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Forecasting in the presence of recent structural change

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Abstract

We examine how to forecast after a recent break. We consider monitoring for change and then combining forecasts from models that do and do not use data before the change; and robust methods, namely rolling regressions, forecast averaging over different windows and exponentially weighted moving average (EWMA) forecasting. We derive analytical results for the performance of the robust methods relative to a full-sample recursive benchmark. For a location model subject to stochastic breaks the relative mean square forecast error ranking is EWMA < rolling regression < forecast averaging. No clear ranking emerges under deterministic breaks. In Monte Carlo experiments forecast averaging improves performance in many cases with little penalty where there are small or infrequent changes. Similar results emerge when we examine a large number of UK and US macroeconomic series.

Key words: Monitoring, recent structural change, forecast combination, robust forecasts.

JEL classification: C10, C59.

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Summary

Forecasting is a central activity for central banks, not least because policy takes effect with a lag. Inevitably, policy is forward looking. Thus in many central banks, including the Bank of England, the published forecast is a key tool in communicating judgements about monetary policy and the economy. The Bank's forecast, published in the *Inflation Report*, represents the judgements of the Monetary Policy Committee and is not mechanically produced by a single model. However, many forecasting models - a 'suite' of models - help the Committee determine its judgement, including simple largely atheoretical models of the type considered in this paper.

One common cause of forecast failure is that structural changes or 'breaks' keep on occurring in the underlying relationships in the economy, and this paper addresses that problem. Dealing with this has two aspects. First, detection; and subsequently the right forecasting strategy. Consequently, there are many papers on the identification of breaks, and forecasting methods that are robust to them. But these are mainly in the context of fairly distant events. The fact that in practice forecasters have to forecast after recent changes has received remarkably little attention. Yet this is a pervasive and profound problem.

Furthermore, in practice we may be continually 'monitoring' for breaks, and this raises a subtle issue. In that case the forecaster inevitably carries out repeated tests. This matters, because if statistical tests are repeated enough times, then even if one never occurs in reality by pure chance they must eventually flag a break. Luckily, there are methods to take care of this. But the subsequent problem of how to then adapt the forecasting strategy has hardly been discussed. We therefore address two important issues. First, we ask whether the forecaster should attempt to detect and react to breaks each period, or instead adopt robust forecasting strategies. Second, we consider two quite different environments. In one case, breaks are unique events (or are rare enough to be treated as such), and in the other they recur.

The monitoring strategy we examine is to look for evidence of breaks and then combine forecasts from models that do and do not use data before the change. And the alternative is simply to use methods that are robust to breaks. We examine several commonly used methods of this type, all of which work by in one way or another giving more weight to recent observations (less likely to

be affected by breaks).

We first derive some analytical results for the forecast performance of the robust methods relative to a benchmark using the full-sample. For random breaks in a simple model we obtain rankings, but not under deterministic breaks. Clearly, it is hard to draw theoretical conclusions. So we experiment with 'Monte Carlo' simulations (creating many randomly drawn artificial data sets) for a variety of cases. The best methods can vary widely according to the particular break and choice of parameters. With the monitoring method we find the gains are small, although equally the costs (in cases where there are small breaks) are also small. Other methods can do much better where there are large breaks. The results make it hard to recommend a single method. But a method based on averaging over many different samples often improves on the full-sample benchmark and rarely comes with a large penalty where there are frequent or small breaks.

Finally, we take the methods to real data. We examine simple forecasting models using about 200 US and UK time series. For the United Kingdom, where there are relatively many breaks identified in the full sample, the best-performing method is forecast averaging, consistent with the Monte Carlo results.

We conclude that monitoring for breaks will not lead to a deterioration in forecast performance relative to using the full sample, but not much benefit either. Instead methods that discount past data in various ways are to be preferred. The averaging method we explore seems to be a useful default choice.

1 Introduction

It is widely accepted that structural change is a crucial issue in econometrics and forecasting. By 'structural change', we mean an irregular, discreet and permanent change in a parameter of interest. Clements and Hendry argue forcefully (in eg 1998a,b) that it is the main source of forecast error; Hendry (2000) argues that the dominant cause of these failures is the presence of deterministic shifts; Stock and Watson (1996) looked at many forecasting models of a large number of US time series, and found evidence for parameter instability in a substantial proportion. Consequently there are many papers on the identification of breaks, and methods that are robust to them. But the fact that forecasters have to forecast after recent, or during, changes has received very little attention. Yet this is a pervasive and profound problem facing forecasters who need to generate projections in real time.

Dealing with breaks in a forecast context has two aspects which have received considerable attention for cases where the break occurred in the relatively distant past. First, there is break detection; and subsequently the right forecasting strategy. Regarding the former, break detection has a long history - the seminal paper testing for a break at a known point was Chow (1960). Andrews (1993) introduced a methodology that allowed for unknown break-points: one influential paper is Bai and Perron (1998). The latter question of how to modify the forecasting strategy then arises. This has been tackled by many authors, but one major recent contribution is by Pesaran and Timmermann (2007), who consider a number of alternative forecasting strategies in the presence of breaks. They conclude that forecast pooling using a variety of estimation windows provides a reasonably good and robust forecasting performance.

But the largely unexplored and critical question remains - how to forecast in the presence of recent breaks?

Standard break tests, by their nature, require some end-of-sample observations to perform the test. Typically, between 5% and 15% of the sample size located at the end of the sample is assumed not to contain a break. So timely real-time detection is simply impossible. Indeed, the definition of the end of the sample is practically controversial. However, this acute real-time problem of break detection (where the hypothesis of interest is that there has been a recent break) has been tackled in the small literature on structural change 'monitoring', pioneered by Chu, Stinchcombe and White (1996). As the forecaster monitors in real time for breaks, she carries out repeated tests. This implies the need for an appropriate asymptotic framework, with critical values that ensure rejection probabilities remain bounded by the significance level when breaks do not occur. This work has been refined by many others, including Zeileis, Leisch, Kleiber and Hornik (2005), Leisch, Hornik and Kuan (2000) and Kuan and Hornik (1995). Groen, Kapetanios and Price (2009) extend the analysis to panel data sets.

Oddly, the subsequent problem of how to adapt a forecasting strategy in the presence of recent breaks has hardly been discussed. Consequently, it forms the main topic of the current paper. We address two important issues. First, we ask whether the forecaster should attempt to detect and react to breaks, or adopt forecasting strategies that do not rely on break detection but are instead robust to them. Second, we recognise that there are potentially two quite different statistical environments. In one case, breaks are unique events (or are rare enough to be treated as such), and in the other they recur. These require different analytical frameworks.

Addressing the first issue, a new strategy that we propose involves monitoring and then combining full-sample and post-break models. Clark and McCracken (2009a) write that 'it is possible that using a sample window based on break test estimates could yield better model estimates and forecasts. In practice, however, difficulties in identifying breaks and their timing may rule out such improvements (see, for example, the results in [Clark and McCracken (2009b)]'. We evaluate this in a systematic way.

The alternative is to use robust models. We examine a set of widely advocated methods for forecasting in the presence of past breaks: model averaging, rolling windows and exponentially weighted moving average (EWMA) models. Modifying Pesaran and Timmermann (2007), we consider the forecasting strategies they analyse in the context of recent breaks. Of all the strategies they consider, only forecast combination translates easily to the current framework. Clark and McCracken (2009a), in their discussion of some related empirical results, write that in a forecast evaluation analysis, after 'aggregating across all models, horizons and variables being forecasted, it is clear that model averaging and Bayesian shrinkage methods consistently perform among the best methods. At the other extreme, the approaches of using a fixed rolling window of observations to estimate model parameters and discounted least squares estimation consistently rank among the worst.'[1] By contrast, rolling regressions are advocated by Giacomini and White (2006). Another related paper by Pesaran and Pick (2008) is motivated by the desire to avoid the need to detect breaks.[2] They find that forecast averaging is superior to a single estimation window in almost all cases. They also consider EWMA estimators, and find the results are sensitive to the EWMA tuning parameter.

In Section 2 we propose a new approach for forecasting in the presence of recent breaks and describe some robust forecasting strategies. We then provide some new analytical results for the performance of the latter Section 3. We consider an extensive Monte Carlo study in which all these strategies are evaluated in Section 4. We apply the methods we examine to a large number of US and UK macroeconomic time series in Section 5, where we find results broadly consistent with the Monte Carlo study. Section 6 concludes. Proofs and detailed empirical results are reported in appendices.

2 Forecasting strategies

Our modelling framework can be summarised by the general model

$$y_t = \beta_t' x_t + \epsilon_t, \qquad t = 1, \ldots, T, \ldots \tag{1}$$

where x_t is a $k \times 1$ vector of predetermined stochastic variables, β_t are $k \times 1$ vectors of parameters and ϵ_t is a martingale difference sequence that is independent of x_t and has finite variance that may depend on t.

We specialise **(1)** by assuming that our entertained model is characterised by multiple structural breaks of the form

$$y_t = \sum_{i=1}^{b} \mathcal{I}\left(\{T_{i-1} < t \leq T_i\}\right) \beta_i' x_t + \epsilon_t, \qquad t = 1, \ldots, T_1, \ldots, T_b, \ldots, T, \ldots \tag{2}$$

[1] We note that our paper has a somewhat different approach to papers such as this, where Clark and McCracken take an exhaustive look at real data and run races between different types of model. Our paper has a rather different objective. We are asking a new econometric question: should we monitor for structural breaks and respond accordingly when breaks are detected, or should we use robust models? We develop some analytical results for specific cases; we examine small sample properties for a range of parameter values within the context of a single AR(1) model; we apply this to a set of data. We thereby obtain results which have some claims for generality, although the model considered is restrictive. We hope that in subsequent work we or others will explore those results in more depth for particular data sets.

[2] Unlike us, they do not consider monitoring. They examine forecasts of random walks subject to one-off breaks in the drift and volatility. This set-up is effectively a location model, whereas in our applications we also examine parameter shifts in AR models. We also explore multiple stochastic breaks. This both allows for a more informative analysis of the realistic scenario of repeated breaks, and provides clear results on the relative performance of competing forecasting methods.

where $\mathcal{I}(\mathcal{A})$ is an indicator variable taking the value one if the event $\mathcal{A}$ occurs and zero otherwise. T denotes the end of the observed sample. Since our main focus is real-time forecasting we implicitly assume the existence of data after T. This straightforward model has been analysed extensively in the literature. The main point of departure from a standard analysis is to assume that some break dates are very close to the end of the sample at time T. The forecaster is aware of the possibility of a break in real time and either actively looks for such a break or wishes to adopt a forecasting strategy that is robust to the occurrence of such a break. This is radically different to standard break detection as such methods cannot detect breaks if $T_b/T \to 1$ as $T \to \infty$.[3] An alternative way to proceed is to disregard the structure in **(2)** and focus on a robust model such as a random walk or double-differenced model that may be biased but will be less affected by breaks, as Hendry (eg, 2000) has often suggested. We ignore this approach in the current paper, as we are focused on the (realistic) case where the forecaster has a specific view about both the structure of the break and the utility of a model that considers x_t.

2.1 Forecasting strategies in the presence of a detected recent break

We propose a strategy where recent breaks have been detected using some monitoring procedure. Our approach is related to Pesaran and Timmermann (2007), who provide a detailed analysis of forecasting strategies when breaks occur in the more distant past. But the problem with recent breaks differs as post-break data are by definition in short supply. As a result the first four of the following strategies suggested by Pesaran and Timmermann are either not straightforwardly applicable or infeasible. For reference, these are listed here: using model **(2)**, estimated over post-window data; trading off the variance against the bias of the forecast by estimating the optimal size of the estimation window; estimating the optimal size of the estimation window using cross-validation;[4] combining forecasts from different estimation windows by using weights obtained through cross-validation as in the previous case; and simple average forecast combination, using equal weights. Our proposal builds on this last suggestion but is tailored to the specific problem. The forecaster monitors for breaks. As long as no breaks are detected, the forecasts are produced using the model estimated over the whole sample.[5] Once the forecaster

[3] Most tests for breaks assume that $T_b/T \to C$ $T \to \infty$, where $C \in (0, 1)$.

[4] Cross-validation holds back observations at the end of the sample for a post-sample exercise, in this case to establish a minimum MSFE estimation window.

[5] Thus we assume that at the start of the monitoring period the forecaster has considered the possibility of past breaks which have been accommodated by some unspecified method, if found present. We accommodate this in the Monte Carlo design by assuming there is at most one break, and that the forecaster knows this.

detects a break, it is assumed that the break has occurred at that point in time. Thus if $\hat{T}_1$ is the date the break is detected, it is also assumed to be the estimated date at which the break occurred.[6]

The forecaster then makes two judgements, operationalised by the choice of two tuning parameters. The first defines the time elapsed before the model can be reliably estimated post-break. This parameter is referred to as $\underline{\omega}$ in Pesaran and Timmermann (2007) and we retain this notation. The second parameter is a window size $\overline{f}$ that the forecaster deems acceptable for the post-break model to be the sole model used for future forecasting. $\overline{f}$ is then chosen to be the period over which the forecasts of the post-break and the no-break models will be combined. In other words, forecasts will be combined for the period $\hat{T}_1 + \underline{\omega}$ to $\hat{T}_1 + \underline{\omega} + \overline{f}$. The forecasts after $\hat{T}_1 + \underline{\omega} + \overline{f}$ will therefore arise only from the post-break model.

There is a question of how the forecasts from the no-break (ie, forecasts using all currently available data and ignoring the break) and post-break (using only post-break data) models are to be combined. It is natural that the post-break model should receive increasing weight as new data arrives. We specify that the no-break model will be the sole model used prior to $\hat{T}_1 + \underline{\omega}$ and the post-break model will be the sole model used after $\hat{T}_1 + \underline{\omega} + \overline{f}$. A simple weighting scheme consistent with this choice is one where the weight for the post-break model increases linearly from zero prior to $\hat{T}_1 + \underline{\omega}$ to unity at $\hat{T}_1 + \underline{\omega} + \overline{f}$. That is, the weight for the post-break model at time $\hat{T}_1 + \underline{\omega} + j$ is $j/\left(\overline{f} + 1\right)$, whereas the weight for the no-break model is $1 - j/\left(\overline{f} + 1\right)$, where $j = 0, \ldots, \overline{f}$.

We assume that the forecaster knows there is only a single break. In practice the forecaster may accommodate the possibility that further breaks occur. One solution would be to start monitoring for a new break as soon as the previous break has been detected by using only the post-break model. Then monitoring proceeds simultaneously with forecast combining. The most relevant scenario may be one where the forecaster stops combining forecasts before a new break is detected.[7]

[6] The delay in break detection is ignored as it is hard to estimate this bias. See Groen *et al* (2009) for evidence on its extent.

[7] It is reasonable to argue that if breaks occur more frequently than assumed here, the model itself must come under scrutiny. A clear path for addressing this is to endogenise the break process into the model following, eg, work by either Kapetanios and Tzavalis (2010) or Pesaran, Pettenuzzo and Timmermann (2007). But an analysis of either course of action is beyond the scope of this paper.

2.2 *Forecasting strategies that are robust to the presence of a recent break*

We recognise monitoring may be problematic. Small breaks are hard to detect; it is not suitable where we expect frequent breaks; breaks are detected with a delay; and estimates of the timing is imprecise.[8] We therefore also consider strategies robust to the presence of recent breaks, essentially by discounting past data.

In one view of parameter instability β_t is time dependent but deterministic. This has a long pedigree in statistics starting with Priestley (1965). More recent examples include Dahlhaus (1996), Robinson (1989), Robinson (1991), Orbe, Ferreira and Rodriguez-Poo (2005), Kapetanios (2008) and Kapetanios (2007). Here β_t is treated as a deterministic process that can be estimated non-parametrically for which standard non-parametric techniques such as kernel-based estimation exist. A practical implementation of this idea is to estimate model **(2)** using a rolling window. The most important question then is to determine the size of the window. A number of considerations can be of use. A cross-validation approach similar to that of Pesaran and Timmermann (2007) may be useful. Alternatively, this problem may be viewed as closely related to determining the bandwidth when estimating β_t by kernel methods. Since there are useful methods for this in non-parametric analysis, they can be used for this problem too. We therefore consider rolling-window estimation as an easy and powerful possibility for the problem we wish to address in this paper.

We consider two other straightforward and easily implementable alternatives. The first is based on estimating coefficients using exponentially weighted moving averages (EWMA). A detailed description may be found in Harvey (1989) but the idea is that, unlike rolling windows where only a subset of available observations receive a non-zero weight in estimation, all available observations receive some weight, but older observations receive less. A parameter controls the rate of decline of weighting older observations, which plays a similar role to the rolling window size. A final alternative, advocated by Pesaran and Timmermann (2007), is to combine forecasts using different estimates of the coefficients where these estimates are obtained using all possible contiguous subsets of observations that include the latest available observation.[9]

[8] See Groen *et al* (2009).

[9] One approach we do not consider is to use time-varying coefficient models as an approximation to the type of repeated discreet change we consider. Here the model **(1)** may be viewed as a measurement equation, augmented by a transition equation in terms of a vector of time-varying parameters, β_t. Thus model **(1)** constitutes a state-space model that can be analysed with widely available methods. In

3 Some theoretical results

We now present some asymptotic results for the robust forecasting strategies presented in Section 2.2, when multiple breaks occur. For tractability, we concentrate on a simple location model. Our Monte Carlo study provides indicative results for more complicated models.

3.1 Stochastic breaks

We begin with a novel stochastic process, based partly on recent work by Koop and Potter (2007) and Kapetanios and Tzavalis (2010). Let the model be

$$y_t = \beta_t + \epsilon_t, \qquad t = 1, \ldots, T, \tag{3}$$

where

$$\beta_t = \sum_{i=1}^{t} \mathcal{I}\left(\nu_i = 1\right) u_i, \tag{4}$$

and ν_i is an i.i.d. sequence of Bernoulli random variables taking the value 1 with probability p and 0 otherwise. ϵ_t and u_i are also i.i.d. series independent of each other and ν_i with finite variance denoted by σ_ϵ^2 and σ_u^2 respectively. This is the simplest model that can accommodate multiple breaks. We are interested in the MSFE of a one-step-ahead forecast based on a model estimated over the whole period

$$\hat{y}_{T+1|T} = \hat{\beta}_T, \qquad \text{where} \quad \hat{\beta}_T = \frac{\sum_{t=1}^{T} y_t}{T} \qquad \text{(Full-sample forecast)}, \tag{5}$$

versus one that is estimated from a method that discounts early data. So we consider three additional forecasts

$$\tilde{y}_{T+1|T} = \tilde{\beta}_T, \qquad \text{where} \quad \tilde{\beta}_T = \frac{\sum_{t=T-m+1}^{T} y_t}{m}, \; m < T, \quad \text{(Rolling forecast)}, \tag{6}$$

$$\bar{y}_{T+1|T} = \frac{1}{T}\sum_{i=1}^{T} \tilde{y}_{T+1|T}^{(i)}, \qquad \text{(Forecast averaging over estimation periods)}, \tag{7}$$

where we denote $\tilde{y}_{T+1|T}$ for a rolling window of size m by $\tilde{y}_{T+1|T}^{(m)}$, and finally

$$\breve{y}_{T+1|T} = \sum_{t=1}^{T} \lambda\left(1-\lambda\right)^{T-t} y_t \qquad \text{(EWMA forecast)} \tag{8}$$

practice this can be a computationally intensive and time-consuming process. In a multivariate setting it may be infeasible. For example, 10 explanatory variables would require 10 distinct unobserved processes for the time-varying coefficients. Specifying and estimating such a model is demanding by most standards and more so if an empirical practitioner is considering several specifications. From a theoretical perspective, that state-space model is bilinear, which may represent a stationary process, rather than one of structural change. Thus the time varying approach goes against the nature of the problem we try to address.

for some $0 < \lambda < 1$. We wish to determine the mean square error of all these forecasts under **(3)**-**(4)**.

The following theoretical results are proved in Appendix A.

Theorem 1 Let the true model be given by **(3)**-**(4)**. Then,

$$E\left(\hat{y}_{T+1|T} - y_{T+1}\right)^2 = \left(\tfrac{(T-1)(2T-1)}{6T} + 1\right) p\sigma_u^2 + \tfrac{(T+1)}{T}\sigma_\epsilon^2 = \frac{1}{3}Tp\sigma_u^2 + o(T)$$

Theorem 2 Let the true model be given by **(3)**-**(4)**. Then,

$$E\left(\tilde{y}_{T+1|T} - y_{T+1}\right)^2 = \left(\tfrac{(m-1)(2m-1)}{6m} + 1\right) p\sigma_u^2 + \tfrac{(m+1)}{m}\sigma_\epsilon^2 = \frac{1}{3}mp\sigma_u^2 + o(m)$$

Theorem 3 Let the true model be given by **(3)**-**(4)**. Then,

$$E\left(\bar{y}_{T+1|T} - y_{T+1}\right)^2 = \frac{7}{54}Tp\sigma_u^2 + o(T)$$

Theorem 4 Let the true model be given by **(3)**-**(4)**. Then,

$$\lim_{T\to\infty} E\left(\check{y}_{T+1|T} - y_{T+1}\right)^2 = O(1)$$

Given the underlying random walk nature of the stochastic breaks model, the MSFE for the full-sample forecast is diverging at rate T. An obvious way this can be counteracted is to allow p to depend on T and specify it as $p_T = pT^{-1}$, thereby ensuring that breaks are rare enough not to induce random walk behaviour to the data. The specification of p does not affect the comparison of forecasts obtained *via* rolling or standard recursive regressions. In particular it is easy to see that for large T and m where $m/T \to 0$, we have that the leading term for recursive regressions is $T/3$ whereas for rolling regressions it is $m/3$ clearly implying that the recursive full-sample regression has a larger unconditional MSFE. The result in Theorem 3 for Pesaran and Timmermann's model averaging over all possible estimation periods suggests that it has an MSFE of the same order but lower than the full-sample forecast MSFE. This MSFE is higher than the MSFE of the rolling forecast. Finally, the EWMA forecast has the lowest MSFE of all the other forecasts.

Our structural break process implies that MSFEs and uncertainty trend with time. We are mainly concerned with discreet and permanent breaks but we will briefly consider a framework without that random walk structure. Consider

$$\beta_t = \begin{cases} \beta + u_t, & \text{if } \nu_t = 1 \\ \beta_{t-1}, & \text{if } \nu_t = 0 \end{cases} \tag{9}$$

where u_t and ν_t are specified as in **(3)**-**(4)**. Without loss of generality we assume that $E(u_t) = 0$.[10] For the full-sample forecast, the forecast error takes the form

$$\begin{aligned} \hat{y}_{T+1|T} - y_{T+1} &= \frac{1}{T}\sum_{t=1}^{T}\beta_t + \frac{1}{T}\sum_{t=1}^{T}\epsilon_t - \beta_{T+1} - \epsilon_{T+1} \\ &= \frac{1}{\upsilon^{1,T}}\sum_{i=1}^{\upsilon^{1,T}}\frac{p_i^{1,T}}{p^{1,T}}u_{t_i^{1,T}} + \frac{1}{T}\sum_{t=1}^{T}\epsilon_t - u_{T+1} - \epsilon_{T+1} \end{aligned} \tag{10}$$

where $\upsilon^{1,T}$ denotes the number of times $\nu_t = 1$ in the period $1, \ldots, T$; $t_i^{1,T}, i = 1, \ldots, \upsilon^{1,T}$, denotes the times at which $\nu_t = 1$; $p_i^{1,T}$ denotes the number of periods between $t_i^{1,T}$ and $t_{i+1}^{1,T}$ where $t_{1+\upsilon^{1,T}}^{1,T} = T$ and $p^{1,T} = \frac{T}{\upsilon^{1,T}}$. It is clear that as $T \to \infty$, $p^{1,T} \xrightarrow{p} p$, and also that

$$\frac{1}{\upsilon^{1,T}}\sum_{i=1}^{\upsilon^{1,T}}\frac{p_i^{1,T}}{p^{1,T}}u_{t_i^{1,T}} = O_p\left(T^{-1/2}\right) \tag{11}$$

Further, it is clear that the first and second terms of the RHS of **(10)** are mutually uncorrelated, and that the third and fourth terms are uncorrelated with the remaining terms.

The forecast errors for the other methods are given by

$$\begin{aligned} \tilde{y}_{T+1|T} - y_{T+1} &= \frac{1}{m}\sum_{t=T-m+1}^{T}\beta_t + \frac{1}{m}\sum_{t=T-m+1}^{T}\epsilon_t - \beta_{T+1} - \epsilon_{T+1} \\ &= \frac{1}{\upsilon^{T-m+1,T}}\sum_{i=1}^{\upsilon^{T-m+1,T}}\frac{p_i^{T-m+1,T}}{p^{T-m+1,T}}u_{t_i^{T-m+1,T}} + \frac{1}{m}\sum_{t=T-m+1}^{T}\epsilon_t - u_{T+1} - \epsilon_{T+1} \end{aligned}$$

$$\bar{y}_{T+1|T} - y_{T+1} = \frac{1}{T}\sum_{j=1}^{T}\left(\frac{1}{\upsilon^{T-j+1,T}}\sum_{i=1}^{\upsilon^{T-j+1,T}}\frac{p_i^{T-j+1,T}}{p^{T-j+1,T}}u_{t_i^{T-j+1,T}} + \frac{1}{j}\sum_{t=T-j+1}^{T}\epsilon_t\right) - u_{T+1} - \epsilon_{T+1}$$

and

$$\begin{aligned} \breve{y}_{T+1|T} - y_{T+1} &= \sum_{t=1}^{T}\lambda(1-\lambda)^{T-t}\beta_t + \sum_{t=1}^{T}\lambda(1-\lambda)^{T-t}\epsilon_t - \beta_{T+1} - \epsilon_{T+1} \\ &= \sum_{t=1}^{T}\lambda(1-\lambda)^{T-t}(\beta_t - \beta) + \sum_{t=1}^{T}\lambda(1-\lambda)^{T-t}\epsilon_t - u_{T+1} - \epsilon_{T+1} + o(1) \end{aligned}$$

[10]We note that there is no memory in the β_t process, which may be considered unsatisfactory.

Similarly to **(11)**, we have that

$$\frac{1}{\upsilon^{T-m+1,T}} \sum_{i=1}^{\upsilon^{T-m+1,T}} \frac{p_i^{T-m+1,T}}{p^{T-m+1,T}} u_{t_i^{T-m+1,T}} = O_p\left(m^{-1/2}\right) \qquad \textbf{(12)}$$

As a result it is clear that for $\hat{y}_{T+1|T}$, $\tilde{y}_{T+1|T}$ and $\bar{y}_{T+1|T}$ the only parts of the forecast error that contribute non-zero terms to the MSFE asymptotically (ie, as $m, T \to \infty$) are u_{T+1} and ϵ_{T+1}. In this sense, the stochastic breaks given by **(9)** are similar to the standard case, where the forecast error variance comes from future shocks rather than parameter estimation. So, all three forecasts have the same first-order MSFE asymptotically. In contrast, for $\breve{y}_{T+1|T}$ and using **(A-7)**, we have

$$E\left(\lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} \epsilon_t\right)^2 = \lambda^2 \sigma_\epsilon^2 \left(\frac{(1-\lambda)^{2T} - 1}{(1-\lambda)^2 - 1}\right) = O(1) \qquad \textbf{(13)}$$

and

$$E\left(\lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} (\beta_t - \beta)\right)^2 \geq \lambda^2 \sigma_u^2 \left(\frac{(1-\lambda)^{2T} - 1}{(1-\lambda)^2 - 1}\right) = O(1)$$

As a result the MSFE of EWMA exceeds that of the other three forecasts for the case of stochastic breaks given by **(9)**. This is in direct contrast to the results of Theorems 1-4 for the stochastic breaks case given by **(4)**.

A possible conclusion is that rolling regressions and forecast averaging have the desirable property that they are either better or as good as full-sample forecasting under a variety of stochastic break scenarios, and might therefore be preferred in the possible presence of structural change.

3.2 Deterministic breaks

An alternative set-up for multiple structural breaks is the conventional approach where breaks are deterministic, which may more accurately reflect small sample settings.

$$y_t = \begin{cases} \beta_1 + \epsilon_t & \text{if } t \leq t_1 \\ \beta_2 + \epsilon_t & \text{if } t_1 < t \leq t_2 \\ \vdots & \quad \vdots \\ \beta_n + \epsilon_t & \text{if } t_{n-1} < t \leq t_n \equiv T+1 \end{cases} \qquad \textbf{(14)}$$

Define $t_i^* = t_i - t_{i-1}$ where $t_0 = 0$. Further, define $\beta_i^* = \beta_i - \beta_n$. Let $t_{n_m - 1} < T - m < t_{n_m}$ for some $n_m \leq n$. Also, define $\tilde{t}_{n_m} = t_{n_m} - T + m$, and $\tilde{t}_i = t_i^*$ for $i > n_m$. Then, it is straightforward to show that for the full-sample and rolling forecast, respectively

$$E\left(\hat{y}_{T+1|T}-y_{T+1}\right)^2=\left(\frac{\sum_{i=1}^{n-1}t_i^*\beta_i^*}{T}\right)^2+\frac{(T+1)\,\sigma_\epsilon^2}{T}=B_1+V_1$$

and

$$E\left(\tilde{y}_{T+1|T}-y_{T+1}\right)^2=\left(\frac{\sum_{i=n_m}^{n-1}\tilde{t}_i\beta_i^*}{m}\right)^2+\frac{(m+1)\,\sigma_\epsilon^2}{m}=B_2+V_2$$

In this case it is clear that there is a trade-off between the squared bias terms B_i, $i=1,2$ and the variance terms V_i, $i=1,2$. Either method may dominate depending on the values of all parameters. Considering a simple case, where $n=n_m=2$ and $t_1=m=T/2$, we have that

$$E\left(\tilde{y}_{T+1|T}-y_{T+1}\right)^2-E\left(\hat{y}_{T+1|T}-y_{T+1}\right)^2<0$$

if

$$\sigma_\epsilon^2<\frac{m\left(\beta_2-\beta_1\right)^2}{2}$$

which of course is satisfied for all $\beta_2-\beta_1\neq 0$ as long as $m\rightarrow\infty$, giving the standard result (eg Pesaran and Timmermann (2007)) in this simple case.

We next look at model averaging over all possible estimation periods. We have that

$$E\left(\bar{y}_{T+1|T}-y_{T+1}\right)^2=\frac{1}{T^2}\sum_{i=1}^{T}\sum_{j=1}^{T}\left\{\left(\frac{\sum_{s=n_i}^{n-1}\tilde{t}_s\beta_s^*}{i}\right)\left(\frac{\sum_{q=n_j}^{n-1}\tilde{t}_q\beta_q^*}{j}\right)+\frac{(\min(i,j)+1)\,\sigma_\epsilon^2}{\min(i,j)}\right\}$$

Finally, for the EWMA estimator we get

$$\breve{y}_{T+1|T}=\sum_{t=1}^{T}\lambda\,(1-\lambda)^{T-t}\,y_t$$

We wish to derive $E\left(\breve{y}_{T+1|T}-y_{T+1}\right)^2$. It is straightforward to show that

$$E\left(\breve{y}_{T+1|T}-y_{T+1}\right)^2=\left(\sum_{i=i}^{n}\sum_{j=t_{i-1}+1}^{t_i}\left(\lambda\,(1-\lambda)^{T-j}\,\beta_i-\beta_n\right)\right)^2+\left(\sum_{t=1}^{T}\lambda\,(1-\lambda)^{T-t}+1\right)\sigma_\epsilon^2$$

$$=\left(\sum_{i=i}^{n}\sum_{j=t_{i-1}+1}^{t_i}\left(\lambda\,(1-\lambda)^{T-j}\,\beta_i-\beta_n\right)\right)^2+\left(\lambda^2\left(\frac{(1-\lambda)^{2T}-1}{(1-\lambda)^2-1}\right)+1\right)\sigma_\epsilon^2$$

Contrary to the stochastic case, the asymptotic results do not offer an unambiguous guide to MSFE rankings, even in restrictive cases.

3.3 Summary of theoretical results

Briefly, although we are able to derive some results there is ambiguity about rankings. EWMA may be the worst or best strategy. Small sample results and more general specifications are therefore explored in the next section.

4 Monte Carlo analysis

In this section we consider the forecasting performance of the forecasting strategies discussed in Section 2. The Monte Carlo study contains three designs. For the first two we consider an autoregressive (order 1) model subject to a structural change, and in the third the location model.

In the first experiment, a single break occurs during the forecast period. This case is designed to explore a situation where the forecaster believes that breaks are rare, and in practise can be considered as unique events. As we argued in Section 2.1, it may then be reasonable to monitor for a break and react after detection by using a forecast combination strategy. Robust forecasting strategies are also applicable. The second design allows frequent breaks to occur. Consequently, monitoring will not be a good strategy and is not considered. The third design replicates the stochastic location model used to derive theoretical results in Section 3, where consequently we have the clearest expectation of the ranking.

For each experiment there are 500 Monte Carlo replications. All forecasts are one-step ahead. The benchmark forecast disregards the possibility of a break and uses an AR(1) model estimated over the whole available sample. We compare the forecasts in relative root MSFE (RRMSFE) terms.

4.1 Design of experiments

For the autoregressive experiments, we use an AR(1) model:

$$y_t = \alpha_t + \rho_t y_{t-1} + \epsilon_t, \qquad t = 1, \ldots, T_0, \ldots, T_1, \ldots, T. \tag{15}$$

4.1.1 *Deterministic single break*

We begin with the specification of the single break case. Forecasting and break monitoring start at T_0, which we set to 100. The break occurs at T_1, which is set to 110, and occurs either in the autoregressive parameter or the intercept. These parameters take the value ρ_1 or α_1 up to T_1 and ρ_2 or α_2 thereafter.

That is, the actual data generation process is

$$y_t = \begin{cases} \alpha_1 + \rho_1 y_{t-1} + \epsilon_t, & t = 1, \ldots, T_1 - 1 \\ \alpha_2 + \rho_2 y_{t-1} + \epsilon_t, & t = T_1, \ldots, T \end{cases} \tag{16}$$

If the intercept or the autoregressive parameter are assumed constant they take the values $\alpha_1 = \alpha_2 = 0$ and $\rho_1 = \rho_2 = 0$ respectively. ρ_1 and ρ_2 take values from the set $\{-0.6, -0.4, -0.2, 0.2, 0.4, 0.6, 0.8\}$, while α_1 and α_2 take values from the set $\{-1.2, -0.4, 0.4, 0.8, 1.2, 1.6\}$. Monitoring is assumed to cease when a break is detected.[11] Forecasting and evaluation (between T_0 and T) stops at $T = 150$. Averaging occurs during $\hat{T}_1 + 5$ to $\hat{T}_1 + \overline{f}$, where $\overline{f}$ is set at 20 or 60 and $\hat{T}_1$ is the date at which the break is detected.[12]

The robust strategies we consider are: a rolling window where the size of the window is set to M at 20 and 60 periods; forecast averaging of forecasts obtained using parameters estimated over all possible estimation windows; and exponential weighted moving average estimation of the parameters.

In the EWMA based least squares estimator of the regression $y_t = \beta' x_t + u_t$, $t = 1, \ldots, T$, is $\hat{\beta}_{EWMA} = \left(\lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} x_t x_t'\right)^{-1} \lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} x_t y_t$, where λ is a decay parameter. The choice of $0 < \lambda < 1$ is usually arbitrary. Harvey (1989) suggests that λ should lie between 0.05 and 0.3. This matters in practice: see for example Pesaran and Pick (2008). We examine two cases. The first sidesteps the choice by averaging forecasts using $\lambda = 0.1, 0.2, 0.3$, and in the second we use a value at the low end of the range, 0.05.[13] We refer to these as EWMAA (where the final 'A' indicates average) and EWMAL ('L' indicates low decay) respectively.

[11] Effectively, we are assuming the forecaster knows the structure of the model (in this as in other respects).

[12] The delay in $\hat{T}_1 + 5$ is set arbitrarily.

[13] Implying that the weight falls below 5% for lags greater than 60, one of the window lengths reported in the rolling and monitoring approaches.

4.1.2 Stochastic multiple breaks in an AR(1) model

For the multiple stochastic case either the autoregressive parameter or the autoregressive model's intercept change as follows:

$$\rho_t = \begin{cases} \rho_{t-1}, & \text{with probability} \quad 1-p \\ \eta_{\rho,t}, & \text{with probability} \quad p \end{cases}$$

$$\alpha_t = \begin{cases} \alpha_{t-1}, & \text{with probability} \quad 1-p \\ \eta_{\alpha,t}, & \text{with probability} \quad p \end{cases}$$

$p = 0.1, 0.05, 0.02, 0.01$ implying that the average duration between breaks varies between 10 and 100 periods. $\eta_{i,t} \sim iidU\left(\eta_{il}, \eta_{iu}\right)$, $i = \rho, \alpha$, where

$$\left\{\eta_{\rho,l}, \eta_{\rho,u}\right\} = \{-0.8, 0.8\}, \{-0.6, 0.6\}, \{-0.4, 0.4\}, \{-0.2, 0.2\}$$

and

$$\left\{\eta_{\alpha,l}, \eta_{\alpha,u}\right\} = \{-2, 2\}, \{-1.6, 1.6\}, \{-1.2, 1.2\}, \{-0.8, 0.8\}, \{-0.4, 0.4\}.$$

When there are breaks in ρ, $\alpha = 0$, whereas for breaks in α, $\rho = 0$ (leaving the unconditional mean unchanged). The sample size is set to $T = 300$ and forecast evaluation starts at $t = 100$. Other aspects of the specification such as rolling-window length are as in the single break case. As there are multiple breaks, only robust forecasting strategies are considered.

4.1.3 Location model

In this simple stochastic case, we use the specification in **(3)**-**(4)**.

$$p = 0.5, 0.33, 0.2, 0.1, 0.05, 0.01$$

implying that breaks occur on average between every 2 and 100 periods. We set $\epsilon_t \sim N(0, 1)$, and $u_t \sim iidU\left(u_l, u_u\right)$, where

$$\{u_l, u_u\} = \{-1, 1\}, \{-0.9, 0.9\}, \{-0.8, 0.8\}, \{-0.7, 0.7\}, \{-0.6, 0.6\}.$$

Other characteristics are the same as for the more general case but the estimated model contains only a constant.

4.2 Results for single breaks

In the single break experiments where we are able to evaluate our monitoring approach, we consider breaks in either persistence or the mean. Table A reports the former for $\alpha = 0$. For monitoring, in some cases there are gains in forecast performance. However, in most cases the gains are more modest than with the other methods. But there are no cases where monitoring leads to worse performance than the benchmark. The implication is that it is a conservative forecasting strategy, in the sense that it would tend to do (marginally) better than the benchmark in some cases but will not lead to large forecast errors. In this set-up, where there are gains, they tend to be greater for the shorter period.

The rolling-window methods perform better than monitoring for large breaks. Where they do well, a short post-break window improves the performance. But where they do worst, the opposite is the case. In general, longer windows offer a more conservative strategy. The forecast averaging method outperforms the longer period rolling window in most cases and where it does worse than the benchmark, does not do so by a large margin. In several cases it is best.

By contrast, although the averaged EWMA (EWMAA) does extremely well for some large changes, it does very badly for small changes or no structural change (along the diagonals). It is a risky strategy. The low-discount EWMA (EWMAL) is not so sensitive to small or large breaks. It lies somewhere between the short and long rolling window.

In Table B we consider a break in α. The results are qualitatively similar to those in Table A.

Table A: RRMSFE for alternative forecasting strategies; Single break in ρ; $\alpha = 0$

$\rho_1 \backslash \rho_2$	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
	Monitoring ($\overline{f} = 20$)								Monitoring ($\overline{f} = 60$)							
-0.6	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.92	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.95
-0.4	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.94	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
-0.2	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.95	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
0	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99
0.2	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
0.4	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Rolling Window ($M = 20$)								Rolling Window ($M = 60$)							
-0.6	1.09	1.06	1.00	0.94	0.84	0.74	0.61	0.48	1.01	1.01	0.99	0.96	0.93	0.88	0.82	0.74
-0.4	1.06	1.09	1.06	1.01	0.94	0.82	0.70	0.54	1.01	1.02	1.01	0.98	0.96	0.91	0.84	0.76
-0.2	0.99	1.07	1.09	1.06	1.01	0.93	0.81	0.64	0.98	1.01	1.02	1.01	0.99	0.95	0.89	0.79
0	0.90	1.01	1.07	1.09	1.08	1.02	0.90	0.74	0.93	0.98	1.01	1.02	1.01	0.98	0.94	0.84
0.2	0.80	0.91	1.01	1.07	1.09	1.08	1.00	0.87	0.88	0.94	0.99	1.01	1.02	1.01	0.98	0.90
0.4	0.70	0.84	0.94	1.02	1.08	1.09	1.08	0.97	0.85	0.91	0.95	0.99	1.01	1.02	1.01	0.95
0.6	0.61	0.74	0.84	0.94	1.02	1.08	1.11	1.07	0.81	0.88	0.92	0.96	0.99	1.01	1.02	1.00
0.8	0.53	0.66	0.76	0.86	0.95	1.01	1.08	1.12	0.81	0.86	0.91	0.94	0.96	0.99	1.01	1.02
	Forecast Averaging								EWMAA							
-0.6	1.01	1.00	0.97	0.94	0.89	0.83	0.75	0.67	1.26	1.23	1.14	1.06	0.90	0.75	0.59	0.41
-0.4	1.00	1.01	1.00	0.97	0.94	0.87	0.80	0.70	1.22	1.26	1.23	1.14	1.05	0.87	0.71	0.51
-0.2	0.96	1.00	1.01	1.00	0.97	0.93	0.85	0.75	1.13	1.24	1.27	1.22	1.15	1.03	0.84	0.62
0	0.91	0.97	1.01	1.01	1.00	0.97	0.91	0.81	1.03	1.16	1.24	1.26	1.22	1.14	0.96	0.74
0.2	0.86	0.92	0.97	1.00	1.01	1.00	0.96	0.88	0.89	1.04	1.16	1.23	1.25	1.22	1.08	0.90
0.4	0.80	0.88	0.93	0.98	1.01	1.01	1.00	0.94	0.75	0.92	1.06	1.16	1.23	1.23	1.18	1.02
0.6	0.75	0.83	0.88	0.93	0.97	1.00	1.02	0.99	0.63	0.79	0.92	1.05	1.15	1.22	1.22	1.14
0.8	0.72	0.79	0.85	0.89	0.93	0.97	1.00	1.01	0.52	0.68	0.81	0.93	1.04	1.12	1.19	1.19
	EWMAL															
-0.6	1.04	1.02	0.98	0.91	0.84	0.76	0.64	0.49								
-0.4	1.02	1.04	1.02	0.97	0.91	0.82	0.71	0.55								
-0.2	0.97	1.02	1.04	1.02	0.98	0.89	0.79	0.63								
0	0.88	0.98	1.02	1.04	1.02	0.97	0.88	0.72								
0.2	0.80	0.91	0.98	1.03	1.04	1.01	0.95	0.82								
0.4	0.72	0.84	0.93	0.99	1.02	1.04	1.01	0.92								
0.8	0.66	0.76	0.86	0.93	0.99	1.02	1.03	0.99								
0.8	0.64	0.75	0.82	0.88	0.94	0.99	1.02	1.03								

Notes: EWMAA: Averaging EWMA forecasts with decay parameters of 0.1, 0.2 and 0.3; EWMAL: EWMA with decay parameter 0.05.

Table B: RRMSFE for alternative forecasting strategies; Single break in α; $\rho = 0$

$\alpha_1 \backslash \alpha_2$	-1.2	-0.8	-0.4	0	0.4	0.8	1.2	1.6	-1.2	-0.8	-0.4	0	0.4	0.8	1.2	1.6
	Monitoring ($\overline{f} = 20$)								Monitoring ($\overline{f} = 60$)							
-1.2	1.00	1.00	0.99	0.95	0.90	0.86	0.83	0.82	1.00	1.00	0.99	0.97	0.95	0.93	0.92	0.91
-0.8	1.00	1.00	1.00	0.98	0.95	0.90	0.85	0.84	1.00	1.00	1.00	0.99	0.97	0.96	0.93	0.92
-0.4	0.98	1.00	1.00	1.00	0.99	0.95	0.90	0.86	0.99	1.00	1.00	1.00	0.99	0.97	0.95	0.93
0	0.95	0.99	1.00	1.00	1.00	0.99	0.95	0.91	0.98	0.99	1.00	1.00	1.00	0.99	0.98	0.95
0.4	0.90	0.95	0.99	1.00	1.00	1.00	0.98	0.95	0.95	0.98	0.99	1.00	1.00	1.00	0.99	0.98
0.8	0.86	0.90	0.95	0.98	1.00	1.00	1.00	0.99	0.93	0.95	0.98	0.99	1.00	1.00	1.00	0.99
1.2	0.83	0.87	0.90	0.95	0.98	1.00	1.00	1.00	0.92	0.93	0.95	0.97	0.99	1.00	1.00	1.00
1.6	0.81	0.83	0.86	0.89	0.94	0.99	1.00	1.00	0.91	0.92	0.93	0.95	0.97	0.99	1.00	1.00
	Rolling Window ($M = 20$)								Rolling Window ($M = 60$)							
-1.2	1.09	1.02	0.88	0.75	0.68	0.62	0.59	0.58	1.02	0.99	0.93	0.87	0.84	0.81	0.79	0.79
-0.8	1.02	1.09	1.03	0.88	0.75	0.67	0.61	0.59	0.99	1.01	0.99	0.93	0.88	0.83	0.81	0.79
-0.4	0.88	1.02	1.09	1.02	0.88	0.76	0.67	0.63	0.93	0.99	1.02	0.99	0.93	0.87	0.84	0.81
0	0.76	0.88	1.02	1.09	1.02	0.88	0.75	0.68	0.88	0.93	0.99	1.02	0.99	0.93	0.88	0.84
0.4	0.67	0.76	0.89	1.03	1.09	1.02	0.88	0.76	0.84	0.88	0.93	0.99	1.02	0.99	0.93	0.88
0.8	0.62	0.67	0.76	0.88	1.03	1.09	1.02	0.88	0.81	0.84	0.88	0.93	0.99	1.02	0.99	0.93
1.2	0.59	0.63	0.67	0.76	0.88	1.02	1.09	1.02	0.80	0.81	0.83	0.87	0.93	0.99	1.02	0.99
1.6	0.57	0.59	0.62	0.67	0.76	0.88	1.02	1.08	0.78	0.79	0.81	0.83	0.87	0.93	0.99	1.02
	Forecast Averaging								EWMAA							
-1.2	1.01	0.98	0.90	0.83	0.79	0.76	0.73	0.73	1.25	1.16	0.97	0.81	0.71	0.63	0.59	0.58
-0.8	0.98	1.01	0.98	0.90	0.84	0.79	0.75	0.74	1.17	1.26	1.17	0.97	0.81	0.70	0.62	0.59
-0.4	0.90	0.98	1.01	0.98	0.90	0.84	0.79	0.76	0.97	1.16	1.26	1.16	0.97	0.81	0.69	0.64
0	0.84	0.91	0.98	1.01	0.98	0.90	0.84	0.79	0.80	0.98	1.17	1.25	1.16	0.97	0.80	0.71
0.4	0.79	0.84	0.91	0.98	1.01	0.98	0.90	0.84	0.70	0.81	0.98	1.17	1.26	1.17	0.97	0.82
0.8	0.76	0.79	0.84	0.90	0.98	1.01	0.97	0.90	0.64	0.70	0.82	0.97	1.16	1.26	1.16	0.98
1.2	0.74	0.76	0.79	0.84	0.90	0.98	1.01	0.98	0.60	0.64	0.69	0.82	0.98	1.16	1.25	1.16
1.6	0.73	0.73	0.75	0.79	0.83	0.91	0.98	1.01	0.57	0.59	0.62	0.70	0.81	0.98	1.17	1.26
	EWMAL															
-1.2	1.04	0.98	0.87	0.77	0.70	0.66	0.64	0.63								
-0.8	0.98	1.04	0.98	0.86	0.77	0.70	0.67	0.65								
-0.4	0.87	0.98	1.04	0.98	0.87	0.77	0.71	0.67								
0	0.77	0.87	0.97	1.04	0.98	0.87	0.77	0.71								
0.4	0.71	0.77	0.87	0.98	1.04	0.98	0.87	0.77								
0.8	0.67	0.70	0.78	0.86	0.98	1.04	0.98	0.87								
1.2	0.64	0.66	0.70	0.77	0.87	0.98	1.04	0.98								
1.6	0.63	0.64	0.67	0.70	0.77	0.86	0.98	1.04								

Notes: EWMAA: Averaging EWMA forecasts with decay parameters of 0.1, 0.2 and 0.3; EWMAL: EWMA with decay parameter 0.05.

4.3 *Results for recurring breaks*

We now examine recurring breaks. We exclude the monitoring method as it is inappropriate in this environment.

4.3.1 Location model

For reference, we begin with the simple location model. Results are reported in Table C. Given our analytical results, we expect that EWMA will perform best, followed by rolling regressions with short windows, rolling regressions with longer windows, forecast averaging and finally forecasting based on the full sample. In fact, this is essentially what we find. The majority of best-performing cases are for the EWMAA. For low probability breaks the EWMAL performs best. The short rolling window is often better than the EWMAL in this parametrisation. Interestingly, the only case where the models fail to beat the full-sample benchmark is with the EWMAA for the most infrequent breaks (average duration between breaks 100 periods) and smallest change.

4.3.2 AR(1) model

Turning to more realistic structures, Table D reports the results for recurring breaks in persistence ρ in an autoregressive model, for constant α. For the largest shifts, a low-order rolling window is generally the best performer. However, as the size of the shift declines, the small rolling-window performance deteriorates so that in most cases it cannot outperform the full-sample estimates. The penalty from a short estimation period outweighs the gain from discounting the pre-break period. The longer window rolling case is more robust, in the sense that it both outperforms the full-sample benchmark at low break probabilities for larger changes and is close to the benchmark for small changes and lower probabilities. In marked contrast to the location model results, the EWMAA never performs well. In all cases it performs worse than the alternative methods, and in many cases much worse. However, the low discount variant EWMAL again performs well for large infrequent breaks, with a small-change penalty intermediate between the short and long rolling windows. But arguably, forecast averaging is dominant. In the best-performing cases (large breaks) it is comparable to the shorter rolling window and generally no worse than the benchmark in the worst cases. Consequently the worst-case cost is small, and

Table C: RRMSFE for forecasting strategies (location model); Recurring breaks

$p \backslash \begin{smallmatrix} u_l \\ u_u \end{smallmatrix}$	-1 / 1	-0.9 / 0.9	-0.8 / 0.8	-0.7 / 0.7	-0.6 / 0.6	-1 / 1	-0.9 / 0.9	-0.8 / 0.8	-0.7 / 0.7	-0.6 / 0.6
	Rolling Window ($M = 20$)					Rolling Window ($M = 60$)				
0.5	0.18	0.22	0.21	0.25	0.30	0.38	0.40	0.40	0.42	0.43
0.33	0.22	0.22	0.27	0.30	0.37	0.39	0.39	0.42	0.47	0.48
0.2	0.27	0.33	0.33	0.39	0.47	0.45	0.46	0.47	0.51	0.57
0.1	0.41	0.45	0.48	0.55	0.61	0.52	0.57	0.59	0.65	0.70
0.05	0.57	0.61	0.68	0.71	0.76	0.64	0.67	0.71	0.77	0.81
0.01	0.88	0.90	0.93	0.95	0.97	0.89	0.89	0.93	0.94	0.96
	Forecast Averaging					EWMAA				
0.5	0.46	0.49	0.48	0.51	0.54	0.13	0.16	0.17	0.21	0.26
0.33	0.48	0.49	0.52	0.53	0.58	0.17	0.18	0.23	0.26	0.34
0.2	0.52	0.56	0.55	0.59	0.65	0.23	0.29	0.30	0.37	0.45
0.1	0.60	0.63	0.65	0.69	0.73	0.38	0.43	0.47	0.54	0.61
0.05	0.71	0.73	0.77	0.79	0.82	0.56	0.61	0.68	0.73	0.78
0.01	0.90	0.91	0.93	0.94	0.96	0.91	0.94	0.97	0.99	1.01
	EWMAL									
0.5	0.23	0.23	0.27	0.29	0.32					
0.33	0.25	0.27	0.29	0.34	0.39					
0.2	0.31	0.35	0.39	0.44	0.50					
0.1	0.42	0.46	0.53	0.56	0.64					
0.05	0.56	0.61	0.66	0.72	0.77					
0.01	0.85	0.87	0.92	0.92	0.96					

Notes: EWMAA: Averaging EWMA forecasts with decay parameters of 0.1, 0.2 and 0.3; EWMAL: EWMA with decay parameter 0.05.

this method could therefore be described as conservative. In many cases it is the best performer. So forecast averaging emerges as a successful strategy.

The results in Table E, where the intercept shifts, reveal less diversity. Overall, the best performer is again arguably the forecast average for similar reasons to those above. It uniformly outperforms the medium rolling window and EWMAA, and most cases in the short rolling window. And in no case does it do worse than the full sample. As in Table D, in no case is the EWMAA best, and tends to be worst, often by wide margins, increasing as the magnitude of changes declines. EWMAL is again best for the larger breaks, but worse than forecast averaging for smaller breaks. We conclude that although no method is unambiguously superior, forecast averaging has the edge over rolling regressions, that for rolling regressions longer windows are more robust in the sense they avoid major errors, that EWMAL is good for larger breaks, and that in most circumstances the EWMAA is a poor forecast model.

Table D: RRMSFE for forecasting strategies (AR model); Recurring breaks in ρ; $\alpha = 0$

$p\backslash$ $\eta_{\rho,l}$	-0.8	-0.6	-0.4	-0.2	-0.8	-0.6	-0.4	-0.2
$\eta_{\rho,u}$	0.8	0.6	0.4	0.2	0.8	0.6	0.4	0.2
	Rolling Window ($M = 20$)				Rolling Window ($M = 60$)			
0.1	0.97	1.04	1.07	1.09	1.00	1.01	1.02	1.02
0.05	0.93	1.01	1.06	1.09	0.96	1.00	1.01	1.02
0.02	0.90	1.00	1.05	1.09	0.93	0.97	1.00	1.02
0.01	0.91	1.02	1.06	1.09	0.91	0.97	1.00	1.02
	Forecast Averaging				EWMAA			
0.1	0.95	0.98	1.00	1.01	1.02	1.14	1.21	1.25
0.05	0.93	0.97	0.99	1.01	1.00	1.12	1.20	1.25
0.02	0.91	0.96	0.99	1.00	0.99	1.12	1.20	1.25
0.01	0.91	0.97	0.99	1.00	1.02	1.16	1.22	1.25
	EWMAL							
0.1	0.92	0.99	1.02	1.04				
0.05	0.89	0.97	1.01	1.04				
0.02	0.87	0.96	1.01	1.03				
0.01	0.90	0.96	1.01	1.04				

Notes: EWMAA: Averaging EWMA forecasts with decay parameters of 0.1, 0.2 and 0.3; EWMAL: EWMA with decay parameter 0.05.

Table E: RRMSFE for forecasting strategies (AR model); Recurring breaks in α; $\rho = 0$

$p\backslash$ $\eta_{\alpha,l}$	-2	-1.6	-1.2	-0.8	-0.4	-2	-1.6	-1.2	-0.8	-0.4
$\eta_{\alpha,u}$	2	1.6	1.2	0.8	0.4	2	1.6	1.2	0.8	0.4
	Rolling Window ($M = 20$)					Rolling Window ($M = 60$)				
0.1	1.04	1.04	1.04	1.05	1.08	1.02	1.01	1.02	1.01	1.02
0.05	0.94	0.95	0.98	1.02	1.07	0.99	0.99	0.99	1.00	1.02
0.02	0.84	0.88	0.92	0.99	1.06	0.91	0.93	0.94	0.97	1.01
0.01	0.84	0.87	0.93	0.99	1.07	0.88	0.89	0.93	0.97	1.01
	Forecast Averaging					EWMAA				
0.1	0.97	0.97	0.98	0.99	1.00	1.06	1.06	1.10	1.16	1.23
0.05	0.93	0.94	0.95	0.97	1.00	0.97	0.99	1.05	1.13	1.22
0.02	0.88	0.90	0.92	0.96	0.99	0.91	0.96	1.02	1.12	1.22
0.01	0.87	0.89	0.92	0.96	1.00	0.93	0.97	1.05	1.13	1.23
	EWMAL									
0.1	0.96	0.97	0.98	1.00	1.03					
0.05	0.91	0.91	0.93	0.98	1.02					
0.02	0.84	0.85	0.90	0.95	1.01					
0.01	0.82	0.86	0.89	0.96	1.01					

Notes: EWMAA: Averaging EWMA forecasts with decay parameters of 0.1, 0.2 and 0.3; EWMAL: EWMA with decay parameter 0.05.

4.4 Summary

Thus we can draw some tentative conclusions. Results are sensitive to parameter choices, except for the average (where we simply use uniform weights over all possibilities). A monitoring and combination strategy will improve forecast performance and is unlikely to lead to major forecast errors relative to the full-sample benchmark; in that sense it is a conservative strategy. But forecast improvements are small. Where we are confident moderately large breaks are likely to occur or are occurring infrequently, rolling windows can be useful. But they may be susceptible to poor forecast performance, the more so the shorter the window. Longer windows make for poorer performance for large breaks but better for small. The averaged EWMAA can provide very large improvements for large breaks but in general is a risky strategy to adopt as it can lead to large errors. The low discount EWMAL is comparable to an intermediate rolling window length or averaging in some respects. Overall, the forecast averaging method emerges as a good compromise between improved forecast performance in the face of large breaks and modest costs in other cases. It also has the advantage of being free of the necessity to make a parameter choice. In general the results are broadly consistent with the analytical results.

Notwithstanding this, it has to be acknowledged that Monte Carlo results are inevitably sensitive to experimental choices. They are specific contexts that are never precisely realised with real data. We therefore move next to assessing performance on a range of macro data.

5 Empirical application

In this section we examine how our methods would have fared when applied to a large range of UK and US quarterly data series.[14] We are not trying to develop the best methods for particular data sets, but instead trying to get an impression of whether the issues identified above are important in practice. In all cases we transform series to stationarity and employ AR(1) forecasting models. For the United Kingdom, we use data on 94 series spanning 1977 Q1 to 2008 Q2, and examine two forecast evaluation sub-periods within this (1992 Q1 to 1999 Q4 and 2000 Q1 to 2008 Q2). For the United States, we have data on 97 series from 1960 Q1 to 2008 Q3, and examine three forecast evaluation sub-periods (1975 Q1 to 1986 Q2, 1986 Q3 to 1997 Q4, and 1998 Q1 to 2008 Q3). For each series, we compare RMSFEs to that from an AR(1) benchmark.

[14] We take no account of real-time data revisions.

The methods we report relate to those in the Monte Carlo study, and are monitoring using 40 and 60-period windows (M40 and M60),[15] rolling-window forecasts using 40 and 60-period windows (R40 and R60), averaging across estimation periods (AV) and the exponentially weighted moving average (EWMA). Detailed results for each series and series descriptions are given in Appendices B and C for the United Kingdom and United States respectively.

5.1 UK results

An obvious prior question to ask is whether there is evidence of structural breaks in the series we examine. So begin by performing Bai and Perron (1998) tests for structural breaks (shifts in either constant or autoregressive parameter for an AR(1) process), reported in Table F. We identify 33 series containing breaks out of the total, so this suggests that structural change was indeed an important issue in the United Kingdom over this period. It should be clear that this test uses the full sample and this information would not be available in real time.

The full set of results is given in Appendix B for the two periods we examine. They are summarised in Table G. We report the mean, the median (giving some indication of skewness), the minimum and maximum, the standard deviation, and skewness of the relative RMSFE. We also report the number of cases in which Diebold-Mariano tests reject equality of performance between a robust method and the full-sample (FS) null at 5% in favour of the robust method (DM(R)), while DM(FS) rejects against the robust method, again at 5% significance level.

The theory for the stochastic case suggested that the average RMSFE should exceed the rolling, which should exceed the EWMA. But in the Monte Carlos, we found the results are sensitive to parameter choice and callibrations. On the mean and median RMSFE criteria in both periods the minima are delivered by the averaging method, followed by the EWMAL. The EWMAA is not only the worst performer, but on average fails to beat the full-sample AR, although in some cases it does extremely well (indicated by the very low values in the 'Minimum' rows). The monitoring method on average beats the benchmark, with a 40-period window outperforming 60 periods. The rolling window does better, especially for the 40-period window. The rolling regressions also deliver low minima, especially for the shorter window. However, if the forecaster gives a high weight to avoiding extreme forecast errors, then using the monitoring method may be the

[15]We use a window of 40 observations rather than 20 for the empirical application as it corresponds to a ten-year estimation period .

Table F: Series with identified breaks: UK data

Private sector output growth	1
GfK index score	1
Stock of net corporate debt	1
Nominal wages per worker	1
Sectoral M4	1
M4 liabilities to private non-financial corporations	1
Net lending to household sector	2
GDP	1
Gross National Income	1
Manufacturing	1
Manufacturing of textile & textile products	1
Manufacturing of leather & leather products	1
Manufacturing of wood & wood products	1
Manufacturing of non-metallic mineral products	1
Manufacturing of basic metals & fabricated prod	2
Manufacturing of electrical & optical equipment	1
Distribution, hotels & catering; repairs	1
Output Index: Total	1
Total adjustment to basic prices	1
GDP at market prices	1
Gross Value Added at factor cost	1
Money stock M4 (end period)	3
Notes & coin in circulation outside Bank of England	1
Total Government benefits paid to household sector	3
General Government: Final consumption expenditure	2
Household final consumption expenditure	2
Durable goods	1
Claimant count rate	1
Whole economy, inc bonus: % change 3-month average	1
Unemployed	3
Economically active	1
Total actual weekly hours worked	2
Imports: Total trade in goods and services excl MTIC fraud	1

Table G: Summary for empirical results for UK

	M40	M60	R40	R60	AV	EWMAA	EWMAL
	First Period (1992 Q1 - 1999 Q4)						
Mean	0.978	0.984	0.957	0.975	0.918	1.054	0.925
Median	1.000	1.000	0.974	0.984	0.951	1.056	0.958
Minimum	0.607	0.692	0.118	0.792	0.155	0.010	0.096
Maximum	1.050	1.031	1.525	1.235	1.265	2.228	1.229
Std. Dev.	0.058	0.043	0.170	0.085	0.157	0.300	0.159
Skewness	-3.783	-4.239	-0.725	0.383	-1.429	0.155	-1.866
DM(R)	14	14	18	16	17	6	16
DM(FS)	2	2	4	4	1	8	2
	Second Period (2000 Q1 - 2008 Q2)						
Mean	0.972	0.980	0.925	0.959	0.903	1.029	0.914
Median	1 1.000	1.000	0.959	0.987	0.949	1.096	0.963
Minimum	0.619	0.737	0.006	0.005	0.047	0.005	0.007
Maximum	1.040	1.025	1.511	1.514	1.301	1.622	1.350
Std. Dev.	0.065	0.044	0.238	0.218	0.189	0.317	0.203
Skewness	-2.806	-2.819	-0.676	-0.636	-1.182	-0.525	-1.101
DM(R)	12	12	16	16	22	8	19
DM(FS)	1	2	2	8	1	9	1

Notes: The table reports summary statistics on the set of Relative RMSFEs for alternative forecasting methods. M60: Monitoring using a 60-period window; M40: Monitoring using a 40-period window; R40: Rolling Forecast using a 40-period window; R60: Rolling Forecast using a 60-period window; AV: Averaging across estimation periods; EWMAA: Exponentially Weighted Moving Average (Averaging 3 EWMA forecasts with decay parameters of 0.1, 0.2 and 0.3); EWMAL: EWMA with decay parameter given by 0.05. DM(R) is the number of series for which the Diebold-Mariano test rejects in favour of the given robust method at the 5% significance level, while DM(FS) is the number of series for which the Diebold-Mariano test rejects against the given robust method at the 5% significance level.

best strategy. The maximum RRMSFE are close to unity in that case, and the variation in the RRMSFE also smallest. The EWMAA, by contrast, is worst on this criterion. On the formal tests, in the first period the rolling 40 period ranks first, followed by the average. With the exception of the EWMAA which is selected only slightly more often than would be expected by chance, the other methods are closely comparable. The EWMAA is significantly outperformed by the full-sample forecasts in more cases than it outperforms: by contrast, except for the rolling cases there are only one or two rejections for the other methods. Similar results hold for the second period. In this race, averaging wins by a head, while EWMAL comes a close second.

We conclude that over these periods averaging would have been a good strategy, although EWMAL, rolling regressions and monitoring would also have improved forecast performance.

However, monitoring would have been a relatively conservative strategy, again in the sense that it would on average offer a small advantage over using the full sample and avoids making large forecast errors, while not offering large improvements in performance. This may well reflect the difficulty of detecting structural breaks.

5.2 US results

For the United States, far fewer breaks are identified (Table H). Consequently, there are fewer gains to using the methods (Table I), although more so in the third period. Forecast averaging no longer unambiguously emerges as the best average performer, but EWMAA remains both the worst on average and the most variable performer, with the best and worse individual forecasts in each period. The monitoring methods remain conservative in the sense we identified in the United Kingdom (small average gains and avoiding very poor performance). Based on the formal DM tests, there was little evidence that any model would have helped forecast these series, with the exception of averaging in the third period, where there is some weak evidence in favour.

6 Conclusions

A common source of forecast failure is the existence of structural breaks in the data generating process. One characterisation of a break is an abrupt parameter shift. In that context, a natural strategy for a forecaster operating in real time might be to monitor for a break, and then to adopt a robust forecasting strategy until enough data exist to allow the break to be modelled or only post-break data be used. However, the intrinsic difficulty is that by the nature of the exercise there are few observations available either to estimate parameters or to evaluate forecasts. For distant breaks, combinations of differently specified models are known to have good forecast properties, and we use a tailored version of this for the post-break period.

Table H: Series with identified breaks: US data

Industrial Production: Consumer Goods	1
Unemployment Rate: All Workers	1
Civilians Unemployed - 15 Weeks & Over	1
1-Year Treasury Constant Maturity Rate	1
Total Reserves of Depository Institutions	1
S&P 500 Finance Total return Index	1

Table I: Summary for empirical results for US

	M40	M60	R40	R60	AV	EWMAA	EWMAL
				First Period (1975 Q1 - 1986 Q2)			
Mean	1.011	1.005	1.033	1.012	1.032	1.221	1.043
Median	1.000	1.000	1.033	1.007	1.034	1.212	1.046
Minimum	0.872	0.905	0.906	0.937	0.889	0.792	0.842
Maximum	1.171	1.106	1.135	1.355	1.291	2.594	1.440
Std. Dev.	0.032	0.020	0.042	0.042	0.057	0.212	0.069
Skewness	0.689	0.077	-0.266	5.515	0.455	2.695	1.594
DM(R)	2	2	0	0	0	0	0
DM(FS)	3	1	7	3	12	10	9
				Second Period (1986 Q3 - 1997 Q4)			
Mean	0.990	0.991	0.999	1.040	0.987	1.145	0.987
Median	1.000	1.000	0.999	1.029	1.008	1.161	1.004
Minimum	0.815	0.870	0.641	0.798	0.711	0.583	0.686
Maximum	1.092	1.054	1.284	1.414	1.113	1.732	1.150
Std. Dev.	0.032	0.024	0.100	0.101	0.070	0.227	0.081
Skewness	-2.027	-2.210	-0.400	0.978	-1.528	0.014	1.019
DM(R)	3	4	2	2	7	3	6
DM(FS)	4	2	1	14	2	3	1
				Third Period (1998 Q1 - 2008 Q3)			
Mean	0.998	0.991	1.002	0.977	0.952	1.307	0.984
Median	1.000	1.000	1.025	0.997	0.969	1.104	0.982
Minimum	0.842	0.877	0.311	0.324	0.513	0.333	0.356
Maximum	1.623	1.052	2.557	1.626	1.113	15.818	2.384
Std. Dev.	0.073	0.028	0.212	0.139	0.093	1.540	0.189
Skewness	6.153	-2.109	3.664	-0.259	-1.595	8.675	3.666
DM(R)	1	3	5	3	13	1	4
DM(FS)	0	0	6	5	0	6	2

Notes: The table reports summary statistics on the set of Relative RMSFEs for alternative forecasting methods. M60: Monitoring using a 60-period window; M40: Monitoring using a 40-period window; R40: Rolling Forecast using a 40-period window; R60: Rolling Forecast using a 60-period window; AV: Averaging across estimation periods; EWMAA: Exponentially Weighted Moving Average (Averaging 3 EWMA forecasts with decay parameters of 0.1, 0.2 and 0.3); EWMAL: EWMA with decay parameter given by 0.05. DM(R) is the number of series for which the Diebold-Mariano test rejects in favour of the given robust method at the 5% significance level, while DM(FS) is the number of series for which the Diebold-Mariano test rejects against the given robust method at the 5% significance level.

But an alternative is to ignore the discreet nature of the hypothesised structural change and pursue some robust forecasting strategy that effectively allows for time variation in a simple but flexible manner. In general, robust methods weight recent observations more than distant. We examine a rolling-window estimator, combination methods and an exponentially weighted moving average estimator, all avoiding the need to monitor for breaks. There is a cost to discarding data when there has not been a break but also advantages: there is no delay in recognising a break has occurred, and they may be robust to varying forms of structural change.

We derive some theoretical results for the robust methods. In a location model with a stochastic break, we obtain a clear ranking for the methods, with EWMA being best, followed by rolling regressions and forecast averaging. But with a break process lacking a memory where the parameter mean reverts, EWMA is worst. Rankings cannot be obtained in a deterministic case.

In our Monte Carlo exercises which examine single and multiple breaks in an AR process and multiple breaks in a location model, the best methods can vary widely according to the particular break and parametrisation. Where we explore the monitoring method (only in the single break case) we find the gains are small, although equally the costs (where there are small breaks) are also small. Other methods can do much better where there are large breaks. The results make it hard to recommend a single method and are sensitive to choice of parameters (eg, the window length or the EWMA decay parameter). However, a version of the EWMA averaging over several decay values is only a good choice in the location model. But the averaging approach, while not always the best, often improves on the full-sample benchmark and rarely comes with a large penalty where there are frequent or small breaks.

When we examine AR(1) models using about 200 US and UK time series, we find that for both countries while the averaged EWMA can occasionally do very well, in general it performs poorly and can perform very badly, consistent with the Monte Carlo results. For the United Kingdom, where there are relatively many breaks identified in the full sample, the best-performing method is forecast averaging, consistent with the Monte Carlo results, although rolling regressions and a low decay EWMA also beat the benchmark. For both countries monitoring brings only a small improvement in mean forecast performance. There is a sense that it is a conservative strategy, as it can deliver improved forecast performance but is unlikely to lead to serious forecast failure relative to the benchmark.

Appendix A: Proofs

A.1 Proof of Theorem 1

We have that

$$\hat{y}_{T+1|T} - y_{T+1} = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i + \frac{1}{T}\sum_{t=1}^{T}\epsilon_t - \sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i - \mathcal{I}(\nu_{T+1} = 1)\,u_{T+1} - \epsilon_{T+1}$$

$$= \frac{1}{T}\sum_{t=1}^{T-1}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i - \frac{1}{T}(T-1)\sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i$$

$$+ \frac{1}{T}\sum_{t=1}^{T}\epsilon_t - \mathcal{I}(\nu_{T+1} = 1)\,u_{T+1} - \epsilon_{T+1}$$

Then,

$$E\left(\hat{y}_{T+1|T} - y_{T+1}\right)^2 = \frac{(T-1)(2T-1)}{6T}p\sigma_u^2 + \frac{(T-1)^2}{T}p\sigma_u^2 - \frac{(T-1)^2}{T}p\sigma_u^2 + \frac{1}{T}\sigma_\epsilon^2 + p\sigma_u^2 + \sigma_\epsilon^2$$

$$= \left(\frac{(T-1)(2T-1)}{6T} + 1\right)p\sigma_u^2 + \frac{(T+1)}{T}\sigma_\epsilon^2 \qquad \textbf{(A-1)}$$

proving the result.

A.2 Proof of Theorem 2

Similarly to Theorem 1,

$$\tilde{y}_{T+1|T} - y_{T+1} = \frac{1}{m}\sum_{t=T-m+1}^{T}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i + \frac{1}{m}\sum_{t=T-m+1}^{T}\epsilon_t$$

$$- \sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i - \mathcal{I}(\nu_{T+1} = 1)\,u_{T+1} - \epsilon_{T+1}$$

$$= \frac{1}{m}\sum_{t=T-m+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i - \frac{1}{m}(m-1)\sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i$$

$$+ \frac{1}{m}\sum_{t=T-m+1}^{T}\epsilon_t - \mathcal{I}(\nu_{T+1} = 1)\,u_{T+1} - \epsilon_{T+1}$$

giving

$$E\left(\tilde{y}_{T+1|T}-y_{T+1}\right)^2=\frac{(T-m)\,(m-1)^2+1/6m\,(2m-1)\,(m-1)}{m^2}p\sigma_u^2+\frac{T\,(m-1)^2}{m^2}p\sigma_u^2$$
$$-\frac{(2T-m)\,(m-1)^2}{m^2}p\sigma_u^2+\frac{\sigma_\epsilon^2}{m}+p\sigma_u^2+\sigma_\epsilon^2$$
$$=\left(\frac{(m-1)\,(2m-1)}{6m}+1\right)p\sigma_u^2+\frac{(m+1)}{m}\sigma_\epsilon^2 \qquad \textbf{(A-2)}$$

A.3 Proof of Theorem 3

We have

$$E\left(\bar{y}_{T+1|T}-y_{T+1}\right)^2=E\left(\frac{1}{T}\sum_{i=1}^{T}\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)^2$$
$$=\frac{1}{T^2}\sum_{i=1}^{T}\sum_{j=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)\left(\tilde{y}^{(j)}_{T+1|T}-y_{T+1}\right)\right]$$
$$=\frac{1}{T^2}\sum_{i=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)^2\right]$$
$$+\frac{1}{T^2}\sum_{i=1,i\neq j}^{T}\sum_{j=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)\left(\tilde{y}^{(j)}_{T+1|T}-y_{T+1}\right)\right]$$

By **(A-2)**,

$$\frac{1}{T^2}\sum_{i=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)^2\right]=\frac{1}{T^2}\sum_{i=1}^{T}\left[\left(\frac{(i-1)\,(2i-1)}{6i}+1\right)p\sigma_u^2+\frac{i+1}{i}\sigma_\epsilon^2\right]$$
$$=p\sigma_u^2\left(\frac{1}{T}+\frac{1}{T^2}\sum_{i=1}^{T}\frac{(i-1)\,(2i-1)}{6i}\right)+\frac{1}{T^2}\sigma_\epsilon^2\sum_{i=1}^{T}\frac{i+1}{i}$$

But,

$$\frac{1}{T^2}\sum_{i=1}^{T}\frac{(i-1)\,(2i-1)}{6i}=1/6+o(1).$$

Further,

$$\frac{1}{T^2}\sum_{i=1}^{T}\frac{i+1}{i}=O(T^{-1}).$$

Next, we need to determine $E\left[\left(\tilde{y}^{(m_1)}_{T+1|T} - y_{T+1}\right)\left(\tilde{y}^{(m_2)}_{T+1|T} - y_{T+1}\right)\right]$ when $m_1 \neq m_2$. Without loss of generality, we assume that $m_1 > m_2$. We have

$$E\left[\left(\frac{1}{m_1}\sum_{t=T-m_1+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i\right)\left(\frac{1}{m_2}\sum_{t=T-m_2+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i\right)\right]$$
$$= \frac{(T-m_2)(m_2-1)^2 + \frac{1}{6}m_2(2m_2-1)(m_2-1)}{m_1 m_2} p\sigma_u^2 + \frac{(T-m_1)(m_1-m_2)(m_2-1)}{m_1 m_2} p\sigma_u^2 +$$
$$\frac{(m_2-1)(m_1-m_2)(m_1-m_2+1)}{2m_1 m_2} p\sigma_u^2$$
$$= \frac{(T-m_2)(m_2-1)^2 + \frac{1}{6}m_2(2m_2-1)(m_2-1)}{m_1 m_2} p\sigma_u^2$$
$$+ \frac{(2T-m_2-m_1+1)(m_1-m_2)(m_2-1)}{2m_1 m_2} p\sigma_u^2$$

$$E\left[\left(\frac{(m_1-1)}{m_1}\sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i\right)\left(\frac{(m_2-1)}{m_2}\sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i\right)\right] = \frac{T(m_1-1)(m_2-1)}{m_1 m_2} p\sigma_u^2$$
$$E\left[\left(\frac{1}{m_1}\sum_{t=T-m_1+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i\right)\left(\frac{(m_2-1)}{m_2}\sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i\right)\right] = \frac{(2T-m_1)(m_2-1)(m_1-1)}{2m_1 m_2} p\sigma_u^2$$
$$E\left[\left(\frac{1}{m_2}\sum_{t=T-m_2+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}(\nu_i = 1)\,u_i\right)\left(\frac{(m_1-1)}{m_1}\sum_{i=1}^{T}\mathcal{I}(\nu_i = 1)\,u_i\right)\right] = \frac{(2T-m_2)(m_2-1)(m_1-1)}{2m_1 m_2} p\sigma_u^2$$

So, overall,

$$E\left[\left(\tilde{y}^{(m_1)}_{T+1|T} - y_{T+1}\right)\left(\tilde{y}^{(m_2)}_{T+1|T} - y_{T+1}\right)\right] = \frac{-m_2^2 + 3m_1 m_2 + 3m_1 + 1}{6m_1} p\sigma_u^2 + \frac{1}{m_2}\sigma_\epsilon^2 + \sigma_\epsilon^2$$

Thus,

$$\frac{2}{T^2}\sum_{m_2=1}^{T}\sum_{m_1=m_2+1}^{T} E\left[\left(\tilde{y}^{(i)}_{T+1|T} - y_{T+1}\right)\left(\tilde{y}^{(j)}_{T+1|T} - y_{T+1}\right)\right] \tag{A-3}$$
$$= \left(\frac{19}{36} - \frac{35}{108T} + \frac{7T}{54} - \frac{1}{3}\sum_{m_2=1}^{T}\frac{1}{m_2}\right) p\sigma_u^2 + \frac{\sigma_\epsilon^2}{m_2} + \sigma_\epsilon^2$$

Overall, then,

$$E\left(\bar{y}_{T+1|T} - y_{T+1}\right)^2 = \frac{7T}{54} p\sigma_u^2 + o(T)$$

A.4 Proof of Theorem 4

For Theorem 4, we consider the EWMA estimator. This is given by

$$\breve{y}_{T+1|T} = \sum_{t=1}^{T}\lambda(1-\lambda)^{T-t}\, y_t$$

We have

$$\breve{y}_{T+1|T} - y_{T+1} = \lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} \left(\sum_{i=1}^{t} \mathcal{I}(\nu_i = 1) u_i \right) + \lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} \epsilon_t - \sum_{i=1}^{T} \mathcal{I}(\nu_i = 1) u_i$$

$$- \mathcal{I}(\nu_{T+1} = 1) u_{T+1} - \epsilon_{T+1} = \lambda \sum_{t=1}^{T-1} (1-\lambda)^{T-t} \left(\sum_{i=1}^{t} \mathcal{I}(\nu_i = 1) u_i \right)$$

$$- (1-\lambda) \sum_{i=1}^{T} \mathcal{I}(\nu_i = 1) u_i + \lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} \epsilon_t - \mathcal{I}(\nu_{T+1} = 1) u_{T+1} - \epsilon_{T+1} \tag{A-4}$$

Then,

$$E \left[\lambda \sum_{t=1}^{T-1} (1-\lambda)^{T-t} \left(\sum_{i=1}^{t} \mathcal{I}(\nu_i = 1) u_i \right) \right]^2 = \lambda^2 p \sigma_u^2 \sum_{t=1}^{T-1} \left(\sum_{i=1}^{t} (1-\lambda)^i \right)^2$$

But,

$$\sum_{t=1}^{T-1} \left(\sum_{i=1}^{t} (1-\lambda)^i \right)^2 = (1-\lambda)^2 \sum_{t=1}^{T-1} \left(\sum_{i=0}^{t-1} (1-\lambda)^i \right)^2 = (1-\lambda)^2 \sum_{t=1}^{T-1} \left(\frac{1-(1-\lambda)^{t-1}}{\lambda} \right)^2$$

$$= \left(\frac{1-\lambda}{\lambda} \right)^2 \sum_{t=1}^{T-1} \left(1 - 2(1-\lambda)^{t-1} + (1-\lambda)^{2(t-1)} \right)$$

$$= \left(\frac{1-\lambda}{\lambda} \right)^2 \left(T - 1 - \frac{2-2(1-\lambda)^{T-1}}{\lambda} + \frac{1-(1-\lambda)^{2(T-1)}}{1-(1-\lambda)^2} \right)$$

So, overall

$$E \left[\lambda \sum_{t=1}^{T-1} (1-\lambda)^{T-t} \left(\sum_{i=1}^{t} \mathcal{I}(\nu_i = 1) u_i \right) \right]^2 \tag{A-5}$$

$$= (1-\lambda)^2 p \sigma_u^2 \left(T - 1 - \frac{2-2(1-\lambda)^{T-1}}{\lambda} + \frac{1-(1-\lambda)^{2(T-1)}}{1-(1-\lambda)^2} \right) = (1-\lambda)^2 T p \sigma_u^2 + O(1).$$

Next,

$$E \left[(1-\lambda) \sum_{i=1}^{T} \mathcal{I}(\nu_i = 1) u_i \right]^2 = (1-\lambda)^2 T p \sigma_u^2. \tag{A-6}$$

Next,

$$E \left(\lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} \epsilon_t \right)^2 = \lambda^2 E \left(\sum_{t=1}^{T} (1-\lambda)^{T-t} \epsilon_t \right)^2 = \lambda^2 \sigma_\epsilon^2 \sum_{t=1}^{T} (1-\lambda)^{2(T-t)} \tag{A-7}$$

$$= \lambda^2 \sigma_\epsilon^2 \frac{(1-\lambda)^{2T} - 1}{(1-\lambda)^2 - 1} = O(1).$$

Next,

$$E\left(\left(1-\lambda\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i\right)\left(\lambda\sum_{t=1}^{T-1}\left(1-\lambda\right)^{T-t}\left(\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\right)$$
$$=\lambda(1-\lambda)\sum_{t=1}^{T-1}t\left(1-\lambda\right)^{T-t}p\sigma_u^2$$

But,

$$\lambda(1-\lambda)\sum_{t=1}^{T-1}t\left(1-\lambda\right)^{T-t}=\frac{\left(1-\lambda\right)\left(\left(1-\lambda\right)^{T+1}+\left(T\lambda-1\right)\left(1-\lambda\right)\right)}{\lambda} \qquad \textbf{(A-8)}$$
$$=(1-\lambda)^2T+O\left(1\right)$$

Overall, combining the results in **(A-5)**-**(A-8)** implies that the term of the highest order when squaring and taking expectations in **(A-4)** is given by $2\left((1-\lambda)^2-(1-\lambda)^2\right)Tp\sigma_u^2=0$. The other terms are all $O(1)$. So, overall we conclude that

$$\lim_{T\to\infty}E\left(\breve{y}_{T+1|T}-y_{T+1}\right)^2=O(1)$$

Appendix B: Detailed UK results

Table J: Relative RMSFE results for UK: first period (1992 Q1-1999 Q4)

	M40	M60	R40	R60	AV	EWMAA	EWMAL
Consumer Price Index	0.988	0.992	0.820	**0.969**	1.005	0.928	0.938
Private sector output annual growth	1.000	1.000	1.015	0.965	1.027	1.090	1.026
Private sector output quarterly growth	1.000	1.000	0.922	0.909	0.839	0.858	0.856
CBI survey: Employment intentions, next 3 months	1.000	1.000	0.865	0.963	0.881	0.981	0.898
UK FTSE All-Share dividend yield	1.000	1.000	1.019	0.998	1.024	1.198	1.025
Bank of England REPO rate	1.000	1.000	1.201	**1.208**	1.037	1.092	1.024
GfK index score	1.000	1.000	1.091	1.029	1.100	1.543	1.120
Nationwide House Price Index	1.000	1.000	**1.073**	0.997	1.017	1.068	1.030
Stock of net corporate debt	1.000	1.000	1.038	1.008	1.031	1.105	1.021
Constant market price imported consumption	1.000	1.000	1.030	1.043	1.012	1.036	1.016
Government procurement of goods and services (including investment)	1.000	1.000	0.990	1.009	1.034	1.240	1.041
Real post-tax labour income, constant prices	1.000	1.000	1.101	1.006	1.000	1.151	1.025
Stock of notes and coin	0.768	0.834	0.930	0.845	0.783	0.781	0.851
Private sector productivity, hours-based measure	1.000	1.000	1.135	1.049	1.059	1.199	1.053
Long-term foreign nominal rate of interest.	1.000	1.000	1.036	1.012	1.024	1.371	1.011
Corporate bond real interest rate	1.000	1.000	1.158	**1.172**	1.001	1.070	0.994
Unit labour costs (private sector measure)	**0.919**	**0.944**	**0.814**	**0.867**	**0.728**	0.780	0.783
Nominal value of the firm	1.000	1.000	0.934	1.032	0.950	1.349	0.924
Tax revenue from corporation tax, current prices	1.000	1.000	1.003	0.896	0.738	0.940	0.821
Total tax payments of household sector	1.000	1.000	0.957	0.991	0.963	1.530	0.962
Real exchange rate	1.000	1.000	0.980	**1.061**	1.042	2.228	1.025
Average private sector weekly hours	1.000	1.000	0.804	0.856	0.840	1.088	0.817
Average whole-economy average hours	1.000	1.000	0.777	0.836	0.795	0.883	0.763
Total private sector compensation spending	**0.942**	**0.960**	0.948	**0.947**	**0.866**	0.841	0.933
Total government compensation spending	0.954	**0.967**	**0.815**	0.863	0.832	0.936	**0.827**
Employment in heads	0.982	0.984	1.032	1.015	1.003	1.544	1.011
Private sector employment in heads	1.000	1.000	0.981	0.989	0.957	1.052	1.015
Nominal wages per worker	0.936	**0.954**	1.045	0.954	0.956	0.939	0.958
Nominal private sector wages per worker	**0.935**	**0.954**	0.961	0.940	0.908	0.860	0.918
Sectoral M4	1.000	1.000	0.896	1.015	0.812	0.815	0.847
Sectoral M4 Lending	1.000	1.000	0.960	0.972	0.871	0.934	0.826
M4 lending (monetary financial institutions' sterling net lending to private sector)	0.924	0.941	0.744	0.930	**0.672**	0.694	0.729
Net lending to private non-financial corporations	1.000	1.000	1.099	1.036	0.917	0.828	0.953
M4 liabilities to private non-financial corporations	1.000	1.000	0.925	1.001	0.891	1.049	0.901
M4 liabilities to other financial corporations	1.000	1.000	0.962	0.957	0.909	1.192	0.891
Net lending to household sector	0.607	0.692	0.974	0.996	0.579	1.415	0.996
Gross Domestic Product	1.000	1.000	**0.617**	**0.792**	**0.597**	0.601	**0.626**
Gross National Income	0.904	0.934	0.830	0.866	0.763	0.914	**0.761**
Changes in inventories including alignment adjustment	1.000	1.000	1.005	1.010	1.029	**1.225**	1.045
IOP: All production industries	1.000	1.000	0.991	1.047	1.009	1.137	1.026
IOP: Mining & quarrying	1.000	1.000	1.193	0.990	1.097	1.162	1.091
IOP: Manufacturing	1.000	1.000	0.979	1.110	1.015	1.168	1.034
IOP: Manufacturing of food, drink & tobacco	1.000	1.000	1.042	1.009	**1.047**	**1.398**	**1.059**
IOP: Manufacturing of textile & textile products	1.000	1.000	0.977	1.042	1.029	1.158	1.031
IOP: Manufacturing of leather & leather products	1.000	1.000	**0.866**	0.929	0.882	0.905	0.889
IOP: Manufacturing of wood & wood products	1.000	1.000	1.003	1.003	1.010	1.255	0.999
IOP: Pulp/paper/printing/publishing industries	1.000	1.000	1.132	**1.105**	1.083	1.263	**1.091**

Notes: Bold entries indicate rejection for the Diebold-Mariano test.

Table K: Relative RMSFE results for UK: first period (1992 Q1-1999 Q4)(cont.)

	M40	M60	R40	R60	AV	EWMA	EWMAL
IOP: Manufacturing coke/petroleum prod/nuclear fuels	1.000	1.000	1.024	1.029	1.034	1.331	1.062
IOP: Manufacturing of rubber & plastic products	0.972	0.980	0.975	1.117	1.004	1.114	1.016
IOP: Manufacturing of non-metallic mineral products	1.000	1.000	1.058	1.004	1.074	**1.411**	1.077
IOP: Manufacturing of basic metals & fabricated prod	1.000	1.000	0.908	0.915	0.953	1.373	0.948
IOP: Manufacturing of machinery & equipment	1.000	1.000	1.030	1.046	1.036	**1.408**	1.044
IOP: Manufacturing of electrical & optical equipment	1.000	1.000	0.909	0.932	0.929	0.989	0.921
IOP: Manufacturing of transport equipment	1.000	1.000	1.017	0.967	0.987	1.171	1.010
IOP: Extraction of oil & gas	0.982	0.988	1.278	0.996	1.144	1.177	1.105
UK Total Wages & Salaries	**0.940**	**0.956**	1.041	**0.934**	0.907	0.868	0.959
Business Investment (excluding exceptional transfer)	1.000	1.000	0.906	0.950	0.947	1.054	0.941
CAPEX: Total excluding exceptional transfer	**1.020**	**1.013**	0.982	0.989	0.970	1.223	0.973
Output Index: Distribution, hotels & catering; repairs	1.000	1.000	1.266	1.208	1.179	**1.591**	1.169
Output Index: Transport storage & communication	1.000	1.000	**0.740**	**0.806**	**0.716**	0.850	**0.698**
Output Index: Total	1.000	1.000	**0.714**	**0.804**	**0.678**	0.660	**0.692**
Total Gross Fixed Capital Formation	1.000	1.000	0.946	0.970	0.962	1.017	0.958
Total adjustment to basic prices (General Government + Rest of the World)	1.000	1.000	0.954	0.969	0.914	1.096	0.894
Gross Domestic Product at market prices	**0.893**	**0.928**	0.831	**0.883**	**0.701**	0.705	**0.712**
Gross Value Added at factor cost	1.000	1.000	0.728	0.793	**0.669**	0.669	**0.695**
Money stock M4 (end period)	**0.941**	**0.960**	0.855	1.008	**0.744**	0.783	0.783
Notes & coin in circulation outside Bank of England	0.992	0.991	0.998	1.013	1.008	**1.420**	1.041
Money Stock: Retail Deposits and Cash in M4	1.000	1.000	**0.118**	0.949	**0.155**	**0.010**	**0.096**
Total Government benefits paid to household sector	**0.901**	**0.932**	**0.777**	**0.856**	**0.789**	0.794	**0.779**
General Government: Final consumption expenditure	1.000	1.000	0.913	0.996	0.945	0.990	0.934
Household final consumption expenditure: National concept	1.000	1.000	**1.426**	0.979	1.169	1.302	1.159
Household final consumption expenditure: National concept IDEF SA 2000=100	**0.929**	**0.953**	**0.729**	**0.904**	**0.755**	**0.640**	**0.694**
Durable goods	1.000	1.000	**1.525**	1.235	1.265	**1.627**	1.229
Semi-durable goods	1.000	1.000	**0.921**	0.941	0.955	1.130	0.951
Durable goods, index	0.947	0.963	**0.781**	**0.880**	**0.819**	0.928	**0.728**
Non-durable goods, index	**0.919**	**0.946**	**0.736**	**0.890**	**0.742**	**0.626**	**0.752**
Services, index	**0.903**	**0.934**	**0.797**	**0.882**	**0.801**	**0.741**	**0.804**
Semi-durable goods, index	**0.805**	**0.868**	**0.702**	**0.852**	**0.591**	**0.312**	**0.434**
Claimant count rate	1.000	1.000	0.968	1.001	1.032	1.159	1.028
Whole economy (incl. bonus)	**0.866**	**0.907**	0.977	**0.902**	0.854	**0.637**	**0.704**
Whole economy (incl. bonus): % change 3-month average	1.000	1.000	0.901	0.992	0.928	1.122	0.961
In employment, aged 16+	1.000	1.000	1.000	0.994	0.976	1.059	1.019
Unemployed, aged 16+	0.997	0.998	**0.962**	0.966	0.904	0.902	0.955
Economically active	1.000	1.000	1.016	0.996	0.932	0.982	0.977
Population aged 16+	1.050	1.031	**1.078**	1.035	1.016	1.022	0.965
Unemployment rate, aged 16+	1.000	1.000	**0.956**	0.974	0.924	0.964	0.963
Total actual weekly hours worked	1.000	1.000	0.902	0.953	0.772	0.606	0.827
PPI: Output of manufactured products	1.000	1.000	1.008	0.989	1.036	**1.359**	1.043
PPI: NSO: All Manufacturing excl duty	1.000	1.000	1.049	0.975	1.010	1.312	1.012
Imports: Total trade in goods and services excluding MTIC fraud, Current prices	1.000	1.000	1.043	1.022	1.043	1.333	1.047
Exports: Total trade in goods and services excluding MTIC fraud	1.000	1.000	**0.844**	0.934	**0.845**	0.866	**0.838**
Imports: Total trade in goods and services excluding MTIC fraud, Constant prices	1.000	1.000	0.996	0.969	0.935	0.983	0.957
Balance of Payments: IM: Finished manufactures	0.984	0.989	**0.849**	**0.906**	0.877	0.902	0.861
Balance of Payments: Total Trade in Goods & Services	1.000	1.000	1.144	1.084	1.039	1.094	1.056

Table L: Relative RMSFE results for UK: second period (2000 Q1-2008 Q2)

	M40	M60	R40	R60	AV	EWMAA	EWMAL
Consumer Price Index	1.027	1.016	1.162	1.115	1.052	1.124	1.078
Private sector output annual growth	1.000	1.000	1.082	1.101	1.072	1.242	1.089
Private sector output quarterly growth	1.000	1.000	1.230	**1.458**	1.150	1.230	1.177
CBI survey: Employment intentions, next 3 months	1.000	1.000	1.129	1.129	1.074	1.398	1.097
UK FTSE All-Share dividend yield	1.000	1.000	1.031	0.973	0.993	1.145	0.996
Bank of England REPO rate	1.000	1.000	0.856	0.980	0.969	1.199	0.965
GfK index score	1.000	1.000	1.066	0.994	1.029	1.143	1.027
Nationwide House Price Index	1.040	1.025	1.088	1.048	1.027	0.845	1.032
Stock of net corporate debt	1.000	1.000	0.924	0.979	0.964	1.362	0.961
Constant market price imported consumption	1.000	1.000	0.991	1.036	0.976	1.295	1.000
Government procurement of goods and services (including investment)	1.000	1.000	1.001	0.975	0.968	1.326	0.968
Real post-tax labour income, constant prices	1.000	1.000	0.934	0.868	0.940	1.160	0.951
Stock of notes and coin	0.983	0.988	1.212	1.025	0.991	1.054	1.039
Private sector productivity, hours-based measure	1.000	1.000	1.033	**1.120**	1.057	**1.355**	1.058
Long-term foreign nominal rate of interest.	1.000	1.000	0.930	0.995	1.001	1.556	1.002
Corporate bond real interest rate	1.000	1.000	0.796	0.915	0.911	1.096	0.907
Unit labour costs (private sector measure)	**0.880**	**0.918**	**0.553**	**0.613**	**0.634**	0.682	**0.621**
Nominal value of the firm	1.000	1.000	**0.754**	**0.788**	**0.780**	0.776	**0.748**
Tax revenue from corporation tax, current prices	1.000	1.000	**0.664**	0.750	**0.690**	0.705	0.708
Total tax payments of household sector	1.000	1.000	0.849	0.869	0.919	1.175	0.923
Real exchange rate	1.000	1.000	0.986	0.947	0.952	1.262	0.957
Average private sector weekly hours	1.000	1.000	0.889	0.881	0.905	1.213	0.914
Average whole-economy average hours	1.000	1.000	0.847	0.839	**0.847**	1.080	0.839
Total private sector compensation spending	0.900	0.933	**0.733**	**0.702**	**0.755**	0.731	**0.741**
Total government compensation spending	**0.953**	**0.968**	1.063	0.897	0.921	1.223	0.943
Employment in heads	1.000	1.000	1.091	1.033	0.995	0.893	1.018
Private sector employment in heads	1.000	1.000	**0.607**	0.914	**0.658**	**0.464**	**0.696**
Nominal wages per worker	**0.748**	**0.827**	**0.571**	**0.639**	**0.625**	**0.535**	**0.601**
Nominal private sector wages per worker	0.778	0.846	**0.581**	**0.634**	**0.628**	0.700	**0.605**
Sectoral M4	1.000	1.000	1.028	1.053	0.950	1.095	0.975
Sectoral M4 Lending	1.000	1.000	1.133	**1.196**	1.029	1.202	1.045
M4 lending (monetary financial institutions' sterling net lending to private sector)	0.927	0.949	0.907	1.022	0.892	0.961	0.934
Net lending to private non-financial corporations	1.000	1.000	1.091	1.187	1.030	1.553	1.042
M4 liabilities to private non-financial corporations	1.000	1.000	0.835	0.983	0.911	1.015	0.923
M4 liabilities to other financial corporations	1.000	1.000	0.970	1.091	0.925	0.898	0.936
Net lending to household sector	**0.897**	**0.931**	0.795	**0.690**	**0.765**	0.781	**0.784**
Gross Domestic Product	1.000	1.000	1.422	**1.512**	1.301	**1.622**	1.339
Gross National Income	**0.876**	**0.913**	**0.533**	**0.612**	**0.592**	0.683	**0.591**
Changes in inventories including alignment adjustment	1.000	1.000	1.054	1.000	1.027	**1.382**	1.063
IOP: All production industries	1.000	1.000	0.925	0.996	0.952	0.979	0.930
IOP: Mining & quarrying	1.000	1.000	0.986	0.992	0.908	1.029	0.874
IOP: Manufacturing	1.000	1.000	0.942	1.068	1.012	1.189	0.999
IOP: Manufacturing of food, drink & tobacco	1.000	1.000	1.017	0.997	0.990	1.125	0.995
IOP: Manufacturing of textile & textile products	1.000	1.000	1.028	**1.044**	1.019	1.142	1.046
IOP: Manufacturing of leather & leather products	1.000	1.000	1.118	1.042	1.038	1.169	1.051
IOP: Manufacturing of wood & wood products	1.000	1.000	0.922	0.943	0.966	1.139	0.974
IOP: Pulp/paper/printing/publishing industries	1.000	1.000	0.886	0.922	0.938	1.046	0.925

Table M: Relative RMSFE results for UK: second period (2000 Q1-2008 Q2)(cont.)

	M40	M60	R40	R60	AV	EWMAA	EWMAL
IOP: Manufacturing coke/petroleum prod/nuclear fuels	1.000	1.000	1.042	1.030	1.056	**1.531**	1.094
IOP: Manufacturing of rubber & plastic products	1.000	1.000	0.965	0.975	0.916	1.075	0.937
IOP: Manufacturing of non-metallic mineral products	1.000	1.000	0.952	1.085	1.011	1.285	0.999
IOP: Manufacturing of basic metals & fabricated prod	1.000	1.000	1.108	1.514	1.157	1.425	1.120
IOP: Manufacturing of machinery & equipment	1.000	1.000	0.916	0.903	**0.878**	0.813	0.869
IOP: Manufacturing of electrical & optical equipment	1.000	1.000	0.946	0.958	0.948	1.191	0.973
IOP: Manufacturing of transport equipment	1.000	1.000	1.031	1.087	1.059	**1.388**	1.096
IOP: Extraction of oil & gas	**0.881**	**0.918**	0.862	0.869	**0.766**	0.818	**0.730**
UK Total Wages & Salaries	0.869	0.904	0.831	0.813	0.822	0.852	0.849
Business Investment (excluding exceptional transfer)	1.000	1.000	1.125	**1.185**	1.069	1.130	1.099
CAPEX: Total excluding exceptional transfer	1.005	0.998	0.962	1.010	0.907	0.892	0.918
Output Index: Distribution, hotels & catering; repairs	1.000	1.000	1.008	1.039	0.986	1.065	1.007
Output Index: Transport storage & communication	1.000	0.999	**1.511**	**1.293**	**1.258**	1.317	**1.350**
Output Index: Total	1.010	1.005	1.416	1.307	1.165	1.530	1.246
Total Gross Fixed Capital Formation	1.000	1.000	1.094	1.061	1.060	**1.453**	1.102
Total adjustment to basic prices (General Government + Rest of the World)	**0.910**	**0.938**	**0.635**	**0.774**	**0.720**	0.892	**0.663**
Gross Domestic Product at market prices	**0.840**	**0.891**	**0.486**	**0.571**	**0.592**	0.618	**0.585**
Gross Value Added at factor cost	1.000	1.000	1.288	**1.384**	1.238	**1.603**	1.267
Money stock M4 (end period)	0.985	0.988	1.162	1.275	0.927	0.861	0.948
Notes & coin in circulation outside Bank of England	0.967	0.977	0.910	0.900	0.843	0.875	0.859
Money Stock: Retail Deposits and Cash in M4	1.000	1.000	**0.006**	**0.005**	**0.047**	**0.005**	**0.007**
Total Government benefits paid to household sector	0.900	0.931	0.814	0.873	0.842	1.126	0.850
General Government: Final consumption expenditure	1.000	1.000	0.684	**0.768**	**0.705**	0.713	**0.637**
Household final consumption expenditure: National concept	1.000	1.000	**1.280**	1.136	1.067	1.129	1.070
Household final consumption expenditure: National concept IDEF SA 2000=100	**0.848**	**0.895**	**0.552**	**0.655**	**0.579**	**0.514**	**0.651**
Durable goods	1.000	1.000	1.105	1.087	0.986	1.109	1.006
Semi-durable goods	1.000	1.000	1.069	1.060	1.012	1.203	1.028
Durable goods, index	**0.905**	**0.936**	**0.648**	**0.766**	**0.699**	0.714	**0.697**
Non-durable goods, index	1.023	1.009	1.103	1.097	0.938	1.044	0.993
Services, index	**0.902**	**0.934**	**0.541**	**0.696**	**0.579**	**0.579**	**0.606**
Semi-durable goods, index	**0.944**	**0.961**	0.837	0.892	0.733	0.583	0.826
Claimant count rate	0.951	0.966	1.032	1.010	0.944	0.978	0.996
Whole economy (incl. bonus)	0.619	0.737	0.447	0.513	0.540	0.319	0.559
Whole economy (incl. bonus): % change 3-month average	1.000	1.000	0.626	0.646	0.633	0.466	0.567
In employment, aged 16+	1.000	1.000	**0.532**	**0.917**	**0.603**	**0.375**	**0.665**
Unemployed, aged 16+	0.921	0.943	0.805	0.887	0.805	0.731	0.796
Economically active	1.000	1.000	0.810	0.959	**0.797**	0.743	**0.791**
Population aged 16+	0.998	0.998	1.042	1.045	0.993	1.029	1.001
Unemployment rate, aged 16+	0.889	0.922	0.803	0.909	0.790	**0.641**	0.803
Total actual weekly hours worked	1.000	1.000	**0.467**	**0.650**	**0.566**	**0.507**	**0.477**
PPI: Output of manufactured products	1.000	1.000	0.996	0.995	0.994	1.164	0.999
PPI: NSO: All Manufacturing excl duty	1.000	1.000	0.956	0.966	0.962	1.162	0.976
Imports: Total trade in goods and services excluding MTIC fraud, Current prices	1.000	1.000	1.106	1.046	1.019	1.180	1.041
Exports: Total trade in goods and services excluding MTIC fraud	1.000	1.000	1.123	1.041	1.064	1.355	1.111
Imports: Total trade in goods and services excluding MTIC fraud, Constant prices	1.000	1.000	1.164	1.052	1.071	**1.388**	1.093
Balance of Payments: IM: Finished manufactures	0.995	0.994	0.870	0.819	0.799	1.058	0.786
Balance of Payments: Total Trade in Goods & Services	1.000	1.000	1.051	1.006	1.007	**1.335**	1.042

Appendix C: Detailed US results

Table N: Relative RMSFE results for US: first period (1975 Q1-1986 Q2)

	M40	M60	R40	R60	AV	EWMAA	EWMAL
Industrial Production: Final Products (Market Group)	1.000	1.000	1.065	0.998	1.055	1.249	1.055
Industrial Production: Consumer Goods	1.000	1.000	1.000	0.978	1.031	1.414	1.01
Industrial Production: Durable Consumer Goods	1.000	1.000	1.022	0.987	1.029	1.478	1.02
Industrial Production: Non-durable Consumer Goods	1.000	1.000	0.953	0.972	0.965	1.276	0.958
Industrial Production: Business Equipment	1.000	1.000	**1.067**	0.998	1.047	1.135	1.053
Industrial Production: Materials	1.000	1.000	1.026	0.992	1.038	1.160	1.057
Industrial Production: Durable Materials	1.000	1.000	1.044	1.003	1.068	1.295	1.096
Industrial Production: Non-durable Materials	1.000	1.000	1.002	0.985	1.024	1.284	1.032
Industrial Production: Manufacturing (SIC)	1.000	1.000	1.040	0.997	1.043	1.207	1.054
Industrial Production Index	1.000	1.000	1.031	0.995	1.031	1.137	1.042
ISM Manufacturing: PMI Composite Index	1.000	1.000	1.078	1.030	1.116	1.536	1.192
Real Disposable Personal Income	1.000	1.000	1.008	1.008	0.943	0.988	0.974
Real Disposable Personal Income Less Transfer Payments	1.000	1.000	1.030	1.001	1.030	1.132	1.044
Civilian Labor Force: Employed, Total	1.020	1.012	1.011	0.998	1.028	1.349	1.038
Unemployment Rate: All Workers	**1.035**	**1.022**	1.124	0.978	1.152	1.197	1.173
Unemployment By Duration: Average Duration In Weeks	1.075	1.036	1.048	0.964	0.999	0.977	1.05
Civilians Unemployed - Less Than 5 Weeks	1.000	1.000	1.036	1.032	**1.071**	**1.391**	1.083
Civilian Unemployed - 5-14 Weeks	1.000	1.000	1.030	1.011	**1.078**	**1.441**	**1.079**
Civilians Unemployed - 15 Weeks & Over	1.000	1.000	1.057	0.999	1.130	**1.400**	1.159
Civilians Unemployed - 15-26 Weeks	1.000	1.000	1.029	1.004	**1.080**	**1.305**	1.106
Total non-farm employment	1.000	1.000	1.040	1.011	**1.071**	1.212	**1.073**
Total private employment	1.027	1.017	**1.036**	1.007	**1.068**	1.273	**1.068**
Goods-producing employment	1.000	1.000	1.033	1.003	**1.068**	**1.257**	**1.071**
Natural resources and mining employment	1.171	1.106	1.113	1.057	1.291	2.594	1.44
Construction employment	1.000	1.000	1.071	1.001	1.109	1.293	1.149
Manufacturing employment	1.000	1.000	1.035	1.005	1.070	1.308	1.074
Surable goods manufacturing employment	1.000	1.000	1.027	1.005	1.071	1.314	1.08
Non-durable goods manufacturing employment	1.000	1.000	1.037	1.001	1.081	1.396	1.078
Service-providing employment	1.014	1.007	**1.059**	1.015	1.067	1.249	1.066
Trade, transportation and utilities employment	1.004	1.001	1.017	1.010	1.047	1.260	1.055
Retail trade employment	1.006	1.002	1.019	1.047	1.040	1.271	1.044
Wholesale trade employment	0.996	0.994	1.038	0.998	1.033	1.151	1.031
Financial activities employment	1.018	1.009	0.963	1.017	1.047	1.192	1.048
Private service-providing employment	1.010	1.006	1.033	1.009	**1.051**	**1.222**	1.052
Government employment	0.995	0.996	0.929	1.015	0.959	1.045	0.974
Manufacturing average weekly hours of production workers	1.000	1.000	1.040	1.012	**1.050**	1.279	**1.068**
Manufacturing average weekly overtime of production workers	1.000	1.000	1.061	1.008	1.081	1.467	1.07
Housing Starts: Total: New Privately Owned Housing Units Started	1.000	1.000	1.070	1.021	**1.057**	1.220	**1.076**
Housing Starts in Northeast Census Region	1.000	1.000	1.077	1.006	1.090	1.586	1.126
Housing Starts in Midwest Census Region	1.000	1.000	1.033	1.016	1.038	1.213	1.071
Housing Starts in South Census Region	1.000	1.000	1.050	1.018	**1.054**	**1.252**	**1.073**
Housing Starts in West Census Region	1.000	1.000	1.022	1.021	1.054	1.213	1.076
New Private Housing Units Authorized by Building Permit	1.000	1.000	1.047	1.007	1.024	1.369	1.058
New Orders, Consumer Goods & Materials	1.000	1.000	1.057	1.014	1.062	1.356	1.102
New Orders, Non-defense Capital Goods	1.000	1.000	1.050	0.982	1.027	1.314	1.035
Dow Jones Industrial Average	1.000	1.000	1.016	1.011	1.027	1.216	1.018
Japanese Yen-United States Dollar Exchange Rate	1.025	1.016	1.002	0.999	1.033	1.165	1.049
Canadian Dollar-United States Dollar Exchange Rate	0.963	0.973	0.989	1.015	0.985	1.135	0.993
Effective Federal Funds Rate	1.025	1.011	1.014	1.001	1.008	1.087	1.032

Table O: Relative RMSFE results for US: first period (1975 Q1-1986 Q2) (cont.)

	M40	M60	R40	R60	AV	EWMAA	EWMAL
3-Month Treasury Bill: Secondary Market Rate	1.028	1.013	1.037	1.020	1.034	1.137	1.067
6-Month Treasury Bill: Secondary Market Rate	1.036	1.018	**1.048**	1.025	1.044	1.162	1.075
1-Year Treasury Constant Maturity Rate	1.037	1.019	**1.059**	1.025	1.051	1.171	1.074
5-Year Treasury Constant Maturity Rate	1.048	1.029	1.097	1.027	1.091	1.237	1.08
10-Year Treasury Constant Maturity Rate	1.043	1.026	1.085	1.016	1.090	1.246	1.062
Moody's Corporate AAA Yield	1.059	1.036	1.135	1.012	1.124	1.353	1.069
Moody's Corporate BAA Yield	1.029	1.017	1.067	1.010	1.038	1.082	1.007
Spread 3M-FF	1.090	1.049	0.984	0.977	1.081	1.451	1.07
Spread 6M-FF	1.054	1.028	0.999	0.984	1.050	1.334	1.074
Spread 1Y-FF	1.049	1.027	1.027	0.999	1.056	1.272	1.063
Spread 5Y-FF	1.039	1.021	1.023	1.001	1.021	1.109	1.035
Spread 10Y-FF	1.044	1.024	1.022	1.001	1.020	1.122	1.035
Spread AAA-FF	0.997	0.998	1.026	1.000	1.020	1.141	1.04
Spread BAA-FF	1.000	1.000	1.019	0.995	1.030	**1.205**	1.035
M1 Money Stock	0.956	0.965	0.906	0.955	0.897	1.002	0.919
M2 Money Stock	0.872	0.905	0.950	1.020	0.911	1.108	0.939
Money Supply - M2	0.997	0.998	0.961	1.027	0.909	0.923	0.939
Monetary base, adj for reserve requirement changes	0.952	0.963	0.933	0.948	0.889	1.104	0.907
Total Reserves of Depository Institutions	1.000	1.000	0.943	0.937	0.950	1.141	0.933
Non-Borrowed Reserves of Depository Institutions	1.042	1.025	1.022	1.019	1.044	1.203	1.031
Consumer Credit Outstanding - Non-revolving	1.000	1.000	**1.061**	1.020	1.004	0.871	1.041
Commercial & Industrial Loans Outstanding	1.000	1.000	1.016	0.997	1.031	1.180	1.037
PPI: Finished Goods	1.040	1.024	1.131	1.077	0.993	0.904	1.003
PPI: Finished Consumer Goods	1.039	1.023	1.107	1.074	0.986	0.892	0.995
PPI: Intermediate Mat. Supplies & Components	1.032	1.019	1.085	**1.082**	1.001	1.039	0.995
PPI: Crude Materials	0.986	0.991	1.072	**1.080**	1.041	1.192	1.015
Consumer Price Index For All Urban Consumers: All Items	1.041	1.022	1.046	1.003	1.009	1.108	1.022
CPI-U: Apparel	1.012	1.003	0.996	1.002	0.949	1.047	0.963
CPI-U: Transportation	1.039	1.019	1.027	1.011	0.988	1.053	1.008
CPI-U: Medical care	1.020	1.005	1.120	1.000	0.936	1.186	0.946
CPI-U: Commodities	1.049	1.029	1.045	1.019	1.010	1.083	1.027
CPI-U: Durables	1.016	0.984	1.081	0.995	0.941	0.950	0.973
CPI-U: All Items Less Food	1.034	1.016	1.037	0.998	1.002	1.048	1.021
CPI-U: All Items Less Shelter	1.043	1.023	1.067	1.023	1.000	1.066	1.016
CPI-U: All Items Less Medical Care	1.043	1.023	1.045	1.007	1.005	1.088	1.021
Spot Market Price Index: BLS & CRB: all commodities	1.000	1.000	1.055	1.027	1.015	1.147	0.998
Construction: average hourly earnings of production workers	0.969	0.974	0.999	1.041	1.004	1.067	0.944
Manufacturing: average hourly earnings of production workers	0.982	0.985	1.046	**1.355**	0.925	0.792	0.842
U. of Michigan Index of Consumer Expectations	1.000	1.000	1.011	1.015	1.031	1.333	1.051
Dow Jones Industrials Total Return Index	1.000	1.000	0.995	1.012	1.023	1.212	1.013
S&P 500 Energy Total Return Index	1.000	1.000	1.051	1.034	1.054	1.524	1.077
S&P 500 Finance Total Return Index	1.000	1.000	1.029	0.994	**1.060**	**1.514**	**1.093**
S&P 500 Total Return Index	1.000	1.000	0.993	1.014	1.035	**1.266**	1.033
S&P 500 Transportation Total Return Index	1.000	1.000	1.002	1.006	1.030	1.317	1.046
S&P 500 Utilities Total Return Index	1.000	1.000	0.970	1.001	0.970	0.954	0.946
Dow Jones Corporate Bond Yield	1.000	1.000	**1.061**	0.997	1.079	1.363	1.104
USA Prime Rate	1.000	1.000	1.031	1.002	**1.067**	1.493	**1.090**
West Texas Intermediate Oil Price (US$/Barrel)	1.000	1.000	1.005	1.018	0.988	1.000	1.001

Table P: Relative RMSFE results for US: second period (1986 Q3-1997 Q4)

	M40	M60	R40	R60	AV	EWMAA	EWMAL
Industrial Production: Final Products (Market Group)	1.000	1.000	1.077	1.024	1.012	1.174	1.046
Industrial Production: Consumer Goods	1.000	1.000	1.034	1.029	1.036	1.385	1.059
Industrial Production: Durable Consumer Goods	1.000	1.000	1.060	1.046	1.037	1.235	1.067
Industrial Production: Non-durable Consumer Goods	1.000	1.000	0.854	0.886	0.911	1.173	0.908
Industrial Production: Business Equipment	1.000	1.000	1.055	0.996	1.011	1.101	1.02
Industrial Production: Materials	1.000	1.000	1.165	1.083	1.087	1.314	1.073
Industrial Production: Durable Materials	1.000	1.000	0.999	1.003	1.010	1.174	0.966
Industrial Production: Non-durable Materials	1.000	1.000	1.038	0.956	0.964	1.228	0.985
Industrial Production: Manufacturing (SIC)	1.000	1.000	1.107	1.027	1.047	1.290	1.06
Industrial Production Index	1.000	1.000	1.152	1.039	1.055	1.260	1.073
ISM Manufacturing: PMI Composite Index	1.000	1.000	1.090	1.034	1.039	1.398	1.076
Real Disposable Personal Income	1.000	1.000	0.960	0.888	**0.888**	1.064	0.877
Real Disposable Personal Income Less Transfer Payments	1.000	1.000	1.004	0.853	0.847	0.848	0.79
Civilian Labor Force: Employed, Total	1.000	1.000	1.002	1.026	1.042	1.334	1.071
Unemployment Rate: All Workers	0.989	0.987	0.985	**1.215**	1.004	0.770	0.992
Unemployment By Duration: Average Duration In Weeks	1.001	1.001	1.065	**1.067**	0.972	0.896	1.019
Civilians Unemployed - Less Than 5 Weeks	1.000	1.000	0.970	0.977	0.982	1.304	0.974
Civilian Unemployed - 5-14 Weeks	1.000	1.000	0.998	1.048	1.012	1.149	1.028
Civilians Unemployed - 15 Weeks & Over	1.000	1.000	1.007	1.056	1.011	1.059	1.042
Civilians Unemployed - 15-26 Weeks	1.000	1.000	0.875	1.012	**0.912**	0.810	0.913
Total non-farm employment	1.000	1.000	0.999	1.055	1.012	1.162	1.026
Total private employment	1.000	1.000	0.974	1.079	1.002	1.171	1.01
Goods-producing employment	1.000	1.000	1.001	1.094	1.013	1.178	1.021
Natural resources and mining employment	1.000	1.000	0.740	1.031	**0.711**	0.760	0.686
Construction employment	1.000	1.000	0.903	**1.096**	0.982	1.380	0.976
Manufacturing employment	1.000	1.000	1.062	1.068	1.017	1.131	1.034
Durable goods manufacturing employment	1.000	1.000	1.013	1.055	0.995	1.121	0.996
Non-durable goods manufacturing employment	1.000	1.000	1.100	1.062	1.031	1.223	1.046
Service-providing employment	1.000	1.000	0.989	1.013	1.025	1.231	1.038
Trade, transportation and utilities employment	1.036	1.018	0.985	1.024	1.021	1.212	1.031
Retail trade employment	1.000	0.994	0.958	0.971	0.993	1.113	0.994
Wholesale trade employment	1.000	1.000	0.978	0.988	1.004	1.161	1.016
Financial activities employment	1.031	1.018	1.012	1.000	1.011	1.105	1.024
Private service-providing employment	**1.092**	**1.054**	0.973	1.042	1.044	1.392	1.034
Government employment	1.003	1.000	0.926	0.825	0.895	1.732	0.939
Manufacturing average weekly hours of production workers	1.000	1.000	1.055	1.053	1.034	1.565	1.085
Manufacturing average weekly overtime of production workers	1.000	1.000	0.996	0.991	0.958	1.103	0.979
Housing Starts: Total: New Privately Owned Housing Units Started	1.000	1.000	1.019	0.984	0.987	1.260	1.006
Housing Starts in Northeast Census Region	1.000	1.000	**1.098**	1.004	1.041	1.331	**1.087**
Housing Starts in Midwest Census Region	1.000	1.000	0.981	1.046	1.023	1.356	1.041
Housing Starts in South Census Region	1.000	1.000	0.995	1.009	0.999	1.167	1.017
Housing Starts in West Census Region	1.000	1.000	0.991	1.005	0.986	1.161	1.004
New Private Housing Units Authorized by Building Permit	1.000	1.000	1.016	1.001	0.990	1.346	1.011
New Orders, Consumer Goods & Materials	1.000	1.000	1.063	1.044	1.039	1.388	1.059
New Orders, Non-defense Capital Goods	1.000	1.000	1.067	1.126	1.079	1.569	1.139
Dow Jones Industrial Average	1.000	1.000	1.006	0.969	1.009	1.389	1.025
Japanese Yen-United States Dollar Exchange Rate	1.000	1.000	1.043	1.013	1.040	1.683	1.105
Canadian Dollar-United States Dollar Exchange Rate	1.000	1.000	1.086	1.029	1.018	1.021	1.048
Effective Federal Funds Rate	0.916	0.928	1.284	**1.322**	1.031	0.853	0.984

Table Q: Relative RMSFE results for US: second period (1986 Q3-1997 Q4) (cont.)

	M40	M60	R40	R60	AV	EWMAA	EWMAL
3-Month Treasury Bill: Secondary Market Rate	0.933	0.941	1.235	**1.291**	1.019	0.897	0.995
6-Month Treasury Bill: Secondary Market Rate	0.959	0.961	1.163	**1.223**	1.018	0.963	0.999
1-Year Treasury Constant Maturity Rate	0.989	0.985	1.130	**1.169**	1.027	1.042	1.012
5-Year Treasury Constant Maturity Rate	1.004	1.001	1.012	1.054	1.027	**1.171**	1.032
10-Year Treasury Constant Maturity Rate	1.003	1.001	1.015	1.050	1.027	1.158	1.032
Moody’s Corporate AAA Yield	0.999	0.998	1.040	1.059	1.027	1.114	1.02
Moody’s Corporate BAA Yield	1.000	0.998	1.043	1.049	1.022	1.106	1.006
Spread 3M-FF	0.947	0.958	1.095	**1.414**	1.113	0.931	0.991
Spread 6M-FF	0.990	0.988	1.153	**1.369**	**1.109**	1.202	1.041
Spread 1Y-FF	1.000	1.000	0.969	1.126	**1.040**	1.196	1.044
Spread 5Y-FF	1.000	1.000	0.953	1.054	1.008	1.141	1.005
Spread 10Y-FF	1.000	1.000	0.959	1.076	0.979	0.979	0.973
Spread AAA-FF	1.000	1.000	0.960	1.067	0.964	0.888	0.942
Spread BAA-FF	0.994	0.996	0.958	1.120	0.985	0.960	1.001
M1 Money Stock	0.909	0.933	1.043	1.083	1.030	1.118	0.946
M2 Money Stock	0.999	0.999	1.167	**1.186**	1.000	0.855	0.883
Money Supply - M2	1.000	1.000	1.139	**1.154**	0.973	0.816	0.841
Monetary base, adj for reserve requirement changes	1.053	1.032	1.110	1.083	1.083	1.289	1.108
Total Reserves of Depository Institutions	0.939	0.955	0.896	0.923	0.939	1.116	0.911
Non-Borrowed Reserves of Depository Institutions	0.945	**0.960**	0.911	0.952	**0.918**	**0.583**	**0.876**
Consumer Credit Outstanding - Non-revolving	1.000	1.000	0.888	**1.126**	0.933	0.982	0.893
Commercial & Industrial Loans Outstanding	1.000	1.000	1.064	1.065	1.000	1.156	0.929
PPI: Finished Goods	0.958	0.970	0.871	0.985	0.911	0.966	0.915
PPI: Finished Consumer Goods	0.966	0.975	0.883	0.991	0.929	1.023	0.93
PPI: Intermediate Mat. Supplies & Components	0.978	0.979	0.953	0.966	0.955	1.232	0.977
PPI: Crude Materials	1.000	1.000	1.055	1.031	1.059	1.697	1.15
Consumer Price Index For All Urban Consumers: All Items	0.971	0.978	0.888	1.004	0.909	0.925	**0.927**
CPI-U: Apparel	0.948	0.965	0.963	0.981	0.915	0.772	0.846
CPI-U: Transportation	0.960	0.971	0.891	0.957	0.937	1.308	0.944
CPI-U: Medical care	0.888	**0.911**	0.971	1.161	1.089	0.942	0.923
CPI-U: Commodities	**0.936**	**0.955**	0.802	**0.931**	**0.870**	0.910	0.856
CPI-U: Durables	0.948	0.961	1.103	**1.216**	0.991	0.868	0.927
CPI-U: All Items Less Food	0.977	0.983	0.877	1.012	0.919	0.991	**0.931**
CPI-U: All Items Less Shelter	0.959	0.970	0.868	0.960	0.892	0.905	0.901
CPI-U: All Items Less Medical Care	0.960	0.972	0.873	0.993	0.898	0.898	**0.920**
Spot Market Price Index: BLS & CRB: all commodities	1.000	1.000	1.019	1.027	1.034	**1.240**	1.061
Construction: average hourly earnings of production workers	0.998	0.999	**0.731**	**0.832**	**0.749**	**0.667**	**0.777**
Manufacturing: average hourly earnings of production workers	**0.815**	**0.870**	**0.641**	0.798	**0.737**	**0.637**	**0.756**
U. of Michigan Index of Consumer Expectations	1.000	1.000	0.978	0.949	0.943	1.114	0.944
Dow Jones Industrials Total Return Index	1.000	1.000	0.959	0.970	0.956	1.365	0.965
S&P 500 Energy Total Return Index	1.000	1.000	0.987	1.013	0.975	1.373	0.976
S&P 500 Finance Total Return Index	1.000	1.000	1.012	0.996	1.018	1.308	1.033
S&P 500 Total Return Index	1.000	1.000	0.953	0.992	0.950	1.280	0.954
S&P 500 Transportation Total Return Index	1.000	1.000	0.976	0.967	0.969	**1.375**	0.988
S&P 500 Utilities Total Return Index	1.000	1.000	1.059	1.099	1.043	1.260	1.093
Dow Jones Corporate Bond Yield	1.000	1.000	1.002	0.999	1.016	1.360	1.045
USA Prime Rate	1.000	1.000	0.935	**1.130**	1.013	1.223	1.047
West Texas Intermediate Oil Price (US$/Barrel)	1.000	1.000	0.907	0.934	0.959	1.488	0.968

Table R: Empirical results for US: third period (1998 Q1-2008 Q3)

	M40	M60	R40	R60	AV	EWMA	EWMAL
Industrial Production: Final Products (Market Group)	1.000	1.000	1.052	0.958	0.968	1.194	1.01
Industrial Production: Consumer Goods	1.000	1.000	0.956	0.953	**0.920**	1.024	0.918
Industrial Production: Durable Consumer Goods	1.000	1.000	0.960	1.041	0.961	0.994	0.946
Industrial Production: Non-durable Consumer Goods	0.978	0.983	0.849	0.873	0.883	1.054	0.896
Industrial Production: Business Equipment	1.000	1.000	1.144	1.091	1.006	1.196	1.093
Industrial Production: Materials	1.000	1.000	1.039	0.982	0.952	1.133	0.994
Industrial Production: Durable Materials	1.000	1.000	1.107	1.010	0.965	1.194	1.024
Industrial Production: Non-durable Materials	0.964	0.976	0.919	0.925	0.944	1.049	0.943
Industrial Production: Manufacturing (SIC)	1.000	1.000	1.021	0.952	0.953	1.007	0.957
Industrial Production Index	1.000	1.000	1.055	0.966	0.957	1.082	0.989
ISM Manufacturing: PMI Composite Index	1.000	1.000	0.989	1.011	1.012	1.272	1.053
Real Disposable Personal Income	1.000	1.000	0.878	0.866	0.909	1.029	0.914
Real Disposable Personal Income Less Transfer Payments	1.000	1.000	**1.475**	1.110	1.049	1.282	**1.120**
Civilian Labor Force: Employed, Total	1.000	1.000	1.019	0.997	1.018	2.059	1.047
Unemployment Rate: All Workers	1.008	1.002	1.093	1.114	1.004	0.931	1.039
Unemployment By Duration: Average Duration In Weeks	0.948	0.957	**1.077**	0.992	0.991	0.952	1.02
Civilians Unemployed - Less Than 5 Weeks	1.000	1.000	1.074	1.053	1.029	1.265	1.072
Civilian Unemployed - 5-14 Weeks	1.000	1.000	1.163	1.076	1.047	1.275	1.131
Civilians Unemployed - 15 Weeks & Over	1.000	1.000	1.090	1.067	1.013	1.116	1.069
Civilians Unemployed - 15-26 Weeks	1.000	1.000	1.185	1.105	1.016	1.153	1.075
Total non-farm employment	1.000	1.000	0.963	0.893	0.910	1.020	0.928
Total private employment	1.000	1.000	0.949	0.863	0.868	1.107	0.908
Goods-producing employment	1.000	1.000	0.950	0.861	0.833	1.083	0.877
Natural resources and mining employment	1.000	1.000	**0.311**	**0.324**	**0.513**	**0.333**	**0.356**
Construction employment	1.000	1.000	0.882	0.913	0.887	0.913	0.86
Manufacturing employment	1.000	1.000	0.923	0.835	0.799	1.175	0.856
Durable goods manufacturing employment	1.000	1.000	0.909	0.846	0.823	1.245	0.875
Non-durable goods manufacturing employment	0.958	**0.970**	0.792	0.759	**0.745**	0.796	0.731
Service-providing employment	1.000	1.000	0.956	0.915	0.939	1.104	0.947
Trade, transportation and utilities employment	1.000	1.000	0.845	0.856	**0.887**	0.949	0.871
Retail trade employment	1.000	1.000	**0.705**	**0.727**	**0.785**	0.989	**0.712**
Wholesale trade employment	1.000	1.000	0.901	0.920	0.932	1.074	0.942
Financial activities employment	1.007	1.002	0.963	0.974	0.968	1.043	0.971
Private service-providing employment	1.000	1.000	0.919	0.864	0.900	1.039	0.913
Government employment	1.052	1.022	0.803	0.857	0.995	2.691	1.145
Manufacturing average weekly hours of production workers	1.000	1.000	**1.108**	**1.071**	1.037	**1.306**	1.072
Manufacturing average weekly overtime of production workers	1.000	1.000	1.071	1.140	1.035	1.334	1.057
Housing Starts: Total: New Privately Owned Housing Units Started	1.000	1.000	1.019	1.169	1.012	1.219	0.982
Housing Starts in Northeast Census Region	1.000	1.000	1.055	1.048	1.017	**1.288**	1.052
Housing Starts in Midwest Census Region	1.000	1.000	0.995	1.058	0.984	1.045	0.953
Housing Starts in South Census Region	1.000	1.000	1.029	1.121	0.969	0.915	0.934
Housing Starts in West Census Region	1.000	1.000	1.044	1.150	0.979	0.937	0.945
New Private Housing Units Authorized by Building Permit	1.000	1.000	1.070	1.135	0.997	0.900	1.02
New Orders, Consumer Goods & Materials	1.000	1.000	0.872	0.929	**0.892**	0.817	0.816
New Orders, Non-defense Capital Goods	1.000	1.000	**1.195**	**1.078**	1.003	0.973	1.04
Dow Jones Industrial Average	1.000	1.000	1.079	1.066	1.048	1.423	**1.101**
Japanese Yen-United States Dollar Exchange Rate	1.000	1.000	0.990	1.013	1.005	**1.316**	1.036
Canadian Dollar-United States Dollar Exchange Rate	1.000	1.000	1.042	1.021	1.012	**1.320**	1.066
Effective Federal Funds Rate	0.970	0.971	0.910	0.897	**0.933**	0.839	0.903

Table S: Empirical results for US: third period (1998 Q1-2008 Q3) (cont.)

	M40	M60	R40	R60	AV	EWMA	EWMAL
3-Month Treasury Bill: Secondary Market Rate	0.967	0.972	0.926	0.918	**0.946**	0.902	0.916
6-Month Treasury Bill: Secondary Market Rate	0.978	0.978	0.937	0.927	0.951	0.936	0.926
1-Year Treasury Constant Maturity Rate	0.981	0.979	0.946	0.926	0.963	0.984	0.926
5-Year Treasury Constant Maturity Rate	0.973	0.977	1.039	0.968	1.022	1.238	0.982
10-Year Treasury Constant Maturity Rate	0.973	0.979	1.024	0.961	1.017	1.154	0.98
Moody's Corporate AAA Yield	1.006	1.001	**1.095**	0.954	1.020	1.200	1.003
Moody's Corporate BAA Yield	1.062	1.037	1.102	0.964	1.032	1.248	1.023
Spread 3M-FF	1.073	1.040	1.045	1.018	1.020	1.248	1.074
Spread 6M-FF	1.064	1.038	1.074	1.022	1.041	1.339	1.094
Spread 1Y-FF	1.000	1.000	1.101	1.053	1.051	1.423	1.147
Spread 5Y-FF	1.000	1.000	1.030	1.032	1.034	1.179	1.079
Spread 10Y-FF	1.000	1.000	1.008	1.006	1.007	1.097	1.039
Spread AAA-FF	1.010	1.001	0.942	0.961	0.958	0.947	0.975
Spread BAA-FF	0.987	0.988	0.945	0.971	0.950	0.903	0.98
M1 Money Stock	1.000	1.000	1.057	1.037	1.025	1.511	1.075
M2 Money Stock	0.949	0.964	1.042	1.013	0.951	1.097	0.959
Money Supply - M2	0.953	0.966	1.055	1.030	0.958	1.121	0.969
Monetary base, adj for reserve requirement changes	1.083	1.052	1.072	1.054	1.070	1.970	1.19
Total Reserves of Depository Institutions	1.000	1.000	1.100	1.074	1.113	3.771	1.152
Non-Borrowed Reserves of Depository Institutions	1.623	0.917	2.557	1.626	0.718	15.818	2.384
Consumer Credit Outstanding - Non-revolving	1.000	1.000	1.147	**1.116**	1.010	1.101	1.048
Commercial & Industrial Loans Outstanding	1.023	1.013	1.033	1.000	0.998	1.234	1.053
PPI: Finished Goods	1.000	1.000	1.065	1.033	0.972	1.214	1.024
PPI: Finished Consumer Goods	1.000	1.000	1.081	1.048	0.991	1.239	1.044
PPI: Intermediate Mat. Supplies & Components	1.000	1.000	1.096	1.054	1.018	1.275	1.078
PPI: Crude Materials	1.000	1.000	1.083	**1.045**	1.044	**1.591**	1.107
Consumer Price Index For All Urban Consumers: All Items	0.866	0.889	0.776	0.820	0.812	0.778	0.784
CPI-U: Apparel	**0.981**	**0.987**	**0.767**	0.869	**0.863**	0.877	**0.768**
CPI-U: Transportation	1.000	1.000	0.935	0.892	0.910	0.980	0.9
CPI-U: Medical care	0.892	0.901	0.851	0.905	**0.873**	0.944	0.895
CPI-U: Commodities	1.000	1.000	0.810	0.811	0.817	0.810	0.786
CPI-U: Durables	0.904	**0.927**	**0.815**	0.891	**0.899**	0.910	0.838
CPI-U: All Items Less Food	0.842	0.877	0.737	0.774	**0.802**	0.782	0.754
CPI-U: All Items Less Shelter	0.910	0.922	0.864	0.871	0.855	0.840	0.836
CPI-U: All Items Less Medical Care	0.879	0.897	0.798	0.827	0.820	0.806	0.796
Spot Market Price Index: BLS & CRB: all commodities	1.000	1.000	1.045	1.035	1.017	1.120	1.039
Construction: average hourly earnings of production workers	0.985	0.979	1.026	1.024	0.832	0.948	0.848
Manufacturing: average hourly earnings of production workers	1.000	1.000	**0.642**	**0.624**	**0.715**	0.696	**0.673**
U. of Michigan Index of Consumer Expectations	1.000	1.000	0.916	0.878	0.920	1.236	0.914
Dow Jones Industrials Total Return Index	1.000	1.000	1.042	1.068	1.023	1.111	1.042
S&P 500 Energy Total Return Index	1.000	1.000	0.921	0.943	0.940	0.958	0.941
S&P 500 Finance Total Return Index	1.000	1.000	**1.107**	1.043	1.004	1.012	0.991
S&P 500 Total Return Index	1.000	1.000	1.067	1.127	1.037	1.127	1.07
S&P 500 Transportation Total Return Index	1.000	1.000	1.025	1.047	1.021	**1.403**	1.061
S&P 500 Utilities Total Return Index	1.000	1.000	1.045	**1.063**	1.017	1.056	1.048
Dow Jones Corporate Bond Yield	1.000	1.000	1.056	1.059	1.031	1.298	1.097
USA Prime Rate	1.000	1.000	0.852	0.863	0.881	1.595	0.946
West Texas Intermediate Oil Price (US$/Barrel)	1.000	1.000	1.136	1.076	1.023	1.271	1.06

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