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Abstract

Changes in monetary policy and shifts in dynamics of the macroeconomy are typically described using empirical models that only include a limited amount of information. Examples of such models include time-varying vector autoregressions that are estimated using output growth, inflation and a short-term interest rate. This paper extends these models by incorporating a larger amount of information in these tri-variate VARs. In particular, we use a factor augmented vector autoregression extended to incorporate time-varying coefficients and stochastic volatility in the innovation variances. The reduced-form results not only confirm the finding that the great stability period in the United Kingdom is characterised by low persistence and volatility of inflation and output but also suggest that these findings extend to money growth and asset prices. The impulse response functions display little evidence of a price puzzle indicating that the extra information incorporated in our model leads to more robust structural estimates.

Key words: FAVAR, great stability, time-varying parameters, stochastic volatility.

JEL classification: E30, E32.

Contents

Summary

After the introduction of inflation targeting in 1992, the United Kingdom experienced a period of low inflation and stable output growth often referred to as the 'great stability'. Recent research into this phenomenon has suggested that this stability had been unmatched since the gold standard. A growing empirical literature has examined this apparent change in the dynamics of the UK economy, perhaps due to shifts in the monetary policy regime. These papers usually employ empirical models that contain a limited amount of macroeconomic variables – typically using systems of equations known as vector autoregressions (VARs): a set of equations where the explanatory variables in each equation are the complete set of lagged variables in the system. GDP growth, inflation and the nominal interest rate are the typical variables included in VARs that describe the transmission mechanism of monetary policy. If, in reality, the central bank examines a wider set of variables when setting policy, estimates of the monetary policy shock derived from these small empirical models may be biased – ie not completely disentangled from non-policy shocks. As a consequence an accurate assessment of structural shifts may be hampered.

This paper therefore explores the dynamics of the United Kingdom's macroeconomy using a VAR model that incorporates a larger amount of economic information than a typical tri-variate model. In particular, we use an extended version of the 'factor augmented VAR' (FAVAR) model recently proposed in the literature. The idea behind the FAVAR model is that the bias created by the difference in the information set of the researcher and the agents described in the model can be alleviated by augmenting the standard VAR with *common factors* that are extracted from a large set of macroeconomic indicators. These common factors summarise the relevant information in the macroeconomic indicators and therefore provide a proxy for the information set of agents in the model.

Our FAVAR model for the United Kingdom contains common factors extracted from data on real activity, inflation, money and credit and asset prices in addition to a short-term nominal interest rate. We allow the coefficients of the model and the variances of the shocks to vary over time. The model is estimated over the period 1970 Q1 to 2004 Q2, thus restricting attention to the period before and during great stability.

In accordance with previous studies, our estimates show a decline in the volatility of shocks to inflation and real activity. In addition, the results suggest that this stability extends to money, credit and asset prices. The average response of the variables in the FAVAR to monetary policy shocks is similar before and after the introduction of inflation targeting. The response of inflation to a (contractionary) monetary policy shock appears to be more plausible than previous studies – in particular not displaying an anomalous (initial) positive response (ie the 'price puzzle'). This may point to the fact that the extra information included in this model improves the identification of the monetary policy shock. Shocks to monetary policy contribute little to inflation and the interest rate during the inflation-targeting period.

1 Introduction

After the introduction of the inflation-targeting regime in 1992, the United Kingdom experienced an unprecedented period of low inflation and stable output growth that is sometimes referred to as the 'great stability', lasting at least until the rise in energy and commodity prices in 2004. Benati (2004) showed that this stability was unmatched since the gold standard.

The onset of this great stability may have indicated a possible change in the dynamics of the UK economy and the transmission and practice of monetary policy. For the United States (where this phenomenon is referred to as the 'great moderation'), this question has received considerable attention since the seminal work of Cogley and Sargent (2002) who report a significant change in the degree of 'activism' of US monetary policy. As in Clarida, Gali and Gertler (2000) the authors argue that the fall in the level and persistence of inflation in the 1980s and the 1990s coincided with an increase in the degree of activism. Some of the subsequent literature has produced different results. For example, the evidence on US policy activism reported in Cogley and Sargent (2005) and based on an extended model is less clear cut than the authors' earlier work. Similar results are reported in Primiceri (2005) and Sims and Zha (2006).

Most of these studies use VAR models, extended to allow for time variation in the coefficients and variances. This methodology is undoubtedly powerful. However, one potential problem is the fact the amount of information incorporated in these models is relatively limited. Typically, the VAR models consist of three variables – a short-term interest rate, output growth and inflation. This feature has two potential consequences. First, missing variables could lead to biases in the reduced-form VAR coefficients. This may imply that reduced-form estimates of persistence and volatility are biased. Second, the omission of some variables could hinder the correct identification of structural shocks. One possible manifestation of these problems is impulse response functions that are at odds with economic theory. A number of recent studies have raised these points. For example, Bernanke, Boivin and Eliasz (2005) argue that if the information set used by the econometrician is smaller than that employed by the monetary authority, then structural shocks and their responses may be mismeasured because the empirical model excludes some variables that the central bank responds to. Similarly, Castelnuovo and Surico (2006), building on Lubik and Schorfheide (2004), argue that during periods of indeterminacy, the dynamics of the economy are characterised by a latent variable. Therefore,

(reduced-form and structural) estimates of the VAR model may be biased when estimation is carried out over these periods.

This paper examines the evolving UK macroeconomy in an empirical framework that incorporates substantially more information than the standard three-variable model used in most studies. In particular, we employ an extended version of the factor augmented VAR introduced in Bernanke *et al* (2005). This model incorporates information from a large number of macroeconomic indicators representing various sectors of the economy. Our extensions include allowing for time variation in the coefficients and the variances of the shocks.

Estimates from our model suggest a fall in persistence and volatility of inflation, real activity, money and asset prices during the inflation-targeting period. In addition, estimates of the monetary policy shock also indicate a decrease in volatility. The extra information incorporated in the model substantially reduces the price puzzle, especially during the 1970s. In particular, the persistent increase in inflation following a monetary tightening over this period (as documented in Castelnuovo and Surico (2006)) is not observed. This points to estimates of the structural shocks that are more robust. Impulse responses from the model indicate little change in the transmission of monetary policy shocks. A decomposition of the volatility of each variable indicates a contribution of the policy shock of around 10% to 20%.

The paper is organised as follows. The next two sections introduce the empirical methodology used in the study. Section 4 describes the basic reduced-form estimates from the model. Section 5 presents and interprets the structural estimates. Section 6 concludes.

2   Factor augmented VARs

Consider the following simple backward-looking model of the economy:

$$\pi_t = \beta \pi_{t-1} + \chi \left(y_{t-1} - y_{t-1}^* \right) + s_t \tag{1}$$

$$y_t = \alpha y_{t-1} + \varpi \left(R_{t-1} - \pi_{t-1} \right) + d_t \tag{2}$$

where the Phillips curve in equation **(1)** relates inflation (π_t) to the deviation of output (y_t) from

potential (y^*) and a supply shock s_t. Equation **(2)** is a standard IS curve that describes the relationship between output and the real interest rate ($R_{t-1} - \pi_{t-1}$) and a demand shock, d_t. Finally, the monetary authority sets interest rates according to a standard Taylor rule:[1]

$$R_t = B\pi_{t-1} + \lambda \left(y_{t-1} - y_{t-1}^* \right) + v_t \tag{3}$$

where v_t is the monetary policy shock.

Bernanke *et al* (2005) argue that assumptions made about the information structure are crucial when deciding whether a standard VAR can describe such a model. In particular, if it is assumed that the variables in the VAR correspond exactly to the model variables and are observed by the central bank *and* the econometrician, then the VAR model provides an adequate description of the theoretical model. However, both these assumptions are difficult to justify. First, measurement error implies that measures of inflation and output are less than perfect proxies for model variables. Of course this problem is much more acute for unobserved variables such as potential output. Second, it is highly likely that the researcher only observes a subset of the variables examined by the monetary authority.

The obvious solution to this problem is to try to include more variables in the VAR. However the degrees of freedom constraint becomes binding quite quickly in standard data sets. Bernanke *et al* (2005) suggest a more practical solution. They propose a 'factor augmented' VAR (FAVAR) model, where factors from a large cross-section of economic indicators are included as extra endogenous variables in a VAR. More formally, let $X_{i,t}$ be a $T \times N$ matrix of economic indicators thought to be in the central bank's information set and let $Y_{j,t}$ denote a $T \times M$ matrix of variables that are assumed to be observed by both the econometrician and the central bank. Then the FAVAR model can be written as:

$$
\begin{aligned}
X_{i,t} &= \Lambda F_t + \Psi Y_{j,t} + e_{i,t}, \\
\begin{pmatrix} F_t \\ Y_{j,t} \end{pmatrix} &= \Phi \begin{pmatrix} F_{t-1} \\ Y_{j,t-1} \end{pmatrix} + v_t,
\end{aligned}
\tag{4}
$$

---

[1] It is not suggested that the UK monetary authorities set interest rates using such a rule, but it is a convenient empirical representation of monetary policy.

where $i = 1, 2..N$, $j = 1, 2..M$,

$$E(e'_{i,t}e_{i,t}) = R$$
$$E(v'_t v_t) = Q$$
$$E(e'_{i,t}v_t) = 0$$

and F_t is $T \times J$ matrix of common factors, Λ is an $N \times J$ matrix of factor loadings and Ψ is a $N \times M$ matrix of coefficients that relate $X_{i,t}$ to $Y_{i,t}$.

The first expression in **(4)** is the observation equation of the system and describes how the observed series are related to the unobserved factors. The second expression (the transition equation) is a VAR(L) in F_t, Y_t (with a $((J + M) \times L) \times ((J + M) \times L)$ coefficient matrix Φ) and is used to describe the dynamics of the economy.

Two identification issues need to be dealt with in this extended VAR model. First, in order to identify the factors, restrictions need to be placed on either the observation or the transition equation. Bernanke *et al* (2005) leave the transition equation unrestricted and impose restrictions on the factor loadings. In particular, the top $J \times J$ block of Λ is assumed to be an identity matrix and the top $J \times M$ block of Ψ is assumed to be zero. Note, however, that these restrictions only offer a normalisation and do not impose a structural interpretation on the factors. This requires further restrictions on the factor loading matrix. These are described in detail in the context of our empirical model.

The second identification issue concerns the identification of shocks to the transition equation. As in the standard VAR literature, this is carried out by imposing restrictions on the covariance of the VAR innovations, Q, or by restricting the sign of the impulse response functions. Once the structural shocks are identified, impulse response functions can be constructed not only for F_t and $Y_{i,t}$ but for all the variables in $X_{i,t}$.

3   A small model of the UK economy

Our model for the UK economy is closely related to the FAVAR model described above. There are, however, two crucial differences. First, as in Belviso and Milani (2005) we impose a structural interpretation on the factors. Second, we allow the parameters of the transition equation to be time-varying. This time variation allows us to examine the United Kingdom's macroeconomic performance over the past three decades.

Consider first the observation equation:

$$
\begin{pmatrix} X_{i,t}^y \\ X_{i,t}^\pi \\ X_{i,t}^m \\ R_t \\ X_{i,t}^a \end{pmatrix} = \begin{pmatrix} \Lambda^y & 0 & 0 & 0 & 0 \\ 0 & \Lambda^\pi & 0 & 0 & 0 \\ 0 & 0 & \Lambda^m & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \Psi^a & \Lambda^a \end{pmatrix} \begin{pmatrix} F_t^y \\ F_t^\pi \\ F_t^m \\ R_t \\ F_t^a \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ 0 \\ e_{4t} \end{pmatrix}
\tag{5}
$$

Here, the superscript y denotes real activity, π denotes inflation, m denotes money and a represents asset prices. $X_{i,t}^y$ is a panel of variables that contain information about real activity in the United Kingdom. Similarly, $X_{i,t}^\pi$, $X_{i,t}^m$ and $X_{i,t}^a$ represent sets of variables that contain information about inflation, money supply and asset price movements respectively. The Λ are the corresponding matrices of factor loadings. As in Bernanke *et al* (2005) we assume that the short-term nominal interest rate R_t is the 'observed factor', ie the variable observed by the econometrician and the monetary authority. The structure of the loading matrix implies that only asset prices $X_{i,t}^a$ are allowed to have a contemporaneous relationship with short-term interest rates.

The four unobserved factors $\left(F_t^y, F_t^\pi, F_t^m, F_t^a \right)$ can now be interpreted as a real activity factor, an inflation factor, a money factor and an asset price factor respectively. We assume that these four factors (along with R_t) capture the relevant information about the United Kingdom's macroeconomy.[2]

As we describe below, time varying dynamics are introduced into the model by allowing for drift

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[2] An alternative approach to identification of the factors would be to consider sign restrictions as in Kose, Otrok and Whiteman (2003). Experimentation with this approach suggested that in the current set-up it leads to a large increase in computation time and makes the estimation algorithm less stable.

in the coefficients and the error covariance matrix of the transition equation. Note that an alternative way of modelling time variation is to allow the factor loadings ($\Lambda's$ and Ψ^a) to drift over time. There are, however, two reasons why we do not adopt this alternative model. First, this model implies that any time variation in the dynamics of each factor *and* the volatility of shocks to each factor is driven entirely by the drift in the associated factor loading. This assumption is quite restrictive, especially as it only allows changes in the mean and persistence of each factor to occur simultaneously with changes in the volatility of the shocks.[3] Second, this model implies a much larger computational burden as the Kalman filter and smoother have to be employed for each underlying series.[4]

The transition equation of the system is a VAR model of the following form:

$$Z_t = \sum_{l=1}^{L} \phi_{l,t} Z_{t-l} + v_t \tag{6}$$

where $Z_t = \{F_t^y, F_t^\pi, F_t^m, R_t, F_t^a\}$ and L is fixed at 2.

We postulate the following law of motion for the coefficients ϕ.

$$\phi_t = \phi_{t-1} + \eta_t$$

The covariance matrix of the innovations v_t is factored as

$$VAR(v_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})' \tag{7}$$

The time-varying matrices H_t and A_t are defined as:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 & 0 \\ 0 & 0 & 0 & h_{4,t} & 0 \\ 0 & 0 & 0 & 0 & h_{5,t} \end{bmatrix} \quad A_t \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 & 0 \\ \alpha_{51,t} & \alpha_{52,t} & \alpha_{53,t} & \alpha_{54,t} & 1 \end{bmatrix} \tag{8}$$

with the $h_{i,t}$ evolving as geometric random walks,

$$\ln h_{i,t} = \ln h_{i,t-1} + v_t$$

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[3] In addition, this model implies that the dynamics of the observed factor are time invariant. Moreover, the impact of the observed factor on the other variables in the transition equation is also assumed to be constant over time. Again, these assumptions are quite restrictive in a model designed to investigate the changing impact of monetary policy.

[4] This is also the main reason why we do not consider time variation in both the observation and the transition equations.

Following Primiceri (2005) we postulate the non-zero and non-one elements of the matrix A_t to evolve as driftless random walks,

$$\alpha_t = \alpha_{t-1} + \tau_t , \tag{9}$$

and we assume the vector $[e'_t, \upsilon'_t, \eta'_t, \tau'_t, \nu'_t]'$ to be distributed as

$$
\begin{bmatrix} e_t \\ \upsilon_t \\ \eta_t \\ \tau_t \\ \nu_t \end{bmatrix} \sim N(0, V), \text{ with } V = \begin{bmatrix} R & 0 & 0 & 0 & 0 \\ 0 & \Omega_t & 0 & 0 & 0 \\ 0 & 0 & Q & 0 & 0 \\ 0 & 0 & 0 & S & 0 \\ 0 & 0 & 0 & 0 & G \end{bmatrix} \text{ and } G = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{bmatrix}
\tag{10}
$$

The model described by equations **(5)** to **(10)** incorporates a large amount of information about the UK economy. In particular, if the factors in equation **(5)** contain relevant information not captured by the three variables in the VAR used in studies such as Primiceri (2005) then one might expect policy shocks identified within the current framework to be more robust. Our flexible specification for the transition equation implies that the model accounts for the possibility of structural breaks in the dynamics that characterise the economy.[5]

3.1 Estimation

The model described by equations **(5)** to **(10)** is estimated using the Bayesian methods described in Kim and Nelson (1999). In particular, we employ a Gibbs sampling algorithm that approximates the posterior distribution. The algorithm exploits the fact that given observations on Z_t the model is a standard time-varying parameter model.

A detailed description of the prior distributions and the sampling method is given in Appendix A. Here we summarise the basic algorithm which involves the following steps:

1. Given initial values for the factors simulate the VAR parameters and hyperparameters

   - The VAR coefficients ϕ_t and the off-diagonal elements of the covariance matrix α_t are

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[5]Note, however, that this model may still not directly capture instability in the factor loadings.

simulated by using the methods described in Carter and Kohn (2004)

- The volatilities of the reduced-form shocks H_t are drawn using the date by date blocking scheme introduced in Jacquier, Polson and Rossi (2004).

- The hyperparameters Q and S are drawn from an inverse Wishart distribution while the elements of G are simulated from an inverse gamma distribution.

2. Given initial values for the factors draw the factor loadings (Λ and Ψ) and the covariance matrix R.

- Given data on Z_t and $X_{i,t}$ standard results for regression models can be used and the coefficients and the variances are simulated from a normal and inverse gamma distribution.

3. Simulate the factors conditional on all the other parameters

- This is done in a straightforward way by employing the methods described in Bernanke *et al* (2005) and Kim and Nelson (1999).

4. Go to step 1.

We use 100,000 Gibbs sampling replications and discard the first 60,000 to reduce the impact of intial conditions. Out of the remaining 40,000 draws, we retain every tenth replication in order to reduce correlation among the draws. We use the last 1,000 draws for inference. Appendix B shows that the autocorrelation of the retained draws is reasonably low providing reasonable evidence of convergence to the ergodic distribution.

3.1.1 Data

We employ quarterly data for the United Kingdom over the years 1970 Q1 to 2004 Q2. A list of all the variables used for each factor is provided in Appendix C. The main variables used for the activity factor include measures of public and private consumption and investment, industrial production across a variety of sectors and the trade balance. Variables for the inflation factor include price indices of goods at various stages of production (ie input and final good prices) and a measure of inflation expectations. The choice of variables for the money and asset price factors

was limited primarily by data availability and the cross-section is somewhat smaller than the corresponding data used by Bernanke *et al* (2005) for the United States. Data on M0, M4 deposits and lending are used for the money factor, while equity prices, dividend yields and house prices enter the panel for the asset price factor. As in Belviso and Milani (2005) all variables are standardised. Note that this implies that the estimated factors are unitless.

4 Reduced-form results

First we assess how our evidence on the persistence and variability of macroeconomic indicators compares with results reported in previous studies.

4.1 Factors

Chart 1 plots the median estimates of the four unobserved factors, along with the one standard deviation confidence band.

The real activity and the inflation factor match prior expectations – ie they indicate lower activity and higher inflation in the mid-1970s, the early 1980s and the early 1990s.

The money growth factor indicates that the inflation-targeting regime was characterised by relatively lower money growth than the preceding two decades.

The estimated asset price factor is quite volatile but clearly indicates the large asset price falls over the periods 1974 Q3 to 1975 Q2, 1979 Q4 to 1980 Q3 and 1987 Q4 to 1991 Q1.

4.2 Persistence

Chart 2 plots the spectral density of the factors at frequency zero. This is calculated as

$$f_{t|T}(\omega) = s(I_5 - \phi_{t|T}e^{-i\omega})^{-1}\frac{\Omega_{t|T}}{2\pi}\left[(I_5 - \phi_{t|T}e^{-i\omega})^{-1}\right]' s'$$

where I_5 denotes a 5×5 identity matrix and s is a selection vector that picks out the coefficients associated with the ith variable in the VAR. The frequency ω is set equal to zero.

It is immediately clear that the inflation-targeting period is associated with the lowest persistence

in all four factors. The results for inflation, in particular, are very similar to those presented in Cogley, Morozov and Sargent (2005) and Osborn and Sensier (2005). The estimate indicates high inflation persistence in the 1970s and the 1980s but suggests that inflation has been very close to a white noise process after the introduction of the inflation-targeting regime in 1992.

4.3 Evolution of stochastic volatilities

Chart 3 plots the standard deviations of the innovations to the transition equation (ie the elements of the H_t matrix). In general the volatilities exhibit the hump shaped pattern reported by Cogley *et al* (2005). That is, the volatility of the shocks hitting each equation was at its highest in the early 1970s and the early 1980s but has declined substantially since. Note that the decline in variance of the shocks to the interest rate and the asset price factor corresponds closely to the start of the inflation-targeting period.

The last panel of the chart plots the 'total prediction variance', a measure of the total noise hitting the system. This is calculated as $\ln |\Omega_{t|T}|$. This measure of volatility has declined significantly. This not only supports the results reported in previous studies but suggests that they apply in a more general VAR that includes a number of additional variables.

5 Structural analysis

As in Bernanke *et al* (2005), we proceed by identifying a monetary policy shock. However, in a departure from Bernanke *et al's* approach we impose sign restrictions on the covariance matrix. This follows Canova and de Nicolo (2002) and Uhlig (2005), among others. Our identification strategy imposes the restriction that the contemporaneous impact of the monetary policy shock be non-negative on the interest rate and non-positive on the other four variables in the VAR. The decision to impose these restrictions on the contemporaneous impact of the shock is driven by two concerns. First, this reduces the computational burden considerably as it does not require the calculation of the impulse response functions for every draw of the Gibbs sampler.[6] Second, contemporaneous restrictions allow us to be relatively 'agnostic' about the impact of the monetary policy shock (beyond the contemporaneous effects) while simultaneously imposing

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[6] In contrast our procedure implies that the impulse response functions are calculated only for those draws of the Gibbs sampler that satisfy the sign restrictions.

more structure than a Cholesky decomposition. In other words, our identification scheme leaves open the possibility that the eventual impact of the shock on some variables may violate our theoretical priors. That is, our identification scheme still allows puzzles such as the price puzzle to arise. As argued below, the degree to which this occurs may be used to determine the relative performance of the model.

The identification scheme is implemented as follows. We compute the time-varying structural impact matrix, $A_{0,t}$, via the procedure recently introduced by Rubio, Waggoner and Zha (2005). Specifically, let $\Omega_t = P_t D_t P_t'$ be the eigenvalue-eigenvector decomposition of the VAR's time-varying covariance matrix Ω_t, and let $\tilde{A}_{0,t} \equiv P_t D_t^{\frac{1}{2}}$. We draw an $N \times N$ matrix K from the $N(0, 1)$ distribution. We take the QR decomposition of K. That is we compute Q and R such that $K = QR$. We then compute a candidate structural impact matrix as $A_{0,t} = \tilde{A}_{0,t} \cdot Q'$. If $A_{0,t}$ satisfies the sign restrictions we keep it. Otherwise we move to the next iteration of the Gibbs sampler.

Chart 4 plots the estimated standard deviation of the monetary policy shock. The chart indicates that the inflation-targeting period has been characterised by the lowest volatility of the structural monetary policy shock.

5.1 Impulse response functions

Chart 5 plots the impulse response functions of the endogenous variables in the VAR for a 100 basis point unanticipated shock to the interest rate.

Following Koop, Pesaran and Potter (1996) these impulse response functions are defined as:

$$ IRF = E\left(Z_{t+k} \backslash \Psi_{t+k}, \mu_{MP}\right) - E\left(Z_{t+k} \backslash \Psi_{t+k}\right) \tag{11} $$

where Φ denotes all the parameters and hyperparameters of the VAR and k is the horizon under consideration. Equation (11) states that the impulse response functions are calculated as the difference between two conditional expectations. The first term in equation (11) denotes a forecast of the endogenous variables conditioned on a monetary policy shock μ_{MP}. The second term is the baseline forecast, ie conditioned on the scenario where the monetary policy shock equals zero.

The impulse responses are computed via Monte Carlo integration for 500 replications of the Gibbs sampler for every four quarters in the sample. Details on the Monte Carlo integration procedure can be found in Koop *et al* (1996).

5.1.1 Price puzzle

Consider the response of the inflation factor. There is some evidence of a price puzzle – ie a positive response of inflation to a contractionary policy shock that is observed one period after the shock – especially during the early 1970s. This anomalous response seems to be limited to the period immediately after the shock and disappears by the early 1980s.

Note that the price puzzle is more muted than in the estimates presented by Castelnuovo and Surico (2006).[7] In our model the positive response only persists for around one quarter after the shock. This result is similar to that reported in Bernanke *et al* (2005) where the FAVAR model produces a substantially smaller price puzzle than a model that excludes factors.

This evidence does suggest that the extra information incorporated in the FAVAR model reduces the severity of the price puzzle. In addition, these results support the analysis of Castelnuovo and Surico (2006) who argue that the price puzzle in structural VARs may be a symptom of omitted variable bias that may arise when the Taylor principle is violated. In particular, Castelnuovo and Surico (2006) show that when the economy is operating under indeterminacy an additional unobserved latent variable characterises the dynamics of the economy. The omission of this variable in standard VARs may lead to the price puzzle and may indicate that the identification scheme has not completely isolated the monetary policy shock. The factors included in our model summarise a large amount of information that may proxy the unobserved latent variable. The fact that we find a reduction in the extent of the price puzzle in the early 1970s (relative to previous studies) lends support to this idea.[8]

What do these observations imply for our estimated policy shock? The fact that a mild price puzzle appears in the earlier part of the sample may indicate that the FAVAR still excludes some

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[7]The price puzzle, in Castelnuovo and Surico (2006) lasts for around six to eight quarters. These estimates are obtained using a fixed-coefficients VAR estimated over the sample 1979-92. See their Figure 6.

[8]It is also interesting to note that the price puzzle in the early part of the sample is accompanied by a small 'liquidity puzzle', ie a positive response of the money growth factor to a contractionary monetary policy shock. As in the case of inflation, this positive response lasts for about one quarter.

information that is relevant for this period. However, the fact that the magnitude of these puzzles is small also indicates that the FAVAR model performs better than standard tri-variate VARs employed in previous studies.

5.2   *Variances and their decomposition*

As a final exercise, we decompose the variance of the factors and evaluate the contribution of the monetary policy shock and its evolution over time. The variance of each endogenous variable in the transition equation can be calculated as

$$vec\left(E(Z_t Z_t')\right) = \left[I - \left(\tilde{\phi}_t' \otimes \tilde{\phi}_t\right)\right]^{-1} vec\left(\tilde{\Omega}_t\right) \tag{12}$$

where $\tilde{\phi}_t$ represents the VAR coefficients in companion form and $\tilde{\Omega}_t$ denotes a matrix conformable with $\tilde{\phi}_t$ consisting of Ω_t in the top 5×5 block and zeros elsewhere. The variance due to the monetary policy shock *only* can be calculated using **(12)** but replacing $\tilde{\Omega}_t$ with $\bar{\Omega}_t$ where

$$\bar{\Omega}_t = \bar{A}_{0,t}' \bar{H}_t \bar{A}_{0,t} \tag{13}$$

where $\bar{A}_{0,t}'$ is a draw of $A_{0,t}$ that satisfies the sign restrictions and is normalised by dividing each column by $diag\left(A_{0,t}\right)$ and $\bar{H}_t$ is a diagonal matrix of the variance of the shocks where the volatility of all shocks except the monetary policy shock is set equal to zero.

The black lines in the top panel of Chart 6 plot the (median) standard deviation of each endogeneous variable in the VAR calculated via equation **(12)**. The volatility of each variable was at it highest during the mid-1970s and the early 1980s but declined with the onset of Thatcher's disinflationary policies. This decline continued until the early 1990s when the standard deviation of the interest rate, money and asset price factors rose again. The volatilities have been low in the post-1992 period.

The red lines in the top panel plot the (median) standard deviation of each variable due to the monetary policy shock and the bar charts in the panels below show the proportion explained by the policy shock. The results indicate that in the case of inflation and the interest rate, the contribution of monetary policy shock has declined relative to the 1970s and is at its lowest over the inflation-targeting period.

Finally, note that the policy shock explains a relatively small proportion of the variance of the

interest rate across our sample. This may suggest that our model adequately captures the information set used by the monetary authority when setting interest rates and deviations from the 'policy rule' incorporated in the model have been unimportant. If, in contrast, variables examined by monetary authorities (when setting policy) were left out of the model, one would expect the contribution of the policy shock to be higher. For example, if policy before 1992 were based on variables other than output growth and inflation one might expect the policy shock in a standard time-varying VAR (estimated using inflation, output growth and interest rates) to have made a larger contribution to interest rate movements.

6 Conclusions

This paper fits a time-varying factor augmented VAR model to UK data from 1970 to the end of the great stability in 2004. The reduced-form results from the model confirm the fall in volatility and persistence of UK output and inflation, seen after 1992, documented in previous studies. In addition, the estimates indicate that this pattern is also seen in money growth and asset prices.

The structural results from the model are based on a scheme that identifies the monetary policy shock. Evidence of a price puzzle in our impulse responses is much more muted than in previous studies. This may suggest that the extra information incorporated in our factors leads to more robust structural estimates. Finally, the estimated variance decomposition indicates that the contribution of the monetary policy shock to inflation and the interest rate has been low over the inflation-targeting period.

Appendix A: Priors and estimation

Consider the time-varying FAVAR model given by equations **(5)** and **(6)**. As shown by Bernanke *et al* (2005) identification requires some restrictions on the factor loadings matrix. Following Bernanke *et al* (2005) we restrict the first element of Λ_i to be one and the first element of Ψ_i to be zero.

Prior distributions and starting values

Factors and factor loadings

Following Bernanke *et al* (2005) we centre our prior on the factors (and obtain starting values) by using a principal components estimator applied to each $X_{i,t}$. In order to reflect the uncertainty surrounding the choice of starting values, a large prior covariance of the states ($P_{0/0}$) is assumed.

Starting values for the factor loadings are also obtained from the PC estimator (with the restrictions given above imposed). The prior on the diagonal elements of R is assumed to be inverse gamma:

$$R_{ii} \sim IG(5, 0.001)$$

In choosing this diffuse prior we closely follow Bernanke *et al* (2005), but employ a slightly higher scale parameter in order to reflect the high volatility of some of the series in the panel.

VAR coefficients

The prior for the VAR coefficients is obtained via a fixed coefficients VAR model estimated over the sample 1955:01 to 1969:04 using data for GDP growth, CPI inflation, M0 growth, the Treasury bill rate and the FTSE All-Share price index. The choice of these variables is dictated primarily by data availability. However, as noted below the results do not depend on the choice of this prior. The variables are scaled to match the panel used in the subsequent estimation. ϕ_0 is therefore set equal to

$$\phi_0 \sim N(\hat{\phi}^{OLS}, V)$$

where V is a diagonal matrix with $\frac{1}{5}$ times the OLS estimates of $var(\hat{\phi}^{OLS})$ on the main diagonal.

Elements of H_t

Let $\hat{v}^{ols}$ denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data described above. The prior for the diagonal elements of the VAR covariance matrix (see **(8)**) is as follows:

$$\ln h_0 \sim N(\ln \mu_0, I_5)$$

where μ_0 are the diagonal elements of the Cholesky decomposition of $\hat{v}^{ols}$.

Elements of A_t

The prior for the off-diagonal elements A_t is

$$A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$$

where $\hat{a}^{ols}$ are the off-diagonal elements of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the diagonal elements set equal to ten times the absolute value of the corresponding element of $\hat{a}^{ols}$.

Hyperparameters

The prior on Q is assumed to be inverse Wishart

$$Q_0 \sim IW\left(\bar{Q}_0, T_0\right)$$

where $\bar{Q}_0$ is assumed to be $var(\hat{\phi}^{OLS}) \times 10^{-4} \times 3.5$ and T_0 is the length of the sample used to for calibration.

The prior distribution for the blocks of S is inverse Wishart:

$$S_{i,0} \sim IW(\bar{S}_i, K_i)$$

where $i = 1..5$ indexes the blocks of S. $\bar{S}_i$ is calibrated using $\hat{a}^{ols}$. Specifically, $\bar{S}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by 10^{-3}.

Following Cogley and Sargent (2005) we postulate an inverse gamma distribution for the

elements of G,

$$\sigma_i^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right)$$

Simulating the posterior distributions

Factors and factor loadings

This closely follows Bernanke *et al* (2005). Details can also be found in Kim and Nelson (1999).

Factors

The distribution of the factors F_t is linear and Gaussian:

$$F_T \backslash X_{i,t}, R_t, \Xi \sim N\left(F_{T \backslash T}, P_{T \backslash T}\right)$$
$$F_t \backslash F_{t+1}, X_{i,t}, R_t, \Xi \sim N\left(F_{t \backslash t+1, F_{t+1}}, P_{t \backslash t+1, F_{t+1}}\right)$$

where $t = T - 1, ..1$, Ξ denotes a vector that holds all the other FAVAR parameters and:

$$
\begin{aligned}
F_{T \backslash T} &= E\left(F_T \backslash X_{i,t}, R_t, \Xi\right) \\
P_{T \backslash T} &= Cov\left(F_T \backslash X_{i,t}, R_t, \Xi\right) \\
F_{t \backslash t+1, F_{t+1}} &= E\left(F_t \backslash X_{i,t}, R_t, \Xi, F_{t+1}\right) \\
P_{t \backslash t+1, F_{t+1}} &= Cov\left(F_t \backslash X_{i,t}, R_t, \Xi, F_{t+1}\right)
\end{aligned}
$$

As shown by Carter and Kohn (2004) the simulation proceeds as follows. First we use the Kalman filter to draw $F_{T \backslash T}$ and $P_{T \backslash T}$ and then proceed backwards in time using:

$$F_{t|t+1} = F_{t|t} + P_{t|t} P_{t+1|t}^{-1}\left(F_{t+1} - F_t\right)$$

$$P_{t|t+1} = P_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t}$$

If more than one lag of the factors appears in the VAR model, this procedure has to be modified to take account of the fact that the covariance matrix of the shocks to the transition equation (used in the filtering procedure described above) is singular. For details see Kim and Nelson (1999).

Elements of R

As in Bernanke *et al* (2005) R is a diagonal matrix. The diagonal elements R_{ii} are drawn from the following inverse gamma distribution:

$$R_{ii} \sim IG\left(\bar{R}_{ii}, T + 0.001\right)$$

where

$$\bar{R}_{ii} = 5 + \hat{e}'_i \hat{e}_i + \Gamma'_i \left[\bar{M}_0^{-1} + \left(F'_{i,t} F_{i,t}\right)^{-1}\right]^{-1} \Gamma_i$$

where $\Gamma_i = \Lambda_i$ or if appropriate $\Gamma_i = [\Lambda_i, \Psi]$ and $\hat{e}_i$ denotes the OLS estimate the ith element of R. As in Bernanke *et al* (2005) $M_0 = I$.

Elements of Λ and Ψ

Letting $\Gamma_i = \Lambda_i$ or $\Gamma_i = [\Lambda_i, \Psi]$ for the appropriate equation, the factor loadings are sampled from

$$\Gamma_i \sim N\left(\bar{\Gamma}_i, R_{ii} \bar{M}_i^{-1}\right)$$

where $\bar{\Gamma}_i = \bar{M}_i^{-1}\left(F'_{i,t} F_{i,t}\right)\hat{\Gamma}_i$, $\bar{M}_i = \bar{M}_0 + \left(F'_{i,t} F_{i,t}\right)$ and $\hat{\Gamma}_i$ represents an OLS estimate.

Time-varying VAR

Given an estimate for the factors, the model becomes a VAR model with drifting coefficients and covariances. This model has become fairly standard in the literature and details on the posterior distributions can be found in a number of papers including Cogley and Sargent (2005), Cogley *et al* (2005) and Primiceri (2005). Here, we describe the algorithm briefly. Details can be found in the papers mentioned above.

VAR coefficients ϕ_t

As in the case of the unobserved factors, the time-varying VAR coefficients are drawn using the methods described in Carter and Kohn (2004).

Elements of H_t

Following Cogley and Sargent (2005), the diagonal elements of the VAR covariance matrix are sampled using the methods described in Jacquier *et al* (2004).

Element of A_t

Given a draw for ϕ_t the VAR model can be written as

$$A_t' \left(\tilde{Z}_t \right) = u_t$$

where $\tilde{Z}_t = Z_t - \sum_{l=1}^{L} \phi_{l,t} Z_{t-l} = v_t$ and $VAR(u_t) = H_t$. This is a system of equations with time-varying coefficients and given a block diagonal form for $Var(\tau_t)$ the standard methods for state-space models described in Carter and Kohn (2004) can be applied.

VAR hyperparameters

Conditional on Z_t, $\phi_{l,t}$, H_t, and A_t, the innovations to $\phi_{l,t}$, H_t, and A_t are observable, which allows us to draw the hyperparameters – the elements of Q, S, and the σ_i^2 – from their respective distributions.

Appendix B: Data

Real Activity

Variable	Source	Transformation
Household Final Consumption	ONS	LD
General Government Final Consumption Expenditure	ONS	LD
Construction	ONS	LD
Volume of Exports	IFS	LD
Volume of Imports	IFS	LD
Export of Goods and Services	IFS	LD
Imports of Goods and Services	IFS	LD
Total Exports	ONS	LD
Total Imports	ONS	LD
Total Gross Fixed Capital Formation	ONS	LD
Gross Domestic Product	ONS	LD
Gross National Income	IFS	LD
Output Index: Transport Storage and Communication	ONS	LD
Output Index: Total	ONS	LD
Output Index: Distribution Hotels and Catering	ONS	LD
Industrial Production: Manufacturing	ONS	LD
Industrial Production: All Production Industries	ONS	LD
Industrial Production: Mining and Quarrying	ONS	LD
Industrial Production: Electricity, Gas and Water Supply	ONS	LD
Industrial Production: Food Drinks and Tobacco	ONS	LD
Industrial Production: coke/petroleum prod/nuclear fuels	ONS	LD
Industrial Production: Chemicals Man-made Fibres	ONS	LD
Industrial Production: Total Production % Change	ONS	N
Gross Capital Formation Changes in Inventories	ONS	N
Changes in Inventories	ONS	N
Changes in Inventories Including Alignment Adjustment	ONS	N

Inflation

Variable	Source	Transformation
Producer Price Index	IFS	LD
RPI: Total Food	ONS	LD
RPI: Total Non-Food	ONS	LD
RPI Total All items other than Seasonal Food	ONS	LD
RPI: Metals	ONS	LD
RPI: Raw Materials	ONS	LD
RPI: Beverages	ONS	LD
RPI: Food	ONS	LD
Oil Price: Average Crude	IFS	LD
Export Price Index	IFS	LD
Import Price Index	IFS	LD
CPI	ONS	LD
GDP Deflator	ONS	LD
RPIX	ONS	LD
Total Wages and Salaries	ONS	LD

Money

Variable	Source	Transformation
Aggregate M4 Level	ONS	LD
Sectoral M4 Level: PNFCs	ONS	LD
Sectoral M4 Level: Households	ONS	LD
Sectoral M4 Level: OFCs	ONS	LD
Aggregate M4 Lending Level	ONS	LD
Sectoral M4 Lending Level: PNFCs	ONS	LD
Sectoral M4 Lending Level: Households	ONS	LD
Sectoral M4 Lending Level: OFCs	ONS	LD
Mortgage Equity Withdrawal	ONS	LD
M0 Level	ONS	LD

Asset Prices

| Variable	Source	Transformation
FTSE All-Share Price Index	BOE	LD
FTSE All-Share Dividend Yield	BOE	LD
FTSE All-Share Price to Earnings Ratio	BOE	LD
Real House Price Index	BOE	LD

ONS denotes the Office for National Statistics in the United Kingdom. IFS refers to the IMF statistical database, International Financial Statistics. BOE denotes Bank of England. LD refers to log differences, while N denotes no transformation. All variables are de-meaned and standardised.

Appendix C: Convergence

The chart below plots 20th order autocorrelations of the draws for some of the main parameters of the model. The autocorrelations are fairly low indicating good convergence properties. We also re-estimated our model using a non-informative prior for the parameters of the transition equation **(6)**. In particular we set

$\phi_0 \sim N(0_{n(n \times L+1)}, I_5 \times 0.01)$, $\ln h_0 \sim N(\ln(1_n \times 0.1), I_5)$, $E(A_0) = 1_{\frac{n \times n - 1}{2}} \times 0.1$ where $0_{n(n \times p+1)}$ is $(n(n \times L + 1)) \times 1$ vector of zeros, 1_n is a $n \times 1$ vector of ones, $1_{\frac{n \times n - 1}{2}}$ denotes a $\frac{n \times n - 1}{2} \times 1$ vector of ones and n denotes the number of endogenous variables in the VAR. The reduced form results (which are available on request) are very similar to those reported in the main text.
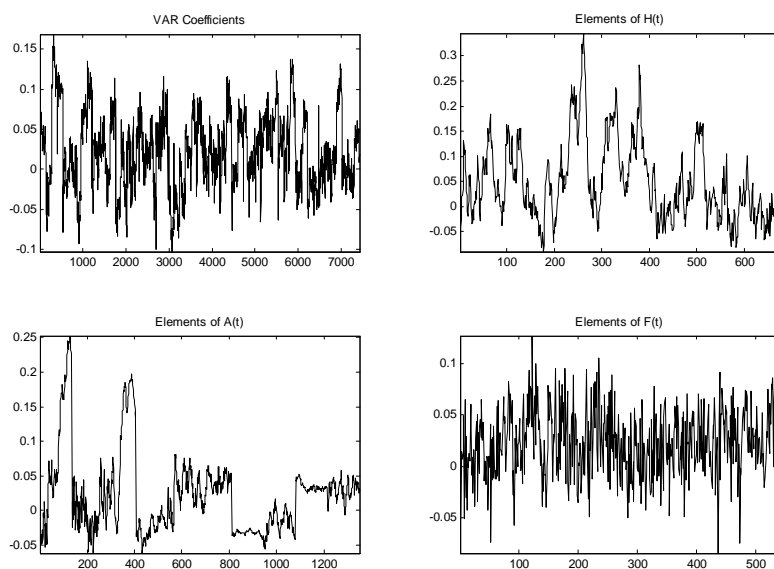
Appendix D: Charts

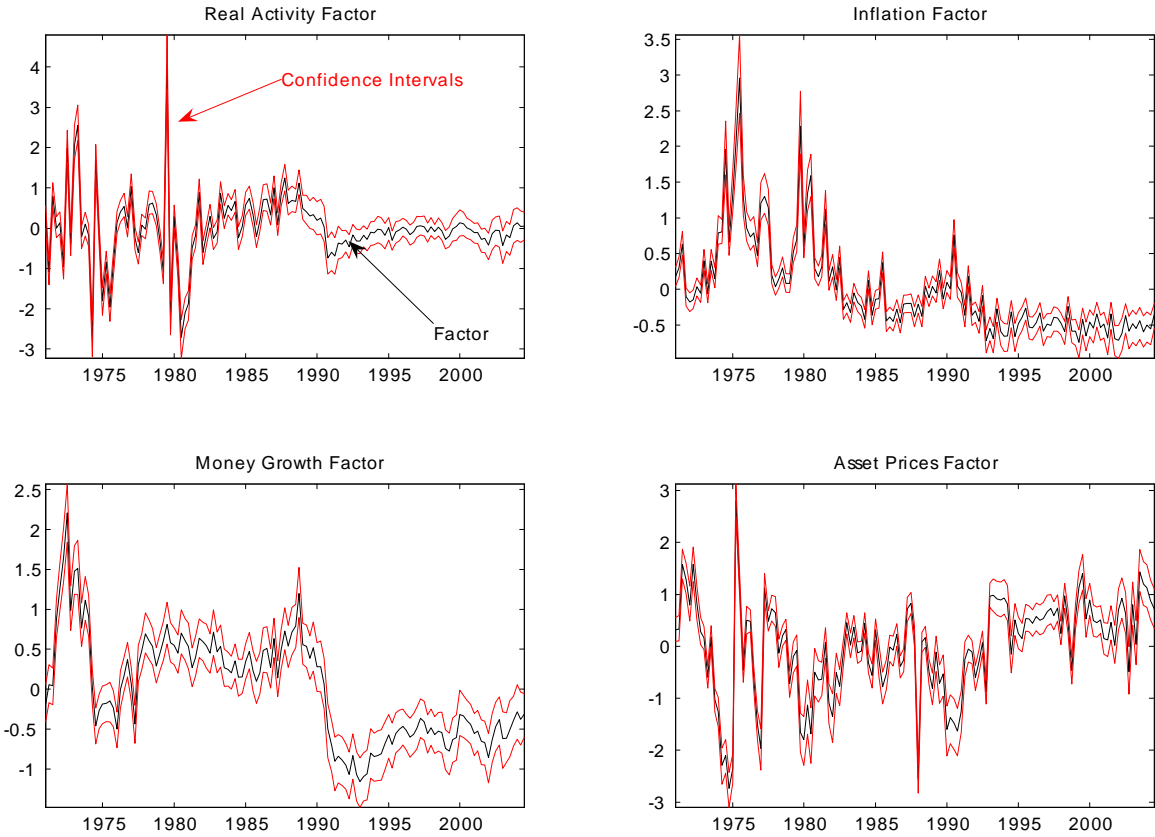## Chart 1: Estimated factors and one standard deviation confidence intervals

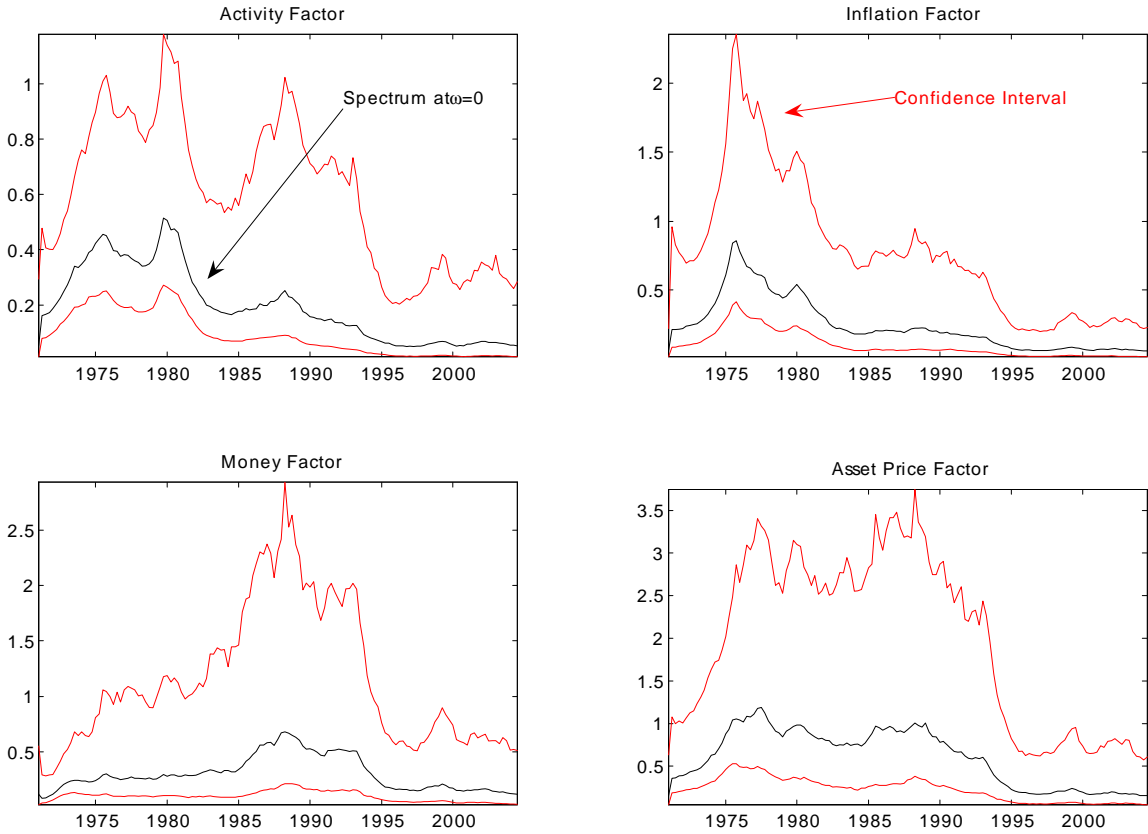# Chart 2: Spectra and one standard deviation confidence intervals

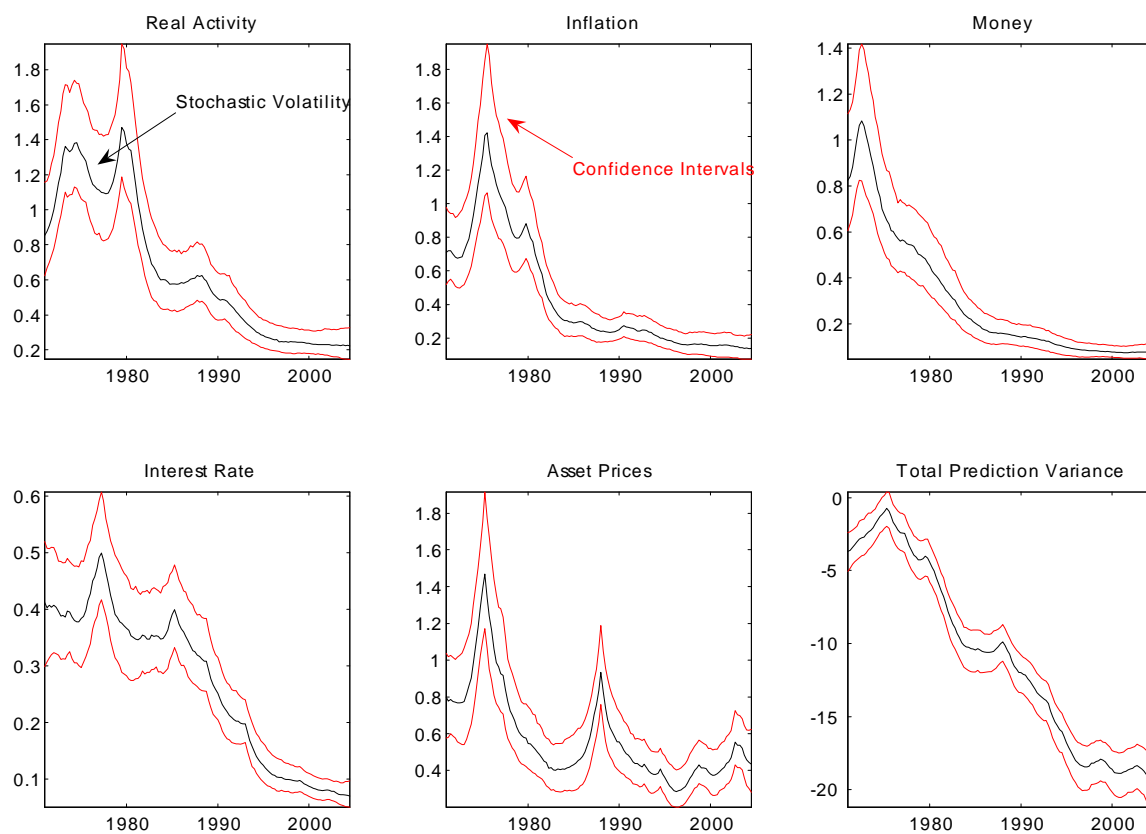**Chart 3: Volatility of the reduced-form residuals and one standard deviation confidence intervals**
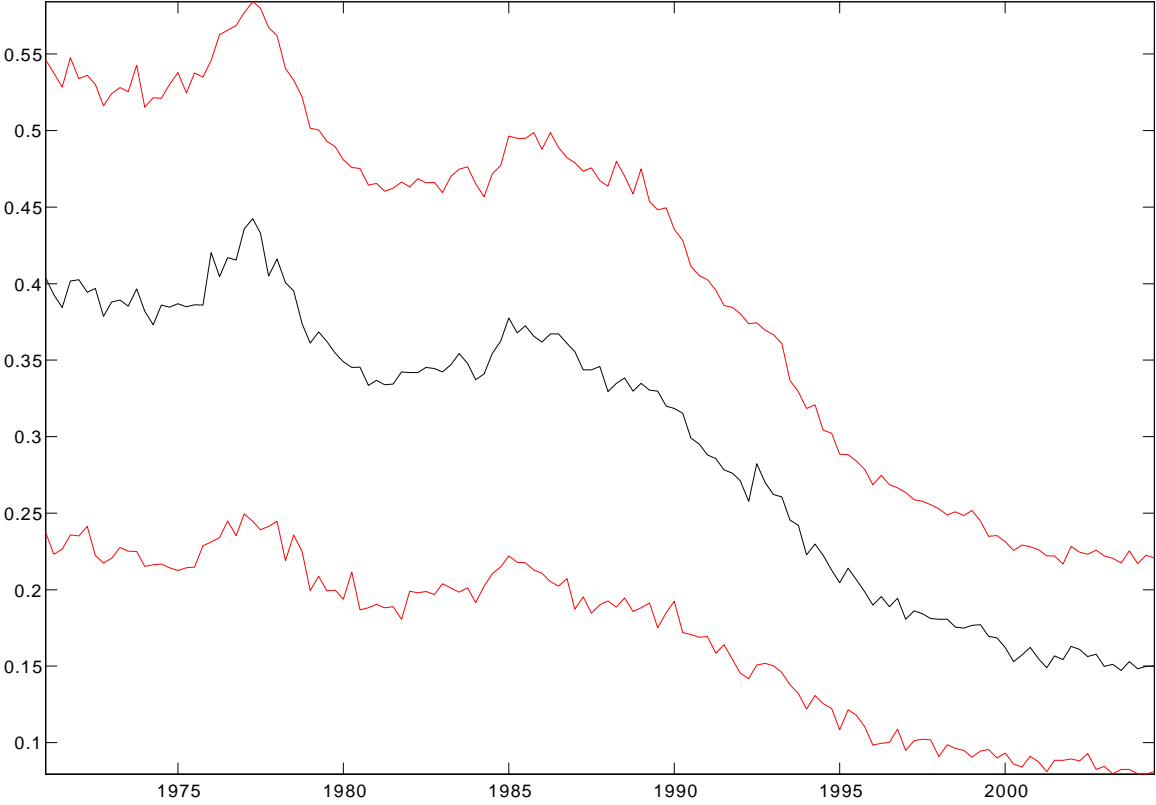
Chart 4: Standard deviation of the monetary policy shock
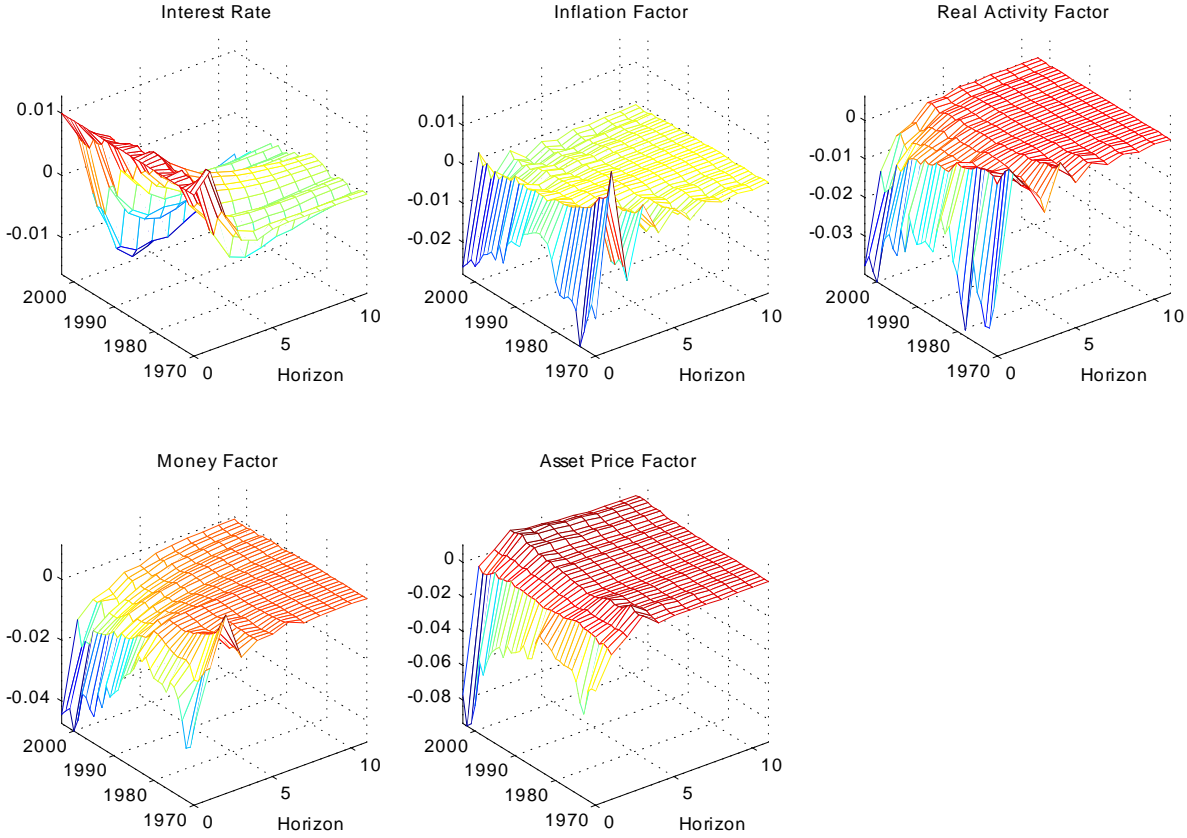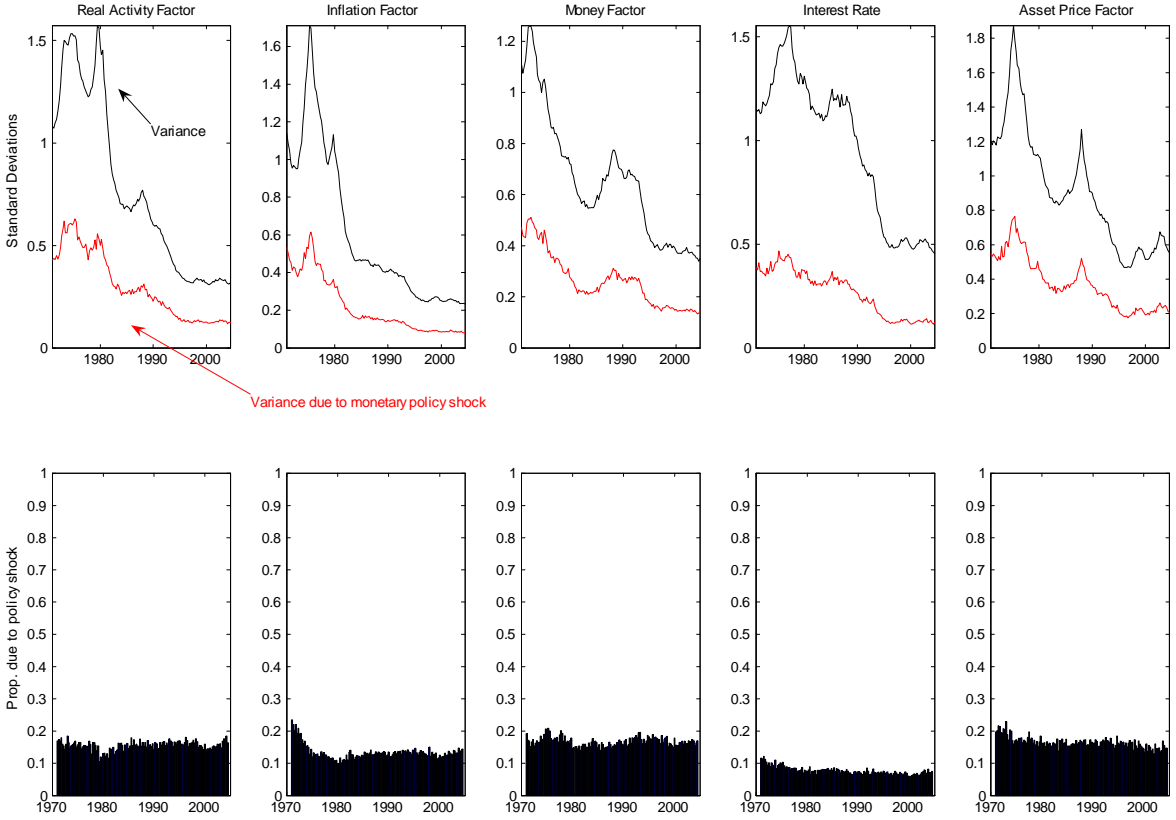
Chart 5: Impulse response functions

Chart 6: Variance decomposition

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