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Abstract

A commonly held view is that the life of a monetary policy maker forced to operate under discretion can be improved by the authorities delegating monetary policy objectives that are different from the social welfare function (including interest rate smoothing, price-level targeting and speed-limit objectives). We show that this holds with much less generality than previously realised. The reason is that in monetary policy models with capital accumulation (or similar variables) there may be multiple equilibria under discretion. Delegating modified objectives to the monetary policy maker does not change this. We find that the best equilbria under delegation are sometimes inferior to the worse ones without delegation. In general the welfare benefits of schemes like price-level targeting must be regarded as ambiguous.

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Summary

In the standard monetary policy model, the monetary authorities face a commitment problem that has been termed the ‘stabilisation bias’. When a shock hits that threatens to push up inflation, the policymaker would like to generate the expectation that inflation will be low in the future, because this will help anchor inflation today, and in so doing allow it to tighten policy by less, which itself is beneficial. To generate this expectation of a muted rise in inflation, the policymaker promises that tight policy will be tight not only today, but also tomorrow. However, when the threat to inflation has waned, tight policy is costly to sustain, and it is better to renege at that point. Anticipating this, observers do not believe the promise of tight policy at the outset, inflation expectations rise, and the authority is forced to tighten policy by more today than would have been necessary if its promise had been believed. Such a policymaker is said to operate under discretion. A policymaker that can commit, (that is, is forced by some means not to reconsider its plans when the threat to inflation abates), can achieve inflation control at the expense of much less variability in the real economy. This is because it does not have to tighten policy so much today, and can instead rely on policy being a little tighter today and tomorrow.

It has been claimed that the benefits of this policy can be obtained even in the absence of a commitment if the monetary authority is handed an objective to follow that is modified with respect to the one that society ultimately prefers. A few schemes have been proposed that do this, but the one that has received most attention and is easiest to explain is the price-level target. This target involves replacing the term in inflation that would normally appear in the policymaker’s objective function with deviations of the price level from some target path. This scheme does its job by making the objective that the discretionary policymaker faces tomorrow depend in part on what happened today. If the inflation rate turns out high today, then, in order to meet the price-level target, inflation needs to be correspondingly lower tomorrow. The expectation that this will happen leads people to forecast that inflation will be low, and this mimics the outcome obtained under commitment.

Our paper shows that the benefits from schemes like price-level targeting obtain with much less generality than previously thought. The analysis sketched above was carried out in the simplest possible monetary policy models that abstract from dynamics caused by features like capital

accumulation. In such models, it was correctly assumed that there was only one possible equilibrium when policymakers were assumed to be operating under discretion. However, in the more realistic model that we deploy which features capital accumulation, we invariably find that there is more than one equilibrium. We show that when we introduce the delegation schemes - such as price-level targeting, but including others too - this feature of having more than one equilibrium survives. The significance of this finding is that in our model it is not possible to say whether using a price-level target (or one of the other schemes) would make a discretionary policymaker better off or not. In some cases, the worst equilibria under the delegation schemes are inferior to the best equilibria when the policymaker tries to maximise the original, unmodified objective function. These results hold for all the delegation schemes we study (price-level targeting, hybrid price-level and inflation targeting, interest rate smoothing, and the speed-limit policy, one which ensures policy pays attention to the change, rather than the level in the gap between actual output and potential). The results also hold for two different variants on our model of capital accumulation.

1 Introduction

This paper revisits the claim that the stabilisation bias in monetary policy making - the welfare loss if a central bank is unable to commit - identified by Svensson (1997) - can be resolved by certain delegation schemes. These schemes assign a modified objective to the central bank, that it is assumed are pursued under discretion, but which impart nonetheless an inertial quality to optimal policy that mimics the character of optimal monetary policy under commitment. Schemes proposed so far include: a price-level target (Svensson (1997) and Vestin (2006)), in which the argument in inflation in the typical central bank mandate is replaced by a price level target; a speed-limit policy, in which the term in the output gap is replaced by a term in the change in the output gap (Walsh (2003)); and an interest rate smoothing policy (Woodford (2003b)), under which the normal central bank mandate is augmented with a term in the change in the interest rate. We demonstrate that the welfare benefits that accrue under these schemes obtain with much less generality than previously thought.

The analysis of these potential solutions to the stabilisation bias were conducted – for the sake of clarity and simplicity – in a simple New Keynesian model that abstracts from persistent endogenous state variables, like stocks of debt or capital. This abstraction was made on the grounds that nothing by way of generality was lost by so doing. However, Blake and Kirsanova (2010) demonstrated that in models with endogenous state variables we should in general expect more than one equilibrium if monetary policy is conducted optimally, and under discretion. We study a version of the familiar sticky price model modified to incorporate capital accumulation. Our main contribution is to show that this multiplicity is pervasive under the delegation schemes so far proposed. The implication of this finding is that rather than there being a unique value for the welfare change accruing to a transition to a delegation scheme, there are many such values. And whether there is a gain associated with a delegation scheme will depend on which equilibrium under the original unmodified objective one starts from, and to which equilibrium one transits to. As we show in the paper, it can be the case that the best equilibrium under the delegation scheme entails lower welfare than the worst equilbrium under inflation targeting, so there is no guarantee of a welfare improvement. Importantly, there is nothing in the model itself to explain how equilibrium selection would occur following a delegation. We study two different models of capital accumulation: a variant with a rental market, and a variant with firm-specific capital. Our basic result holds for both, although it is a little less stark in the case of the firm-specific capital model.

The paper is organised as follows. In the next section we outline the model and discuss the calibration. Section 3 recapitulates the analysis in Blake and Kirsanova (2010) in the case of our sticky price model with capital, showing how multiplicity arises when monetary policy is conducted under discretion and under an unmodified monetary policy objective. Section 4 compares and contrasts three delegation regimes: interest rate smoothing, speed-limit policy and inflation targeting. We study the extent to which multiplicity pervades under these delegation schemes. Section 5 assesses welfare under the three delegation schemes for the different equilibria that we find. Section 6 concludes.

2 Model

We use a New Keynesian model with capital accumulation and complete markets, as presented in Sveen and Weinke (2005) and Woodford (2005). This model has monopolistic competition and sticky prices in goods markets. Capital accumulation is assumed to take place at the firm level and the additional capital resulting from an investment decision becomes productive with a one period delay. We assume a convex capital adjustment cost at the firm level. Since the details of the model are discussed in Sveen and Weinke (2005) we proceed directly to the equations that result from linearising the equilibrium conditions around the steady state.

2.1 Linearised equilibrium conditions

We linearise around a zero inflation steady state. All variables are expressed in terms of log deviations from their steady-state values.

From the standard household's optimisation problem we obtain, respectively, an Euler equation and a labour supply equation

$$c_t = \mathcal{E}_t c_{t+1} - \frac{1}{\sigma}(i_t - \mathcal{E}_t \pi_{t+1} - \rho) \quad \textbf{(1)}$$

$$w_t = \phi n_t + \sigma c_t \quad \textbf{(2)}$$

where parameter $\rho = -\log \beta$ is the time discount rate, σ is the household's relative risk aversion or, equivalently, the inverse of the intertemporal elasticity of substitution and ϕ is the inverse of the Frisch labour supply elasticity. We denote the nominal interest rate at time t as $i_t = \log R_t$, and $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$ is inflation. We also denote aggregate consumption as c_t, n_t the aggregate labour supply and w_t the average real wage. $\mathcal{E}_t$ is the expectations operator conditional on information available through time t.

The law of motion of capital is obtained from averaging and aggregating optimal investment decisions on the part of firms. This implies

$$\Delta k_{t+1} = \beta \mathcal{E}_t \Delta k_{t+2} + \frac{1}{\varepsilon_\psi}\left(\left(1-\beta\left(1-\delta\right)\right)\mathcal{E}_t ms_{t+1} - \left(i_t - \mathcal{E}_t \pi_{t+1} - \rho\right)\right) \quad \textbf{(3)}$$

where aggregate capital is denoted by k_t and $ms_t = w_t - k_t + n_t$ measures the average real marginal return to capital. Parameter β is the subjective discount factor, parameter δ is the rate of depreciation and parameter ε_ψ measures the capital adjustment cost at the firm level. The average real marginal return to capital is measured in terms of marginal savings in labour costs since firms are demand-constrained in this model.

The linearised aggregate production function is given by:

$$y_t = \alpha k_t + (1-\alpha) n_t \quad \textbf{(4)}$$

where parameter α denotes the capital share.

The inflation equation takes the standard form:

$$\pi_t = \beta \pi_{t+1} + \kappa mc_t + \eta_t$$

where $mc_t = w_t - y_t + n_t$ denotes the average real marginal cost. If capital can be rented then parameter $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ where parameter θ gives the probability that a firm does not re-optimise its price in an given period. If the rental market does not exist, then $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{(1-\alpha)}{(1-\alpha+\varepsilon\alpha)}\frac{1}{\xi}$ where parameter ε denotes the elasticity of substitution between the differentiated goods, while parameter ξ is a function of the model's structural parameters which is computed numerically using the method developed in Woodford (2005). Finally, and additionally to the setting in Sveen and Weinke (2005), we introduce a cost-push shock to the model. It is common to interpret this shock as a temporary discretionary change in firms' desired margins.

The goods market clearing condition can be written as:

$$y_t = \zeta c_t + \frac{1-\zeta}{\delta}\left(k_{t+1} - (1-\delta) k_t\right) \quad \textbf{(5)}$$

where ζ is the steady-state consumption to output ratio, $\zeta = 1 - \frac{\delta\alpha(\varepsilon-1)}{\varepsilon(\rho+\delta)}$.

2.2 Monetary policy

2.2.1 Social welfare

We assume that the central bank uses the nominal short-term interest rate i_t as an instrument and acts under discretion. We also assume that the social welfare function is well captured by the following discounted quadratic loss function:

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(\pi_s^2+\omega y_s^2\right), \tag{6}$$

where we use $\omega = \kappa/\varepsilon$.[1] This welfare function has been shown by Woodford (2003a), Chapter 6, to approximate the aggregate of individual utility functions in a model without capital, but otherwise identical to the one we work with. In our model, this approximation will not hold up to the second order and so our policy objective function is to some degree *ad hoc*. However, as King and Wolman (2004) and Blake and Kirsanova (2010) argue, multiplicity under discretion is not a consequence of a particularly 'unfortunate' form of social welfare, but rather a general property of discretionary policy, as the private sector and the policymaker make decisions based on forecast of each other's actions.[2] In what follows we simply refer to this objective as *the* social objective. We also label the regime with social policy objective as 'inflation targeting'. Note that we do this for convenience and not to take a stand on the optimality or the precise nature of inflation-targeting regimes as practiced in real life.

2.2.2 Discretion

Our definition of discretionary policy is conventional and is widely used in the monetary policy literature, see eg Backus and Driffill (1986), Oudiz and Sachs (1985), Clarida, Galí and Gertler (1999), and Woodford (2003a). Following Blake and Kirsanova (2010) we define discretionary policy for our model.[3]

[1] This relative weight is given in Woodford (2003a), Chapter 6 as a microfounded weight for the most simple model.

[2] We discuss how our results are affected by the chosen form of objectives later.

[3] To save space we work with a deterministic version in this section. The model is certainty equivalent so adding shocks is straightforward.

We assume that the policymaker knows the law of motion **(1)-(5)** of the aggregate economy when it formulates policy. The policymaker's decision problem is to find the best policy for every period, knowing that future policymakers have the freedom to change policy, and knowing that future policymakers face the same problem.

We assume that the policymaker acts in a discretionary way in the following sense. At every point t in time the private sector observes the policy that reacts only to the current state, so can be written in the form[4]

$$i_t = \imath_k k_t. \tag{7}$$

The private sector expects that the future policymakers will apply the same decision process and will react to the contemporary state only, ie will implement policy **(7)**. We assume that the aggregate decision of the private sector, taken after the policymaker has acted, can be written as the linear feedback function

$$k_{t+1} = k_k k_t \tag{8}$$

$$c_t = c_k k_t \tag{9}$$

$$\pi_t = \pi_k k_t. \tag{10}$$

At any time t, the policymaker reacts to the current state **(7)**, knows that the private sector observes its action, and knows that the private sector expects all future policymakers will apply the same decision process and implement policy **(7)**. Henceforth we shall refer to parameters that define the behaviour of the policymaker and the private sector, k_k, c_k, π_k and $\imath_k$, as 'decisions'.

We lead **(8)-(10)** one period and use **(1)-(5)** to write private sector decisions as a response to the state and response to policy

$$k_{t+1} = k_S(k_k, c_k, \pi_k) k_t + k_P(k_k, c_k, \pi_k) i_t, \tag{11}$$

$$\pi_t = \pi_S(k_k, c_k, \pi_k) k_t + \pi_P(k_k, c_k, \pi_k) i_t, \tag{12}$$

$$c_t = \tilde{c}_S(k_k, c_k, \pi_k) k_t + c_P(k_k, c_k, \pi_k) i_t, \tag{13}$$

where we use the subscripts S for state and P for policy. Written this way, **(11)-(13)** isolate the 'instantaneous' influence of policy on private sector decisions. We report the exact form of the coefficients in the appendix.

[4] We restrict ourselves to the 'memoryless' or Markov equilibria, where agents' decisions are functions of the current state only. We also assume a linear contemporaneous relationship.

We now complete the definition of discretion. Policy determined by **(7)** is discretionary if the policymaker finds it optimal to continue to follow it in every period $s > t$, given the private sector (i) knows that in every period $s > t$ future policymakers re-optimise and use the same decision process, (ii) observes the current policy, (iii) anticipates policy **(7)** to be implemented in all future periods.[5]

We can write the criterion for optimality, the Bellman equation, as

$$\begin{aligned} Sk_t^2 &= \min_{i_t} \left((\pi_S k_t + \pi_P i_t)^2 + \omega \left(\left(\zeta c_S + \frac{1-\zeta}{\delta} (k_S - (1-\delta)) \right) k_t \right.\right. \\ &\left.\left. + \left(\zeta c_P + \frac{1-\zeta}{\delta} k_P \right) i_t \right)^2 + \beta \tilde{S} (k_S k_t + k_P i_t)^2 \right), \end{aligned} \quad \textbf{(14)}$$

where we take the intraperiod leadership of the policymaker into account by substituting in constraints **(11)-(13)**. Differentiation of **(14)** with respect to i_t yields the optimal policy response

$$i_t = -\frac{\left(\pi_P \pi_S + \left(\zeta c_P + \frac{1-\zeta}{\delta} k_P\right) \omega \left(\zeta c_S + \frac{1-\zeta}{\delta} (k_S - (1-\delta))\right) + \beta S k_P k_S\right)}{\left(\pi_P^2 + \omega \left(\zeta c_P + \frac{1-\zeta}{\delta} \tilde{k}_P\right)^2 + \beta S k_P^2\right)} k_t = \imath_k k_t. \quad \textbf{(15)}$$

The coefficient $\imath_k$ in **(15)** determines the optimal policy feedback on the predetermined state, k_t, with the feedback coefficient a function of S. In order to find the equilibrium value function S we substitute the optimal solution **(15)** into the Bellman equation **(14)** and obtain the following quadratic equation with positive leading coefficient and a negative constant term

$$\beta S^2 + \mu S - \omega \left(\frac{(1-\zeta)}{\delta} \left(\pi_S - (k_S - (1-\delta)) \frac{\pi_P}{k_P} \right) + \zeta \frac{(\pi_S c_P - \pi_P c_S)}{k_P} \right)^2 = 0,$$

where coefficient $\mu = \mu(k_k, c_k, \pi_k)$ is given in the appendix. This equation has only one nonnegative solution for S.

In order to obtain the optimal policy we substitute S into **(15)** to give

$$\imath_k = \imath_k (k_k, c_k, \pi_k). \quad \textbf{(16)}$$

By construction, for every triplet $\{k_k, c_k, \pi_k\}$ that describes a time-invariant private sector response we obtain a *unique* $\imath_k$ that describes the policy decision.

[5] In the language of game theory we restrict our attention to time-consistent feedback equilibria with intraperiod leadership, see eg de Zeeuw and van der Ploeg (1991), Oudiz and Sachs (1985) and Cohen and Michel (1988). Here and below we simply call such equilibria as 'discretionary'.

We substitute equation **(7)** into **(11)-(13)** and, after some manipulations, obtain the following system that describes time-invariant optimal response of the private sector

$$k_k = \frac{1}{\beta\nu_k + \lambda_k\nu_r}\left(\left(\beta\left(1-\nu_o k_k\right)+\lambda_o\nu_r\right)k_k - \nu_r\pi_k + \left(\lambda_c\nu_r - \beta\nu_c\right)c_k + \beta\nu_r\iota_k\right), \quad \textbf{(17)}$$

$$\pi_k = \left(\beta\pi_k + \lambda_o\right)k_k + \lambda_c c_k - \lambda_k, \quad \textbf{(18)}$$

$$c_k = \frac{1}{\beta + \sigma\lambda_c}\left(\left(\beta c_k - \sigma\lambda_o\right)k_k + \sigma\pi_k + \sigma\left(\lambda_k - \beta\iota_k\right)\right). \quad \textbf{(19)}$$

where all coefficients ν are given in the appendix.

The set of coefficients $\mathcal{D} = \{\iota_k, k_k, c_k, \pi_k\}$ describes the solution to the discretionary optimisation problem outlined above. It uniquely defines the trajectories $\{i_t, k_t, \pi_t, c_t\}_{t=0}^{\infty}$ for any given $k_0 = \bar{k}$. Conversely, if the sequence $\{i_t, k_t, \pi_t, c_t\}_{t=0}^{\infty}$ solves the discretionary policy outlined above then there is a unique set $\mathcal{D} = \{\iota_k, k_k, c_k, \pi_k\}$ that satisfies **(7)**, **(11)-(13)** and **(14)**. We call the set of coefficients $\mathcal{D} = \{\iota_k, k_k, c_k, \pi_k\}$ a *discretionary equilibrium*.

2.2.3 Delegation regimes

Our paper is a reassessment of the gains associated with various delegation schemes for monetary policy, so we need to make these regimes concrete. They are associated with particular penalty functions assigned to the central bank that differ from the benchmark welfare function. These schemes are all nested within the following objective function:

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(\omega_\pi\pi_s^2 + \omega_y y_s^2 + \omega_p p_s^2 + \omega_{sl}\Delta y_s^2 + \omega_i\left(\Delta i_s\right)^2\right). \quad \textbf{(20)}$$

Using this objective function, we can implement four modifications to inflation targeting by the following choice of the weights ω:

Delegation scheme	Constraints	
Strict price-level targeting	$\omega_\pi = \omega_{sl} = \omega_i = 0$	$\omega_p > 0, \omega_y = \kappa/\varepsilon$
Hybrid price-level and inflation targeting	$\omega_{sl} = \omega_i = 0$	$\omega_p > 0, \omega_\pi = 1, \omega_y = \kappa/\varepsilon$
Interest rate smoothing	$\omega_{sl} = \omega_p = 0$	$\omega_y = \kappa/\varepsilon, \omega_\pi = 1, \omega_i > 0$
Speed-limit policy	$\omega_i = \omega_p = \omega_y = 0$	$\omega_\pi = 1, \omega_{sl} > 0$

To put this into words, briefly: price-level targeting, the focus of Svensson (1999) and Vestin (2006), is implemented by replacing the term in the inflation rate in the social welfare function by a term in the price level. Hybrid price-level and inflation targeting, whose optimality under discretion was studied by Roisland (2006), is implemented by *adding* to the social welfare function a term in the price level. Interest rate smoothing is implemented by *adding* to the social welfare function a term in the change in the interest rate. And finally, the speed-limit policy, proposed by Walsh (2003), is achieved by *replacing* the term in the level of the output gap in the social welfare function with a term in the change in the output gap.

2.3 Calibration

We use the same calibration as in Sveen and Weinke (2005). We set the capital share $\alpha = 0.36$. Our choice for the risk aversion parameter σ is 2, and a unit elasticity of labour supply is assumed, $\phi = 1$. The elasticity of substitution between goods, ε, is set to 11. The rate of capital depreciation, δ, is assumed to be 0.025 and we set $\varepsilon_{\psi} = 3$. Finally, our value for the Calvo price stickiness parameter, θ, is 0.75. This implies a mean duration for a particular price of 3 quarters.

3 Multiple discretionary equilibria under inflation targeting

Blake and Kirsanova (2010) solve system **(16)-(19)** and find three discretionary equilibria under inflation targeting. It is straightforward to demonstrate that only two of them are Iterative Expectations (IE)-stable under joint eductive learning; see Dennis and Kirsanova (2009).[6] In what follows we shall only consider IE-stable equilibria under joint learning.[7]

The existence of multiple equilibria means that, following a shock, the economy can follow one of several transition paths, each of which satisfies the conditions for time-consistency. Chart 1 shows the two transition paths that correspond to the two IE-stable equilibria. The solid and the dashed lines denote responses to a unit cost-push shock in the two equilibria. In both equilibria the interest rate rises in response to a positive cost-push shock, but the amount by which interest rate rises is substantially greater in one than the other. As a consequence, in one equilibrium we see a larger fall in the output gap and a smaller rise in inflation than in the other equilibrium. We call the first equilibrium 'seemingly dry' and the second 'seemingly wet', as it looks like the

[6] Standard iterative algorithms, routinely used in the literature (Oudiz and Sachs (1985), Backus and Driffill (1986), Currie and Levine (1993) and Söderlind (1999)), will discover these equilibria.

[7] We assume that such equilibria are equally plausible. Future research will have to establish if any other reasonable criteria can unambiguously select the best equilibrium.

central bank has greater determination to combat the cost-push shock in the first equilibrium. Chart 1 also suggests that the seemingly dry policymaker implements the nearly optimal solution as its actions are more similar to the ones under commitment than the actions of the seemingly wet policymaker. For brevity we shall refer to these equilibria as to just ‘dry’ and ‘wet’ correspondingly, but we bear in mind that these two equilibria are generated by the same policy objective.

In order to understand the multiplicity recall that firms choose current-period prices based on marginal cost. The marginal cost can be written as:

$$mc_t = \left(\zeta\left(\frac{(\phi+1)}{(1-\alpha)}-1\right)+\sigma\right)c_t + \frac{(1-\zeta)}{\delta}\left(\frac{(\phi+1)}{(1-\alpha)}-1\right)\left(k_{t+1}-(1-\delta)\,k_t\right) - \alpha\frac{(\phi+1)}{(1-\alpha)}k_t.$$

It is apparent that for a given interest rate policy higher consumption raises inflation but it also makes profit optimising firms increase next-period capital stock in order to meet anticipated increased demand, and higher next-period capital raises current-period inflation too. The decisions to raise consumption and to increase the next-period capital stock are dynamic complements as defined, for example, in Cooper and John (1988). Multiplicity of the policy-induced private sector equilibria becomes a likely outcome: the private sector may choose to react in several possible ways – here they are ‘slow’ and ‘fast’ – each of which is consistent with a corresponding policy forecast.[8] Of course, the policymaker will react differently in response to different private sector actions, but it will find it optimal to react either slow or fast, consistent with its rational beliefs about the beliefs of the private sector. We end up with two discretionary equilibria where the policymaker validates beliefs in each particular equilibrium. In presence of two equilibria co-ordination failure happens: the agents can co-ordinate on any of the two. An exogenous event may decide which one will realise.

More generally, the presence of capital accumulation in the model necessarily implies sluggish adjustment of the economy back to the steady state. In contrast, if we did not include the capital accumulation process into the model, all adjustment would happen within a single time period. No different beliefs about the different speed of adjustment would be possible and no multiple equilibria would arise.[9] When sluggish adjustment becomes possible, the conventional requirement of eventual convergence to the steady state does not determine the speed of

[8] See King and Wolman (2004) and Blake and Kirsanova (2010) for a discussion of policy-induced private sector equilibria.

[9] This is a rather general fact, see eg Proposition 1 in Blake and Kirsanova (2010), which demonstrates that a model without endogenous predetermined state variables can only have one discretionary equilibrium.

convergence. Several paths become possible and co-ordination failure happens: the agents can co-ordinate on any of several possible paths. Any random exogenous event can decide which equilibrium will realise.

Before we proceed to our analysis of how delegation schemes work, we demonstrate how the multiplicity depends on parameters that define the capital adjustment process. We explore this aspect of the model because it is precisely capital accumulation that differentiates this model from previous studies in which the authors assumed, rightly, that there was a unique equilibrium under discretion.

The top row of pictures in Charts 2, 3 and 4 are identical and demonstrate areas of multiplicity for the model with rental market for capital under the inflation-targeting regime. The 'x' in each chart marks the spot of our baseline calibration. The top left chart fixes the adjustment costs parameter at its baseline value and varies the depreciation rate and the capital share. The other two charts in this row fix first the depreciation rate, then the capital share, allowing the other two to vary. The shaded regions in the charts are the regions of unique dry equilibrium. It is apparent that multiplicity is widespread and we are likely to have a unique dry equilibrium only if parameters of interest are pushed to their extreme and unrealistic values.

In the model with firm-specific capital, there is also a large area in which there is a unique wet equilibrium (the results are plotted in the top row of the bottom half of Charts 2-4). The model with firm-specific capital behaves as the model with rental markets but with greater price stickiness, see Sveen and Weinke (2005) and Woodford (2005). With greater price stickiness it becomes difficult to control the economy in a dry way for large enough capital share, α. If α is not close to one but relatively large then an efficient control of inflation in the dry equilibrium requires more aggressive monetary policy. But a higher capital share requires less aggressive interest rate movements as they cause bigger investment and consumption movements and so stabilisation of capital stock may become problematic. These two requirements conflict with each other. At some large value of α the seemingly dry solution does not exist. It is reinstated if α becomes even bigger, but still not very close to one. The effect of α on coefficient on marginal cost in the Phillips curve, κ, is non-linear and the need to raise interest rate to control inflation in a dry way becomes smaller with very large α. The conflict lessens and two equilibria reappear.

The baseline calibration of the model with firm-specific capital places the economy into the area of unique wet equilibrium, but in its boundary zone, which means that a slight change of parameters is likely to move the economy into the area of multiplicity.

Out of these results arises the central question of this paper: how do delegation schemes – like speed-limit policies, interest rate penalties, or price-level targeting – affect the likelihood of obtaining multiple equilibria? If multiplicity survives – and, as we have already stated, it does – how to quantify the welfare implications of choosing one of these delegation schemes?

4 Pervasive multiple equilibria under policy delegation

In this section, we present our findings about the equilibria that arise under the three policy delegation schemes. We shall use the inflation-targeting regime as the benchmark case. Because of the similarities in the results, we consider together interest rate smoothing and speed-limit targeting regimes on the one hand, and then hybrid and strict versions of price-level targeting on the other.

We classify the equilibria that arise as follows. Denote the array of parameters that characterise a delegation scheme as $\boldsymbol{\omega} = \left(\omega_\pi, \omega_y, \omega_p, \omega_{sl}, \omega_i\right)$. The inflation-targeting regime we denote as $\boldsymbol{\omega}_0 = (1, \kappa/\varepsilon, 0, 0, 0)$. Any discretionary equilibrium can be characterised by policy functions and responses of the private sector. We denote this set of equilibrium reactions – which uniquely corresponds to a path of adjustment of the economy following a shock – as $\mathcal{D}$, see Section 2.2.2. Of course, $\mathcal{D}$ is a function of parameters of the system that include policy parameters $\boldsymbol{\omega}$. Suppose we discover an equilibrium $\mathcal{D}(\boldsymbol{\omega})$. We say that $\mathcal{D}$ is either a wet or dry discretionary equilibrium, if $\lim_{\boldsymbol{\omega} \to \boldsymbol{\omega}_0} \mathcal{D}(\boldsymbol{\omega})$ is either a wet or dry equilibrium under the inflation-targeting regime.

4.1 Interest rate smoothing and speed-limit policy

For the model with rental capital, the top rows of figures in Charts 2 and in 3 show a large region of multiplicity and the baseline calibration under inflation targeting places the economy in the middle of it.[10] This large region is preserved when we implement either an interest rate smoothing policy or a speed-limit policy, which can be seen from the second row of figures in

[10]We plot these charts assuming policy weights $\boldsymbol{\omega}$ such that they maximise welfare for the baseline calibration in the best equilibrium.

Charts 2 and 3. The equilibria that we find for these delegation regimes are the dry and wet equilibria discussed in the previous section. Under inflation targeting, for firm-specific capital, the baseline calibration puts the economy on the boundary between the area of a single wet equilibrium and multiple equilibria: delegation of either preserves this (in the case of an interest rate smoothing policy) or tips the economy firmly into the multiple equilibria region (in the case of the speed-limit policy).

Thus far, delegation would not seem to have conferred any advantages. But this is not the whole story. For the model with firm-specific capital (lower parts of Charts 2 and in 3) both delegation regimes shrink the area of a unique wet equilibrium substantially. A requirement of smooth policy, either with an explicit requirement to operate interest rate smoothly, or with requirement to avoid large changes in output and thus all interrelated economic variables, lessens the conflict between the control of inflation and ensuring stability of capital stock. This can properly be regarded as a potential benefit of delegation. In a similar vein, notice that for the interest rate smoothing policy, delegation increases a little the region of the parameter space for which there is a unique dry equilibrium. This could likewise be regarded as a benefit. Taken together these findings suggest that the chance of finding oneself in a uniquely dry equilibrium is improved by delegation, and the chance of finding oneself in a uniquely wet one is reduced.

4.2 Hybrid and strict price-level targeting

4.2.1 Hybrid price-level targeting (HPLT)

There are two broad points we make about our results under HPLT. The first is to note that this regime does not get rid of the problem of multiplicity under discretion. The second is that new equilibria are created by this scheme, which we will describe below.

Beginning with the pervasiveness of multiplicity under HPLT, the second row of figures in the top part of Chart 4 shows that the economy with rental market for capital is likely to be in the large area of multiplicity, an area that is slightly larger even than the one that obtains under inflation targeting. The same holds for the model with firm-specific capital.

Turning to the effect that HPLT has on the qualitative nature of the different equilibria, there are important differences in properties of the two equilibria we find here. We find that the dry equilibrium is related in the way discussed earlier to the dry equilbrium under inflation targeting, namely that $\lim_{\boldsymbol{\omega}\to\boldsymbol{\omega}_0} \mathcal{D}(\boldsymbol{\omega})$ *is* the dry equilibrium under inflation targeting. However, the second equilibrium, that is discovered under the hybrid price-level targeting, does not exist for inflation targeting with $\omega_p = 0$. We call this second equilibrium 'passive', as it is characterised by a fall in interest rates in response to a positive cost-push shock.[11]

This result shows, importantly, that *new* equilibria can arise under delegation policies. This serves to underline the theme of this paper, namely, the unpredictability of the effect of delegation.

In order to understand this result recall again that the stationarity of the price level under any degree of price-level targeting ($\omega_p > 0$) requires inflation overshooting when it converges back to the steady state. It is possible to achieve inflation overshooting in two ways. A policymaker operating under commitment raises the interest rate and keeps it high for longer to ensure negative marginal cost, ie below its steady-state level. Negative marginal cost means inflation should *rise* while converging to the steady-state zero level. Following a cost-push shock and an interest rate rise inflation falls in the first period, overshoots the zero level and then converges to the steady state from below, rising. In the dry equilibrium the policymaker tries to repeat this policy, but under a time-consistency constraint. Similarly to the commitment case, marginal cost is kept below zero for most of the periods, inflation overshoots the zero steady-state level and the price level is stationary.

However, the policymaker can keep marginal cost below zero by keeping capital stock sufficiently high for some time. This is achieved by lowering interest rate sharply in the response to a cost-push shock, see the dashed line scenario in the first column of plots in Chart 5. Following a sharp fall in the interest rate and thus a negative real interest rate, future consumption falls below its steady-state level. An initial rise in investment leads to higher stock of capital one period later. As all its components fall, the marginal cost also falls below the steady-state level one period after the shock; it also stays below for several consequent periods

[11] Although we compute all limits numerically, and it is difficult to argue discontinuity because of this, we shall see that there are striking differences between the properties of 'wet' equilibrium with $\omega_p = 0$ and the 'passive' equilibrium with $\omega_p = 10^{-14}$.

and this ensures inflation overshooting and stationarity of the price level. Stabilisation of the economy and, thus, the capital stock back to the steady-state level requires a small disinvestment over a long period. Consumption also stays below the steady-state level but rises as interest rates remain low. All this ensures inflation remains negative for a long time while the economy adjusts towards the steady state.

The second column of charts demonstrates that as ω_p becomes smaller then there is less need to bring the price level back to the steady state. Additionally, controlling inflation variability gains the priority. Therefore, monetary policy wants to bring the price level down slowly. This, however, is impossible to do smoothly, under a time-consistent policy. For any given $\omega_p > 0$ monetary policy still has to ensure inflation overshooting. But if inflation stays below zero for a long time, as we have seen for $\omega_p = 0.5$, then the price level falls too quickly for a small penalty ω_p and inflation cost dominates the loss. So, inflation might need to rise quicker and even to return to the positive area again. To achieve this, capital cannot *stay* high; it should fall to increase marginal costs, and this would allow inflation to rise. The interest rate has to go *down* to allow this increase in marginal cost. The second column of plots in Chart 5 suggests that when ω_p becomes smaller ($\omega_p = 0.002$) all variables have to change direction of movements three times (up-down-up) before they monotonically convergence to the steady state. Further reduction in penalty ω_p requires a 'zig-zag' dynamics for all economic variables. This ensures slow convergence of price level and (relatively) small inflation cost.[12]

For the baseline calibration the passive solution does not exist if ω_p is close to zero. With a near unit-root dynamic process for the price level, inflation 'zig-zags' should be nearly symmetric with respect to zero inflation line, but this cannot be achieved in an economy with investment and a positive depreciation rate. (If we reduce the depreciation rate then this solution survives for smaller penalties ω_p.) The smaller effect of marginal cost on inflation also reduces the problem: in the model with firm-specific capital for our baseline calibration the passive equilibrium survives for extremely small values $\omega_p > 0$, as we checked numerically. When $\omega_p < 10^{-14}$ then the passive equilibrium disappears but the wet equilibrium is reinstated.

This explains the existence of the dry and passive equilibria. The wet equilibrium, however, cannot exist for any $\omega_p > 0$ (numerically the threshold is 10^{-14}). The reason for this is again the

[12] Neither Batini and Yates (2003) nor Roisland (2006) study this conflict of targets.

need of inflation overshooting. If $\omega_p = 0$ then the seemingly wet policymaker initially rises interest rate in order to lower it sharply in the consequent period so that the resulting higher investment corrects capital to the steady state quickly. Moreover, it lowers the interest rate by more than the dry policymaker does. Under this regime there is no additional requirement of controlling an additional stock variable, the price level. With an additional requirement to stabilise the price level, the second-period reduction in interest rate is not helpful: it does not generate inflation overshooting and so does not lead to the stationarity of the price level. That is why any small $\omega_p > 0$ that would, by continuity, lead to smaller fall in interest rate in the second and consequent periods, would not correspond to any price-stationary equilibrium.

4.2.2 Strict price-level targeting (PLT)

We also investigate not only hybrid price-level targeting, but its pure form, strict price-level targeting (PLT) where we reduce the relative weight on inflation, ω_π, to zero as was originally proposed by Vestin (2006). The results for this scheme are very similar to those for HPLT. Namely: multiplicity survives under delegation, and for both the rental and firm-specific models of capital; the wet equilibrium is replaced by a different 'passive' equilibrium as explained above. One very slight difference in this case is that the region of the parameter space for which we find a single dry equilibrium is shrunk a little further.

5 Welfare disparities across different equilibria

The existence of multiple equilibria under inflation targeting and under delegation would not matter if they did not involve substantive differences in welfare. As our chart of impulse responses under inflation targeting revealed, however, the different equilibria may imply very different variabilities in inflation and the output gap, so it will be no surprise to find that multiplicity does matter for welfare.

Our next set of results quantify welfare under the different equilibria we find for particular parameterisations of our inflation-targeting and delegation regimes. For each we look at the two different models of capital accumulation: the rental market, and the firm-specific capital model. These results are collected in Table A below.

	Rental capital			Firm-specific capital		
Interest Rate Smoothing	$\omega_i = 0$	$\omega_i^* = 0.01$	$\bar{\omega}_i = 99.8$	$\omega_i = 0$	$\omega_i^* = 0.01$	$\bar{\omega}_i = 26.1$
Dry	1.126	1.106	3.151	–	1.2515	1.7961
Wet	7.523	7.514	6.863	2.3961	2.3880	2.2640
Speed-Limit Policy	$\omega_y = \omega_y^s$ $\omega_{sl} = 0$	$\omega_y = 0$ $\omega_{sl}^* = 0.005$	$\omega_y = 0$ $\bar{\omega}_{sl} = 6.1$	$\omega_y = \omega_y^s$ $\omega_{sl} = 0$	$\omega_y = 0$ $\omega_{sl}^* = 0.001$	$\omega_y = 0$ $\bar{\omega}_{sl} = 0.243$
Dry	1.126	1.020	6.809	–	1.045	2.170
Wet	7.523	7.517	7.331	2.3961	2.392	2.336
Hybrid Price-Level Targeting	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 1$ $\omega_p^* = 0.5$	$\omega_\pi = 1$ $\underline{\omega}_p = 0.0013$	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 1$ $\omega_p^* = 0.5$	$\omega_\pi = 1$ $\underline{\omega}_p = 10^{-14}$
Dry	1.126	1.052	1.116	–	1.0917	–
Wet	7.523	–	–	2.3961	–	–
Passive	–	11.95	1053.7	–	3.984	152.7
Strict Price Level Targeting	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 0$ $\omega_p^* = 1.7$	$\omega_\pi = 0$ $\underline{\omega}_p = 0.023$	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 0$ $\omega_p^* = 1.2$	$\omega_\pi = 0$ $\underline{\omega}_p = 10^{-14}$
Dry	1.126	1.00007	4.079	–	1.0002	–
Wet	7.523	–	–	2.3961	–	–
Passive	–	16.789	10.90	–	5.0613	4.424

Table A: Social loss of delegation regimes relative to that under commitment of benevolent policymaker

The figures in this table express social losses as a ratio to the figures obtained under the commitment solution of a benevolent policymaker **(6)**.

For each regime we give three sets of results. In the first column we have welfare values for the regime of inflation targeting under discretion. If an equilibrium does not exist for a particular set of parameters we label it with a dash. For example, the wet equilibrium is the only equilibrium for the baseline calibration for the model with firm-specific capital. Also, the passive equilibrium does not exist either under inflation targeting or for the interest rate smoothing or the speed-limit policies. In the second column we give the optimal weight on the corresponding policy objective, under which the loss in the best equilibrium is minimal. We label such weight with an asterisk. The third column contains one other representative result, which is only for comparison of welfare values. Typically we choose an extreme value of parameter of interest. If this parameter is close to a lower boundary of the set of possible parameters, we label it with a line underneath; if an upper boundary, we label it with line over the parameter.

The results here prompt us to ask whether, despite the presence of multiple equilibria under discretion, we could still establish that popular delegation schemes would improve welfare. Suppose we began in the dry equilibrium under the benevolent policy, and we were to impose a delegated interest rate smoothing policy with weight ω_i^*, we would anticipate moving to the 'corresponding' dry equilibrium under the delegation policy. This would generate a small welfare gain by cutting losses from 1.126 to 1.106, relative to the optimum of 1. Analogously, we might be tempted to reason that if we began in the wet equilibrium under the benevolent policy, we would shift to the corresponding wet equilibrium under the interest rate smoothing policy, cutting losses from 7.523 to 7.514. The same welfare gains might be apparent from inspecting the corresponding pairs of welfare outcomes for the wet and dry equilibria under the speed-limit and price-level targeting policies.

However, this reasoning is hazardous. First, although by arguments of continuity these pairs of equilibria are related, when we announce a new delegation policy it is a regime change, so we cannot rule out that we might start from a wet equilibrium and move to a dry under delegation, or *vice versa*. As discretionary equilibria are Markov-perfect, transitions do not depend on past states and switches between equilibria may rather be governed by exogenous events than policy announcements. The economy could jump from the dry equilibrium to wet equilibrium even though this would be undesirable for the policymaker. So no gains may arise from the delegation. The wet equilibrium under benevolent policy gives lower welfare than the wet equilibrium for the interest rate smoothing policy; similarly for the dry equilibrium. But it does not follow that we can assert that interest rate smoothing improves welfare. To scrutinise this more closely would require taking a stand on how private agents would process the news about the new regime: how they would learn about its existence and durability.

Similarly, and as we have already noted, in the case of strict and hybrid price-level targeting, we can, by varying the appropriate penalty, cause the wet equilibrium to disappear and a passive one to appear in its place. For such choices of penalty functions it is not possible to jump from one wet equilibrium to another.

If we accept that it is possible to jump from a dry equilibrium under inflation targeting to a wet or a passive equilibrium under delegation, then we must admit the possibility that delegation *worsens* welfare. Table A shows that in every case, the wet equilibria under delegation give worse welfare outcomes than the dry equilibrium under inflation targeting, this holding for both models of capital accumulation.

Table A documents another finding that bears on our central question of how delegation affects welfare. Each delegation scheme involves a spectrum of possible regimes defined by different values for the corresponding penalty parameter: in the case of the speed limit policy the different regimes correspond to different values for ω_y; for the interest rate smoothing regime we have a spectrum of regimes corresponding to different values for ω_i; and for the hybrid and strict price-level targeting regimes the possibilities are spanned by values for ω_π and ω_p. Some regimes in these spaces give better outcomes than others. There are optimal values for the penalty parameters for each regime. What we find is that these optima are different for the dry and wet/passive equilibria. For example: take price-level targeting for our rental capital model, (bottom left section of Table A). Conditional on finding oneself in a dry regime, the optimal value for ω_p in a strict price-level target is 1.7. However, conditional on ending up in a passive regime, this value can be improved upon by setting $\omega_p = 0.023$. What this shows is that the nature of the optimal regime depends on which kind of equilibrium the economy will finish up in.

We have taken the liberty in this paper of talking about 'welfare' when we have been using what is, strictly, an *ad hoc* loss function for this model with capital accumulation. To check whether our results were robust to assuming other loss functions, we ran several experiments with different policy objectives. We kept the order of magnitude ω the same as in the baseline case, but modified ωy_t^2 term into $\omega_y y_t^2 + \omega_c \zeta^2 c_t^2 + \omega_i \left(k_{t+1} - (1-\delta) k_t\right)^2$, as either one might expect the social welfare metric to look like, or if one just wants to pin down consumption and investment separately. We then varied ω_y, ω_c and ω_i between zero and some numbers of order κ/ε. There were only negligibly small quantitative changes to our simulations. The existence and pervasiveness of multiple equilibria and the relative performance of different delegation schemes survives.

6 Summary of results and conclusions

This paper re-examined results that speed-limit, interest rate smoothing or price-level targeting delegation schemes can mimic inflation targeting under commitment and in this sense reduce the stabilisation bias in monetary policy models. Those previous results were derived using models in which there were no endogenous state variables, and in which there were guaranteed to be unique equilibria under discretion. This paper uses a more general setting of a conventional New Keynesian model with capital accumulation, and which generates multiple equilibria under discretionary policy. We showed that multiplicity survives under the most common delegation schemes. This was broadly true for both the rental and firm-specific capital model that we studied, although the results were more clear-cut for the former. We also showed that qualitatively new equilibria can arise under delegation, which can be worse in welfare terms than the worst equilibrium under inflation targeting. We deduce therefore that the conclusion that delegation improves life for the discretionary policymaker holds with much less generality than previously noted, and in fact should be regarded as a special case applying to the least realistic of monetary policy models. In particular, it does not hold for a small but realistic modification to the basic New Keynesian framework to incorporate capital accumulation. We infer that the welfare consequences of delegation are ambiguous, since in order to quantify the benefits one needs to know from which of the multiple equilibria the economy starts under inflation targeting and to which it will move under delegation. The worst equilibria under delegation are inferior to the best equilibria under inflation targeting. And there is nothing in the model to rule out a welfare reducing transition if a delegation scheme is implemented. Although we have focused on adding capital as an endogenous state variable, note that it is highly likely that these results would obtain if instead or in addition we included other similar variables like debt.

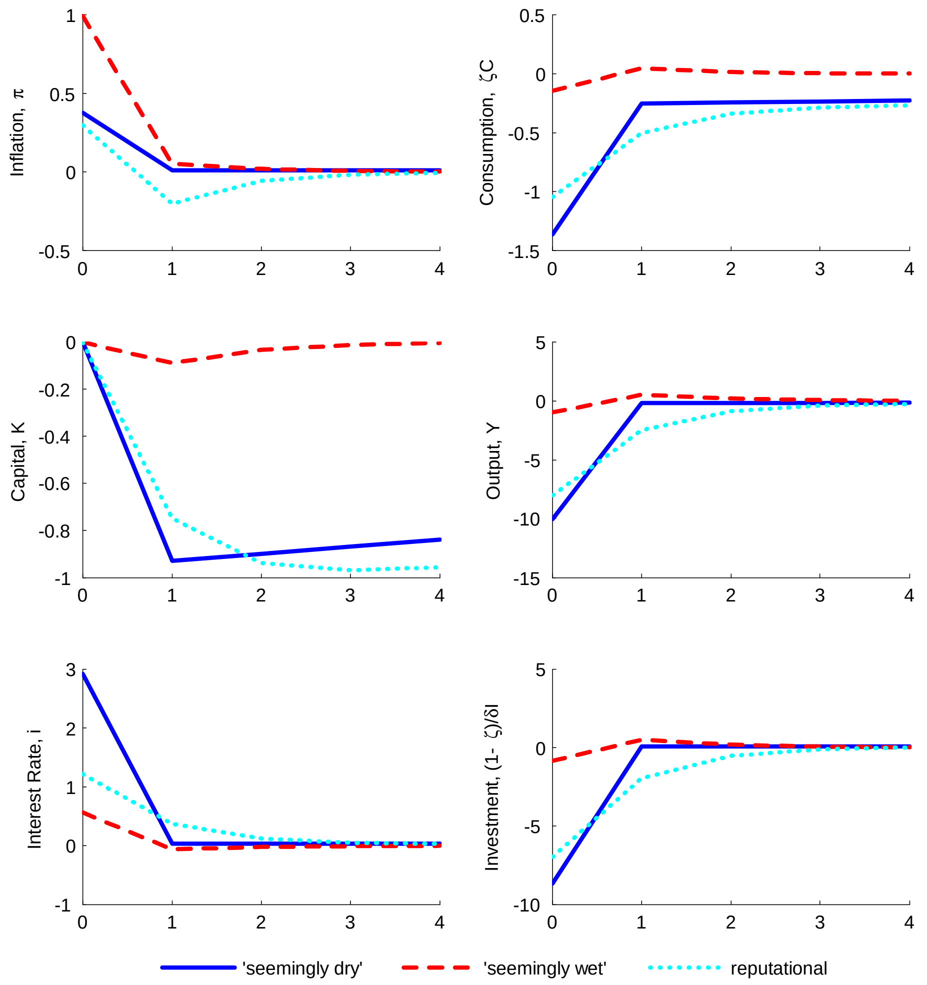

Chart 1: Impulse responses to a unit cost-push shock under three different regimes

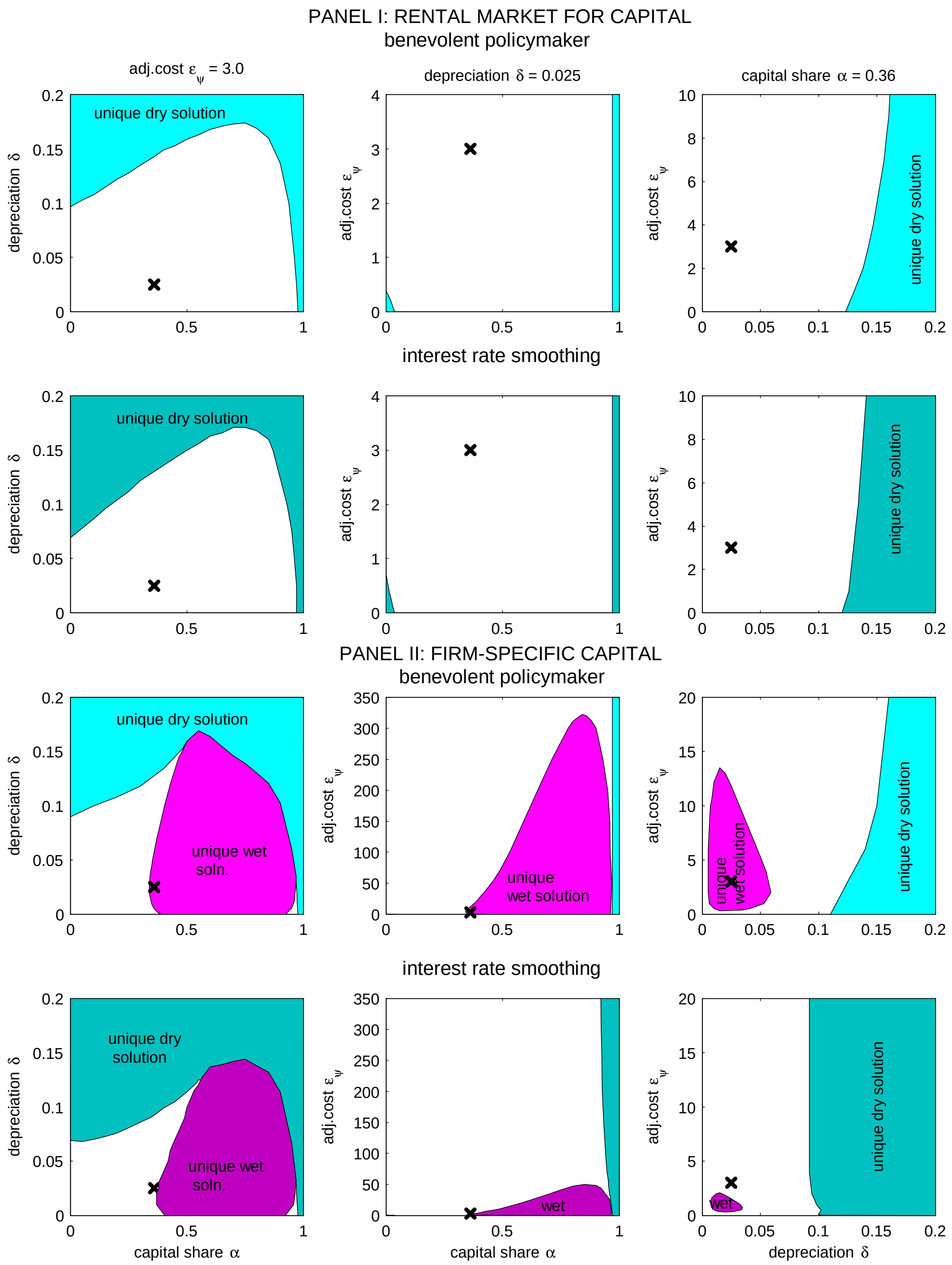

Chart 2: Regions of multiplicity for interest rate smoothing policy

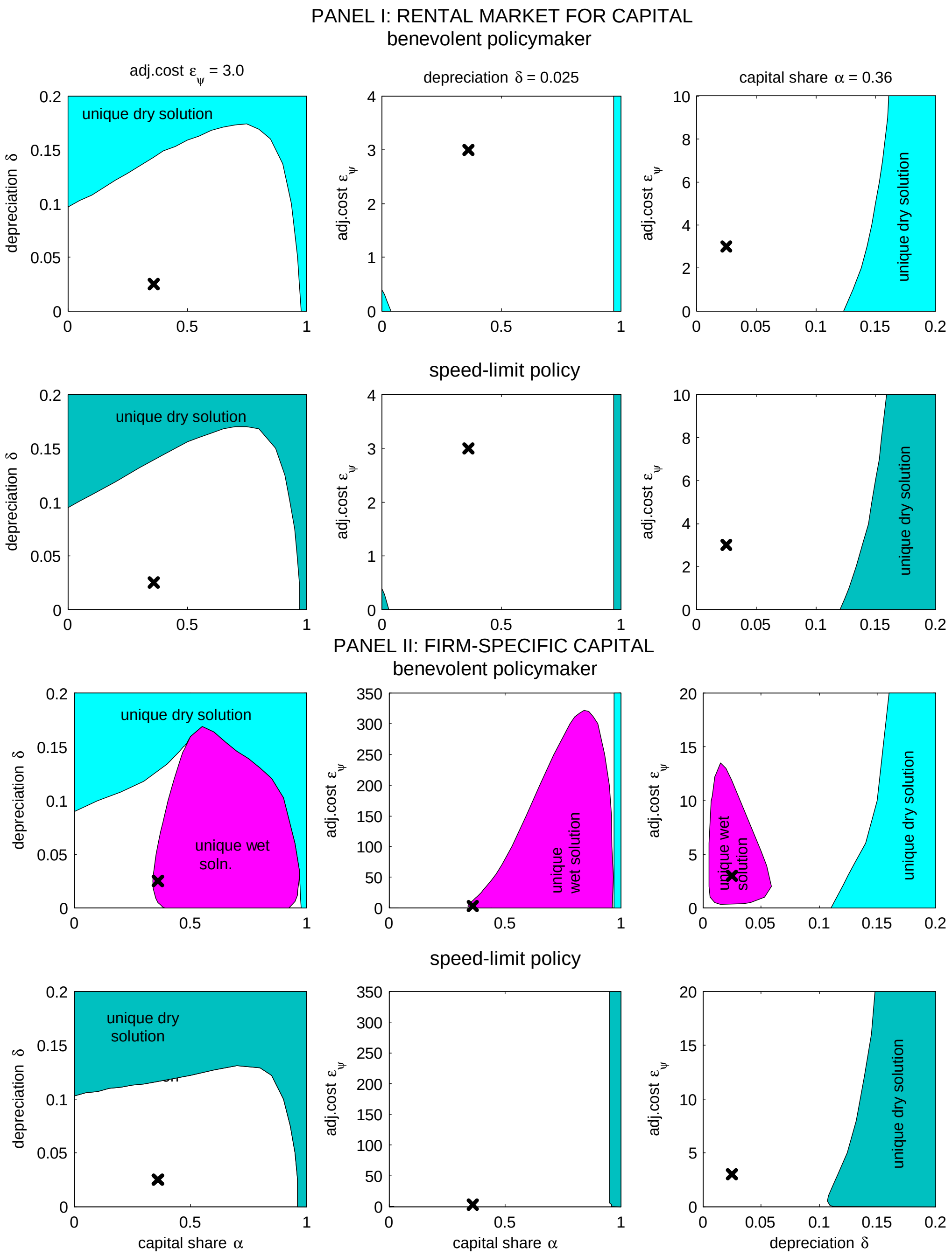

Chart 3: Regions of multiplicity for speed-limit policy

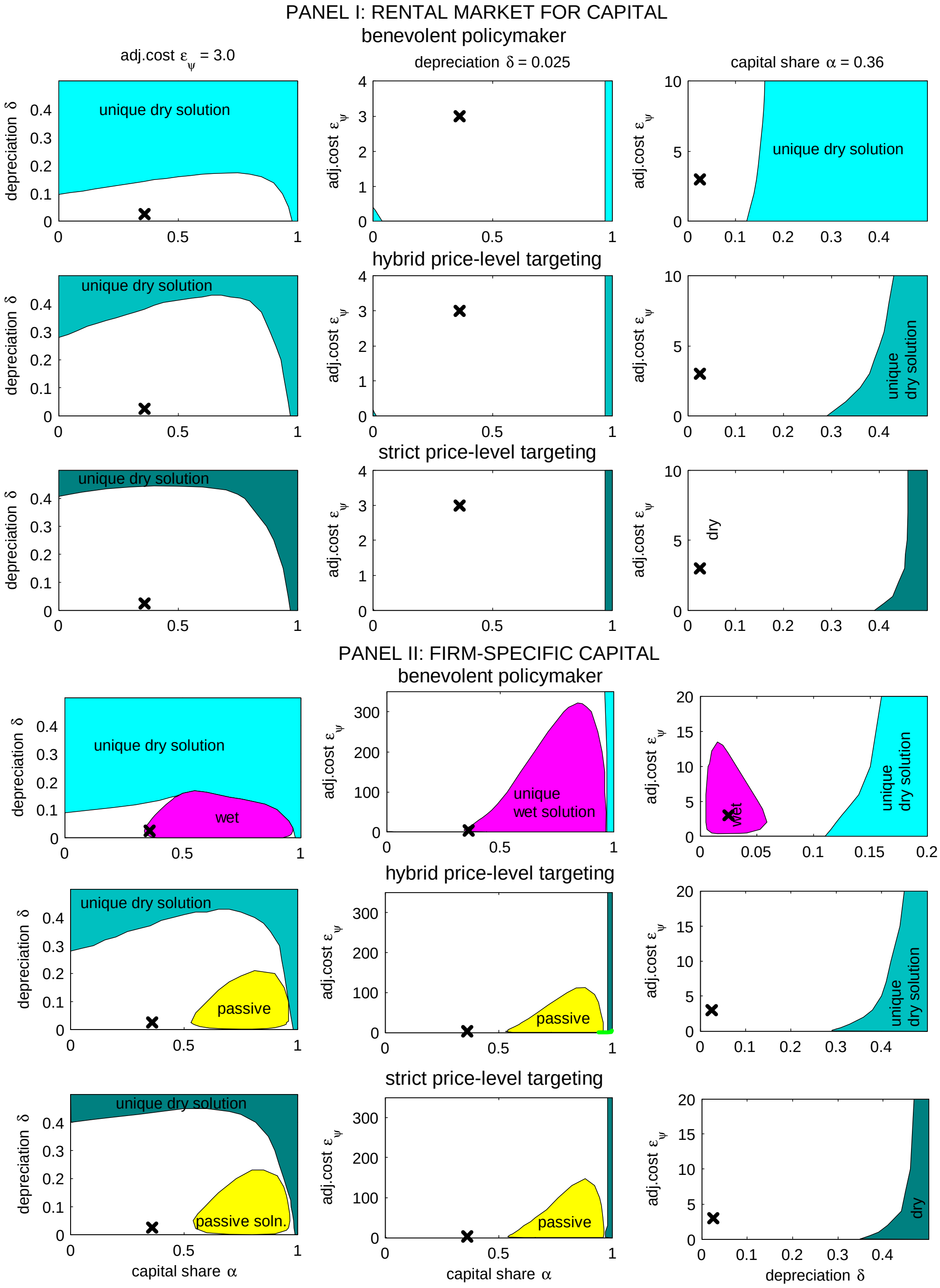

Chart 4: Regions of multiplicity for the price-level targeting

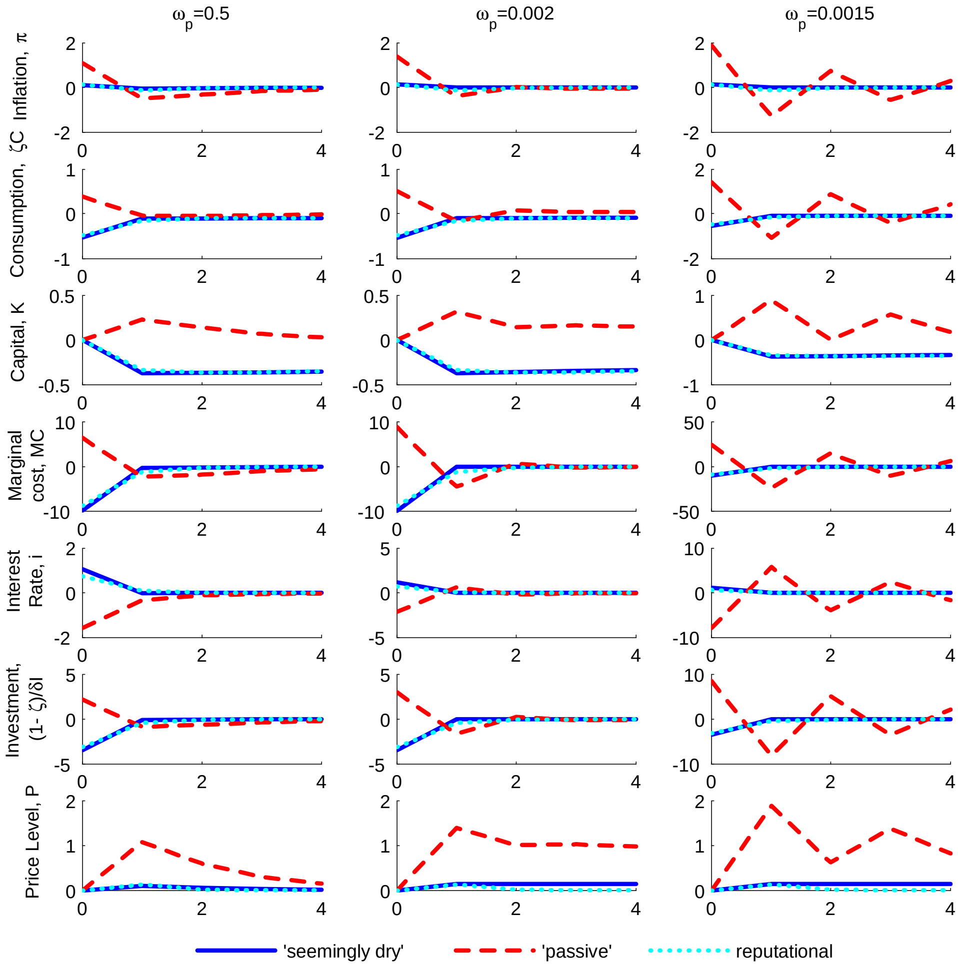

Chart 5: Impulse responses to a unit cost-push shock under hybrid price-level targeting for three different penalties ω_p. Rental market for capital

Appendix: Parameters of the New Keynesian model with capital

In the main text, for notational convenience we labelled certain convolutions of parameters in the linearised model with a single symbol; here we define those symbols in terms of the convolutions of parameters in the original model.

$$
\begin{aligned}
\lambda_c &= \kappa\left(\frac{(\phi+\alpha)\zeta}{1-\alpha}+\frac{1}{\sigma}\right), \lambda_o = \kappa\frac{(\phi+\alpha)(1-\zeta)}{(1-\alpha)\delta}, \lambda_k = \kappa\left(\frac{(\phi+\alpha)(1-\zeta)(1-\delta)}{\delta(1-\alpha)}+\frac{\alpha(1+\phi)}{(1-\alpha)}\right) \\
\zeta &= 1-\frac{\delta\alpha(\varepsilon-1)}{\varepsilon(\delta-\ln\beta)}, \bar{\nu} = \left(\varepsilon_\psi(1+\beta)+\frac{(1-\beta(1-\delta))}{1-\alpha}\left(\frac{(\phi+1)(1-\zeta)(1-\delta)}{\delta}+\alpha\phi+1\right)\right)^{-1}, \\
\nu_o &= \left(\varepsilon_\psi\beta+\frac{(1-\beta(1-\delta))(\phi+1)(1-\zeta)}{(1-\alpha)\delta}\right)\bar{\nu}, \nu_c = (1-\beta(1-\delta))\left(\frac{(\phi+1)\zeta}{1-\alpha}+\frac{1}{\sigma}\right)\bar{\nu}, \\
\nu_r &= \left(1-(1-\beta(1-\delta))\left(\frac{(\phi+1)\zeta\sigma}{(1-\alpha)}+1\right)\right)\bar{\nu}, \nu_k = \varepsilon_\psi\bar{\nu}, \xi = 1-(\nu_r+\sigma\nu_c)\pi_k-\nu_c c_k-\nu_o k_k, \\
k_S &= \frac{\nu_k}{\xi}, k_P = -\frac{(\nu_r+\sigma\nu_c)}{\xi}, c_S = \frac{\nu_k c_k+\nu_k\sigma\pi_k}{\xi}, c_P = \frac{-\sigma-\nu_r c_k+\sigma\nu_o k_k}{\xi}, \\
\pi_S &= \frac{1}{\xi}\left(-\lambda_k+\nu_k\lambda_o+((\beta+\sigma\lambda_c)\nu_k+(\nu_r+\sigma\nu_c)\lambda_k)\pi_k+(\lambda_c\nu_k+\nu_c\lambda_k)c_k+\lambda_k\nu_o k_k\right), \\
\pi_P &= -\frac{1}{\xi}\left(\sigma\lambda_c+\lambda_o\nu_r+\sigma\nu_c\lambda_o+\beta(\nu_r+\sigma\nu_c)\pi_k+\lambda_c\nu_r c_k-\sigma\lambda_c\nu_o k_k\right) \\
\mu &= \left(\frac{\pi_P^2}{k_P^2}+\omega\left(\eta\frac{c_P}{k_P}+\gamma\right)^2-\beta\left(\left(\pi_P\frac{k_S}{k_P}-\pi_S\right)^2+\omega\left(\eta\left(c_S-c_P\frac{k_S}{k_P}\right)-\gamma(1-\delta)\right)^2\right)\right)
\end{aligned}
$$

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