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# Working Paper No. 436
## Systemic capital requirements

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## Abstract

The credit risk that an individual bank poses to the rest of the financial system depends on its size, the type of exposures it has to the real economy, and its obligations to other institutions. This paper describes a system-wide risk management approach to calibrating individual banks' capital requirements that takes into account these factors and which correspond to a policymaker's chosen target for systemic credit risk. The optimisation strategy identifies the minimum level of aggregate capital for the system and its distribution across banks that are consistent with a chosen objective for systemic credit risk. This parameterises a trade-off between efficiency and stability.



**Key words:** Financial stability, systemic risk, capital requirements, structural credit risk model, financial networks, non-linear constrained optimisation.

**JEL classification:** C61, C63, G01, G21, G28.


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**Summary**

Banking regulation has historically focused on making a detailed assessment of risk at the level of individual banks' balance sheets. But it is possible that, in an interconnected system, banks that appear sufficiently healthy when viewed individually may collectively present a material threat to the solvency of the system as a whole. First, there may be similarities between banks' asset exposures that generate a tendency for banks' solvency positions to deteriorate and improve together. This can leave the system vulnerable to common shocks to the macroeconomy or to capital markets. Second, losses at an individual bank that are sufficient to cause it to default may trigger contagious failures of other banks in the system if they have extended it loans. Such contagious failures could trigger further rounds of contagious defaults in the banking system. System-wide losses could then far exceed the size of the initial shock.

Vulnerabilities of the system as a whole that cannot be identified by focusing narrowly on the health of individual banks suggest that a change in the way that risks to the banking system are assessed and prudential requirements for banks are calibrated could be beneficial. For example, capital requirements for banks could be set with the goal of achieving a level of systemic credit risk that a policymaker is willing to tolerate. This paper describes a system-wide risk management approach to deriving capital requirements for banks that reflect the impact their failure would have on the wider banking system and the likelihood of contagious losses occurring. These are referred to in this paper as 'systemic capital requirements'.

At the centre of the approach is the policymaker's optimisation problem. The policymaker is assumed to be interested in ensuring that the probability of banking system insolvency over a given time horizon is less than a chosen target level. This reflects the policymaker's systemic risk tolerance. The target could, of course, be achieved in all states of the world by setting very high systemic capital requirements. But the policymaker may also want to limit the potential inefficiency costs associated with regulatory capital requirements. If equity capital is more expensive than debt because of market frictions, higher capital requirements could, for example, increase the cost of bank lending to non-bank borrowers. The possible trade-off between financial stability and financial efficiency motivates a constrained optimisation problem, where a policymaker seeks to identify systemic capital requirements for individual banks that minimise the total level of capital in the banking system, subject to meeting their chosen systemic risk target. In other words, a policymaker sets banks' capital requirements to maximise efficiency subject to achieving a preferred level of stability. The solution of the constrained optimisation problem is a unique level of capital in the banking system and its distribution across banks.

Nested inside the policymaker's constrained optimisation problem is a simple structural model of a banking system in which shocks to banks' non-bank assets can cause insolvency. The

underlying model further allows such shocks that originate outside the banking system to be transmitted and amplified through a network of interbank loans, so that credit losses spill over onto other banks when one or more banks become insolvent. The model captures two important drivers of systemic risk: (i) correlations between banks' assets (as a result of common exposures to non-banks), which may lead to multiple banks becoming insolvent simultaneously; and (ii) the potential for contagious bank defaults to occur because of losses on interbank lending.

The model is calibrated to resemble the major UK banks. It is used to illustrate how assessing risks only at the level of individual banks' balance sheets can lead a policymaker to underestimate the level of systemic risk in the banking system as a whole. The probability of very large losses crystallising in the banking system is greater when the potential for interbank contagion is taken into account, particularly when a number of banks have their balance sheet simultaneously weakened by losses on loans to non-banks.

The modelling choices in this paper reflect a trade-off between realism (complexity) and pragmatism (simplicity) in the description of credit risks facing an interconnected banking system. The paper uses a simplified description of the evolution of banks' balance sheets so that computational effort can be focused on solving the constrained optimisation problem faced by the systemic policymaker, taking into account the interlinkages between banks. As such, the primary focus of the paper is to obtain general insights into the properties of risk-based systemic capital requirements, rather than to calibrate precise nominal amounts that may be required to achieve particular risk targets in practice.

Systemic capital requirements for individual banks, determined as the solution to the policymaker's optimisation problem, depend on the structure of banks' balance sheets (including their obligations to other banks) and the extent to which banks' asset values tend to move together. Generally, banks' systemic capital requirements are found to be increasing in: balance sheet size relative to other banks in the system; interconnectedness; and, materially, contagious bankruptcy costs.

The paper illustrates, however, that risk-based systemic capital requirements would decrease during economic upswings and increase during downswings in tandem with measures of bank credit risk that are based on contemporaneous financial market prices, other things being equal. This procyclicality can be smoothed, to some extent, by using through-the-cycle measures of the riskiness of banks' assets. Nevertheless, the effect of such smoothing on the distribution of system credit losses is modest relative to the effect of cyclical changes to the composition of banks' balance sheets (leverage), suggesting a role for explicitly countercyclical capital requirements.

## 1 Introduction

Banks are heterogeneous in many respects. They differ in terms of their size, their exposures to the real economy and in their obligations to other participants in the financial system. As a result, individual bank failures can have substantially different impacts on the solvency of other institutions in the system. Prudential regulation has historically focused on making a detailed assessment of risk at the level of individual bank balance sheets. For example, under the advanced approach of Basel II, a bank's capital requirement depends on the riskiness of its assets. The financial crisis has highlighted the need for regulation to more explicitly take into account risks to the system as a whole – for example, tailoring individual banks' capital requirements to their contributions to systemic risk (Acharya *et al* (2010), Brunnermeier *et al* (2009) and Bank of England (2009)).

This paper describes an approach by which a policymaker can derive capital requirements for individual banks that reflect the impact their failure would have on the wider financial system, taking into account the likelihood of different outcomes ('systemic capital requirements'). At the heart of the approach is the policymaker's optimisation problem. The policymaker is assumed to be interested in limiting the probability of the banking system becoming insolvent over a given horizon – this determines their systemic risk target, a choice variable. But the policymaker is also concerned about the potential costs associated with raising capital requirements for the system in aggregate, assuming that equity (capital) is socially more expensive than debt. This trade-off motivates a constrained optimisation problem: the policymaker seeks to minimise the total level of capital in the banking system subject to being able to meet their chosen systemic risk constraint. In other words, the policymaker aims to set banks' individual capital requirements so as to maximise aggregate efficiency, conditional on achieving a target level of financial stability. The output of the optimisation problem is an appropriate level *and* distribution of capital across banks. The policymakers' systemic risk target parameterises the chosen trade-off between systemic instability and inefficiency.

In this paper, the constrained optimisation problem faced by the systemic policymaker is combined with a structural model of the banking system to determine risk-based systemic capital requirements for individual banks. The evolution of banks' balance sheets and the manner in which interbank (or 'network') exposures between firms are cleared follows Elsinger, Lehar and Summer (2006), in the spirit of Merton (1974). This captures two drivers of systemic risk: (i) the correlations between banks' assets that may lead to multiple banks becoming fundamentally insolvent simultaneously; and (ii) the potential for contagious failures to occur, as losses from fundamentally insolvent banks are transmitted and amplified in the wider system via defaults on interbank obligations.

The banks in this paper are calibrated to resemble major UK banks. First, the model is used to illustrate the properties of systemic risk in that benchmark network. Second, the paper presents comparative static exercises to illustrate how banks may increase credit risk at the system level when their balance sheets are larger, when their financial obligations to other banks are greater, and when the deadweight costs of contagious defaults are higher. These experiments are followed by the results of the policymakers' optimisation problem – namely, the level and distribution of systemic capital requirements across banks associated with a particular measure of systemic risk and a chosen target for this measure, under both the benchmark calibration and the counterfactual cases.

The model used in this paper makes a number of key simplifying assumptions, some of which are strong in the case of banks. For example, it abstracts from the possibility of wholesale funding market closure. Moreover, because contagious default losses are fundamentally abrupt and non-linear (as in the real world), the results are sensitive to the model calibration. The precise quantitative results in this paper should therefore be interpreted with caution. The paper is intended as an illustration of how a policymaker could in principle use structural credit risk models to help calibrate policies to measure and mitigate threats to the system. The paper also assumes a very specific form of an objective function that a systemic policymaker could adopt, centred solely on resilience. A broader modelling framework and objective function might also include measures of cyclical imbalances in the economy including, for example, deviations of bank credit availability from a measure of equilibrium. These considerations are beyond the scope of this paper.

The paper is structured as follows. Section 2 outlines approaches recently proposed in the literature for determining systemic capital requirements for banks. Details of the model used in this paper, its calibration and the iterative process used to solve the policymakers' constrained optimisation problem can be found in Section 3. Illustrative results for a benchmark calibration of the model are presented in Section 4. Comparative static exercises are performed in Section 5. The constrained optimisation problem is solved in Section 6 to determine systemic capital requirements for individual banks. Section 7 concludes.

## 2 Comparison with other approaches

This paper offers an alternative to the approaches that have been proposed in the literature following the financial crisis to setting systemic capital requirements for banks.

Brunnermeier *et al* (2009) and Acharya *et al* (2010) suggest similar approaches in which banks' contributions to systemic risk are measured according to Value-at-Risk (VaR) type methodologies. Brunnermeier *et al* propose using the Conditional Value-at-Risk (CoVaR) measure of banks' contributions to systemic risk developed by Adrian and Brunnermeier (2009), which quantifies the extent to which tail risks faced by banks move together. Specifically, a bank's $\alpha$% CoVaR is the $\alpha$% VaR for the banking system conditional on that bank making losses equal to its $\alpha$% VaR. The difference between a bank's CoVaR and the unconditional $\alpha$% VaR for the system is interpreted as that bank's contribution to systemic risk. But while Brunnermeier *et al* focus on banks holding capital to protect others in the financial system, Acharya *et al* suggest instead that banks should pay a premium to reflect the insurance that they might enjoy in a systemic crisis. They propose basing a premium on a bank's $\alpha$% VaR conditional on the banking system making losses equal to its $\alpha$% VaR (a bank's marginal VaR).

A drawback common to both of these approaches is that they do not explain how measures of systemic risk should actually be mapped into banks' capital requirements. One possible solution is proposed by Gauthier *et al* (2010), who use CoVaR and component VaR (a way of decomposing portfolio risk into the risks associated with each portfolio component) to allocate a fixed amount of capital among a set of banks in a structural credit risk model. For example, an individual bank's capital requirement could be the total capital requirement for the system multiplied by its CoVaR-based contribution to systemic risk divided by the sum of banks' CoVaR-based contributions. Like this paper, the approach in Gauthier *et al* seeks to ensure that banks that have a relatively larger impact on system stability have higher capital requirements. But Gauthier *et al* focus on sharing rules for a fixed pool of capital and do not simultaneously seek to determine the overall level of capital in the banking system. The identification strategy for the appropriate level and distribution of capital is the key contribution of this paper.

Tarashev *et al* (2010) is similar in its objective to the approach outlined in this paper. The authors measure a bank's contribution to systemic risk using the Shapley value concept from co-operative game theory (Shapley (1953)). For each and every subset of the banking system in which a bank is present, a bank's contribution to risk in the subset is the difference between the $\alpha$% VaR for the subset inclusive of the bank and the $\alpha$% VaR for the subset excluding the bank. In this example, VaR is the so-called 'characteristic function' though alternatives such as (conditional) expected loss can also be used. A bank's Shapley value is then a function of a bank's contributions to risk in each of the subsets. The paper thus seeks to allocate systemic risk fairly across banks: Tarashev *et al* set up a constrained optimisation problem in which a policymaker equalises banks' Shapley values subject to achieving a target level for the expected shortfall of assets to liabilities at the level of the system.

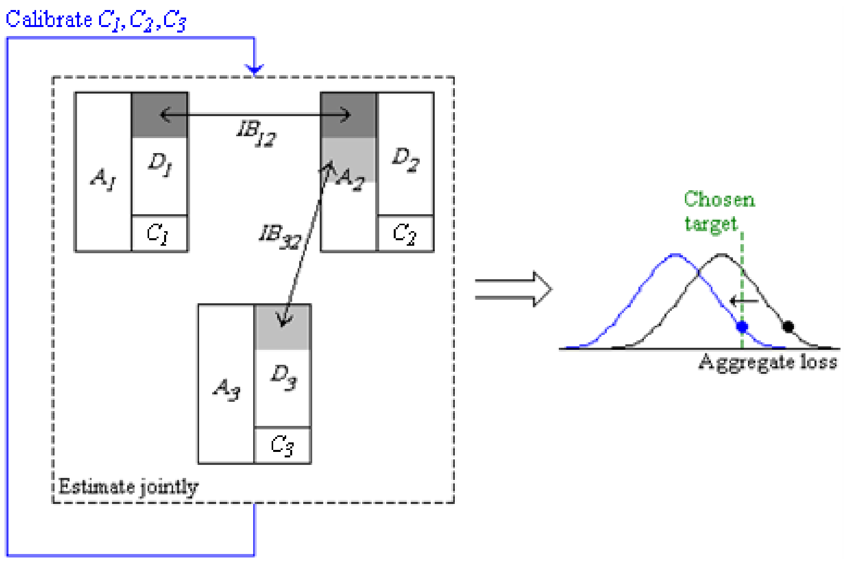
## 3 Modelling systemic solvency risk

This paper uses the Merton-style structural credit risk model described by Elsinger *et al* (2006) and deployed by Gauthier *et al* (2010) to quantify risks to the solvency of an interconnected banking system. The model captures two channels of systemic credit risk: (i) the risk that banks fail simultaneously because of correlations between the values of their assets;[(1)] and (ii) the risk that banks fail contagiously because of direct balance sheet interlinkages between banks, assuming that contagious bankruptcy carries a fixed deadweight cost (as a proportion of assets) in addition to the loss given default associated with fundamental insolvency. These costs include, for example, the losses that may be incurred because assets are sold at fire-sale prices.

The model can be thought of as a panel of correlated Merton (1974) balance sheet models, jointly estimated using observed bank equity returns, and combined with a network of interbank exposures that is cleared using the algorithm described by Eisenberg and Noe (2001).[(2)] It can be used to quantify various joint risk measures for the banking system in aggregate, in addition to risk measures for banks individually. And because the model explicitly incorporates the structure of banks' balance sheets, it can be used to perform comparative static experiments that answer questions like 'what if the capital ratio of bank *A* went from *x* to *y*?'. Such thought experiments can be used to identify the configuration of capital across banks that correspond to a particular level of credit risk in the banking system.

Figure 1 illustrates the broad set-up of the model used to quantify systemic risk in this paper (inside the hatched border) and the type of comparative static exercises and numerical optimisations performed (blue). The asset value dynamics of the combined balance sheet model can be used to produce the distribution of asset shortfalls below promised debt liabilities for the system as a whole – which are hereafter called *system losses*.

**Figure 1: A general balance sheet model for quantifying systemic credit risk[(a)]**




(a) *A*, *D* and *C* refer to assets, debt and capital respectively.

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[(1)] These correlations reflect banks' exposures to common (aggregate) shocks, which generate a tendency for the creditworthiness of banks in the system to deteriorate and improve in tandem.
[(2)] The Eisenberg and Noe algorithm has been used in several papers that analyse interbank contagion; for example, see Van Lelyveld and Liedorp (2006), Wells (2004) and Upper (2007) for a review of these analyses.

*3.1 Overview of the model*

Each bank $\{i\}_{i=1,2,\ldots,n}$ holds assets outside of the banking system of $A_i^O$ and is assumed to have a single issue of zero-coupon debt outstanding to non-banks with a face value of $D_i^O$ that falls due for repayment at time $\tau_i = \tau$. In addition, each bank $\{i\}_{i=1,2,\ldots,n}$ may have an aggregate interbank asset against the other banks in the system of $A_i^I$ and an aggregate interbank liability of $D_i^I$. Like debt to non-banks, interbank debt is also assumed to have a maturity $\tau_i = \tau$.[(3)] The configuration of interbank obligations is described by an $n \times n$ matrix *M:*

$$M = \begin{bmatrix} 0 & D_{1,2}^I & \cdots & D_{1,n}^I \\ D_{2,1}^I & 0 & D_{2,n-1}^I & D_{2,n}^I \\ \vdots & \vdots & \ddots & \vdots \\ D_{n,1}^I & D_{n,2}^I & \cdots & 0 \end{bmatrix} \quad \textbf{(1)}$$

where $D_i^I = \sum_{j \neq i} D_{ij}^I$ and $A_i^I = \sum_{j \neq i} D_{ji}^I$. Banks are also partly financed by equity: bank *i* has a capital ratio $c_i \equiv C_i / A_i$, where $C_i$ is the nominal value of capital issued by a bank. In this paper, capital is assumed to be comprised exclusively of common equity.

Total system assets are given by $A \equiv \sum_i A_i = \sum_i (A_i^O + A_i^I)$ and the total face value of debt liabilities is $D \equiv \sum_i D_i = \sum_i (D_i^O + D_i^I)$.

Each bank's assets $A_i$ evolve according to a geometric Brownian motion with *ex-ante* fixed coefficients $\{\mu_i, \sigma_i\}$:

$$\frac{dA_i}{A_i} = \mu_i dt + \sigma_i dW_i^P \qquad \text{for } i = 1,2,\ldots,n \quad \textbf{(2)}$$

and where shocks may be correlated across banks, $dW_i^P dW_j^P = \rho_{ij} dt \neq 0$.

Based on these correlated asset dynamics, the solvency positions of banks $\{i\}_{i=1,2,\ldots,n}$ are checked at date $\tau_i = \tau$. There are two types of default in the model, labelled 'fundamental' and 'contagious'. If, after simulating forward the above diffusion process, the assets of any given bank, *X*, are below its (fixed) debt liabilities at time $\tau_X = \tau$, bank *X* is declared fundamentally insolvent. In this case, its loss given default is endogenously given by the difference between the level of assets at the point at which solvency is assessed and the face value of its debt falling due. But losses for the system do not end here. The fundamental default of bank *X* triggers

[(3)] In practice, interbank debt is often of short maturity. For simplicity, this paper assumes a single point at which bank solvency is assessed, parameterised to match the weighted average maturity of liabilities for UK banks. See also Section 3.4.

losses for other banks in the network that have extended it interbank loans. In some cases, clearing of the interbank network $M$ may result in a second bank, $Y$, defaulting – even though it may be above the solvency threshold if it had not made this loss on its interbank exposure to bank $X$. This represents a contagious failure of bank $Y$. In this case, the assets of bank $Y$ are marked down from the level reached under the diffusion process in equation **(2)** by an exogenously chosen contagious bankruptcy cost of 10%.[4] The interbank positions of other banks in the network are then re-evaluated. This process is repeated until there are no further rounds of contagious default in the banking network. It presents a mechanism by which losses initially borne by one bank can be transmitted and amplified through an interconnected banking system. Denoting the value of each bank's assets after network clearing by $\tilde{A}_i$, total losses in the banking system at debt maturity $\tau_i = \tau$ are thus given by:

$$L = \sum_i (D_i - \tilde{A}_i(\tau)) \tag{3}$$

Other, more realistic, descriptions of banks' balance sheets could, in principle, be used inside the network model described above. For example, the liability side of banks' balance sheets could also be made dynamic and banks could seek to target a long-run leverage ratio in response to asset and liability shocks. The choice made in this paper to base the evolution of banks' balance sheets on Merton (1974) is pragmatic. As described in the remainder of this section, the policymaker's optimisation problem – namely, to achieve a tolerable measure of systemic risk for the lowest compatible level of capital in the system as a whole – is solved numerically. Because measures of systemic risk are an output of the model rather than an input,[5] finding the minimal configuration of banks' capital that achieves exactly the policymaker's target for systemic risk is computationally time consuming. Adding greater complexity to the stochastic behaviour of individual banks' balance sheets would further increase computation time, potentially substantially. This paper therefore seeks to obtain general insights into the properties of risk-based systemic capital requirements rather than calibrate precise nominal amounts that may be required to achieve particular targets for credit risk at the system level.

### 3.2 *Quantifying risks to systemic solvency*

This section describes the algorithm by which the model outlined above is estimated and implemented for a stylised representation of the UK banking system. The equations in this section are derived in Appendix 1, following closely Elsinger *et al* (2006).

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[4] This assumption is consistent with Alessandri *et al* (2009), based on empirical evidence in James (1991).
[5] That is, systemic risk depends on the vector of capital across banks, over which the policymaker's optimisation is performed.

The asset value dynamics in equation **(2)** imply a log-normal distribution of asset returns:

$$x_{i,t} \equiv \ln\left(\frac{A_{i,t+\Delta t}}{A_{i,t}}\right) \overset{P}{\sim} N\left(\left(\mu_i - \frac{1}{2}\sigma_i^2\right)\Delta t, \sigma_i^2 \Delta t\right) \tag{4}$$

An estimate of the asset drift rate $\hat{\mu}_i$ can be obtained from the mean of this distribution:

$$\hat{\mu}_i = \frac{E[\ln A_{i,t+\Delta t}] - E[\ln A_{i,t}]}{\Delta t} + \frac{1}{2}\sigma_i^2 \tag{5}$$

Using the well-known functional form of the Normal distribution, it is possible to derive a log-likelihood function for banks' asset returns, $l(\sigma_i)$ , the value of which is ultimately determined by a single unknown parameter, $\sigma_i$:

$$l(\sigma_i) = -\frac{1}{2\sigma_i^2 \Delta t}\sum_{t=1}^{T}\left(x_{i,t} - \left(\mu_i - \frac{1}{2}\sigma_i^2\right)\Delta t\right)^2 - \frac{T}{2}\ln\left(\sigma_i^2 \Delta t\right) - \frac{T}{2}\ln(2\pi) \tag{6}$$

This likelihood function can be used to estimate the vector of drift rates on banks' assets $\{\hat{\mu}_i\}_{i=1,2,\ldots,n}$ and the unknown contemporaneous variance-covariance structure between banks' asset returns $\{\Sigma_{ij}\}_{i,j=1,2,\ldots,n}$ (Stage 1). These parameters can, in turn, be used to simulate the distributions of losses borne by individual banks and at the level of the system overall at the time when debt is due to be repaid (Stage 2). The calculations involved in these two stages are described below. They are summarised as a flow diagram in Appendix 2.

<u>Stage 1 (model calibration)</u>

For each bank $\{i\}_{i=1,2,\ldots,n}$:

- Choose exogenously an initial value of the unobserved volatility of asset returns $\sigma_i$.
- Using time series of observed bank equity prices and the risk-free spot interest rate of maturity $\tau_i = \tau$ over a given time interval $t \in [0, T]$, invert the standard Merton (1974) model (Appendix 3) to back-out a corresponding time series of asset values $A_{i,t}$ over interval $t \in [0, T]$.
- Calculate an estimate for the drift rate of assets for bank $i$, $\mu_i = \hat{\mu}_i$ , from equation **(5)**.
- Compute the value of the log-likelihood function in equation **(6)** from $\sigma_i$, $A_{i,t}(\sigma_i)$ and $\hat{\mu}_i(A_{i,t}(\sigma_i))$.
- Numerically solve for $\sigma_i$ such that $\max_{\sigma_i}\left(l(\sigma_i)\right)$ is obtained. This yields the maximum likelihood estimate of the diffusion parameters $\{\mu_i, \sigma_i\}$ for each bank viewed separately.

Stage 2 (simulating the distribution of system losses)

- Calculate the realised variance-covariance matrix $\Sigma$ between banks' asset returns and corresponding correlation structure (Appendix 4) from the estimates of $A_{i,t}$ over period $t \in [0, T]$ recovered in Stage 1.
- Re-compute asset values for each bank $\{i\}_{i=1,2,\ldots,n}$ using the new variances (diagonal elements) of the full variance-covariance matrix $\Sigma$ , $\sigma_i = \sqrt{diag_i(\Sigma)}$.
- Simulate forward the correlated asset value distribution for the system of banks $\{i\}_{i=1,2,\ldots,n}$.[6]
- For each bank, calculate the distribution of asset shortfalls below promised debt liabilities (the endogenous loss given default) based on its fundamental solvency position before accounting for default cascades.
- Clear the network of interbank exposures using the Eisenberg and Noe (2001) algorithm and markdown the assets of any contagiously failing banks from the value reached under the diffusion process in equation **(2)** by an exogenously chosen 10%. Iterate around the network clearing algorithm until no further contagious losses occur. The extent to which losses are transmitted and amplified across the system at this point is determined in part by whether interbank assets and liabilities are netted:
  - With netting – smaller payments are required between banks to clear the network.
  - Without netting – larger payments are required between banks to clear the network.

  This paper investigates the no-netting case, as it has the strongest implications for systemic risk. In principle, with full and immediate netting of all interbank contracts, an equal exposures matrix of any size – ie of the form $M = c\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 0 \end{bmatrix}$ – could never trigger contagious defaults because interbank asset and liabilities are equal for all banks, $\sum_i M_{ij} = \sum_i M_{ji}$.
- Identify the zth percentile of the resulting loss distribution for the system as a whole, aggregating across banks, using numerical integration (Simpson's rule). The percentile may be chosen, for example, to achieve some probability that assets are below liabilities for the consolidated balance sheet of the banking system ('system insolvency').

### 3.3 *Optimising over banks' capital to achieve an exogenous systemic risk target*

If capital is more expensive than debt,[7] the efficiency of the banking system is a decreasing function of the total capital held across banks, $\sum_i C_i$. Capital could be more expensive than debt for various reasons, including because of principal agent problems between managers and shareholders (Jensen (1986) and Jensen and Meckling (1976)) or because of information

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[6] A convenient way to do this, used in this paper, is to first perform a Cholesky decomposition of $\Sigma$.
[7] That is, assuming the Modigliani-Miller (1958) theorem does not hold.

asymmetries between insiders and external investors (Myers and Majluf (1984)).[8] A policymaker interested in mitigating systemic risk, however defined, while limiting inefficiency in the banking system might therefore seek to achieve a chosen systemic risk objective for the lowest compatible level of capital in aggregate. They confront a non-linear constrained optimisation problem of the form:

$$\min_{\{C_i\}_{i=1,2,\dots,n}} \left(\sum_i C_i\right) \quad s.t.\ \text{systemic risk objective} \tag{7}$$

In this paper, the systemic risk objective is defined in terms of a target for the location of the zth percentile of the distribution of system losses relative to promised debt liabilities. In this case the constrained optimisation problem is:

$$\min_{\{C_i\}_{i=1,2,\dots,n}} \left(\sum_i C_i\right) \quad s.t.\ VaR_z^{system}\left(\{C_i\}_{i=1,2,\dots,n}\right) = 0 \tag{8}$$

The constraint in equation **(8)**, $VaR_z^{system}\left(\{C_i\}_{i=1,2,\dots n}\right) = 0$, can also be expressed as $Pr(\sum_i A_i(C_i) < \sum_i D_i) = 1 - z$. In other words, the policymaker tries to minimise inefficiency (total capital) subject to the banking system remaining solvent with a chosen target probability (the probability of system assets being below system liabilities being equal to $1 - z$).[9] The parameter $z$ reflects the trade-off between the systemic risk and efficiency objectives. For example, a high value of $z$ might be suitable if there is a relatively shallow trade-off between systemic risk and economic efficiency, ie in the case that capital is not materially more expensive than debt.

The optimal level of capital in the banking system in aggregate and the optimal distribution of capital across banks are not separable. This complicates solving the constrained optimisation problem in equation **(8)**. For example, consider a system of two banks. Each has the same fundamental uncertainty about asset value returns going forward, but bank *X* starts with a much larger cushion of capital than bank *Y*. In this case, an extra £1 of capital given to bank *X* has a smaller impact on its asset shortfall distribution than if it were given to bank *Y*. Bank *X* is already very safe and so perhaps it goes from having two in 10,000 draws from nature where its assets are insufficient to meet liabilities, to one in 10,000. Bank *Y*, meanwhile, might go from having 1,000 in 10,000 draws where assets fall below liabilities to 800 in 10,000. If, in addition,

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(8) The extent to which capital is relatively more expensive than debt for banks nevertheless remains an open question. For example, see Admati, DeMarzo, Hellwig and Pfeiderer (2010), as cited in the speech given by the Governor of the Bank of England at the Second Bagehot Lecture Buttonwood Gathering in New York City on 25 October 2010.

(9) The assumption that the policymaker sets banks' capital requirements to limit the probability of system insolvency is conservative. Other assumptions could be made. For example, the policymaker could seek to limit the probability that the shortfall of assets below debt liabilities for the system does not exceed a particular (non-zero) value, to reflect the investment capital that may be available from investors prior to systemic default.

bank $X$ (idiosyncratically safe) has a large interbank asset against bank $Y$ (idiosyncratically risky), giving an extra £1 to bank $Y$ materially reduces the contingent-default risk of bank $X$ – and, consequently, for the two-bank system in aggregate. Increasing the aggregate level of capital by £1 therefore has little impact on systemic risk *unless* it is given to bank $Y$. In other words, the optimal level and distribution of capital are fundamentally interrelated.

The optimisation strategy used in this paper to solve the constrained optimisation problem in equation **(8)** is as follows:

(a) Optimise over the total level of system capital $\sum_i C_i$, holding fixed the relative shares held by each bank, to achieve the policymakers' chosen systemic risk constraint, in this paper $VaR_z^{system}(\{C_i\}_{i=1,2,\ldots,n}) = 0$. This is done by taking banks' observed capital levels and increasing (decreasing) $\sum_i C_i$ if $VaR_z^{system}(\{C_i\}_{i=1,2,\ldots,n}) > (<)0$. This step aims to anchor the approximate level of capital that the system requires overall to be sufficiently robust.[10] In this paper, this is done using a simple grid search algorithm.

(b) Adjust the share of aggregate capital held by each bank, $\{C_i\}_{i=1,2,\ldots,n} \rightarrow \{\tilde{C}_i\}_{i=1,2,\ldots,n}$, such that the chosen measure of systemic risk is reduced, $VaR_z^{system}\left(\{\tilde{C}_i\}_{i=1,2,\ldots,n}\right) < 0$. If this is possible, the allocation $\{\tilde{C}_i\}_{i=1,2,\ldots,n}$ must be superior to $\{C_i\}_{i=1,2,\ldots,n}$ because systemic risk is lowered for the same level of efficiency since $\sum_i \tilde{C}_i = \sum_i C_i$. In this paper, the reallocation of capital across banks is done using simulated annealing to try to ensure that the global solution of the constrained optimisation problem is obtained.[11]

(c) Reduce the level of system capital by a small amount $\varepsilon$, allocated pro-rata across banks, and perform the optimisation in the previous step again.[12]

(d) Repeat steps (b)-(c) until it is no longer possible to further reduce system capital and simultaneously achieve the policymakers' chosen tolerable level of systemic risk, such that $VaR_z^{system}\left(\{\tilde{C}_i\}_{i=1,2,\ldots,n}\right) \rightarrow 0$ from below. This yields the minimum level of aggregate capital that can be allocated across banks and simultaneously meet the chosen systemic risk constraint.

(10) The optimised level of system capital is approximate at this stage because it may ultimately be possible to achieve the same systemic risk constraint using a lower level of aggregate capital by choosing carefully its allocation across banks (steps (b), (c) and (d)).

(11) Simulated annealing is a non-gradient based optimisation technique that is less likely to converge on a local (rather than global) minimum than standard gradient-based algorithms, while generally requiring fewer computations than an exhaustive grid search. For a description of simulated annealing, see Cerny (1985).

(12) The size of the perturbation used in this step reflects a trade-off between solution accuracy (spanning the parameter space in less coarse increments) and computational feasibility (ensuring model run-times are manageable). In this paper, total system capital is reduced in increments of 1% of its initial level. Smaller perturbations were found to have little impact on the results under the benchmark calibration of the model.

Optimising over the overall level of capital in the system and its distribution across banks is the key contribution of this paper. Provided the reallocation of capital between banks in step (b) is performed with sufficient care, the four-step algorithm described above is very likely to yield the global solution of the stability/efficiency trade-off (parameterised by $z$). In principle, the objective function in equation **(8)** could be augmented with a minimum capital requirement restriction to reflect narrowly microprudential standards. However, these are unlikely to bind for plausible tolerances of systemic credit risk, absent other concerns that could enter the systemic policymaker's objective function such as the ability of the banking system to supply credit to the real economy in a downturn. Alternatively, a modified version of this optimisation strategy could be used to allocate a fixed amount of capital across banks according to chosen metrics of systemic risk, similar to Gauthier *et al*.

### *3.4 Calibration details*

The model is calibrated using UK data for the period 2004 H1 to 2009 H1. Banks' balance sheets are based on information from five major UK banks' published accounts. 'Debt' is interpreted as total liabilities excluding large exposures to the other four banks, minus shareholders' funds excluding minority interests.[13] 'Large exposures' are defined as those exposures that exceed 10% of eligible capital,[14] which UK banks have to report to their supervisors. The interbank network, $M$ in equation **(1)**, is calibrated to these large exposures.[15] When banks fail because of losses on interbank exposures (contagious default) rather than in response to the value of their non-bank assets (fundamental default), this paper assumes that assets are marked down by 10% from the level reached endogenously through the diffusion process in equation **(2)**.

It is assumed that banks have a weighted-average debt liability of one year, at which point their solvency is assessed. Other parameters of the model are estimated following the procedure described in Section 3.2. In particular, banks' observed equity prices are used to estimate the contemporaneous expected return on banks' assets and the variance-covariance structure between banks' asset returns.

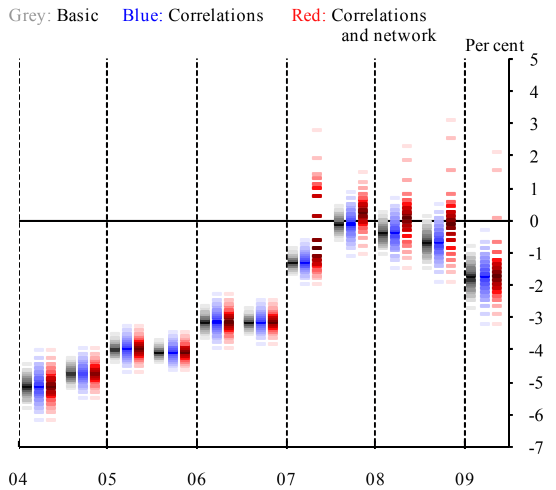
By way of illustration, the systemic policymaker is assumed to want to locate the 95$^{th}$ percentile of the loss distribution for the system at zero ($z = 0.05$). At each stage of the policymaker's optimisation problem in equation **(8)**, distributions are constructed from 50,000 simulations of the underlying interconnected Merton models.

[13] This is approximately equivalent to assuming that Tier 1 capital is sufficiently loss absorbing.
[14] Defined as Tier 1 plus Tier 2 capital.
[15] This relaxes the maximum entropy approach adopted in Elsinger *et al* (2006).

## 4 Benchmark results

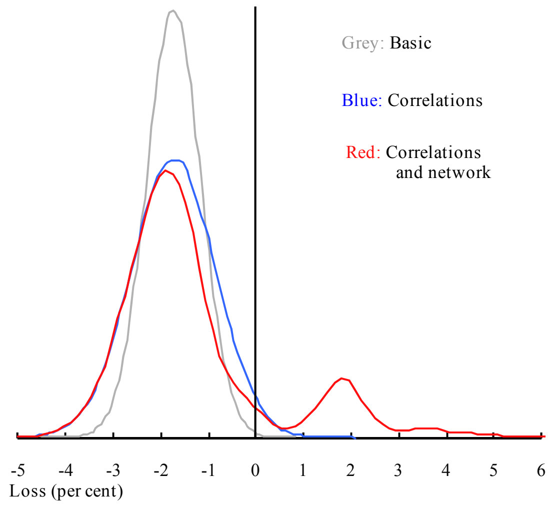
Chart 1 shows the implied distribution of losses for the system in aggregate using the calibrated model. Considering the evolution of banks' balance sheets in isolation and aggregating across institutions (Chart 1, grey bars) gives an artificially benign picture of risks to the system as a whole because correlations between banks' asset values and the possibility of adverse loss spirals occurring through the interbank network are ignored. Incorporating the possibility of time-varying correlations between banks' asset values widens the distribution of losses at the system level (Chart 1, blue bars) because the estimated correlations are positive (Appendix 4), and so banks' solvency positions tend to improve and deteriorate in tandem. The system loss distribution widens further if the potential for contagion via direct balance sheet exposures between banks is also incorporated (Chart 1, red bars). The tail of the distribution is fatter when stress in the system is higher, as during the financial crisis. From 2007 H1, the distribution of losses at the level of the system becomes significantly bi-modal, corresponding to a splitting of outcomes for the system – one where bank defaults are contained and one where contagious defaults spread through the system (Chart 2).

**Chart 1: Evolution of aggregate losses in the banking system over time(a)(b)**




(a) Percentiles of aggregate loss distribution across a panel of UK banks: based on stand-alone balance sheets (grey); accounting for correlation between asset returns across banks (blue); and accounting for asset return correlation and explicit interbank exposures between firms, assuming contagious default carries a deadweight cost of 10% of assets (red).
(b) Loss expressed as a fraction of system-wide debt liabilities.

**Chart 2: Distribution of aggregate losses in the banking system (2009 H1)(a)(b)**




(a) Aggregate loss distribution across a panel of major UK banks: based on stand-alone balance sheets (grey); accounting for correlation between asset returns across banks (blue); and accounting for asset return correlation and explicit interbank exposures between firms, assuming contagious default carries a deadweight cost of 10% of assets (red).
(b) Loss expressed as a fraction of system-wide debt liabilities.

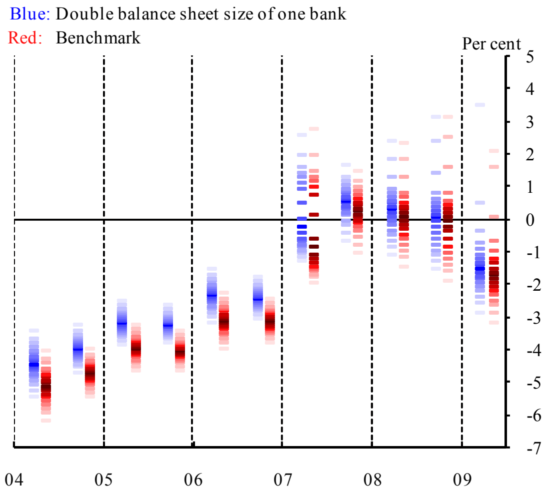
## 5 Comparative static exercises: changing the structure of the banking system

Shifts in the structure of the banking system may change the impact that a given bank has on the solvency of the system overall when it fails. This section illustrates the effect on systemic credit risk in the model of counterfactual changes in: the size of banks in the system; interconnectedness between banks; the deadweight cost of contagious bankruptcy; and measured risk, as implied by market prices. All changes are relative to the benchmark representation described in the Section 4 under the case where the effects of asset correlations between banks and interbank exposures are both included.

### *5.1 Increasing the size of banks*

Larger banks have a greater impact on the solvency of the system as a whole when they fail than do smaller banks, other things being equal. Chart 3 shows the effects on the system loss distribution of doubling the balance sheet size of bank 1 (Chart 3, blue) relative to the benchmark calibration shown in Section 4 (Chart 3, red). Mechanically, this increases the nominal-amount loss given default associated with a fundamental default of bank 1. This tends to push up the scale of first-round losses borne by other banks in the network on their lending to bank 1 (shared pro-rata), leading to a greater prevalence of first-round contagious defaults in response to the same fundamental shocks. This, in turn, raises the prevalence of cascades of contagious default, pushing out the tail of the system loss distribution (Chart 4), in addition to mechanically moving the centre of the distribution to the right.

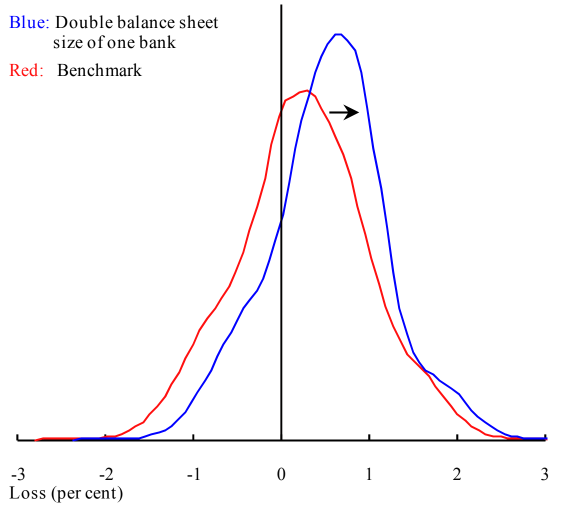
**Chart 3: Evolution of aggregate losses in the banking system over time when one bank doubles in size[(a)(b)]**




(a) Percentiles of aggregate loss distribution across a panel of UK banks.

(b) Loss expressed as a fraction of system-wide debt liabilities.

**Chart 4: Distribution of aggregate losses in the banking system when one bank doubles in size (2007 H2)[(a)]**

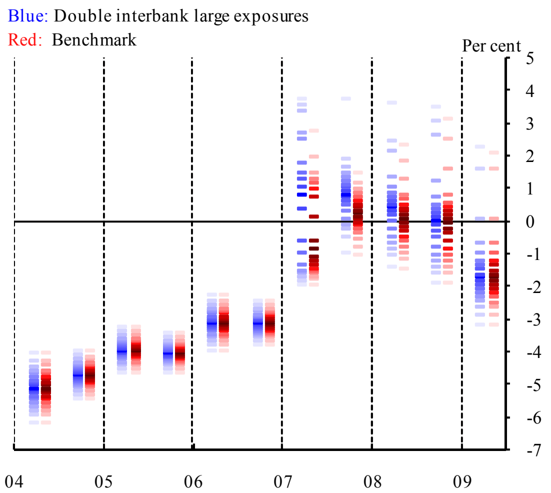



(a) Loss expressed as a fraction of system-wide debt liabilities.

## 5.2 *Increasing interconnectedness of the banking network*

A more interconnected banking system is generally more prone to contagious defaults, other things being equal. Chart 5 shows the effect of doubling the values of all interbank exposures between banks, compared to the benchmark case. When banks in the system are individually and collectively well capitalised relative to the riskiness of their assets, increasing interconnectedness has relatively little impact on the risks posed to the system overall. In this case, there are very few states of the world in which banks become fundamentally insolvent at the same time that other institutions are sufficiently weak to suffer contagious failure (see the 2004-06 period in Chart 5). But there is a tipping point at which the capital in the system overall is sufficiently low relative to the riskiness of assets that increasing interconnectedness between banks does have a significant effect on the risk of contagion (see the 2007-09 period in Chart 5).[16] The marginal impact of interconnectedness varies according to the state of the banking network. In some states of the world, increasing interconnectedness can substantially alter risks in the system as a whole and may lead to the emergence of a second mode corresponding to waves of contagious defaults (Chart 6). This has been called the robust-yet-fragile property of financial networks.[17]

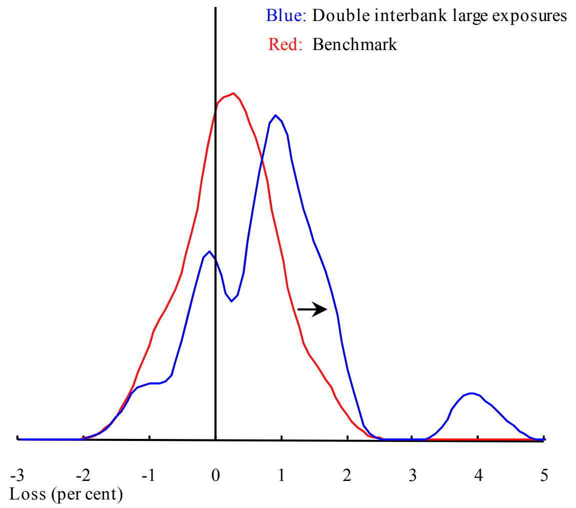
**Chart 5: Evolution of aggregate losses in the banking system over time when interbank exposures double in size**[(a)(b)]




(a) Percentiles of aggregate loss distribution across a panel of UK banks.

(b) Loss expressed as a fraction of system-wide debt liabilities.

**Chart 6: Distribution of aggregate losses in the banking system when interbank exposures double in size (2007 H2)**[(a)]




(a) Loss expressed as a fraction of system-wide debt liabilities.

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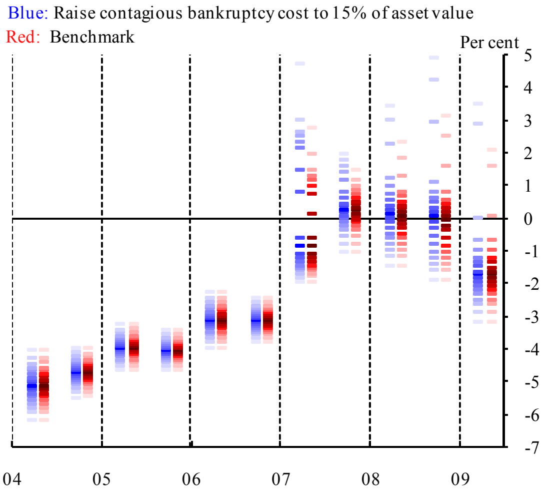
[16] The property that interconnectedness may flip the financial system between periods of shock-absorption and shock-amplification is discussed in Haldane (2009).

[17] See, for example, Gai *et al* (2007).

### 5.3 *Increasing the deadweight cost of contagious default*

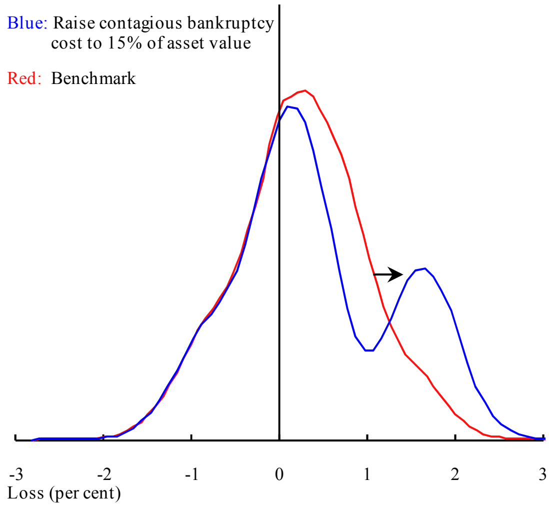
Raising the deadweight cost of defaults triggered by losses on interbank exposures materially increases the fragility of the system as a whole. It becomes easier for contagion to take hold, and contagion generally spreads further in response to the same fundamental shock. Charts 7 and 8 suggests that even small changes to contagious bankruptcy costs can have large implications for the system, particularly when the balance sheets of multiple banks are relatively weak, as during the financial crisis.

**Chart 7: Evolution of aggregate losses in the banking system over time when raising the cost of contagious default[(a)(b)]**




(a) Percentiles of aggregate loss distribution across a panel of UK banks.
(b) Loss expressed as a fraction of system-wide debt liabilities.

**Chart 8: Distribution of aggregate losses in the banking system when raising the cost of contagious default (2007 H2)[(a)(b)]**




(a) Loss expressed as a fraction of system-wide debt liabilities.
(b) Cost of contagious default raised to 15% of asset value, from 10% in the benchmark case.

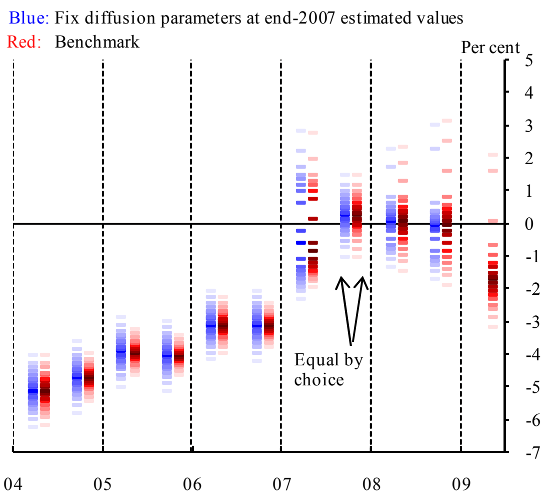
### 5.4 *Holding fixed all asset diffusion parameters*

It is widely understood that capital requirements linked to measures of system-wide risks at any point in time are inherently procyclical,[(18)] including at the system level if banks have similar asset portfolios. The measures of risk shown in Sections 4 and 5.1-5.3 exhibit this property – rising in the run-up to the 2007-08 financial crisis and falling back slightly in 2009. Broadly, these results can be traced to changes over time in the composition of banks' balance sheets and in the estimated statistical properties of banks' asset returns. To what extent is the relative narrowness of the systemic loss distribution in the period pre-crisis an artefact of benign macroeconomic and financial market conditions? Chart 9 holds fixed, over the full sample, the expected return on banks' assets and the variance-covariance structure between banks' asset returns at their estimated levels corresponding to 2007 full-year balance sheets. The results are

[(18)] See, for example, Chapter 2 of the April 2010 *Global Financial Stability Report* published by the IMF.

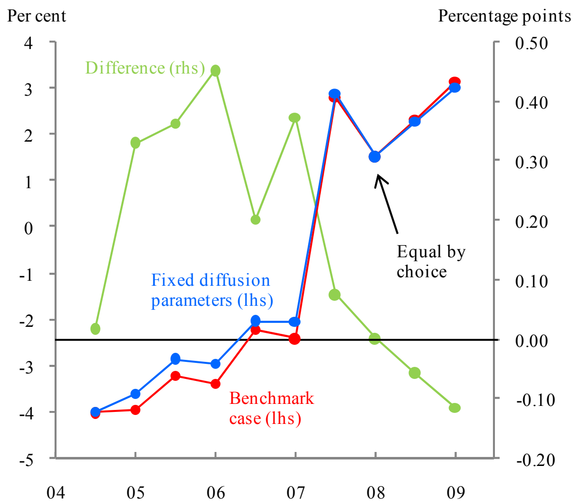
intuitive. Relative to the benchmark case, the distribution of system losses widens pre-crisis (2004-06) when measured risk was low, and narrows slightly during 2008 when measured risk was high, including in the tail (Chart 10). Nevertheless, the effect on the likelihood of system insolvency of making these adjustments to the asset diffusion parameters is modest relative to the effect on system solvency of observed changes to the composition of banks' balance sheets over time. This suggests an explicitly countercyclical role for systemic capital requirements that lean against increases in bank leverage in an economic upswing and *vice versa*, in addition to reflecting changes in the riskiness of asset exposures over time.

**Chart 9: Evolution of aggregate losses in the banking system over time, holding fixed statistical characteristics of assets**[(a)(b)]




(a) Percentiles of aggregate loss distribution across a panel of UK banks.
(b) Loss expressed as a fraction of system-wide debt liabilities.

**Chart 10: Tail of the distribution of aggregate losses, holding fixed statistical characteristics of assets**[(a)]




(a) Loss expressed as a fraction of system-wide debt liabilities.

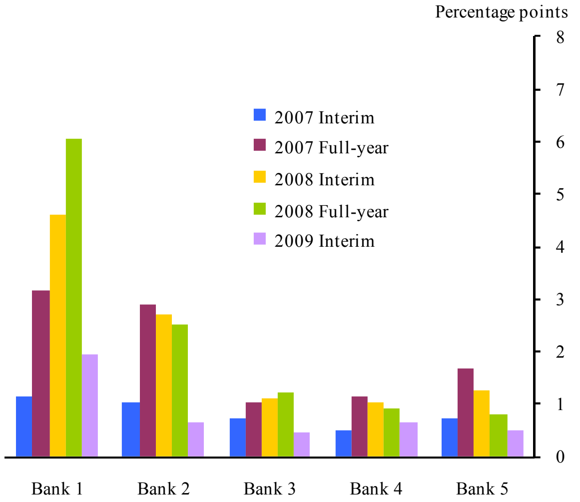
## 6 Systemic capital requirements

Adjusting the total level of capital in the system and its distribution across banks alters the shape and location of the loss distribution for the system as a whole because, in consequence, this alters the frequency with which fundamental defaults and cascades of contagious defaults occur. In this section, the policymaker's constrained optimisation problem is solved to obtain banks' systemic capital requirements. The results are shown as marginal changes in capital, or surcharges, relative to banks' observed capital.

### *6.1 Systemic capital surcharges under the benchmark case*

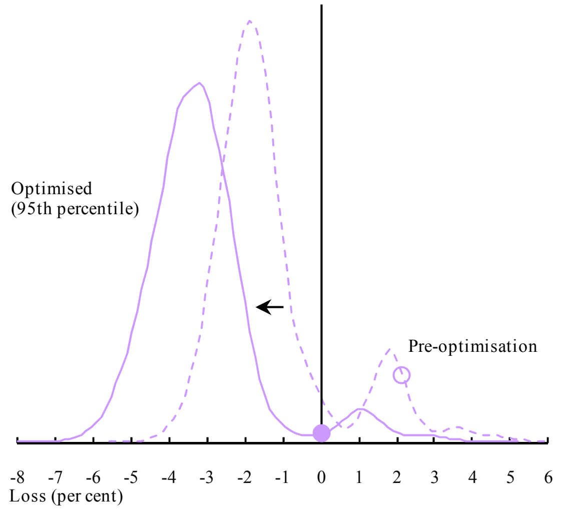
Chart 11 shows, for the benchmark case, the required change in individual banks' capital to solve the policymaker's optimisation problem – that is, those surcharges that sufficiently reduce the probability of system insolvency for the minimum compatible level of capital held in the system in total. Systemic capital surcharges differ across banks because losses incurred by the rest of the system differ according to which bank(s) fail, and because the probabilities of such losses crystallising vary. By design of the policymaker's objective function (equation **(8)**), the corresponding distributions of system losses under the optimised configurations of bank capital shift to the left. Chart 12 shows the particular case for 2009 H1.

**Chart 11: Systemic capital surcharges**[a]




(a) Change in the ratio of capital to assets for each bank in the network following the optimisation in equation **(8)**.

**Chart 12: System loss distributions pre and post-optimisation (2009 H1)**[(a)(b)(c)]




(a) Accounting for asset correlation and explicit interbank exposures between firms, assuming contagious default carries a deadweight cost of 10% of assets.

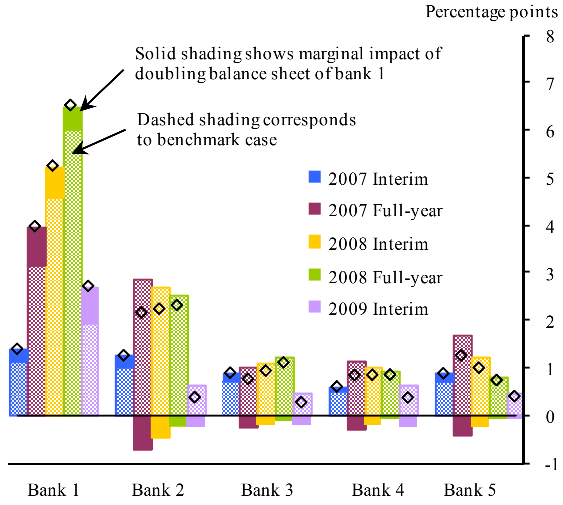
(b) Following the optimisation in equation **(8).** Circles show location of 95th percentile.

(c) Loss expressed as a fraction of system-wide debt liabilities.

## *6.2 Systemic capital surcharges with increased bank size*

Chart 13 shows the marginal impact on banks' systemic capital surcharges, relative to the benchmark case, when doubling the balance sheet size of bank 1. Systemic capital surcharges for bank 1 are higher than in the benchmark case to offset the larger impact its failure has on the rest of the system through the interbank network, as described in Section 5.1. This is actually sufficient, in a number of instances, to reduce systemic capital surcharges for the other banks in the network because there are fewer scenarios under which they experience interbank losses large enough to cause them to contagiously fail, which would otherwise further transmit and amplify losses. Outcomes for system as a whole are nevertheless lumpier than in the benchmark case (Chart 14) because although the likelihood of contagion spreading has been lowered as the solution to the policymaker's optimisation problem, contagion is stronger when it does take hold.

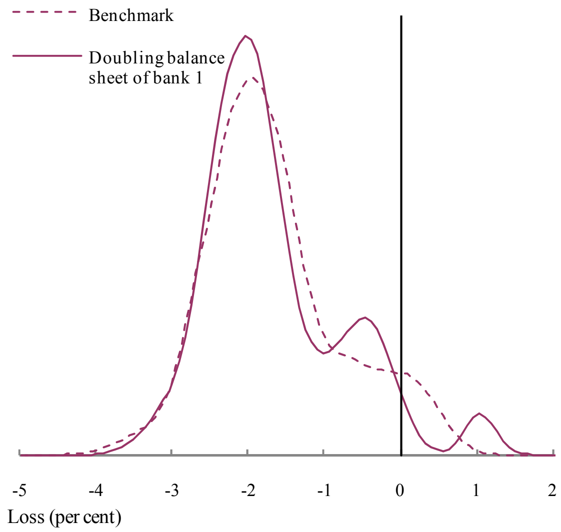
**Chart 13: Systemic capital surcharges when one bank doubles in size(a)(b)**




(a) Change in the ratio of capital to assets for each bank in the network following the optimisation in equation **(8)**.

(b) Diamonds show total change in capital requirement.

**Chart 14: System loss distribution when one bank doubles in size (2007 H2)(a)(b)**



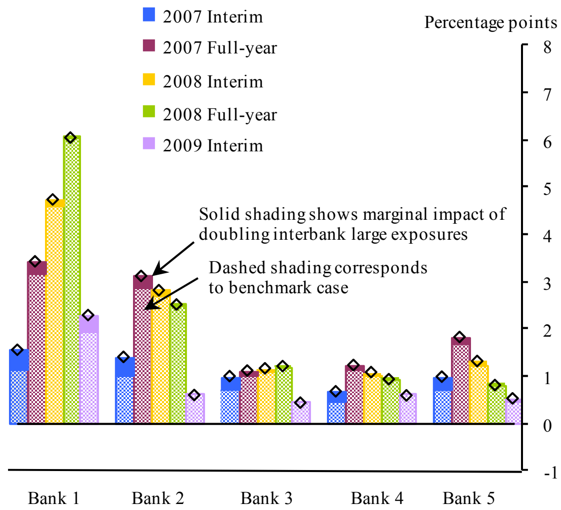

(a) Following the optimisation in equation **(8)**.

(b) Loss expressed as a fraction of system-wide debt liabilities.

### *6.3 Systemic capital surcharges with increased interconnectedness*

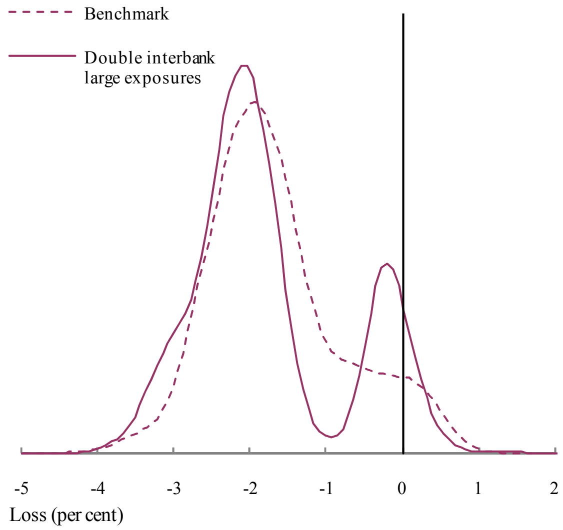
Increasing interconnectedness between banks pushes up the capital required by the system as a whole to withstand the same fundamental shocks. Chart 15 shows the marginal impact on systemic capital surcharges of doubling direct balance sheet exposures between banks, relative to the benchmark case. These results further illustrate the robust-yet-fragile property of financial networks discussed in Section 5: the system as a whole can in some circumstances be made substantially more robust for a relatively small increase in aggregate capital, notwithstanding differences in capital surcharges across banks. For example, even in 2007, the model suggests that the potential effect of contagion through the interbank network was particularly acute (compare the red and blue bars in Chart 1). This is mirrored in the marginal impact on systemic capital requirements in 2007 when doubling interconnectedness relative to the smaller surcharges later in the period (Chart 15). While the total amount of additional capital that is sufficient to offset the effect of greater interconnectedness is relatively modest across the period, the resulting network remains relatively more prone to contagion than the benchmark case, as reflected in a more bi-modal distribution of system losses (Chart 16).

**Chart 15: Systemic capital surcharges when interconnections double in size(a)(b)**




(a) Change in the ratio of capital to assets for each bank in the network following the optimisation in equation **(8)**.
(b) Diamonds show total change in capital requirement.

**Chart 16: System loss distribution when interconnections double in size (2007 H2)(a)(b)**



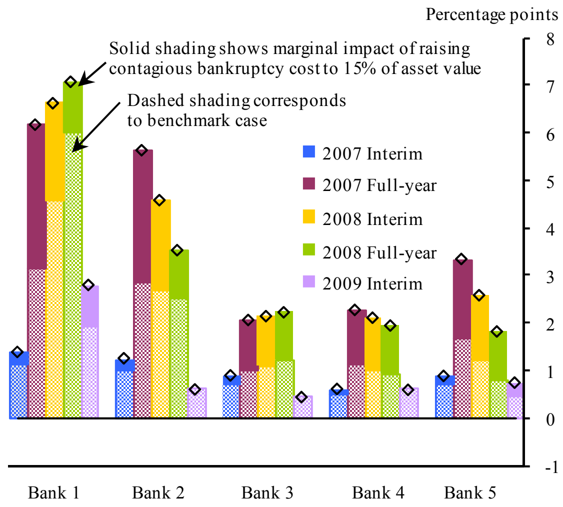

(a) Following the optimisation in equation **(8)**.
(b) Loss expressed as a fraction of system-wide debt liabilities.

### *6.4 Systemic capital surcharges with higher deadweight cost of contagious default*

As described in Section 5.3, even modest increases in the fixed cost of contagious default[19] can have a large impact on the shape of the system loss distribution. This is reflected in the marginal impact on systemic capital requirements (Chart 17) required to relocate the system loss distribution as desired by the policymaker (Chart 18). Nevertheless, although the tail of the system loss distribution under the case of higher contagious bankruptcy costs can be significantly fatter than in the benchmark case (Chart 7), the additional capital required to achieve the policymaker's objective (equation **(8)**) prior to the crisis, including in 2007 H1, is modest compared to that required between 2007 H2 and 2008 H2 (Chart 17). This is because even by 2007 H2, the systemic capital surcharges determined by the policymaker need to shift a significantly greater mass of the distribution to the left than in 2007 H1 to meet the chosen risk constraint.

[19] Recalling that these are over and above those associated with fundamental default, which are, in turn, endogenously determined from the value of assets at debt maturity relative to the face value of debt liabilities (equation **(2)**).

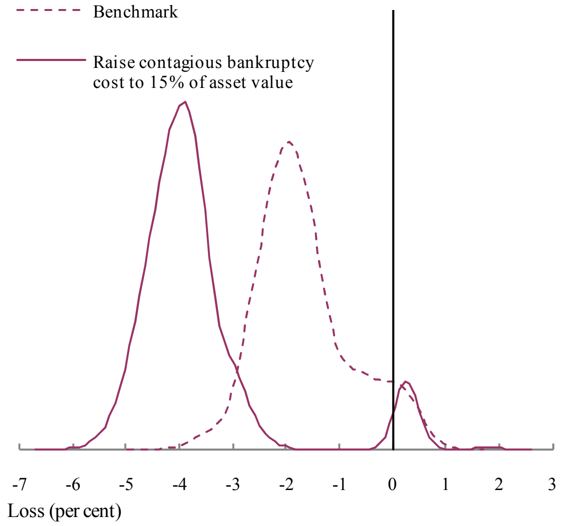
**Chart 17: Systemic capital surcharges when raising the cost of contagious default**[a][b]




(a) Change in the ratio of capital to assets for each bank in the network following the optimisation in equation **(8)**.

(b) Diamonds show total change in capital requirement.

**Chart 18: System loss distribution when raising the cost of contagious default (2007 H2)**[a][b]



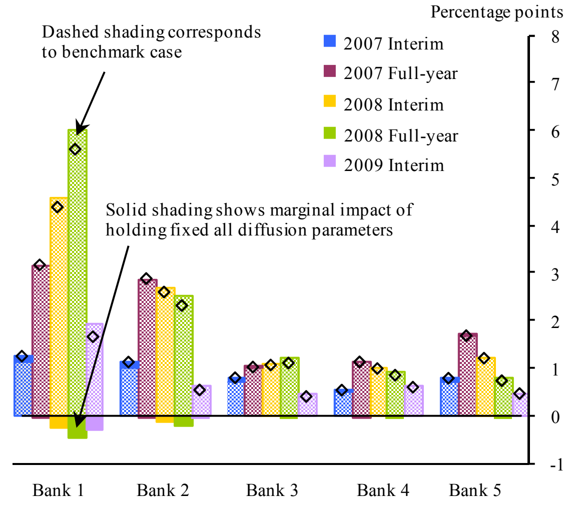

(a) Following the optimisation in equation **(8)**.

(b) Loss expressed as a fraction of system-wide debt liabilities.

## 6.5 *Cyclicality of systemic capital surcharges*

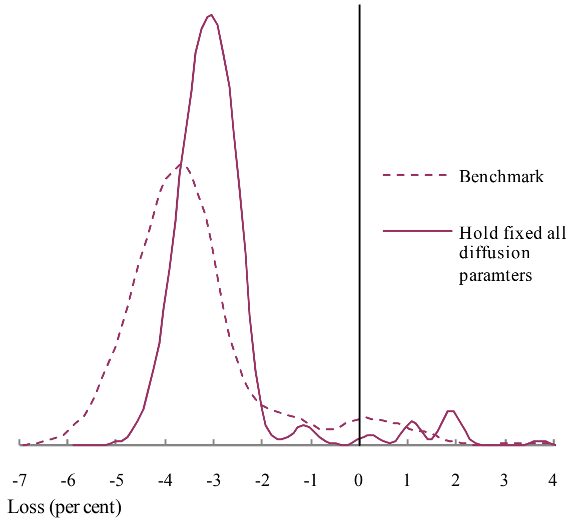
By comparison to the impact on the system loss distribution, relative to the benchmark case, of changes in the composition of banks' balance sheets over time, the impact of holding fixed the statistical properties of asset values in the model appears small (as described in Section 5.4). The marginal impact on systemic capital surcharges is therefore also small (Chart 19). Surcharges are nevertheless slightly lower through the crisis than in the benchmark case because banks' asset risk/returns parameters are held constant over time at their 2007 H2 values – whereas contemporaneous measures of risk during the crisis are higher.

**Chart 19: Systemic capital surcharges, holding fixed all diffusion parameters**[(a)(b)]




(a) Change in the ratio of capital to assets for each bank in the network following the optimisation in equation **(8)**.
(b) Diamonds show total change in capital requirement.

**Chart 20: System loss distribution, holding fixed all diffusion parameters (2008 H2)**[(a)(b)]




(a) Following the optimisation in equation **(8)**.
(b) Loss expressed as a fraction of system-wide debt liabilities.

## 7 Conclusions

The financial crisis has led to calls for banks' capital requirements to be set, in part, to reflect the impact that their failure would have on the rest of the financial system. This paper has presented a potential approach to informing the calibration of systemic capital requirements. A policymaker interested in mitigating risks to the solvency of the system in aggregate sets individual bank capital requirements to ensure that a chosen target level of systemic risk is achieved (stability) while at the same time minimising the overall level of capital in the banking system (efficiency). Using a calibrated model of a stylised banking system, the results in this paper suggest that banks' systemic capital requirements are increasing in balance sheet size and in the value of their interbank obligations, other things being equal. But care is required in drawing firm conclusions from models of this type. In particular, even assuming a more sophisticated, perfectly specified and calibrated model – including, for example, multiple classes of asset and liability and accounting for the possibility of wholesale funding market closure – a policymaker would need to make allowance for procyclicality in model estimates of systemic risk and, consequently, in their assessment of banks' appropriate systemic capital requirements through the cycle. Calibrating input parameters of such a model through the cycle offers a partial solution.

This paper assumes a particular form of an objective function that a systemic policymaker could adopt, centred solely on resilience of the banking system as a whole. A broader modelling framework and objective function could also include measures of cyclical imbalances including, for example, deviations of bank credit availability from equilibrium.

## Appendix 1: Log-likelihood function for asset returns

The underlying asset value for each bank $i$ follows a geometric Brownian motion:

$$\frac{dA_{i,t}}{A_{i,t}} = \mu_i dt + \sigma_i dW_{i,t}^P \qquad \text{for } i = 1,2,\dots,n \tag{A1}$$

Using Itô's lemma, the log asset diffusion is therefore:

$$d(\ln A_{i,t}) = \frac{\partial(\ln A_{i,t})}{\partial A_{i,t}} dA_{i,t} + \frac{1}{2} \cdot \frac{\partial^2(\ln A_{i,t})}{\partial {A_{i,t}}^2} VaR_t^P(dA_{i,t}) \tag{A2}$$

$$= \left(\mu_i - \frac{1}{2}\sigma_i^2\right) dt + \sigma_i dW_{i,t}^P \tag{A3}$$

Integrating this between times $t_1$ and $t_2$ gives:

$$x_{i,t} \equiv \ln\left(\frac{A_{i,t_2}}{A_{i,t_1}}\right) = \left(\mu_i - \frac{1}{2}\sigma_i^2\right)(t_2 - t_1) + \sigma_i\left(W_{i,t_2}^P - W_{i,t_1}^P\right) \tag{A4}$$

Defining $\Delta t \equiv t_2 - t_1$, the distribution of asset value returns is therefore lognormal:

$$x_{i,t} \overset{P}{\sim} N\left(\left(\mu_i - \frac{1}{2}\sigma_i^2\right)\Delta t, \sigma_i^2 \Delta t\right) \tag{A5}$$

Using the well-know functional form for the Normal distribution, the corresponding density function at each point in time is:

$$f(x_{i,t}) = \frac{1}{\sqrt{2\pi\sigma_i^2 \Delta t}} \exp\left(-\frac{1}{2} \cdot \frac{\left(x_{i,t} - \left(\mu_i - \frac{1}{2}\sigma_i^2\right)\Delta t\right)^2}{\sigma_i^2 \Delta t}\right) \tag{A6}$$

If draws from this distribution over time are i.i.d, the likelihood function $L = \prod_{t=1}^{T} f(x_{i,t})$ is:

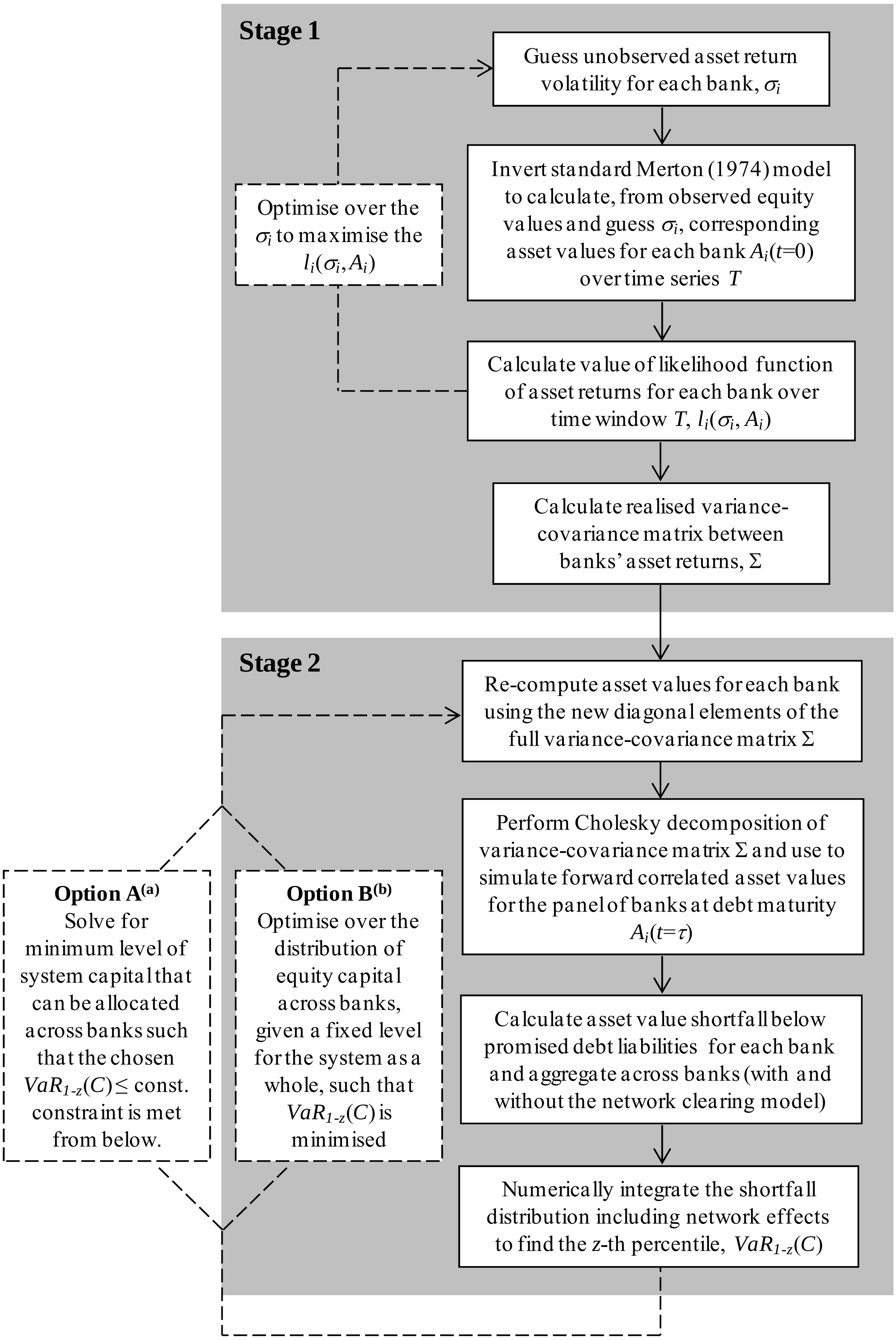
$$L = (2\pi)^{-T/2}(\sigma^2 \Delta t)^{-T/2} \exp\left(-\frac{1}{2\sigma^2 \Delta t}\sum_{t=1}^{T}\left(x_{i,t} - \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t\right)^2\right) \tag{A7}$$

and so the log-likelihood function is:

$$l \equiv \ln L = \ln\left\{(2\pi)^{-\frac{T}{2}}(\sigma^2\Delta t)^{-\frac{T}{2}}\exp\left[-\frac{1}{2\sigma^2\Delta t}\sum_{t=1}^{T}\left(x_{i,t}-\left(\mu-\frac{1}{2}\sigma^2\right)\Delta t\right)^2\right]\right\} \quad \textbf{(A8)}$$

$$= -\frac{1}{2\sigma_i^2\Delta t}\sum_{t=1}^{T}\left(x_{i,t}-\left(\mu-\frac{1}{2}\sigma^2\right)\Delta t\right)^2-\frac{T}{2}\ln(\sigma_i^2\Delta t)-\frac{T}{2}\ln(2\pi) \quad \textbf{(A9)}$$

## Appendix 2: Flow diagram of key steps in the model




(a) Option A is the optimisation investigated in this paper.

(b) Option B is closer in spirit to Gauthier *et al* (2010), but is not investigated in this paper.

## Appendix 3: Merton (1974) structural credit risk model

The standard Merton (1974) model is as follows:

$$E = AN(d_1) - F\exp(-r\tau)N(d_2) \quad \textbf{(A10)}$$

$$d_1 = \frac{ln\frac{A}{F} + \left(r + \frac{1}{2}\sigma_A^2\right)\tau}{\sigma_A\sqrt{\tau}} \quad \textbf{(A11)}$$

$$d_2 = d_1 - \sigma_A\sqrt{\tau} \quad \textbf{(A12)}$$

So given observed values for equity (*E*), debt (*F*), and the risk-free interest rate (*r*), it is possible to obtain the value of assets (*A*) as a function of asset volatility ($\sigma_A$ ).

## Appendix 4: Estimated correlation matrices between banks' asset returns

**2004**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.42 | 0.45 | 0.55 | 0.30 |
| **2** | 0.42 | 1.00 | 0.50 | 0.58 | 0.41 |
| **3** | 0.45 | 0.50 | 1.00 | 0.55 | 0.42 |
| **4** | 0.55 | 0.58 | 0.55 | 1.00 | 0.40 |
| **5** | 0.30 | 0.41 | 0.42 | 0.40 | 1.00 |

**2005**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.68 | 0.53 | 0.50 | 0.43 |
| **2** | 0.68 | 1.00 | 0.61 | 0.58 | 0.59 |
| **3** | 0.53 | 0.61 | 1.00 | 0.57 | 0.48 |
| **4** | 0.50 | 0.58 | 0.57 | 1.00 | 0.41 |
| **5** | 0.43 | 0.59 | 0.48 | 0.41 | 1.00 |

**2006**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.68 | 0.63 | 0.65 | 0.57 |
| **2** | 0.68 | 1.00 | 0.73 | 0.60 | 0.51 |
| **3** | 0.63 | 0.73 | 1.00 | 0.59 | 0.53 |
| **4** | 0.65 | 0.60 | 0.59 | 1.00 | 0.51 |
| **5** | 0.57 | 0.51 | 0.53 | 0.51 | 1.00 |

**2007**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.76 | 0.77 | 0.80 | 0.62 |
| **2** | 0.76 | 1.00 | 0.77 | 0.80 | 0.70 |
| **3** | 0.77 | 0.77 | 1.00 | 0.78 | 0.73 |
| **4** | 0.80 | 0.80 | 0.78 | 1.00 | 0.77 |
| **5** | 0.62 | 0.70 | 0.73 | 0.77 | 1.00 |

**2008**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.48 | 0.59 | 0.59 | 0.29 |
| **2** | 0.48 | 1.00 | 0.57 | 0.62 | 0.57 |
| **3** | 0.59 | 0.57 | 1.00 | 0.49 | 0.33 |
| **4** | 0.59 | 0.62 | 0.49 | 1.00 | 0.45 |
| **5** | 0.29 | 0.57 | 0.33 | 0.45 | 1.00 |

**2009 (interim)**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.61 | 0.56 | 0.51 | 0.43 |
| **2** | 0.61 | 1.00 | 0.60 | 0.59 | 0.55 |
| **3** | 0.56 | 0.60 | 1.00 | 0.51 | 0.40 |
| **4** | 0.51 | 0.59 | 0.51 | 1.00 | 0.43 |
| **5** | 0.43 | 0.55 | 0.40 | 0.43 | 1.00 |

## References

**Acharya, V V, Pedersen, L H, Philippon, T and Richardson, M (2010)**, 'Measuring systemic risk', *mimeo*.

**Admati, A R, DeMarzo, P M, Hellwig, M F and Pfeiderer, P (2010)**, 'Fallacies, irrelevant facts, and myths in the discussion of capital regulation: why bank equity is not expensive', *mimeo*.

**Adrian, T and Brunnermeier, M (2009)**, 'CoVaR', *mimeo*.

**Alessandri, P, Gai, P, Kapadia, S, Mora, N and Puhr, C (2009)**, 'Towards a framework for quantifying systemic stability', *International Journal of Central Banking*, Vol. 5(3), pages 47–81.

**Bank of England (2009)**, *The role of macroprudential policy*, November.

**Brunnermeier, M, Crockett, A, Goodhart, C, Persaud, A D and Shin, H (2009)**, 'The fundamental principles of financial regulation', *Geneva Reports on the World Economy*.

**Cerny, V (1985)**, 'A thermodynamical approach to the travelling salesman problem: an efficient simulation algorithm', *Journal of Optimization Theory and Applications*, Vol. 45, pages 41–51.

**Eisenberg, L and Noe, T H (2001)**, 'Systemic risk in financial systems', *Management Science*, Vol. 47, No. 2, pages 236–49.

**Elsinger, H, Lehar, A and Summer, M (2006)**, 'Using market information for banking system risk assessment', *International Journal of Central Banking*, Vol. 2(1), pages 137–66.

**Gai, P, Jenkinson, N and Kapadia, S (2007)**, 'Systemic risk in modern financial systems: analytics and policy design', *Journal of Risk Finance*, Vol. 8, pages 156–65.

**Gauthier, C, Lehar, A and Souissi, M (2010)**, 'Macroprudential regulation and systemic capital requirements', *Bank of Canada Working Paper No. 2010–4*.

**Haldane, A (2009)**, 'Rethinking the financial network', speech at the Financial Student Association, Amsterdam, April.

**James, C (1991)**, 'The losses realized in bank failures', *Journal of Finance*, Vol. 46(4), pages 1,223–42.

**Jensen, M C (1986)**, 'The agency costs of free cash flow: corporate finance and takeovers', *American Economic Review*, Vol. 76(2), pages 323-29.

**Jensen, M C and Meckling, W H (1976)**, 'The theory of the firm: managerial behavior, agency costs and ownership structure', *Journal of Financial Economics*, Vol. 3, pages 305-60.

**Merton, R (1974)**, 'On the pricing of corporate debt: the risk structure of interest rates', *Journal of Finance*, Vol. 29, pages 449–70.

**Modigliani, F and Miller, M (1958)**, 'The cost of capital, corporation finance and the theory of investment', *American Economic Review*, Vol. 58, pages 261–97.

**Myers, S C and Majluf, N S (1984)**, 'Corporate financing and investment decisions when firms have information that investors do not have', *Journal of Financial Economics*, Vol. 13, pages 187-221.

**Shapley, L S (1953)**, 'A value for n-person games', reprinted in Roth, A E (ed.), *The Shapley value*, Cambridge University Press.

**Tarashev, N, Borio, C and Tsatsaronis, K (2010)**, 'Attributing systemic risk to individual institutions', *Bank for International Settlements Working Paper No. 308*.

**Upper, C (2007)**, 'Using counterfactual simulations to assess the danger of contagion in interbank markets', *Bank for International Settlements Working Paper No. 234*.

**Van Lelyveld, I and Liedorp, F (2006)**, 'Interbank contagion in the Dutch banking sector: a sensitivity analysis', *International Journal of Central Banking*, Vol. 2(2), pages 99-132.

**Wells, S (2004)**, 'Financial interlinkages in the United Kingdom's interbank market and the risk of contagion', *Bank of England Working Paper No. 230*.