BANK OF ENGLAND

Working Paper No. 379

Household debt, house prices and consumption in the United Kingdom: a quantitative theoretical analysis

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March 2010

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Abstract

Household debt and house prices in the United Kingdom rose substantially between 1987 and 2006. In this paper we use a calibrated overlapping generations model of the household sector to examine the extent to which changes in demographics, lower inflation, and a lower long-run real interest rate may explain the build-up of debt and the rise in house prices over that period. Our model suggests that lower real interest rates were particularly important. If households expected lower real interest rates to persist, then the model can more than explain the rise in debt and can explain most of the rise in house prices. However, the model leaves a puzzle because it predicts that an unanticipated fall in real interest rates should lead to a consumption boom that did not materialise in the data.

Key words: Consumption, housing market, collateral constraints, life cycle, OLG.

JEL classification: E21, R31.

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The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England. The authors wish to thank Andrew Benito, Andy Blake, Bob Chirinko, Iain De Weymarn, Giulio Fella, Richard Harrison, Winfried Koeniger, Lavan Mahadeva, Kalin Nikolov, James Proudman, Peter Westaway, Garry Young, seminar participants at Nottingham University and Queen Mary London, and participants at the Bank of England Workshop on 'Consumption, Housing and Consumer Credit' (January 2008) for useful comments on an earlier version of this paper. We are also grateful to Jon Aldous and Mike Hawkins for help with the inheritance data, and to Wouter Den Haan and Dirk Krueger for helpful conversations prior to the start of our work. This paper was finalised on 1 September 2009.

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ISSN 1749-9135 (on-line)

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Summary

Between 1987 and 2006 household debt and residential house prices in the United Kingdom increased substantially. During this period total household debt and house prices as a percentage of income both grew by more than 50%. These phenomena were accompanied by some large changes to the macroeconomic environment faced by UK households, which may have been seen as potentially long-lasting. In particular, the inflation rate fell to a low and stable level; long-run real interest rates fell both in the United Kingdom and internationally; and the population became older with the ageing of the baby-boom generation. Lower inflation eliminates the so-called front-end loading of mortgage repayments which means that, for a given initial level of repayments, consumers can borrow more as a fraction of their income than when inflation and nominal interest rates are higher. A lower long-run real interest rate lowers the current and future expected cost of housing as well as the cost of borrowing, all else equal. These two changes, together with the passing of the baby-boom generation through middle age, are likely to have boosted the demand for housing and other assets, and therefore to have been important determinants of the observed rise in house prices and debt over that period. However, it is not clear, *a priori*, how quantitatively important these changes were in terms of their ability to explain the increases in house prices and debt, or how they would have continued to affect the household sector balance sheet and the housing market in the subsequent years or decades. Moreover, while there did not appear to be a large amount of uncertainty about the inflation target and the future inflation rate, there was considerable uncertainty about the persistence of the low level of the real interest rate.

In this paper we set out to investigate these issues. Specifically, we are interested in understanding to what extent changes in demographics, lower inflation, and the lower real interest rate can explain the observed rise in UK house prices and debt over the two decades after 1987. It should of course be clear that, although these are all matters of great interest to policy makers, the long-run real interest rate is a structural aspect of the real economy which is unaffected by monetary policy. We are also interested in how these factors might affect the long-run equilibrium of the economy, and how the economy might adjust to that equilibrium. However, we do not attempt to explain the behaviour of house prices and consumption in the extreme conditions faced by households in 2008 and 2009.

The workhorse of our analysis is an 'overlapping generations' model calibrated to the UK economy. It recognises that households do not live forever, and that at any one time there are households in different parts of their lives – some young, some middle aged, and some retired. The model incorporates: housing and non-durable consumption; financial wealth; loan to value and loan to income borrowing constraints; realistic demographics; and bequests. The results suggest that demographic factors can be important in explaining the evolution of the household sector's aggregate balance sheet, but are not alone capable of producing the size of the movements in debt and house prices that we have observed in the data. Moreover, the effects of demographic change are too gradual to account for the sharp rises in debt and house prices that occurred during the second half of that period. What instead emerges from our analysis is that the main driver of the rise in house prices and debt is the decline in the real interest rate, most of which occurred after the turn of the century, and which was an international phenomenon. Crucial to that conclusion is the assumption that households perceived low global real interest rates as being permanent. In that case, the model can explain the rise in debt and much of the rise in house prices.

However, it should be noted that the extent to which the model can quantitatively explain the rise in house prices depends in large part on the period of comparison. For example, the model can more than explain the rise in UK house prices between 1992-96 and 2002-06, but not between 1997-2001 and 2002-06. In addition, comparisons are further complicated by the division of model time periods into five -year chunks (which helps to ensure that the computational demands of our exercises are not excessive). The model only more than explains the rise in house prices between 1992-96 and 2002-06 if the level of house prices in 2002-06 is taken to be the average prevailing over that period. If instead, the level of house prices in 2002-06 is taken to be that prevailing in 2006, the model cannot explain all of the rise in UK house prices over that period. All of that suggests that care should be taken not to draw precise quantitative conclusions from our analysis. Nevertheless, and consistent with standard economic theory, one implication of our results is that the level of long-term real interest rates is a crucial factor in determining the equilibrium level of debt and house prices.

A by-product of a fall in the real interest rate is strong consumption and a corresponding decline in financial wealth. So, we are unable to explain some features of the data. That is, that the increase in house prices was not accompanied by a consumption boom, but was instead accompanied by an accumulation of both financial assets and financial liabilities. This failing, together with the abstract nature of our model and its reliance on assumptions about unobservable parameters, means that there is some uncertainty around the conclusion that the rise in debt and house prices observed at the end of 2006 was to be expected.

1 Introduction

Between 1987 and 2006 household debt and residential house prices in the United Kingdom increased substantially. During this period total household debt and house prices as a percentage of income grew by more than 50%.

These phenomena were accompanied by some large changes to the macroeconomic environment faced by UK households, which may have been viewed as being potentially long lasting. In particular, the inflation rate fell to a low and stable level; global real interest rates fell; and the population became older with the ageing of the baby-boom generation. Lower inflation eliminates the so-called front-end loading of mortgage repayments which means that, for a given initial repayment, consumers can borrow more as a fraction of their income than when inflation and nominal interest rates are higher. A lower long-run real interest rate lowers the current and expected future real cost of housing as well as the cost of borrowing, all else equal. These two changes, together with the passing of the baby-boom generation through middle age, are likely to have boosted the demand for housing and other assets, and are likely to have been important determinants of the observed rise in house prices and debt over that period. However, it is not clear, *a priori*, how quantitatively important these changes were in terms of their ability to explain the increases in house prices and debt and how they would have continued to affect the household sector balance sheet and the housing market in the subsequent years or decades. Moreover, while there did not appear to be a large amount of uncertainty about the inflation target and the future inflation rate, there was considerable uncertainty about the persistence of the low level of the real interest rate.

In this paper we set out to investigate these issues. Specifically, we are interested in understanding to what extent changes in demographics, lower inflation, and the lower real interest rate can explain the observed rise in UK house prices and debt over the two decedes after 1987. It should of course be clear that, although these are all matters of great interest to policy makers, the long-run real interest rate is a structural aspect of the real economy which is unaffected by monetary policy. We are also interested in how these factors might affect the long-run equilibrium of the economy, and how the economy might adjust to that equilibrium. However, we do not attempt to understand the fall in UK house prices that began in 2008, following the banking crisis that began in late 2007.

The workhorse of our analysis is a quantitative, endowment overlapping generations (OLG) model of life-cycle decisions calibrated to the UK economy. The model incorporates: housing and non-durable consumption; financial wealth; loan to value (LTV) and loan to income (LTI) borrowing constraints; realistic demographics; accidental and voluntary bequests. The house price is endogenously determined, whereas the interest rate is taken as given.[1] The assumption about the interest rate is consistent with the United Kingdom being a small open economy and real interest rates being determined globally. As this is an endowment economy, we also take the life-time income profile of consumers as well as the (elastic) supply of housing as given.[2]

In studying the possible evolution of the economy we favour a model-based approach over the estimation of reduced forms. Time-series estimation would inevitably encompass different policy regimes and so would be unlikely to lead to stable estimates, and reduced forms estimations are hard to interpret. An alternative approach would be to exploit cross-country variation. But international comparisons are also problematic because the evolution of house prices and debt in any particular country most likely depends on the idiosyncrasies of the credit market institutions in that country. Most importantly, working with a theoretical model has the advantage of allowing us to experiment with different assumptions about the future as well as with counterfactual scenarios. Hence we can gauge the relative importance of various drivers of house prices and debt.

A number of papers have incorporated housing choices in models of household behaviour to address a variety of important issues. However, these analyses are usually limited to steady states. For instance, Li and Yao (2005) analyse the aggregate wealth effect of a house price shock in a partial equilibrium model. Fernandez-Villaverde and Krueger (2005) examine the role of durables in explaining the hump-shape pattern of non-durable consumption and the fact that the young do not hold liquid assets. Recent relevant work also includes – among others – Gervais (2002), Yang (2005), Chambers, Garriga and Schlagenhauf (2004), Jeske and Krueger (2005), and Hintermaier and Koeniger (2007). Unfortunately, there is almost no work of which we are aware that looks at the dynamic transition towards a new level of debt as the outcome of lower

[1]Besides the house price, bequests are also endogenously determined in such a way that the total wealth accidentally and voluntarily left by the dieing agents in one period matches the total wealth inherited by the surviving agents.

[2]As such, our analysis is similar to that adopted in Poterba (1984), who investigates the effects of higher inflation and changes in the tax code on house prices and residential capital. In doing so, he uses an asset market model and, like us, he assumes that income and interest rates are exogenous.

real and nominal interest rates and changing demographics. Our paper is meant to fill that gap. To the best of our knowledge, there is only one paper which looks at the transitional dynamics in a life-cycle model with housing – Cerny, Miles and Schmidt (2005).[3] That paper, however, addresses a somewhat different question: the impact on the housing market of different pension arrangements. Previous Bank of England work (Tudela and Young (2005)) has also studied the evolution of household behaviour using a life-cycle model, but that model is simpler than the one employed here in that it does not account for credit constraints, bequests and, most importantly, takes the house price as exogenous.[4] Our paper can be regarded as a continuation of that line of work.[5]

To anticipate our findings, the model suggests that demographic factors can be important in explaining the future evolution of the household sector's aggregate balance sheet, but do not seem capable of producing the size of the movements in debt and house prices that we observed in the data between 1987 and 2006. Moreover, the effects of demographic change are too gradual to account for the sharp rises in debt and house prices that occurred during the second half of that period. What emerges from our analysis is that the main driver of the rise in house prices and debt was the decline in the real interest rate, most of which occurred after 2000. Crucial to that conclusion is the assumption that households perceived low real interest rates as being here to stay. In that case, the model can more than explain the rise in debt and can explain much of the rise in house prices. One conclusion of our analysis is therefore that the level of the long-term real interest rate is a crucial factor in determing the levels of debt and house prices observed at the end of 2006. But other consequences of a fall in the real interest rate in the model are strong consumption and a decline in gross financial wealth. As such, the model is unable to explain some features of the data: that is, that the increase in house prices was not accompanied by a consumption boom, but was instead accompanied by an accumulation of both financial assets and financial liabilities. This failure of the model suggests that its quantitative predictions should be treated with caution. Moreover, the model's predictions are not applicable

[3]Campbell and Hercowitz (2006) examine 'the transition to a high-debt economy' in a model with infinitely lived agents, who are characterised as having different discount rates which either makes them perpetual borrowers or perpetual lenders. The heterogeneous discount factor assumption makes the model analytically tractable, but it is not realistic and might not be an innocuous assumption. After this paper was started, recent work in progress by Kiyotaki, Michaelides and Nikolov (2007) also examines transition dynamics caused by changes in the economic environment. Our model differs from theirs – besides the different application – because our model focuses on the life cycle and demographics, whereas their model provides a more complete description of the supply side of the economy.

[4]See also Hamilton (2003) for an analysis of households' secured debt using an accounting framework.

[5]The model can also be extended in a number of ways (eg by incorporating income uncertainty and a rental sector) and therefore can be regarded as a starting point for studying a number of household sector issues.

to the unusual circumstances following the financial market turbulence that began in mid-2007.

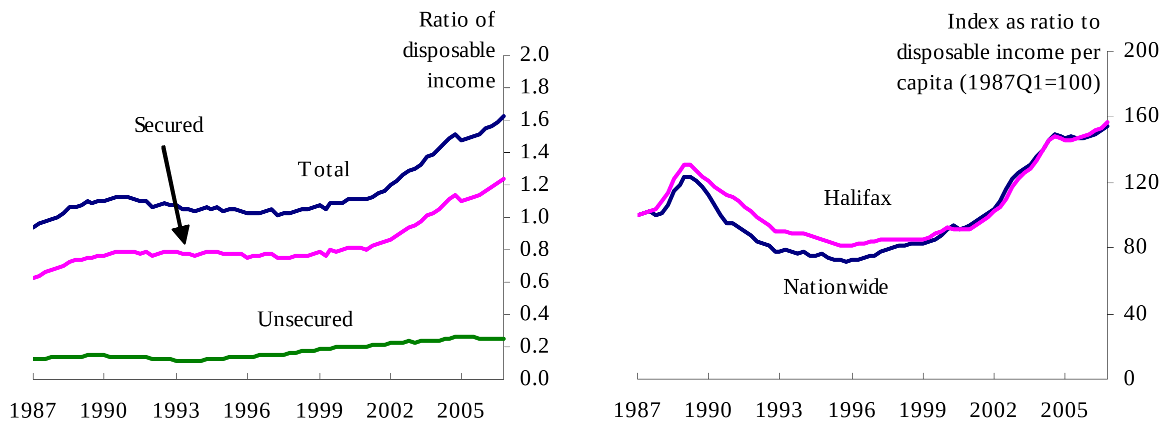
The rest of this paper is organised as follows. Section 2 illustrates the stylised facts which motivate our analysis. Section 3 outlines the model and the solution method. Section 4 describes the calibration. Section 5 reports the outcome of a number of experiments designed to understand the potential impact of the macroeconomic changes described in Section 2. Section 6 concludes.

2 Stylised facts

2.1 Aggregate trends

Between 1987 and 2006 total household debt in the United Kingdom more than quadrupled to over £1.25 trillion. Primarily, this rapid increase in debt reflected a rise in borrowing secured on housing (although unsecured borrowing also grew rapidly). Over the same period, Nationwide data suggests that residential house prices have also more than quadrupled – by the end of 2006 the price of an average house had grown to over £170,000 compared to around £40,000 in 1987. Partly these changes can be explained by an increase in household disposable income over the same period. But, even after adjusting for higher household income, total debt (left-hand panel in Chart 1) and house prices (right-hand panel in Chart 1) increased by more than 50%.

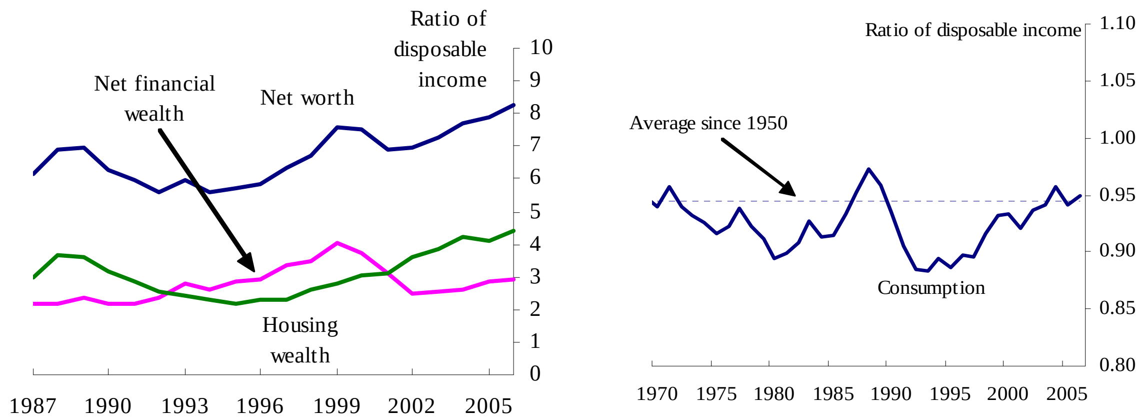
Chart 1: Household debt and house prices



Sources: Left – Bank of England (VZRI) and Office for National Statistics (NNPP, NNRP).
Right – Halifax and Nationwide.

Despite the rapid increase in household debt, the total net worth of UK households grew over this period (left-hand panel Chart 2). Partly that was accounted for by the sharp rise in house prices and consequently gross housing wealth, but it was also accounted for by an increase in net financial wealth over that period. The latter reflected both an increase in the value and quantity of financial assets owned by households. In fact, in aggregate over this period, households accumulated financial assets at almost the same rate as financial liabilities. The result is that, with the exception of the late 1980s and early 1990s when the UK economy experienced a boom followed by a recession, consumption as a share of income increased only gradually and was marginally above its historical average by the end of 2006 (right-hand panel of Chart 2).[6]

Chart 2: Household wealth and consumption



Sources: Left – ONS (CGRC, NNML-NNPP, CGRI). Right – ONS (NFVO, NFYS).

In sum, for the purpose of our analysis, the aggregate picture shows that between 1987 and 2006: (i) household debt and house prices grew rapidly; (ii) household wealth also increased; (iii) consumption as a share of income did not change much in that households as a whole were saving nearly as much as they were borrowing.

But the aggregate picture does not tell the whole story because it is likely that there was significant heterogeneity among households. In particular, it seems unlikely that the households accumulating the additional financial assets were the same as those accumulating additional liabilities. For example, the 2005 British Household Panel Survey (BHPS) suggests that only around 40% of households simultaneously held financial assets and liabilities; and only 26%

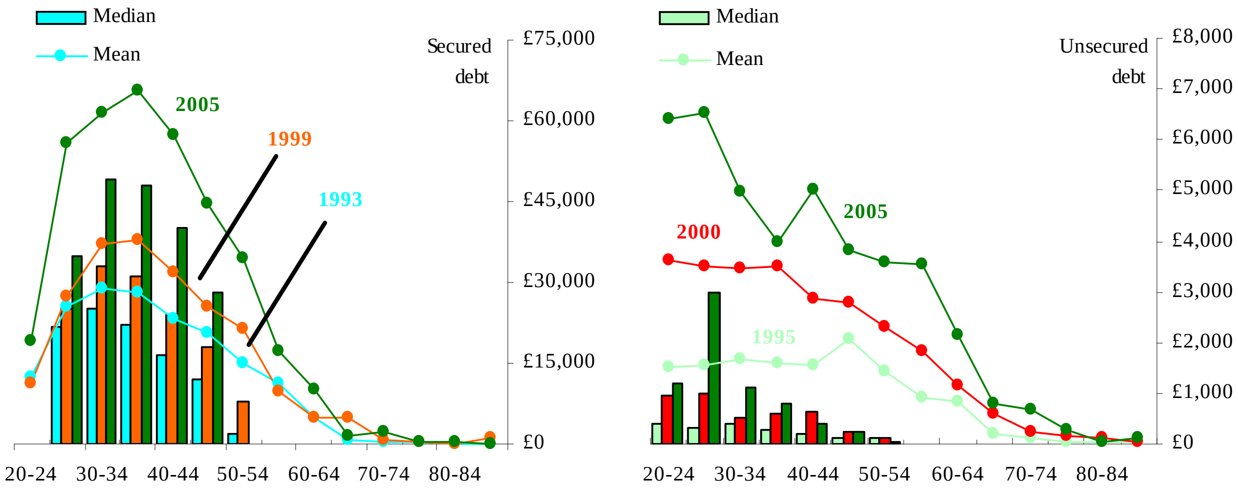
[6] During the late 1980s and early 1990s, the United Kingdom experienced a boom followed by a recession in which borrowing and the housing market suffered. See Attanasio and Weber (1994) or King (1994) for a discussion.

(15%) of households simultaneously held more than £1,000 (£5,000) of both assets and debts. Given these facts, it is vital to examine the dissaggregate picture.

2.2 Disaggregate trends

Chart 3 plots the median (the bars) and mean levels (the lines) of secured and unsecured debt by age from the BHPS.[7] There was significant heterogeneity within and between age groups, both in terms of the level of debt taken on by different households and the extent to which this changed over time.[8] The distribution of unsecured debt was particularly skewed. It was predominantly held by younger households[9] and, for most age groups, the mean comfortably exceeded the median. By contrast, most secured debt was held by middle-aged households, who are more likely to be homeowners and have mortgages. The increased indebtedness of these households likely accounts for most of the rise in aggregate secured debt. Consistent with the aggregate data shown in Chart 1, the largest rise in secured debt took place after 1999.

Chart 3: Secured and unsecured debt



Sources: BHPS (collected by the ISER Research Centre at the University of Essex) and Bank calculations.

Chart 4 shows that middle-aged and older households were more likely to be homeowners and tended to hold more housing wealth than younger or older households. Moreover, housing

[7]The BHPS data used in this subsection were constructed along the lines described in Redwood and Tudela (2004). These data have not been adjusted for inflation over the period. Doing so, does not materially alter the charts or conclusions drawn from them.

[8]Secured debt data are available in each BHPS since 1993, whereas unsecured debt data are available in the 1995, 2000 and 2005 surveys.

[9]The sharp rise in younger households' unsecured debt might partly reflect increased use of student loans. The 2005 BHPS survey shows that around 8% of people aged 18 to 25 had a student loan, compared to around 2% in the 2000 survey.

wealth boomed between 1999 and 2005 to the advantage, in particular, of older generations. Homeownership rates for younger households were lower and declined between 1991 and 2005, which probably explains why younger households collectively did not enjoy such large increases in housing wealth.[10]

Chart 4: Housing wealth and homeownership rates

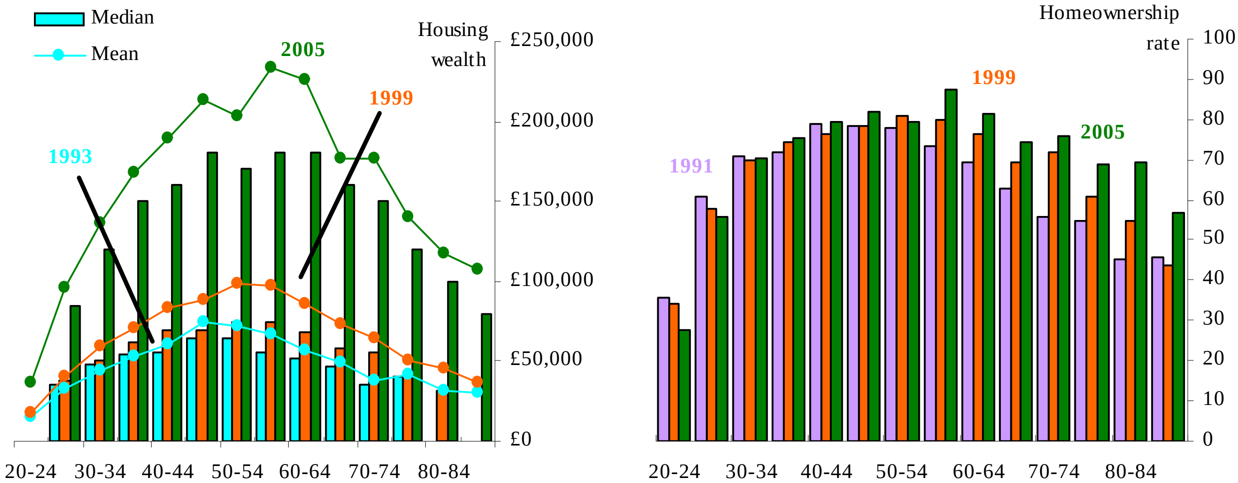
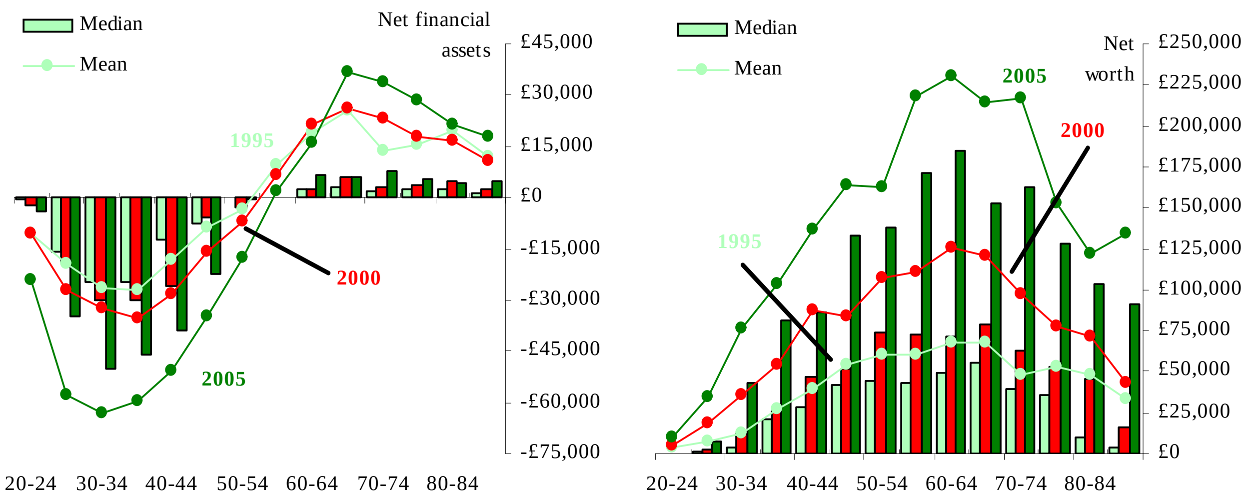


Chart 5: Net financial assets and net worth



Sources: BHPS (collected by the ISER Research Centre at the University of Essex) and Bank calculations.

The extent to which the growth in debt might have posed repayment problems for individual households depends in part on how their overall asset and wealth position evolved. Chart 5 shows that net financial wealth – the difference between a household's financial assets and

[10]Chart 4 also suggests that the distribution of housing wealth has become more skewed, which might be associated with the more pronounced decline in homownership rates among younger, poorer, and less well educated groups.

liabilities – was negative for the majority of younger households and positive for the majority of older households.[11] Chart 5 also shows that these differences became more pronounced since 2000. But including housing wealth, most households (82%) had positive wealth.

In sum, for the purpose of our analysis, the disaggregate picture shows that: (i) secured debt increased the most among young and middle-aged households; (ii) middle-aged and older households owned the most and enjoyed the largest increases in housing wealth; (iii) middle-aged households' net worth increased the most.

2.3 Potential drivers

In this subsection we describe three macroeconomic changes that we believe may have been important drivers of the aggregate and disaggregate trends described above and on which our modelling work will focus. These are: the fall in the real interest rate, the fall in the inflation rate, and the ageing of the baby-boom generation. This list is by no means exhaustive. For example, lower macroeconomic volatility, higher earnings dispersion, more competitive unsecured borrowing rates, higher household formation rates, the abolition of mortgage payment tax relief, and more flexible lending criteria might also have been significant explanatory factors. While incorporating all these factors in an OLG model in a satisfactory manner is not a straightforward task, we believe that we are focusing on potentially the most important ones.

2.3.1 Lower real interest rates

Real interest rates fell between 1987 and 2006. For example, the three-year spot rate derived from the UK index-linked government bond market fell by more than 1 percentage point over that period.[12] This was a widely documented global phenomenon that coincided with a global savings glut – predominantly in emerging market economies – which necessitated a fall in real interest rates to bring desired global saving and investment into line (see eg Bernanke (2005)).

A lower real interest rate could have supported higher equilibrium levels of debt and higher

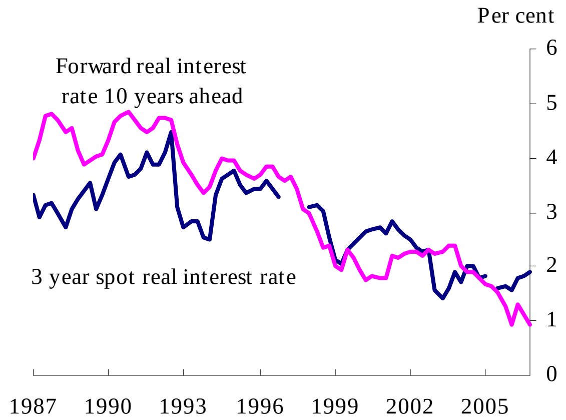
[11] The financial wealth data exclude household wealth associated with their future pension entitlements.

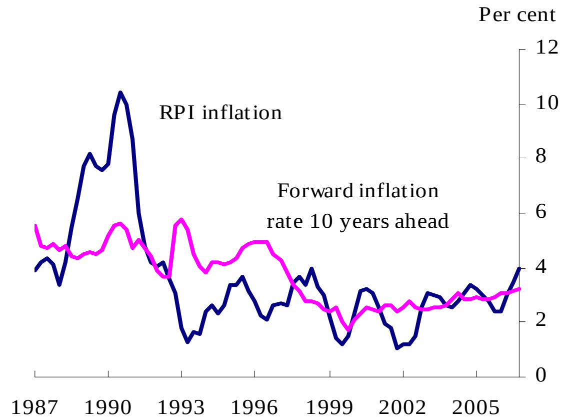
[12] See Anderson and Sleath (1999) for a discussion of real and nominal UK government yield curve calculations.

house prices by lowering the cost of borrowing (reducing the incentive to save)[13] and by lowering the real cost of housing, all else equal.[14] Importantly, measures of expectations for real interest rates in the future – the ten-year instantaneous forward rate in the left-hand panel of Chart 6 – also fell to the same low levels, implying that households might have expected low real interest rates to persist.

It is possible that the fall in long-term real government bond yields overstates any fall in households' real interest rate expectations. In particular, market participants attribute part of the decline in long-term real yields to exceptionally strong institutional demand from pension and life assurance funds for long-dated index-linked government bonds, which may exist regardless of their interest rate expectations (see eg Joyce, Sorensen and Weeken (2008)). But simpler measures of real interest rates (like Bank Rate minus the annual change in the consumer prices index) have also fallen over this period. That may have been accompanied by a decline in households' interest rate expectations if, for example, households attach some weight to current data in forming their expectations.

Chart 6: Real interest rates and inflation





Sources: Left – Bank of England. Right – Bank of England and ONS (CHAW).

[13] More accurately, there are substitution, income, and wealth effects. See Attanasio (1999) for a discussion.

[14] Weeken (2004) uses a dividend discount framework to show that a fall in the real interest rate could have accounted for a large part of the rise in house prices.

2.3.2 Lower inflation and more relaxed borrowing constraints

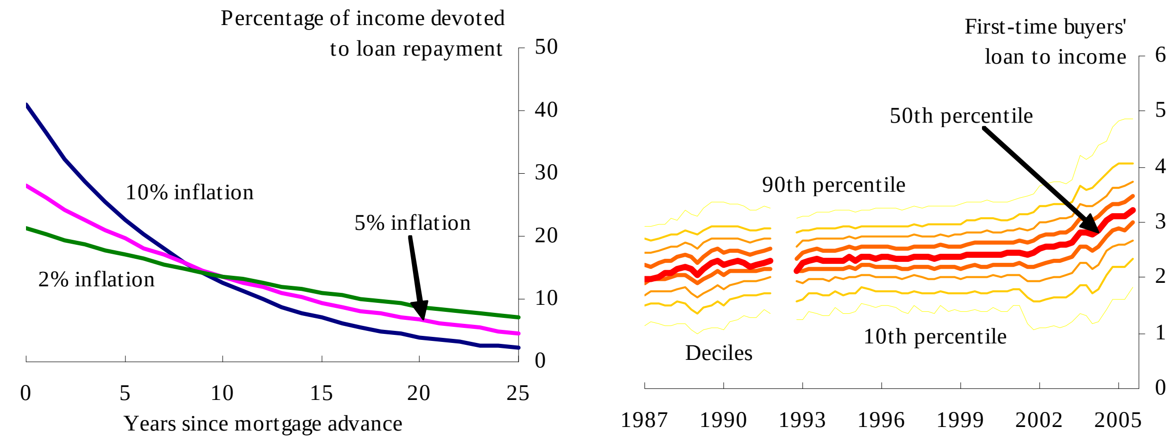
Inflation (as measured by the retail prices index) also fell by over 1 percentage point between 1987 and 2006 and by over 5 percentage points from its local peak in 1990 (right-hand panel Chart 6). In addition, and in common with the fall in the real interest rate, inflation expectations fell. To see why lower inflation might increase the equilibrium levels of debt and house prices consider a typical mortgage contract. Borrowers are required to make constant *nominal* payments over the life of the loan. When inflation and nominal interest rates are higher, repayments as a percentage of income are larger initially. That initial high repayment burden can restrict the amount that households can afford to borrow. For example, a new variable rate mortgagor would have to devote around twice as much of their income to make the first repayment at 10% inflation than at 2% (left-hand panel of Chart 7). Equivalently, for a given initial repayment the household could borrow around twice as much in the lower inflation world.[15]

If lower inflation had facilitated more borrowing, one would expect households to have borrowed more relative to their incomes than previously. The right-hand panel of Chart 7 plots each decile of the LTI ratio for first-time buyers at the time they took out their mortgage. That is, at each point in time each decile shows the percentage of first-time buyers with lower LTIs than indicated by that decile. The chart shows that first-time buyers' LTIs drifted up during the late 1980s and 1990s at all points in the distribution. This suggests that either lenders were prepared to lend households more relative to their incomes or that households chose to borrow more as their initial repayments were reduced in line with lower nominal interest rates.[16]

[15] See eg Campbell and Cocco (2006) and Barnes and Thwaites (2005) for a fuller discussion.

[16] First-time buyers' LTIs had been growing more rapidly in the period up to the end of 2006. However, to weigh against the latter development, LTV ratios had fallen, perhaps reflecting a trade-off between the LTIs and LTVs that lenders were prepared to offer. See Fernandez-Corugedo and Muellbauer (2006) for a discussion.

Chart 7: Credit constraints and inflation

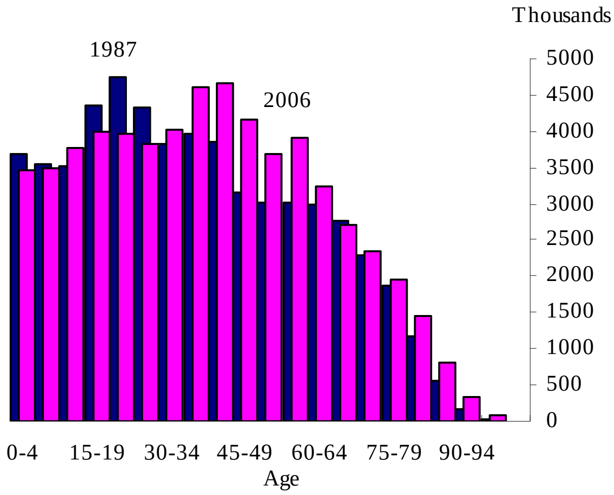


Sources: Left – authors' calculations. Right – Survey of Mortgage Lenders.

2.3.3 Demographics

Changes to the distribution of the population might have contributed to stronger housing demand. Chart 4 suggests that middle-aged households tend to demand more housing than younger or older households. So aggregate housing demand might also have been strengthened by higher demand from the baby-boom generation as they moved through middle age (Chart 8).[17]

Chart 8: Population distribution



Source: ONS

[17] Another demographic factor which may have been important is the increase in the number of *households*. Higher divorce rates and lower marriage rates increased the rate of household formation. This ought to have increased the demand for housing because larger households can share living space and so benefit from economies of scale. So, for a given number of individuals, the demand for housing would likely be higher if the average size of a household were smaller. We do not explicitly consider that here, but see Barker (2004) for a discussion.

3 The model

The workhorse for our analysis is a quantitative, endowment OLG model of life-cycle decisions over non-durable consumption, housing consumption, and financial wealth. Housing plays three roles in the model. It provides a flow of consumption services, it acts as a store of value, and it can be used as collateral for borrowing. Agents are restricted in the amount they can borrow through LTV and LTI constraints – the sort of constraints observed in the real world. The only risk agents face in the model is mortality risk. This has the effect of generating accidental bequests because we assume that agents cannot insure themselves against longevity risk (ie there are no annuity markets). We also assume that agents care, to a degree, about the utility of the following generations so that agents also have a motive for leaving bequests. For simplicity, we assume that the total amount of wealth left over (accidentally or voluntarily) by the agents that die in a given period are redistributed uniformly to the surviving agents in the subsequent period.

We make the assumption that agents supply labour inelastically and that the economy is small and open. That is, we take the wage rate and interest rate as given. In addition, the aggregate supply of housing is also taken as exogenous to the model, although it is assumed to be responsive to the house price, which is endogenous. In other words, our model is primarily a model of the demand side.[18] Finally, agents are endowed with perfect foresight; that is, they are able to anticipate the market clearing path of future house prices as well as their amount of bequests they will receive when making their decisions. Below we provide a detailed description of the model's building blocks.

3.1 Demographics

At each date t the population M_t is partitioned into I age cohorts made up of homogeneous individuals. The evolution of M_t over time is modelled through the recursive process **(1)**, which takes into account both the survival and fertility rates of the existing population as well as the arrival of new immigrants:

$$M_{t+1} = \Gamma M_t + \Omega M_t \tag{1}$$

[18] Modelling the supply side would have involved both analytical and computational hurdles which are hard to overcome. And it may also have involved sacrificing the relatively rich description of household behaviour, which we think is one of the advantages of the model. See Kiyotaki, Michaelides and Nikolov (2007) for a heterogeneous agent model with a housing and goods supply side.

$$\Gamma = \begin{bmatrix} \kappa_1 & \kappa_2 & \kappa_3 & . & . & \kappa_I \\ \zeta_1 & 0 & 0 & . & . & 0 \\ 0 & \zeta_2 & 0 & . & . & 0 \\ 0 & 0 & \zeta_3 & . & . & 0 \\ . & . & . & . & & . \\ 0 & 0 & 0 & . & \zeta_{I-1} & 0 \end{bmatrix} \quad \Omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ . \\ \omega_I \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & . & 1 \end{bmatrix} \qquad \textbf{(2)}$$

The first matrix in **(2)**, Γ, characterises the rate and dynamics of natural population change, where κ_i indicates the fertility rate of cohort i and ζ_i the probability that someone of age i reaches age $i+1$. We assume that individuals do not live beyond age I, so that $\zeta_I = 0$. Since the population in our model is large, we can make the standard assumption that the survival and fertility rates are not only the probabilities of an individual surviving or giving birth, but also the deterministic fraction of each cohort that survives and gives birth. The second matrix in **(2)**, Ω, determines the rate of immigration into each cohort, ω_i. It implies that the (net) number of immigrants arriving in any given period is proportional to the size of the population in the previous period.[19] Starting from an initial condition, the population structure M will change over time until it reaches a steady state, in which the population structure replicates itself over time and the overall size of the population grows at the same constant rate. It can be shown that the steady-state structure of the population and the steady-state growth rate of the population G^n are given, respectively, by the eigenvector and the associated eigenvalue of the population projection matrix $\Gamma + \Omega$.[20]

Agents become economically active at some exogenously specified age i_0 and until that time are supported by their parents. We assume that those parents have higher consumption needs while their children remain part of the household. We follow the literature (eg Fernandez-Villaverde and Krueger (2005)) in not modelling how adults form households. Clearly, this is not an innocuous assumption. For example, household formation might be an important determinant of an individual's housing choice. However, a formal modelling of household formation would substantially raise the size of the model, potentially making the solution computationally

[19] For simplicity, we assume that the fertility and survival rates are time invariant (Lee (1974)). The projections obtained are, however, remarkably close to those published by the Government Actuary Department. (We will return to this below. See also Appendix B.)

[20] In other words, it can be shown that, asymptotically, the population evolves according to $M_t = \lambda^t q$, where λ is a time-invariant scalar representing the rate at which the population grows and q is a time-invariant vector. It follows from **(1)** that $\lambda^{t+1} q = (\Gamma + \Omega)\, \lambda^t q$ so that λ is an eigenvalue of the population projection matrix and q the associated eigenvector. By application of the Perron-Frobenius theorem, it is possible to show that the relevant eigenvalue λ is the dominant eigenvalue (see eg Caswell (1996)).

infeasible.[21] Similarly, since we have modelled the fertility and immigration processes as being deterministic, we do not take into account the fact that fertility and immigration are to a degree endogenous and uncertain. Hence the population process specified above should not be viewed as fully capturing the interaction between the economy and demographic choice. Instead, it should be viewed as being a parsimonious way of modelling the evolution of the size and distribution of the population over time.

3.2 Preferences

Agents choose a plan for non-durable goods consumption $C \geq 0$, housing capital $H \geq 0$ (which generates H units of housing services or housing consumption), and the amount of (risk-free) financial assets ($A > 0$) or debt ($A < 0$) to maximise their utility over consumption and housing given the total amount of wealth available for spending S (also called 'cash on hand'):

$$S_{i,t} = Y_{i,t} + B_{i,t} + R_{t-1}A_{i-1,t-1} + Q_t\,(1-\delta)\,H_{i-1,t-1} \qquad \textbf{(3)}$$

where i denotes the agent's age and t the time period. $S_{i,t}$ is the sum of earnings $Y_{i,t}$, bequests received $B_{i,t}$, financial wealth (or debt) inherited from the previous period $A_{i-1,t-1}$ gross of any interest paid on it (R_{t-1} is the gross nominal interest rate), and depreciated housing wealth evaluated at today's house price Q_t (δ is its depreciation rate). We can write the household's maximisation problem recursively as **(4)**, where β denotes the discount factor, U is the period utility function, and W is a function describing the utility agents attach to the possibility of leaving a bequest:

$$V_{i,t}(S_{i,t}) = \max_{C_{i,t},H_{i,t}} \{U(C_{i,t}, H_{i,t}; z_{i,t}) + \beta[\zeta_i V_{i+1,t+1}(S_{i+1,t+1}) + (1-\zeta_i)W(S_{i+1,t+1})]\} \qquad \textbf{(4)}$$

The period utility function U describes the utility agents associate with non-durable consumption and the consumption of housing services. This also depends on the number of children belonging to that agent (which is captured by z). The functional form of U is assumed to be:

$$U(C_{i,t}, H_{i,t}; z_{i,t}) = \frac{z_{i,t}}{1-\gamma}\left[\left(\frac{C_{i,t}}{z_{i,t}}\right)^{\alpha}\left(\frac{H_{i,t}}{z_{i,t}}\right)^{1-\alpha}\right]^{1-\gamma} \qquad \textbf{(5)}$$

[21] See Cubeddu and Rios-Rull (2003) for a formal treatment of the effects of stochastic household formation and break-up.

Here we make a number of assumptions. First, we follow Tudela and Young (2005) in assuming that it is of the constant relative risk aversion family with an intertemporal elasticity of substitution of γ^{-1}. Second, we follow Fernandez-Villaverde and Krueger (2005) in assuming that households aggregate over housing consumption and non-durable consumption using a Cobb-Douglas function. The Cobb-Douglas function is a special case of the constant elasticity of substitution (CES) aggregator, where the elasticity of substitution is equal to one. This form can be justified on the basis of the empirical evidence discussed in Fernandez-Villaverde and Krueger.[22] Third, we follow Lazear and Michael (1980) in assuming that the effect of demographics on household utility can be summarised by a household equivalence scale $z_{i,t}$.

The bequest function W describes the utility agents derive from bequeathing financial and housing assets. Notice that if $W(S_{i+1,t+1}) = 0$, households have no bequest motive. However, aggregate bequests will not equal zero because we assume that no market for annuities exists so that households also leave bequests accidentally if they die before the maximum age I. In the general case, where $W(S_{i+1,t+1}) \neq 0$, households weight their own future utility and bequests according to their probability of survival, ζ. This implies that older agents, who have a lower probability of survival, attach a higher weight to the utility derived from the possibility of leaving a bequest than younger agents.

We model bequests to be broadly consistent with the utility function of the recipient as in

De Nardi (2004):

$$W(S_{i+1,t+1}) = \frac{\xi_1}{1-\gamma}\left[\left(Y^p_{t+1} + \frac{R_t A_{i,t} + Q_{t+1}(1-\delta)H_{i,t}}{\xi_2}\right)\left(\frac{\alpha}{P^c_{t+1}}\right)^{\alpha}\left(\frac{1-\alpha}{P^h_{t+1}}\right)^{1-\alpha}\right]^{1-\gamma} \quad \textbf{(6)}$$

The term ξ_1 controls the strength of the motive and ξ_2 controls the degree to which bequests are a luxury. The utility recipients derive from the bequest depends on: the amount that they would have consumed had they not received the bequest (which is proxied by the level of permanent income Y^p_{t+1}); the size of the bequest received (gross financial wealth, $R_t A_{i,t}$, plus depreciated housing wealth, $Q_{t+1}(1-\delta)H_{i,t}$); and the relative prices of non-durable consumption (P^c_{t+1}) and housing consumption (the user-cost of housing – P^h_{t+1}).

[22] The Cobb-Douglas utility function is also consistent with a balanced growth path steady state. That is, a steady state in which all variables remain stationary or grow at a constant rate over time and remain strictly positive for all the time.

3.3 Constraints

Households maximise their expected lifetime utility subject to four constraints: a standard intertemporal resource constraint, an end-of-life solvency constraint, a LTV constraint, and a LTI constraint.

At age i the intertemporal budget constraint is given by :

$$S_{i,t} = P_t C_{i,t} + Q_t H_{i,t} + A_{i,t} \quad \textbf{(7)}$$

That is, 'cash on hand' must be equal to nominal consumption and housing expenditure, $P_t^c C_{i,t} + Q_t H_{i,t}$, plus the amount of financial assets chosen (or minus the amount of debt taken on), $A_{i,t}$.[23]

Households who reach the maximum age, and therefore die at the end of the period with certainty, must be solvent:

$$Q_{t+1}(1-\delta)H_{I,t} + R_t A_{I,t} \geq 0 \quad \textbf{(8)}$$

The maximum amount of debt that the oldest agents can leave after death (plus interest owed on that debt) must be covered by the value of their house. As well as these two standard resource constraints, we also assume that LTV and LTI borrowing constraints exist:

$$-A_{i,t} \leq (1-\phi)\, Q_t H_{i,t} \quad \textbf{(9)}$$

$$-A_{i,t} \leq \theta_i\,(R_t)\, Y_{i,t} \quad \textbf{(10)}$$

The LTV constraint **(9)** is standard and controls the amount agents can borrow against their homes. The parameter ϕ indicates the minimum size of deposit that the household has to put down.[24] We assume that the maximum LTI ratio θ in **(10)** depends both on age and the nominal interest rate. This captures the idea that lenders tend to ration credit on the basis of total loan size compared to current income and on the basis of the affordability of the loan compared to current income. The latter would depend on the servicing cost of the loan and hence the nominal interest rate. As discussed in Section 2, that implies that a fall in the inflation rate can effectively loosen borrowing constraints.

[23] Note that this setup implies that households will never hold both financial assets and debt at the same time. This is not completely unrealistic since, as briefly discussed in Section 2, very few households hold even a small amount of both financial assets and debts.

[24] Since agents can adjust their debt without cost, the LTV ratio will have to be satisfied in every period. Put differently, agents in our model will never go into negative equity. In order to allow for that possibility we would have to formally model mortgage contracts with a refinancing cost.

3.4 Endowment

An individual's income $Y_{i,t}$ varies by age and time:

$$Y_{i,t} = Y_t^p y_t y_i \tag{11}$$

$$\frac{Y_{t+1}^p}{Y_t^p} = G^p G^y \tag{12}$$

We assume that both real (at rate G^y) and nominal (at rate $G^p G^y$) incomes grow at deterministic rates in the long run **(12)** and that real income varies deterministically over the life cycle, as reflected in y_i (y_t is the stationary component of income which is common to all age groups). The latter assumption is designed to capture tendencies for productivity and labour supply to increase over the earlier parts of an individual's life and then fall over later parts of an individual's life.

3.5 Housing supply

In the long run (steady state) we assume that the housing stock grows in line with the population (at rate G^n). This assumption, together with Cobb-Douglas preferences in the utility function, implies that long-run house price inflation (G^q) is equal to the long-run rate of nominal income growth per capita **(13)**.

$$\frac{Q_{t+1}^p}{Q_t^p} \equiv G^q = G^p G^y \tag{13}$$

$$q_t = \frac{Q_t}{Q_t^p} \tag{14}$$

In the short run, we assume that housing supply is responsive (with some elasticity χ) to deviations of the price of housing from its long-run trend **(14)**. This assumption is consistent with the findings by Poterba (1984) that changes in the price of housing are sufficient statistics for explaining changes in the supply of new houses.[25] We also assume that the supply of housing does not shrink if house prices fall. Putting the assumptions about the short and long run together gives the housing supply equation:

$$\frac{H_{t+1}^s - H_t^s}{H_t^s} = \max\left\{G^n\left(\frac{q_{t+1} - q_t}{q_t}\right)^{\chi}, 0\right\} \tag{15}$$

[25] Cerny, Miles, and Schmidt (2005) also make this assumption.

3.6 Aggregation and equilibrium

Aggregate variables in our economy can be computed by summing the decisions of individual agents. For example, aggregate non-durable consumption can be defined as **(16)**, where $M_{i,t}$ is the number of households at age i in period t

$$\overline{C}_t = \sum_i M_{i,t} \cdot C_{i,t} \quad \textbf{(16)}$$

We define any other aggregate quantity in the same way.

In each period t, an equilibrium in our economy is a value function and a policy rule for each household, a path for the price of housing, $\{Q_t\}_{t=1,2,...}$, and a set of bequests received, $\left\{\left\{B_{i,t}\right\}_{i=1,..,N}\right\}_{t=1,2,..}$, such that, given initial conditions for the population $\left\{M_{i,1}\right\}_{i=1,..,N}$, the state $\left\{S_{i,1}\right\}_{i=1,..,N}$, and housing supply H_1^s, and paths for all the exogenous variables (eg $\{R_t\}_{t=1,2,..}$):

(1) $V_{i,t}$ solve the functional equation **(4)** and $C_{i,t}(S_{i,t})$, $H_{i,t}(S_{i,t})$, $A_{i,t}(S_{i,t})$ are the associated policy rules that satisfy the constraints **(7)-(10)**.

(2) The housing market clears. That is, the total demand for housing is equal to the total supply H_t^s:

$$\sum_i M_{i,t} H_{i,t}(S_{i,t}) = H_t^s \quad \textbf{(17)}$$

where H_t^s evolves according to **(15)**.

(3) Bequests evolve according to:

$$\sum_i M_{i,t+1} B_{i,t+1} = \sum_i M_{i,t}(1-\zeta_i)\left\{R_t A_{i,t}(S_{i,t}) + (1-\delta) Q_{t+1} H_{i,t}(S_{i,t})\right\} \quad \textbf{(18)}$$

and are redistributed uniformly among the surviving agents, ie $B_{i,t+1} = \bar{B}_{t+1}$.

(4) The population evolves according to **(1)-(2)**.

A stationary equilibrium or steady state is defined as above, but prices and the population are assumed to grow at constant rates over time and the population distribution is assumed to have converged to its long-run state.

3.7 Solution method

An analytical solution to this problem does not exist and computing the approximate solution to this model is not straightforward. Appendix A contains the main details, but the algorithm can be understood by the following step-by-step procedure. The procedure is similar to the algorithm described in Auerbach and Kotlikoff (1987).

1. Reduce the number of state variables by transforming the model economy into an equivalent and stationary version.
2. Take a guess at the trend stationary time path for house prices and the level of bequests received by individual agents.
3. Solve the dynamic programming problem implied by the agent maximisation problem to derive the optimal life-cycle housing and non-durable consumption profiles conditional on these guesses.
4. Aggregate over individuals' consumption choices to derive a time-path for total housing demand and bequests. Given aggregate housing demand, define a new time path for house prices and bequests received using a tatonnement algorithm (see Judd (1998) for a description). If the new set of house prices and bequests are close enough to the old set, go to step 5. If not, update the guesses and repeat steps 3 and 4.
5. Check that the economy has converged to a steady state and that the solution is sufficiently accurate. That is, that the housing market has cleared and that the errors in approximating the dynamic programming problem are acceptably small.

4 Calibration

A period in the model is set to represent five years. This should be short enough to allow for a fairly smooth variation in behaviour across the life cycle, but not so short that the computational demands become extreme. The model is parameterised by splitting the parameters into two sets. The first set comprises those parameters whose values have been estimated in many previous studies or estimated directly from the data without the use of our model. The second set of parameters are selected so that our model roughly matches some key features of the data. More specifically, we want the debt to income and house price to income ratios from the first period in the model's demographic transition (under the assumption that the levels of the nominal interest rate and inflation rate would remain at the same levels as observed on average between 1987 and 1991) to match the average of the ratios observed in the data between 1987 and 1991 as closely as possible.[26] The reason why we focus on a demographic transition rather than a steady state in our calibration is that the demographics forces are such that the population and hence our model would reach a steady state only after several decades. Using such a steady state as a baseline to analyse the effects of recent changes in the exogenous variables (eg interest rates) could therefore have led to misleading conclusions.

The parameters of the model are summarised in Table A. We turn now to describing our choices in detail.

Table A: Parameters of the model

	i. Fixed parameters:			Source:
Γ	See Table B		Fertility and survival rates	ONS (2006), Shaw (2006)
Ω	See text		Immigration rates	ONS (2006)
z_i	See Table B		McClements' equivalence scales	ONS (2004)
G^y	0.025		Real per capita income growth (per annum)	National Accounts
y_i	See Table B		Age-related earnings	Campbell and Cocco (2006), OECD (2005)
α	0.91		Share of non-housing consumption	National Accounts – see text
δ	0.02		Housing stock depreciation rate (per annum)	National Accounts
χ	0.5		Price elasticity of supply	Barker (2004)
ϕ	0.90		Maximum LTV ratio	SML – see text
θ	See Table C		Maximum LTI ratio	See text
	ii. Calibrated parameters:			
γ^{-1}	1/2		Intertemporal elasticity of substitution	
β	1.015		Discount factor (annual)	
ξ_1	4.7		Strength of bequest motive	
ξ_2	10		Degree to which bequests are luxury	

Demographics: Adult life begins at 20 ($i_0 = 5$ in model time) and ends at 90 ($I = 18$), so there are $I - i_0 + 1 = 14$ different adult age cohorts. We calibrate the fertility rates κ_i and conditional

[26]We also need to specify an initial condition for the age distribution of wealth, which is the state variable in the consumers' maximisation problems $\{S_{i,1}\}_{i=1,..,N}$. The literature identifies two broad approaches to calibrating a wealth distribution of this sort. The first is to use micro data. The second is to use the distribution from a 'pseudo steady state' of the model whereby the population structure observed in the period 1987-91 would have been taken as being stationary (even if in reality it was not). Since we do not have micro data on the age distribution of wealth prior to 1995, we use the second method.

survival probabilities ζ_i using data from the Government's Actuary Department (GAD).[27] Annual fertility and survival rates are averaged over the period 1987 to 2006 to give the time-invariant rates collected in matrix Γ in **(2)** (Table B reports these values together with other age-related parameters, which will be described later).[28]

We make a strong assumption about immigration. We assume that all migrants arrive at age 20 (ie that $\omega_i = 0$ for all i except $i = 5$) and that they are identical to members of the native population. This assumption is not particularly realistic, but it means that we do not have to separately compute immigrants' decisions. It can also be defended on the grounds that the bulk of immigrants arrive at a young age (see ONS (2006)) and are likely to carry little or no wealth with them. Since we do not allow for emigration, we calibrate the immigration rate so that it matches total net immigration into the United Kingdom.[29]

Given an initial condition μ_1 for the population distribution (ie the population by age group in 1987-91), and net immigration rates for every period, we can use **(1)** and **(2)** to generate the population distribution in all successive periods. The population distributions thus generated are very close to the latest projections provided by GAD. Details and comparisons are provided in Appendix B.

Preferences: We set the share of non-durable consumption in the utility function, α, so that the model share of expenditure on housing is consistent with the intratemporal optimality condition for the relationship between housing consumption and non-durable consumption given an estimate for the user cost of housing and the observed UK data for non-durable consumption, housing wealth, goods prices, interest rates and an estimate of housing depreciation.[30] For the

[27] The projections are reported in ONS (2006). See also Shaw (2006) for a summary.

[28] These fertility and survival rates imply a negative rate of steady state population growth of -2.53% per year (the largest stable eigenvalue from the matrix Γ). But with a moderate amount of immigration (see below), the steady state population growth is small and positive. In our case, it is 0.39% per year.

[29] The net immigration rates are the following: 0.44% for 1991-96; 1.17% for 1997-2001; 1.54% for 2002-06; and 1.20% for all subsequent periods. These were chosen on the basis of the actual data available from ONS (2006) and the assumptions embodied in the latest Government Actuary Department population projections. More precisely, the GAD projections assume a constant net inflow of 145,000 persons as of 2007-08. We translated this figure into a flow of 1.20% of the total population. We then assumed that this percentage flow would remain constant in the future. This means that the net inflow expressed in number of persons is assumed to increase with the total size of the population, unlike in the GAD projections, which assumes the net inflow to remain constant at 145,000. We do this for simplicity since it allows us to determine the steady-state growth rate of the population directly, using the matrix $\Gamma + \Omega$. In practical terms, the assumption makes little difference at least in terms of a comparison between the projections made using our model and the projections GAD have made (see Appendix C for details).

[30] An unconstrained consumer's intratemporal optimality condition is the following:

Table B: Age-related parameters					
Age (i)	Fertility rates (κ_i)	Survival rates (ζ_i)	Initial popul. (μ_1)	Equiv. scale (z_i)	Earnings (y_i)
0-4	0.0000	0.9938	0.066	-	-
5-9	0.0000	0.9995	0.063	-	-
10-14	0.0000	0.9992	0.060	-	-
15-19	0.0727	0.9976	0.071	-	-
20-24	0.1770	0.9970	0.080	1.0093	1.0197
25-29	0.2426	0.9966	0.080	1.0390	1.3876
30-34	0.2377	0.9960	0.070	1.0882	1.6825
35-39	0.1115	0.9946	0.067	1.1527	1.8516
40-44	0.0215	0.9913	0.071	1.1897	1.8843
45-49	0.0011	0.9864	0.058	1.1725	1.8066
50-54	0.0000	0.9789	0.054	1.1194	1.6626
55-59	0.0000	0.9645	0.052	1.0477	1.4967
60-64	0.0000	0.9443	0.051	1.0086	1.3431
65-69	0.0000	0.9101	0.050	1.0004	0.6237
70-74	0.0000	0.8481	0.039	1.0000	0.6085
75-79	0.0000	0.7633	0.033	1.0000	0.5937
80-84	0.0000	0.6387	0.021	1.0000	0.5792
85-89	0.0000	0.4763	0.010	1.0000	0.5651

equivalence scale z_i we use the McClements equivalence scale (eg ONS (2004)) (Table B).[31] We calibrate the intertemporal elasticity of substitution γ^{-1}, the discount factor β, and the bequest function coefficients ξ_1 and ξ_2 jointly so that the initial aggregate ratios (debt to income and house price to income ratios) match up reasonably closely with the data.[32]

Constraints: Borrowing constraints are chosen on the basis of the Survey of Mortgage Lenders' (SML) data shown in Section 2. In particular, we set the LTV to be no larger than 0.9 ($\phi = 0.1$).

$$\frac{H_{i,t}}{C_{i,t}} = \frac{\alpha}{1-\alpha}\frac{P_t^c}{P_t^h}$$

where P_t^c is the price of non-housing consumption (normalised to one) and P_t^h is the user cost of housing, ie $P_t^h = Q_t\left(1 - E_t\frac{Q_{t+1}}{Q_t}\frac{(1-\delta)}{R_t}\right)$. So, the above expression can be rewritten as:

$$\frac{\alpha}{1-\alpha} = \frac{Q_t H_{i,t}}{P_t^c C_{i,t}}\left(1 - E_t\frac{Q_{t+1}}{Q_t}\frac{(1-\delta)}{R_t}\right)$$

In the period 1987-2006 the average ratio of housing wealth to nominal consumption ($Q_t H_{i,t}/P_t^c C_{i,t}$) was 3.17. Over the same period the nominal interest rate was on average 6.6% ($R_t = 1.066$). Assuming that housing depreciation was equal to 2% ($\delta = 0.02$) and expected capital gains from housing were the same as our model's long-run assumption about house price inflation ($Q_{t+1}/Q_t = 1.053$ or 5.3% per year), then we get that $\alpha = 0.91$.

[31] We have to make some adjustments because children in our model fall into five-year age brackets which do not exactly coincide with those proposed in the McClements scale. Specifically, we take a weighted average across the relevant age brackets. For example, to calculate the scaling factor for age 1 children (0-4 years) we multiply the 0-1 year old scale (0.09) by 0.4 and add it to 0.6 times the 2-4 year old scale (0.18), giving 0.144. Overall, the equivalence scale is thus calculated as: 1+0.144*number of children aged 0-4+0.218*number of children aged 5-9+0.254*number of children aged 10-14+0.342*number of children aged 15-19.

[32] The rate of time preference implied by the calibrated value of the discount factor in our model falls within the two point estimates provided by Hurd (1989).

Roughly 70% of new mortgagors have LTVs less than 90% and those with larger LTVs might face higher interest rates. We set the the LTI constraint by assuming that lenders would not allow the initial repayment to exceed 30% of households' disposable incomes were they to borrow using a standard credit-foncier (variable-rate) mortgage contract with a duration of 25 years.[33] From this we can calculate a LTI constraint for any given nominal interest rate. Table C reports the values obtained from these calculations conditional on the nominal interest rate in the first four periods of the model.[34]

Table C: Loan to income constraints

Period	Nominal rate	LTI constraint
t	R_t	$\theta(R_t)$
1987-91	9.80%	2.77
1992-96	5.96%	3.84
1997-2001	5.67%	3.95
2002-06	4.32%	4.54

Endowment: We calibrate the real rate of per capita income growth to be 2.5% per annum. For pre-retirement individuals, we calibrate the age-profile of income from estimates by Campbell and Cocco (2006) using data from the Family Expenditure Survey.[35] For individuals who have reached retirement age (age 10, equivalent to 65-69), we calibrate the income they receive in retirement to 47.6% of the income they received in the period before retirement (ie age 13 or 60-64). That is the average replacement ratio for the United Kingdom as reported by the OECD (2005). During retirement, we assume that income grows at the rate of inflation. The age profile of income is shown in Table B.[36]

Housing supply: We calibrate the depreciation rate of housing to be 2% a year. We calibrate the

[33] The exception to this is that we do not allow an agent to take on a contract which would last for longer than the maximum amount of time he or she can live. For example, an agent aged 65-69 can live at most to age 85-89 in our model. We assume that for this agent the credit-foncier contract cannot last more than 20 years (or 4 periods in model units). This implies that for a given income and interest rate, the maximum this agent can borrow is less than a younger agent. The LTI ratios for age 10, 11, 12, 13 and 14 (65–69, 70-74, 75-79, 80-84, 85-89) agents are 2.59, 2.31, 1.86, 1.14, 0 in period 1, 3.44, 2.91, 2.21, 1.26, 0 in period 2, 3.53, 2.97, 2.24, 1.27, 0 in period 3, and 3.97, 3.27, 2.4, 1.32, 0 in period 4.

[34] The data shown in the last row of Table C were correct at the time the model was calibrated and the simulations were carried out. Since then the observation for 2006 Q4 has become available and some revisions have occured. However, the differences are small and do not have any material implications for the simulations and analysis presented below.

[35] The estimation procedure is described in Attanasio (1999). In summary, the natural logarithm of household labour income is regressed on age dummies, cohort dummies, time dummies, and demographic variables. A third-order polynomial in age is then fitted to the resulting age dummies and the age-profile for income is obtained by adding the age effects from the polynomial over the five-year age groups.

[36] The profile in Table A excludes the effects of growth (ie y_t^p has been removed and y_t has been set to one). As a result, the income profile reported in this table declines at the rate of real income growth through retirement. When nominal income growth is added back in, that profile would grow at the rate of inflation.

price elasticity of supply χ to be 0.5, in line with the average of all the estimates for the price elasticity of housing supply contained in the Barker review (2004) of housing supply. Most of these estimates for the elasticity of housing supply were made using data from the entire post-war period. It is possible that the elasticity was lower between 1987 and 2006. To account for that possibility, we also examine how sensitive our results are to the elasticity being half the size (ie $\chi = 0.25$).[37] Finally, we calibrate the initial condition for housing supply H_1^s so that the housing supply to income ratio matches the average in the data between 1987 and 1991.

Table D: Aggregate ratios in the baseline model

	Actuals (1987-91)	First Period Baseline
House prices	12.11	13.02
Consumption	0.94	0.90
Debt	0.71	0.70
Financial wealth	1.76	2.04
Net financial wealth	1.05	1.34
Housing wealth	3.27	3.55
Bequests	0.06	0.06

Note: See footnote 39 about the house price to income ratio.

The calibrated model: Table D summarises the main outcomes of the calibration exercise.[38] The calibrated model does a good job at matching the house price, housing wealth, bequests, and debt to income ratios. By contrast, the model overstates aggregate gross financial wealth and, as a result, also overstates aggregate net financial wealth.

5 Model simulation and experiments

In this section we use the calibrated model to gain some insights into the impact that the changes in the population and in the macroeconomic environment presented in Section 2 might have had on the household sector and the housing market. The first experiment is meant to illustrate the potential effects of changing demographics over time assuming that the inflation rate and the real interest rate remain at the same levels as observed on average between 1987 and 1991, the initial

[37] Malpezzi and Maclennan (2001) show the ranges within which plausible estimates of the price elasticity of housing supply should fall.

[38] Note that the house price to which we calibrate the model is obtained by dividing nominal housing wealth by the real housing stock as calculated by the ONS. The house price to income ratio is obtained by dividing that house price by total disposable income. The resulting ratio takes values which may look unfamiliar (eg one way to compute some of the more familiar published ratios is to take the average value of a home and divide it by average household income). The scale of the ratio is not in itself of much interest in our analysis, since we are mainly interested in the relative variations brought about by changes in the economic environment faced by households.

period of our analysis. We call this the baseline model because it is under these assumptions that we have calibrated the model.

The subsequent experiments are meant to illustrate the potential effects of the sharp disinflation and the fall in the real interest rate experienced from 1992 onwards. In our experiments we are particularly interested in three things: the size of the response; the length of time the economy takes to reach steady state; and the comovements between variables.

5.1 Demographic transition to the steady state

Here we report the transition to the steady state of the baseline model given the actual and the projected structures of the population. These projections are reasonably consistent with the official projections from the Government Actuary Department (GAD) (see Appendix B for details). Chart 9a reports the transition in ratios of aggregate variables to aggregate earnings, while Chart 9b reports the transition in per-capita variables. Long-run trends are stripped out from all variables. In other words, the charts describe the deviations of the ratios and per-capita variables from their balanced growth paths (see Appendix A for details).

There are several results worth noting. First, the house price to earnings ratio, and consequently the housing wealth to earnings ratio, falls during the demographic transition (Chart 9a). House prices themselves actually increase during transition (Chart 9b), but at a slower rate than aggregate income, which is why the ratio falls.[39] The rise in house prices is explained by the increase in the size of the population which boosts demand for housing.

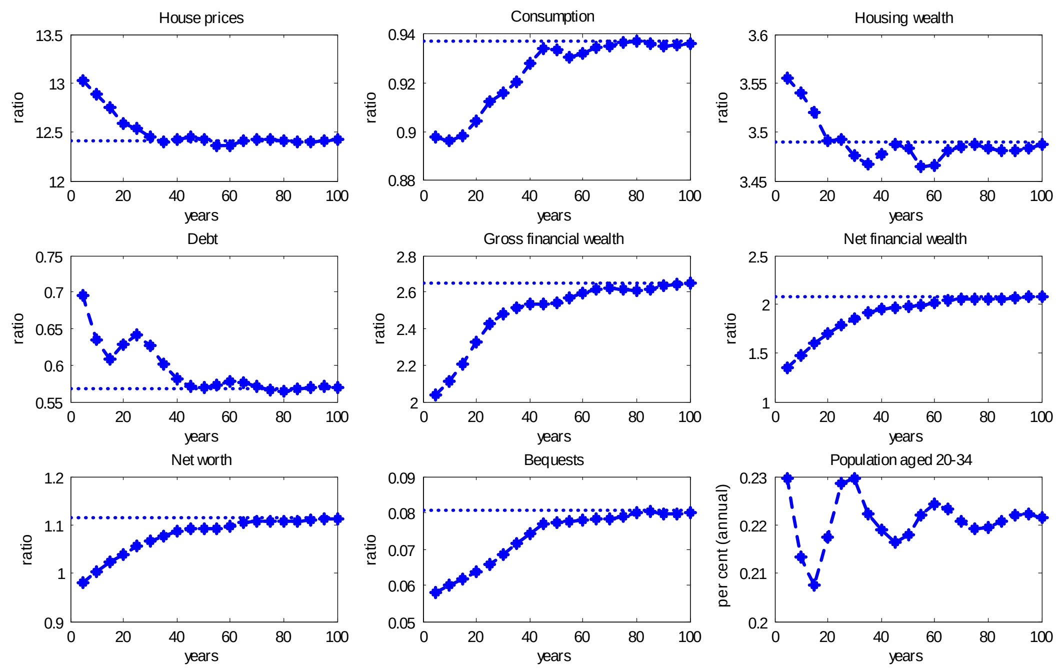
Second, the debt to income ratio falls and displays an oscillatory pattern (Chart 9a). This needs some explanation. Consumption is smoother over the life cycle than financial wealth, as agents accumulate debt in the early stages of their life, repay it, and then start accumulating positive financial wealth. This means that if there is a larger cohort of people moving through time, this would have a larger impact on debt and financial wealth than on consumption. Indeed, the panel in the bottom right of Chart 9a shows that the oscillatory pattern of debt roughly corresponds to the share of the youngest cohorts (20-34 year olds) in the population since it is these cohorts that

[39]Changes in aggregate earnings in the model reflect changes in both the size and distribution of the population. During the demographic transition the (detrended) size of the population increases. The result is that aggregate earnings are approximately 11% higher in the steady state than at the start of the transition.

hold most of the debt. Per-capita consumption changes very little over the transition (Chart 9b). If agents smooth consumption over their lives we would not expect a change in the size and structure of the population to have as large an impact on per-capita consumption as on wealth. And this is indeed the case as shown in Chart 9b.

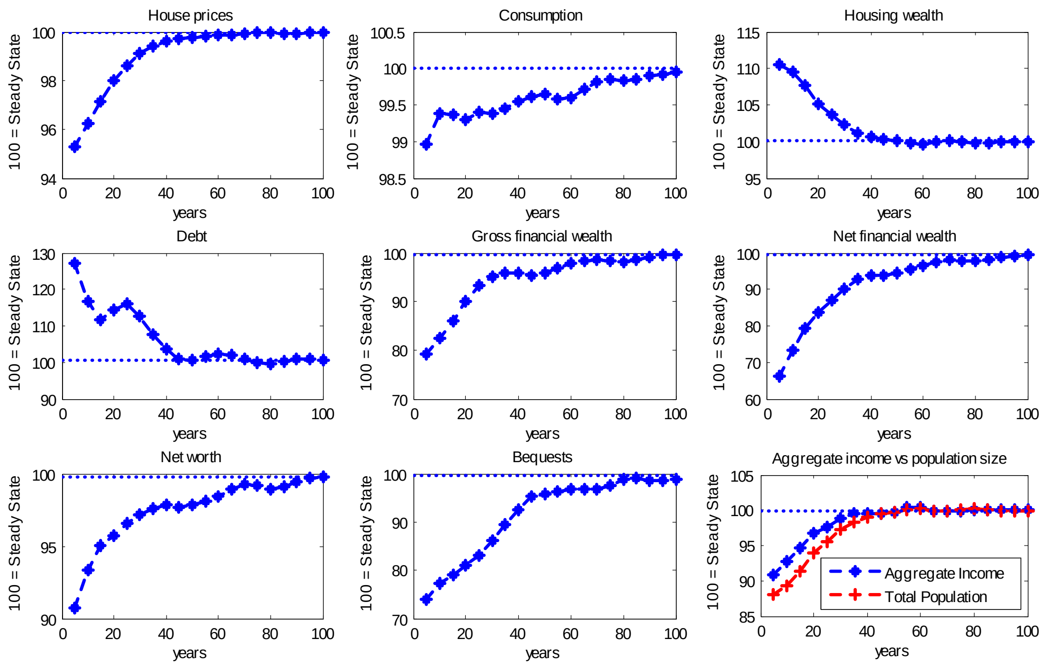
Third, the non-housing consumption, the net financial wealth and the net worth ratios all rise (Chart 9a). These patterns are consistent with the changes in the size and the structure of the population (Chart 9c). Over the transition, the relative number of middle-aged to older households increases and, in our model, middle-aged and older households tend to hold less housing wealth but more financial wealth. So, a larger number of middle-aged to older households would tend to boost aggregate financial wealth at the expense of housing wealth. Overall, the model suggests that although demographic factors can have a relatively large effect on the household sector's aggregate balance sheet, they do not seem capable of producing the movements in debt and house prices that we have observed in the data. In particular, house prices increase by less than 5% during the demographic transition, far less than observed in the data between 1987 and 2006 (see Chart 1).

Chart 9a: Aggregate ratios during the demographic transition – baseline economy



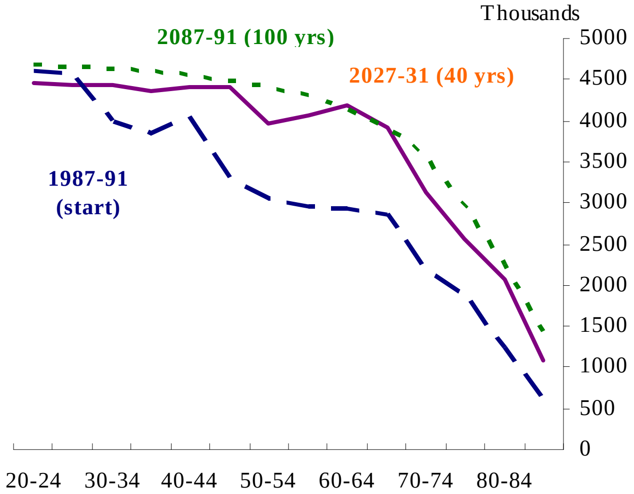
Note: Ratios obtained by scaling aggregate variables by aggregate earnings.

Chart 9b: House prices and per-capita variables along the demographic transition – baseline economy



Note: Per capita variables obtained by scaling aggregate variables by population size.

Chart 9c: Model-generated structure of the population



5.2 *Shocks to the baseline economy*

In this section we carry out a number of experiments to understand how the economy reacts to the type of macroeconomic changes observed since the beginning of the 1990s: a sizable fall in the inflation rate and a fall in the real interest rate. The experiments consist of 'shocking' the baseline economy along its demographic transition to the steady state. That is, we evaluate the impact of unanticipated changes in the inflation rate and the real interest rate, while also taking into account the evolution of the population. The magnitudes of the changes (shocks) are set to reflect those observed in the data between 1992 and 2006 (Table E; figures are averages of five-year periods).[40] While we assume the change in inflation to be permanent, we consider two different scenarios about the future behaviour of and expectations for the real interest rate. In the first we assume that the real rate remains at the same low level as observed between 2002 and 2006. In the second we assume that the real interest rate returns gradually to its level in the first period of the model (1987-91) following an autoregressive process of order one with autoregressive coefficient of 0.5.

Table E: Actual aggregate ratios and prices

	1987-91	1992-96	1997-2001	2002-06
Real interest rate	3.16%	3.14%	3.02%	1.73%
Inflation rate	6.44%	2.73%	2.57%	2.55%
Nominal interest rate	9.8%	5.96%	5.67%	4.32%
House prices	12.11	9.03	9.69	14.97
Consumption	0.94	0.88	0.92	0.95
Debt	0.73	0.77	0.79	1.06
Financial wealth	1.76	1.83	2.14	1.90
Net financial wealth	1.03	1.06	1.35	0.84
Housing wealth	3.15	2.34	2.68	3.91
Total wealth	4.91	4.16	4.83	5.81
Net worth	4.18	3.39	4.04	4.75

There are three assumptions implicit in our experiments that are worth noting. First, the observed changes in the inflation rate and in the real interest rate are unanticipated by agents at the time that they occur. Second, the type of shocks we consider here are 'zero-probability' shocks as, by construction, agents do not factor aggregate uncertainty into their decisions. Third, once a shock is realised agents are assumed to be able to correctly predict the path of market-clearing prices which arise as a result of that shock (perfect foresight). Yet they cannot predict any subsequent shocks. Put differently, any perfect-foresight path will be valid until a new shock arrives, at which time agents revise their optimal decisions consistently with a new perfect-foresight path.

[40]The values reported in the last column of Table E were correct at the time the analysis was conducted. See also footnote 31.

5.2.1 A permanently lower real interest rate

Chart 10 and Chart 11 show the results of the experiment in which the real interest rate fall is permanent. Chart 10 shows the effects of the fall in real interest rate and inflation rate on the aggregate variables, expressed as percentage deviations from the baseline economy's adjustment path,[41] while Chart 11 provides a comparison of the cross-section profiles of the main variables at different points in time.

A first interesting finding is that the house price to earnings ratio rises little in the first ten years of the transition when the only quantitatively significant change is the fall in the inflation rate. The fall in inflation and the nominal interest rate acts by increasing the maximum LTI ratio at which households can borrow (with the aim of capturing reduced front-end loading – see Section 2.3.2). As a result, it only directly affects the behaviour of the youngest cohorts, who were credit constrained in the high inflation, high nominal interest rate world. By contrast, the house price to earnings ratio rises substantially once the real interest rate falls in period 4. It rises by about 50%, overshooting its new equilibrium which is about 40% higher than the initial steady state. Other variables also react strongly to the fall in the real interest rate. Debt jumps by about one and a half times its baseline level and then continues rising towards its new equilibrium, where it is almost four times as high as in the high interest rate world. It takes a relatively long time for debt to reach its steady state level, reflecting gradual turnover in the housing market as young consumers, who take on more debt than their predecessors, replace older consumers.

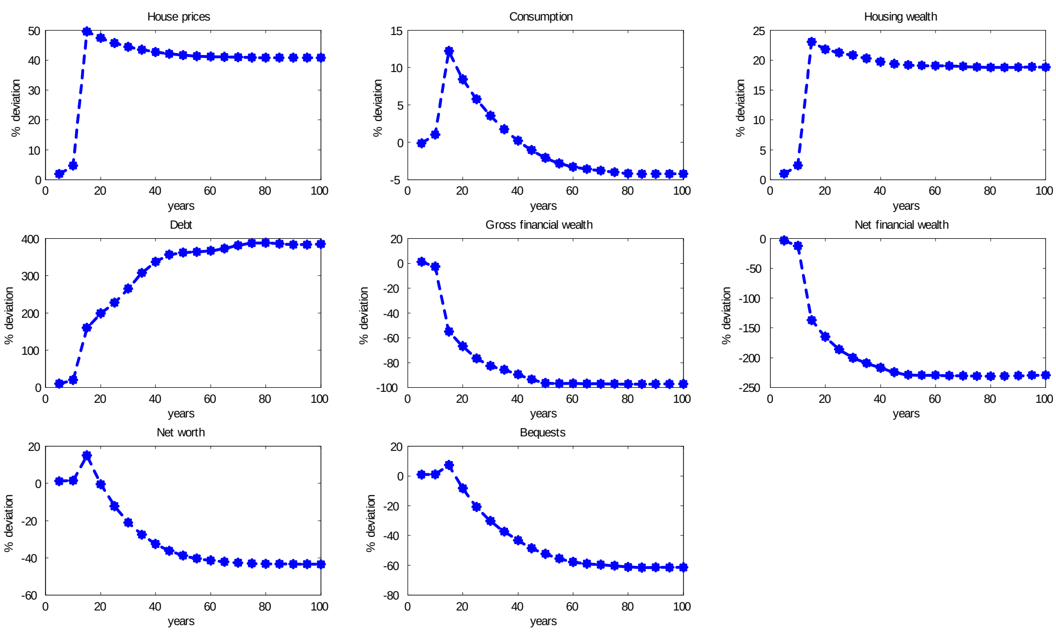
A by-product of the lower real interest rate is an initial consumption boom (about 12% above trend). Both housing consumption and non-durable consumption increase in response to the fall in real interest rates, but the response across different age groups varies, as shown in Chart 11. The latter compares the transition's first period in the baseline model economy (the orange line) to different periods in the 'shocked' model. The first cross-section in the 'shocked' model corresponds to the large fall in the inflation rate at the start of the transition (the blue line). The second cross-section corresponds to the large fall in the real interest rate in the 2002-06 period (the green line). And, finally, the third cross-section is the steady state one (the red line) to which the cross-sections converge over time. The chart shows that housing consumption

[41] As the economy converges in both experiments to new steady states, the deviations will not necessarily settle down to zero since the steady state of the shocked economy has a different inflation rate and/or real interest rate to the baseline economy.

increases for all groups apart from the very old and the very young. The old are made better off by the rise in house prices and choose to substitute away from housing consumption into non-durable consumption. By contrast, the young are made worse off and consume less of both housing and non-durable goods. The changes in housing wealth by age are broadly consistent with those from the data discussed in Section 2.

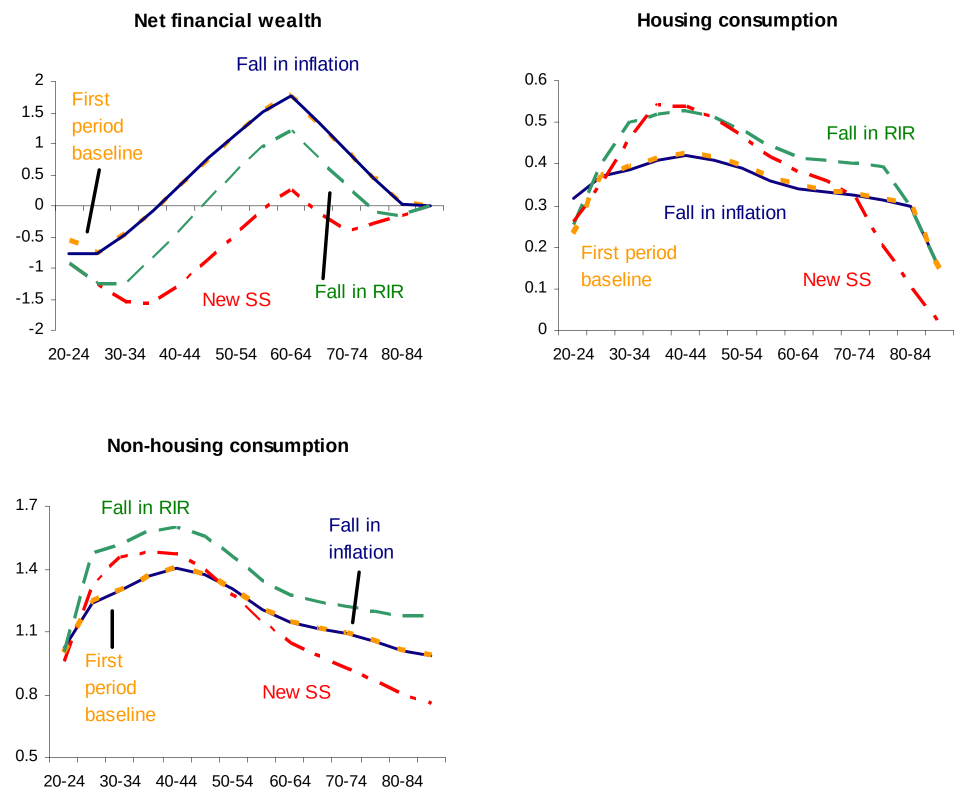
As time goes by, and older generations are replaced by younger generations, the aggregate consumption boom subsides. Eventually, consumption falls below its initial level. That is because it is optimal for younger consumers to consume more in the lower real interest rate world. That has the effect of reducing the resources left available for consumption later in life. In other words, the lifetime consumption profile becomes tilted towards the early part of life. Appendix C complements the results in this section by displaying standard impulse responses from the steady state of the baseline economy of a 1% fall in the real interest rate and a 4% fall in the inflation rate. These impulse responses help to clarify the impact of lower inflation and lower real interest rates separately.

Chart 10: Shock to baseline – permanent reduction in the real interest rate



The counterpart to the strong rise in debt and consumption is a sharp fall in gross and net financial wealth. Chart 11 shows that all age groups reduce their net financial wealth in response to lower real interest rates. That is, the youngest groups borrow more and the older groups run down part or all of their financial wealth. These results fail to match both the aggregate and dissaggregate data discussed in Section 2. That analysis showed that net financial wealth and consumption has remained broadly stable in the aggregate data, while more borrowing by younger generations has been partly offset by increased net financial wealth of older generations.

Chart 11: Shock to baseline – cross-sections (permanent reduction)



During the transition, the net financial wealth profile continues to gradually shift down towards the 'new steady-state' cross-section. In this process, the age at which agents switch from being debtors to creditors gradually rises. The result is that positive financial wealth almost disappears in the new steady state (Chart 11).

5.2.2 A temporarily lower real interest rate

We now turn to what we call the 'temporary scenario'.[42] We consider the same sequence of observed shocks as above, but we assume that the real interest rate returns to its original value

[42] See also Appendix C for a complementary impulse response analysis.

according to an autoregressive process with coefficient of 0.5. That is, the initial deviation of the real interest rate from its original (baseline) value is halved every five years (ie every model period). There are two main points to note. First, relative to the permanent shock changes in the variables are much smaller (Chart 12). For example, the house price to income ratio rises above equilibrium by about 7% while consumption rises by less than 2%. Debt rises by about 26% and falls slowly towards its new steady state value which is some 12% higher than the baseline economy's steady state. Relatively larger is the fall in gross and net financial assets. Second, as the real interest rate returns to its original level, middle-aged consumers substitute out of housing consumption and into financial wealth (Chart 13). Only the youngest and oldest cohorts consume more housing services in the new steady state, in which only inflation has changed. The relaxation of borrowing constraints implied by lower inflation and a lower nominal interest rate allows the youngest consumer to borrow more and consume more housing.

The main message from the 'temporary scenario' experiment is that the expectation or belief that observed changes in the real interest rate are permanent is crucial in generating house price and debt increases of comparable magnitude to those documented in Section 2.

Chart 12: Shock to baseline – temporary reduction in the real interest rate

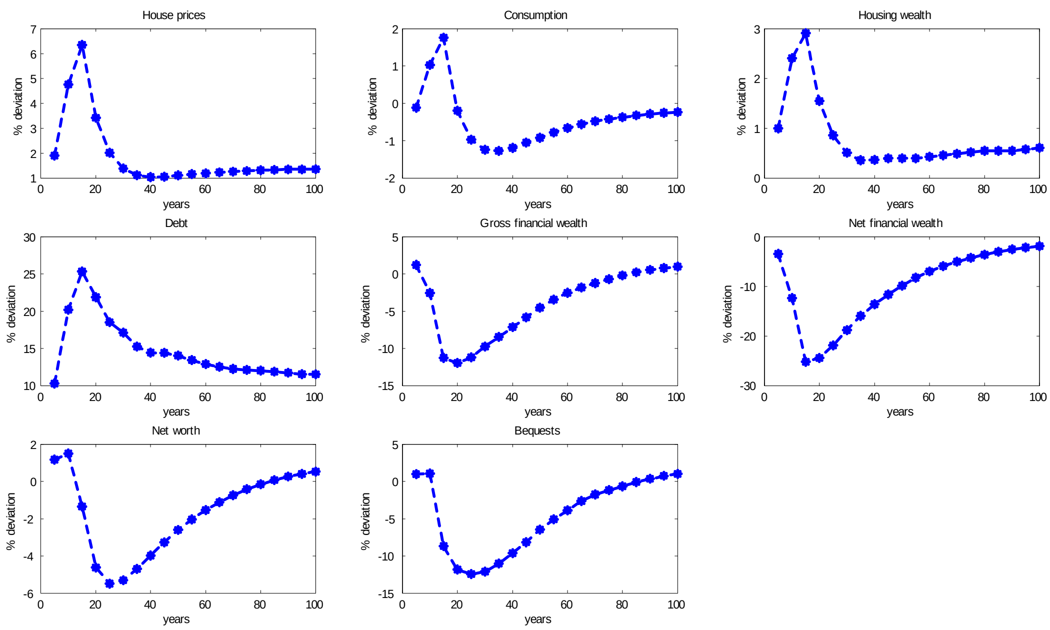
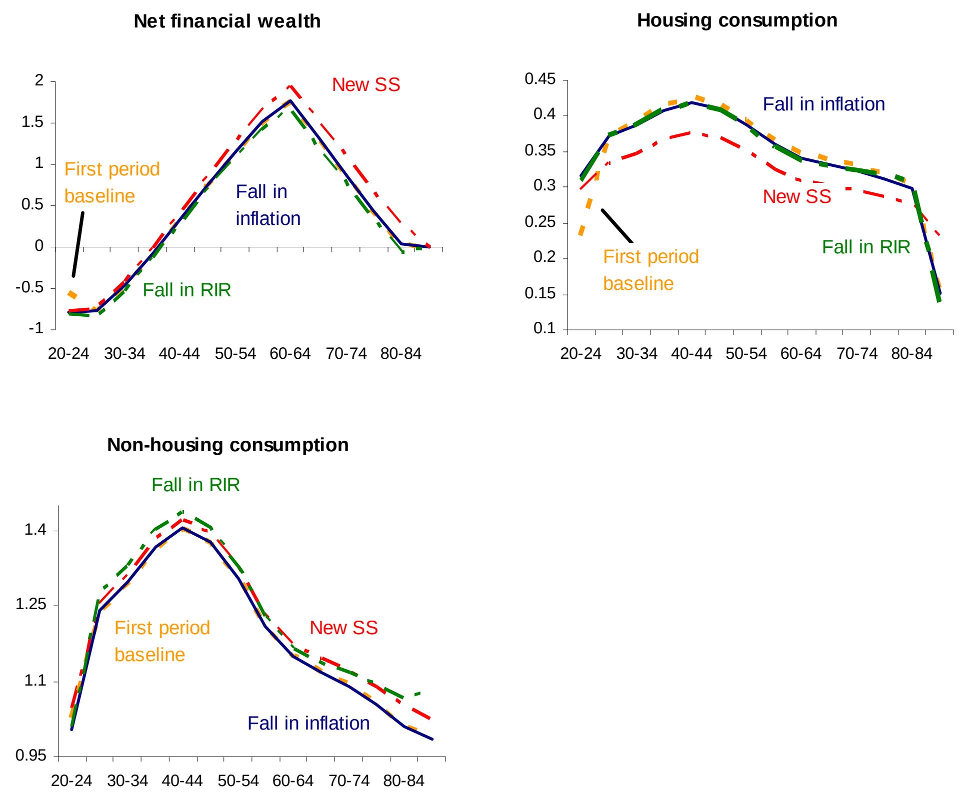


Chart 13: Shock to baseline – cross-sections (temporary reduction)



5.2.3 A summary of the perfect-foresight experiments

Having traced out the effects over time of disinflation and lower real interest rates, we now look at whether the model can account for the rise in house prices, debt and consumption observed in the data between 1987 and 2006. The previous analysis has shown that it is the real interest rate that has contributed the most to changes in house prices. So here we focus on the 2002-06 period in which the largest fall in the real interest rate occurred. It is important to bear in mind that the model has been built with the purpose of shedding light on medium and long-term developments in the household sector. Therefore, it is not well suited to matching cyclical fluctuations in the data. For example, the anlaysis has not explained the bust in the housing market that occurred in the early 1990s; nor could it be expected to explain the dramatic developments in the housing market and wider economy that occurred in 2008 and 2009, following the onset of the banking crisis in 2007. But the model can say something about more long-lived adjustments triggered by demographic changes, a drop in the real interest rate and permanently lower inflation. In other words, it provides a benchmark against which to assess the observed changes in the variables documented in Section 2. Any large deviation from it would prompt thinking about what other factors not captured by the model might explain the differences.

Table F shows that the model suggests that the observed decline in the real interest rate can explain most of the rise in house prices that took place after 2001, provided that agents expected the current low level of the real interest rate to persist indefinitely. In that case, the model more than predicts the increase in housing wealth and net worth. Furthermore, it largely overpredicts the rise in debt and underpredicts the rise in financial wealth. The counterpart to these facts is that the model overpredicts consumption. The persistence of the real interest rate, as perceived by households, is obviously of great importance. If agents expect the low real interest rate to be temporary and to revert back to its original level, the model can hardly explain any of the rise in house prices or debt.

Table F: Perfect-foresight experiments

	HP	C	HW	D	FW	NFW	NW
Actual: change since 1997-2001	54.5%	3.3%	45.9%	34.2%	-11.2%	-37.8%	17.6%
Model:							
Permanent scenario change since 1997-2001	44.1%	11.1%	70.1%	107.0%	-51.8%	-145.5%	15.4%
Temporary scenario change since 1997-2001	2.5%	.07%	1.2%	-.03%	-4.9%	-7.6%	-1.0%
Low housing elasticity case: Permanent scenario change since 1997-2001	54.5%	11.9%	69.9%	89.2%	-43.3%	-119.9%	21.2%
Temporary scenario change since 1997-2001	2.2%	0.8%	0.4%	-1.1%	-4.2%	-5.9%	-1.2%

HP: house price; C: non-durable consumption; HW: housing wealth; D: debt;
FW: financial wealth; NFW: net financial wealth; NW: net worth.

These results may hinge on the calibration of the price elasticity of the housing supply. To check how the results are dependent on this parameter, we recomputed the model with the value of the elasticity halved ($\chi = .25$). With this lower value (and assuming that agents expect the real interest rate to remain low permanently) the model can explain all of the increase in house prices. The model overpredicts the rise in debt and the fall in financial wealth by less than with the original elasticity, but it overpredicts net worth by more.

6 Conclusion

We have built an OLG model of household behaviour and calibrated it to the UK economy with the aim of understanding to what extent changes in demographics, lower inflation, and a lower real interest rate can explain the observed rise in debt and house prices between 1987 and 2006, before the events unleashed by the financial turbulence that began in 2007.

Our model suggests that demographic factors can be an important determinant of the household sector's aggregate balance sheet, but do not seem capable of producing the size of the movements in debt and house prices observed in the data. Moreover, the effects of demographic change are very gradual and so could not account for the sharp rise in house prices observed towards the end of the period considered.

Instead, our analysis suggests that the main driver of the rise in house prices and debt was the decline in the real interest rate, most of which occurred at the start of the millennium. For the decline in real interest rate to be the driving factor, our analysis suggests that it is essential that households perceived it as permanent. In that case, the model can more than explain the rise in debt and can explain most of the rise in house prices. If housing supply is less price elastic than has been assumed in the central case, then the model can explain all of the rise in house prices. But this conclusion depends on the period of comparison. For example, under the central case, the model overexplains the rise in house prices since 1991, but does not explain all of the rise since 1996. In addition, house prices were substantially higher by the end of 2006 than they were on average between 2002 and 2006. If we were to take the end of 2006 as our benchmark, the model explains less of the rise in house prices. So, overall, the results can be viewed as suggesting that the increase in house prices between 1987 and 2006 was broadly consistent with other changes to the UK macroeconomy over that period.

Moreover, another consequence of the fall in the real interest rate is that consumption booms, which did not in fact happen to any marked extent over the period in question. As such, we are unable to explain one stylised fact from the data. That is, that the boom in house prices was not accompanied by a boom in consumption; but was instead accompanied by an accumulation of both financial assets and financial liabilities. Exploring the factors behind that stylised fact would be an interesting avenue for future research. But this failure of the model also suggests that our results should be interpreted with caution, especially with regard to what the precise equilibrium levels of house prices and debt were by the end of 2006.

There are a number of ways the analysis in this paper could be extended and improved. Extending the analysis in any of these directions would very likely alter the conclusions to a greater or lesser extent. First, the model was primarily of the demand side. Adding a supply side would be an important robustness check since choosing the amount of labour to provide, for example, would give households another means through which to adjust to changes in the economic environment. Moreover, adding a supply side would also allow for a proper accounting of income. In particular, the assumption that increasing the size of the population has no effect on the income that individual consumers earn is a strong one.

Second, there are some features of the demand side that we abstract from. In the real world, housing is costly to adjust. The result is that individuals tend to move home fairly infrequently. That infrequent adjustment could partly explain the higher persistence of house price inflation in the data than in our model. However, the fact that units of time in our model span five years should minimise the implications of our abstraction, given that five years is likely to be a long enough period of time for much of the influence of housing adjustment costs on aggregate household behaviour to have dissipated. Of more importance is likely to be our abstraction from income uncertainty which, at the household level, is an important determinant of household saving. That we have not accounted for income uncertainty (or uncertainty of any other sort), could partly explain why the response of debt to lower real interest rates in our model is so large.

Third, we assumed that individuals faced no uncertainty over the future paths of inflation and real interest rates. Without this assumption, it would have been extremely difficult to solve the model. However, the assumption is unlikely to be innocuous. The conclusion that the levels of debt and house prices observed between 2002 and 2006 were consistent with other parts of the UK economy depended crucially on households expecting real interest rates to remain permanently low. Although they may have believed that that was the most likely scenario, it seems unlikely that they would not have take into account the possibility that interest rates might change at some point in the future.

Appendix A. Solving the model

This appendix describes how to solve the model. The first step is to transform the model economy to a stationary equivalent. Working with a trend stationary economy is more convenient because all variables are stationary in the steady state. That means that there is no need to keep track of how variables grow over time (ie time is eliminated as a state variable). In the model outlined above, variables grow over time because prices grow, income grows, and the population grows. The first step in transforming the model is to strip out population growth. This can be done by normalising the matrices Γ& $\mathbf{\Omega}$ by the largest eigenvalue of $\Gamma + \mathbf{\Omega}$. The second step is to strip out growth in prices and in incomes. Define the following transformed variables **(A-1)**.

$$y_{i,t} = \frac{Y_{i,t}}{Y_t^p},\ s_{i,t} = \frac{S_{i,t}}{Y_t^p},\ a_{i,t} = \frac{A_{i,t}}{Y_t^p},\ b_{i,t} = \frac{B_{i,t}}{Y_t^p},\ c_{i,t} = \frac{P_t^{cp} C_{i,t}}{Y_t^p},\ h_{i,t} = \frac{Q_t^p H_{i,t}}{Y_t^p} \qquad \textbf{(A-1)}$$

Now, given that agents' utility functions are of the Cobb-Douglas variety, we can define the following transformed maximisation problem **(A-2)-(A-6)**.

$$v_{i,t}(s_{i,t}) = \max_{c_{i,t},h_{i,t}} \{u(c_{i,t}, h_{i,t}; z_{i,t}) + \widetilde{\beta}[\zeta_i v_{i+1,t+1}(s_{i+1,t+1}) + (1-\zeta_i) w(s_{i+1,t+1})]\} \qquad \textbf{(A-2)}$$

$$V_{i,t}(S_{i,t}) = \left(\frac{P_t^y}{\left(P_t^p\right)^{\alpha} \left(P_t^q\right)^{1-\alpha}} \right)^{1-\gamma} v_{i,t}(s_{i,t}) \qquad \textbf{(A-3)}$$

$$\widetilde{\beta} = \left(\frac{G_t^y}{\left(G_t^p\right)^{\alpha} \left(G_t^q\right)^{1-\alpha}} \right)^{1-\gamma} \beta \qquad \textbf{(A-4)}$$

$$u(c_{i,t}, h_{i,t}; z_{i,t}) = \frac{z_{i,t}}{1-\gamma} \left[\left(\frac{c_{i,t}}{z_{i,t}}\right)^{\alpha} \left(\frac{h_{i,t}}{z_{i,t}}\right)^{1-\alpha} \right]^{1-\gamma} \qquad \textbf{(A-5)}$$

$$w(s_{i+1,t+1}) = \frac{\xi_1}{1-\gamma} \left[\left(1 + \frac{\frac{R_t}{G^y} a_{i,t} + \frac{G^q}{G^y} q_{t+1}(1-\delta) h_{i,t}}{\xi_2} \right) \left(\frac{\alpha}{p_{t+1}^c}\right)^{\alpha} \left(\frac{1-\alpha}{p_{t+1}^h}\right)^{1-\alpha} \right]^{1-\gamma} \qquad \textbf{(A-6)}$$

Similarly, we can define the following transformed constraints.

$$y_{i,t} + b_{i,t} + \frac{R_{t-1}}{G^y} a_{i-1,t-1} + \frac{G^q}{G^y} q_t (1-\delta) h_{i-1,t-1} = p_t^c c_{i,t} + q_t h_{i,t} + a_{i,t} \quad \textbf{(A-7)}$$

$$G^q q_{t+1}(1-\delta) h_{I,t} + R_t a_{I,t} \geq 0 \quad \textbf{(A-8)}$$

$$-a_{i,t} \leq (1-\phi)\, q_t h_{i,t} \quad \textbf{(A-9)}$$

$$-a_{i,t} \leq \theta_i (R_t)\, y_{i,t} \quad \textbf{(A-10)}$$

These transformations also lead to the following stationary equivalent housing supply equation.

$$\frac{h_{t+1}^s - h_t^s}{h_t^s} = \max\left\{ \left(\frac{q_{t+1} - q_t}{q_t} \right)^{\chi}, 0 \right\} \quad \textbf{(A-11)}$$

The transformed household maximisation problem has two endogenous state variables $s_{i,t} = \{a_{i-1,t-1}, h_{i-1,t-1}\}$ and two choice variables $x_{i,t} = \{c_{i,t}, h_{i,t}\}$. The number of state variables can be reduced by noting that since there are no costs to adjusting either financial assets or housing assets, the composition of agents' balance sheets at the start of the period is irrelevant to them. All that matters is the total amount of resources available to them. That observation allows us to define a single state variable which, following Carroll and Deaton, we call cash-on-hand.

$$s_{i,t} = y_{i,t} + b_{i,t} + \frac{R_{t-1}}{G^y} a_{i-1,t-1} + \frac{G^q}{G^y} q_t (1-\delta) h_{i-1,t-1} \quad \textbf{(A-12)}$$

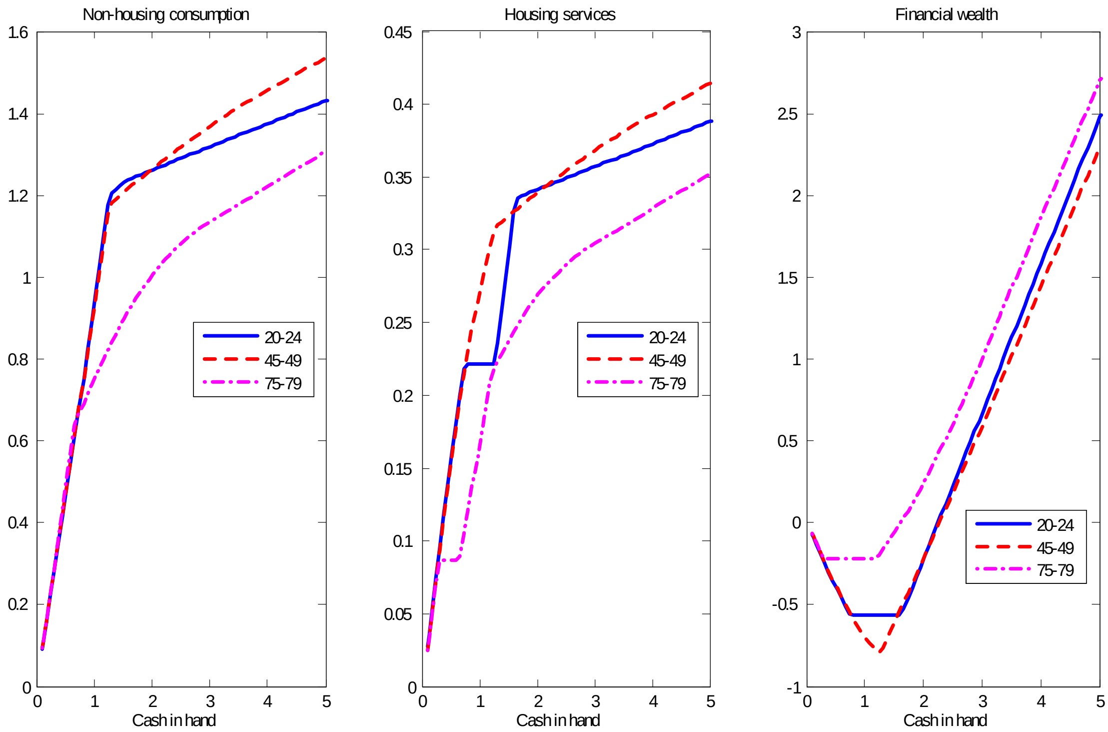
Having defined the state and control variables, we solve the dynamic programming problem using value function iteration and interpolation. Starting from the last period of life (age I), we know that the value function depends only on utility of consumption in that period and from bequests left behind after death **(A-13)**.

$$v_{I,t}(s_{I,t}) = \max_{c_{I,t}, h_{I,t}} \{ u(c_{I,t}, h_{I,t}; z_{I,t}) + \widetilde{\beta} w(s_{I+1,t+1}) \} \quad \textbf{(A-13)}$$

This simplifies the problem. However, the value function in **(A-13)** is still an infinite dimensional object. Following the literature, we approximate that infinite dimensional object using a discrete grid of points across the state space $s_{i,t}$ For each point on that grid, the problem is reduced to a static constrained maximisation problem, which can be solved using any constrained optimisation software. (We use the MATLAB toolbox function FMINCON.) Having solved the maximisation problem at each point on the grid, we store the terminal values.

We then work our way back through the agent's life storing the values and the decision rules. At each age, the stored values are used to interpolate between grid points on next period's value function using a cubic spline. After reaching the first adult age, we have decision rules for agents at every age. The following figures plot some selected decision rules from our model's steady state.

Chart A.1: Decision rules



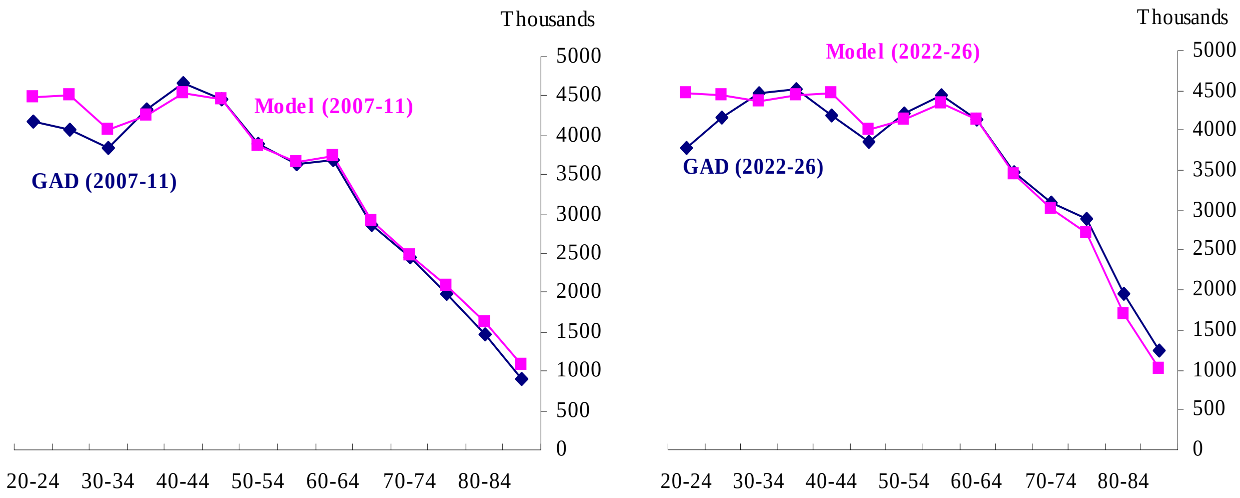
With the decision rules in hand, we can solve forward to determine agents' optimal life-cycle non-durable consumption, housing, and financial wealth demand conditional on a guess at the equilibrium path for house prices and bequests. We can then iterate on the guess at the market clearing prices until we find the approximate equilibrium. The calibration section shows what these look like in the first period of the model's transition.

Appendix B. Population projections

This appendix provides a comparison of the population projections generated by our model and those generated by the Government Actuary Department (GAD). An alternative to using a model to forecast the population would have been to use GAD's projections. There are two advantages to using a model in the way we have done. First, it gives us the flexibility to forecast the population in any year or time period we choose. GAD only publish projections for particular years. Second, the population is guaranteed to settle on a stable distribution with a constant growth rate. That allows the population part of the model to interact seamlessly with the rest of the model in terms of its ability to generate a balanced growth path steady state.

Having said that, we have made some simplifying assumptions. In particular, we assumed that the fertility, mortality and net immigration rates are constant. Inevitably, the GAD projections incorporate more sophisticated assumptions about the evolution of fertility, mortality and net immigrations rates in the future and, as such, are likely to be better forecasts. Therefore, it is important to check that our model population projections are not too far away from the latest projections made by GAD in 2004.

Chart B.1: Model population projections compared to GAD



Sources: Government Actuary Department 2004 population projections and Bank calculations.

Chart B.1 compares the model's population projections to GAD's projections averaged over two different five-year periods. The chart on the left compares the two projections for the period 2007-11 and the chart on the right does the same for 2022-26. There are two things to take away from these charts. First and most importantly, the model's projections are very similar to GAD's, even as far as 20 years ahead. Second, our model predicts a larger 20-30 year old population than GAD. That reflects the fact that we are lumping all net immigration into the youngest age group (20-24 year olds). These differences become slightly more pronounced as the projections are rolled forward.

Overall, what this appendix makes clear is that the population projections made with our model are quite similar to official projections. Moreover, given the uncertainty associated with any population projection (King (2004)) or any other type of projection, our projections seem reasonable.

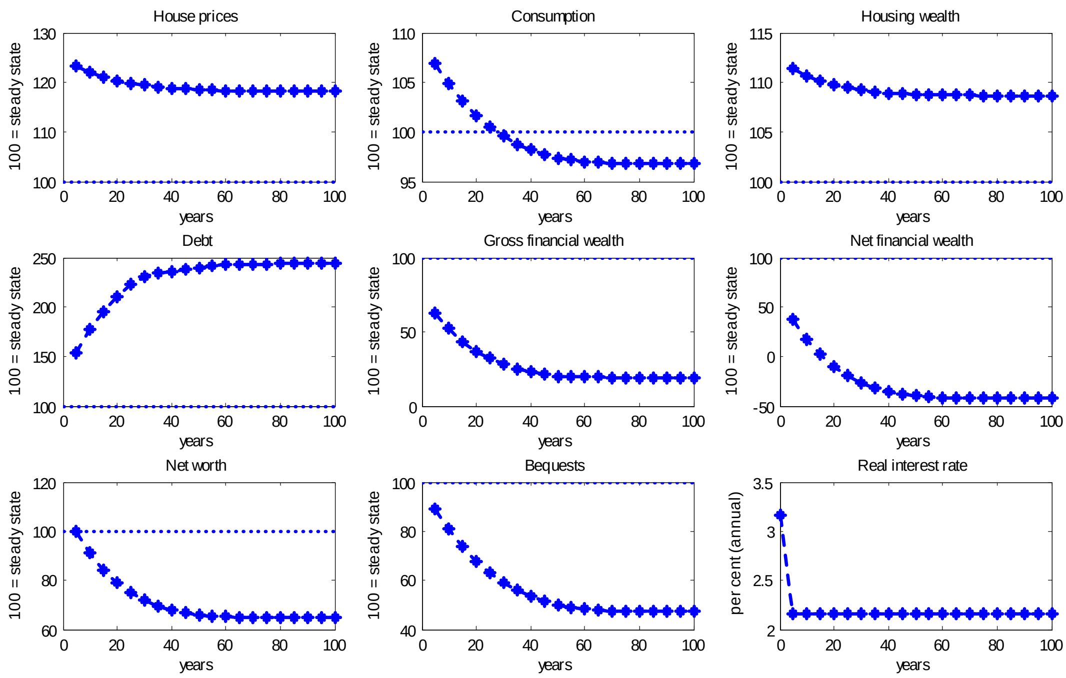
Appendix C. Impulse response analysis

In this appendix we report a number of experiments designed to understand how the economy works. In each, we start from the steady state and compute the model's response to unanticipated zero probability shocks to the real interest rate and inflation. By assuming that the population also remains in its stationary state we are able to isolate the effects of each shock.

Permanent fall in the real interest rate

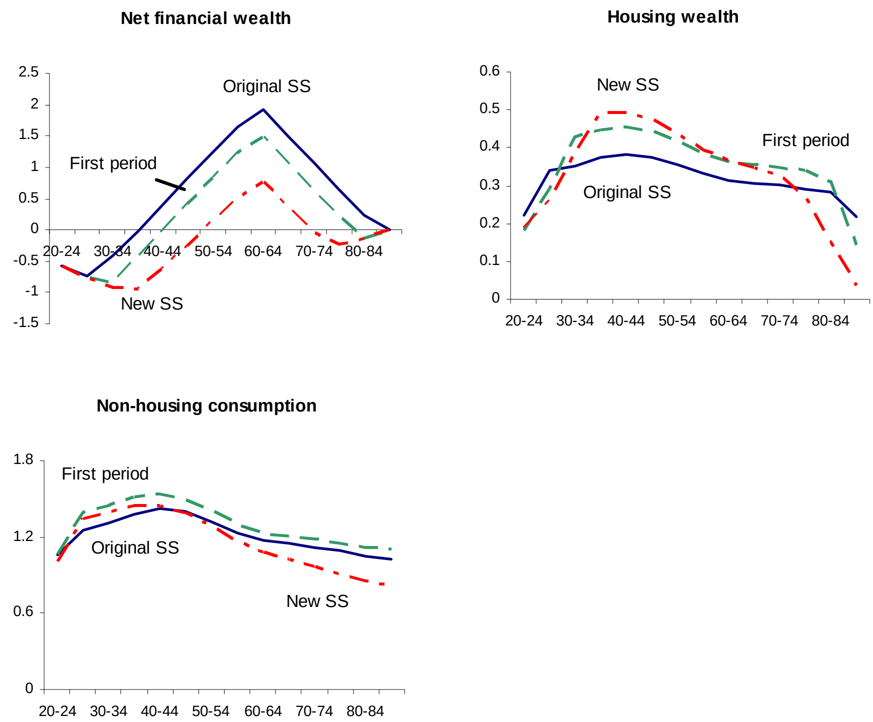
In this experiment, we consider a permanent 100 basis points (1 percentage point) fall in the real interest rate (Chart C.1). We also assume that the parallel reduction in the nominal interest rate does not alter the LTI constraints faced by households. In other words, we keep the borrowing constraints unchanged so that the response solely reflects changes in the real interest rate.

Chart C.1: Permanent 100 basis points fall in the real interest rate



The house price to income ratio rises by almost 25% on impact, overshooting its new equilibrium, which is around 18% higher than the initial steady state. Most of the adjustment takes place within 20 years. The consumption to income ratio jumps up by 7% initially as all agents apart from the youngest two cohorts (who are constrained) react to the lower interest rate by consuming more (Chart C.2). Consumption takes much longer to reach its new steady state level than house prices, implying that the correlation between consumption and house prices weakens during the transition. When consumption eventually reaches its new steady state, it is lower than in the higher real interest rate steady state. That is because it is optimal for agents to borrow and consume more when they are younger. The result is that they have less resources available for consumption later in life (Chart C.2).

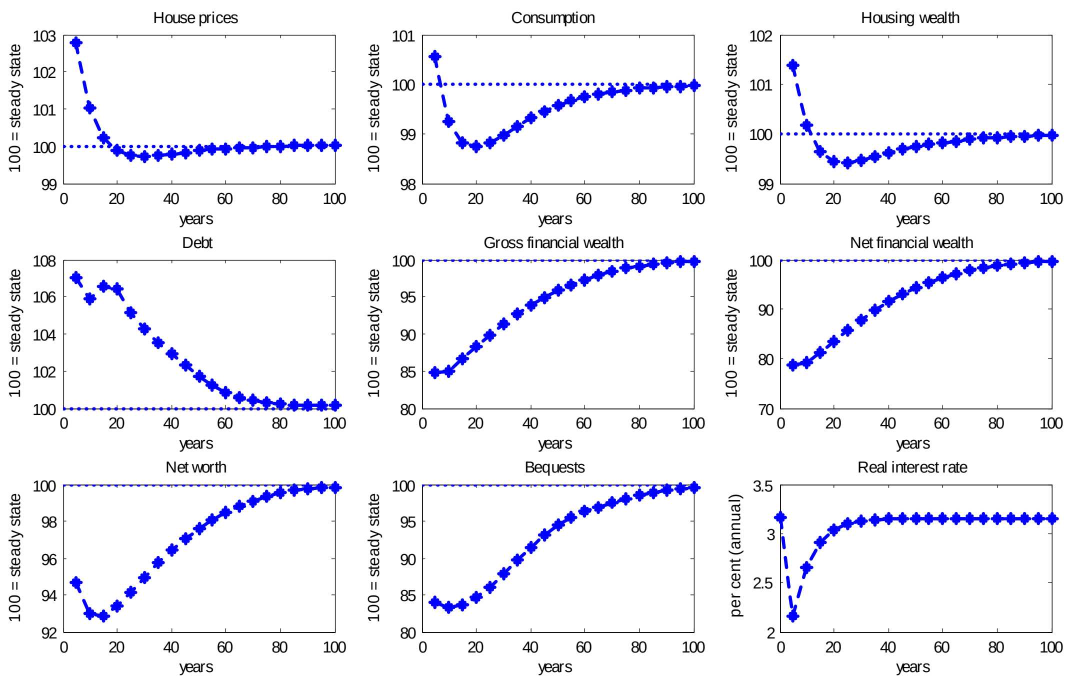
Chart C.2: Cross-sections: permanent 100 basis points fall in the real interest rate



Transitory fall in the real interest rate

Here we consider a shock to the real interest rate of the same magnitude as above, but we assume that it is temporary (Chart C.3). Specifically, the real interest rate is assumed to revert back to its original value according to a first order autoregressive process with coefficient of 0.5. There are two main points to note. First, relative to the permanent shock changes in the variables are much smaller. For example, the house price rises above equilibrium by about 3% while consumption rises by less than 1%. Again, the two youngest cohort cannot take advantage of the lower real interest rate because they are constrained (Chart C.4). Second, the positive comovement between consumption and house price is stronger for more of the transition than in the permanent shock case.

Chart C.3: Temporary 100 basis points fall in the real interest rate (AR = .5)

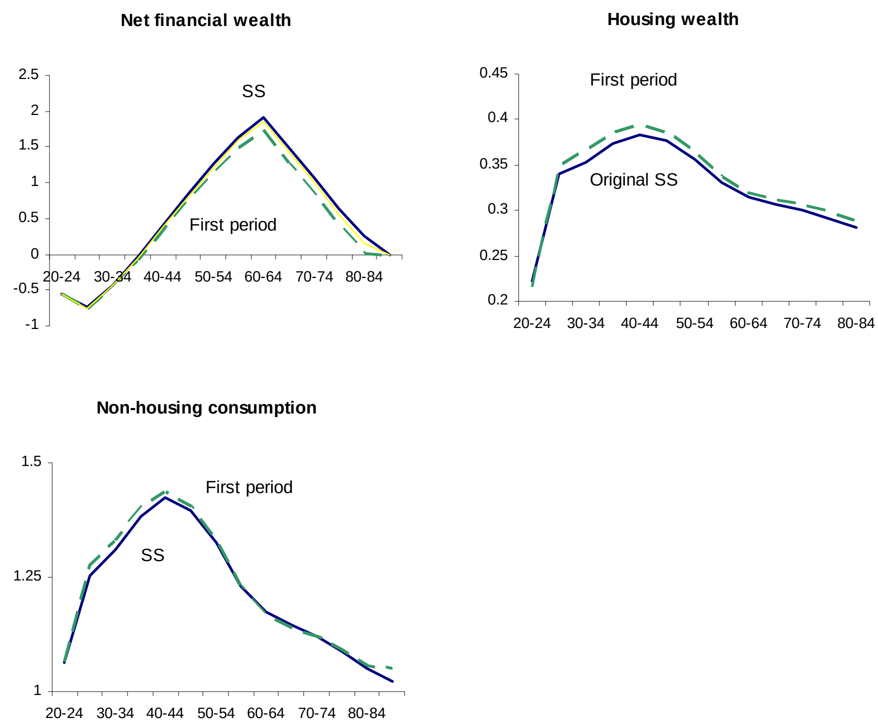


Permanent fall in the inflation rate

Here we consider a permanent fall in the inflation rate of 400 basis points, roughly the change observed over the 1990s (Chart C.5). Lower inflation in our model has a real impact because it lowers the nominal interest rate, which in turn relaxes the LTI constraint. Putting an upper bound of 30% on the fraction of disposable income that agents employ to serve their mortgage debt and assuming a constant nominal repayment schedule through a credit-foncier contract implies different maximum LTI ratios for different levels of the nominal interest rate. (See Section 3 for a description of the constraint and Section 4 for the calibration.) Accordingly, in our model a reduction in the nominal interest rate of 400 basis points raises the LTI constraint from 2.77 to 3.95.

An interesting finding is that the impact of the shock is very small for most of the variables. In particular, aggregate consumption is virtually unchanged,[43] while debt rises by slightly more than 10%. Inspection of the cross-section distributions (not shown here) reveal that the relaxation of the borrowing constraint only benefits the youngest cohort which is now able to borrow significantly more. The new available resources are then spent mainly on housing services, thereby causing the house price to increase. All the other generations, except the very old, accumulate more financial resources, although the change in their financial position is very small. That only the youngest cohort benefits from a relaxation of the borrowing constraint should not be surprising. Despite the fall in inflation, the real interest rate does not change, so it is not optimal for other cohorts to borrow more even if they are allowed to do so. For a lower real interest rate, however, young consumers are constrained and so would not be able to expand their borrowing as they would like.[44]

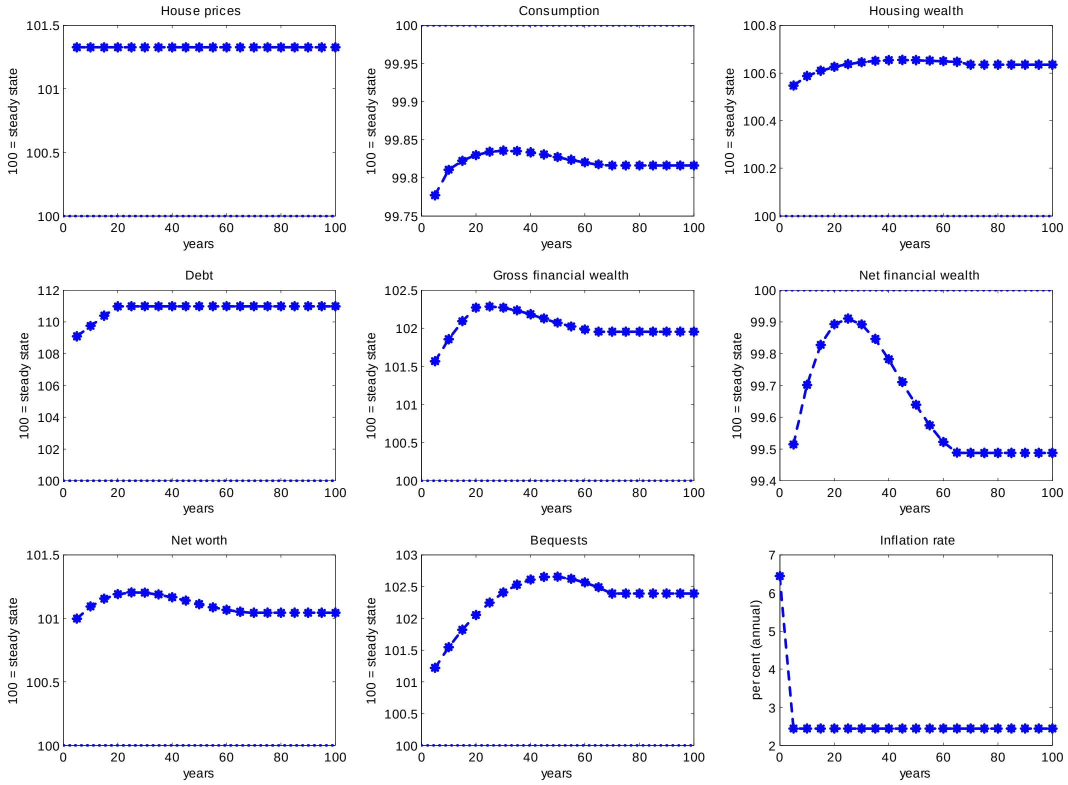
Chart C.4: Cross-sections: temporary 100 basis points fall in the real interest rate



[43] The small deviations from the steady state might be due to approximation error.

[44] Note that the model includes heterogeneity only among cohorts, not within cohorts. This means that the model is likely, to some degree, to underestimate the increase in debt, as low-income households would probably be the ones which could have benefited the most from a relaxation of borrowing constraints. On the other hand, the poorest households are also likely to borrow less.

Chart C.5: Permanent 400 basis points fall in the inflation rate



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