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Evolving macroeconomic dynamics in a small open economy: an estimated Markov-switching DSGE model for the United Kingdom

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Philip Liu[(1)] and Haroon Mumtaz[(2)]

Abstract

This paper carries out a systematic investigation into the possibility of structural shifts in the UK economy using a Markov-switching dynamic stochastic general equilibrium (DSGE) model. We find strong evidence for shifts in the structural parameters of several equations of the DSGE model. In addition, our results indicate that the volatility of structural shocks has also changed over time. However, a version of the model that allows for a change in the coefficients of the Taylor rule and shock volatilities provides the best model fit. Estimates from the selected DSGE model suggest that the mid-1970s were associated with a regime characterised by a smaller reaction by the monetary authorities to inflation developments.

Key words: Markov switching, DSGE, Bayesian estimation.

JEL classification: E23, E32.

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Summary

The United Kingdom has experienced major structural and economic changes over the past three decades. In a large empirical literature, researchers have argued that these changes have manifested themselves as shifts in the dynamics of macroeconomic variables, with a number of papers focusing on documenting these changes. An understanding of what lies behind and the consequences of these changes is obviously important for the conduct of monetary policy.

However, much of the work on the UK economy is subject to a number of criticisms. Among these, first, studies are typically formulated in a closed economy setting. This is surprising given the fact that the United Kingdom is a small open economy and international developments have become increasingly important. Second, they typically employ vector autoregressions (VARs), systems of regression equations which simply specify each variable of interest as a function of past values of all variables included in the model. Although VARs have the distinct advantage of simplicity and flexibility, they do not always deliver a clear economic interpretation of shocks hitting the economy.

The aim of this paper is to investigate structural changes in the United Kingdom using a model where these criticisms are mitigated. We examine the evolving structure using an estimated open economy dynamic stochastic general equilibrium model (DSGE) where the parameters of key structural equations are allowed to change periodically over time. DSGEs are models where all the dynamic linkages between variables are transparently explained in terms of the behaviour of firms, households or the policymaker. The 'stochastic' part means that unexpected shocks continually hit the economy. So unlike VARs, the DSGE model explicitly incorporates expectations of agents (for example, the public and the central bank) into the modelling process and provides a clear interpretation of shocks that are assumed to hit the economy at any given time. We estimate several different versions of this model – ie, versions that allow parameters of different structural equations to change over time. We then use statistical criteria to test how well each version of the model fits UK data. The changing dynamics of the UK economy are examined using the best-fitting model.

This turns out to be a very plausible one. One feature is that periods of turbulence come and go,

but were infrequent between 1992 and the recent past, although the results towards the end of our sample in 2007/08 and early 2009 are characterised by high volatility. Moreover, these estimates from the chosen model suggest that the mid 1970s were characterised by small reactions by the monetary authorities to inflation. As a consequence, output, inflation and the real exchange were more volatile then than the recent past.

1 Introduction

The United Kingdom has experienced major structural and economic changes over the past three decades. A large empirical literature has argued that these changes have manifested themselves as shifts in the dynamics of macroeconomic variables. For example Benati (2008) shows that the 1970s and the 1980s were characterised by volatile inflation and output growth. In addition, the persistence of inflation was estimated to be high during this period. In contrast the period after the introduction of inflation targeting in 1992 was associated with low inflation and output volatility and low inflation persistence.

A related strand of this research has focused on the task of trying to establish the extent to which changes in inflation and output dynamics can be linked to the change in the operation of monetary policy in the United Kingdom. Using a time-varying structural vector autoregression (VAR) model, Benati (2008) argues that a fall in the volatility of demand and supply shocks can explain most of the recent stability in the United Kingdom's output and inflation. A small number of VAR-based studies have focused on investigating the possibility of changes in the transmission of monetary policy in the United Kingdom. Castelnuovo and Surico (2006) show that the impact of (contractionary) monetary policy shocks on inflation was substantially different in the pre and post-inflation targeting period, with the inflation response large and positive in the earlier subsample and small and negative after 1992. Results in Benati (2008) suggest a similar conclusion – the fall in inflation as a result of an increase in interest rates was smaller in the 1970s and 1980s relative to the post-1992 period. The estimated output response, however, displays the opposite pattern, with GDP growth falling by less in the post-1992 period.

This literature on the possibility of changes in the UK monetary transmission mechanism is subject to a number of criticisms. First, (with the exception of Mumtaz and Sunder-Plassmann (2010)), these empirical studies are typically formulated in a closed economy setting. This is surprising given the fact that the United Kingdom is a small open economy and international developments have become increasingly important especially during the recent financial crisis. Second, although VAR-based studies have the distinct advantage of simplicity and flexibility, identification of shocks is not uncontroversial.

The aim of this paper is to investigate structural changes in the UK economy using an empirical model where these criticisms are mitigated. In particular, we examine the evolving structure of the UK economy using an estimated open economy DSGE model where the parameters of key structural equations are assumed to be subject to regime shifts, ie evolve as a Markov-switching process. As in Davig and Leeper (2007) and Farmer, Waggoner and Zha (2008) the model is solved under rational expectations and estimated using Bayesian methods. We examine different versions of this model – ie versions that allow different structural equations to 'switch' and examine the dynamics of the UK economy using the version that provides the best fit to our data.

This analysis contributes to the literature on the UK transmission mechanism along three dimensions. First, to our knowledge this is the first application of a Markov-switching DSGE model to UK data. The study therefore complements the analysis in Davig and Doh (2008) and Bianchi (2009) for the United States. Second, the estimated model allows us to examine evolving UK macroeconomic dynamics in a *structural* DSGE setting. This is in contrast to the existing empirical literature which has primarily approached this issue using VAR models. The main advantage of our approach, therefore, is the fact that we can gauge the sources of structural change in a systematic manner and link it to possible changes in deep parameters. This is a distinct advantage over VAR-based studies where this exercise is conditional on the identification scheme used. Third (to our knowledge) this paper is the first to estimate an open economy Markov-switching DSGE model. Therefore, we extend the split sample analysis in Lubik and Schorfheide (2007) to a framework where agents have rational expectations about the possibility of regime shifts.

We estimate four versions of the Markov-switching DSGE model: (*i*) a version that allows a regime shift in shock volatilities, (*ii*) a version allowing for (independent) shifts in shock volatility and parameters of the domestic Phillips curve, (*iii*) allowing for (independent) shifts in shock volatility and parameters of the process for import price inflation and (*iv*) allowing for (independent) shifts in shock volatility and parameters of the monetary policy rule. We find that all of these models are preferred to a fixed parameter DSGE model. The model that allows for shift in shock volatility and the parameters of the Taylor rule provides the best model fit among the Markov-switching models. Estimates from this model suggest that the mid-1970s were characterised by a low reaction by the monetary authorities to inflation. In contrast the central bank reacted more to the output gap and the change in the nominal exchange rate over this period. We find that this change in the policy rule had important consequences for the dynamics of macroeconomic variables such as output and inflation:

- Output, inflation and the real exchange rate responded more to monetary policy shocks during the mid-1970s regime. Similarly, the response of inflation to cost-push and technology shocks was significantly larger during this regime.
- A historical decomposition exercise suggests that monetary policy shocks are important for inflation fluctuations during the mid-1970s while technology shocks appear to drive all the main downturns in output.

The paper is arranged as follows: Section 2 describes the linearised DSGE model, the solution method for forward-looking Markov-switching models and the algorithm to estimate the parameters of the model. Section 3 presents the parameter estimates for the various Markov-switching models we consider. In Section 4 we consider which of the estimated models provides the best fit and present impulse responses and a historical decomposition based on the selected model.

2 A small open economy DSGE model

The model that we analyse is taken from Justiniano and Preston (2010) and is a generalisation of the models developed in Galí and Monacelli (2005) and Monacelli (2005). In particular, Justiniano and Preston (2010) introduce incomplete asset markets, habit formation and indexation of prices to past inflation. We refer the reader to Justiniano and Preston (2010) for a detailed derivation of the model. Here we provide a brief description of the key equations of the log-linearised model, Table A provides a complete list of the linearised model equations.

By solving the households' intertemporal utility maximisation problem gives the following linearised Euler equation

$$(1+h)c_t = hc_{t-1} + E_t c_{t+1} - \frac{1-h}{\sigma}(i_t - E_t \pi_{t+1}) + \frac{1-h}{\sigma}(\epsilon_{g,t} - \rho_g \epsilon_{g,t}) \qquad \textbf{(1)}$$

where current consumption c_t depends on past, future consumption and the real rate of interest

$i_t - E_t \pi_{t+1}$. h denotes the degree of habit persistence, σ denotes the inverse elasticity of intertemporal substitution and $\epsilon_{g,t}$ is a preference shock.

The Phillips curve for domestic price inflation is defined as

$$(1 + \beta\delta_H)\pi_{H,t} = \delta_H \pi_{H,t-1} + \beta E_t \pi_{H,t+1} + \frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H} mc_t \quad \textbf{(2)}$$

and the marginal cost mc_t is given by

$$mc_t = \varphi y_t - (1+\varphi)\epsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \quad \textbf{(3)}$$

where $\pi_{H,t}$ is domestic price inflation, y_t denotes domestic output, s_t denotes terms of trade, $\epsilon_{a,t}$ is an exogenous technology shock, φ is the inverse elasticity of labour supply and α is the import share. Equation **(2)** states that domestic inflation is related to expected future inflation via the discount factor β , inflation lagged one period through the degree of indexation δ_H and to marginal cost via $\gamma = \frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H}$, where θ_H is the fraction of firms that cannot optimally adjust their price every period. In the small open economy setting, domestic price inflation depends on several sources of foreign disturbances through the marginal cost term in equation **(3)**. In particular, there is a direct effect from the terms of trade and an indirect effect that operates through the goods market clearing condition

$$y_t = (1-\alpha)c_t + \alpha[\eta(s_t + q_t) + y_t^*] \quad \textbf{(4)}$$

where y_t^* denotes foreign output and q_t denotes the real exchange rate. Equation **(4)** shows that domestic production is the sum of domestic consumption plus exports to the rest of the world.

As in Monacelli (2005), import retailers are assumed to be monopolistic competitors that introduces deviations from the law of one price for imported goods. This generates deviations from purchasing power parity (PPP) in the short run. Solving the retailers optimisation problem gives the following Phillips curve for import price inflation

$$(1 + \beta\delta_F)\pi_{F,t} = \delta_F \pi_{F,t-1} + \beta E_t \pi_{F,t+1} + \frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F} \psi_{F,t} + \epsilon_{cp,t} \quad \textbf{(5)}$$

where $\pi_{F,t}$ denotes domestic currency import price inflation, $\psi_{F,t} = q_t - (1-\alpha)s_t$ denotes deviations from the law of one price and $\epsilon_{cp,t}$ is the exogenous cost-push shock. Equation **(5)** states that import price inflation depends on its lag via the indexation parameter δ_F, expected future inflation and marginal cost captured by the law of one price gap. As the fraction of importing firms that cannot optimally adjust price (θ_F) tends to 0, the deviations from PPP becomes smaller.

Justiniano and Preston (2010) introduce incomplete asset substitution between domestic and foreign bonds which gives the following uncovered interest rate parity condition

$$E_t q_{t+1} - q_t = (i_t - \pi_{t+1}) - (i_t - \pi^*_{t+1}) + \chi a_t + \epsilon_{\phi,t} \tag{6}$$

where a_t denotes the level of foreign assets position, χ is the debt elasticity with respect to the interest rate premium and $\epsilon_{\phi,t}$ is the risk premium shock. The foreign assets budget constraint is simply defined as $c_t + a_t = \frac{1}{\beta} a_{t-1} - \alpha(q + \alpha s_t) + y_t$.

The model is closed by assuming the behaviour of the monetary authority is described by the following Taylor-type interest rate rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left[\lambda_1 \pi_t + \lambda_2 y_t + \lambda_3 \Delta e_t + \sigma_m \eta_{m,t}\right] \tag{7}$$

where Δe_t is the change in the nominal exchange rate, $\eta_{m,t}$ is the interest rate shock, ρ_t is the degree of interest rate smoothing, λ_1, λ_2 and λ_3 are the reaction coefficients to inflation, output and the change in the nominal exchange rate.

The model consists of 22 state variables X_t (including 4 expectation terms) and 8 exogenous processes Z_t. These 8 exogenous processes constitute the structural shocks included by Justiniano and Preston (2010) : a preference shock $\epsilon_{g,t}$, a technology shock $\epsilon_{a,t}$, an import cost push shock $\epsilon_{\phi,t}$, a risk premium shock $\epsilon_{\phi,t}$, a monetary policy shock $\eta_{m,t}$, a foreign output shock $\epsilon_{Y^*,t}$, a foreign inflation shock $\epsilon_{\pi^*,t}$, a foreign interest rate shock $\epsilon_{I^*,t}$. The model can be rewritten in matrix form as

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi Z_t + \Pi \eta_t \tag{8}$$

Under rational expectation and no regime shifts, the model in equation **(8)** can be solved using standard rational expectation algorithm such as the *Gensys* solution method proposed in Sims (2002). This returns the solution in the form of a first-order VAR

$$X_t = G(\Phi) X_{t-1} + A(\Phi) Z_t \tag{9}$$

where Φ denotes the structural parameters of the model. Equation **(9)** can be combined with an observation equation of the form:

$$Y_t = \mu + H X_t$$

where Y_t represents a data matrix.[1] The Kalman filter algorithm can then be used to evaluate the

[1] We have assumed no measurement errors.

Table A: Equations of the log-linearised model

Euler equation	$(1+h)c_t = hc_{t-1} + E_t c_{t+1} - \frac{1-h}{\sigma}(i_t - E_t\pi_{t+1})$
	$+\frac{1-h}{\sigma}(\epsilon_{g,t} - \rho_g\epsilon_{g,t})$
Market clearing	$y_t = (1-\alpha)c_t + \alpha[\eta(s_t + q_t) + y_t^*]$
Law of one price	$\psi_{F,t} = q_t - (1-\alpha)s_t$
Terms of trade	$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t}$
Change in the nominal ER	$\Delta e_t = q_t - q_{t-1} + \pi_t - \pi_t^*$
Domestic price inflation	$(1+\beta\delta_H)\pi_{H,t} = \delta_H\pi_{H,t-1} + \beta E_t\pi_{H,t+1}$
	$+\frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H}\left[\varphi y_t - (1+\varphi)\epsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1})\right]$
Import price Inflation	$(1+\beta\delta_F)\pi_{F,t} = \delta_F\pi_{F,t-1} + \beta E_t\pi_{F,t+1}$
	$+\frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F}\psi_{F,t} + \epsilon_{cp,t}$
Domestic CPI inflation	$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}$
Uncovered interest parity	$E_t q_{t+1} - q_t = (i_t - E_t\pi_{t+1}) - (i_t^* - E_t\pi_{t+1}^*) + \chi a_t + \epsilon_{\phi,t}$
Foreign asset budget constraint	$c_t + a_t = \frac{1}{\beta}a_{t-1} - \alpha(q + \alpha s_t) + y_t$
Interest rate reaction function	$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\lambda_1\pi_t + \lambda_2 y_t + \lambda_3\Delta e_t + \sigma_m\eta_{m,t}\right]$
Preference shock	$\epsilon_{g,t} = \rho_g\epsilon_{g,t-1} + \sigma_g\eta_{g,t}$
Technology Shock	$\epsilon_{a,t} = \rho_a\epsilon_{a,t-1} + \sigma_a\eta_{a,t}$
Import cost-push shock	$\epsilon_{cp,t} = \rho_{cp}\epsilon_{cp,t-1} + \sigma_{cp}\eta_{cp,t}$
Risk premium shock	$\epsilon_{\phi,t} = \rho_\phi\epsilon_{\phi,t-1} + \sigma_\phi\eta_{\phi,t}$
Foreign interest rate	$i_t^* = \rho_{i^*}i_{t-1}^* + \sigma_{i^*}\eta_{i^*,t}$
Foreign output	$y_t^* = \rho_{y^*}y_{t-1}^* + \sigma_{y^*}\eta_{y^*,t}$
Foreign inflation	$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\eta_{\pi^*,t}$

likelihood function and estimate the underlying parameters.

2.1 Solution and estimation of the Markov-switching model

We consider the possibility of structural change by allowing key equations in Table A to be subject to regime shifts. In particular we estimate three versions of the model that allow for (*i*) Markov switching in the policy rule (equation **(7)**), (*ii*) Markov switching in the domestic price inflation Phillips curve (equation **(2)**) and (*iii*) Markov-switching process for import price inflation (equation **(5)**). In each case we allow for independent regime switching in the volatility of the structural shocks. We compare these estimated models with restricted specifications that either only allow a regime switch in the volatility or rules out regime shifts all together.

The versions of the model in cases *i, ii* and *iii* are of interest as they represent the key dimensions along which the structure of the UK economy may have changed. For example, a large (mostly reduced form) literature has argued that the parameters of the UK monetary policy rule changed after the introduction of inflation targeting in 1992 (see for example Nelson (2001)). Similarly, several studies focus on changes in inflation persistence (eg Benati (2004)) and the possibility of a change in exchange rate pass-through (Mumtaz, Oomen and Wang (2006)). Our specification allows us to approach these issues within a structural framework.

In order to specify the Markov-switching DSGE model, partition the parameter vector Φ into three blocks

$$\Phi = \{\Phi^{S};\ \Sigma^{s};\ \bar{\Phi}\}$$

where Φ^{S} denotes the parameters subject to regime shifts (ie the parameters of the policy rule, the domestic price Phillips curve and the equation for import price inflation), Σ^{s} denotes the volatility of the structural shocks that are subject to regime shifts, while $\bar{\Phi}$ are time-invariant parameters. The superscript S denotes the unobserved regime associated with the parameters and is assumed to take on the discrete values $S = 1, 2$. The superscript $s = 1, 2$ denotes the unobserved regime associated with the volatilities Σ and evolves independently of S. The two state variables S and s are assumed to follow a first-order Markov-chain with the following transition probability matrices respectively:

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$$

$$Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$$

where $P_{ij} = P\left(S_t = j | S_{t-1} = i\right)$ and $Q_{ij} = P\left(s_t = j | s_{t-1} = i\right)$.

Note that an alternative approach is to model time-variation by allowing for drift in the structural parameters within a non-linear DSGE model (as in Fernandez-Villaverde and Rubio-Ramirez (2007)). However, as shown in Fernandez-Villaverde and Rubio-Ramirez (2007) the computational burden inherent in this approach implies that time-variation can only be introduced one parameter at a time. This constraint is quite limiting in our context where the interest lies in possible shifts in structural equations and the contributions of those shifts to macroeconomic dynamics. For example, while it may be interesting to document shifts in one parameter in the Phillips curve or the policy rule, one of the main aims of our analysis is to gauge the contribution of shifts in the (entire) policy rule or equations governing domestic and import price dynamics. Therefore, the Markov-switching approach is preferred for our application.

The regime-switching DSGE model for regime S can written as

$$\overset{\Gamma_0}{\begin{pmatrix} \Gamma_{0,1}^S \\ \Gamma_{0,2} \end{pmatrix}} X_t = \overset{\Gamma_1}{\begin{pmatrix} \Gamma_{1,1}^S \\ \Gamma_{1,2} \end{pmatrix}} X_{t-1} + \overset{\Psi}{\begin{pmatrix} \Psi_1^S \\ 0 \end{pmatrix}} Z_t + \overset{\Pi}{\begin{pmatrix} 0 \\ \Pi \end{pmatrix}} \eta_t \qquad \textbf{(10)}$$

We follow the method described in Farmer *et al* (2008) to solve the model in **(10)**. The appendix provides more details of the solution method. In short, Farmer *et al* (2008) proceed by rewriting the Markov-switching DSGE model as a fixed parameter model in an expanded state vector:

$$\bar{\Gamma}_0 \bar{X}_t = \bar{\Gamma}_1 \bar{X}_{t-1} + \bar{\Upsilon} u_t + \bar{\Pi} \eta_t \qquad \textbf{(11)}$$

where the parameter matrices $\bar{\Gamma}_0$, $\bar{\Gamma}_1$, $\bar{\Upsilon}$ and $\bar{\Pi}$ are functions of structural parameters and the transition probabilities. Farmer *et al* (2008) define a minimum state variable (MSV) solution to the system **(11)** and provide a diagnostic to check for existence and uniqueness of the solution. Moreover, they prove that the MSV solution to the expanded system **(11)** is an MSV solution to the original model in **(10)**. If a unique solution exists then this can be written as a Markov-switching VAR

$$X_t = G^S X_{t-1} + A^S Z_t \qquad \textbf{(12)}$$

Alternative solution methods are described in Davig and Leeper (2007) and Svensson and Williams (2007). However as discussed in Farmer *et al* (2008), these methods do not provide a diagnostic to check for uniqueness of the solution. We find that Farmer *et al* (2008)'s solution method works well even in our relatively large-scale model with the iterative procedure converging rapidly in most cases.

Combining equation **(12)** with an observation equation results in the following state-space model with Markov switching:

$$\begin{aligned} Y_t &= H X_t \qquad \textbf{(13)} \\ X_t &= G^S X_{t-1} + A^S Z_t \\ Z_t &\sim N(0, Q^s) \end{aligned}$$

where the Markov states S and s evolve independently with transition probability matrices $P\,(S_t = j | S_{t-1} = i) = P$ and $P\,(s_t = j | s_{t-1} = i) = Q$ respectively. The presence of the unobserved DSGE states X_t *and* the unobserved Markov states implies that the standard Kalman filter can no longer be used to provide inference on X_t and to calculate the value of the likelihood. The Kalman filter provides inference on the state vector X_t given information up to time t. Note, however, that the presence of the unobserved Markov states implies that the inference has to be conditioned on the value of S and s in the current period and in the past. As noted by Kim and Nelson (1999) each iteration of the Kalman filter therefore implies an M fold increase in the number of cases to consider (where M denotes the number of regimes) with the computation becoming intractable fairly rapidly. Kim and Nelson (1999) propose an approximation which makes this filter operational. The key feature of this approximation is that a limited number of states are carried forward in the Kalman filter iterations each period and these are then 'collapsed' at the end of each iteration. To apply this algorithm in our setting we start by defining a new state variable S_t^* which indexes both S_t and s_t and has a four state transition matrix given by $P^* = P \otimes Q$. Following Kim and Nelson (1999) and Davig and Doh (2008) we track S_t^*, S_{t-1}^* and S_{t-2}^* which implies we account for $4^3 = 64$ possible paths for the DSGE states. Kim and Nelson (1999) algorithm (detailed in the appendix) involves running the Kalman filter for each of the tracked paths and then taking a weighted average where the weights are given by the probability assigned to each path by the filter proposed in Hamilton (1989).

We adopt a Bayesian approach to model estimation. In particular, we combine the approximate likelihood function (obtained via the procedure described above) with prior distributions for the parameters and use a Markov chain Monte Carlo (MCMC) algorithm to approximate the posterior.

The prior distributions along with the lower and upper bounds for the model parameters are summarised in Table B. These are based on the prior distributions in Justiniano and Preston (2010) and Lubik and Schorfheide (2007). We calibrate the degree of openness parameter α to be 0.185 which is equal to the average share of imports and exports share for the United Kingdom. We assume the degree of risk aversion (inverse of the intertemporal elasticity) σ follows a Gamma distribution with a mean of 2 and standard deviation of 0.2. Similarly, the inverse Frisch elasticity of labour supply φ and elasticity of substitution between domestic and foreign goods η are both assumed to have a mean of 1.5 and moderately large standard deviation of 0.75, because of wide range of these estimates in the literature. The degree of habit persistence h follows a Beta distribution with a mean of 0.8 and standard deviation of 0.1. Both of the Calvo pricing parameters θ_H and θ_F are assumed to follow a Beta distribution centred around 0.5 and with a standard deviation of 0.1. The indexation parameters are found to be crucial in fitting the dynamics of inflation; here we adopt a fairly agnostic view by specifying very loose priors for δ_H and δ_F (mean centred around 0.5 and standard deviation of 0.25). The debt elasticity with respect to the interest rate premium χ is assumed to follow a Gamma distribution with a mean of 0.01 and standard deviation of 0.02. We follow Justiniano and Preston (2010) in setting the priors for the parameters of the Taylor rule.

The autoregressive parameter for the exogenous stochastic disturbances (risk premium, technology, preference, import cost-push, foreign inflation, foreign output and foreign interest rate shocks) are all assumed to follow a Beta distribution with a mean of 0.5 and standard deviation of 0.15. The priors for the standard deviation of these shocks follow an inverse Gamma distribution with very wide variance (mean of 0.5 and standard deviation of 10). Finally, we follow Sims and Zha (2006) in specifying a Dirichlet prior for the transition probabilities, with the scale matrix chosen to reflect the belief that regimes are persistent. The parameters for the Dirichlet distribution are assumed to be $\alpha_1 = 18$ and $\alpha_2 = 1$, giving the probability of staying in the same regime to be 0.95.[2] We also assume the priors on the model's structural parameters are symmetric across the different regimes.

A random walk Metropolis Hastings algorithm is used to approximate the posterior. This algorithm is implemented by the following steps:

1. We combine the prior distributions and the likelihood obtained via Kim and Nelson (1999)'s algorithm to obtain an approximation of the posterior. A combination of numerical maximisers is used to find the mode of the posterior. In particular, we refine our starting values using the simplex algorithm. These are then used as input into Chris Sims' optimisation routine CSMINWEL (see http://sims.princeton.edu/yftp/optimize/).
2. We use the posterior mode as the starting value for an initial Metropolis Hastings run of 100,000 iterations. The variance of the proposal distribution is set equal to the scaled inverse Hessian obtained from the numerical maximisation. The scaling parameter is chosen to ensure an acceptance rate of 20% to 40%.
3. The posterior mean and variance from the last 5,000 iterations of this initial MCMC run is then used to initialise the main Metropolis algorithm. We run the MCMC algorithm for 200,000

[2]Let $\alpha_0 = \alpha_1 + \alpha_2$, the mean of the Dirichlet($\alpha_1 + \alpha_2$) distribution is $E(x_i) = \frac{\alpha_i}{\alpha_0}$.

Table B: Prior Distributions

Parameters	Distribution	Mean	Standard Deviation	Lower Bound	Upper Bound
$\alpha^{(1)}$	Point mass	0.19	-	-	-
σ	Gamma	1.20	0.40	0	10
φ	Gamma	1.50	0.75	0	10
θ^H	Beta	0.50	0.10	0	1
θ^F	Beta	0.50	0.10	0	1
η	Gamma	1.50	0.75	0	10
h	Beta	0.50	0.25	0	1
δ^H	Beta	0.50	0.25	0	1
δ^F	Beta	0.50	0.25	0	1
ρ_i	Beta	0.50	0.25	0	1
λ_1	Gamma	1.50	0.25	0	10
λ_2	Gamma	0.25	0.13	0	10
λ_3	Gamma	0.25	0.13	0	10
χ	Gamma	0.01	0.02	0	10
ρ_a	Beta	0.50	0.15	0	1
ρ_g	Beta	0.50	0.15	0	1
ρ_ϕ	Beta	0.50	0.15	0	1
ρ_{cp}	Beta	0.50	0.15	0	1
$\rho_{\pi*}$	Beta	0.50	0.15	0	1
ρ_{Y*}	Beta	0.50	0.15	0	1
ρ_{i*}	Beta	0.50	0.15	0	1
$\sigma_{\pi*}$	Inverse Gamma	0.50	10.0	0	10
σ_{Y*}	Inverse Gamma	0.50	10.0	0	10
σ_{i*}	Inverse Gamma	0.50	10.0	0	10
σ_a	Inverse Gamma	0.50	10.0	0	10
σ_{mp}	Inverse Gamma	0.50	10.0	0	10
σ_g	Inverse Gamma	0.50	10.0	0	10
σ_ϕ	Inverse Gamma	0.50	10.0	0	10
σ_{cp}	Inverse Gamma	0.50	10.0	0	10
P_{11}	Dirichlet(2)	18.0	1.00	-	-
P_{22}	Dirichlet	18.0	1.00	-	-
Q_{11}	Dirichlet	18.0	1.00	-	-
Q_{22}	Dirichlet	18.0	1.00	-	-

(1) The degree of openness α is calibrated to be equal to the imports and exports share for the United Kingdom.

(2) The Markov transition probabilities are assumed to follow a Dirichlet distribution with $\alpha_1 = 18$ and $\alpha_2 = 1$.This gives the probability of staying in the same regime to be 0.95.

MCMC iterations and save every 20^{th} draw. The cumulative mean plots in the appendix show relatively little change in the posterior over the retained draws providing evidence for convergence.[3]

We estimate the baseline rational expectations with no regime switching and four versions of the Markov-switching DSGE model:

- Model 0: Rational expectations with no regime switching.
- Model 1: Rational expectations with two-state Markov switching in the volatility of the structural shocks (this does not require any adjustments to the standard rational expectation solution algorithm because certainty equivalence holds in our linear framework).
- Model 2: In addition to the switching in the volatility of the shocks, we also allow for the parameters of the domestic price inflation Phillips curve (θ_H and δ_H in equation **(2)**) to follow an independent two-state Markov process. This is useful to assess whether the domestic structural Phillips curve have changed over time.
- Model 3: Similar to Model 2, the third case allows for regime switching in the import price inflation equation (θ_F and δ_F in equation **(5)**). This can be used to assess whether exchange rate pass-through have changed over time.
- Model 4: The fourth version of the estimated model considers regime switching in the open economy Taylor rule (ρ_i, λ_1, λ_2, and λ_3 in equation **(7)**) to assess changes to the monetary policy reaction function.
- Model 5: The final version of the model allows two regimes for all structural parameters in the model but assumes that agents do not form expectations about the possibility of a regime shift. Instead, the model is solved in each regime independently (using a standard solution methods). This model proxies 'extreme beliefs' on the part of model agents.

[3]Note that Bianchi (2009) describes a Gibbs sampling algorithm to estimate the Markov-switching DSGE model. Bianchi (2009)'s approach involves sampling from the conditional distributions of the Markov states using the Multi-Move Gibbs sampler proposed in Kim and Nelson (1999) and then using a standard Kalman filter to compute the likelihood of the DSGE model conditional on the draw of the Markov states. This method has the advantage that it avoids an approximation of the likelihood function. However, in computational terms the Gibbs sampling algorithm proved prohibitively demanding for our model with one issue emerging from the analysis. The issue concerned the step in the algorithm where the Markov states are drawn from their distribution. In particular, this step may be unsuccessful when one of the states is not visited (ie only one value of S_t^* is drawn for the entire sample). As noted by Sims and Zha (2006) (in the context of VAR models), this implies that in the next step of the sampler, the data are not informative for the redundant state and the sampling algorithm is unable to proceed.

In Section 4 we conduct a formal model selection exercise to assess which of these models provide the best fit for the data.

2.1.1 Data description

UK data from 1970 Q1 to 2009 Q1 was used for the estimation of the model. Quarterly observations on GDP (y_t), real effective exchange rate (q_t), import price deflator ($P_{F,t}$), quarterly nominal interest rate (i_t), overall CPI (P_t) are taken from the Office for National Statistics and Bank of England databases. US quarterly nominal interest rate (i_t^*), overall CPI (P_t^*) and GDP (y_t^*) are taken from the St. Louis Fed FRED database. We take logs of all the series apart from the nominal interest rates. The first difference of the import price deflator, UK CPI and US CPI series is used to approximate import price inflation, domestic inflation and foreign inflation. To compute the domestic and foreign output gaps, we use the HP filter to detrend the data.[4] Lastly, all variables are rescaled to have a zero mean over the sample.

3 Parameter estimates

3.1 Time-invariant rational expectation model

Table C presents the mean of the posterior parameter estimates across the five models we consider and the 95% probability intervals are shown in parenthesis. The first column presents the baseline time-invariant rational expectation model. The estimated value for the intertemporal elasticity of substitution σ is 1.8, slightly higher than the value reported in Lubik and Schorfheide (2007) for the United Kingdom and smaller than Justiniano and Preston (2010) found for other small open economies. The posterior estimate for the inverse elasticity of labour supply φ is higher than the range of estimates reported in Justiniano and Preston (2010). This may reflect the fact that our sample covers periods where labour market conditions in the United Kingdom were more rigid. The mean estimate for the elasticity of substitution between foreign and domestic goods η is 0.9 which reflects United Kingdom's small open economy position and the estimate is consistent with other SOE.

[4]As pointed out in Canova (1998), estimation results can be sensitive to the chosen detrending method. We find similar results for our main model (Model 4) when the GDP series are first differenced instead.

The estimated value of the Calvo parameters θ^H and θ^F suggest home goods prices are optimised around every two quarters and import prices adjust less frequently at around seven quarters. In contrast to other studies for the United Kingdom, we find consumption habits and inflation indexation play a limited role in the model. This finding is consistent with the results reported in Justiniano and Preston (2010) where the authors argue that this is because of the set of autoregressive shocks chosen for the model. The interest rate smoothing coefficient in the policy rule is estimated to be 0.8. The inflation reaction coefficient is estimated to be 1.5, which is in line with the results reported in previous studies. However, the coefficient on output is found to be relatively smaller, around 0.01. In contrast to the result reported by Lubik and Schorfheide (2007), we find the coefficient on the exchange rate to be quite small, around 0.02. This may reflect the wider coverage of our data sample which includes the 1970s and the most recent inflation-targeting experience. In addition to this, our model finds strong evidence of imperfect exchange rate pass-through. This suggests changes in the exchange rate takes time to feed through to inflation, therefore reducing the central bank's incentive to actively respond to exchange rate changes in its interest rate decisions.

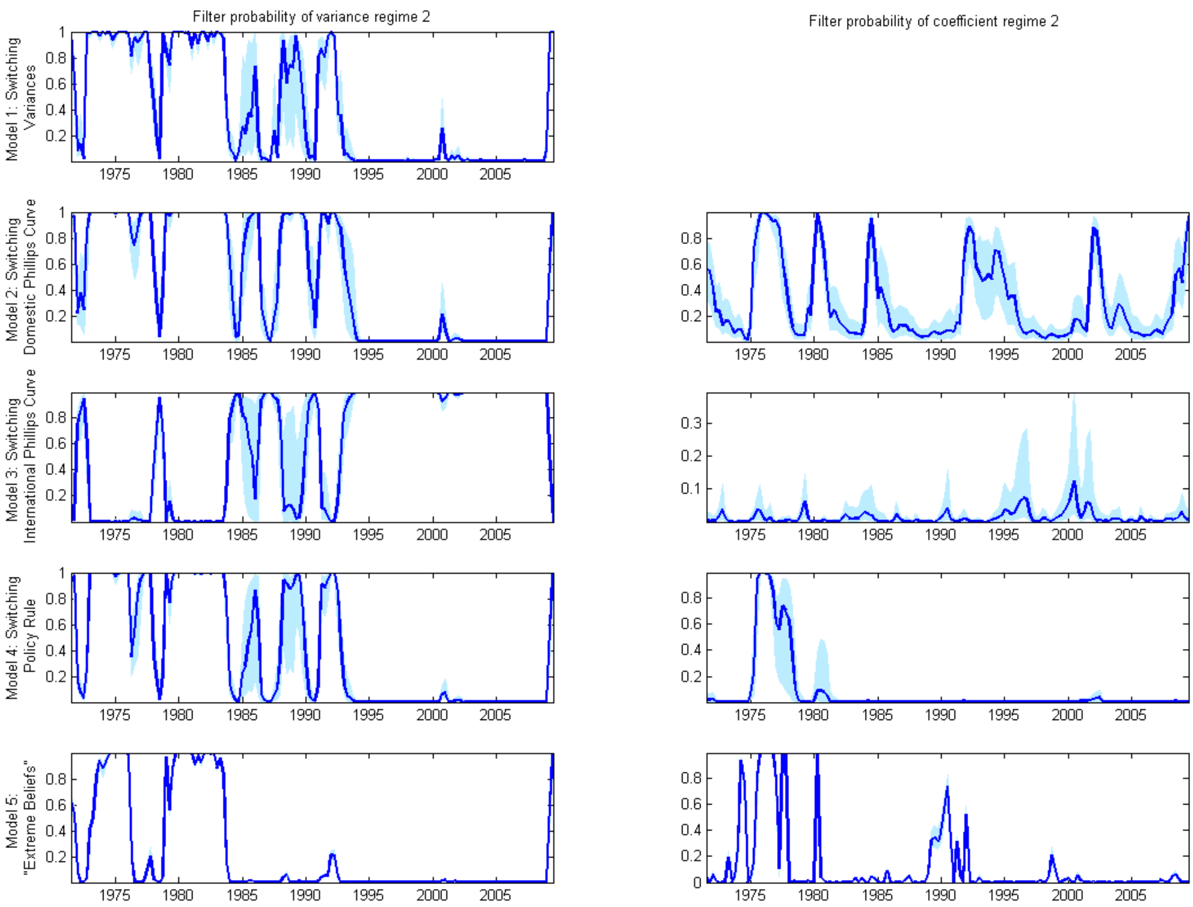
One noticeable aspect of the baseline estimation is the very high degree of persistence for the domestic shocks. The autoregressive coefficients for technology (ρ_a) preference (ρ_g) and risk premium shocks (ρ_ϕ) are very close to unit root, this partly explains the low degree of 'intrinsic' persistence the model captures. On the other hand, the persistence of the foreign shocks are similar to the ones reported in Justiniano and Preston (2010).

3.2 The model with switching variances

The second and third column in Table C present the estimated parameters of our first Markov-switching specification M_1– ie the model that only allows the volatility of shocks to switch across the two regimes. Although this specification is fairly restrictive in that it does not allow the structure of the economy to change, it is instructive to consider the parameter estimates from this model as a comparison to the more flexible specifications presented below.

Consider the time-invariant parameter estimates. These estimates are similar to those obtained in the fixed parameter specification above. There are a few noticable differences, however. In particular, the estimated elasticity of substitution between domestic and foreign goods is

Chart 1: Filter probabilities for the Markov-switching models



significantly smaller at 0.7 while reaction to inflation in the policy rule is somewhat larger at 1.7. Moreover, the estimated autocorrelation of structural shocks is generally smaller than the fixed parameter specification. The bottom panels of the table present the estimates of the variance of shocks in the two regimes and the associated transition matrix Q. The estimates of Q indicate that both regimes are fairly persistent lasting, on average, for about four years. Regime 2 is the high volatility state, with the posterior estimate of the shock variances substantially larger. The difference across regimes is largest for foreign shocks, with Regime 2 estimates three to four times larger than Regime 1. Within the domestic shocks, σ_{CP} , σ_{ϕ} and σ_a show a significant change in volatility with no overlap across regimes in the 95% confidence bands. Consistent with

VAR-based studies the variance of the monetary policy shock σ_M displays a large change across the sample, with the Regime 1 estimate around half of the estimate in Regime 2.

The top left panel of Chart 1 plots the posterior mean of the filter probabilities associated with the high variance regime.[5] The chart shows that the high volatility state, Regime 2 was dominant during the 1970s and the early 1980s. The mid-1980s saw a brief switch to Regime 1, but the probability of the high volatility state increased again towards the end of the 1980s and beginning of the 1990s. The inflation-targeting period was largely associated with the low volatility state. However, the recent crisis was clearly associated with the high volatility state.

[5]The left panels of the chart plot the filter probability of variance Regime 2 while the right panel plots the probability of Regime 2 for structural parameters.

Table C: Posterior parameter estimates of Markov-switching models

Para	Model 0	Model 1		Model 2		Model 3		Model 4		Model 5	
		Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
σ	1.83 (1.65, 1.97)	1.77 (1.63, 1.92)	–	1.76 (1.62, 1.89)	–	1.78 (1.64, 1.93)	–	1.78 (1.64, 1.94)	–	1.76 (1.75, 1.77)	2.23 (2.22, 2.24)
φ	4.40 (2.70, 5.50)	4.07 (2.77, 5.69)	–	3.28 (2.29, 4.62)	–	4.15 (2.69, 6.10)	–	3.93 (2.64, 5.39)	–	1.60 (1.59, 1.62)	1.11 (1.09, 1.13)
θ^H	0.55 (0.43, 0.63)	0.59 (0.52, 0.66)	–	0.70 (0.60, 0.76)	0.41 (0.27, 0.53)	0.59 (0.51, 0.67)	–	0.57 (0.48, 0.65)	–	0.46 (0.46, 0.46)	0.32 (0.30, 0.33)
θ^F	0.87 (0.83, 0.91)	0.81 (0.76, 0.87)	–	0.81 (0.76, 0.86)	–	0.79 (0.74, 0.83)	0.60 (0.48, 0.74)	0.80 (0.75, 0.84)	–	0.82 (0.82, 0.83)	0.84 (0.83, 0.84)
η	0.90 (0.80, 1.02)	0.67 (0.61, 0.74)	–	0.63 (0.57, 0.69)	–	0.65 (0.60, 0.72)	–	0.68 (0.60, 0.76)	–	0.61 (0.60, 0.61)	0.89 (0.88, 0.91)
h	0.14 (0.07, 0.21)	0.16 (0.10, 0.21)	–	0.16 (0.11, 0.22)	–	0.14 (0.10, 0.19)	–	0.14 (0.10, 0.20)	–	0.11 (0.10, 0.12)	0.75 (0.73, 0.76)
δ^H	0.03 (0.01, 0.05)	0.00 (0.00, 0.01)	–	0.11 (0.02, 0.25)	0.17 (0.02, 0.57)	0.06 (0.01, 0.13)	–	0.06 (0.01, 0.15)	–	0.22 (0.20, 0.22)	0.29 (0.27, 0.30)
δ^F	0.01 (0.00, 0.02)	0.02 (0.01, 0.04)	–	0.05 (0.01, 0.12)	–	0.04 (0.01, 0.10)	0.45 (0.09, 0.86)	0.03 (0.01, 0.06)	–	0.02 (0.01, 0.03)	0.23 (0.22, 0.24)
ρ_i	0.80 (0.72, 0.83)	0.76 (0.72, 0.80)	–	0.81 (0.76, 0.85)	–	0.77 (0.72, 0.82)	–	0.74 (0.68, 0.80)	0.91 (0.85, 0.96)	0.80 (0.78, 0.81)	0.84 (0.84, 0.85)
λ_1	1.56 (1.35, 1.77)	1.71 (1.50, 1.95)	–	1.74 (1.53, 1.98)	–	1.73 (1.48, 2.05)	–	1.81 (1.56, 2.11)	1.51 (1.20, 1.82)	2.72 (2.70, 2.73)	1.35 (1.34, 1.35)
λ_2	0.01 (0.00, 0.02)	0.04 (0.01, 0.07)	–	0.07 (0.02, 0.15)	–	0.04 (0.01, 0.08)	–	0.04 (0.01, 0.08)	0.21 (0.07, 0.38)	0.03 (0.02, 0.03)	0.26 (0.25, 0.27)
λ_3	0.02 (0.01, 0.04)	0.08 (0.03, 0.11)	–	0.20 (0.15, 0.29)	–	0.11 (0.06, 0.15)	–	0.11 (0.05, 0.19)	0.26 (0.08, 0.45)	0.26 (0.25, 0.26)	0.12 (0.10, 0.14)
χ	0.00 (0.00, 0.00)	0.00 (0.00, 0.00)	–	0.00 (0.00, 0.00)	–	0.00 (0.00, 0.00)	–	0.00 (0.00, 0.00)	–	0.00 (0.00, 0.00)	0.00 (0.00, 0.00)
ρ_a	0.99 (0.98, 1.00)	0.94 (0.88, 0.98)	–	0.86 (0.75, 0.94)	–	0.94 (0.86, 0.98)	–	0.93 (0.87, 0.98)	–	0.95 (0.94, 0.96)	0.63 (0.61, 0.64)
ρ_g	0.88 (0.82, 0.92)	0.71 (0.63, 0.78)	–	0.71 (0.65, 0.76)	–	0.71 (0.64, 0.78)	–	0.68 (0.59, 0.76)	–	0.68 (0.66, 0.69)	0.61 (0.60, 0.63)
ρ_ϕ	0.94 (0.92, 0.96)	0.87 (0.84, 0.89)	–	0.86 (0.84, 0.88)	–	0.87 (0.84, 0.89)	–	0.87 (0.84, 0.89)	–	0.87 (0.87, 0.88)	0.87 (0.87, 0.88)
ρ_{cp}	0.73 (0.63, 0.81)	0.84 (0.75, 0.91)	–	0.85 (0.77, 0.91)	–	0.88 (0.82, 0.92)	–	0.84 (0.77, 0.90)	–	0.83 (0.82, 0.83)	0.86 (0.86, 0.87)
ρ_{π^*}	0.72 (0.68, 0.77)	0.69 (0.64, 0.74)	–	0.70 (0.64, 0.75)	–	0.71 (0.66, 0.76)	–	0.68 (0.63, 0.73)	–	0.69 (0.68, 0.71)	0.73 (0.72, 0.75)

Continued on next page …

Table C – continued from previous page

Para	Model 0	Model 1		Model 2		Model 3		Model 4		Model 5	
		Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
ρ_{Y^*}	0.73 (0.64, 0.82)	0.63 (0.52, 0.73)	–	0.63 (0.55, 0.72)	–	0.63 (0.52, 0.74)	–	0.59 (0.48, 0.70)	–	0.45 (0.44, 0.46)	0.46 (0.45, 0.48)
ρ_{i^*}	0.88 (0.85, 0.90)	0.84 (0.79, 0.88)	–	0.83 (0.79, 0.87)	–	0.84 (0.78, 0.88)	–	0.83 (0.79, 0.87)	–	0.85 (0.85, 0.86)	0.51 (0.50, 0.51)
σ_{π^*}	0.47 (0.40, 0.56)	0.27 (0.22, 0.32)	0.62 (0.44, 0.86)	0.27 (0.23, 0.33)	0.53 (0.38, 0.72)	0.59 (0.43, 0.81)	0.27 (0.22, 0.32)	0.28 (0.24, 0.34)	0.51 (0.37, 0.74)	0.34 (0.33, 0.35)	1.22 (1.22, 1.23)
σ_{Y^*}	2.82 (2.19, 3.35)	1.12 (0.68, 1.70)	7.32 (4.99, 9.49)	0.98 (0.65, 1.41)	8.34 (6.53, 9.75)	7.40 (5.15, 9.53)	1.24 (0.73, 1.88)	1.06 (0.67, 1.54)	7.53 (4.62, 9.67)	1.55 (1.55, 1.56)	1.90 (1.89, 1.92)
σ_{I^*}	1.39 (1.22, 1.55)	0.39 (0.30, 0.50)	1.95 (1.32, 2.85)	0.40 (0.31, 0.53)	1.83 (1.26, 2.63)	2.09 (1.37, 3.27)	0.40 (0.30, 0.54)	0.38 (0.30, 0.48)	1.97 (1.33, 2.81)	0.58 (0.57, 0.59)	1.96 (1.93, 1.97)
σ_a	0.99 (0.84, 1.16)	0.51 (0.38, 0.69)	1.21 (0.78, 1.79)	0.51 (0.38, 0.69)	1.22 (0.84, 1.69)	1.28 (0.82, 1.88)	0.54 (0.39, 0.71)	0.54 (0.39, 0.73)	1.36 (0.84, 1.69)	0.61 (0.60, 0.62)	1.03 (1.02, 1.04)
σ_M	0.44 (0.36, 0.54)	0.38 (0.29, 0.48)	0.77 (0.52.1.09)	0.36 (0.28, 0.45)	0.72 (0.53, 0.95)	0.75 (0.51, 1.08)	0.37 (0.29, 0.47)	0.37 (0.29, 0.46)	0.76 (0.51, 1.06)	0.32 (0.31, 0.33)	1.39 (1.39, 1.41)
σ_g	0.30 (0.27, 0.33)	0.23 (0.19, 0.27)	0.32 (0.26, 0.41)	0.24 (0.20, 0.29)	0.29 (0.24, 0.36)	0.23 (0.19, 0.28)	0.23 (0.19, 0.27)	0.23 (0.19, 0.28)	0.33 (0.26, 0.41)	0.21 (0.20, 0.21)	0.63 (0.61, 0.64)
σ_ϕ	0.55 (0.50, 0.60)	0.29 (0.24, 0.35)	0.59 (0.44, 0.78)	0.31 (0.26, 0.37)	0.63 (0.52, 0.75)	0.28 (0.24, 0.33)	0.29 (0.23, 0.36)	0.28 (0.24, 0.33)	0.67 (0.59, 0.73)	0.27 (0.26, 0.28)	0.98 (0.97, 0.99)
σ_{CP}	0.80 (0.73, 0.88)	0.34 (0.28, 0.42)	0.97 (0.71, 1.32)	0.36 (0.30, 0.45)	0.91 (0.65, 1.25)	0.35 (0.28, 0.42)	0.34 (0.28, 0.40)	0.35 (0.28, 0.42)	0.99 (0.74, 1.32)	0.38 (0.37, 0.39)	1.66 (1.65, 1.68)
P_{11}	–	–	–	0.94 (0.88, 0.98)	–	0.99 (0.99, 1.00)	–	0.99 (0.99, 1.00)	–	0.96 (0.95, 0.97)	–
P_{22}	–	–	–	0.88 (0.77, 0.96)	–	0.92 (0.81, 0.98)	–	0.92 (0.81, 0.98)	–	0.83 (0.82, 0.84)	–
Q_{11}	–	0.94 (0.89, 0.98)	0.93 (0.87, 0.98)	0.95 (0.91, 0.98)	–	0.94 (0.89, 0.97)	–	0.94 (0.89, 0.97)	–	0.99 (0.98, 1.00)	–
Q_{22}	–	–	–	0.96 (0.90, 0.99)	–	0.92 (0.87, 0.97)	–	0.92 (0.87, 0.97)	–	0.97 (0.95, 0.98)	–

(1) The 95% probability intervals are indicated in parenthesis.

3.3 The model with the switching domestic Phillips curve

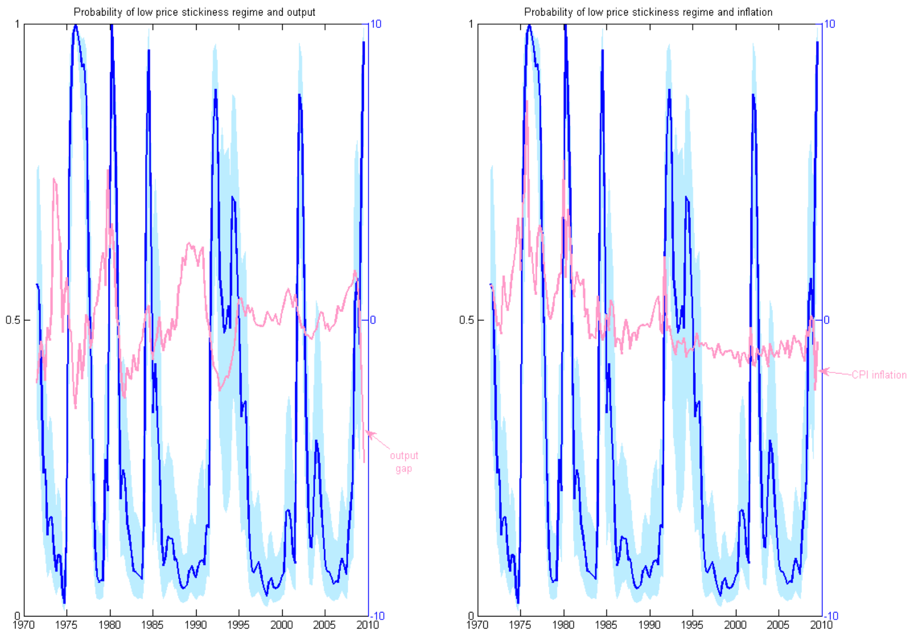
The second Markov-switching specification that we investigate allows regime shifts along two dimensions. First, as in the specification described in Section 3.2, the variance of the structural shocks is regime dependent. Secondly, we allow the parameters of the Phillips curve (equation **(2)**) to follow an independent Markov process. The fourth and fifth column in Table C summarise the posterior parameter estimates for this model denoted as M_2. The time-invariant estimates are very similar to those obtained for M_1 and M_0. In particular, the estimated parameters for the policy rule (equation **(7)**) are virtually identical to the first specification. The estimated parameters for the equation for import price inflation (equation **(5)**) are also very similar to the variances-only model.

The regime-dependent estimate of the indexation parameter δ^H indicates a slightly higher value of indexation in Regime 2. However the relatively large error bands in both regimes indicate that evidence for a systematic shift in this parameter is weak. In contrast there appears to be a significant shift in the the Calvo parameter θ^H. The Regime 1 estimate of this parameter is almost twice as big as the Regime 2 estimate. Note that the 95% error bands are quite tight in both regimes with no overlap across regimes. This indicates strong evidence for a systematic shift in this parameter. The Regime 1 estimate indicates that in this regime prices are optimised around every 3.5 quarters. In contrast, in Regime 2 the frequency of price changes is higher with prices being optimised every two quarters.

The second-row right-panel of Chart 1 shows the filter probability of the second regime associated with the switching domestic Phillips curve parameters Pr $(S_t = 2)$ which can be interpreted as the 'low price stickiness' regime. This probability is high in the mid-1970s, the early and the mid-1980s, the early 1990s and finally during the recent recession in 2008.

In Chart 2 we plot this probability along with the output gap and CPI inflation for the United Kingdom. It is interesting to note that Regime 2 is clearly associated with low values of GDP growth. Similarly, the fluctuations in CPI inflation are higher in this regime. That is, this regime is associated with the Great Inflation of the mid and late 1970s, the inflationary episodes during the mid-1980s and the early 1990s and fluctuations in inflation seen during the past year. These results imply that the degree of price stickiness is lower during periods characterised by recession

Chart 2: Filter probability for the model with switching Phillips curve parameters, output gap and demeaned quarterly CPI inflation



and/or large changes in inflation. This pattern of change in the degree of price stickiness matches the predictions of the literature on time-dependent pricing (see for example Dotsey, King and Wolman (1999)). With time-dependent pricing firms face menu costs on adjusting prices. However, the cost associated with keeping prices fixed becomes larger in periods associated with higher inflation variability resulting in a decrease in price stickiness during these periods. Our results are therefore more in line with the time-dependent pricing specification than Calvo pricing that underpins the model presented in Section 2. In general our results are consistent with those presented for the United States (for the Calvo parameter) in Fernandez-Villaverde and Rubio-Ramirez (2007) and indicate large changes in the Phillips curve across time.

The bottom panels of Table C present the estimate regime switching shock variances. As before Regime 2 is the high volatility state with both the foreign and domestic shocks substantially more volatile. The second-row left-panel of Chart 1 indicates that the high volatility regime was prevalent during the 1970s, the early 1980s, the early 1990s and during the recent recession.

3.4 The model with switching import price dynamics

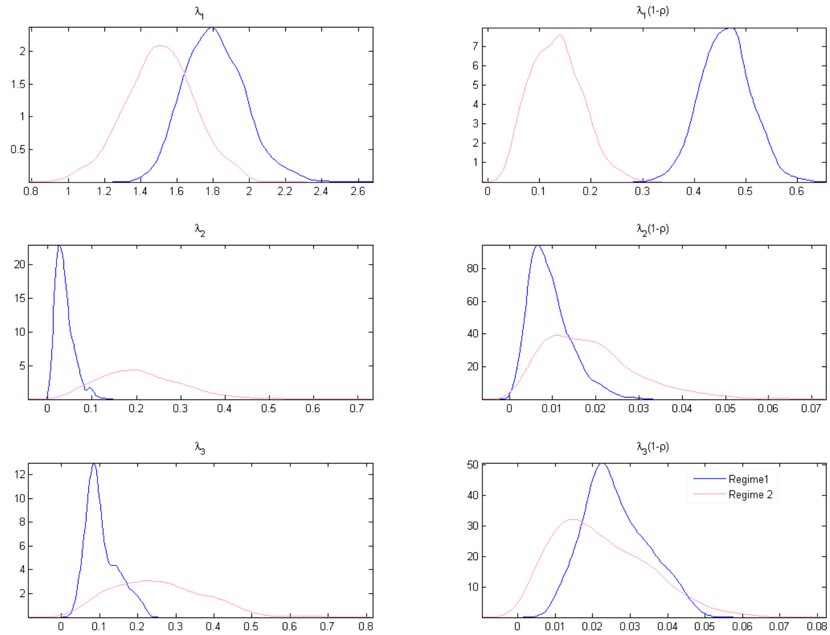
Next, we allow for regime switching in the Phillips curve for import price inflation (equation **(5)**), the posterior estimates are shown in the sixth and seventh column of Table C. The estimate of θ^F is higher in Regime 1 suggesting that this regime is associated with higher degree of import price stickiness. Note, however, that the error bands around the estimates of this parameter in both regimes are large with little evidence that there is a statistically significant shift in this parameter. Similarly, while the point estimate of δ^F is substantially higher in Regime 2, the large error bands do not support a systematic shift in this parameter. Further support for this can be seen in the third-row right-panel of Chart 1 which shows the estimated filter probability of Regime 2 associated with the structural parameters. The sample is dominated by Regime 1 with Pr $(S_t = 2)$ close to zero over most of the sample period. Although many reduced-form VAR studies have documented a decline in import price pass-through for the United Kingdom, we find only weak evidence of a systematic switch in the structural parameters of the import price Phillips curve.

In contrast, the left panel of the third row of Chart 1 shows clear evidence of shift in the shock variances. Note that for this model Pr $(s_t = 2)$ corresponds to the low volatility regime. The chart shows that, as before, the 1970s and the 1980s and the last few quarters of the sample were characterised by high shock volatility.

3.5 The model with the switching Taylor rule

We investigate possible changes in the monetary policy rule by allowing the coefficients of equation **(7)** to be regime dependent (along with shock variances as above). The eight and ninth column of Table C present both the estimated regime-dependent and time-invariant parameters. The estimated value of P_{11} suggests that the first regime is highly persistent. This regime is characterised by a strong reaction to inflation with the posterior mean of λ_1 estimated at 1.8. In contrast, the mean estimate of λ_1 in Regime 2 is slightly lower at 1.5. Note that the second regime is also characterised by significantly higher interest rate smoothing. The point estimates of λ_2 and λ_3 suggest that Regime 2 was associated with a stronger reaction to the output gap and exchange rate changes. Note, however, that the difference across regimes is not significant with the confidence intervals overlapping.

Chart 3: The estimated posterior distribution of inflation reaction coefficient in both regimes



In Chart 3 we explore the difference in λ_1 across regimes further. The top left panel of the chart plots the estimated posterior distribution of λ_1 in the two regimes. The chart clearly shows that the mass of the estimated Regime 1 distribution lies to the right of the Regime 2 distribution. Note that while the Taylor principle does not apply directly in this open economy Markov-switching setting, it is interesting to note that the Regime 2 distribution of λ_1 includes values below 1 in its lower tail pointing to the fact that this regime was associated with a weaker reaction of the central bank to inflation. Note that as there is a significant shift in the degree on interest rate smoothing across regimes, it is also useful to compare the distribution of the impact coefficient on inflation in the policy rule $\lambda_1(1-\rho)$. The right panel of Chart 3 plots the posterior distribution of $\lambda_1(1-\rho)$ in both regimes. It is immediately clear that once interest rate smoothing is accounted for, Regime 1 is clearly associated with a significantly higher reaction to inflation by the monetary authority.[6]

There are a number of different ways of interpreting the fact that Regime 2 is associated with higher interest rate smoothing. First, following conventional wisdom the high estimate of ρ may

[6]Note, however, that the long-run coefficient is key for inflation stabilisation.

reflect the fact that this regime saw a gradual adjustment of interest rates by the monetary authority. Alternatively as suggested by Rudebusch (2002) higher smoothing may reflect more persistent shocks during this regime. Note that an increase in ρ could also potentially reflect the fact that the 'true' policy rule that actually applies during this regime has terms other than output, inflation and the exchange rate on the right-hand side, with the higher estimate of ρ reflecting the absence of these (unobserved) factors.

The fourth row and right panel of Chart 1 shows the estimated filter probability of the coefficient regimes. The sample is dominated by Regime 1 with Regime 2 concentrated in the mid-1970s. In particular, $\Pr(S_t = 2) > 0.5$ over the period 1975 Q2 to 1978 Q1, suggesting that these three years were associated with a weaker reaction by monetary authorities to inflation. The left panel of the fourth row of Chart 1 shows that the high variance regime (Regime 2) dominated the pre-inflation targeting period, with high volatility again returning in the last few quarters.

In order to check the sensitivity of these parameter estimates to the prior distribution for the policy rule parameters, we re-estimate the model using a larger prior variance for the inflation coefficient. In particular, we double the prior variance to 0.5. The aim is to check if under this alternative prior, the Regime 2 estimate of the inflation reaction coefficient is estimated at a lower value. However, our results are relatively unchanged when this alternative prior is used. In particular (as in our benchmark specification), the estimated inflation reaction is larger than 1 in both regimes.

A similar result is reported for the United States in Davig and Doh (2008) who find that once they allow for a change in the volatility of the shocks, the inflation reaction coefficient is estimated to be greater than 1 in both regimes of their DSGE model. As argued by these authors, the estimation algorithm 'fits' the change in inflation volatility (over the 1970s and the subsequent decades) via the estimated change in policy rule coefficients *and* the change in shock volatility. If shock volatilities are restricted to be the same across the sample, Davig and Doh (2008) then find that the estimated inflation reaction coefficient in the passive regime becomes smaller than 1. However, as shown below, there is strong evidence to support a regime switch in the volatility of structural shocks for our data and model. Therefore, we do not consider the possibility of homoscedastic shocks in our application.

Table C: Log marginal likelihood for each estimated model

Model	Log marginal likelihood
M_0	-2035.76
M_1	-1687.86
M_2	-1548.33
M_3	-1564.59
M_4	-1456.69
M_5	-1490.53

4 Model selection and time-varying dynamics of the UK economy

In this section we consider the relative fit of the estimated models discussed above. Following Bianchi (2009) and Davig and Doh (2008), we estimate the marginal likelihood for each version of the estimated DSGE model. The marginal (or the integrated) likelihood is defined as

$$p\left(Y_t\right) = \int_{\theta} f\left(Y_t \backslash \theta\right) g\left(\theta\right) \qquad \textbf{(14)}$$

where θ denotes the DSGE parameters, $f\left(Y_t \backslash \theta\right)$ is the likelihood function while $g\left(\theta\right)$ represents the prior distributions. The marginal likelihood can be approximated using the modified harmonic mean (MHM) method which employs the following theorem

$$\frac{1}{p\left(Y_t\right)} = \int_{\Theta} \frac{h\left(\theta\right)}{f\left(Y_t \backslash \theta\right) g\left(\theta\right)} p\left(\theta \backslash Y_t\right) d\theta \qquad \textbf{(15)}$$

where $h\left(\theta\right)$ denotes a weighting function, ie a probability density function whose support is in Θ. Numerically the integral in equation **(15)** can be evaluated as

$$\frac{1}{p\left(Y_t\right)} = \sum_{i=1}^{N} \frac{h\left(\theta^i\right)}{f\left(Y_t \backslash \theta^i\right) g\left(\theta^i\right)}$$

where $i = 1..N$ indexes the draws from the MCMC sampler. Geweke (1999) suggests a normal density as the weighting function, with the mean and the variance of the density constructed using output from the posterior simulator. The normal density is truncated to ensure that its support lies in the support of the posterior. Sims, Waggoner and Zha (2008) argue that while the implementation of the MHM estimator using the methods proposed in Geweke (1999) works well for standard DSGE models, the addition of time-variation in the model may render the choice of a normal density for $h\left(\theta\right)$ less appropriate. Sims *et al* (2008) argue that this is a result of the fact that the posterior of a time-varying DSGE model can be highly non-Gaussian. Instead of the truncated normal density they propose the use of an elliptical density as the weighting function. However, as in Davig and Doh (2008) we find that in our application this estimator of the marginal likelihood is unstable and highly sensitive to draws far away from the posterior mean. Therefore following Davig and Doh (2008) we use the estimator proposed by Geweke (1999) to compute the marginal likelihood.[7]

Table D presents the estimated value of the log marginal likelihood for each version of the model that we consider. It is clear from the table that the time-invariant DSGE model is strongly rejected by the data. Within, the Markov-switching DSGE models, Model M_1 has the lowest marginal likelihood which suggests that it is not simply the volatility of structural shocks that has changed over time. Model M_2 and Model M_3 are fairly similar in terms of the estimated marginal likelihood. However, Model M_4 which allows the policy rule to switch is clearly preferred. This suggests that for the UK economy a change in the policy rule as well as a change in shock variances is a crucial feature of the data.

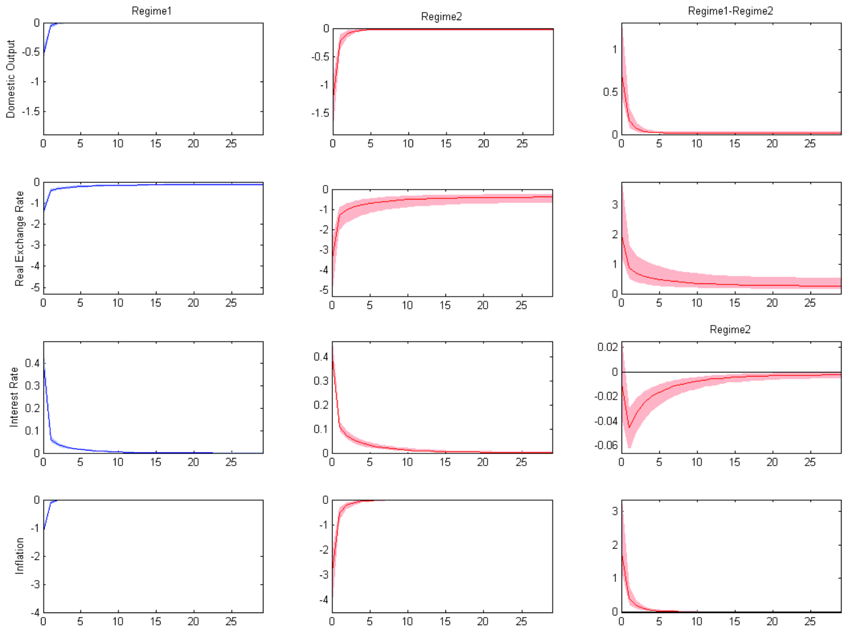
The final row of Table D presents the marginal likelihood from Model M_5. This model allows all parameters of the model to switch. However, it assumes that agents do not form expectations about the the possibility of regime shifts. Instead, the model is solved seperately in each regime (using a standard rational expectations solution method). The last two columns of Table C show that the estimates from this model suggest: (a) a large change in the policy rule, with Regime 2 (prevalent in the mid-1970s – see last row of Chart 1) associated with a smaller reaction to inflation and (b) a large change in the volatility of structural shocks. Note, however, that this model is not preferred to Model M_4 and this suggests that the assumptions inherent in this approach are not supported by the data that we employ.

4.1 Evolving dynamics of the UK economy

In this section we use our preferred Markov-switching DSGE model (Model M_4) to characterise possible changes in UK macroeconomic dynamics. We do this by considering the possible evolution in impulse response functions (to key structural shocks). In addition, we calculate the historical decomposition in order to evaluate the role of different structural shocks in driving key variables.

[7]Note that this MHM estimator is implemented for our Markov-switching model as described in the technical appendix to Sims and Zha (2006).

Chart 4: Impulse response to a monetary policy shock for Model 4



4.1.1 Impulse response functions

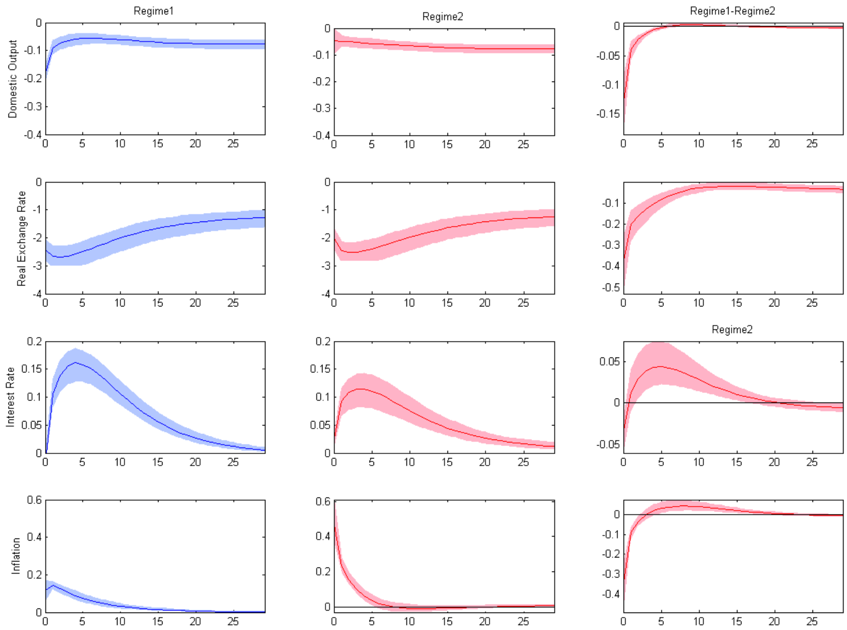
Chart 4 presents the impulse response to a monetary policy shock in each regime of the chosen model.[8] The estimated impulse responses indicate important differences across the two regimes. The final column of the chart shows that the Regime 2 response of output, inflation and the real exchange rate is significantly larger than the estimate in Regime 1. This suggests that the policy rule prevalent during the mid-1970s resulted in larger fluctuations in the economy. This result is similar to that presented in Boivin and Giannoni (2006) for the United States.

The regime-dependent response to a cost-push shock is presented in Chart 5.[9] The role of changing monetary policy is also evident from the response of inflation to this shock. The Regime 2 response of inflation is three times larger (on impact) than the Regime 1 response. In

[8] As in Sims and Zha (2006), the impulse response functions are estimated for each regime seperately. We do not, for example, take into account the possibility of a regime switch once a shock occurs.

[9] The first column in the chart presents the response in Regime 1, ie the regime characterised by a higher reaction to inflation in the policy rule. The second column presents the response in Regime 2, ie the regime characterised by a lower reaction to inflation in the policy rule. The final column presents the difference between the two. The solid lines are medians while the shaded area represents the 64% error band.

Chart 5: Impulse response to a cost-push shock for Model 4

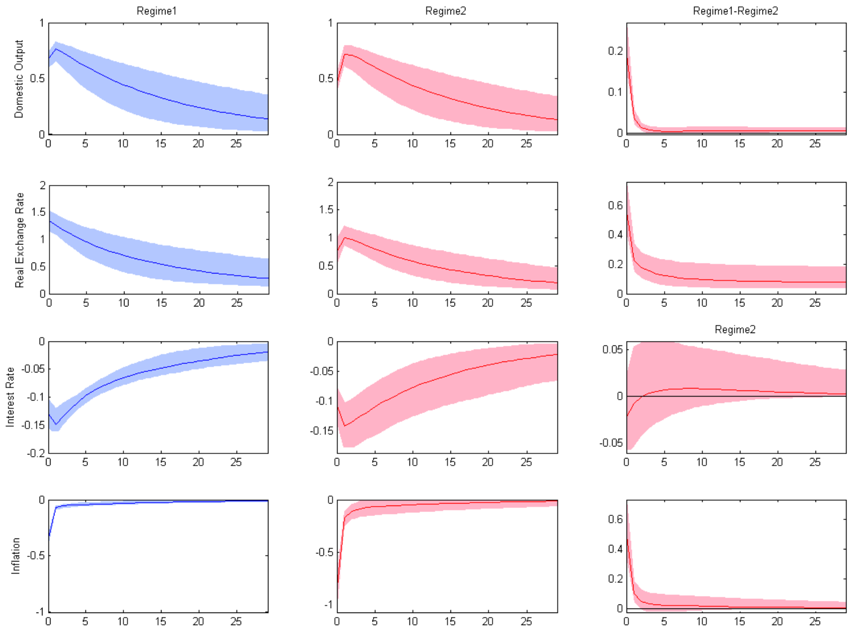


Regime 2, the nominal and the real interest rate increases by a smaller amount and subsequently the resulting real exchange rate appreciation and fall in output is smaller.

Chart 6 shows that the response of inflation to a technology shock was significantly larger during the mid-1970s (Regime 2). In contrast, both output and the real exchange rate increase somewhat more in response to this shock in Regime 1.

In general, the estimated impulse response functions point to significant changes in the dynamics of inflation, with the change in the policy rule (to the Regime 1 rule) resulting in smaller changes in inflation in the face of structural shocks. Note that similar results are evident from the response of inflation to the other shocks in the model which are not presented in the interest of brevity. Full results are available from the authors.

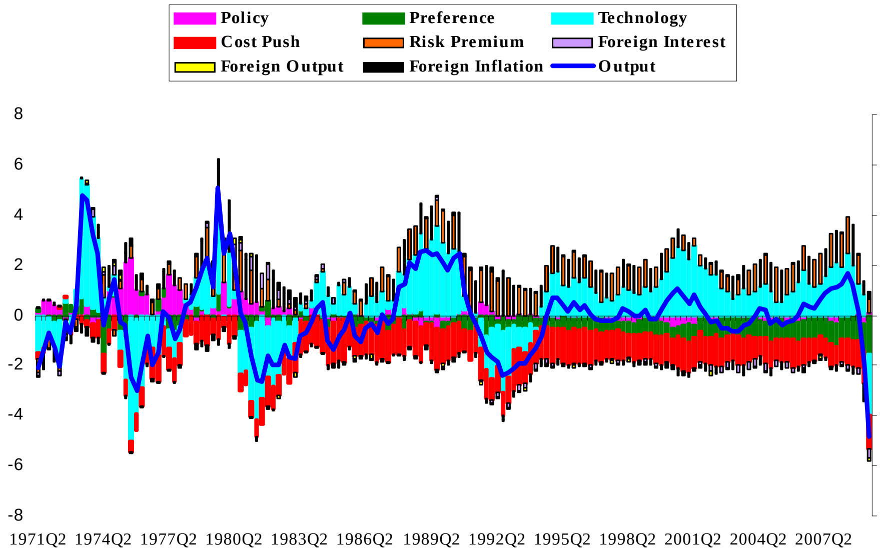
Chart 6: Impulse response to a technology shock for Model 4



4.1.2 Historical decomposition

Charts 7, 8, 9 and 10 present the historical decomposition of output, the real exchange rate, the policy rate and the inflation rate respectively. In order to calculate the historical decomposition we apply the approximate smoothing algorithm described in Kim and Nelson (1999) to the state space in equation **(13)** in order to estimate the smoothed structural shocks. Chart 7 shows that the technology shock is crucial during periods characterised by large fluctuations in output. This is especially true during the recession of the mid-1970s and the early 1980s, the boom of the late 1980s and then in the downturn that characterised the early 1990s. Note that the recent contraction in output is explained largely by the preference, technology and cost-push shocks. It is clear from Chart 10 that the policy shock played the largest role in driving the high inflation during the mid-1970s and the early 1980s. Therefore, as in time-varying VAR-based studies (see Benati (2008)) fluctuations in non-systematic policy played an important role in determining inflation outcomes during the 1970s. Note that preference shocks and policy shocks made a negative contribution to inflation during the 1990s with the sharp drop in inflation in recent quarters attributed to the former shock. Large depreciations in the real exchange rate during the

Chart 7: Historical decomposition of de-trended output

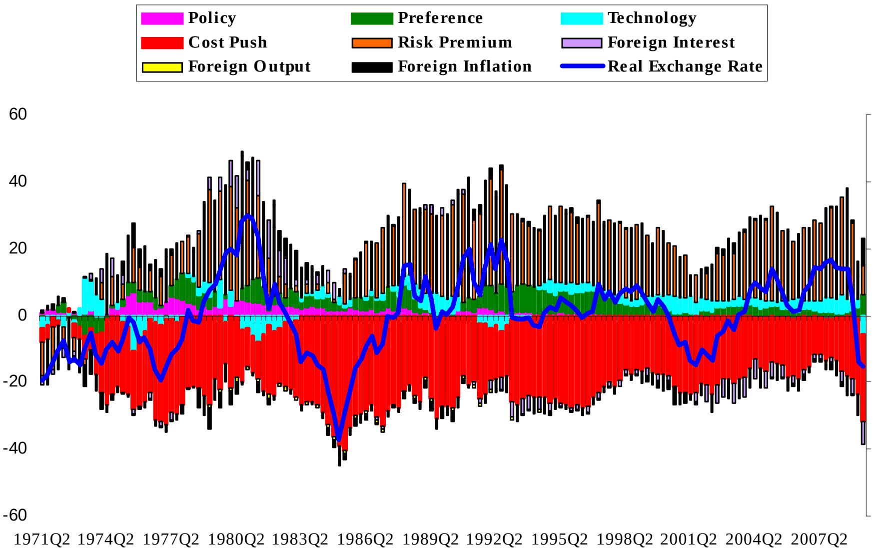


early 1980s and the recent recession appear to be largely driven by the cost-push shock (see Chart 8). Chart 9 shows that the policy shock made a negative contribution to the policy rate during the mid and late 1970s with the cost-push shock making a positive contribution over that period. Fluctuations in the policy rate over the inflation-targeting period are dominated heavily by the preference shock.

5 Conclusions

This paper investigates the possibility of regime changes in an open economy DSGE model estimated using data for the United Kingdom. We estimate versions of the model that allow for a regime switch in the parameters of the Phillips curve, the process for import price inflation and the Taylor rule as well as allowing for (independent) changes in shock volatilities and compare these to a DSGE model with fixed parameters. The model comparison exercise indicates that UK data is best described by the model with the switching Taylor rule and changing shock volatility. Estimates from this model suggest that the mid-1970s were associated with a regime characterised by a smaller reaction by the monetary authorities to inflation. In contrast, the monetary authority reacted more to fluctuations in the output gap and the growth of the exchange rate. Periods of high shock volatility come and go, but were infrequent between 1992 and the

Chart 8: Historical decomposition of demeaned real exchange rate



recent past, although the results towards the end of our sample in 2007/08 and early 2009 are characterised by high volatility. Using this selected model, we show that the response of macroeconomic variables to structural shocks has changed substantially over the sample period.

Chart 9: Historical decomposition of demeaned policy rate

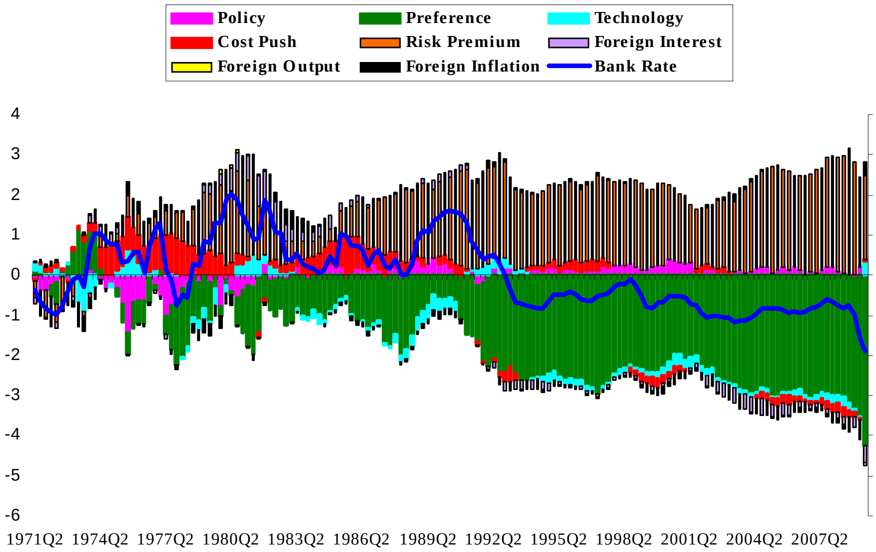
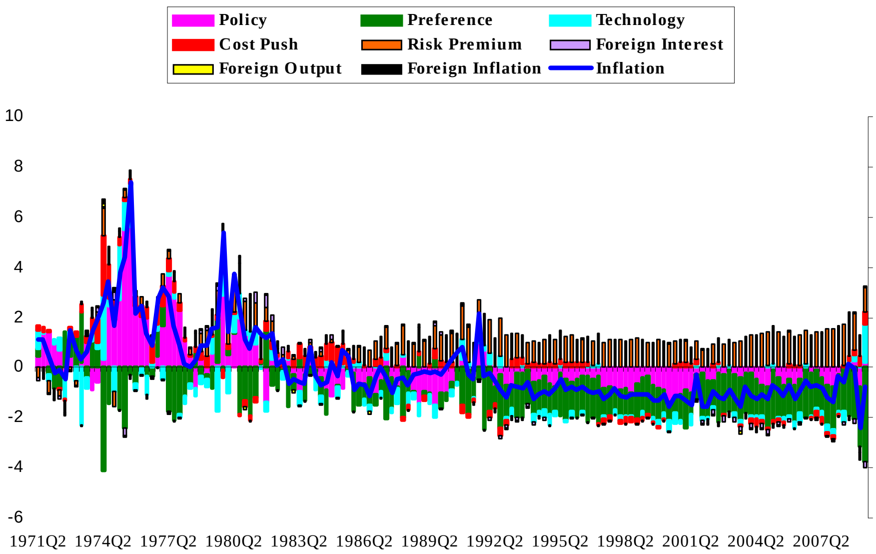


Chart 10: Historical decomposition of demeaned inflation rate



Appendix: Solution and estimation of the Markov-switching DSGE model

Model solution

Consider the Markov-switching model in equation **(10)**. The first step in the solution method involves rewriting the Markov-switching DSGE model as a fixed parameter model in an expanded state vector:

$$\bar{\Gamma}_0 \bar{X}_t = \bar{\Gamma}_1 \bar{X}_{t-1} + \bar{\Upsilon} u_t + \bar{\Pi} \eta_t \qquad \textbf{(A-1)}$$

where

$$\bar{\Gamma}_0 = \begin{pmatrix} diag\left(\Gamma_{0,1}^1, \Gamma_{0,1}^2\right) \\ \Gamma_{0,2} \\ \Phi \end{pmatrix} \qquad \textbf{(A-2)}$$

$$\bar{\Gamma}_1 = \begin{pmatrix} diag\left(\Gamma_{1,1}^1, \Gamma_{1,1}^2\right) \\ \Gamma_{1,2} \\ 0 \end{pmatrix} \qquad \textbf{(A-3)}$$

$$\bar{\Upsilon} = \begin{pmatrix} I & diag(\Psi_1^1, \Psi_1^2) \\ 0 & 0 \end{pmatrix} \qquad \textbf{(A-4)}$$

$$\bar{\Pi} = \begin{pmatrix} 0 \\ \Pi \\ 0 \end{pmatrix} \qquad \textbf{(A-5)}$$

$$\Phi = e_2 \otimes \Phi_{S=2}$$

The expanded state vector $\bar{X}_t$ is defined as

$$\bar{X}_t = \begin{pmatrix} X_t^{S=1} \\ X_t^{S=2} \end{pmatrix} \qquad \textbf{(A-6)}$$

The shocks u_t are defined as

$$u_t = \begin{pmatrix} \Xi^S \left(e_{S_{t-1}} \otimes (1 \otimes I)\right) \bar{X}_{t-1} \\ e_{S_t} \otimes Z_t \end{pmatrix} \qquad \textbf{(A-7)}$$

with $\Xi^S = \left(diag\left[\Gamma_{1,1}^1, \Gamma_{1,1}^2\right]\right) \times ((e_i 1' - P) \otimes I)$ where $e_i, i = 1, 2$ is the i'th column of an identity matrix. As explained in Farmer *et al* (2008), equation **(A-7)** consists of two types of shock. The top element of u_t represent the switching shocks. These turn on and off appropriate blocks of the model to represent the Markov-switching dynamics. The bottom element of u_t are the normal shocks that hit the structural equations distributed to the appropriate block of the model. Farmer *et al* (2008) show that both shocks are zero in expectation.

Farmer *et al* (2008) define the solution to the Markov-switching model as a stochastic process $\{x_t, \eta_t\}_{t=1}^{\infty}$ such that

1. η_t satisfies the property that $E_{t-1}(\eta_t) = 0$.
2. x_t is bounded in expectation $\|E_t(x_{t+s})\| < M_t$ for all $s > 0$.
3. $\{x_t, \eta_t\}$ satisfy equation **(11)**.

Farmer *et al* (2008) argue that the class of solutions that satisfy these conditions is large with sunspot solutions dominant. They therefore focus on MSV solutions and prove that the MSV solution to the expanded system **(11)** is an MSV solution to the original model. The matrices Φ play a key role in the definition of the MSV solution. In order to ensure that the stochastic process $\{x_t, \eta_t\}_{t=1}^{\infty}$ is bounded the solution is constrained to lie in the linear subspace. Farmer *et al* (2008) achieve this by defining a matrix z such that $z'X_t = 0$. In other words, while the definition of equation **(11)** ensures that $X_t^{S=2} = 0$ when Regime 1 occurs, the matrix z is required to ensure that restrictions are imposed on $X_t^{S=1}$ when Regime 1 occurs. A definition of Φ which imposes this restriction is as follows: Start with an intial value for $\Phi_{S=2}$ $\left(\Phi_{S=2}^{0}\right)$ and define the matrices $\bar{\Gamma}_0^0$ and $\bar{\Gamma}_1$ where the superscript denotes the steps of an iterative procedure. Compute the QZ decomposition of $\left\{\bar{\Gamma}_0^0, \bar{\Gamma}_1\right\} : Q^0 T^0 Z^0 = \bar{\Gamma}_1$ and $Q^0 S^0 Z^0 = \bar{\Gamma}_0^0$. Order the matrices T^0 and S^0 such that the ratio of the diagonal elements T_{ii}^0 / S_{ii}^0 are arranged in increasing order. Let q be the integer such that $T_{ii}^0 / S_{ii}^0 < 1$ is $i < q$ and $T_{ii}^0 / S_{ii}^0 > 1$ if $i > q$. Let z_u be the last $np - q$ rows of $Z^0 = \{z_1, z_2\}$. Set $\Phi_{S=2}^1 = z_2$ and repeat until convergence. If this iterative procedure converges, then the solution to equation **(11)** is a solution to the original model. Then *Gensys* can be used to check for existence, uniqueness and compute the solution to equation **(11)**. If a unique solution exists then this can be written as a Markov-switching VAR

$$X_t = G^S X_{t-1} + A^S Z_t \qquad \textbf{(A-8)}$$

Calculating the likelihood function

The filtering algorithm of Kim and Nelson (1999) proceeds as follows. The first step of the algorithm involves running the following Kalman filter recursions conditional on $S_t^* = j$, $S_{t-1}^* = i$ and $S_{t-2}^* = h$ where S_t^* indexes both S_t and s_t.

$$X_{t,t-1}^{(h,i,j)} = G^j X_{t,t-1}^{(i,j)} \quad \textbf{(A-9)}$$

$$P_{t|t-1}^{(h,i,j)} = G^j P_{t-1|t-1}^{(i,j)} G^{j\prime} + A^j Q^j A^{j\prime} \quad \textbf{(A-10)}$$

$$\eta_{t|t-1}^{(h,i,j)} = Y_t - H X_{t|t-1}^{(h,i,j)} \quad \textbf{(A-11)}$$

$$f_{t|t-1}^{(h,i,j)} = H P_{t|t-1}^{(h,i,j)} H' \quad \textbf{(A-12)}$$

$$X_{t|t}^{(h,i,j)} = X_{t,t-1}^{(h,i,j)} + P_{t|t-1}^{(h,i,j)} H' \left(f_{t|t-1}^{(h,i,j)}\right)^{-1} \eta_{t|t-1}^{(h,i,j)} \quad \textbf{(A-13)}$$

$$P_{t|t}^{(h,i,j)} = (I - P_{t|t-1}^{(h,i,j)} H' \left(f_{t|t-1}^{(h,i,j)}\right)^{-1}) H P_{t|t-1}^{(h,i,j)} \quad \textbf{(A-14)}$$

$X_{t,t-1}^{(h,i,j)}$ denotes an inference of the DSGE states conditional on information at time $t-1$ and $S_t^* = j$, $S_{t-1}^* = i$ and $S_{t-2}^* = h$. Similarly $P_{t|t-1}^{(h,i,j)}$ and $f_{t|t-1}^{(h,i,j)}$ is the covariance of $X_{t,t-1}^{(h,i,j)}$ and $\eta_{t|t-1}^{(h,i,j)}$. $X_{t,t-1}^{(i,j)}$ refers to the collapsed states and denotes an inference on the X_t conditional on information at time $t-1$ and $S_t^* = j$, $S_{t-1}^* = i$. Kim and Nelson (1999) propose the following approximation for $X_{t,t-1}^{(i,j)}$:

$$X_{t,t}^{(i,j)} = \sum_{h=1}^{M} \Psi_{i,j,h} \times X_{t,t}^{(h,i,j)} \quad \textbf{(A-15)}$$

The covariance $P_{t/t}^{(i,j)}$ is approximated as

$$P_{t/t}^{(i,j)} = \sum_{h=1}^{M} \Psi_{i,j,h} \times \left[\left(P_{t/t}^{(h,i,j)}\right) + \left(X_{t,t}^{(i,j)} - X_{t,t}^{(h,i,j)}\right)\left(X_{t,t}^{(i,j)} - X_{t,t}^{(h,i,j)}\right)'\right] \quad \textbf{(A-16)}$$

The weighting terms $\Psi_{i,j,h}$ are the following probabilities:

$$\Psi_{i,j,h} = \frac{\Pr\left(S_{t-2}^* = h, S_{t-1}^* = i, S_t^* = j\right)}{\Pr\left(S_{t-1}^* = i, S_t^* = j\right)} \quad \textbf{(A-17)}$$

The Hamilton filter (see Hamilton (1989)) is used to update the estimates for elements $\Psi_{i,j,h}$. The Hamilton filter recursion consists of two steps: Given an initial value for $\Pr\left(S_{t-1}^* = i, S_t^* = j\right)$ at time $t-1$, a new estimate for $\Pr\left(S_{t-2}^* = h, S_{t-1}^* = i, S_t^* = j\right)$ is obtained as

$$\Pr\left(S_{t-2}^* = h, S_{t-1}^* = i, S_t^* = j | \Xi_{t-1}\right) = \Pr\left(S_{t-1}^* = i, S_t^* = j\right) \times P^* \quad \textbf{(A-18)}$$

where Ξ denotes the information set. Once the data is observed at time t, this estimate can be updated using

$$\Pr\left(S_{t-2}^{*}=h,\, S_{t-1}^{*}=i,\, S_{t}^{*}=j\,|\,\Xi_{t}\right)=$$
$$\frac{f\left(X_{t}|S_{t-2}^{*}=h,\, S_{t-1}^{*}=i,\, S_{t}^{*}=j,\, \Xi_{t-1}\right)\times\Pr\left(S_{t-2}^{*}=h,\, S_{t-1}^{*}=i,\, S_{t}^{*}=j\,|\,\Xi_{t-1}\right)}{\sum_{h=1}^{M}\sum_{i=1}^{M}\sum_{j=1}^{M}f\left(X_{t}|S_{t-2}^{*}=h,\, S_{t-1}^{*}=i,\, S_{t}^{*}=j,\, \Xi_{t-1}\right)\times\Pr\left(S_{t-2}^{*}=h,\, S_{t-1}^{*}=i,\, S_{t}^{*}=j\,|\,\Xi_{t-1}\right)} \quad \textbf{(A-19)}$$

where the conditional densities are defined as

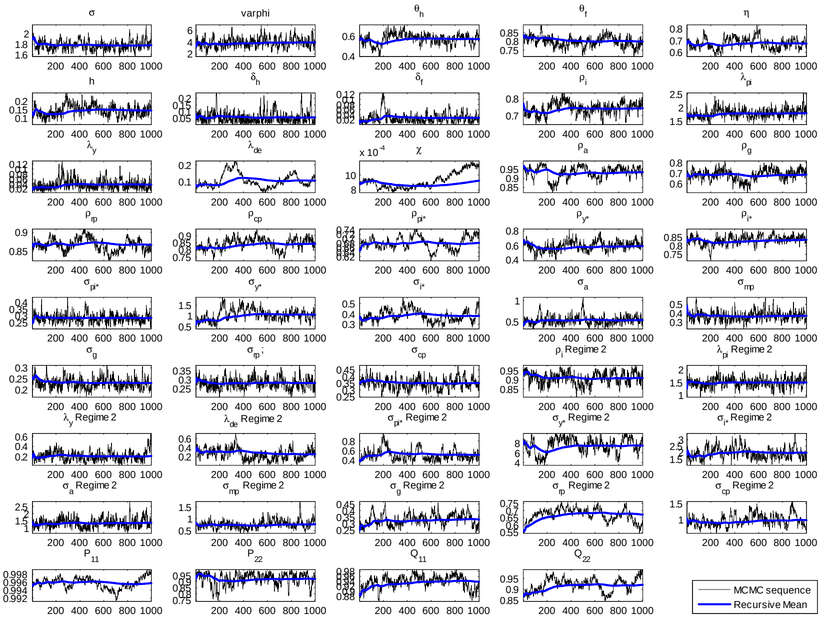
$$f\left(X_{t}|S_{t-2}^{*}=h,\, S_{t-1}^{*}=i,\, S_{t}^{*}=j,\, \Xi_{t-1}\right)=2\pi^{-n/2}\left|f_{t|t-1}^{(h,i,j)}\right|^{-1/2}\exp\left(-\frac{1}{2}\eta_{t|t-1}^{(h,i,j)\prime}\left(f_{t|t-1}^{(h,i,j)}\right)^{-1}\eta_{t|t-1}^{(h,i,j)\prime}\right) \quad \textbf{(A-20)}$$

Note that the denominator of equation **(A-19)** is the marginal density of X_t, $f\left(X_t|\Xi_{t-1}\right)$. The approximate log-likelihood of the model is given by

$$\ln L=\sum_{t=1}^{T}\ln f\left(X_{t}|\Xi_{t-1}\right) \quad \textbf{(A-21)}$$

Convergence

The chart below presents the sequence and recursive means of the (last 1,000) Metropolis Hastings draws for the chosen model (Model 4). The sequence of draws for most parameters appears to fluctuate around a relatively stable mean providing some evidence of convergence to the posterior distribution.


MCMC sequence
Recursive Mean

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