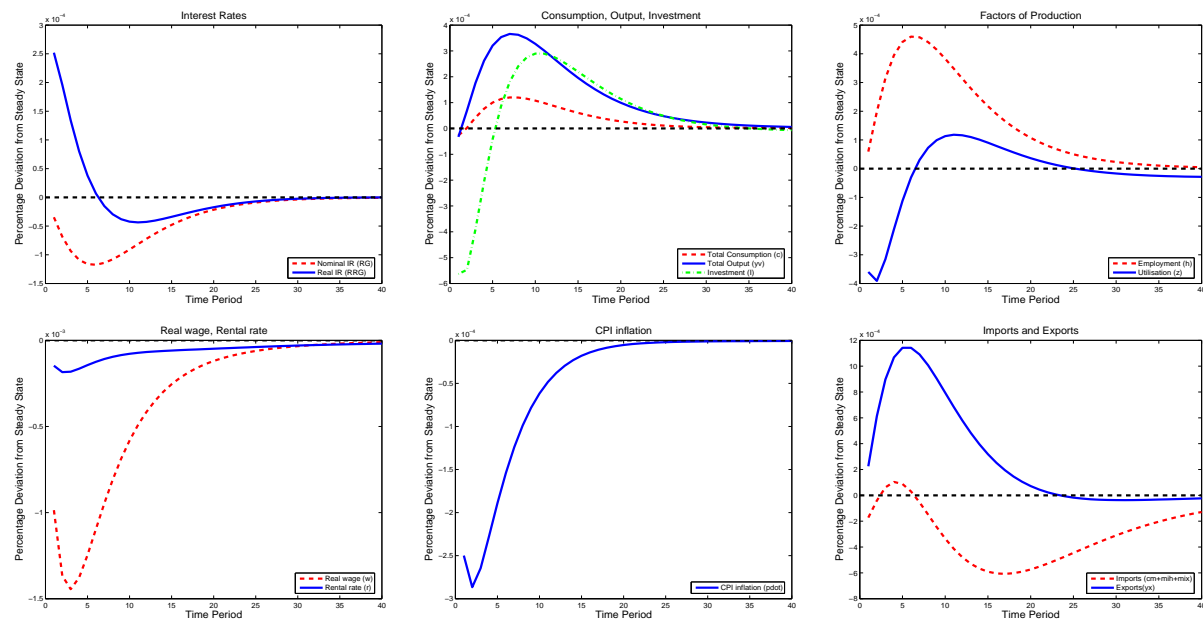


Note. The chart shows the coherence function between output and consumption, hours worked, investment, inflation, nominal interest rate, and real wage respectively. Grey bands denote the 95% confidence interval of the coherence function of the data. The solid green line is the coherence function of the calibrated baseline model and the dashed blue line is the coherence function of the baseline model after it has been augmented with additional structural shocks and estimated. For details refer to Sections 3.3.2 and 3.4.

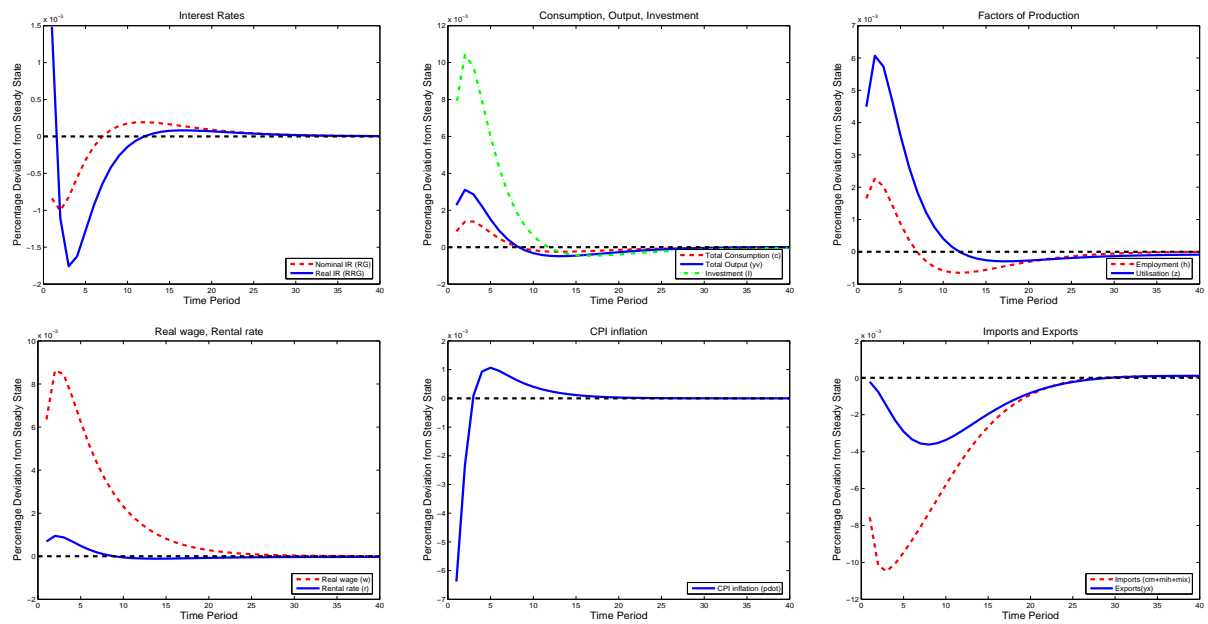
## Chart 6: Positive productivity shock



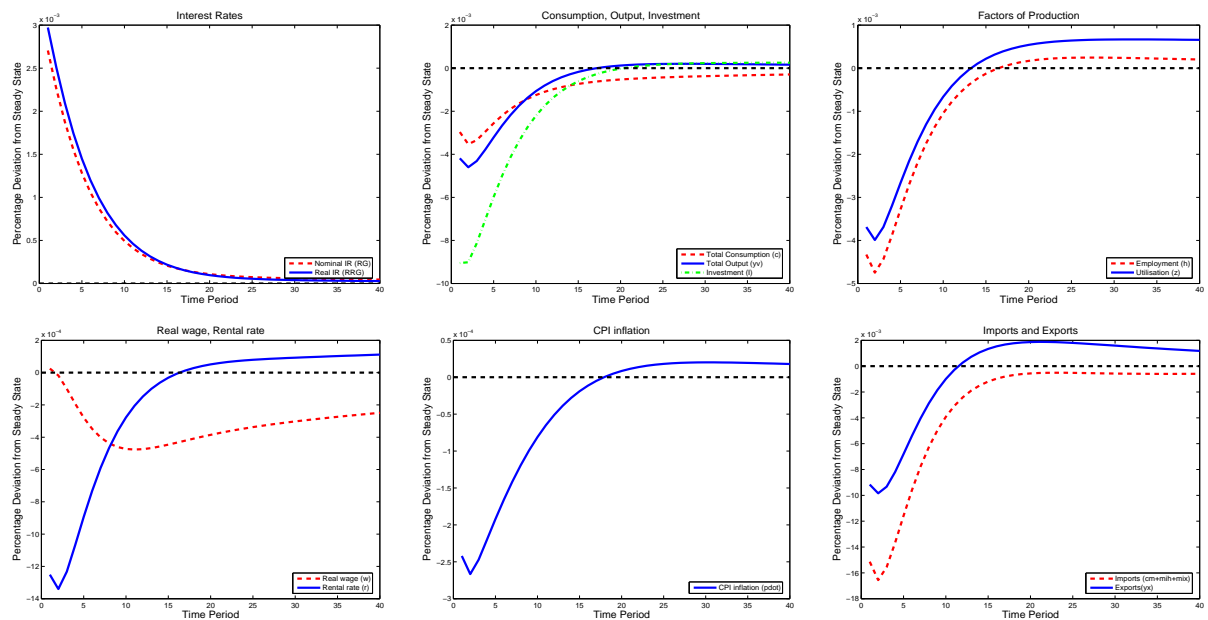
## Chart 7: Positive labour supply shock



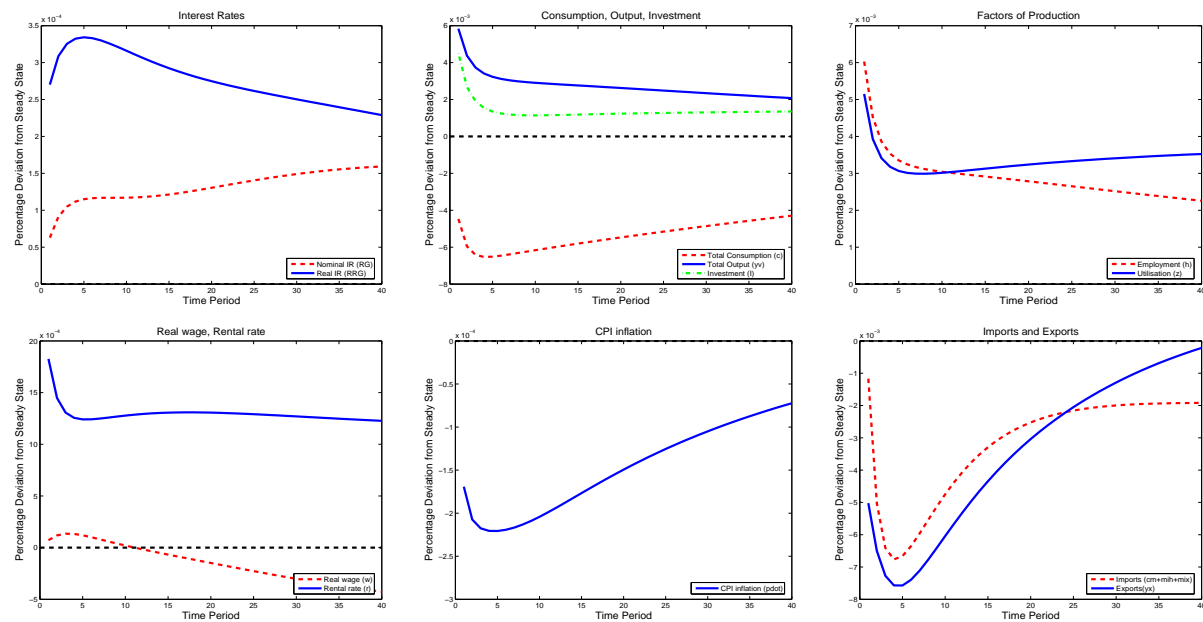
## Chart 8: Positive price mark-up shock



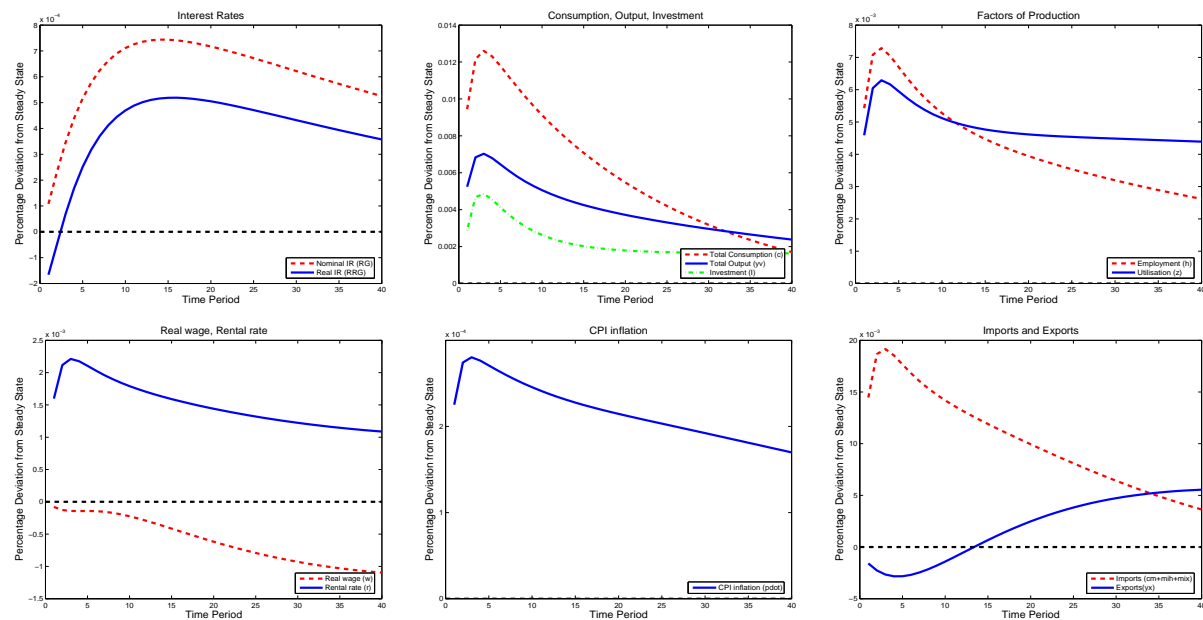
## Chart 9: Contractionary monetary policy shock



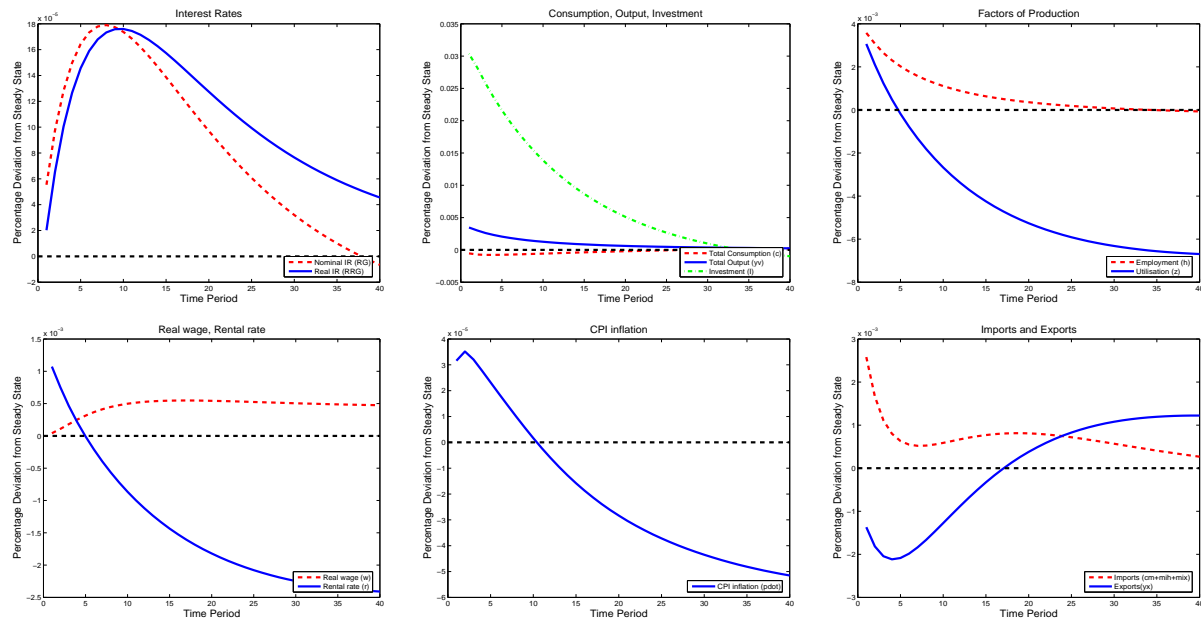
## Chart 10: Positive government spending shock



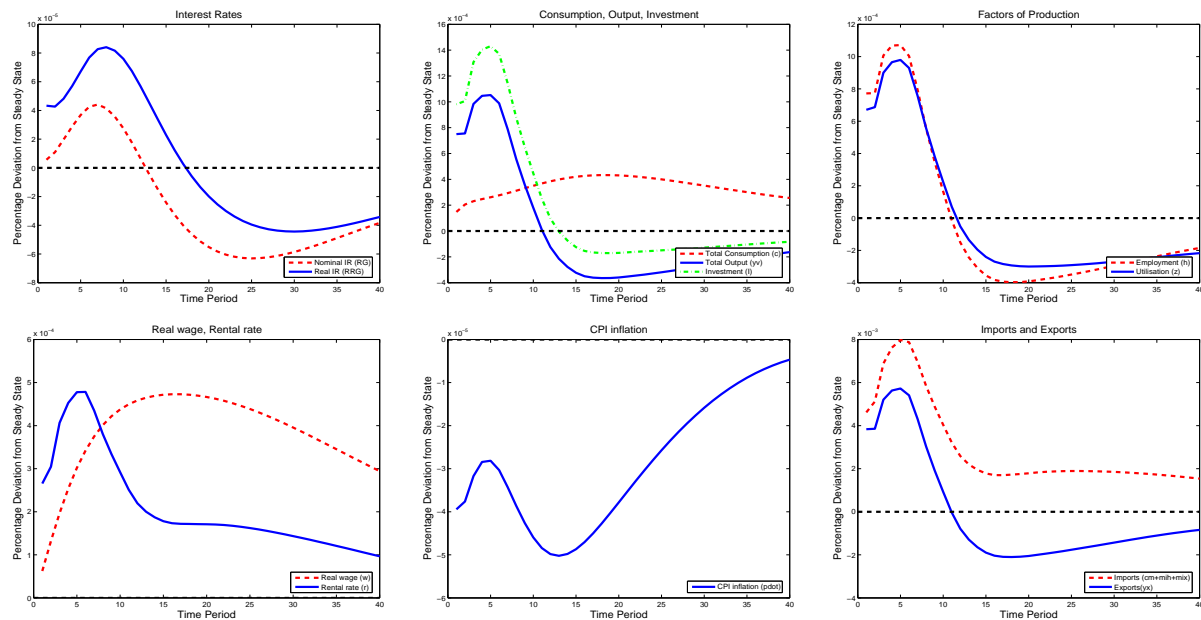
## Chart 11: Positive consumption shock



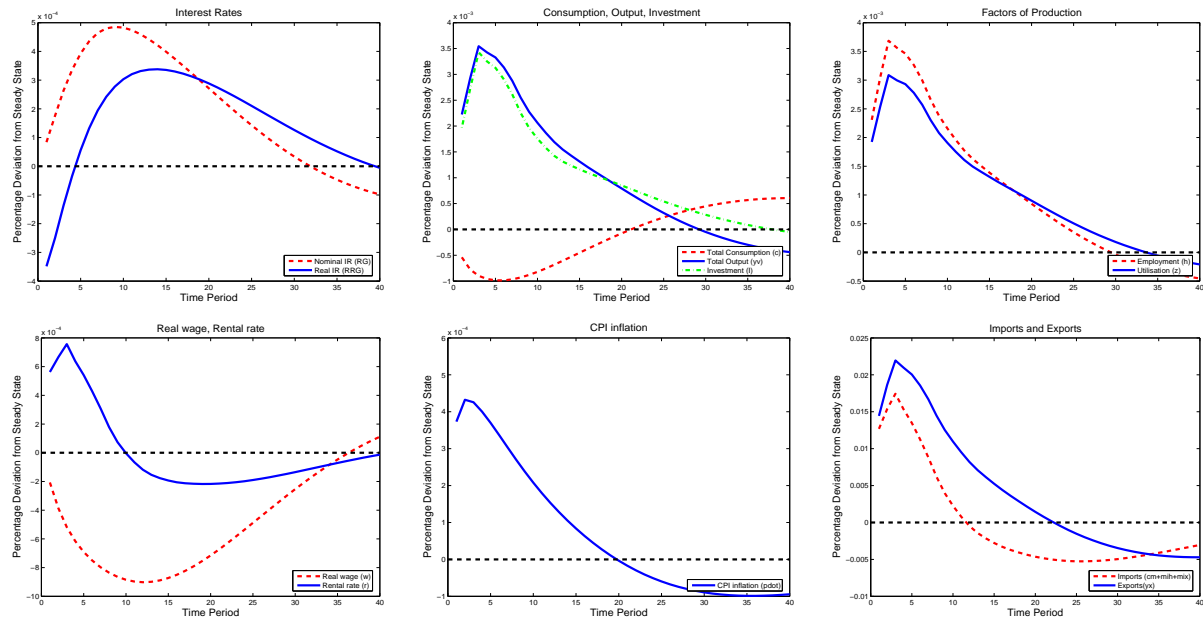
**Chart 12: Positive investment shock**



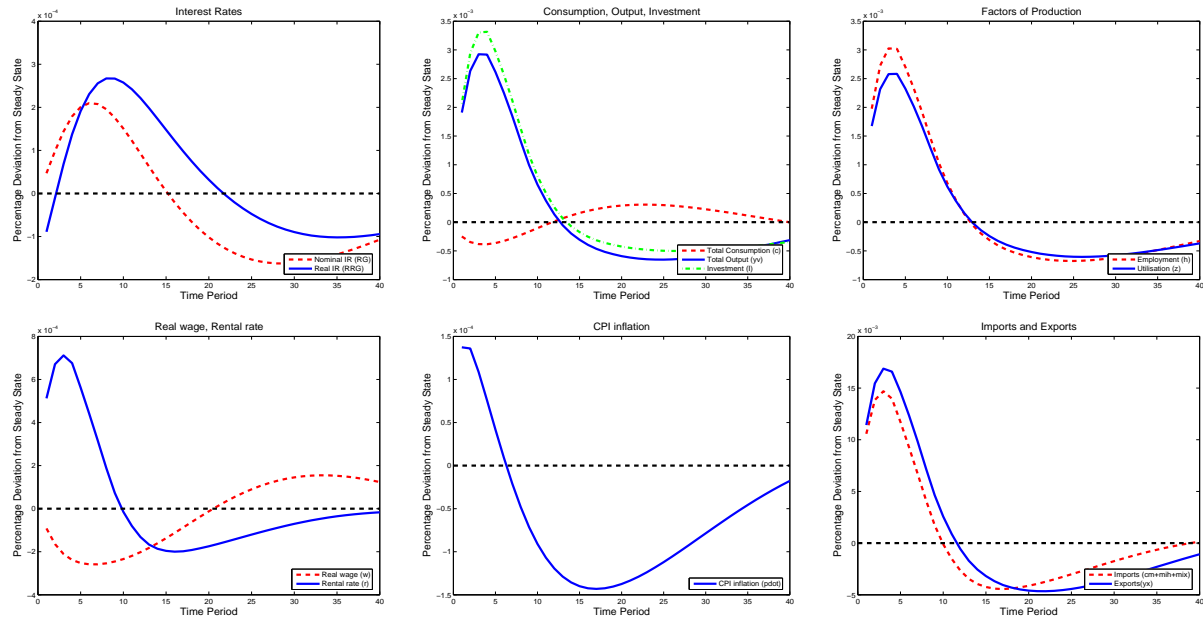
**Chart 13: Positive world demand shock**



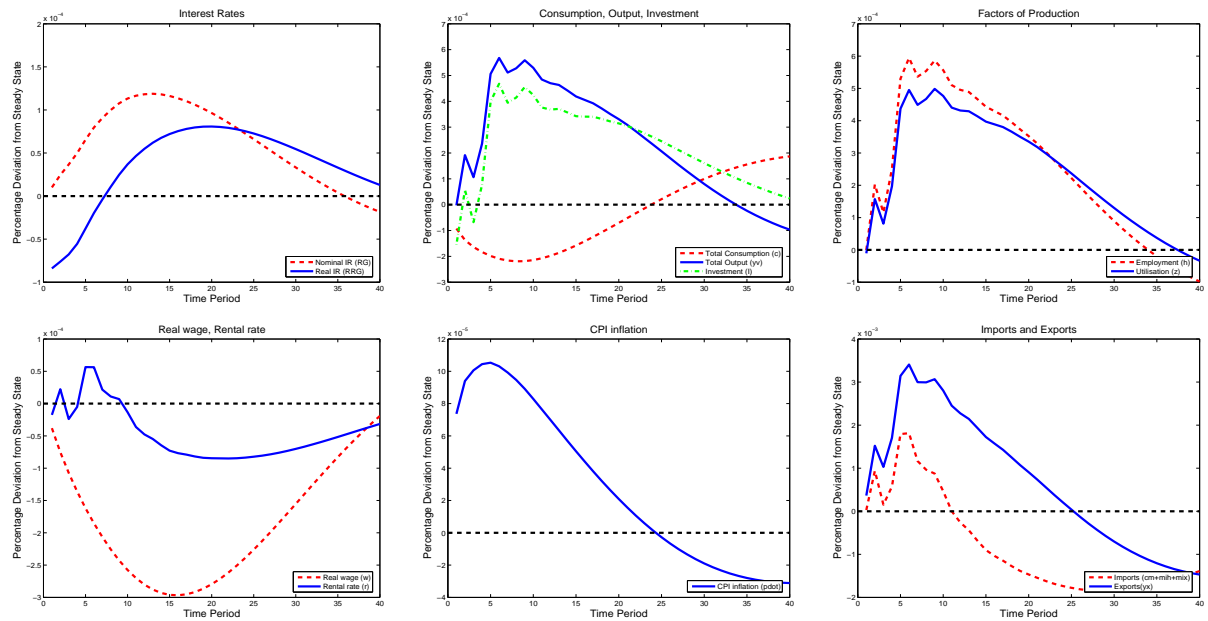
**Chart 14: Positive shock to world export prices**



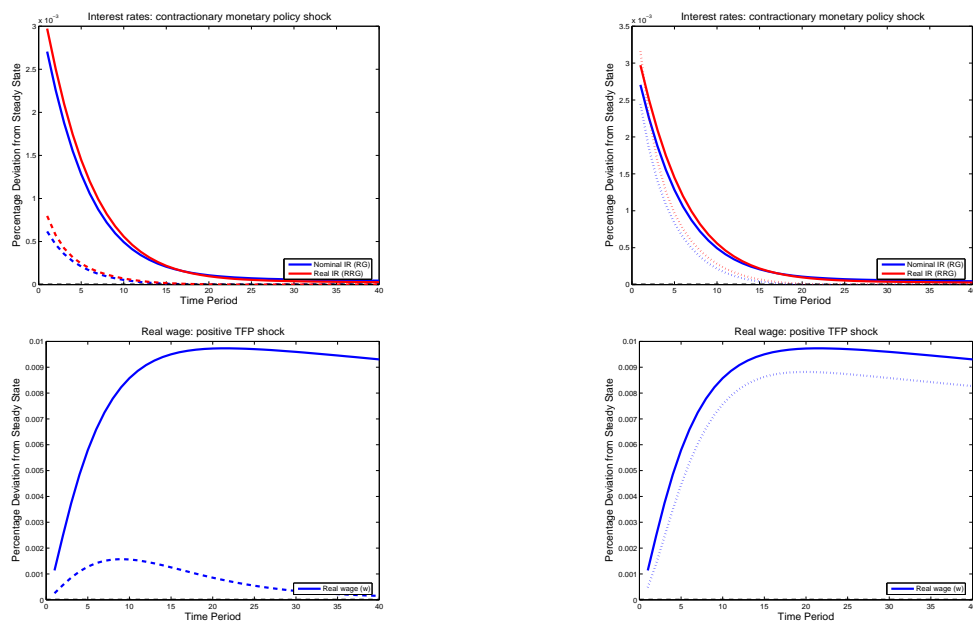
**Chart 15: Positive shock to world nominal interest rate**



**Chart 16: Positive shock to world inflation**



**Chart 17: Impulse response comparison: calibrated versus estimated model**





**Table D: The log-linearised model equations**

	FOCs
PPI	$\begin{aligned} p\hat{h}v_t &= \hat{\Psi}_t^{hv} + \hat{m}c_t \\ p\hat{w}_t &= \hat{\Psi}_t^{wv} + \hat{m}c_t \\ p\hat{x}v_t &= \hat{q}_t + p\hat{x}v_t \\ p\hat{h}_t &= \kappa^{hv} \frac{p\hat{h}v_t}{p\hat{x}v_t} p\hat{h}v_t + (1 - \kappa^{hv}) \frac{p\hat{m}^{ss}}{p\hat{h}^{ss}} \hat{p}\hat{m}_t \\ p\hat{x}_t &= \kappa^{xv} \frac{p\hat{x}v_t}{p\hat{x}v_t} p\hat{x}v_t + (1 - \kappa^{xv}) \frac{p\hat{m}^{ss}}{p\hat{x}^{ss}} \hat{p}\hat{m}_t \\ p\hat{t}_t &= \kappa^{tv} \frac{p\hat{t}v_t}{p\hat{t}v_t} p\hat{t}v_t + (1 - \kappa^{tv}) \frac{p\hat{m}^{ss}}{p\hat{t}^{ss}} \hat{p}\hat{m}_t \\ p\hat{t}_t &= \kappa^{tv} \frac{p\hat{t}v_t}{p\hat{t}v_t} p\hat{t}v_t + (1 - \kappa^{tv}) \frac{p\hat{m}^{ss}}{p\hat{t}^{ss}} \hat{p}\hat{m}_t \end{aligned}$
MP	$\hat{p}_t^m = \kappa^{mv} \frac{p\hat{m}^{ss}}{p\hat{m}^{ss}} p\hat{m}_t + (1 - \kappa^{mv}) \frac{p\hat{m}^{ss}}{p\hat{m}^{ss}} \hat{p}\hat{m}_t$
MU	$\hat{p}_t^m = \kappa^{mv} \frac{p\hat{m}^{ss}}{p\hat{m}^{ss}} p\hat{m}_t + (1 - \kappa^{mv}) \frac{p\hat{m}^{ss}}{p\hat{m}^{ss}} \hat{p}\hat{m}_t$
DMD	$\begin{aligned} \hat{c}_t &= \frac{ch^{ss}}{c^{ss}} p\hat{h}^{ss} (p\hat{h}_t + \hat{c}h_t) + \frac{cm^{ss}}{c^{ss}} p\hat{m}^{ss} (p\hat{m}_t + \hat{c}m_t) \\ \hat{c}h_t &= \hat{c}_t - \sigma^m p\hat{h}_t \\ \hat{c}m_t &= \hat{c}_t - \sigma^m p\hat{m}_t \\ \delta \hat{I}_t &= \hat{I}_t - (1 - \delta) \hat{I}_{t-1} + \chi^z \hat{z}_t \\ \hat{x}_t &= \hat{c}f_t - \sigma^x (\hat{q}_t + p\hat{x}_t - p\hat{x}f_t) \end{aligned}$
SUP	$\hat{y}_t^v = \hat{t}f_t p_t + \frac{(1 - \alpha)(h^{ss})}{\sigma^y - 1} \frac{\sigma^y - 1}{\sigma^y} \hat{h}_t + \frac{\alpha(k^{ss})}{[(1 - \alpha)(h^{ss}) \frac{\sigma^y - 1}{\sigma^y} + \alpha(k^{ss}) \frac{\sigma^y - 1}{\sigma^y}]} \hat{k}_t^s$
MUC	$\begin{aligned} \hat{y}_t^h &= \hat{y}_t^{hv} = \hat{m}h_t \\ \hat{y}_t^x &= \hat{y}_t^{xv} = \hat{m}x_t \\ \hat{U}_{c,t} &= \left( \frac{\psi^{hab} - \sigma^c \psi^{hab}}{\sigma^c} \right) \hat{c}_{t-1} - \frac{1}{\sigma^c} \hat{c}_t \end{aligned}$
MRS	$\hat{m}^{st} = \frac{1}{\sigma^h} \hat{h}_t - \left( \frac{\psi^{hab} - \sigma^c \psi^{hab}}{\sigma^c} \right) \hat{c}_{t-1} + \frac{1}{\sigma^c} \hat{c}_t$
INFs	$\begin{aligned} p\hat{h}v_t &= \hat{p}_t + p\hat{h}v_t - p\hat{h}v_{t-1} \\ p\hat{x}v_t &= \hat{p}_t^f + p\hat{x}v_t - p\hat{x}v_{t-1} \\ p\hat{t}_t &= \hat{p}_t + p\hat{t}_t - p\hat{t}_{t-1} \\ \hat{w}_t &= \hat{p}_t + \hat{w}_t - \hat{w}_{t-1} \end{aligned}$
FOCs	$\begin{aligned} \hat{U}_{c,t+1} &= \hat{U}_{c,t} - R\hat{R}G_t \\ \hat{m}\hat{m}_t &= -\sigma^c (\hat{U}_{c,t} + \frac{\beta}{1 - \beta} \hat{R}G_t) \\ p\hat{h}_t &= \chi^k (\hat{k}_t - \hat{k}_{t-1} - \epsilon^k (\hat{k}_{t+1} - \hat{k}_t - \epsilon^k (\hat{k}_t - \hat{k}_{t-1}))) + R\hat{R}G_t = \beta \chi^z \hat{r}_{t+1} \\ + \beta (1 - \delta) p\hat{h}_{t+1} + \beta \chi^k (\hat{k}_{t+1} - \hat{k}_t - \epsilon^k (\hat{k}_t - \hat{k}_{t-1})) \\ \hat{r}_t &= p\hat{h}_t + \sigma^z \hat{z}_t \\ \hat{w}_t - \hat{p}^{ss} &= \frac{\psi^w (1 - \psi^w)}{(1 - \psi^w)(1 + \beta \epsilon^w)} (1 + \frac{\sigma^w}{\sigma^h})^{-1} (\hat{m}\hat{r}_t - \hat{w}_t) + \frac{\beta}{(1 + \beta \epsilon^w)} (\hat{w}_{t+1} - \hat{p}^{ss}) \\ + \frac{\epsilon^w}{(1 + \beta \epsilon^w)} (\hat{w}_t - 1 - \hat{p}^{ss}) \\ \hat{h}_t &= -\sigma^y \hat{r}_t + \sigma^y \hat{m}c_t + \hat{y}_t^v + (\sigma^y - 1) \hat{t}f_t p_t \\ \hat{k}_t^s &= -\sigma^y \hat{r}_t + \sigma^y \hat{m}c_t + \hat{y}_t^v + (\sigma^y - 1) \hat{t}f_t p_t \\ \hat{k}_t^s &= \hat{z}_t + \hat{k}_{t-1} \\ \hat{q}_{t+1} - \hat{q}_t &= \chi^{bf} (n\hat{f}_{a_t} - n\hat{f}a^{ss}) = R\hat{R}F_t - R\hat{R}G_t \\ n\hat{f}_{a_t} &= \frac{1}{\beta} n\hat{f}_{a_{t-1}} + \frac{1}{\beta} n\hat{f}a^{ss} \hat{R}F_{t-1} - \frac{1}{\beta} n\hat{f}a^{ss} \hat{p}_t^f \\ + \frac{1}{\beta} n\hat{f}a^{ss} \hat{q}_{t-1} - \frac{1}{\beta} n\hat{f}a^{ss} \hat{q}_t + p\hat{x}^{ss} \hat{y}_t^{ss} (\hat{p}x_t + \hat{y}_t^x) \\ - pm^{ss} (cm^{ss} + m\hat{h}^{ss} + m\hat{t}^{ss}) \hat{p}\hat{m}_t - pm^{ss} cm^{ss} \hat{c}m_t \\ - pm^{ss} m\hat{h}^{ss} \hat{m}h_t - pm^{ss} m\hat{t}^{ss} \hat{m}t_t \\ \hat{r}_t &= \theta^{rg} \hat{r}_{t-1} + (1 - \theta^{rg}) [\theta^p (\hat{p}_t - \hat{p}^{ss}) + \theta^y (\hat{y}_t^v - \hat{t}f_t p_t)] \\ \hat{y}_t^v &= \frac{\kappa^{hv} ch^{ss}}{y^{v,ss}} \hat{c}h_t + \frac{\kappa^{hv} f^{ss}}{y^{v,ss}} \hat{f}_t + \frac{\kappa^{xv} g^{ss}}{y^{v,ss}} \hat{g}_t + \frac{\kappa^{xv} x^{ss}}{y^{v,ss}} \hat{x}_t \\ \hat{y}_t^v &= \frac{y^{v,ss}}{y^{v,ss}} \hat{y}^{hv} + \frac{y^{v,ss}}{y^{v,ss}} \hat{y}^{xv} \\ \hat{y}_t^x &= \hat{x}_t \\ \hat{R}G_t &= R\hat{R}G_t + (E_t \hat{p}_{t+1} - \hat{p}^{ss}) \end{aligned}$
MPR	$\hat{r}_t = \theta^{rg} \hat{r}_{t-1} + (1 - \theta^{rg}) [\theta^p (\hat{p}_t - \hat{p}^{ss}) + \theta^y (\hat{y}_t^v - \hat{t}f_t p_t)]$
GMC	$\hat{y}_t^v = \frac{\kappa^{hv} ch^{ss}}{y^{v,ss}} \hat{c}h_t + \frac{\kappa^{hv} f^{ss}}{y^{v,ss}} \hat{f}_t + \frac{\kappa^{xv} g^{ss}}{y^{v,ss}} \hat{g}_t + \frac{\kappa^{xv} x^{ss}}{y^{v,ss}} \hat{x}_t$
FPC	$\hat{R}G_t = R\hat{R}G_t + (E_t \hat{p}_{t+1} - \hat{p}^{ss})$

Note. [1] Abbreviations are used for Consumer Price Index (CPI), Producer Price Index (PPI), Import Price (MP), Mark-Up (MU), Demand (DMD), Supply (SUP), Marginal Utility of Consumption (MUC), First-Order Conditions (FOCs), Utilisation Rate (UR), Real UIP (RUIP), Net Foreign Assets (NFA), Monetary Policy Rule (MPR), Goods Market Clearing (GMC), Fisher Parity Condition (FPC), and Inflation Equations (INFs). [2]  $\hat{R}G_t \cong \hat{r}_t^f$ ,  $\hat{R}F_t \cong \hat{r}_t^f$ ,  $\hat{R}R_{G_t} \cong \hat{r}_t^f$ ,  $\hat{R}R_{F_t} \cong \hat{r}_t^f$ ,  $\hat{r}_t^f \cong n\hat{f}_{a_t} - n\hat{f}a^{ss}$

**Table E: Watson's statistic for model fit**

	Equal weight	Weight on C	Weight on HW	Weight on INV	Weight on INF	Weight on NIR	Weight on RW	Weight on Y
<b>PANEL A. BASELINE MODEL</b>								
<i>C</i>	0.339 [0.001,0.495]	0.817 [0.631,0.953]	-1.077 [-1.755,-0.733]	0.101 [-0.584,0.147]	-0.322 [-0.890,0.000]	-0.251 [-0.999,0.148]	-0.387 [-1.080,-0.057]	-0.534 [-1.080,-0.312]
<i>HW</i>	0.124 [0.046,0.224]	-0.288 [-0.444,-0.200]	0.355 [0.251,0.526]	-0.034 [-0.110,0.095]	0.095 [-0.271,-0.015]	-0.133 [-0.252,-0.027]	0.099 [-0.036,0.137]	0.037 [-0.036,0.110]
<i>INV</i>	0.186 [0.124,0.292]	0.0489 [0.000,0.075]	0.016 [-0.028,0.060]	0.192 [0.130,0.307]	-0.049 [-0.094,-0.016]	-0.033 [-0.082,0.011]	0.069 [0.018,0.086]	0.064 [0.041,0.103]
<i>INF</i>	-0.041 [-0.114,0.037]	-0.039 [-0.120,0.045]	-0.097 [-0.160,-0.027]	-0.098 [-0.153,-0.053]	0.441 [0.368,0.515]	-0.013 [-0.106,0.078]	-0.197 [-0.265,-0.124]	-0.126 [-0.161,-0.071]
<i>NIR</i>	-0.224 [-0.666,0.005]	-0.432 [-0.996,-0.155]	-0.667 [-1.029,-0.371]	-0.570 [-1.017,-0.352]	-0.312 [-0.873,-0.016]	0.753 [0.642,0.822]	-0.630 [-1.135,-0.321]	-0.596 [-0.985,-0.429]
<i>RW</i>	0.397 [0.275,0.538]	-0.141 [-0.407,0.045]	0.056 [-0.107,0.217]	0.092 [-0.058,0.232]	-0.447 [-0.655,-0.277]	-0.138 [-0.447,0.049]	0.499 [0.360,0.692]	-0.045 [-0.221,0.067]
<i>Y</i>	0.376 [0.242,0.529]	-0.185 [-0.362,-0.113]	0.079 [-0.078,0.158]	0.126 [0.039,0.221]	-0.145 [-0.381,-0.078]	-0.145 [-0.369,-0.078]	0.074 [-0.217,0.051]	0.570 [0.424,0.747]
<b>PANEL B. FLEXIBLE PRICE MODEL</b>								
<i>C</i>	0.762 [0.168,0.829]	0.951 [0.668,0.973]	0.449 [-0.518,0.552]	0.152 [-1.023,0.473]	-0.865 [-2.220,-0.272]	-0.318 [-2.226,0.059]	-1.2165 [-2.748,-0.561]	-0.121 [-1.153,0.283]
<i>HW</i>	0.288 [0.147,0.384]	0.300 [0.232,0.380]	0.435 [0.317,0.618]	0.207 [0.003,0.289]	-0.0260 [-0.242,0.124]	0.082 [-0.224,0.164]	-0.263 [-0.516,-0.132]	0.195 [0.051,0.300]
<i>INV</i>	0.469 [0.343,0.635]	0.231 [0.090,0.352]	0.225 [0.026,0.322]	0.489 [0.351,0.690]	-0.133 [-0.354,0.048]	0.020 [-0.353,0.142]	-0.011 [-0.228,0.118]	0.353 [0.258,0.430]
<i>INF</i>	-0.106 [-0.390,0.068]	-0.760 [-1.238,-0.469]	-0.880 [-1.391,-0.568]	-0.907 [-1.394,-0.600]	0.776 [0.680,0.829]	0.818 [-1.334,-0.448]	-1.213 [-1.618,-0.928]	-0.841 [-1.272,-0.566]
<i>NIR</i>	-1.533 [-3.027,-0.775]	-1.799 [-3.269,-0.887]	-1.861 [-3.444,-1.110]	-1.866 [-3.471,-1.012]	-1.607 [-3.146,-0.765]	0.513 [-0.149,0.761]	-2.209 [-3.597,-1.340]	-1.852 [-3.171,-0.946]
<i>RW</i>	-0.022 [-0.242,0.139]	-0.199 [-0.440,-0.023]	-0.324 [-0.580,-0.182]	-0.026 [-0.229,0.123]	-0.432 [-0.629,-0.271]	-0.147 [-0.410,0.057]	0.493 [0.348,0.695]	-0.045 [-0.261,0.081]
<i>Y</i>	0.710 [0.551,0.754]	0.084 [-0.288,0.345]	0.317 [-0.225,0.422]	0.510 [0.232,0.534]	-0.443 [-1.028,-0.222]	-0.141 [-0.324,0.082]	-0.226 [-0.962,-0.048]	0.850 [0.665,0.974]
<b>PANEL C. RBC TYPE MODEL</b>								
<i>C</i>	0.392 [-0.218,0.587]	0.928 [0.753,0.972]	0.584 [0.044,0.640]	-0.130 [-0.841,0.315]	-0.435 [-1.409,-0.018]	-0.193 [-1.324,0.260]	-0.891 [-1.903,-0.377]	0.163 [-0.570,0.454]
<i>HW</i>	0.251 [-0.061,0.405]	0.425 [0.312,0.497]	0.597 [0.452,0.791]	0.129 [-0.222,0.296]	-0.1565 [-0.521,0.096]	0.007 [-0.471,0.200]	-0.413 [-0.837,-0.210]	0.245 [0.003,0.401]
<i>INV</i>	0.677 [-0.075,0.807]	-0.705 [-2.687,-0.031]	-0.683 [-2.701,-0.031]	0.686 [-0.056,0.813]	-2.523 [-4.941,-1.531]	-1.969 [-4.570,-0.868]	-0.609 [-2.852,-0.048]	-0.230 [-1.937,0.179]
<i>INF</i>	-0.145 [-0.315,0.047]	-0.234 [-0.543,-0.087]	-0.398 [-0.613,-0.168]	-0.555 [-0.747,-0.391]	0.756 [0.660,0.830]	-0.248 [-0.500,-0.018]	-0.645 [-0.818,-0.480]	-0.458 [-0.624,-0.263]
<i>NIR</i>	-1.593 [-2.785,-0.906]	-1.258 [-2.422,-0.568]	-1.381 [-2.545,-0.740]	-1.701 [-2.838,-1.013]	-1.288 [-2.433,-0.616]	0.256 [-0.303,0.532]	-1.679 [-2.727,-0.895]	-1.246 [-2.395,-0.573]
<i>RW</i>	0.189 [-0.139,0.354]	-0.362 [-0.709,-0.101]	-0.462 [-0.855,-0.250]	0.282 [-0.108,0.358]	-0.605 [-0.885,-0.387]	-0.170 [-0.600,0.097]	0.630 [0.471,0.802]	-0.005 [-0.388,0.156]
<i>Y</i>	0.610 [0.354,0.742]	0.179 [-0.606,0.411]	0.267 [-0.607,0.469]	0.366 [-0.262,0.451]	-0.991 [-1.905,-0.577]	-0.367 [-1.557,0.085]	-0.505 [-1.439,-0.086]	0.945 [0.750,0.981]

Note. The table reports Watson's statistic for model fit. Numbers in parentheses are 95% confidence intervals. For details refer to Section 3.3.2.

**Table F: Modified log-linearised model equations**

Shock	
PANEL A: NON-LINEAR EQUATIONS	
Consumption	$U_t(c) = \zeta_t^{\text{con}} \frac{\sigma^c}{\sigma^c - 1} \left[ \frac{c_{t+r}(j)}{c_{t+r-1}} \right]^{\frac{\sigma^c - 1}{\sigma^c}}$
Labour supply	$-U_t(h) = [\zeta_t^{hw}]^{-1} (\kappa^h)^{-\frac{1}{\sigma^h}} \frac{\sigma^h}{\sigma^h + 1} [h_{t+r}(j)]^{\frac{\sigma^h + 1}{\sigma^h}}$
Domestic market mark-up	$\Psi_t^{hv} = \zeta_t^{\text{mup}} \sigma^{hb} [(\zeta_t^{\text{mup}} \sigma^{hb} - 1) + \chi^{hv} \Upsilon_t^{hv}]^{-1}$
Capital adjustment cost	$\Delta_t^k(j) = \frac{\chi^k}{2} \frac{[k_t(j) - (\frac{k_{t-1}}{k_{t-2}})^{\epsilon^k} k_{t-1}(j) + \frac{k_{t-1}}{\chi^k} \zeta_t^{\text{inv}}]^2}{k_{t-1}}$
PANEL B: LOG-LINEARISED EQUATIONS	
Consumption	$\hat{c}_t = \frac{1}{1 + \psi^{hab} - \psi^{hab} \sigma^c} E_t \hat{c}_{t+1} + \frac{\psi^{hab} - \psi^{hab} \sigma^c}{1 + \psi^{hab} - \psi^{hab} \sigma^c} \hat{c}_{t-1} - \frac{\sigma^c}{1 + \psi^{hab} - \psi^{hab} \sigma^c} r \hat{r}_t g_t + \frac{\sigma^c}{1 + \psi^{hab} - \psi^{hab} \sigma^c} (\hat{\zeta}_t^{\text{con}} - E_t \hat{\zeta}_{t+1}^{\text{con}})$
Labour supply	$\hat{m} \hat{r}_t s_t = \frac{1}{\sigma^h} \hat{h}_t - \left( \frac{\psi^{hab} - \sigma^c \psi^{hab}}{\sigma^c} \right) \hat{c}_{t-1} + \frac{1}{\sigma^c} \hat{c}_t - \hat{\zeta}_t^{hw} - \zeta_t^{hw} + \frac{e^{hv}}{1 + \beta \epsilon^{hv}} (\hat{p}_{t-1}^{hw} - \hat{p}_t^{ss}) - \frac{1}{1 + \beta \epsilon^{hv}} \hat{\zeta}_t^{\text{mup}}$
Domestic market mark-up	$\hat{p}_t^{hv} - \hat{p}_t^{ss} = \frac{\beta}{1 + \beta \epsilon^k} (E_t \hat{k}_{t+1} - \hat{k}_t) + \frac{e^k}{1 + \beta \epsilon^k} (\hat{k}_{t-1} - \hat{k}_{t-2}) + \frac{\beta \chi^z}{\chi^k (1 + \beta \epsilon^k)} E_t \hat{r}_{t+1} + \frac{1}{\chi^k (1 + \beta \epsilon^k)} \hat{p}_t^{hv}$
Capital adjustment cost	$\hat{k}_t - \hat{k}_{t-1} = - \frac{1}{\chi^k (1 + \beta \epsilon^k)} r \hat{r}_t g_t - \frac{1}{\chi^k (1 + \beta \epsilon^k)} (\hat{\zeta}_t^{\text{inv}} - \beta E_t \hat{\zeta}_{t+1}^{\text{inv}})$

Note. The model shows the model's log-linearised equations after incorporating structural shocks to consumption, labour supply, mark-up in domestic and export markets, and cost of capital adjustment. For details refer to Section 2. We assume that all shocks follow a first-order autoregressive process with an i.i.d. normal error term with zero mean and unit variance, that is  $\hat{\zeta}_t^i = \rho_i \hat{\zeta}_{t-1}^i + \sigma_i \eta_t^i$ , where  $i = \text{con}, \text{hw}, \text{mup}, \text{inv}$ .

**Table G: Bayesian estimation: prior and posterior**

	Prior			Posterior			
	Dist	Mean	Std dev	Mode	Mean	Confidence interval	
$\psi^{hab}$	B	0.690	0.050	0.404	0.417	0.353	0.469
$\psi^w$	B	0.210	0.050	0.177	0.152	0.075	0.207
$\psi^{pm}$	B	0.400	0.150	0.081	0.090	0.042	0.139
$\sigma^c$	N	0.660	0.198	0.153	0.143	0.084	0.194
$\sigma^h$	N	0.430	0.107	0.145	0.142	0.085	0.188
$\epsilon^{hv}$	B	0.260	0.100	0.241	0.279	0.097	0.465
$\epsilon^{xv}$	B	0.140	0.050	0.107	0.122	0.049	0.193
$\epsilon^w$	B	0.580	0.145	0.571	0.526	0.385	0.673
$\epsilon^m$	B	0.170	0.050	0.164	0.178	0.092	0.262
$\epsilon^k$	B	0.500	0.250	0.019	0.048	0.0004	0.093
$\chi^{hv}$	N	326	97.800	227.678	264.973	140.376	399.285
$\chi^{xv}$	N	43	12.500	52.029	52.714	34.764	70.768
$\chi^k$	N	201	60.300	181.784	200.397	121.484	265.159
$\sigma^z$	N	0.560	0.168	0.346	0.410	0.185	0.633
$\theta^{rg}$	B	0.870	0.050	0.873	0.881	0.859	0.906
$\theta^p$	N	1.870	0.281	1.081	1.130	1.040	1.211
$\theta^y$	N	0.110	0.027	0.116	0.123	0.089	0.161
$\rho_{tfp}$	B	0.890	0.050	0.996	0.996	0.994	0.998
$\rho_g$	B	0.960	0.025	0.982	0.981	0.975	0.989
$\rho_{hw}$	U	0.500	0.288	0.001	0.038	0.001	0.083
$\rho_{con}$	U	0.500	0.288	0.966	0.959	0.944	0.978
$\rho_{inf}$	U	0.500	0.288	0.295	0.258	0.014	0.446
$\rho_{inv}$	U	0.500	0.288	0.932	0.924	0.888	0.964
$\sigma_g$	IG	0.009	2*	0.048	0.048	0.044	0.052
$\sigma_{hw}$	IG	0.010	2*	1.866	2.491	1.282	5.399
$\sigma_{mp}$	IG	0.001	2*	0.003	0.003	0.002	0.003
$\sigma_{tfp}$	IG	0.006	2*	0.009	0.009	0.009	0.010
$\sigma_{con}$	IG	0.025	2*	0.366	0.383	0.224	0.554
$\sigma_{inf}$	IG	0.006	2*	0.007	0.008	0.006	0.009
$\sigma_{inv}$	IG	0.060	2*	0.355	0.392	0.203	0.535

Distributions: N = Normal; B = Beta; U = Uniform; IG = Inverse-Gamma.

\*: Degrees of freedom reported for inverse Gamma

**Table H: Watson's statistic for model fit**

	<i>Equal weight</i>	<i>Weight on C</i>	<i>Weight on HW</i>	<i>Weight on INV</i>	<i>Weight on INF</i>	<i>Weight on NIR</i>	<i>Weight on RW</i>	<i>Weight on Y</i>
<b>PANEL A. ESTIMATED MODEL</b>								
<i>C</i>	-0.452 [-2.592,0.007]	0.477 [-1.359,0.839]	-2.909 [-5.542,-1.519]	-1.675 [-5.402,-1.100]	-2.342 [-5.555,-1.152]	-2.797 [-6.041,-1.513]	-3.092 [-6.157,-1.652]	-1.785 [-4.301,-0.428]
<i>HW</i>	0.658 [0.327,0.677]	-0.615 [-1.638,-0.384]	0.921 [0.716,0.947]	-0.490 [-1.459,-0.048]	-0.393 [-1.862,-0.267]	-1.152 [-1.862,-0.364]	-0.156 [-1.427,-0.098]	-0.039 [-1.221,0.095]
<i>INV</i>	0.867 [0.678,0.923]	-0.268 [-1.430,-0.126]	-0.374 [-1.242,0.000]	0.907 [0.719,0.965]	-0.317 [-1.394,-0.079]	-0.900 [-1.562,-0.230]	-0.223 [-1.405,-0.099]	0.165 [-0.712,0.176]
<i>INF</i>	0.214 [0.016,0.290]	-0.487 [-0.798,-0.284]	-0.445 [-0.818,-0.368]	-0.439 [-0.768,-0.290]	0.839 [0.706,0.927]	-0.583 [-0.851,-0.348]	-0.407 [-0.749,-0.234]	-0.268 [-0.616,-0.130]
<i>NIR</i>	-1.149 [-2.387,-0.620]	-2.413 [-4.172,-1.510]	-2.541 [-4.130,-1.699]	-2.488 [-4.095,-1.558]	-2.294 [-4.137,-1.437]	0.341 [-0.577,0.681]	-2.576 [-4.270,-1.608]	-2.284 [-3.931,-1.444]
<i>RW</i>	0.544 [-0.115,0.682]	-1.401 [-2.777,-0.619]	-0.439 [-1.884,-0.153]	-1.011 [-2.263,-0.064]	-0.380 [-1.979,0.150]	-1.179 [-2.462,-0.292]	0.952 [0.501,0.977]	-0.424 [-1.930,0.076]
<i>Y</i>	0.237 [-1.201,0.482]	-1.068 [-3.436,-0.359]	-1.364 [-3.569,-0.512]	-0.961 [-3.213,-0.349]	-1.041 [-3.629,-0.271]	-2.658 [-4.717,-1.181]	-1.269 [-4.061,-0.655]	0.537 [-0.764,0.814]
<b>PANEL B. BASELINE MODEL</b>								
<i>C</i>	0.339 [0.001,0.495]	0.817 [0.631,0.953]	-1.077 [-1.755,-0.733]	0.101 [-0.584,0.147]	-0.322 [-0.890,0.000]	-0.251 [-0.999,0.148]	-0.387 [-1.080,-0.057]	-0.534 [-1.080,-0.312]
<i>HW</i>	0.124 [0.046,0.224]	-0.288 [-0.444,-0.200]	0.355 [0.251,0.526]	-0.034 [-0.110,0.095]	-0.095 [-0.271,-0.015]	-0.133 [-0.252,-0.027]	0.099 [-0.053,0.137]	0.037 [-0.036,0.110]
<i>INV</i>	0.186 [0.124,0.292]	0.0489 [0.000,0.075]	0.016 [-0.028,0.060]	0.192 [0.130,0.307]	-0.049 [-0.094,-0.016]	-0.033 [-0.082,0.011]	0.069 [0.018,0.086]	0.064 [0.041,0.103]
<i>INF</i>	-0.041 [-0.114,0.037]	-0.039 [-0.120,0.045]	-0.097 [-0.160,-0.027]	-0.098 [-0.153,-0.053]	0.441 [0.368,0.515]	-0.013 [-0.106,0.078]	-0.197 [-0.265,-0.124]	-0.126 [-0.161,-0.071]
<i>NIR</i>	-0.224 [-0.666,0.005]	-0.432 [-0.996,-0.155]	-0.667 [-1.029,-0.371]	-0.570 [-1.017,-0.352]	-0.312 [-0.873,-0.016]	0.753 [0.642,0.822]	-0.630 [-1.135,-0.321]	-0.596 [-0.985,-0.429]
<i>RW</i>	0.397 [0.275,0.538]	-0.141 [-0.407,0.045]	0.056 [-0.107,0.217]	0.092 [-0.058,0.232]	-0.447 [-0.655,-0.277]	-0.138 [-0.447,0.049]	0.499 [0.360,0.692]	-0.045 [-0.221,0.067]
<i>Y</i>	0.376 [0.242,0.529]	-0.185 [-0.362,-0.113]	0.079 [-0.078,0.158]	0.126 [0.039,0.221]	-0.145 [-0.381,-0.078]	-0.145 [-0.369,-0.078]	0.074 [-0.217,0.051]	0.570 [0.424,0.747]
<b>PANEL C. BASELINE MODEL WITH ESTIMATED PARAMETERS FOR SHOCK PROCESSES</b>								
<i>C</i>	-0.292 [-2.765,0.239]	0.533 [-1.332,0.843]	-3.356 [-6.912,-2.194]	-1.691 [-4.765,-0.367]	-2.409 [-5.597,-1.085]	-2.954 [-5.735,-0.807]	-2.342 [-6.201,-1.343]	-1.440 [-4.531,-0.511]
<i>HW</i>	0.128 [-0.144,0.270]	-0.640 [-1.092,-0.401]	0.704 [0.540,0.871]	0.182 [-0.497,0.225]	-0.349 [-0.772,-0.102]	-0.241 [-0.841,-0.115]	-0.207 [-0.583,0.031]	-0.014 [-0.427,0.175]
<i>INV</i>	0.577 [0.427,0.757]	0.046 [-0.145,0.240]	0.169 [-0.213,0.206]	0.596 [0.440,0.797]	-0.259 [-0.441,0.010]	-0.165 [-0.481,0.085]	0.061 [-0.230,0.221]	0.413 [0.297,0.483]
<i>INF</i>	0.010 [-0.073,0.077]	-0.086 [-0.134,-0.005]	-0.052 [-0.151,-0.036]	-0.073 [-0.155,-0.030]	0.516 [0.435,0.593]	-0.093 [-0.186,-0.023]	-0.185 [-0.271,-0.116]	-0.09 [-0.174,-0.056]
<i>NIR</i>	-0.762 [-1.535,-0.369]	-1.062 [-1.945,-0.634]	-1.512 [-2.233,-0.971]	-1.176 [-1.951,-0.605]	-1.137 [-1.944,-0.680]	0.925 [0.715,0.967]	-1.214 [-2.050,-0.776]	-1.094 [-1.944,-0.715]
<i>RW</i>	0.426 [-0.367,0.679]	-0.912 [-2.236,-0.376]	-0.450 [-1.432,-0.017]	-0.054 [-1.383,0.270]	-1.466 [-2.177,-0.686]	-0.948 [-2.047,-0.184]	0.928 [0.608,0.948]	-0.233 [-1.329,0.298]
<i>Y</i>	0.375 [-1.223,0.697]	-1.446 [-3.861,-0.444]	-1.167 [-3.975,-0.481]	-0.179 [-2.166,0.422]	-2.416 [-4.915,-1.201]	-2.205 [-4.802,-0.937]	-1.318 [-4.041,-0.422]	0.508 [-0.888,0.791]

Note. Panel A reports Watson's statistics for model fit for the baseline model after it has been augmented with additional structural shocks and estimated. Panel B reports Watson's statistics for the baseline model. Panel C reports Watson's statistics for model fit for the baseline model with estimated parameters for shock processes. For details refer to Sections 3.3.2, 3.4, and 3.5.

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