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Time-varying dynamics of the real exchange rate. A structural VAR analysis
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Abstract

The aim of this paper is to explore the evolution of real exchange rate dynamics over time. We use a time-varying structural vector autoregression to investigate the role of demand, supply and nominal shocks and consider their impact on, and contribution to fluctuations in, the real exchange rate, output growth and inflation in four major economies over the past four decades. Our analysis therefore extends recent empirical research on evolving macroeconomic dynamics which has primarily focused on inflation and output and the time-varying impact of monetary policy on these variables. In addition we generalise recent VAR studies on exchange rate dynamics where the analysis is limited to a time-invariant setting. Our main results are as follows. The transmission of demand, supply and nominal shocks to the real exchange rate, output and inflation has changed substantially over time. Demand shocks have a larger impact on the real exchange rate after the mid-1980s for the United Kingdom, euro area and Japan and after the mid-1990s for Canada. Nominal shocks had a larger impact on output and inflation during the 1970s relative to the recent past. The forecast error variance of the real exchange rate is explained mainly by demand shocks with a smaller role for nominal shocks.

Key words: Real exchange rate, time-varying VAR, sign restrictions, Bayesian estimation.

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Summary

Recent empirical studies have provided strong evidence to suggest that the persistence and volatility of macroeconomic variables has evolved over time in industrialised countries. In particular, this literature (albeit conducted on data prior to the onset of the financial crisis and consequent recession) shows that inflation was less volatile and persistent in recent years than in the 1970s. Moreover, measures of real economic activity were also less volatile. A strand of this literature also suggests that the transmission of shocks to the macroeconomy may have changed over time. One limiting feature of these studies, however, is the absence of any role for the real exchange rate in the models used. Instead, exchange rate dynamics have been investigated in an alternative strand of research using empirical models that do not allow for changes in the transmission of shocks. This is surprising given the weight of evidence that indicates a change in the dynamics of macroeconomic fundamentals such as output and inflation.

The aim of this paper is to reconcile these two empirical strands of the literature. We estimate a system of equations (a vector autoregression) to capture the relationship between the real exchange rate and output and inflation for four industrialised economies – United Kingdom, Japan, euro area and Canada vis-à-vis the United States. Our model allows this dynamic relationship to change over time. In addition it allows the volatility of shocks hitting these economies to change over time.

Our results are as follows. The effect of demand shocks on the real exchange rate has increased over our sample for the United Kingdom, euro area and Canada, with the current response (using data to 2008 Q4) larger than in the 1970s and the early 1980s. Similarly, nominal shocks (defined as an appreciation of the real exchange rate that leads to a fall in output and inflation) have a larger impact on the real exchange rate after the mid-1980s. A model that keeps the relationship between the real exchange rate and the macroeconomy fixed is unable to capture these changes in real exchange rate dynamics. There is also evidence that the relative importance of these shocks has changed over time. Nominal shocks are important for inflation in the late 1980s but less so in the more recent period. Supply shocks appear to have a limited role in explaining real exchange rate fluctuations. For Canada and Japan demand shocks have become a more important source of output fluctuations over the past ten years. Demand shocks have been the most important factor
for the real exchange rate for all countries and throughout the sample, accounting for around 80% of exchange rate fluctuations on average.
1 Introduction

Recent empirical analysis has shown that the dynamics and volatility of inflation and output have changed dramatically for industrialised countries over the past three decades. For example Cogley, Primiceri and Sargent (2008) show that the persistence of inflation in the United States has changed significantly after Paul Volcker took over the chairmanship of the Federal Reserve. Similar results are reported in Cogley and Sargent (2005) who also show that the volatility of shocks to US output and inflation has declined significantly. Results in Primiceri (2005) and Sims and Zha (2006) suggest that identified monetary policy, demand and supply shocks (in a structural vector autoregression (VAR) framework) also display this feature – ie their volatility is much smaller in the post-Volcker period relative to the 1970s.

One feature of the papers mentioned above is the absence of the exchange rate in the models used in these studies. Instead, exchange rate behaviour has been empirically investigated in a different context in another strand of this literature. Examples of papers that use structural VARs to examine the causes and consequences of exchange rate fluctuations include Farrant and Peersman (2006), Clarida and Gali (1994) and Kim (2001). Both Farrant and Peersman (2006) and Clarida and Gali (1994) focus on decomposing movements in the exchange rate into contributions from demand, nominal and supply shocks using a fixed-coefficient structural VAR. While Clarida and Gali (1994) find that exchange rate movements are predominantly driven by what they classify as a demand shock, Farrant and Peersman (2006) find some role for a ‘nominal’ shock.\(^1\)

But these studies do not consider the possibility that either or both the volatility of structural shocks in the economy and the transmission of these shocks may have changed over time. Therefore it is not clear whether the results reported on the importance of different shocks for exchange rate fluctuations reflect the current state of the economy or an average over the past. For example, the recent economic turmoil has been accompanied by large movements in exchange rates in a number of countries. Fixed-coefficient models are unable to account for the possibility that this crisis may have brought about a change in the transmission of shocks to the exchange rate and other macroeconomic variables. Similarly the 1970s and the 1980s were characterised by substantial differences in monetary and/or exchange rate regimes for several

\(^1\)Kim (2001) examines a slightly different question – he investigates the effect of foreign monetary expansions on domestic activity – output and the trade balance.
countries and it is unclear whether fixed-coefficient VARs are appropriate when using data that spans these years. In addition, there exists univariate evidence that real and nominal exchange rate dynamics may have changed over time. See for example Engel and Hamilton (1990), Taylor, Peel and Sarno (2001) and Engel and Kim (1999) for analyses that attempt to model non-linearities, time-variation in reduced-form empirical models that describe the exchange rate.

This paper attempts to address this criticism by examining the dynamics of the exchange rate within a time-varying structural VAR framework. In particular we estimate time-varying VAR models for the real exchange rate, real output and inflation for the United Kingdom, euro area, Japan and Canada relative to the United States and identify asymmetric demand, supply and nominal shocks. The aim of our analysis is to consider: (a) potential changes across time in response of the exchange rate to these shocks; and (b) the relative importance over time of the shocks we identify.

Our results suggest the following main conclusions:

- There is strong evidence that the transmission of demand, supply and nominal shocks to output growth, inflation and the real exchange rate has changed over time.
  - Nominal shocks had a strong impact on inflation and output growth in the 1970s and the 1980s that has generally become smaller over the sample period for the countries considered in our study. There is evidence to suggest that these shocks have had a larger impact on the real exchange rate over the past two decades.
  - For the United Kingdom, the United Kingdom vis-à-vis the euro area, the euro area vis-à-vis the United States and Japan the impact of demand shocks has risen post-1985. In contrast, the impact of this shock on the real exchange rate of Canada increases during the 1990s.
  - The sign of the impact of the supply shock on the real exchange rate, which we left unrestricted, varies with country and time. For the United Kingdom and the euro area positive supply shocks lead to a small appreciation (rise in our definition) of the real exchange rate. In contrast, they lead to a real depreciation in Japan, especially over the recent past.
– A comparison of the results with a fixed-coefficient VAR suggests that ignoring
time-variation can lead to substantially biased inference.

- There is evidence that the contribution of these shocks to the forecast error variance of output,
prices and the real exchange rate has changed over time.

– For the United Kingdom and the euro area, nominal shocks were the primary factor behind
relative price movements against the United States prior to the end of the 1990s, but supply
shock have become somewhat more important after that. For Japan, nominal shocks
explained 60% to 70% of prices, as well as output, during the 1980s. This shock explains
about 20% of real exchange rate forecast error variance for all countries, and generally less
of it at the beginning of the sample when the Bretton Woods system of fixed nominal
exchange rates was still in place.

– For Canada and Japan, relative demand shocks have become a more important source of
relative output forecast error variance over the past ten years. This shock has been the most
important factor for the real exchange rate for all countries and throughout the sample,
accounting for around 80% of exchange rate fluctuations on average.

– Supply shocks have become less important over time for Canadian inflation and somewhat
more important for relative euro-area inflation. They generally account for a large
proportion of relative output forecast error variability, but appear to have a limited role in
explaining real exchange rate forecast error variance.

The paper is organised as follows. The econometric model is described in Section 2. Section 3
presents reduced form results, while Sections 4 and 5 focus on structural results for each country.
Finally, Section 6 concludes.

2 Empirical model

Our empirical model is closely related to the specifications presented in Farrant and Peersman
(2006) and Clarida and Gali (1994). We estimate the following VAR model

\[ Z_t = \sum_{i=1}^{L} \phi_{jit} Z_{t-i} + \nu_t, \]  

(1)
where $Z_t = \{ \Delta y_t, \Delta p_t, \Delta q_t \}$ and $L$ is fixed at 2. Here $y_t$ denotes log real home country output relative to the United States, $p_t$ denotes log consumer prices relative to the United States and $q_t$ denotes the log real exchange rate with United States as the base country. We analyse the United Kingdom, Japan, the euro area and Canada as our candidate ‘home’ economies. Note that the use of relative variables in the model is related to our shock identification and is discussed in the section below.

The main departure from the VAR specification in Farrant and Peersman (2006) and Clarida and Gali (1994) is the fact that our model allows for time-variation in the VAR coefficients and the covariance of the residuals. We postulate the following law of motion for the coefficients $\phi$

$$\phi_t = \phi_{t-1} + \eta_t.$$  

As in Cogley and Sargent (2005), the covariance matrix of the innovations $v_t$ is factored as

$$VAR (v_t) = \Omega_t = A_t^{-1} H_t (A_t^{-1})'.$$  

The time-varying matrices $H_t$ and $A_t$ are defined as:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 \\ 0 & h_{2,t} & 0 \\ 0 & 0 & h_{3,t} \end{bmatrix}, \quad A_t \equiv \begin{bmatrix} 1 & 0 & 0 \\ a_{21,t} & 1 & 0 \\ a_{31,t} & a_{32,t} & 1 \end{bmatrix}.$$  

with the $h_{i,t}$ evolving as geometric random walks

$$\ln h_{i,t} = \ln h_{i,t-1} + v_t.$$  

Following Primiceri (2005), we postulate the non-zero and non-one elements of the matrix $A_t$ to evolve as driftless random walks

$$a_t = a_{t-1} + \tau_t,$$  

and we assume the vector $[v_t', \eta_t', \tau_t', v_t']$ to be distributed as

$$\begin{bmatrix} v_t \\ \eta_t \\ \tau_t \\ v_t \end{bmatrix} \sim N (0, V), \quad \text{with } V = \begin{bmatrix} \Omega_t & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \quad \text{and } G = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}.$$  

### 2.1 Identification of shocks

Clarida and Gali (1994) motivate the identification scheme used in their structural VAR by using
the implications of a stochastic two-country open economy model in the spirit of Dornbusch (1976). As shown in Clarida and Gali (1994) the model consists of the following equations:

\[ y_t^d = d_t + \eta q_t - \sigma \left[ i_t - E_t (p_{t+1} - p_t) \right] \]  \hspace{1cm} (6)

\[ p_t = (1 - \theta) E_{t-1} p^e_t + \theta p^e_t \]  \hspace{1cm} (7)

\[ m^s_t - p_t = y_t - \lambda i \]  \hspace{1cm} (8)

\[ i_t = E_t (s_{t+1} - s_t) \]  \hspace{1cm} (9)

where all variables except the interest rate are in logs and denote home relative to foreign levels.

Equation (6) represents an IS curve where relative output demand \( y_t^d \) depends positively on a demand shock \( d_t \) and the real exchange rate \( q_t = s_t - p_t \) and negatively on the relative real interest rate. The price-setting equation (7) states that the relative price at time \( t \) is a weighted average of the expected market clearing price \( E_{t-1} p^e_t \) and the actual market clearing price \( p^e_t \).

Equation (8) is a standard LM curve where real money balances depend on output and the nominal interest rate. The final equation (9) represents uncovered interest parity with interest rate differential \( i_t \) determined by the expected change in the nominal exchange rate.

Clarida and Gali (1994) complete the specification by introducing three stochastic shocks: (a) supply shock \( \mu_s \); (b) demand shock \( \mu_d \) and (c) a nominal shock \( \mu_n \). As the derivation of the (long-run flexible price) equilibrium in Clarida and Gali (1994) shows, these shocks drive the long-run dynamics of relative output, relative prices and the real exchange rate in a systematic way. Only supply shocks affect relative output in the long run. Both supply and demand shocks can influence the long-run level of the real exchange rate, while all three shocks have an impact on relative prices in the long run. Clarida and Gali (1994) use these implications to impose a triangular structure on the long-run impact matrix of the VAR and map the reduced form residuals to the three structural shocks.

Farrant and Peersman (2006), instead, focus on the short-run implications of this model. They
argue that the three structural shocks (supply, demand and nominal) have clear implications for short-run dynamics of the endogenous variables and these can be used to identify the shocks in the VAR model. More specifically, a positive supply shock is expected to boost relative output and reduce relative prices. A positive demand shock increases both output and prices and leads to a real exchange rate appreciation. A positive nominal shock likewise raises relative output and relative prices but has the opposite effect on the real exchange rate.

In our analysis we follow Farrant and Peersman (2006) and adopt these short-run sign restrictions and a specification of the VAR in relative variables as our benchmark identification strategy. In particular, we identify a supply, demand and nominal shock by imposing the restrictions summarised in Table A on the contemporaneous impact of the endogenous variables in our time-varying VAR.

Note that the specification of the model in terms of relative variables implies that it can only uncover asymmetric shocks that affect the ratio of the variables and indeed the real exchange rate.\(^2\) There are two reasons why the specification using relative variables may be attractive in our framework. First, it leads to a more parsimonious VAR model (ie foreign and domestic variables do not have to be included separately). This is a particularly important consideration in a time-varying VAR framework where additional endogenous variables make it harder to impose a stability constraint on the VAR coefficients (see Del-Negro (2003)). Second, as one of the main aims of this paper is to investigate real exchange rate fluctuations, symmetric shocks may be less important from this perspective.

This benchmark identification scheme has a number of potential advantages over those based on zero restrictions. First, contemporaneous sign restrictions allow us to be relatively unrestrictive about the impact of structural shocks beyond the contemporaneous effects. Second, the sign

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\(^2\)Peersman (2007) analyses asymmetric and symmetric shocks in a fixed-coefficient VAR by including domestic and foreign variables separately.
restrictions are consistent with a wide class of theoretical models. They carry over to standard microfounded DSGE models, for example. Kollmann (2001) shows in a small open economy model with nominal rigidities that an increase in the money supply boosts output, prices and depreciates the real exchange rate, while a rise in productivity raises output, lowers prices and also depreciates the real exchange rate. In Enders, Müller and Scholl (2008) a demand shock in the form of a positive government spending shock raises relative output and inflation and appreciates the real exchange rate for calibrations with elasticities between home and foreign goods between one and two.

To test the robustness of our results we consider two alternative identification schemes. First, as in Clarida and Gali (1994) we use long-run restrictions to identify the three shocks. Second, as in Farrant and Peersman (2006) we extend the sign identification scheme in Table A to identify a monetary policy shock. In particular, the extended identification scheme adds relative policy interest rates to the model and identifies a monetary policy and an exchange rate shock, in addition to the supply and demand shocks. The monetary policy and exchange rate shocks are introduced in place of the nominal shock. The monetary policy shock is identified by imposing the constraint that this shock increases the relative interest rate on impact, leads to a real exchange rate appreciation and reduces relative output and relative prices. The exogenous exchange rate shock that leads to a real exchange rate depreciation, increases relative output, relative prices and the relative interest rate.

### 2.2 Estimation

The model described by equations (1) to (5) is estimated using the Bayesian methods described in Kim and Nelson (1999). In particular, we employ a Gibbs sampling algorithm that approximates the posterior distribution. A detailed description of the prior distributions and the sampling method is given in Appendix B. Here we summarise the basic algorithm which involves the following steps:

1. The VAR coefficients $\phi_i$ and the off-diagonal elements of the covariance matrix $\alpha_i$ are simulated by using the methods described in Carter and Kohn (2004).
2. The volatilities of the reduced-form shocks $H_t$ are drawn using the date-by-date blocking

3. The hyperparameters \( Q \) and \( S \) are drawn from an inverse Wishart distribution while the elements of \( G \) are simulated from an inverse gamma distribution.

We use 200,000 Gibbs sampling replications and discard the first 195,000 as burn-in. To assess convergence we compute recursive means of the retained draws. As Appendix B shows, the recursive means are stable providing evidence of convergence.

3 Reduced-form results

The estimation results are presented in the following two sections. In this section we discuss estimates of time-varying volatilities of the endogenous variables derived from the VAR models. The next section presents structural results focusing particularly on the impulse response and forecast error variance decomposition of the endogenous variables to structural shocks.

In this section we summarise the reduced-form posterior estimates of \( \phi_t \) and \( \Omega_t \). Charts 1 to 5 plot the estimated unconditional volatility (standard deviation) and the spectral density of relative output growth, relative inflation and the real exchange rate for the United Kingdom, Japan, Canada and the euro area. The spectral density is calculated as:

\[
f_{iiT}(\omega) = s(I_3 - \phi_{1T}e^{-i\omega})^{-1}\frac{\Omega_{iiT}}{2\pi}[(I_3 - \phi_{1T}e^{-i\omega})^{-1}]'s' \tag{10}
\]

where \( \omega \) represents the frequency and \( s \) is a selection vector. The unconditional volatility is defined as

\[
\int_{\omega} f_{iiT}(\omega) \, d\omega, \tag{11}
\]

where \( f_{iiT}(\omega) \) denotes the diagonal elements of the spectral density matrix in equation (10).

Chart 1 displays the results for the United Kingdom (relative to the United States). The unconditional volatility is shown in the top panel, the spectral density is displayed in the panel below. The volatility of the real exchange rate shows a smooth increase starting in the mid-1970s, reaching its peak in the early 1990s which coincided with the exit of the pound from the ERM. The start of the inflation-targeting regime in 1992 coincided with a drop in volatility.
Recent months, however, have seen volatility (of relative inflation and the real exchange rate) increasing again. Chart 2 shows the same results for the United Kingdom vis-à-vis its main trading partner, the euro area. Relative inflation and output volatility fell substantially post 1990 whereas this is true only to a lesser extent for the real exchange rate. All three volatilities show an increase over the last few quarters.

Chart 3 displays the results for the euro area relative to the United States. Real exchange rate volatility remained fairly stable over the sample period, with a modest decline at the long-run frequency after 1990. The volatility of relative output growth and inflation declined fairly steadily since the mid-1980s. However, the more recent period has seen a sharp increase in the estimated volatility of relative output growth and inflation.

The increase in the volatility of Canada-US real exchange rate in recent years is especially striking (see Chart 4). After remaining virtually flat for three decades the volatility increased in 2000 and reached its peak in recent quarters. Canadian relative inflation shows a recent increase in volatility, but as in the case of the United Kingdom, volatility of output growth has been stable over the past decade.

Results for Japan relative to the United States are displayed in Chart 5. The estimated standard deviation of the real exchange rate rose in the early 1980s and remained elevated until the early 1990s with a noticeable spike towards the end of the sample. An increase in volatility in recent months can also be seen for output growth with previous volatile episodes concentrated in the early and mid-1980s and the early 1990s. Relative inflation volatility was high until the mid-1980s and has increased again over the last few quarters.

It is also interesting to consider the comovement between the real exchange rate and relative output. A vast literature has highlighted the ‘disconnect’ between exchange rates and real variables, as nominal and real exchange rates are substantially more volatile than ‘fundamentals’. Chart 6 shows the estimated time-varying coherence between the change in the real exchange and relative output growth for each country. The coherence can be interpreted as the degree to which the two series are jointly influenced by cycles of frequency \( \omega \), or loosely speaking the
correlation between the variables at different frequencies. The coherence is calculated as

\[ h_{i,j}(\omega) = \frac{[c_{ij}(\omega)]^2 + [q_{ij}(\omega)]^2}{f_{i,j}^{ii}(\omega)f_{i,j}^{jj}(\omega)} \]  

(12)

where \( c_{ij}(\omega) \) denotes the co-spectrum (the real component of the off-diagonal elements of the spectrum) while \( q_{ij}(\omega) \) is quadrature spectrum (the imaginary component of the off-diagonal elements of the spectrum). ³ Two conclusions are immediately apparent from Chart 6. First, there is little association between real exchange rates and relative output growth at high frequencies with the bulk of the density concentrated at the long-run frequency. Secondly, the estimates show time-variation, with the coherence higher towards the end of the sample period. For example, for the United Kingdom and the euro area, the coherence is low during the 1970s but increases after the mid-1980s. The increase in Canada occurs after the mid-1990s. These results provide tentative evidence that the disconnect is not an intrinsic feature of the data.

4 Structural results

In this section we use the identification scheme set out in Section 2.1 to identify three structural shocks for each of the countries included in our analysis – a supply, demand and nominal shock. Appendix B describes the algorithm used to implement the identification scheme. We summarise the structural estimates via impulse responses, forecast error variance decomposition (FEVD), counterfactual experiments and present the estimated time-series of structural shocks. The aim of this analysis is twofold: (a) to investigate changes in the transmission of these shocks, mainly focusing on changes in impulse responses over time; and (b) to compare the relative importance of these shocks for fluctuations in output growth, inflation and the real exchange rate, using a time-varying FEVD.

Following Koop, Pesaran and Potter (1996) the impulse response functions are defined as:

\[ IRF = E(Y_{t+k}\mid \Psi_{t+k}, \mu) - E(Y_{t+k}\mid \Psi_{t+k}) \]  

(13)

where \( \Psi \) denotes all the parameters and hyperparameters of the VAR, \( k \) is the horizon under consideration and \( \mu \) denotes the shock. Equation (13) states that the impulse response functions are calculated as the difference between two conditional expectations. The first term in equation (13) denotes a forecast of the endogenous variables conditioned on one of the structural shock \( \mu \).

³The co-spectrum measures the covariance between the series at difference frequencies while the quadrature spectrum incorporates evidence for phase shifts. See Hamilton (1994) page 275.
The second term is the baseline forecast, ie conditioned on the scenario where the shock equals zero. The computation is based on 500 Gibbs sampling iterations using 100 Monte Carlo replications.

As the impulse responses are calculated for each quarter, it is not possible to disentangle changes in the volatility of the shock and its transmission when considering un-normalised impulse response functions. Shocks of different magnitude lead to different size initial responses. To focus on the change in the transmission mechanism, we can normalise the shock to lead to a given initial change, say 1%, in one of the endogenous variables on impact. In our results, we report both responses to a standard deviation shock that is not normalised in any way, as well as responses that are normalised to yield, in turn, a 1% change in the first, second and third variable on impact in each period. Note that it is arbitrary which variable we normalise the response on, but different normalisations will help to visualise certain results more clearly than others.

In the second exercise we focus on changes in the volatility of shocks, rather than the transmission mechanism, over time. We compute the time-varying decomposition of the forecast error variance to do so. This is defined as

\[
FEVD_t = \frac{VAR\left(\tilde{y}_{t+k} - Y_{t+k} | \Psi_{t+k}, \mu\right)}{VAR\left(\hat{y}_{t+k} - Y_{t+k} | \Psi_{t+k}\right)}
\]  

(14)

where the term in the denominator denotes the total variance of the forecast error \(\tilde{y}_{t+k} - Y_{t+k}\) while the numerator is the variance due to shock \(\mu\). As in the case of equation (13), Monte Carlo integration is used to compute the forecasts \(\hat{y}_{t+k}\) for each quarter in the sample. Note that the time-varying nature of our model implies an intuitive interpretation of \(FEVD_t\). That is, \(\hat{y}_{t+k}\) can be regarded as forecast at time \(t\) based on information on the economy as summarised by the drifting VAR parameters \(\Psi_t\). Therefore the forecast and the forecast error reflect agents’ beliefs about the economy and the policy regime at that particular point in time rather than on average of the past.

The counterfactual experiments are conducted in the following way. We start from the assumption that the time-varying covariance matrix of the reduced form shocks is given by

\[
\Omega_{t,T} = A_t' H^* A_t.
\]

Here \(H^*\) denotes a diagonal matrix which holds the estimated variance of one structural shock at a time. For example, if we are interested in the nominal shock, then...
We then generate artificial reduced form shocks from the normal distribution with variance given by $\Omega_{ij}$ and generate artificial data for the endogenous variables using equation (1). Note that the counterfactual experiments combine the effects of both time-variation in the parameters and the volatility of the shocks. The time series of the structural shocks is calculated as $A_{0,i}^{-1} \nu_t$, where $A_{0,i}$ denotes the structural impact matrix obtained by imposing the sign restrictions in Table A.

4.1 United Kingdom

4.1.1 Impulse responses

Chart 7 plots the accumulated time-varying impulse responses of relative output growth, relative inflation and changes in the real exchange rate to a relative supply shock for the UK-US VAR. The top panel shows the responses after a 1 standard deviation shock, the bottom three panels represent the accumulated response after four quarters, with the shock normalised to raise relative output, prices and the real exchange rate, respectively, by 1% on impact. The black line in these charts represents the estimated normalised responses from a fixed-coefficient VAR (using the same identification scheme).

The chart shows that the effect of supply shocks on relative inflation increased during the mid-1970s, the early 1990s and in the last few quarters – focus on the second row to see this. Note, however, that there is little indication of a statistically significant change in the inflation impulse response function. There is little evidence to indicate that supply shocks have a significant impact on the UK-US real exchange rate, the estimated impulse response is largely insignificant over the sample.

Unlike supply shocks, Chart 8 suggests that demand shocks have a significant and long-lasting positive effect on the exchange rate throughout the sample. In addition, the response to this shock shows substantial time-variation. Consider the second row of the chart. During the mid-1970s, the real exchange rate appreciated by less than 10% in response to a demand shock that raised relative output growth by 1% on impact. Since the 1990s, this response to a shock of the same
magnitude increased to around 30%. Moreover, the estimated probability that the real exchange rate response in 2009 Q1 is larger than the response in 1975 Q1 is estimated to be 99% suggesting a systematic shift in the impulse response function.\footnote{This probability is calculated as the proportion of the distribution of the cumulated impulse response function (at the one-year horizon) in 2009 Q1 which is larger than its distribution in 1975 Q1.} Note that this change in the response of the real exchange rate coincides with the removal of nominal exchange controls during the early 1980s. The last row of the chart presents the response normalised on the exchange rate. These results indicate that demand shocks had a larger impact on inflation in the pre-inflation targeting period, especially during the mid-1970s. A comparison of these estimates with those from the fixed-coefficient VAR indicates that ignoring time-variation may lead to substantially biased inference – for e.g the fixed coefficient VAR underestimates the current response of the real exchange rate to a demand shock by more than 100%.

Chart 9 shows that the pattern in the response of the real exchange rate to nominal shocks displays similar time-variation as its response to the demand shock. Prior to the 1990s, the exchange rate depreciated by less than 5% in response to a nominal shock that raised output by 1%. The magnitude of the response is closer to 10% more recently (with the probability of 99% that the 2009 response is more negative than the estimated response in 1975). This picture is similar for relative inflation, with larger responses more recently. The implication is that relative output growth responds by less to a given change in the exchange rate that was brought about by a nominal shock in the post-1992 period. This is also clear from the last row of Chart 9 which shows that both relative output and inflation currently decline by a smaller amount in response to a contractionary nominal shock when compared to the 1970s. Note that this shift is statistically important with the probability of larger output (inflation) response during the mid-1970s estimated at 87% (94%). This result is similar to that reported in Castelnuvo and Surico (2006) for the United Kingdom and Boivin and Giannoni (2006) for the United States, where the latter link the change in impulse response functions to an improvement in the practice of monetary policy. As above, our results suggest that the estimated response from the fixed-coefficient VAR provides incorrect inference for the current impact of this shock.

Overall these results provide strong evidence that the transmission of supply, demand and nominal shocks to inflation, output growth and the real exchange rate has changed substantially over the past two decades. They suggest that fixed-coefficient models describing the joint
dynamics of these variables may be misspecified and lead to incorrect inference.

4.1.2 Forecast error variance decomposition

In order to analyse changes in the volatility of the shocks, we decompose the forecast error variance of relative output, prices and the real exchange implied by our VAR into contributions from the three identified shocks. The results from this exercise are shown in Chart 10, which shows important variation across time in the role of each of the shocks, but relative stability over the forecast horizon.

For relative output, the supply shock is important throughout the sample and most important at the beginning in the 1970s, with a contribution of almost 80%. Supply shocks contribute around 45% to relative inflation forecast error variance throughout the sample, and its contribution to the real exchange rate is minimal. It is interesting to note that the role of the nominal shock in explaining inflation changes around the time of the introduction of inflation targeting in the United Kingdom, with the contribution falling from around 50% in the mid-1980s to closer to 35% more recently. In the mid-1970s, 25% of inflation forecast error variance is explained by the demand shock, but its role declined over the rest of the sample.

Demand shocks are the dominant driver of the real dollar-pound exchange rate, with a contribution ranging between 70% and 80%. We find some importance for nominal shocks in driving the real exchange rate (around 20% except for the mid-1970s where it fell to below 10%), but this has fallen somewhat to around 15% in the recent past. Note that these results for the exchange rate are more ‘moderate’ than in Clarida and Gali (1994) who find that the demand shock is of almost exclusive importance, with a contribution in excess of 80% at all horizons for the United Kingdom. On the other hand, our results suggest a somewhat smaller role for nominal shocks than Farrant and Peersman (2006) who find a contribution of more than 30% and up to 50% at short horizons.

4.1.3 Counterfactual experiments and time series of structural shocks

Chart 12 considers the relative importance of each shock for the level of the variables included in the VAR. Each row of the chart shows data generated using the estimated VAR parameters under
the counterfactual assumption that only one structural shock is active. For example, the first row generates data for each endogenous variable under the assumption that only supply shocks are operational. This counterfactual data is shown as the red lines and we compare this with actual historical data (blue line), all in annual growth rates.

The top two panels of the chart show that the supply shock was the most important in driving output growth and inflation in the mid-1970s. In particular the shock captures the large falls in relative output in 1975 and 1979 and the large increase in inflation in 1975. In addition, the recession of the mid-1980s and the early 1990s is also attributed largely to this shock. The supply shock also accounts for the increase in inflation during the early 1990s and interestingly the recent increase seen in 2008. The top right panel of the chart shows that the supply shock has not been an important driving force of the change in the real exchange rate.

Demand shocks capture some of the movement in output growth during the 1970s. In particular, the spike in growth during the late 1970s is explained well by the demand shock. But our estimates do not suggest that the Great Inflation of the mid-1970s was a demand-driven phenomenon. In contrast, the demand shock captures most of the fluctuations in the real exchange rate, capturing the large falls in the early and mid-1980s and the mid-1990s, as well as a large proportion of the depreciation seen over the past year. The nominal shock explains part of the increase in inflation during the mid-1970s, the early 1990s and in 2008. It can also account for part of the decline in the (growth of) the real exchange rate during the early 1980s and in 2008.

Chart 11 shows the median estimate of the time series of structural shocks. Note that the chart presents the four-quarter moving average of the estimated shocks. The estimates indicate large negative supply shocks and positive nominal and demand shocks during the 1970s. Large negative demand shocks coincide closely with the early and mid-1980s and the early 1990s. According to our estimates, the current period is associated with negative demand and supply shocks and positive nominal shocks. The demand shocks are the largest since the 1990s recession.
4.1.4 UK relative to the euro area

We also consider a VAR model where UK variables are expressed relative to euro-area variables. This is of interest given the importance of the euro area as the United Kingdom’s main trading partner.

Chart 13 plots the response to a supply shock. The top row suggests that a 1 standard deviation supply shock increased output by up to 1% to 1.5% before the early 1990s, with a fall of similar magnitude in inflation and the real exchange rate. The response has become smaller subsequently. The normalised responses show that this change is largely due to a fall in shock volatility. For example, when the supply shock is normalised to increase output by 1% (on impact) in each quarter, the inflation and real exchange rate response show little time-variation apart from changes at the beginning and the end of the sample. Similarly, the magnitude of the real exchange rate depreciation is roughly stable across the sample.

The responses to a demand shock suggest a somewhat different picture, with the normalised responses changing over time (Chart 14). The response of the real exchange rate to the demand shock (that increases relative output growth by 1%) has increased over the sample period, with the median response around twice as large now when compared to the 1970s and the 1980s. This change occurred after the early 1990s and therefore coincided with the collapse of the ERM. Note that the last row of the chart suggests that a demand shock that appreciates the real exchange rate by 1% has a smaller impact on relative inflation now than in the earlier part of the sample. These results are similar to those obtained for the UK-US VAR.

Chart 15 shows the impulse responses to a nominal shock. This presents a broadly similar picture to the estimates from the UK-US VAR for the real exchange rate (see Chart 9), with the response increasing after the 1980s (focus on the second and last rows to see this).

The FEVD is presented in Chart 16. As in the case of the UK-US VAR, real exchange rate volatility is driven primarily by demand shocks over the sample period, with supply shocks making a minor contribution in the 1970s and nominal shocks explaining around 20% over the rest of the sample period. However, supply shocks are the most important source of fluctuations in relative output growth and inflation, and contribute around 25% to exchange rate forecast error.
variability in the first few years of the sample.

4.2 Euro area

Chart 17 depicts the estimated impulse responses of the euro area-US variables to relative supply shocks. The estimates suggest that the impact of supply shocks on inflation has increased somewhat since 2000. In the second row of the chart, a supply shock that raises output by 1% on impact is now estimated to reduce relative inflation by approximately three times as much as pre-2000, but as before this change is not generally statistically significant. Note that the median impact of this shock on the real exchange reached its peak during the late 1990s but has declined after the adoption of the euro. Supply shocks have a minimal impact on the real exchange rate, with the impulse response estimated to be close to zero over most of the sample.

It is again demand shocks that lead to a large and time-varying response of the real exchange rate, shown in Chart 18. For the case of a demand shock that raises output by 1% (the second row), the impact on the exchange rate is estimated to vary between 10% during the mid-1970s and around 30% since the mid-1980s.5 The response of relative inflation, on the other hand, starts to increase following the introduction of the euro at the end of the 1990s and is at its highest at the end of the sample.

The impulse responses to nominal shocks are shown in Chart 19. The results are similar to those obtained for a demand shock. Since 2000, inflation is estimated to increase more to a nominal shock that increases output by 1% (ie the median response of inflation is larger) while the negative response of the real exchange rate has been larger since the mid-1980s with a probability of 90% that the 2009 response is more negative than the estimate for 1975.

4.2.1 Forecast error variance decomposition

Chart 20 shows the FEVD for the euro area-US model. Supply shocks are important for output throughout the sample, but in the early 1990s, the contribution of supply shocks drops to 60% at longer horizons before rising again in the last two years of the sample to the levels seen in the

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5We estimate a probability of 92% that the estimate response of the real exchange rate to the demand shock in 2009 is larger than the estimated response in 1975.
1970s (around 80%). There are large changes in the contribution of the nominal shock to inflation ranging from 35% to 65%, with a sharp drop coinciding with the introduction of the euro. As above, the nominal shock accounts for around 20% of real exchange rate forecast error variance, with the demand shock again accounting for most of the rest.

4.2.2 Counterfactual experiments and time series of structural shocks

The counterfactual experiments for the euro area-US VAR (see Chart 22) suggest similar conclusions as in the UK model. As our results from the impulse responses and variance decomposition would suggest, relative output levels seem to be mainly driven by the supply shock, with the counterfactual tracking the actual series closely. While this shock explains some of the increase in relative prices during the mid-1970s and 80s, it accounts for very little of the movements in the real exchange rate. The counterfactual experiment with only the demand shock operational suggests that this shock has been the main driver behind real exchange rate fluctuations throughout the sample. The sharp swings in inflation during the late 1970s and early 1980s are tracked closely by the nominal shock, which also contributes to the increase in relative inflation seen over the past two quarters.

The estimated time series of structural shocks for the euro area-US model, shown in Chart 21, suggests that the 1970s and early 1980s were associated with large positive nominal shocks. The late 1990s saw large negative demand and nominal shocks, while the start of the next decade was associated with positive supply shocks. The last few quarters in the sample are associated with negative supply and demand shocks that are generally small relative to the 1970s.

4.3 Canada

4.3.1 Impulse responses

The impulse responses for Canada are broadly similar to our results for the UK model. The second row of Chart 23 shows that a supply shock (calibrated to increase relative output growth by 1%) reduces relative prices by around 1% over most of the sample period, but by more than that recently (around 1.5%). The real exchange rate response, initially a depreciation, has become smaller in magnitude, changed sign and become statistically insignificant over recent years.
Chart 24 presents the estimated impulse response to a demand shock. When the response is normalised on relative output, a similar pattern emerges for each variable. The impulse response of the exchange rate shows the largest variation, with the magnitude of the cumulated response increasing after the introduction of inflation targeting in Canada in the early 1990s.\(^6\) The last row of the chart shows that when the demand shock is scaled to appreciate the real exchange rate by 1%, the response of relative output and inflation declines over the sample period, with the response the smallest in the period after the introduction of inflation targeting (for a given change in the exchange rate).

The response to the nominal shock is presented in Chart 25. The pattern of the relative exchange rate response to the nominal shock follows the one observed for the United Kingdom, with a given change in relative output associated with a significantly larger depreciation towards the end of the sample. The final row of the chart again shows that a 1% appreciation of the real exchange rate due to this shock had a smaller impact on output and inflation over the past decade when compared to the 1970s and the 1980s.

4.3.2 Forecast error variance decomposition

The picture emerging from the FEVD is generally quite different for Canada compared to the other countries. Chart 26 shows the contribution of the structural shocks to the forecast error variance of the endogenous variables. The supply shock is important for exchange rate forecast error volatility during the 1970s – with a contribution of around 20% in the early 1970s. This shock explains 60% of the forecast error variance of inflation during the early and late 1970s with the contribution declining to around 40% over the rest of the sample period. The supply shock is important for relative output at all horizons, except towards the end of the sample when the contribution declines.

The contribution of the nominal shock to output growth shows a similar pattern, with this shock explaining less towards the end of the sample period. Similarly the contribution of this shock to relative inflation is smaller towards the end of the sample period, especially at long horizons. The nominal shock explains about 20% of real exchange rate forecast error variance over the sample period.

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\(^6\)This shift in the response of inflation and the real exchange rate appears to be systematic with a probability in excess of 0.95 that the 2009 response to these shocks is larger than the 1975 response.
It is interesting to note that, unlike in the other models, the contribution of the demand shock to relative output and inflation is at its highest at the end of the sample period. Therefore this shock is clearly an important factor behind recent relative output growth fluctuations. Apart from the early 1970s, the demand shock is the main contributor to real exchange rate forecast error variance with a contribution of 70% over the 1980s, 1990s and in the past decade.

4.3.3 Counterfactual experiments and time series of structural shocks

In Chart 28, we show the estimated counterfactual paths of the endogenous variables under the assumption that only one of the identified shocks is operational at a time. The top panel of the chart shows that a counterfactual scenario with only supply shocks present explains a large proportion of relative output and relative price fluctuations. During the 1970s, supply shocks track the main recessions. The nominal shock can partly explain the recession of the late 1970s suggesting that a combination of nominal and supply shocks was important for Canada over that period. The increase in relative inflation in 2008 is primarily driven by relative demand shocks. This shock is also the main factor behind past and recent fluctuations in the real exchange rate.

Chart 27 plots the smoothed time series of structural shocks. The estimates suggest large positive relative demand and relative nominal shocks in the early and late 1970s. The mid-1970s and the 1980s were associated with large negative supply shocks, but demand and nominal shocks dominated again in the early 1990s. The large positive relative demand shock at the end of the sample was accompanied by negative nominal shocks.

4.4 Japan

4.4.1 Impulse responses

Chart 29 presents the impulse response functions to a supply shock for the Japan-US VAR. Focusing on the second row that normalises the shock on output, the results suggest that this shock led to a 1.5% fall in relative inflation during part of the 1970s. This is in line with estimates from the other countries. The response during the 1980s and the 1990s was smaller,
estimated at 1%. In contrast to the results presented for some of the other countries, the supply shock has a significant impact on the real exchange rate over part of the sample period, with the exchange rate depreciating by around 1% towards the beginning and end of the sample (second row of the chart).

The response to a demand shock is quite different from that found in other countries (shown in Chart 30). The second row of the chart shows that there is limited evidence of significant time-variation in the response of the real exchange rate. The final row of the chart shows that a demand shock (calibrated to increase the real exchange rate by 1%) had a larger impact on relative inflation in the pre-1980 period.

Chart 31 plots the time-varying response to a nominal shock. During the 1970s the exchange rate depreciated by around 2% in response. The magnitude of the median response has doubled since the mid-1980s. Note also that these estimates continue the pattern of responses to nominal shocks observed for countries such as the United Kingdom and Canada – a larger depreciation is required to bring about a given change in relative output and inflation in the second part of the sample.

It is interesting to note that evidence for time-variation in the impulse responses is less pronounced for Japan, with the estimates from the fixed-coefficient VAR closer to the time-varying estimates.\(^7\)

### 4.4.2 Forecast error variance decomposition

Chart 32 presents the results from the forecast error variance decomposition. Supply shocks explained about 10% of real exchange rate forecast error volatility during the 1970s, but they were more important for relative output growth and inflation volatility, explaining around 40% over the entire sample. The role of supply shocks for output appears to be much lower than for other countries. The contribution of the nominal shock shows strong time-variation. They were important for relative output over the 1980s. The 1990s saw a decline in the contribution of these shocks to output. However, note that nominal shocks have become more important for output

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\(^7\)Note that the estimated probability for a systematic shift in the real exchange rate impulse response function is smaller for Japan when compared to the United Kingdom, euro area and Canada. For example, the probability that the real exchange rate responds more to nominal shocks in 2009 Q1 (compared to 1975 Q1) is 85%.
over the past few quarters again. These shocks contributed little to relative inflation in the past decade. In contrast, their contribution during the 1980s was around 60% for both variables.

It is interesting to note that the demand shock was quite important in driving output fluctuations in the 1990s suggesting that this shock played a vital role. The contribution of the demand shock to inflation forecast error variance shows a sharp increase in the early 1990 coinciding with the asset price collapse. The large contribution of this shock continued throughout the current decade indicating that relative demand shifts may have played a large role in driving Japan’s deflation experience. Note that as in the case of other countries, this shock was the main contributor to real exchange rate forecast error variance.

4.4.3 Counterfactual experiments and time series of structural shocks

Chart 34 shows the counterfactual estimates of the endogenous variables. When the supply shock is assumed to be the only operational structural shock, the counterfactual estimate of relative output growth tracks the large negative movement in growth just before 1975 pointing to the importance of this shock over that time period. Similarly, the recession in the early 1990s is also explained partly by the supply shock. This shock also appears to explain the large increase in relative inflation during the mid-1970s and the early 1980s. Most of the fluctuations in the exchange rate are explained by the demand shock. Nominal shocks are important in explaining the fall in output growth during the mid-1980s and mid-1990s. These shocks also partly explain the increase in relative inflation during the mid-1970s, the early 1980s and the early 1990s.

The estimated (smoothed) structural shocks are shown in Chart 33. The early 1970s were dominated by positive demand shocks with negative supply shocks appearing in the middle of this decade. The second half of the 1980s, generally associated with strong economic activity in Japan, saw large positive demand shocks. This appeared to reverse in the early 1990s with negative demand, nominal and supply shocks dominating the decade. The last year of the sample has been associated with negative nominal and supply shocks offset by positive relative demand shocks.
4.5 *Alternative identification schemes*

In this section we test the robustness of our results by considering two variations on the benchmark identification scheme: identification via long-run restrictions and a four-variable VAR that includes the interest rate differential along with relative output growth, relative inflation and the real exchange rate and uses sign restrictions to identify a monetary policy and an exchange rate shock as well as a supply and demand shock (see Section 2.1 for details). In general we find that these two identification schemes deliver results similar to the benchmark model. Note that in this sensitivity analysis we focus mainly on the real exchange rate. Full results are available on request.

4.5.1 *United Kingdom*

Chart 35 plots the results for the United Kingdom. The top three panels of Chart 35 show the estimated impulse responses (to a 1 standard deviation shock) under the benchmark and additional identification schemes. The bottom panels display the forecast error variance decomposition under the three different identification schemes. In the following discussion we focus on differences to the benchmark scheme.

The responses to supply and nominal shocks is generally very similar to the benchmark scheme. One important difference is that in the long-run identification scheme, the nominal shock does not affect the exchange rate in the long run by assumption. This feature is reflected in the estimated impulse response reverting to zero after a few quarters. This is in contrast to the benchmark model where we find a permanent change in the exchange rate in response to the nominal shock. As pointed out in Farrant and Peersman (2006), this difference may reflect the fact that responses derived under long-run identification represent ‘extreme’ solutions and lie in the tails of the distribution of impulse responses with the consistent sign.

The third row of the chart considers the impulse response when the interest rate differential is added to the VAR and an exogenous exchange rate shock is identified in addition to the monetary policy shock. The pattern across time of the response to supply and demand shocks is very similar to the benchmark results for the exchange rate. The estimated pattern of the response to the monetary policy and exchange rate shocks mirrors that of the response to nominal shocks in
the benchmark model.

When long-run restrictions are used for shock identification, nominal shocks make a much smaller contribution to the real exchange rate forecast error variance. The demand shock is the dominant driver of exchange rate volatility, explaining almost 100% of the forecast error variance. When the extended sign restriction scheme is used the estimated FEVD is virtually unchanged from the benchmark case. The demand shock explains around 80% of the exchange rate variance, with the remainder explained by the monetary policy and the exchange rate shock. Note that the combined proportion explained by the monetary policy and exchange rate shocks is similar in magnitude to the contribution of the nominal shock in our benchmark model, with the share of the monetary policy shock smaller than the exchange rate shock.

4.5.2 Euro area

Chart 36 displays the sensitivity analysis for the euro area-US VAR. The time profile of the response of the real exchange rate to supply and demand shocks under the benchmark and long-run identification schemes is very similar. In contrast, the two schemes deliver different time-varying responses to a nominal shock, with the long-run scheme indicating a stronger depreciation at the end of the sample period. Considering the extended sign restriction scheme, depreciation as a result of monetary policy is smaller towards the end of the sample period, which mimics the pattern seen in the response to the nominal shock in the benchmark euro area model.

The forecast error variance decomposition obtained under long-run identifying restrictions points to conclusions similar to the results for the United Kingdom above, with the demand shock making by far the largest contribution to exchange rate volatility over the sample period. Under the extended sign restriction scheme, the demand shock is again the most important contributor to exchange rate forecast error variance, explaining about 80% with most of the remainder explained by the monetary policy and the exchange rate shock.

4.5.3 Canada

Sensitivity analysis for the Canada-US model is presented in Chart 37. The response to the supply, demand and nominal (monetary policy) shocks under the two alternative identification
schemes is very similar to the benchmark case.

Demand shocks emerge as the most important source of exchange rate fluctuations in the FEVD. However, in contrast to the benchmark case, supply shocks play an important role at the end of the sample period when using the long-run identification scheme (contributing around 15%). In the extended sign restriction model, the forecast error variance decomposition suggests that supply shocks were important for exchange rate volatility during the 1970s, but demand shocks again dominant over most of the sample.

4.5.4 Japan

Chart 38 plots the sensitivity analysis for the Japan-US VAR that are broadly similar to our benchmark identification scheme. One difference is that the impulse responses from the extended sign identification scheme suggest that monetary policy shocks lead to a smaller appreciation in the post-1990 period (demand and supply shocks have an impact similar to the benchmark case).

The variance decomposition indicates that, in contrast to the benchmark model, supply shocks are important for the real exchange rate in the late 1990s (contributing around 15%). Under the extended sign restriction scheme, monetary policy shocks contributed little to exchange rate volatility with the bulk of the variance driven by demand and exchange rate shocks.

5 Some tentative explanations

The structural results from the time-varying VAR model show that the dynamics of the real exchange rate have changed substantially. There are several potential explanations for this phenomenon. At the simplest level, the change in the real exchange rate impulse response function is consistent with a change in the degree of deviations from purchasing power parity (PPP). In particular, a fall in the pass-through of the nominal exchange rate to import prices would (other things being equal) lead to larger fluctuations in the real exchange rate in the face of nominal and real shocks. To explore this and other scenarios further we consider the open economy DSGE model presented in Monacelli (2005) which builds on Gali and Monacelli (2005) by introducing imperfect exchange rate pass-through. We refer the reader to Monacelli (2005) for a detailed explanation of the model. It is, however, instructive to consider the
specification of some key equations.

Consider first the price of domestic imports. The model assumes that local importers import differentiated goods, where the law of one price holds at the dock. Monacelli (2005) shows that the domestic price of these imports is set by solving an optimal mark-up problem which generates deviations from PPP in the short run. The solution to this problem yields the following aggregate supply curve for imports

\[ \pi_{F,t} = \beta E_t \left( \pi_{F,t+1} \right) + \lambda_F \psi_{F,t} \]  

(15)

where \( \pi_{F,t} \) denotes import price inflation, \( \beta \) is the discount factor, \( \lambda_F = \frac{(1-\theta_F)(1-\beta_F)}{\theta_F} \), and \( \psi_{F,t} \) represents the deviation of the world price of imports from domestic currency price (referred to as the law of one price gap by Monacelli (2005)). When \( \theta_F = 0 \), import prices are perfectly flexible and exchange rate pass-through is perfect. Values of \( \theta_F \) closer to 1 represent a situation with imperfect exchange pass-through.

Domestic output price inflation \( \pi_{H,t} \) is determined by the following equation

\[ \pi_{H,t} = \beta E_t \left( \pi_{H,t+1} \right) + \lambda_H \left( \varphi + \sigma \frac{\partial x}{\partial x} \right) x_t + \lambda_H \left( 1 - \frac{\sigma_y}{\sigma_s} \right) \psi_{F,t} \]  

(16)

where \( \sigma_s = 1 + \alpha (2 - \alpha) (\sigma \eta - 1) \), \( \sigma_y = 1 + \alpha (\sigma \eta - 1) \), \( x_t \) denotes the output gap, \( \alpha \) is the share of foreign goods in the CPI basket, \( \sigma \) is the intertemporal elasticity of substitution, \( \eta \) is the elasticity of substitution between domestic and foreign goods, \( \varphi \) denotes the elasticity of labour supply while \( \lambda_H = \frac{(1-\theta_H)(1-\beta_H)}{\theta_H} \), where \( \theta_H \) denotes the proportion of firms that keep prices fixed every period.

We use this model to simulate the impact of a contractionary monetary policy shock (as an example of a nominal shock) and a positive foreign output shock (as an example of a real shock). We consider two variations on the benchmark calibration of this model. First, we simulate the impact of these shocks for values of \( \theta_F \) ranging from 0.0001 to 0.9 to proxy a change in the rate of exchange rate pass-through. Second, we vary the degree of domestic price-stickiness \( \theta_H \).

Chart 39 shows the cumulated response of the real exchange rate to these shocks simulated for these values of \( \theta_F \). It is clear from the chart that as exchange rate pass-through declines, the real
exchange rate appreciates more in response to these shocks. For example, the response of the real exchange rate to the policy shock more than double for values of $\theta_F$ bigger than 0.5. The response to the foreign output shock shows a similar (albeit less dramatic) increase. Under imperfect pass-through, the response of the nominal exchange rate is larger, while domestic prices respond less resulting in a pronounced response of the real exchange rate. Existing empirical evidence is supportive of the idea that countries such as the United Kingdom experienced a decline in exchange rate pass-through over the sample period considered in this study. For example, using a time-varying pass-through regression Mumtaz, Oomen and Wang (2006) show that pass-through of the exchange rate into UK import prices fell over the period 1987-2004. Similar evidence is presented in Sekine (2006) and Takhtamanova (2008). Taking this empirical evidence together with the simulation in Chart 39 suggests one possible explanation for the result (presented in Section 4) that the response of the real exchange rate has changed over time.

Chart 40 presents the impulse responses for values of $\theta_H$ ranging from 0.0001 to 0.9. The left panel of the chart shows that the response of the real exchange rate to monetary policy shocks is larger as $\theta_H \to 1$ and prices become more sticky. The response to the foreign output shock is largely unaffected. Again, there is some empirical evidence to suggest that price stickiness (as measured by $\theta_H$) may have been lower during the high-inflation years of the 1970s and may have increased with the onset of the Great Moderation after the mid-1980s and the early 1990s (see for example Fernandez-Villaverde and Rubio-Ramirez (2007) for the United States). One possible intuition behind this result relies on the predictions of the literature on time-dependent pricing (see for example Dotsey, King and Wolman (1999)). With time dependent pricing firms face menu costs on adjusting prices. However, the cost associated with keeping prices fixed becomes larger in periods associated with higher inflation variability resulting in a decrease in price stickiness during these periods.

It should be noted that the simple model we employ ignores the endogenous reaction of exchange rate pass-through and domestic price stickiness to policy changes (see for example Devereux, Engel and Storgaard (2004)). Therefore we are unable to characterise the role of policy fully within this framework.
6 Conclusions

The aim of this paper is to investigate the time-varying joint dynamics of output growth, inflation and the real exchange rate for the United Kingdom, euro area, Japan and Canada. We use a Bayesian time-varying autoregression to identify (relative) demand, supply and nominal shocks and consider their impact on and contribution to fluctuations in these variables. Our results suggest that the transmission of these shocks has changed significantly over time. Nominal shocks appear to have a different impact during the 1990s when compared to the 1970s – ie such a shock now leads to a larger depreciation but with a smaller corresponding change in output growth. Demand shocks generally lead to a larger appreciation after the 1980s.

Demand shocks appear to be the primary factor behind real exchange rate fluctuations with nominal shocks playing a secondary role. Demand and supply shocks are important in explaining the forecast error variance of inflation and output during the 1970s and the 1980s.

In general our results suggest that it is essential to consider time-variation in models that attempt to describe international macroeconomic dynamics. The results we derived in this very simple model are likely to be relevant in more complex models that are designed to provide quantitative estimates of the transmission mechanism, which may be unreliable if time-variation is not taken into account.
Appendix A: Data

Quarterly, seasonally adjusted real GDP and CPI series are taken from: the Office for National Statistics (United Kingdom), Eurostat and the OECD ( euro area), the Japanese Cabinet Office and Statistics Bureau of MIC (Japan), the Bureau of Economic Analysis and Bureau of Labor Statistics (United States) and CANSIM Canadian Socio-Economic Information Management System (Canada). Global Financial Data have been used for historical series further back. Quarterly nominal bilateral exchange rate data are taken from the IMF’s International Financial Statistics database. The sample runs from 1957:2 (1961:4 for models involving the euro area) until 2008:4.
Appendix B: Estimation and priors

Prior distributions and starting values

VAR coefficients

The initial conditions for the VAR coefficients $\phi_0$ are obtained via an OLS estimate of a fixed-coefficient VAR using the first 50 observations of the sample period.

Elements of $H_t$

Let $\hat{\sigma}_{\text{ols}}$ denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data described above. The prior for the diagonal elements of the VAR covariance matrix (see (3)) is as follows:

$$\ln \sigma_0 \sim N(\ln \mu_0, I_3)$$

where $\mu_0$ are the diagonal elements of the Cholesky decomposition of $\hat{\sigma}_{\text{ols}}$.

Elements of $A_t$

The prior for the off-diagonal elements $A_t$ is

$$A_0 \sim N(\hat{\sigma}_{\text{ols}}, V (\hat{\sigma}_{\text{ols}}))$$

where $\hat{\sigma}_{\text{ols}}$ are the off-diagonal elements of $\sigma_{\text{ols}}$, with each row scaled by the corresponding element on the diagonal. $V (\hat{\sigma}_{\text{ols}})$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{\sigma}_{\text{ols}}$.

Hyperparameters

The prior on $Q$ is assumed to be inverse Wishart

$$Q_0 \sim IW (\tilde{Q}_0, T_0)$$

where $\tilde{Q}_0$ is assumed to be $\text{var}(\hat{\phi}^{\text{OLS}}) \times 10^{-4} \times 3.5$ and $T_0$ is the length of the sample used for calibration.
The prior distribution for the blocks of $S$ is inverse Wishart:

$$S_{i,0} \sim IW(\tilde{S}_i, K_i)$$

where $i = 1..3$ indexes the blocks of $S$. $\tilde{S}_i$ is calibrated using $\hat{a}^{ols}$. Specifically, $\tilde{S}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by $10^{-3}$.

Following Cogley and Sargent (2005) we postulate an inverse-gamma distribution for the elements of $G$,

$$\sigma_i^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right)$$

**Simulating the posterior distributions**

**Time-varying VAR coefficients**

The distribution of the time-varying VAR coefficients $\phi_t$ conditional on all other parameters and hyperparameters is linear and Gaussian:

$$\begin{align*}
\phi_t \setminus X_{i,t}, \Xi & \sim N\left(\phi_{T\setminus T}, P_{T\setminus T}\right) \\
\phi_{t+1} \setminus X_{i,t}, \Xi & \sim N\left(\phi_{t+1 \setminus t+1, \phi_{t+1}}, P_{t+1 \setminus t+1, \phi_{t+1}}\right)
\end{align*}$$

where $t = T - 1, ..1$, $\Xi$ denotes a vector that holds all the other VAR parameters and:

$$\begin{align*}
\phi_{T\setminus T} & = E\left(\phi_T \setminus X_{i,t}, \Xi\right) \\
P_{T\setminus T} & = Cov\left(\phi_T \setminus X_{i,t}, \Xi\right) \\
\phi_{t+1 \setminus t+1, \phi_{t+1}} & = E\left(\phi_{t+1} \setminus X_{i,t}, \Xi, \phi_{t+1}\right) \\
P_{t+1 \setminus t+1, \phi_{t+1}} & = Cov\left(\phi_{t+1} \setminus X_{i,t}, \Xi, \phi_{t+1}\right)
\end{align*}$$

As shown by Carter and Kohn (2004) the simulation proceeds as follows. First we use the
Kalman filter to draw $\phi_{T \setminus t}$ and $P_{T \setminus t}$ and then proceed backwards in time using:

$$
\phi_{t|t+1} = \phi_{t|t} + P_{t|t} P_{t+1|t}^{-1} (\phi_{t+1|t} - \phi_t)
$$

$$
\phi_{t|t+1} = \phi_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t}
$$

*Elements of $H_t$*

Following Cogley and Sargent (2005), the diagonal elements of the VAR covariance matrix are sampled using the Metropolis Hastings algorithm in Jacquier et al (2004). Given a draw for $\phi_t$, the VAR model can be written as

$$
A_t' \left( Z_i \right) = u_t
$$

where $Z_i = Z_i - \sum_{l=1}^I \phi_{l|l} Z_{i-l} = v_t$ and $VAR \left( u_t \right) = H_t$. Jacquier et al (2004) note that conditional on other VAR parameters, the distribution $h_{it}$, $i = 1..3$ is given by

$$
f \left( h_{it}/h_{it-1}, h_{it+1}, u_{it} \right) = f \left( u_{it}/h_{it} \right) \times f \left( h_{it}/h_{it-1} \right) \times f \left( h_{it+1}/h_{it} \right)
$$

$$
= h_{it}^{-0.5} \exp \left( \frac{-u_{it}^2}{2 h_{it}} \right) \times h_{it}^{-1} \exp \left( -\frac{(\ln h_{it} - \mu)^2}{2 \sigma_{h_i}} \right)
$$

where $\mu$ and $\sigma_{h_i}$ denote the mean and the variance of the log-normal density $h_{it}^{-1} \exp \left( -\frac{(\ln h_{it} - \mu)^2}{2 \sigma_{h_i}} \right)$. Jacquier et al (2004) suggest using $h_{it}^{-1} \exp \left( -\frac{(\ln h_{it} - \mu)^2}{2 \sigma_{h_i}} \right)$ as the candidate generating density with the acceptance probability defined as the ratio of the conditional likelihood $h_{it}^{-0.5} \exp \left( \frac{-u_{it}^2}{2 h_{it}} \right)$ at the old and the new draw. This algorithm is applied at each period in the sample.

*Element of $A_t$*

Given a draw for $\phi_t$, the VAR model can be written as

$$
A_t' \left( Z_i \right) = u_t
$$

where $Z_i = Z_i - \sum_{l=1}^I \phi_{l|l} Z_{i-l} = v_t$ and $VAR \left( u_t \right) = H_t$. This is a system of equations with time-varying coefficients and given a block diagonal form for $Var(\tau_t)$ the standard methods for state-space models described in Carter and Kohn (2004) can be applied.
VAR hyperparameters

Conditional on $Z_t$, $\phi_{t,t}$, $H_t$, and $A_t$, the innovations to $\phi_{t,t}$, $H_t$, and $A_t$ are observable, which allows us to draw the hyperparameters – the elements of $Q$, $S$, and the $\sigma_i^2$ – from their respective distributions.

Convergence

Recursive means of Gibbs draws

For each country this MCMC algorithm is applied using 200,000 iterations discarding the first 195,000 as burn-in. The chart above plots recursive means calculated using intervals of 20 draws
for 1,000 retained draws of the main VAR parameters. These show little fluctuation providing evidence for convergence of the algorithm.

**Imposing the sign restrictions**

In the benchmark identification scheme we identify three shocks: a supply shock, a demand shock and a nominal shock via contemporaneous sign restrictions. Our procedure works as follows. Specifically, let \( \Omega_t = P_t'P_t \) be an arbitrary decomposition of the VAR covariance matrix \( \Omega_t \), and let \( \tilde{A}_{0,t} \equiv P_t \). We draw an \( N \times N \) matrix, \( J \), from the \( N(0, 1) \) distribution. We take the \( QR \) decomposition of \( J \). That is, we compute the matrices \( Q \) and \( R \) such that \( J = QR \). This gives us a candidate structural impact matrix as \( A_{0,t} = \tilde{A}_{0,t}Q \). We check if the rows of the \( A_0 \) matrix are consistent with the restrictions in Table A. If this is the case we store \( A_{0,t} \) and repeat the procedure until we have 50 \( A_{0,t} \) matrices that satisfy the sign restrictions. Out of these 50 stored \( A_{0,t} \) matrices we retain the matrix with elements closest to the median across these 50 estimates. If the sign restrictions are not satisfied, we draw another \( J \) and repeat the above.
Charts

Chart 1: Unconditional volatility of relative output growth, relative inflation and changes in the real exchange rate for the UK-US VAR
Chart 2: Unconditional volatility of relative output growth, relative inflation and changes in the real exchange rate for the UK-euro area VAR
Chart 3: Unconditional volatility of relative output growth, relative inflation and changes in the real exchange rate for the euro area-US VAR
Chart 4: Unconditional volatility of relative output growth, relative inflation and changes in the real exchange rate for the Canada-US VAR
Chart 5: Unconditional volatility of relative output growth, relative inflation and changes in the real exchange rate for the Japan-US VAR
Chart 6: Coherence for real exchange rate and relative output
Chart 7: Impulse response to a supply shock for the UK-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 8: Impulse response to a demand shock for the UK-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 9: Impulse response to a nominal shock for the UK-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 10: FEVD for the UK-US VAR
Chart 11: Time series of structural shocks (four-quarter moving average) for the UK-US VAR
Chart 12: Counterfactual estimates for the UK-US VAR
Chart 13: Impulse response to a supply shock for the UK-euro area VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 14: Impulse response to a demand shock for the UK-euro area VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 15: Impulse response to a nominal shock for the UK-euro area VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 16: FEVD for the UK-euro area VAR
Chart 17: Impulse response to a supply shock for the euro area-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 18: Impulse response to a demand shock for the euro area-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 19: Impulse response to a nominal shock for the euro area-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 20: FEVD for the euro area-US VAR
Chart 21: Time series of the structural shocks (four-quarter moving average) for the euro-US VAR
Chart 22: Counterfactual estimates for the euro area-US VAR
Chart 23: Impulse response to a supply shock for the Canada-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 24: Impulse response to a demand shock for the Canada-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 25: Impulse response to a nominal shock for the Canada-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 26: FEVD for the Canada-US VAR
Chart 27: Time series of structural shocks (four-quarter moving average) for the Canada-US VAR
Chart 28: Counterfactual estimates for the Canada-US VAR
Chart 29: Impulse response to a supply shock for the Japan-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 30: Impulse response to a demand shock for the Japan-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 31: Impulse response to a nominal shock for the Japan-US VAR. The top row of the chart presents the estimates for a 1 standard deviation shock. The second row normalises the shock such that the initial increase in relative output is 1% throughout the sample. The third row normalises the shock such that the initial increase in relative inflation is 1% throughout the sample. The fourth row normalises the shock such that the initial increase in the real exchange rate is 1% throughout the sample. The bottom three rows show the response after four quarters. The solid black line represents the impulse response from a fixed-coefficient VAR.
Chart 32: FEVD for the Japan-US VAR
Chart 33: Time series of structural shocks (four-quarter moving average) for the Japan-US VAR
Chart 34: Counterfactual estimates for the Japan-US VAR
Chart 35: Sensitivity analysis for the UK-US VAR
Chart 36: Sensitivity analysis for the euro area-US VAR
Chart 37: Sensitivity analysis for the Canada-US VAR
Chart 38: Sensitivity analysis for the Japan-US VAR

[Diagram showing various graphs and charts related to the sensitivity analysis for the Japan-US VAR]
Chart 39: Cumulated impulse response of the real exchange rate to a monetary contraction and an expansion of foreign output for different degrees of exchange rate pass-through using the Monacelli (2005) model. The real exchange rate is defined such that an increase is an appreciation.
Chart 40: Cumulated impulse response of the real exchange rate to a monetary contraction and an expansion of foreign output for different values of the domestic price stickiness parameter using the Monacelli (2005) model. The real exchange rate is defined such that an increase is an appreciation.
References


