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Estimating the impact of the volatility of shocks: a structural VAR approach

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Abstract

A large empirical literature has examined the transmission mechanism of structural shocks in great detail. The possible role played by changes in the volatility of shocks has largely been overlooked in vector autoregression based applications. This paper proposes an extended vector autoregression where the volatility of structural shocks is allowed to be time-varying and to have a direct impact on the endogenous variables included in the model. The proposed model is applied to US data to consider the potential impact of changes in the volatility of monetary policy shocks. The results suggest that while an increase in this volatility has a statistically significant impact on GDP growth and inflation, the relative contribution of these shocks to the forecast error variance of these variables is estimated to be small.

Key words: Vector autoregression, stochastic volatility, particle filter.

JEL classification: E30, E32.

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Summary

A large body of empirical work has focused on estimating the impact of structural shocks on the economy. A large proportion of these studies employ vector autoregressions (VARs) – a system of equations where each variable depends on the lags of all variables included in the model. However, in their current form VAR models cannot directly incorporate the possible role played by the change in the volatility of the structural shocks as this is assumed not to have a direct affect on the variables included in the model. As shown in recent theoretical work, however, changes in shock volatility and uncertainty can have a direct impact on macroeconomy. For example an increase in uncertainty may cause firms to pause hiring and investment decisions thus affecting real activity.

This paper proposed an extended VAR model which incorporates two additional features. First it allows the volatility of structural shocks to be time-varying. Second it allows for a direct impact of this time-varying volatility on the level of the variables included in the model. The paper describes an econometric method to estimate this extended VAR model.

We use the proposed model to estimate the possible impact of changes in the volatility of monetary policy shocks on the US economy. The monetary policy shock is identified from the data using two methods: (1) by assuming that these shocks have no impact on output growth and inflation for one quarter due to policy lags; and (2) by assuming that when these shocks lead to an increase in the federal funds rate this results in a contemporaneous reduction in output and inflation. In both cases, we estimate that the volatility of the monetary policy shock was high during the mid-1970s, the early 1980s and during the recent recession.

In order to gauge the impact of the volatility of the monetary policy shock, the model is simulated under the scenario where this volatility is assumed to double and no other shocks hit the economy. Under these assumptions, this change in volatility is estimated to reduce US GDP growth by 0.2% and inflation by 0.3%. However, once the importance of this volatility shock is considered relative to other shocks hitting the economy, its contribution is found to be small. This suggests that, in relative terms, changes in the volatility of monetary policy shocks are not economically significant.

1 Introduction

A vast body of empirical research has focused on estimating the impact of structural economic shocks on the economy. In very broad terms this literature has focused on the effects of (a) domestic shocks such as shocks to monetary and fiscal policy and to technology; and (b) shocks originating from the rest of the world – ie the impact of changes in foreign demand and monetary policy.[1] Structural vector autoregressions ((S)VARs), originally proposed by Sims (1980), have featured prominently in this literature as they offer a flexible data-driven approach to modelling the transmission mechanism. Results from these models have been used as a benchmark for the performance of more structural economic models such as dynamic stochastic general equilibrium models (DSGEs).

While the transmission mechanism of these shocks has been studied deeply, the role played by changes in the volatility of shocks has been ignored in the structural VAR literature. Most of the adopted SVAR models assume homoscedastic shocks. Studies that do allow for time-varying shock volatility (see, for example, Primiceri (2005)) do not incorporate a direct impact of the shock variance on the endogenous variables. The omission of this transmission channel is a potential problem because of three considerations.

First, a growing number of studies have demonstrated that the volatility of structural shocks such as monetary policy, supply and demand has fluctuated substantially in industrialised countries. For example, the estimated volatility of the (US) monetary policy shock in Primiceri (2005) increases by more than 100% during the early 1980s. Similar results are presented in Benati and Mumtaz (2007) and Mumtaz and Sunder-Plassmann (2010). Second, the recent financial crisis has highlighted the fact that macroeconomic volatility cannot be regarded as a 'pre-great moderation phenomenon'. In other words, fluctuations in macroeconomic volatility and their potential impact is a relevant concern for policymakers. Third, there is a growing body of theoretical work that has identified channels through which changes in volatility can affect the real economy. For example, Bloom (2009) presents simulations from a model where higher uncertainty causes firms to pause their hiring and investment leading to a drop in real activity. Using a non-linear small open economy DSGE model, Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2009) suggest a channel through which changes in real interest rate volatility

[1] This literature is too vast to do it complete justice. However, prominent papers include the studies by Christiano, Eichenbaum and Evans (1996), Uhlig (2005) and Bernanke, Boivin and Eliasz (2005) that explore the effects of monetary policy shocks on the US economy. The impact of foreign shocks is explored by Cushman and Zha (1997), Kim (2001), Scholl and Uhlig (2006) and Mumtaz and Surico (2009) among others. Seminal papers by Blanchard and Perotti (2002) and Mountford and Uhlig (2009) focus on the impact of shocks to fiscal policy.

can affect open economies that use foreign debt to smooth consumption and to hedge against idiosyncratic productivity shocks. As real interest rate volatility increases and as countries are increasingly exposed to variations in marginal utility, they reduce the level of foreign debt by cutting consumption. Investment falls as foreign debt becomes a less attractive hedge for productivity shocks leading to a fall in real activity.

This paper proposes an extension to the SVAR model by: (1) allowing for time-varying variance of structural shocks via a stochastic volatility specification; and (2) by allowing a dynamic interaction between the level of the endogenous variables in the VAR and this time-varying volatility. This extended VAR model can therefore be used to not only gauge the effect of shocks such as the monetary policy shock but also the impact of changes in the volatility of the shock in question.

The methods used in the paper are a data-driven approach to estimating the impact of volatility shocks. They therefore compliment the more structural analyses in Bloom (2009) and Fernández-Villaverde *et al* (2009). The advantage of the extended SVAR is that it retains the flexibility of the standard homoscedastic SVAR – ie it is applicable to a variety of identified shocks and has the potential to fit the data better than a DSGE model which is subject to more restrictions. In other words (in an analogous manner to standard VARs), the extended SVAR can be used to provide estimates of the impact of changes in structural shock volatility without making strong assumptions about the driving force behind such effects. This flexibility and potential for better data fit, of course, comes with the cost that the model has little to say about the various channels of shock transmission.

The paper demonstrates the use of the extended SVAR model via a simple application to US data. We consider the impact of changes in the volatility of monetary policy shocks on GDP growth, inflation and the federal funds rate. This question has been largely ignored in the SVAR literature. Using the framework proposed in this paper, we are able to directly calculate the impulse response of the level of the endogenous variables to changes in the volatility of the monetary policy shock. Our results suggest that doubling the volatility of the policy shock leads to a fall in annual GDP growth and CPI inflation of 0.2% at the two-year horizon. However, the contribution of this volatility shock to the forecast error variance is small in magnitude.

2 Extended SVAR model with stochastic volatility

The proposed VAR model with stochastic volatility is given by the following equation

$$Z_{it} = c + \sum_{j=1}^{P} \beta_j Z_{it-j} + \sum_{j=1}^{P} \gamma_j \tilde{h}_{it-j} + \Omega_t^{1/2} e_{it}, e_{it} \sim N(0,1) \quad \textbf{(1)}$$

where

$$\Omega_t = A_t H_t A_t^{'} \quad \textbf{(2)}$$

In equation **(1)** Z_t denotes the $i = 1..N$ macroeconomic variables (GDP growth, inflation and the federal funds rate in our application below), while $\tilde{h}_t = [h_{1t}, h_{2t}...h_{Nt}]$ refers to the log volatility of the structural shocks in the VAR. This latter feature can be seen more clearly by considering an example where $N = 3$. The structure of H_t in equation **(2)** is then given by

$$H_t = \begin{pmatrix} \exp(h_{1t}) & 0 & 0 \\ 0 & \exp(h_{2t}) & 0 \\ 0 & 0 & \exp(h_{3t}) \end{pmatrix} \quad \textbf{(3)}$$

The structure of the A_t matrix is chosen by the econometrician to model the contemporaneous relationship among the reduced form shocks. A particularly simple specification is given by the following example that considers a lower triangular structure:

$$A_t = \begin{pmatrix} 1 & 0 & 0 \\ a_{1t} & 1 & 0 \\ a_{2t} & a_{3t} & 1 \end{pmatrix} \quad \textbf{(4)}$$

One may consider a more general structure for A_t. We consider two examples in the empirical application shown below. The transition equation for the stochastic volatility is given by:

$$h_{it} = \sum_{j=1}^{P} \kappa_j Z_{t-j} + \sum_{j=1}^{P} \theta_j \tilde{h}_{t-j} + \eta_{it}, \eta_{it} \sim N(0, Q_i), E\left(e_{it}, \eta_{it}\right) = 0 \quad \textbf{(5)}$$

and the elements of A_t follow a first-order AR process:

$$a_{jt} = \rho_j a_{jt-1} + q_j^{1/2} v_{jt}, j = 1..n \quad \textbf{(6)}$$

There are three noteworthy features about the complete system defined by equations **(1)**, **(2)** and **(5)**. First, equation **(1)** allows the volatility of the *structural* shocks h_{it} to have a (lagged) impact on the endogenous variables Z_{it}.[2] Second, note that the structure of the matrix A_t in equation **(2)** determines the

[2]In our specification the log volatility enters the VAR equations rather than its level. This is primarily because the former specification proved to be substantially more computationally stable than the latter in our experiments. In particular, the level specification is sensitive to the scaling of the variables with the possibility of overflow whenever the scale of the variables is somewhat large.

interpretation of structural shocks and hence their volatility h_{it}. In the three variable example above with Z_{it} containing GDP growth, inflation and the federal funds rate (in that order), the lower triangular structure for A_t (see equation **(4)**) implies that one can interpret h_{3t} as the log volatility of the monetary policy shock, where this shock is restricted to have no affect on GDP growth and inflation in the current period. The ability to place an economic intepretation on some or all of the shocks is important as it allows the model to tackle the analysis of the impact of volatility in a theoretically consistent manner. Choosing a different structure for A_t would therefore imply a different intepretation of the volatilities h_{it}. For example, the non-recursive identification scheme used in Sims and Zha (1998) can be employed to identify a money demand as well as a money supply shock. Alternatively one may consider inequality or sign restrictions on the off-diagonal elements of A_t as a device to identify the shocks. Third, the transition equation **(5)** allows for a dynamic interaction among the volatilities h_{it} and the endogeneous variables Z_{it} and therefore captures any feedback effects that may be present in the data. Equation **(5)** sets out a general specification of the volatility equation. This general specification may be preferred in some circumstances. For example, it can be argued that changes in macroeconomic variables may influence the fluctuation in the deviations from systematic monetary policy as seen, for example, during the 1970s in the United States – a period characterised by large changes in the level and volatility of macroeconomic variables *and* more volatile monetary policy shocks. However, if interest centres on structural shocks that are believed to be unrelated to macroeconomic developments, then a simpler specification for the volatility equation may be more appropriate (for example a specification that sets $\kappa_j = 0$ in equation **(5)**). In our empirical application we test alternative specifications and select the best-fitting model.

Note that equation **(5)** makes the simplifying assumption that the shocks to the volatility equation η_{it} and the observation equation e_{it} are uncorrelated and Q_i is a diagonal matrix. With these assumption in place, one can interpret an innovation in η_{it} as a shock to volatility of the structural shock of interest and the calculate the response of h_{it} and Z_{it}. On the other hand, if these assumptions are relaxed, further identifying restrictions are required to distinguish among the volatility shocks and to seperate the innovation to the volatility from the innovation to the level. Note that in this more general scenario (ie with a full covariance matrix among the volatility and level innovations), identification of the volatility shocks is substantially more involved. In particular, there is no simple way to assign h_{it} to a particular structural shock (as done in the proposed model above) and the researcher has to take a stand on the restrictions to place on the contemporaneous relationships among the volatilities. In contrast, the assumptions in equation **(5)** allows the use of standard identification schemes (that apply to the contemporaneous relationships among of the *level* of the reduced form shocks rather than their volatility).

To retain this ease of intepretation of h_{it} we incorporate the assumption of a diagonal Q_i and no correlation among e_{it} and η_{it} in the proposed empirical model.[3]

The model proposed above is related to a number of recent contributions. For example, the structure of the stochastic volatility model used above closely resembles the formulations used in time-varying VAR models (see Primiceri (2005)). Our model differs from these studies in that it allows a direct impact of the volatilities on the level of the endogenous variables. The model proposed above can be thought of as a multivariate extension of the stochastic volatility in mean model proposed in Koopman and Uspensky (2000). In addition, our model has similarities with the stochastic volatility models with leverage studied in Asai and McAleer (2009). However, unlike these contributions, the model proposed above is formulated with the aim of characterising the dynamic effects of volatility of structural shocks.

3 Estimation

The state-space model consisting of the observation equation **(1)** and transition equation **(5)** is estimated using a Markov Chain Monte Carlo (MCMC) algorithm. The non-linear interaction of the volatility and levels in equation **(1)** makes the evaluation of the likelihood difficult. Following Fernández-Villaverde *et al* (2009) we use a particle filter to calculate the log-likelihood of the model which is then used in a random walk Metropolis-Hastings algorithm to characterise the likelihood function.

3.1 Particle filter

An excellent detailed description of particle filtering and its application to macroeconomic models can be found in Fernández-Villaverde and Rubio-Ramirez (2007) and the references cited therein. Below we provide an intuitive description of the filter as applied to our VAR model.

Consider the distribution of the state variables in the VAR model $\Phi_t = \{h_{it}, A_t\}$ conditional on information up to time t (denoted by z_t)[4]

$$f\left(\Phi_t \backslash z_t\right) = \frac{f\left(Z_t, \Phi_t \backslash z_{t-1}\right)}{f\left(Z_t \backslash z_{t-1}\right)} = \frac{f\left(Z_t \backslash \Phi_t, z_{t-1}\right) \times f\left(\Phi_t \backslash z_{t-1}\right)}{f\left(Z_t \backslash z_{t-1}\right)} \quad \textbf{(7)}$$

[3] In some applications it may be important to consider the possibility that volatility shocks have a contemporaneous impact on macroeconomic variables. An obvious example is any implication that deals with fast-moving financial data. Such a contemporaneous affect can be easily incorporated by including the current value of h_{it} in equation **(1)**.

[4] Note that for simplicity we suppress the dependence of these conditional distributions on the parameters of the state-space model.

Equation **(7)** says that this density can be written as the ratio of the joint density of the data and the states $f(Z_t, \Phi_t \backslash z_{t-1}) = f(Z_t \backslash \Phi_t, z_{t-1}) \times f(\Phi_t \backslash z_{t-1})$ and the likelihood function $f(Z_t \backslash z_{t-1})$ where the latter is defined as

$$f(Z_t \backslash z_{t-1}) = \int f(Z_t \backslash \Phi_t, z_{t-1}) \times f(\Phi_t \backslash z_{t-1})\, d\Phi_t \tag{8}$$

Note also that the conditional density $f(\Phi_t \backslash z_{t-1})$ can be written as

$$f(\Phi_t \backslash z_{t-1}) = \int f(\Phi_t \backslash \Phi_{t-1}) \times f(\Phi_{t-1} \backslash z_{t-1})\, d\Phi_{t-1} \tag{9}$$

These equations suggest the following filtering algorithm to compute the likelihood function:

Prediction Given a starting value $f(\Phi_0 \backslash z_0)$ calculate the predicted value of the state

$$f(\Phi_1 \backslash z_0) = \int f(\Phi_1 \backslash \Phi_0) \times f(\Phi_0 \backslash z_0)\, d\Phi_0$$

Update Update the value of the state variables based on information contained in the data

$$f(\Phi_1 \backslash z_1) = \frac{f(Z_1 \backslash \Phi_1, z_0) \times f(\Phi_1 \backslash z_0)}{f(Z_1 \backslash z_0)}$$

where $f(Z_1 \backslash z_0) = \int f(Z_1 \backslash \Phi_1, z_0) \times f(\Phi_1 \backslash z_0)\, d\Phi_1$ is the likelihood for observation 1. By repeating these two steps for observations $t = 1...T$ the likelihood function of the model can be calculated as $\ln lik = \ln f(Z_1 \backslash z_0) + \ln f(Z_2 \backslash z_1) + ... \ln f(Z_T \backslash z_{t-1})$

In general, this algorithm is inoperable because the integrals in the equations above are difficult to evaluate. The particle filter makes the algorithm feasable by using a Monte-Carlo method to evaluate these integrals.

Prediction In particular, the partical filter approximates the conditional distribution $f(\Phi_1 \backslash z_0)$ via M draws or particles using the transition equation of the model. In other words, given Z_{i0}, h_{i0}, a_{j0} and the knowledge of the variance of η_{it}, the prediction step of the filter involves simulating $m = 1...M$ values of h_{i1}^m using the transition equation **(5)** and a_{j1}^m using equation **(6)**. For each draw for h_{i1}, the conditional likelihood $W^m = f(Z_1 \backslash z_0)$ is evaluated. Conditional on the draw for the state variables, the predicted value for the variables $\hat{Z}_{i1}^M$ can be computed using the observation equation and the prediction error decomposition is used to evaluate the likelihood W^m.

Update The update step involves a draw from the density $f(\Phi_1 \backslash z_1)$. This is done by sampling with replacement from the sequence of particles h^m_{i1}, a^m_{j1} with the re-sampling probability given by $\frac{W^m}{\sum_{m=1}^M W^m}$. This re-sampling step updates the draws for Φ based on information contained in the data for that time period. By the law of large numbers the likelihood function for the observation can be approximated as $\ln lik_t = \ln \frac{\sum_{m=1}^M W^m}{M}$.

Repeating these two steps for $t = 1...T$ one can calculate the approximate likelihood function as $\ln lik = \sum_{t=1}^T \ln lik_t$ for a given value of the VAR parameters. Note that this estimate of the likelihood can be combined with a prior on the parameters to derive the posterior density.

3.2 Metropolis-Hastings algorithm

A random walk Metropolis-Hastings (MH) algorithm is used to approximate the likelihood function or the posterior density of the VAR model. Given a set of starting values for all the parameters of the VAR model collected in a $k \times 1$ vector Ψ^{old}, the algorithm proceeds in the following steps

1. Draw a candidate value for the parameters Ψ^{new} using the following random walk

$$\Psi^{new} = \Psi^{old} + P'\varepsilon$$

   where ε is a $k \times 1$ vector from the standard normal distribution and P is a scaling matrix.
2. Compute the likelihood (or the posterior if using priors) of the model via the particle filter described above at the new and old set of parameters, $\ln lik^{new}$ and $\ln lik^{old}$
3. Calculate the acceptance probability $accept = \min\left[\exp\left(\ln lik^{new} - \ln lik^{old}\right), 1\right]$. If the acceptance probability is greater than u, a draw from the standard uniform distribution, set $\Psi^{old} = \Psi^{new}$. Otherwise retain the old draw.

These steps are repeated R times with the last r draws used to compute statistics of interest such as impulse responses and variance decomposition. The scaling matrix P is chosen to ensure that the proportion of accepted draws is between 20%-40%.[5]

[5]The MCMC approach is particularly useful as it provides a convenient way to approximate the uncertainty surrounding the statistics of interests. This approach also allows the user to circumvent the issues arising from the fact that the likelihood function calculated via the particle filter is not

3.3 Impulse response functions to the volatility of structural shocks

With the contemporaneous correlation between volatility equation error η_{it} and the observation equation error e_{it} restricted to be zero and with constant lag coefficients, the calculation of the response to the volatility shocks is carried out via standard methods – ie via a simulation of the system given by equations **(1)** and **(5)**.

Note that the calculation of the response to the level of structural shocks e_{it} is more involved due to the time-variation in the matrix A_t. The impulse response functions can be estimated in two ways. First, one can assume that elements of A_t are fixed across the impulse response horizon and use standard methods to estimate the impulse response at each point in time (as done in Primiceri (2005)). Alternatively, future uncertainty in A_t can be accounted for via the Monte Carlo integration procedure described in Koop, Pesaran and Potter (1996).

4 Empirical results: monetary policy shock uncertainty and the macroeconomy

A large and growing (SVAR based) literature has examined the impact of the monetary policy shocks on the real economy. However, little attention has been placed on the possible impact of changes in the volatility of policy shocks. There are a number of reasons why the volatility of the policy shock may be important. First, existing evidence clearly suggests that the volatility of this shock for the United States has been characterised by large fluctuations (see, for example, Primiceri (2005)). These fluctuations have typically coincided with large changes in the level of inflation and GDP growth (for example during the late 1970s, as reported in Primiceri (2005)). The model proposed in the paper provides a systematic way to examine the impact of the volatility of the policy shock on macroeconomic variables. Note that one way to interpret this exercise is as an attempt to quantify the impact of uncertainty about monetary policy on the economy. The monetary policy shock represents the deviation of the central bank rate from that prescribed by the policy rule included in the model. The VAR model proposed below tries to establish if an increase in uncertainty about this deviation has macroeconomic effects. To our knowledge no direct attempt has been made to quantify this effect. The exercise below tries to fill this gap in the literature for the US economy.

continuous with respect to the model parameters – a feature that renders gradient based optimisation algorithms unsuitable for this application. However, an optimisation algorithm that is useful in these circumstances is simulated annealing (see Goffe, Ferrier and Rogers (1994) for details).

We consider the following SVAR model

$$\begin{aligned}
Z_t &= c + \beta_1 Z_{t-1} + \gamma_1 \tilde{h}_{t-1} + \Omega_t^{1/2} e_{it} \\
\Omega_t &= A_t H_t A_t^{'} \\
\tilde{h}_{it} &= \kappa_1 Z_{t-1} + \theta_1 \tilde{h}_{t-1} + Q_i^{1/2} \eta_{it} \\
a_{jt} &= \rho_j a_{jt-1} + q_j^{1/2} v_{jt}
\end{aligned} \qquad \textbf{(10)}$$

where Z_t contains annualised percentage GDP growth, annualised quarterly inflation and the central bank interest rate, $\tilde{h}_t = [h_{1t}, h_{2t}, h_{3t}]$ represents the log of $i = 1..3$ diagonal elements of H_t and a_{jt} represent $j = 1..3$ non-zero elements of A_t. Note that we limit the number of lags to one to reduce the computational burden inherent in the estimation approach.

$$H_t = \begin{pmatrix} \exp(h_{1t}) & 0 & 0 \\ 0 & \exp(h_{2t}) & 0 \\ 0 & 0 & \exp(h_{3t}) \end{pmatrix}$$

We consider two possible specifications for A_t. First, assume that A_t is lower triangular (and order the variables in Z_{it} as GDP growth, inflation and the central bank rate) implying that the third shock is interpreted as a monetary policy shock based on this recursive structure:

$$A_t = \begin{pmatrix} 1 & 0 & 0 \\ a_{1t} & 1 & 0 \\ a_{2t} & a_{3t} & 1 \end{pmatrix},$$

Using this specification, our empirical exercise focuses on the response of Z_t to shocks to h_{3t}, the volatility of the policy shock. The second specification for A_t imposes sign restrictions on the contemporaneous impact of the policy rate on GDP growth and inflation. With the variables ordered as the central bank rate, GDP growth and inflation, A_t is assumed to have the following structure

$$A_t = \begin{pmatrix} 1 & 0 & 0 \\ a_{1t}^{-} & 1 & 0 \\ a_{2t}^{-} & a_{3t}^{\times} & 1 \end{pmatrix}$$

where the superscript '$-$' denotes the fact that a_{1t} and a_{2t} are restricted to be less than or equal to zero, while '$\times$'denotes unconstrained estimation. This specification identifies the monetary policy shock as one that leads to a contemporanoeus fall in GDP growth and inflation. Note that the lower triangular structure is retained for A_t in this specification for simplicity. In general one can allow for a full A_t matrix if identification of additional shocks is needed.

The specification for volatility shown in equation **(10)** allows the volatility of each structural shock to depend on the lagged macroeconomic variables and the lagged volatilities. This implies, for example, that the volatility of shocks hitting the inflation and GDP growth equation is affected by the volatility of policy shock. We favour this general specification as it captures some of the regularities seen in the evolution of these volatilities highlighted in previous work. For example, it is generally reported that the volatility of these shocks was high at similar points in time – ie during the mid and late 1970s.

One caveat of these specifications should be noted, however. The model implicitly assumes an interest rate rule under which the Federal Funds rate responds to inflation and GDP growth only and thus ignores the impact of any other variable that may be important from the point of view of policy makers. This may mean that the model overestimates the change in the volatility of the monetary policy shock over periods where policy makers respond to variables other than output and inflation. The recent period which is associated with unconventional monetary policy measures undertaken by the Fed in response to the financial crisis provides an example of a policy stance not necessarily captured by the simple rule implicit in the VAR model.

The MH algorithm is implemented in the following steps:

- To generate starting values for the algorithm we estimate univariate stochastic volatility models for elements of Z_{it} and compute an initial guess for h_{it}. The starting values for the VAR coefficients and hyperparameters are estimated via OLS estimation of the model (on a training sample of 40 observations) using this initial estimate for h_{it}. We use this estimate of h_{it} and the OLS estimates of the VAR to generate a value for the mean of the distribution of the initial state $f\left(\Phi_0 \backslash z_0\right)$. The variance of the initial state is set to 0.01 reflecting the belief that the univariate estimates of stochastic volatility provide a reasonable value for the initial state in the multivariate VAR model.
- The likelihood function of the VAR is computed using M=5,000 particles. This choice is based on the observation that the value of the likelihood function does not appear to change significantly as M is increased beyond 1,000.[6] The appendix shows the estimated log-likelihood (using data simulated from

[6]Implementation of the MH algorithm requires repeated calculation of the likelihood via the particle filter. The particle filter is computationally

the benchmark model) for values of M ranging from 100 to 50,000. The largest change in the likelihood appears as M increases from 100 to 500 with subsequent changes relatively small. We use 25,000 iterations for the MH algorithm and base inference on the last 6,000 draws. We choose the scaling factor for the shock to the proposal density to ensure that the acceptance rate is between 20% to 40%. Initially, the scaling factor is set to be a diagonal matrix with diagonal elements to be 0.01. This is adjusted periodically to maintain the acceptance rate between 20% and 40%. The appendix presents the cumulated means of the retained draws. These show little fluctuation providing some evidence for convergence.

- We compare the fit of the benchmark model in equation **(10)** with a restricted version that imposes the condition that $\gamma_1 = 0$, thus eliminating the direct impact of h_{it-1}. The model comparison is based on the estimated marginal likelihood. The marginal (or the integrated) likelihood is defined as

$$p(Z_t) = \int_{\Xi} f(Z_t \backslash \Xi)\, g(\Xi) \tag{11}$$

where Ξ denotes all the VAR parameters, $f(Z_t \backslash \Xi)$ is the likelihood function while $g(\Xi)$ represents the prior distributions. The marginal likelihood can be approximated using the modified harmonic mean (MHM) method which employs the following theorem

$$\frac{1}{p(Z_t)} = \int_{\Theta} \frac{h(\Xi)}{f(Z_t \backslash \Xi)\, g(\Xi)} p(\Xi \backslash Z_t)\, d\Xi \tag{12}$$

where $h(\Xi)$ denotes a weighting function, ie a probability density function whose support is in Θ. Numerically the integral in equation **(12)** can be evaluated as

$$\frac{1}{p(Z_t)} = \sum_{i=1}^{N} \frac{h(\Xi^i)}{f(Z_t \backslash \Xi^i)\, g(\Xi^i)}$$

where $i = 1..N$ indexes the draws from the MCMC sampler. Geweke (1999) suggests a normal density as the weighting function, with the mean and the variance of the density constructed using output from the posterior simulator. The normal density is truncated to ensure that its support lies in the support of the posterior.

intensive as it requires the evaluation of the conditional likelihood of the model for M particles for each time period in the sample. In this VAR framework, the computation across the particles involves matrix operations (to calculate the multivariate normal density) and is therefore difficult to vectorise and has to be implemented in a loop. This, along with the update step (which requires M draws from a discrete distribution), increases the computational burden significantly.

We code the particle filter in Fortran 95 programming language. For a reasonably large number of particles, the code delivers an estimate of the likelihood function in around 10 seconds. The code is available from the author on request.

Table A: Log marginal likelihood for each estimated US model

Model	Log marginal likelihood
Benchmark (recursive)	-718.040
Restricted (recursive)	-1047.207
Benchmark (sign)	-627.404
Restricted (sign)	-755.803

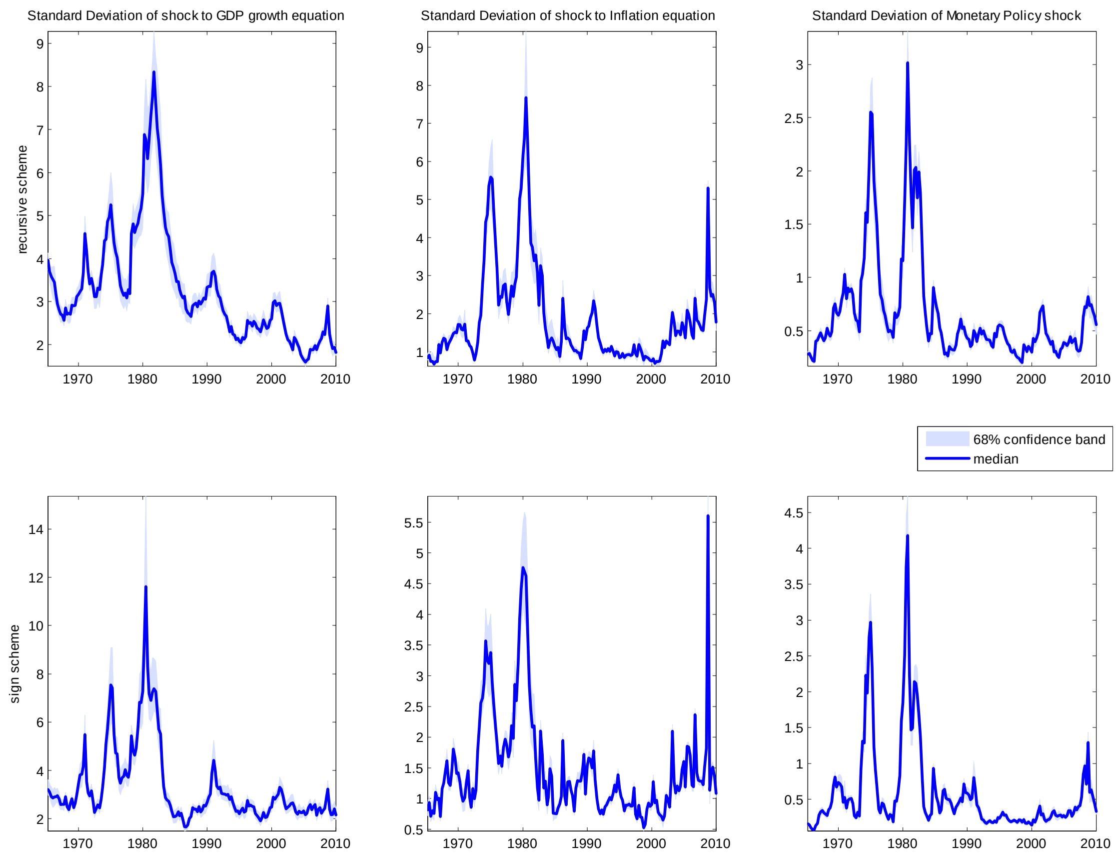
4.1 Empirical results

The model is estimated on quarterly US data spanning the period 1955 Q1 to 2010 Q1. The first 40 observations are employed as the training sample to generate starting values for the MH algorithm. Note that the data are obtained from the FRED database (http://research.stlouisfed.org/fred2/). GDP growth is defined as the annualised log difference of real GDP (series name GDPC96). Inflation is defined as the annualised log difference of CPI (series name CPIAUCSL) while the central bank rate is the federal funds rate. In this application we do not use prior distributions for the VAR parameters and focus entirely on information about the impact of volatility shocks contained in the data.

Table A presents the estimated marginal likelihood for the benchmark models and the restricted alternatives which sets $\gamma_1 = 0$ for the two identification schemes. The restricted model is overwhelmingly rejected for both specifications, suggesting that volatility of shocks have a direct statistically significant impact on the variables included in our model. Note also that the model with sign restrictions is preferred to the VAR with the recursive identification scheme.

The estimated shock volatilities are presented in Chart 1. The top panel presents results from the VAR with the recursive scheme. The bottom panel shows the estimates from the sign restriction model. The first two columns present the standard deviation of the shocks to the GDP and inflation equation in the VAR – ie shocks that we do not place a direct economic interpretation on. The volatility of the GDP equation shock is highest in the pre-1985 period reaching its peak during the early 1980s. The post-1985 period contains smaller peaks at the time of the first Gulf war during the early 1990s, the recession of 2000 and then towards the end of the sample coinciding with the recent financial crisis. The profile for the volatility of the shock to the inflation equation is similar with the highest variance concentrated in the pre-1985 sample. One noticeable feature, however, is the large increase in the volatility of this shock during the recent crisis, almost to the level seen during the 1970s and 1980s. Note that this increase is more pronounced in the sign restriction model. The final column in Chart 1 presents the estimated

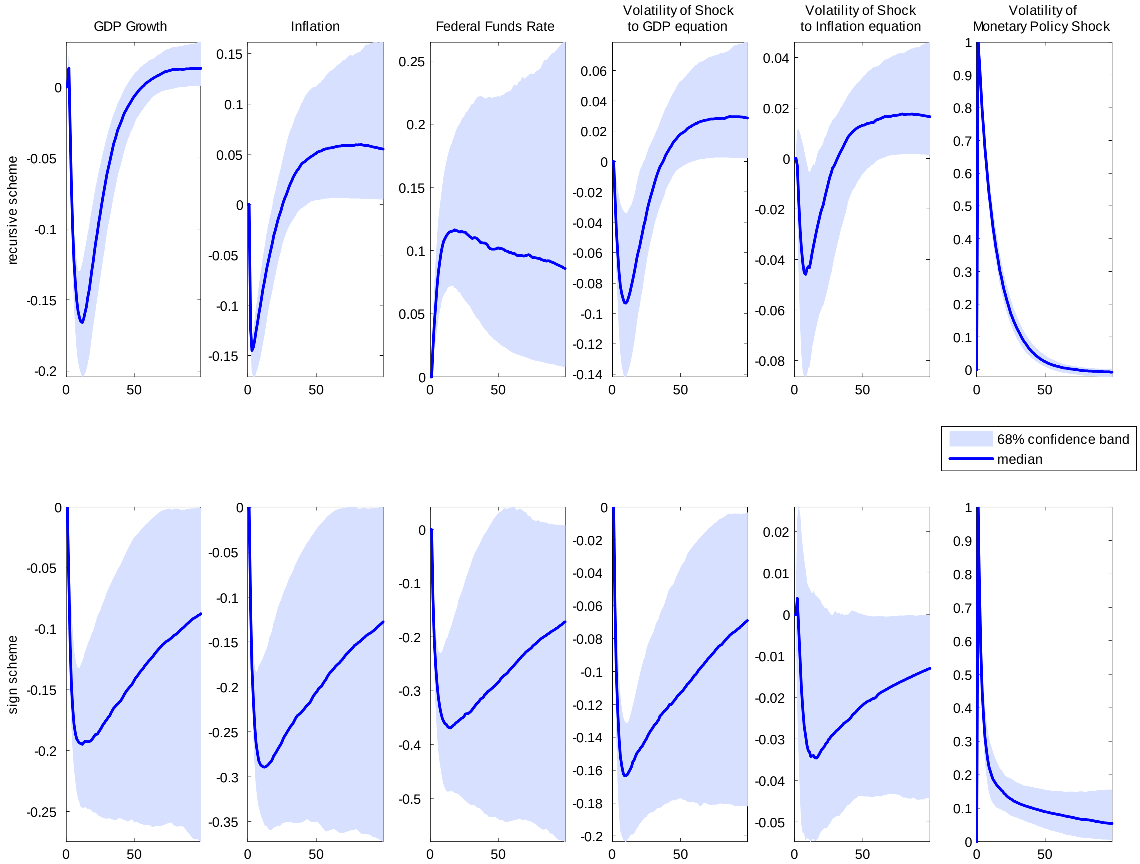
Chart 1: Standard deviation of VAR shocks



volatility of the monetary policy shock. The evolution of the volatility is very similar to the estimate in Benati and Mumtaz (2007), with large increases during the great inflation of the mid-1970s and then during Paul Volcker's experiment of targeting non-borrowed reserves at the end of the 1970s. Note that the great moderation period – starting from the mid-1980s – was associated, on the whole, with less volatile policy shocks. Two exceptions to this stability are the recessions of 2000 and 2009.

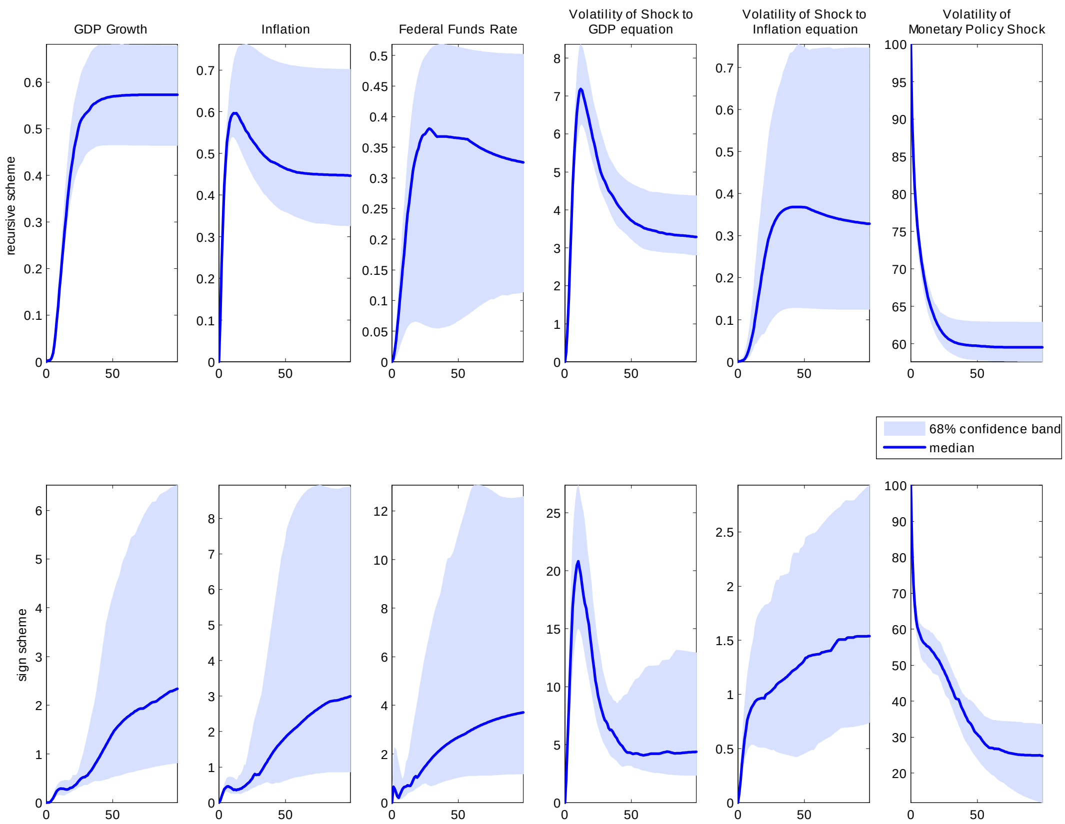
Chart 2 presents the impulse response to a shock to the volatility of monetary policy shock. As before, the first row shows the results from the recursive scheme while the second row displays the estimates when sign restrictions are employed. The shock is calibrated to increase this volatility by 100% (ie a unit increase in the log of the variance). The magnitude of the shock matches the approximate estimated

Chart 2: Impulse response to the volatility of the monetary policy shock



increase in the volatility of this shock between 2007 and 2009. The response of the volatility of the policy shock is estimated to be quite persistent, leading to persistent effects on the macroeconomic variables. In response to the volatility shock, annual GDP growth falls by 0.15%-0.2% at the ten-quarter horizon, with the sign restriction scheme associated with the larger point estimate. The response of inflation is similar, with the volatility shock reducing annual inflation around 0.15% to 0.3% at the one-year horizon. This shock has long-lasting effects on the federal funds rate. The long-lasting effect on the interest rate is driven largely by the fact that both the interest rate and the volatility of its shock are estimated to be quite persistent over the sample period. However, note that the sign of the impact differs across the two identification schemes. Using the recursive scheme the central bank interest is higher (by 0.1%) for an extended period. In contrast, under sign identification, the estimated impact of the volatility shock on the central bank rate is negative, leading to a reduction of around 0.3% at the two-year horizon. This difference again highlights the importance of the identification scheme in determining the estimated volatility of the structural shock and its impact on the endogenous variables. For this data set and application, the marginal likelihood suggests that the sign identification scheme is preferred and therefore provides evidence to support a negative impact of this shock on the central bank rate.

Chart 3: Forecast error variance decomposition



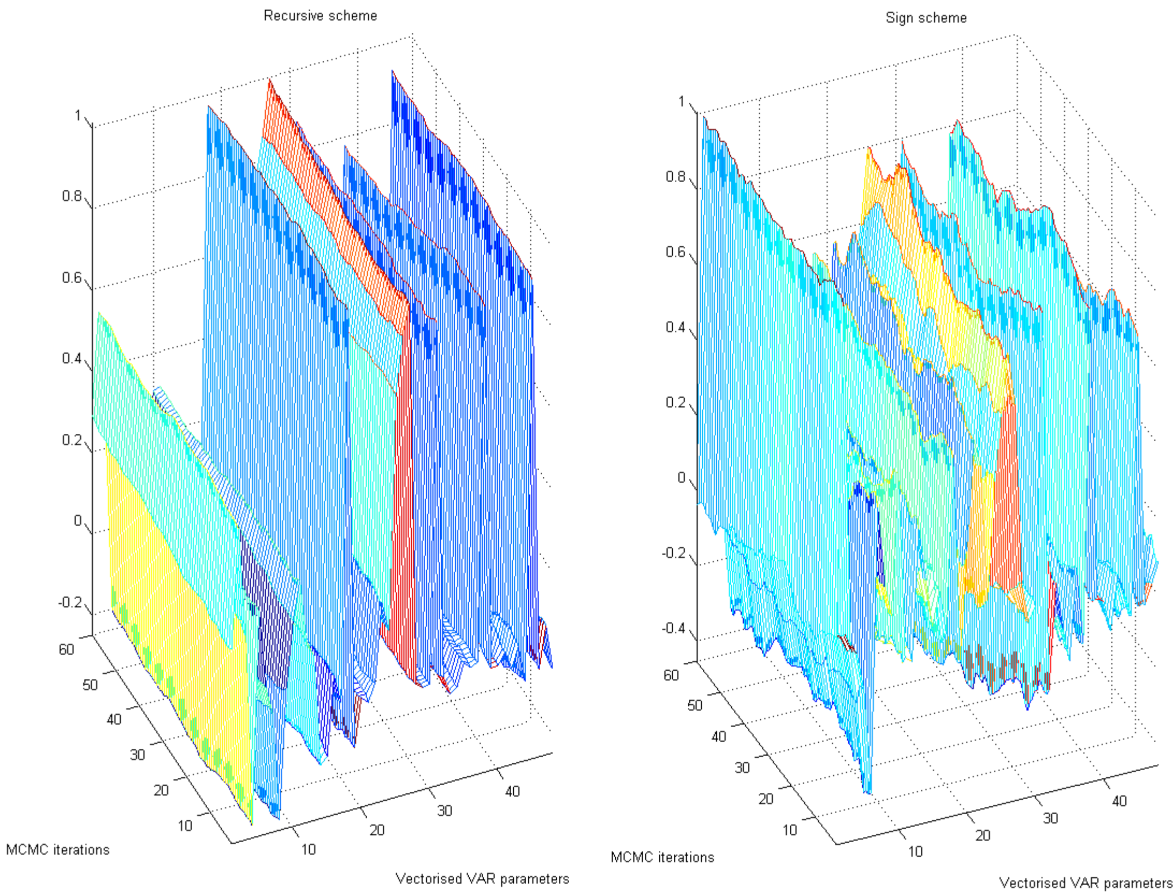
Results from the impulse response analysis indicate that monetary policy volatility shocks can have long-lasting and significant effects on macroeconomic variables. However, it is not clear if this volatility shock is important in relative terms. In order to explore this further, Chart 3 presents the contribution of this volatility shock to the forecast error variance of each endogenous variable using the two identification schemes. We make two simplifying assumptions when estimating the forecast error variance decomposition. First, the chart presents the average estimate across time – the forecast error variance decomposition is estimated at each point in time, taking into account the time-variation in A_t and then averaged. Second, when estimating the decomposition we assume that the elements of A_t are fixed across the forecast horizon – ie for simplicity we do not take possible future variation in A_t into account. This simplifying assumption (which reduces the computational burden significantly by eliminating the need for a Monte Carlo simulation for each MCMC iteration at each point in time) has been made in several related studies that allow for time-variation in VAR coefficients. See, for example, Primiceri (2005) and Sims and Zha (2006). The estimated forecast error variance decomposition clearly indicates that the monetary policy volatility shock explains a very small proportion of the forecast error variance of the macroeconomic variables. Using the recursive identification scheme, the proportion of the forecast error variance of GDP growth, inflation and the central bank rate explained by the volatility shock is less than 1%. Under the sign identification scheme the point estimate of this contribution is slightly higher but the wide error bands indicate that this difference is not significant.

5 Conclusion

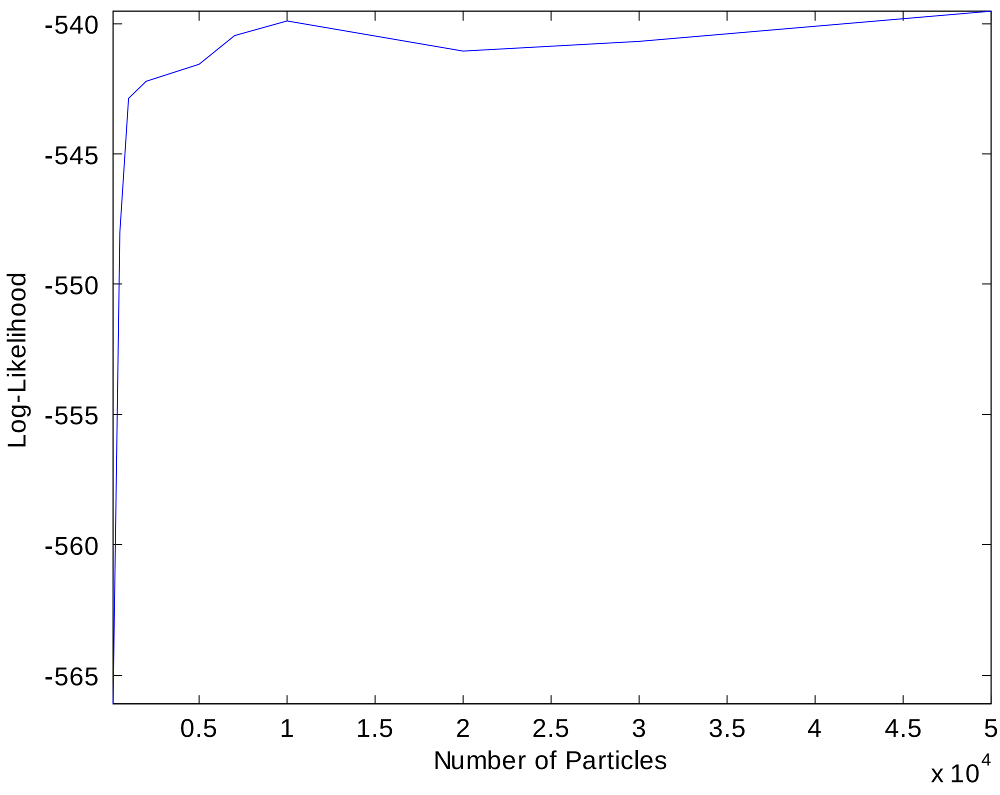
This paper contributes to the growing literature on the impact of volatility by proposing an extended SVAR that allows the variance of structural shock to have a direct effect on the endogenous variables included in the model. The proposed approach is data driven and semi-structural and thus complements the structural analyses in recent work on this issue. The proposed model can be estimated via likelihood-based methods, with the likelihood function evaluated via particle filtering. We demonstrate the application of the model by considering the possible macroeconomic impact of a change in the volatility of monetary policy shocks in the United States. A unit increase in the log of the volatility of this shock has a statistically significant impact. In relative terms, however, the impact of this shock is small with a negligible contribution to the forecast error variance of GDP growth and inflation.

Appendix: Recursive means of MCMC sequence of VAR parameters

The figure below presents the mean of the retained MCMC draws for each estimated model. The means are calculated every 100 draws. The recursive means show little fluctuation across the 6,000 retained draws providing evidence for the convergence of the MCMC algorithm.



The figure below presents the value of the log likelihood for the number of particles M={100, 500, 1,000, 2,000, 5,000, 7,000, 10,000, 20,000, 30,000, 50,000}. The underlying simulated data is based on a sample of 180 observations (the length of the actual sample used in the empirical study) for three endogenous variables.


-540
-545
-550
-555
-560
-565
Log-Likelihood
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Number of Particles
x 10^4

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