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Cyclical risk aversion, precautionary saving and monetary policy

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Abstract

This paper analyses the conduct of monetary policy in an environment in which cyclical swings in risk appetite affect households' propensity to save. It uses a New Keynesian model featuring external habit formation to show that taking note of precautionary saving motives justifies an accommodative policy bias in the face of persistent, adverse disturbances. Equally, policy should be more restrictive following positive shocks.

Key words: Precautionary saving, monetary policy, cyclical risk aversion, macro-finance, DSGE models.

JEL classification: E32, G12.

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Summary

Monetary policy making in central banks calls for an understanding of how the economy responds to shocks. Economists work with models to achieve this. One type of model that has become increasingly used is the dynamic stochastic general equilibrium framework. Theory is used to describe how all the actors in the economy behave, and to spell out the dynamic evolution of the interconnected economy. The 'stochastic' part indicates that there is a fundamental uncertainty pervading the economy.

Most such policy analyses are conducted using linear models. That is, the underlying decision rules, which will often be non-linear, are approximated by 'first-order' linear relationships. These can be very good approximations, but while they may be able to replicate salient features of macroeconomic dynamics, there are important areas where their ability to 'match data' is less satisfactory. In particular, all such models ignore the impact of uncertainty on the transmission mechanism of shocks.

Specifically, there are two important aspects of household behaviour that cannot be captured in linear models. First, there is no reason for households to require compensation for holding risky assets, in contrast to reality. Second, there is no 'precautionary' motive for saving – meaning that the models ignore households' desire to build up reserves of wealth to buffer them against the possibility of episodes of bad luck. So to the extent that precautionary savings are a clear feature of macroeconomic data and that risk premia are significant determinants of asset price data, using models so badly misspecified along these dimensions could result in systematically biased policy recommendations. This paper investigates the issue in more depth.

To address these points, our framework allows uncertainty to affect saving. This channel is ruled out by assumption in (first-order) linear models but is incorporated in our solution method which accounts for (higher-order) uncertainty effects. We assume that the utility households get from consumption is driven by 'external habits'. That is, they value consumption according to the difference between it and a slow-moving reference value. This introduces some cyclical variation into attitudes to risk. The critical thing for the policymaker is that these cyclical swings in risk-attitudes affect the cyclical behaviour of the 'natural' rate of interest.

We find that properly accounting for swings in risk appetite and the desire to save in this way reduces the optimal size of monetary policy responses to productivity shocks. Following a positive productivity shock central bankers striving to maintain price stability cut rates to boost demand and prevent falls in the price level. However, since a persistent positive productivity shock also reduces households' desire to save, the cut in rates required to boost demand is smaller – ie, the desire to save to smooth consumption is partially offset by the desire to save for precautionary reasons. Conversely, given that a positive demand shock merits interest rate hikes to prevent inflation rising - and since associated falls in precautionary motives exacerbate the increases in demand - policy needs to respond more strongly once changes in precautionary savings are accounted for. Overall, the precautionary channel introduces a 'contractionary bias' during booms and an accommodative slant during downturns. The model is highly stylised and illustrates rather than estimates the size of these effects, but helps to clarify the mechanism.

1 Introduction

Much of modern policy analysis is conducted using linear, or linearised, models. While these may be able to replicate salient features of macroeconomic dynamics, there are important areas where their ability to 'match data' is less satisfactory. In particular, all such models ignore the impact of uncertainty on the transmission mechanism of shocks.[1] Agents in these models do not require compensation for holding risky assets, neither do they save for precautionary reasons. To the extent that risk premium is a significant determinant of asset price data and precautionary savings are a clear feature of macroeconomic data, using models so badly misspecified along these dimensions could result in systematically biased policy recommendations. In what follows we investigate this issue in more depth.

The impact of non-linearities and risk on economic dynamics has recently been the subject of considerable attention (eg Rubio-Ramirez and Fernández-Villaverde (2005); Andreasen (2008); Rudebusch and Swanson (2008); Hordahl, Tristani and Vestin (2008); Ravenna and Seppala (2006)). Rather than trying to analyse many aspects of uncertainty, we start by focusing on just one – precautionary savings.[2] The importance of precautionary motives has long been recognised with some estimates suggesting that they account for 40% of all wealth accumulation.[3] Moreover, since we are interested in monetary policy implications of uncertainty, incorporating a channel which directly affects equilibrium interest rates seems fundamental. Finally, focusing on a single aspect of risk makes it easier to establish traction with standard models used for policy analysis and allows us to derive our results analytically.

We believe that a model's ability to match the dynamics of risk-premia is a good diagnostic of whether it accounts for risk correctly. Since the benchmark macro model fails to do so, we extend it by introducing persistent external habits – whose appeal in the asset-pricing context was demonstrated by Campbell and Cochrane (1999). In our framework, external habits generate

[1] Our framework is free of model uncertainty with shocks being the only stochastic component. All linear models belonging to this class are 'certainty-equivalent' meaning that coefficients of their policy functions are independent of uncertainty (shock volatility).

[2] There has been some ambiguity as to what exactly precautionary savings are – see also Floden (2008). Our usage of the term is closest to that in Kimball (1990) and implies that, absent uncertainty, there would be no precautionary savings. Our model assumes complete financial markets and precautionary savings do not arise from borrowing or liquidity constraints as in Deaton (1991) or Huggett and Ospina (2001).

[3] See also Carroll and Samwick (1998). Other papers highlighting the importance of precautionary savings include the seminal contributions of Leland (1968) and Sandmo (1970) as well as the papers of Carroll (1992), Kazarosian (1997) or Ludvigson and Michaelides (2001).

cyclical swings in risk aversion, which translate into fluctuations in the desire to save for precautionary reasons. Crucially, to make this channel relevant, we consider a non-linear approximation to the consumption-Euler equation – explicitly allowing for a state-dependent precautionary-saving motive.

Our first contribution is to characterise factors determining the cyclical properties of precautionary savings. We show that these factors match those driving the dynamics of risk premia. Accordingly, a model in which risk premia vary in line with the data is likely to generate countercyclical precautionary saving motives. We also find that a countercyclical coefficient of risk aversion, which is a standard feature of all habit models, might not be sufficient to generate such dynamics. What is necessary is that the persistence of shocks and habits is sufficiently high – ie agents must expect a fall in living conditions to persist in order for higher risk aversion to translate into a greater desire to save.[4]

We then analyse policy implications of such swings in precautionary saving motives. We derive expressions for the 'natural' rate of interest – ie the one that would prevail if prices were fully flexible – both in a linear, 'certainty-equivalent' setup and in a world in which agents save for precautionary reasons. In doing so, we characterise monetary policy consistent with price stability in both setups.[5]

We find that properly accounting for swings in risk appetite and the desire to save reduces the appropriate monetary policy response to productivity shocks. Following a positive productivity shock central bankers striving to maintain price stability cut rates to boost demand and prevent falls in the price level. However, since a persistent positive productivity shock also reduces agents' desire to save for precautionary reasons, the cut in rates required to boost demand is smaller – ie the intertemporal substitution effect is partially offset by swings in the precautionary motive. Conversely, given that a positive demand shock merits interest rate hikes – and since associated falls in precautionary motives exacerbate the increases in demand – policy needs to respond more strongly once changes in precautionary savings are accounted for. Overall, the

[4]These conditions closely mirror those for risk premium countercyclicality derived in De Paoli and Zabczyk (2008). As a result, and as discussed in that paper, our model is likely to exhibit desirable asset pricing properties.

[5]Amato and Laubach (2004) find that price stability is not fully optimal in the presence of habits. However, such a policy is not quantitatively far from the social optimum – see also footnote 13. For this reason, and to obtain analytical results, we focus on such a policy.

precautionary channel introduces a contractionary bias during booms, and an accommodative slant during downturns.

Our analytical expressions show that the size of the 'precautionary correction' is increasing in the degree of shock volatility. The implication is that ignoring the impact of swings in risk appetite and precautionary behaviour would tend to lead to larger systematic policy mistakes in highly turbulent times, when shock volatility is large.

The remainder of the paper is structured as follows. In the next section we present the model. We also characterise the linearised system of equilibrium conditions and the corresponding natural rate of interest. In Section 3, we incorporate the precautionary savings channel and analyse its implications for the natural rate of interest and thus monetary policy. We then use simulations to illustrate our results and inspect their robustness before summarising and highlighting possible extensions.

2 Model

Our model economy is inhabited by a continuum of consumer-producers living on the unit interval (and indexed by $j \in [0, 1]$). Agents are assumed to maximise expected utility, which is given by

$$U^j = E \sum_{t=0}^{\infty} \beta^{-t} \left(\frac{\xi_{d,t} \left(C_t^j - hX_t \right)^{1-\rho} - 1}{1-\rho} - \frac{\xi_{y,t}^{-\eta} y_t(j)^{\eta+1}}{\eta+1} \right) \quad \textbf{(1)}$$

where C_t^j denotes agent j's consumption, X_t is the level of habits and $\xi_{d,t}$ is a preference shock. The second term in the large bracket captures the disutility of producing $y_t(j)$ units of the differentiated output good given productivity denoted by $\xi_{y,t}$.[6]

The coefficient of relative risk aversion is defined as[7]

$$\vartheta(C_t, X_t) := -C_t \cdot \frac{U_{cc}(C_t, X_t)}{U_c(C_t, X_t)} = \frac{\rho}{S_t} \quad \textbf{(2)}$$

[6]Given the Calvo price-setting specification that we subsequently adopt, households' production income could be different depending on the type of good produced. In the remainder, as in Woodford (2003), we assume that there exist competitive financial markets in which these risks are efficiently shared.

[7]As noted in Campbell, Lo and MacKinlay (1997) 'risk aversion may also be measured by the normalized curvature of the value function [...] or by the volatility of the stochastic discount factor [...] While these measures of risk aversion are different from each other in this model, they all move inversely with S_t.'

where surplus consumption S_t is given by

$$S_t := \frac{C_t - hX_t}{C_t}. \qquad \textbf{(3)}$$

and $U_y(\cdot,\cdot)$ denotes the partial derivative of utility function $U(\cdot,\cdot)$ with respect to y. Since this coefficient measures agents' willingness to enter pure consumption gambles, given habits equal to X_t, it can be referred to as consumption risk aversion. It is easy to show that $\vartheta(C_t, X_t)$ is countercyclical, when – as in Campbell and Cochrane (1999) – S_t is used as a measure of cyclical stance.

We assume that habits X_t are 'external'– ie individual agents treat them as exogenous. We adopt a slow-moving habit specification under which

$$x_t = (1-\phi)c_{t-1} + \phi x_{t-1} \qquad \textbf{(4)}$$

where ϕ controls the persistence of the habit process and small letters denote log-deviations from steady state. We further assume that both preference and productivity shocks are autoregressive processes given by

$$\varepsilon_{d,t+1} = \gamma_{dem}\varepsilon_{d,t} + \epsilon_{d,t+1} \qquad \varepsilon_{y,t+1} = \gamma_{prod}\varepsilon_{y,t} + \epsilon_{y,t+1}$$

with $\varepsilon_{x,t} \equiv \log(\xi_{x,t})$ and the disturbances $\epsilon_{x,t+1}$ being mean zero, uncorrelated $i.i.d.$ random variables with variance given by $\sigma_x^2,\ x \in \{d, y\}$.

Aggregate consumption and price indices, C_t and P_t, are defined as

$$C_t = \left[\int_0^1 c_t(z)^{\frac{\sigma-1}{\sigma}}\,dz\right]^{\frac{\sigma}{\sigma-1}} \qquad P_t = \left[\int_0^1 p(z)^{1-\sigma}\,dz\right]^{\frac{1}{1-\sigma}}$$

where $\sigma > 0$ is the elasticity of substitution between the differentiated varieties. Conditional on the specification above, we can characterise agents' intratemporal and intertemporal decisions. Optimality implies, respectively

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\sigma} Y_t \qquad \textbf{(5)}$$

$$1 = R_t E_t\left[\beta\frac{\xi_{d,t+1}(C_{t+1} - hX_{t+1})^{-\rho}}{\xi_{d,t}(C_t - hX_t)^{-\rho}}\right]. \qquad \textbf{(6)}$$

Alternatively, we could rewrite the consumption Euler equation as

$$1 = R_t E_t \mathcal{M}_{t+1} \qquad \textbf{(7)}$$

where the stochastic discount factor $\mathcal{M}_{t+1}$ is defined as

$$\mathcal{M}_{t+1} \equiv \beta \frac{\xi_{d,t+1}(C_{t+1} - hX_{t+1})^{-\rho}}{\xi_{d,t}\,(C_t - hX_t)^{-\rho}}. \quad \textbf{(8)}$$

Prices are assumed to follow a partial adjustment rule à la Calvo (1983). Producers of differentiated goods know the form of their individual demand functions, given by **(5)**, and maximise profits taking aggregate demand Y_t and the price level P_t as given. In each period, a fraction $\alpha \in [0, 1)$ of randomly chosen producers is not allowed to change the nominal price of their output. The remaining fraction of firms, given by $(1 - \alpha)$, chooses prices optimally by maximising the expected discounted value of profits. The optimal choice of producer j allowed to reset his price at time t can be shown to satisfy

$$E_t \sum_{T=t}^{+\infty} \frac{y_{t,T}(j)}{(\alpha\beta)^{t-T}} \left[\frac{\widetilde{p}_t(j)}{P_T} U_c(C_T, X_T, \xi_{d,T}) + \frac{\sigma}{(\sigma - 1)} U_y\left(y_{t,T}(j), \xi_{y,T}\right) \right] = 0 \quad \textbf{(9)}$$

where $y_{t,T}(j)$ is producer j's time t estimate of demand for his good at time T, should he be unable to reset his price $\widetilde{p}_t(j)$ before period T. It can be proved that equation **(9)** implies that the price index evolves according to

$$(P_t)^{1-\sigma} = \alpha P_{t-1}^{1-\sigma} + (1 - \alpha)\,(\widetilde{p}_t)^{1-\sigma} \quad \textbf{(10)}$$

where we exploit the fact that all producers who reset prices at time t equate them to $\widetilde{p}_t$.

Finally, we close the model with a monetary policy rule. In fact, most of the analysis to follow assumes that the central bank follows a targeting rule that ensures price stability – ie it sets $\pi_t \equiv \log(P_{t+1}/P_t) = 0$ for every t as to replicate the flexible price allocation. But such a target could be implemented using an instrumental rule (eg Gali (2008), Chapter 4). This allows us to draw conclusions for a central bank that uses interest rates to achieve price stability (arguably, a more realistic representation of how monetary policy is conducted).[8] In the final section of the paper we explicitly consider the case in which the monetary authority follows a Taylor-type rule.

The system is closed by the market clearing condition $C_t = Y_t$. To keep the model parsimonious and allow for analytical representation of the results our framework abstracts from capital, there is no storage technology, and agents are homogenous – as in the canonical New Keynesian model. Effectively, this means that in equilibrium there are no savings. Nevertheless, agents

[8]The reason why we choose to solve the model using targeting rules rather than instrumental rules is that in this case the order of approximation of the Euler equation is irrelevant for the dynamics of other variables.

willingness to save does affect dynamics and is reflected in the Euler equation. We can now summarise the log-linearised system of equilibrium conditions as

$$\begin{cases} r_t = & E_t(\rho\,(1-h)^{-1}\,(\Delta y_{t+1} - h\,\Delta x_{t+1}) - \Delta\varepsilon_{d,t+1}) \\ \pi_t = & k\left(\kappa_0\,(1-h)^{-1}\,y_t - \rho\,(1-h)^{-1}\,h x_t - \eta\varepsilon_{y,t} - \varepsilon_{d,t}\right) + \beta E_t \pi_{t+1} \\ x_t = & (1-\phi) y_{t-1} + \phi\, x_{t-1}. \end{cases}$$

where $k = (1-\alpha\beta)(1-\alpha)/(\alpha(1+\sigma\eta))$.

From the system above we can derive the equilibrium interest rate consistent with price stability in a linear world

$$r_t^* = \kappa_1 E_t(\Delta\varepsilon_{y,t+1} - (1-h)\Delta\varepsilon_{d,t+1} - h\,\Delta x_{t+1}^*) \qquad \textbf{(11)}$$

where x^* is the flexible-price level of habits and where

$$\kappa_0 = (1-h)\eta + \rho \qquad \text{and} \qquad \kappa_1 = \rho\,\eta\,\kappa_0^{-1}.$$

Expression **(11)** shows that the interest rate consistent with full price stability falls (rises) following a positive supply (demand) shock – with the magnitude of the response, on impact, given by κ_1 $[(1-h)\kappa_1]$.

3 Cyclical risk aversion and precautionary saving

We now consider the minimum departure from a linear model in which we can analyse the impact of cyclical swings in risk aversion and precautionary saving motives on economic dynamics. As a first step, we retain the linear specification of equilibrium conditions other than the Euler equation **(7)** – an approach that is similar to the one commonly used in the macro-finance literature.[9] This allows us to single out the effect of precautionary savings in the model's dynamics. Also, under this assumption we can obtain analytical solutions for the determinants of precautionary behaviour. Nevertheless, as a robustness check, we relax this assumption later in the paper and assess the implications of having a non-linear approximation to the entire model.

So to capture the precautionary savings motive we exploit the fact that under conditional

[9] Many macro-finance papers consider a linearised macro model while Euler equations for asset prices, such as the risk-free rate, are determined recursively and approximated non-linearly (see, for example, Jermann (1998)).

log-normality of $\mathcal{M}_t$ the Euler condition **(7)** becomes[10]

$$-r_t = \underbrace{E_t(m_{t+1})}_{\text{Intertemporal substitution effect}} + \frac{1}{2} \underbrace{var_t(m_{t+1})}_{\text{Precautionary savings effect}} \quad \textbf{(12)}$$

where $m_{t+1} \equiv \log(\mathcal{M}_{t+1}/\bar{\mathcal{M}})$ and the upper bar denotes steady-state values. So, although in our framework there are no actual savings, the interest rate that clears the bond market is affected by agents' willingness to save both for precautionary and intertemporal smoothing reasons.

While linear models capture the intertemporal substitution effect, they ignore the term $var_t(m_{t+1})$. This term summarises how uncertainty affects interest rates through changes in agents' willingness to amass precautionary savings. Accordingly, to analyse how the precautionary savings channel affects the transmission mechanism of shocks, we need to understand the determinants of $var_t(m_{t+1})$. In particular, we would like to evaluate how such precautionary motives change over the cycle. Defining

$$\widetilde{\mathcal{M}}_{t+1} = \beta \frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}} \quad \textbf{(13)}$$

we can write

$$var_t(m_{t+1}) = var_t(\tilde{m}_{t+1}) + cov_t(\tilde{m}_{t+1}, \Delta\varepsilon_{d,t+1}) + \sigma_d^2. \quad \textbf{(14)}$$

As shown in the appendix, the covariance term in equation **(14)** is necessarily countercyclical. And given that shocks are homoskedastic (ie σ_d^2 is constant), the cyclical properties of $var_t(m_{t+1})$ depend solely on $var_t(\tilde{m}_{t+1})$. Accordingly, we approximate this term to third-order, the lowest which allows for time variation in $var_t(\tilde{m}_{t+1})$[11]

$$var_t(\tilde{m}_{t+1}) = \kappa_1^2(\eta^{-2}\sigma_d^2 + \sigma_y^2)(1 - \kappa_y\varepsilon_{y,t} - \kappa_d\varepsilon_{d,t} + \kappa_x x_t) \quad \textbf{(15)}$$

where

$$\kappa_{y/d} = \frac{2h\eta(h\gamma(1-\phi) + \kappa_0(\gamma_{prod/dem} + \phi - 1))}{\kappa_0^2} \quad \textbf{(16)}$$

$$\kappa_x = \frac{2h(\rho+\eta)(\kappa_0 + \rho h(1-\phi))}{\kappa_0^2}. \quad \textbf{(17)}$$

Equation **(15)** highlights three channels through which uncertainty affects investors' behaviour: the overall level of macroeconomic volatility – given by σ_y^2 and σ_d^2; investors' risk aversion – given by ρ which in turn determines κ_1; current and past economic conditions – as summarised by the state variable x_t and shocks $\varepsilon_{y,t}$, and $\varepsilon_{d,t}$.

[10] This equation holds up to second-order without any distributional assumptions on the stochastic discount factor.

[11] Note that rather than take the full third-order approximation to the Euler equation here we only approximate the variance term. This restriction – which permits us to single out the precautionary savings effect and allows us to derive tractable analytical results – is relaxed in our numerical exercises.

Equation **(15)** demonstrates that as long as investors are risk averse uncertainty affects their consumption decisions ($\rho > 0 \Rightarrow \kappa_1 > 0$). It also illustrates that without habit formation ($h = 0$) the strength of the precautionary saving motive would not vary over the cycle ($\kappa_y = \kappa_d = \kappa_x = 0 \Rightarrow var_t(\tilde{m}_{t+1})$ is constant).[12] Furthermore, inspecting expression **(16)** reveals that

$$\gamma_{prod} + \phi > 1 \Rightarrow \kappa_y > 0 \text{ and } \gamma_{dem} + \phi > 1 \Rightarrow \kappa_d > 0 \quad \textbf{(18)}$$

which means that if shocks affecting economic activity are sufficiently persistent and habits adjust slowly, then $var_t(\tilde{m}_{t+1})$ changes countercyclically. Accordingly, investors will increase their willingness to engage in precautionary saving following bad shocks if they expect future economic conditions to remain poor (consumption to remain persistently close to the habit level).

If, on the other hand, the expectation is for an improvement in economic prospects, then negative shocks might not translate into higher precautionary savings – even if the coefficient of risk aversion given by **(2)** increases. This is because if habits are fast moving and consumption recovers quickly, investors faced with the bad shock will rapidly get used to lower levels of consumption while at the same time, the latter quickly recovers. This means that investors actually expect consumption to be far above its habit level in the future and therefore might be less inclined to engage in precautionary savings. As discussed in De Paoli and Zabczyk (2008), similar conditions are necessary to ensure that risk premia are countercyclical.

4 Precautionary saving and monetary policy

The implications of precautionary saving for interest rates will, therefore, depend on the structural characteristics of the economy. Absent consumption habits, with time-invariant risk aversion, the presence of uncertainty will affect the average level of the natural interest rate, but not its dynamics. In this case, the response of the natural interest rate to shocks would not be affected by buffer-stock saving motives. In the general case, however, changes in perceived uncertainty (captured by changes in $var_t(m_{t+1})$) would generate fluctuations in the equilibrium interest rate – with ramifications for the conduct of monetary policy.

Amato and Laubach (2004) show that in a New Keynesian model with external habits, similar to ours, 'despite the fact that stabilisation is not complete $V[\pi]$ (the optimal volatility of inflation)

[12] Section A.1 in the appendix shows that, in this case, not only is $var_t(\tilde{m}_{t+1})$ constant, but $var_t(m_{t+1})$ is also time invariant.

is quite low'.[13] So in what follows, we consider the case in which the central bank's goal is to maintain price stability. Crucially, if the monetary authority uses interest rates as an instrument to achieve this goal, then knowing the behaviour of the natural rate of interest would be key – as policy rates that ensure price stability track this rate. But how does precautionary behaviour affect the natural rate and, thus, the appropriate policy response to shocks?

Equation **(11)** implies that the magnitude of responses of the natural rate to a productivity shock in a 'linear' world is given by κ_1. When accounting for uncertainty, the size of these responses also depends on the cyclicality of precautionary savings. If shocks and habits are persistent, and thus precautionary behaviour is countercyclical, then the response to shocks is dampened. That is, the precautionary savings effect (captured by $\kappa_y > 0$ in equation **(15)**) counterbalances the intertemporal substitution effect (captured in equation **(11)**). A negative productivity shock increases the perceived riskiness of the economic environment, which raises investors' willingness to save and puts downward pressure on interest rates. As a result, the equilibrium interest rate that is consistent with stable prices will be lower than in a linear economy. These results thus suggest that interest rates should respond less to productivity shocks when precautionary savings are taken into account. A similar conclusion would be reached if we were considering other types of supply shocks – such as mark-up shocks.[14]

Note that our analysis evaluates the implications of allowing for higher-order approximations (as to capture the precautionary savings effect) in a model with habits. This is different to the analysis of Amato and Laubach (2004) who assess the differences in the dynamics of interest rates in a linear model with and without habits. Their findings suggest that habits increase the size of interest rates fluctuations because it increases the intertemporal substitution effect. This is also found in Campbell and Cochrane (1999). Our analysis, on the other hand, focuses on the fact that, in a model *with* habits, allowing time-varying risk aversion to affect agents willingness to save reduces the response of the natural rate to productivity shocks.

We now consider the case of preference shocks. Condition **(18)** shows that when uncertainty is introduced in a model that features persistent shocks and habits, negative preference shocks also

[13] In Amato and Laubach (2004), Table 1, final column – which considers the calibration closest to ours – the optimal volatility of inflation is less that 0.2%.

[14] Mark-up shocks would enter the system of equilibrium conditions in a similar way to productivity shocks, except with a different sign.

lead to higher precautionary savings. But in this case incorporating uncertainty magnifies the impact of the shock. That is, the precautionary savings effect (captured by $\kappa_d > 0$ in equation **(15)**) reinforces the intertemporal substitution effect (captured in equation **(11)**). Accordingly, in such settings, policymakers striving for price stability should respond more aggressively to demand shocks.

Higher precautionary savings can be thought of as introducing an extra negative demand shock – both following negative productivity and preference shocks. Since productivity and demand shocks call for opposite interest rate reactions (when policymakers' aim is to maintain price stability) these results suggest that depending on the source of the shock, policy that ignores precautionary savings will either undershoot or overshoot its appropriate level. But the general prescription is that following expansionary shocks (ie a positive demand or supply shock) monetary policy should be more restrictive than in a certainty equivalent world, while when the shocks are contractionary monetary policy should be more accommodative.

5 Quantitative analysis

The analysis developed so far offered an analytical representation of the monetary policy transmission mechanism. However, a numerical illustration of differences in policy responses is of independent interest since it sheds some light on the quantitative relevance of our results. The numerical simulations also allow us to assess the implications of relaxing the assumption of linearity in the supply condition.

5.1 Some numerical simulations

For our calibration we define one period as a quarter and set $\beta = 0.99$ to yield a 4% annualised steady-state real interest rate. As in Campbell and Cochrane (1999) the parameter ρ is equal to 2.37 and the degree of habit persistence ϕ is set to 0.97. Following Canzoneri, Cumby and Diba (2007) we assume a value of 6 for the inverse of the elasticity of labour supply η and set $\alpha = 0.66$ to obtain an average length of price contracts of three quarters.[15] The elasticity of substitution between differentiated goods σ is assumed to take the value of 10 in line with

[15] There is a lot of uncertainty surrounding the correct value for eta. It ranges from 0.47 (Rotemberg and Woodford (1997)) to 7 (Canzoneri *et al* (2007)). We found that in our model consumption habits tend to make consumption 'too smooth' relative to the data and thus opted for a value of eta which accounted for their existence.

Benigno and Woodford (2005). Similar to Juillard, Karam, Laxton and Pesenti (2006) and Banerjee and Batini (2003) we calibrate the habit size parameter to $h = 0.85$. As in Smets and Wouters (2003, 2007), the persistence of productivity and preference shocks is set to 0.997 and 0.9 respectively, and the variance of productivity shocks is 3.5 times higher than that of preference shocks. Finally, we calibrate the overall level of shock volatility to match the standard deviation of consumption growth of 0.75% (consistent with official UK Office for National Statistics quarterly data for consumption of non-durables and services from 1976 Q1 to 2007 Q3). The values of all parameters are summarised in Table A.[16]

Noticeably, our benchmark calibration follows closely the ones chosen in models that try to match risk premium dynamics. We believe that a model that correctly captures agents' attitude towards risk and uncertainty should reproduce basic risk premia observations. So, following the insights of Campbell and Cochrane (1999) and De Paoli and Zabczyk (2008), we assume a high level for the habit parameter and a very slow-moving process for the habits. We also assess the implications of considering different parameter values.

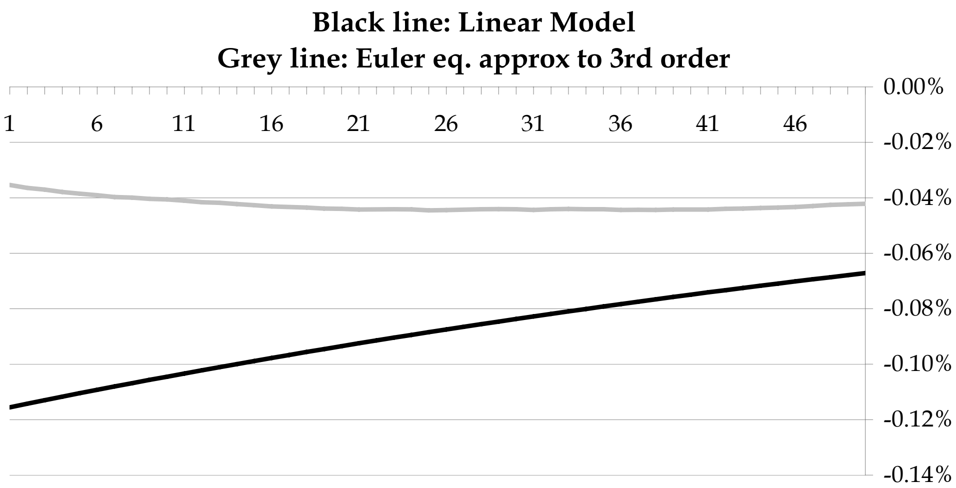
Table A: Parameter values used in the quantitative analysis

Parameter	Value	Notes:
β	0.99	To yield a 4% steady-state real interest rate
η	6	As in Canzoneri *et al* (2007)
ρ	2.37	Following Campbell and Cochrane (1999)
α	0.66	Length of average price contract three quarters
σ	10	Following Benigno and Woodford (2005)
h	0.85	Juillard *et al* (2006) and Banerjee and Batini (2003)
ϕ	0.97	Following Campbell and Cochrane (1999)
γ_{dem}	0.9	Following Smets and Wouters (2003)
γ_{prod}	0.997	Following Smets and Wouters (2007)
σ_y^2 / σ_d^2	3.5	Following Smets and Wouters (2003)
$\sigma_{\Delta c}$	0.75%	UK ONS data from 1976 Q1 to 2007 Q3

We begin the quantitative part of our investigation by comparing the level of the natural rate of

[16]Some of the values used in our calibration are based on microeconomic data while others come from linear, general equilibrium models. Arguably, the fact that we allow for non-linearity could justify amending some of these parameters. For example, while a β of 0.99 implies a deterministic steady-state value of the interest rate equal to 4%, the stochastic mean would be below that. For this reason we verified that our results continue to fold for lower values of beta – including β of 0.982, which yields an ergodic mean of the interest rate equal to 4%. A sensitivity analysis for other parameters is presented below.

Chart 1: Natural rate of interest following a one standard deviation positive productivity shock (annualised, in percentage points)



interest in a linear world with the one that would prevail if the precautionary savings channel was additionally taken into account. First, in line with the theoretical part, we consistently maintain a linearised version of the Phillips curve while alternating between first and third-order approximations to the Euler equation to switch the precautionary channel off and on respectively.[17]

Chart 1 illustrates the response of the natural rate of interest to a one standard deviation productivity shock in a linear model (black line) and in a model that allows for a third-order approximation of the Euler equation and so incorporates precautionary savings (grey line). The chart shows that the fall in the natural rate is smaller once the precautionary saving motive is incorporated. More specifically, once the decreased desire to save is taken into account, the magnitude of the change in interest rates that would be required to boost demand sufficiently to prevent falls in prices is more than halved (from approximately 45 basis points annualised to less than 15 basis points on impact) under our benchmark calibration. Charts 3-4 also illustrate that the differences in the responses are smaller when we reduce the habit parameter (h), the volatility of the productivity shock (σ_y^2) or the habit persistence parameter (ϕ).

[17] In particular, we compute a third-order approximation of the Euler equation using perturbation methods as implemented in *Dynare++* and *Perturbation AIM*. As mentioned previously, third-order is the lowest which allows us examine changes in the precautionary saving motive.

Chart 2: Sensitivity of results to changes in habit 'size' h

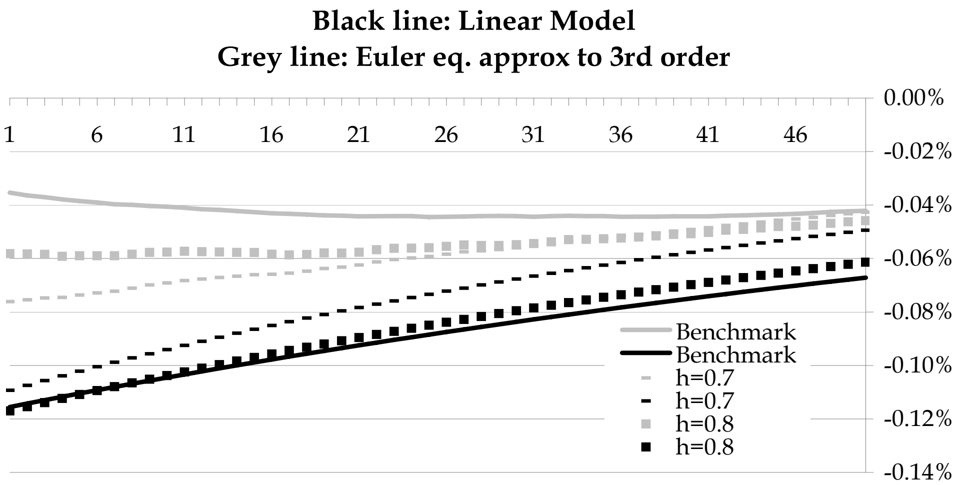


Chart 3: Sensitivity of results to changes in productivity shock volatility σ_y

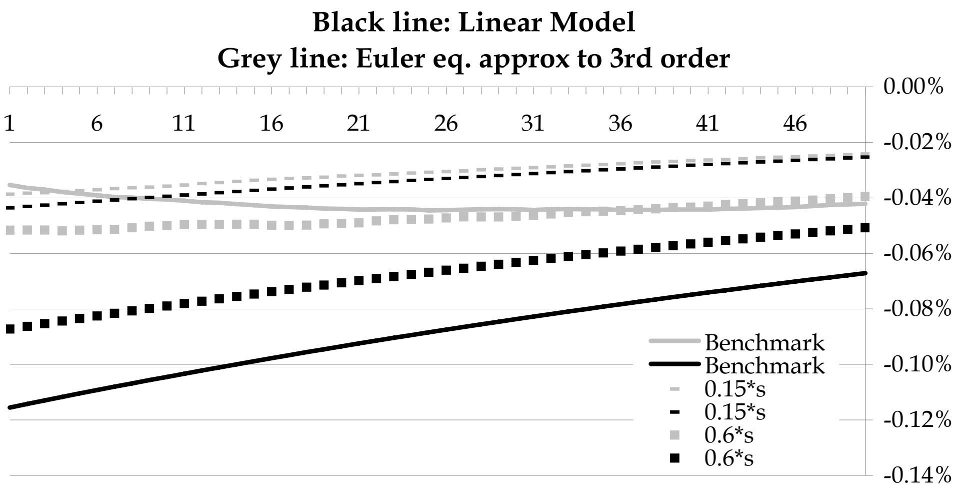


Chart 4: Sensitivity of results to changes in habit persistence ϕ

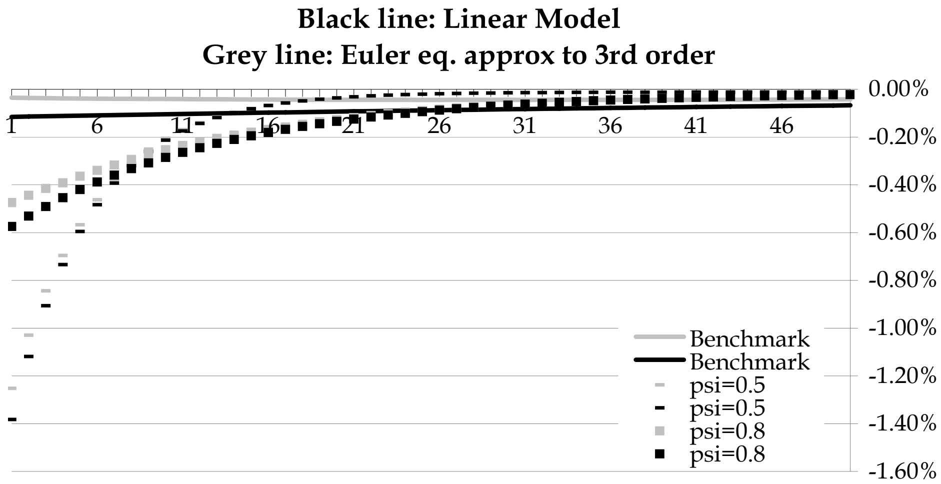


Chart 1 compares the natural rate under two different orders of approximation to the Euler equation. When going beyond the linear approximation, the solution incorporates the pure effect of uncertainty but also corrections coming from non-linearities, which are unrelated to risk. Our argument above hinges on the assumption that the reported differences in impulse responses reflect uncertainty. To verify whether this is the case, Chart **??** considers two solutions. One maintains the assumption that the Euler equation is approximated to third-order, while the other eliminates the effect of uncertainty in this approximation. So, the black line would be equivalent to simulations from a perfect foresight model in which the Euler equation is approximated to third-order.[18] The similarity of the black line in Charts 1 and 5 confirms that the results reported above are indeed driven by uncertainty or, more specifically, precautionary behaviour.

So, in line with the analytical results, the simulations suggest that a central bank following an interest rate rule should be less aggressive in the face of productivity shocks. If one believes that the benchmark calibration correctly captures how uncertainty affect economic dynamics, our exercise would additionally suggest that the effects of precautionary savings can be quantitatively relevant.

Chart 6 demonstrates numerically that the response of the natural rate to a negative preference

[18]More formally the black line is generated by a policy function from which the second order $\sigma\sigma$ correction as well as the third order $\sigma\sigma x$ terms have been deleted. See also Schmitt-Grohe and Uribe (2004) for more details.

Chart 5: Natural rate of interest following a one standard deviation positive productivity shock (annualised, in percentage points): full third-order approximation vs third-order approximation excluding the stochastic corrections

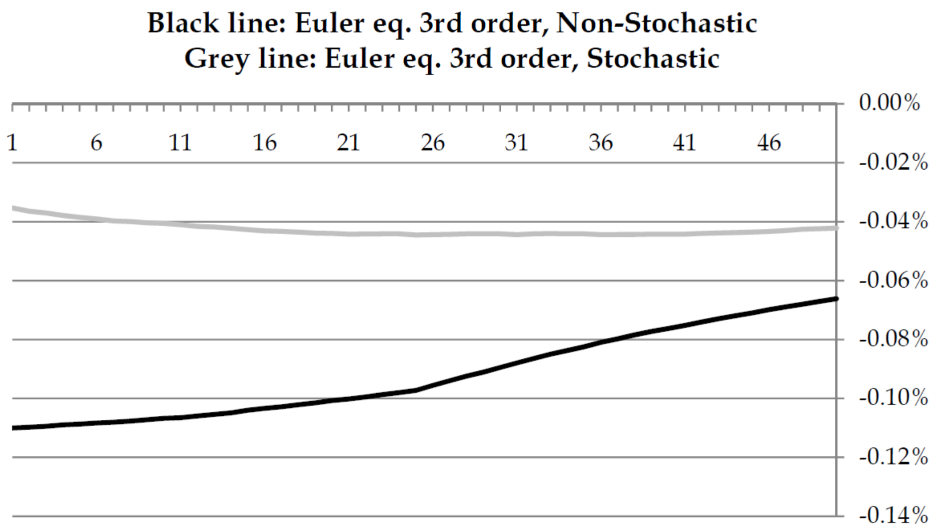
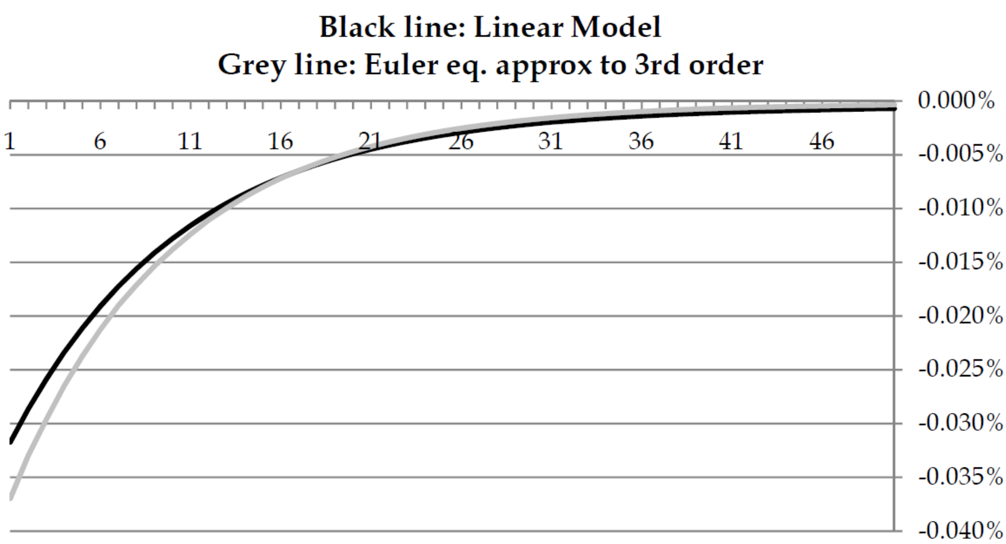


Chart 6: Natural rate of interest following a one standard deviation negative preference shock (annualised, in percentage points)



shock is magnified when the precautionary savings channel is taken into account. Agents' increased desire to save exacerbates the initial shock and calls for more accommodative policy – ie bigger cuts in rates. But the quantitative impact of the precautionary saving effect on the natural rate is smaller than in the case of productivity shocks. This is a reflection of the lower persistence of such shocks relative to productivity shocks.

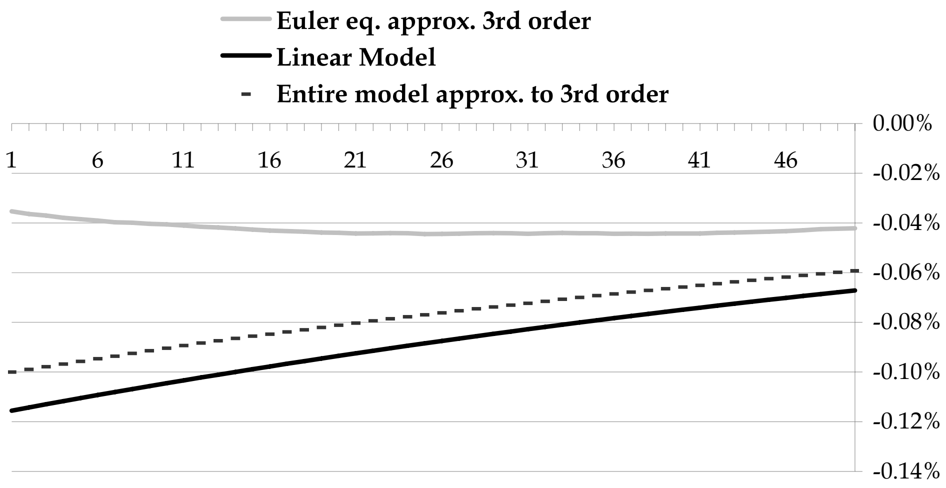
5.2 Allowing for a fully non-linear approximation of the model

Until now we had focused on the precautionary savings channel through which uncertainty affects the behaviour of interest rates. Nevertheless, other channels are present if we consider a fully non-linear approximation of our model. So far we have considered a linear labour-leisure decision,[19] but if we compare a first and third-order approximation of this condition (see Section A.2 in the appendix) we can see that excess consumption – ie consumption relative to the habit level – reacts by more to changes in consumption when third-order moments are incorporated. This implies that agents' marginal rate of intertemporal substitution is also more sensitive to changes in consumption. As a result, agents' desire to smooth consumption will be larger at third-order. So non-linearities in the labour-leisure decision (more specifically, non-linearities in excess consumption) increase the intertemporal substitution effect.

Chart 7 illustrates this result for the case of a positive productivity shock – with the dashed line representing the case in which the entire model is approximated to third-order. As we can see, the natural rate in this model reacts by more than in the case in which only the Euler equation is approximated to higher order. Nevertheless, the response of interest rates under a third-order approximation of the entire model is still smaller than in the fully linear model. That is, the stronger intertemporal substitution effect partially offsets the precautionary savings effects that come about once we allow for non-linearities. But our result that overall precautionary savings dampen the response of the natural rate to productivity shock remains.

[19]Given that we assume that the central bank targets price stability, the labour-leisure decision is the relevant condition to be approximated to higher order.

Chart 7: Sensitivity of results to adopting a fully non-linear specification



5.3 Assessing the performance of a misspecified Taylor rule

We now investigate policy errors which a central bank would make if it incorrectly ignored changes in the strength of agents' precautionary savings motive when setting interest rates. More specifically, we assume that the central bank follows a Taylor rule given by

$$r_t^n = r_t^* + \phi_\pi \pi_t + \phi_\pi (y_t - y_t^*) \quad \textbf{(19)}$$

where r_t^n is the nominal interest rate, y_t^* is the flexible price allocation of output, and r_t^* is the natural rate of interest defined in equation **(11)** – ie one consistent with price stability in a 'linear', risk-free world.

Table B shows the implications of this policy for inflation and the output gap.[20] We see that whereas the Taylor rule given by equation **(19)** ensures zero inflation and output gap volatility in a linear world, where the natural rate is driven purely by the 'intertemporal-substitution' channel, this is no longer the case when uncertainty influences agents' behaviour. More specifically, in that case, the wrong policy increases the standard deviation of the output gap and inflation by 0.4 percentage points.

While our numerical results suggest that the implications of 'policy mistakes' are not large, this

[20]Note that in this exercise we use a nominal version of the Euler equation, given that the central bank is assumed to control the level of the nominal interest rate.

is partially driven by consumption in our model being very insensitive to changes in the interest rate. If we were to reduce the elasticity of intertemporal substitution and consider the case of log utility, this sensitivity would increase and with it the standard deviation of the output gap and inflation. Furthermore, in our calibration the Phillips curve is quite flat, so even if the Taylor rule does not fully stabilise the output gap, this does not translate into a volatile inflation rate (see fifth column of Table B) – ie under a slightly changed calibration these policy errors could become larger.

The concluding observation is that decreasing the level of uncertainty would also reduce the volatilities of inflation and the output gap. That is, lower uncertainty would decrease the size of policy mistakes. Thus, in these settings if central banks confront a stable economic environment, this also translates into small policy mistakes. This result, which is consistent with Chart 3, is illustrated in column 6 of Table B.

Table B: Policy exercise (values annualised and in percentage points)

Moment	Linear model	Incorporating precationary saving Benchmark	$\rho = 1$	$\&\kappa = 0.1$	$\&\sigma_{\Delta c} = 1.5\%$
σ_{π}	0	0.40	0.56	0.80	0.80
σ_{ygap}	0	0.40	0.96	0.40	1.60

6 Conclusion

Our results show that, following persistent, adverse shocks policymakers might be well advised to aim off the predictions of linear models and conduct more accommodative policy – particularly in highly turbulent periods. Equally, when demand and supply conditions are improving, taking note of the precautionary saving motives justifies more restrictive policy than would otherwise be the case. Since the size of the precautionary correction is increasing in the degree of volatility, mistakenly ignoring this channel would be most costly during highly turbulent periods.

In order to obtain intuitive results and single out the precautionary savings channel, our analysis proceeded in a stylised model. An investigation of the impact of other risk channels in a fully fledged dynamic stochastic general equilibrium model might also be of interest. Moreover, formally accounting for stochastic volatility in a model with investment (in light of the analysis in Bloom (2009)) and enriching the framework by considering Epstein-Zin preferences (as in Bansal and Yaron (2004)) would both make for interesting extensions.[21]

[21] Epstein-Zin preferences would be of particular interest since, with this specification it is possible to calibrate separately the precautionary and the intertemporal smoothing motives for savings.

Appendix A: Derivations

The logarithm of the stochastic discount factor is given by

$$\log(\widetilde{\mathcal{M}}_{t+1}) = \log\left(\frac{(C_{t+1}-X_{t+1})^{-\rho}}{(C_t-X_t)^{-\rho}}\right)$$

$$= \log\left(\frac{(C_{t+1}-X_{t+1})^{-\rho}}{C_{t+1}^{-\rho}}\frac{C_t^{-\rho}}{(C_t-X_t)^{-\rho}}\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}\right) = \log\left(S_{t+1}^{-\rho}S_t^{\rho}C_{t+1}^{-\rho}C_t^{\rho}\right)$$

$$= -\rho\big(\log C_{t+1} - \log C_t + \log S_{t+1} - \log S_t\big) \quad \textbf{(A-1)}$$

and so

$$\widetilde{m}_{t+1} = -\rho\big(c_{t+1} - c_t + s_{t+1} - s_t\big).$$

It thus follows that

$$\mathrm{var}_t\widetilde{m}_{t+1} = \mathbf{E}_t\left(\widetilde{m}_{t+1} - \mathbf{E}_t\widetilde{m}_{t+1}\right)^2 = \rho^2\mathbf{E}_t\Big((c_{t+1} - \mathbf{E}_t c_{t+1}) + (s_{t+1} - \mathbf{E}_t s_{t+1})\Big)^2$$

$$= \rho^2\Big(\mathrm{var}_t c_{t+1} + 2\mathrm{cov}_t(c_{t+1}, s_{t+1}) + \mathrm{var}_t s_{t+1}\Big) \quad \textbf{(A-2)}$$

as the conditional expectations of all t-dated variables can be eliminated.

Up to a second-order approximation (which is all we need to compute a third-order accurate expression for $\mathrm{var}_t\widetilde{m}_{t+1}$) we get

$$s_{t+1} = \Psi_1\big(c_{t+1} - \frac{1}{2}(1-h)^{-1}c_{t+1}^2 - \widetilde{x}_t + c_{t+1}\widetilde{x}_t(1-h)^{-1} - \frac{1}{2}(1-h)^{-1}\widetilde{x}_t^2\big) - \log(1-h)$$

where we used the fact that the habit at time $t+1$ depends only on variables known at time t and so we changed the notation of x_{t+1} to $\widetilde{x}_t$. We also defined $\Psi_1 := h/(1-h)$.

We can now compute a third-order approximation to the $\mathrm{var}_t s_{t+1}$ and to that of $\mathrm{cov}_t(c_{t+1}, s_{t+1})$. From the definition

$$\mathrm{var}_t s_{t+1} = \Psi_1^2\mathrm{var}_t\big(c_{t+1} - \frac{1}{2}(1-h)^{-1}c_{t+1}^2 + c_{t+1}\widetilde{x}_t(1-h)^{-1}\big) \quad \textbf{(A-3)}$$

where again expectations of all t-dated variables were eliminated. It is easy to see that

$$\text{var}_t s_{t+1} = \frac{\Psi_1^2}{(1-h)^2}\Bigg((1-h)^2\text{var}_t\big(c_{t+1}\big) + \frac{1}{4}\text{var}_t(c_{t+1}^2) + \widetilde{x}_t^2\text{var}_t\big(c_{t+1}\big)$$
$$- (1-h)cov_t(c_{t+1}, c_{t+1}^2) + 2\widetilde{x}_t(1-h)\text{var}_t c_{t+1} - \widetilde{x}_t\text{cov}_t(c_{t+1}, c_{t+1}^2)\Bigg). \quad \textbf{(A-4)}$$

Similarly

$$\text{cov}_t(c_{t+1}, s_{t+1}) = \Psi_1\text{cov}_t\big(c_{t+1}, c_{t+1} - \frac{1}{2}(1-h)^{-1}c_{t+1}^2 + c_{t+1}\widetilde{x}_t(1-h)^{-1}\big)$$
$$= \Psi_1\big(\text{var}_t c_{t+1} - \frac{1}{2}(1-h)^{-1}\text{cov}_t\big(c_{t+1}, c_{t+1}^2\big) + \widetilde{x}_t(1-h)^{-1}\text{var}_t c_{t+1}\big). \quad \textbf{(A-5)}$$

Consider the case in which shocks follow an AR(1) process, ie

$$\varepsilon_{y,\,t+1} = \gamma_{\,prod}\varepsilon_{y,\,t} + \epsilon_{y,\,t+1} \qquad\qquad \varepsilon_{d,\,t+1} = \gamma_{\,dem}\varepsilon_{d,\,t} + \epsilon_{d,\,t+1}.$$

where ε_y and ε_d are independent (cross-sectionally and intertemporally). As shown above, to compute the variance of $\widetilde{m}_{t+1}$ we need expressions for

$$\text{var}_t c_{t+1} \text{ and } \qquad\qquad cov_t(c_{t+1}, c_{t+1}^2).$$

We know that

$$c_t = y_t = \big(\rho(1-h)^{-1} + \eta\big)^{-1}\big(\rho(1-h)^{-1}h\widetilde{x}_{t-1} + \varepsilon_{d,t} + \eta\varepsilon_{y,t}\big) \quad \textbf{(A-6)}$$

and so

$$\text{var}_t c_{t+1} = \text{var}_t\big(\rho(1-h)^{-1}h\widetilde{x}_t + \epsilon_{d,t+1} + \gamma_{\,dem}\varepsilon_{d,t} + \eta\epsilon_{y,t+1} + \eta\gamma_{\,prod}\varepsilon_{y,t}\big)$$
$$\cdot\big(\rho(1-h)^{-1} + \eta\big)^{-2} = \Psi_2^{-2}\big(\text{var}_t\epsilon_{d,t+1} + \eta^2\text{var}_t\epsilon_{y,t+1}\big) = \Psi_2^{-2}\big(\sigma_d^2 + \eta^2\sigma_y^2\big) \quad \textbf{(A-7)}$$

where $\Psi_2 = \big(\rho(1-h)^{-1} + \eta\big)$ and where $\text{cov}_t(\epsilon_{d,t+1}, \epsilon_{y,t+1}) = 0$.

By a similar token

$$\text{cov}_t(c_{t+1}, c_{t+1}^2) = \Psi_2^{-3}\text{cov}_t\Bigg(\big(\rho\,\Psi_1\,\widetilde{x}_t + \epsilon_{d,t+1} + \gamma_{\,dem}\varepsilon_{d,t} + \eta\epsilon_{y,t+1} + \eta\gamma_{\,prod}\varepsilon_{y,t}\big),$$
$$\big(\rho\,\Psi_1\,\widetilde{x}_t + \epsilon_{d,t+1} + \gamma_{\,dem}\varepsilon_{d,t} + \eta\epsilon_{y,t+1} + \eta\gamma_{\,prod}\varepsilon_{y,t}\big)^2\Bigg)$$

Since the shocks are assumed Gaussian and uncorrelated, we can write

$$\text{cov}_t(c_{t+1}, c_{t+1}^2) = 2\Psi_2^{-3}\Big(\rho\,\Psi_1\,\widetilde{x}_t + \gamma_{\,dem}\varepsilon_{d,t} + \eta\gamma_{\,prod}\varepsilon_{y,t}\Big)\Big(\sigma_d^2 + \sigma_y^2\eta^2\Big). \quad \textbf{(A-8)}$$

Using equalities **(A-7)** and **(A-8)** in equation **(A-4)** yields

$$\begin{aligned}\text{var}_t s_{t+1} &= \Psi_1^2\Bigg(\Psi_2^{-2}\big(\sigma_d^2+\eta^2\sigma_y^2\big)-(1-h)^{-1}2\Psi_2^{-3}\big(\sigma_d^2+\sigma_y^2\eta^2\big)\\ &\quad\cdot\big(\rho\,\Psi_1\,\widetilde{x}_t+\gamma_{\,dem}\varepsilon_{d,t}+\eta\gamma_{\,prod}\varepsilon_{y,t}\big)+2\widetilde{x}_t(1-h)^{-1}\Psi_2^{-2}\big(\sigma_d^2+\eta^2\sigma_y^2\big)\Bigg)\\ &= \frac{2\Psi_3^2\big(\sigma_d^2+\eta^2\sigma_y^2\big)}{(1-h)}\left(\frac{(1-h)}{2}+\big(1-\rho\Psi_3\big)\widetilde{x}_t-\frac{\big(\gamma_{\,dem}\varepsilon_{d,t}+\eta\gamma_{\,prod}\varepsilon_{y,t}\big)}{\Psi_2}\right) \qquad \textbf{(A-9)}\end{aligned}$$

where $\Psi^3 := \Psi_1\Psi_2^{-1} = h/\big(\rho+(1-h)\eta\big)$. Similarly, plugging **(A-7)** and **(A-8)** into equation **(A-5)** and denoting $\Psi_4 := \Psi_1\Psi_2^{-2} = \big(h(1-h)\big)/\big(\rho+\eta(1-h)\big)^2$ we can write down

$$\begin{aligned}\text{cov}_t(c_{t+1},s_{t+1}) &= \frac{\Psi_4\big(\sigma_d^2+\eta^2\sigma_y^2\big)}{(1-h)}\\ &\quad\cdot\left(1-h+(1-\rho\,\Psi_3)\widetilde{x}_t-\Psi_2^{-1}\left(\gamma_{\,dem}\varepsilon_{d,t}+\eta\gamma_{\,prod}\varepsilon_{y,t}\right)\right). \qquad \textbf{(A-10)}\end{aligned}$$

We can then use equations **(A-7)**, **(A-9)** and **(A-10)** in **(A-2)** to obtain

$$\text{var}_t\widetilde{m}_{t+1} = \frac{\rho^2\big(\sigma_d^2+\eta^2\sigma_y^2\big)}{(\rho+\eta(1-h))^2}\left(1+\frac{2h(\rho+\eta)}{(\rho+\eta(1-h))}\widetilde{x}_t-\frac{2h\big(\gamma_{\,dem}\varepsilon_{d,t}+\eta\gamma_{\,prod}\varepsilon_{y,t}\big)}{(\rho+\eta(1-h))}\right).$$

Given that $\widetilde{x}_t = x_{t+1}$ and so, $\widetilde{x}_t = c_t(1-\phi)+\phi x_t$ we get

$$\begin{aligned}\text{var}_t\widetilde{m}_{t+1} &= \frac{\rho^2\big(\sigma_d^2+\eta^2\sigma_y^2\big)}{(\rho+\eta(1-h))^2}\left(1-\frac{2h\gamma_{\,dem}}{(\rho+\eta(1-h))}\varepsilon_{d,t}-\frac{2h\eta\gamma_{\,prod}}{(\rho+\eta(1-h))}\varepsilon_{y,t}\right.\\ &\qquad\left.+\frac{2h(\rho+\eta)(1-\phi)}{(\rho+\eta(1-h))}c_t+\frac{2h(\rho+\eta)\phi}{(\rho+\eta(1-h))}x_t\right).\end{aligned}$$

Recalling the definition of c_t – equation **(A-6)**

$$c_t = \big(\rho(1-h)^{-1}+\eta\big)^{-1}\big(\rho(1-h)^{-1}hx_t+\varepsilon_{d,t}+\eta\varepsilon_{y,t}\big) \qquad \textbf{(A-11)}$$

and plugging it into the expression derived above yields, after simplifying

$$\begin{aligned}\text{var}_t\widetilde{m}_{t+1} &= \left(1-\frac{2h((1-h)(\rho+\eta)(\phi-1)+(\rho+\eta(1-h))\gamma_{\,dem})}{(\rho+\eta(1-h))^2}\varepsilon_{d,t}\right.\\ &\qquad-\frac{2h\eta((1-h)(\rho+\eta)(\phi-1)+(\rho+\eta(1-h))\gamma_{\,prod})}{(\rho+\eta(1-h))^2}\varepsilon_{y,t}\\ &\qquad\left.+\frac{2h(\rho+\eta)((1-h)\eta+\rho(1-h(\phi-1)))}{(\rho+\eta(1-h))^2}x_t\right)\cdot\frac{\rho^2\big(\sigma_d^2+\eta^2\sigma_y^2\big)}{(\rho+\eta(1-h))^2}.\end{aligned}$$

which is the expression reported in the body of the text.

A.1 The covariance term cov$_t(\widetilde{m}_{t+1}, \Delta\varepsilon_{d,t+1})$

In line with the reasoning of the previous section, we can write

$$\text{cov}_t(\widetilde{m}_{t+1}, \Delta\varepsilon_{d,t+1}) = -\frac{\rho\sigma_d^2}{(\rho+\eta(1-h))}\Bigg(1 + \frac{h(1-h)(\rho+\eta)(1-\phi)}{(\rho+\eta(1-h))^2}\cdot\varepsilon_{d,t}$$
$$+\frac{h(1-h)(\rho+\eta)(1-\phi)\eta}{(\rho+\eta(1-h))^2}\cdot\varepsilon_{y,t} + \frac{h(\rho+\eta)\big((\rho+\eta)\phi + h(\rho^2(1-\phi)-\eta\phi)\big)}{(\rho+\eta(1-h))^2}\cdot x_t\Bigg).$$

Note that the coefficients on $\varepsilon_{y,t}$ and $\varepsilon_{d,t}$ are negative, so the covariance term always moves countercyclically. The coefficient multiplying x_t is negative when $\rho^2(1-\phi) > \eta\phi$, but given that x_t is predetermined, this would not affect the countercyclicality of the covariance term.

A.2 A third-order approximation of the labour-leisure decision

Under flexible prices, or if the central bank successfully stabilises inflation, equation **(9)** can be written as

$$\xi_{d,T}\,(C_T - hX_T)^{-\rho} = \frac{\sigma}{\sigma-1}\left(y_{t,T}(j)/\xi_{y,T}\right)^{\eta} \qquad \textbf{(A-12)}$$

or, given that $Y_T = C_T$ and defining $C_T^e := C_T - hX_T$

$$\xi_{d,T}\left(C_T^e\right)^{-\rho} = \frac{\sigma}{\sigma-1}\left(C_T/\xi_{y,T}\right)^{\eta} \qquad \textbf{(A-13)}$$

Simply taking logs yields

$$\log(\xi_{d,T}) - \rho\log\left(C_T^e\right) = \log\left(\frac{\sigma}{\sigma-1}\right) + \eta\log C_T - \eta\log(\xi_{y,T}). \qquad \textbf{(A-14)}$$

Given the steady-state condition for C_T and C_T^e, this simplifies to

$$\varepsilon_{d,t} - \rho\log\left(C_T^e/\bar{C}^e\right) = \eta\log(C_t/\bar{C}) - \eta\varepsilon_{y,t} \qquad \textbf{(A-15)}$$

or

$$\varepsilon_{d,t} - \rho c_t^e = \eta c_t - \eta\varepsilon_{y,t}. \qquad \textbf{(A-16)}$$

In order to obtain a third-order approximation of the labour-leisure decision in terms of consumption, we need to expand excess consumption c_t^e to third-order. It follows that

$$c_t^e \simeq (1-h)^{-1}(c_t - hx_t)\cdot$$
$$\left\{1 + c_t^2\frac{1}{3(1-h)^2} - c_t x_t\frac{2h}{3(1-h)^2} + x_t^2\frac{h^2}{3(1-h)^2} - c_t\frac{1}{2(1-h)} + x_t\frac{h}{2(1-h)}\right\}$$

which can further be simplified to

$$c_t^e \simeq (1-h)^{-1}(c_t - hx_t) \cdot \left\{1 - \frac{1}{2(1-h)}(c_t - hx_t) + \frac{1}{3(1-h)^2}(c_t - hx_t)^2\right\}.$$

So, the labour-leisure decision can be written as

$$\eta c_t + \rho(1-h)^{-1}(c_t - hx_t) \cdot \left\{1 - \frac{(c_t - hx_t)}{2(1-h)} + \frac{(c_t - hx_t)^2}{3(1-h)^2}\right\} = \eta\varepsilon_{y,t} + \varepsilon_{d,t}. \qquad \textbf{(A-17)}$$

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