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The impact of permanent energy price shocks on the UK economy

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Abstract

This paper outlines the properties of one of the models used at the Bank of England for analysing the impact of energy prices on the UK economy. We build a dynamic general equilibrium model that includes a variety of channels through which energy prices affect demand and supply. On the demand side we model household consumption of final energy goods (petrol and utilities) separately from other goods and services. On the supply side, we model the production of final energy goods and the way that they enter the production process of other goods and services. We calibrate the model using UK data and examine how the various channels in the model contribute to the responses to permanent energy price shocks of a similar magnitude to those observed in the recent data. We show the effects of such shocks have important implications for monetary policy.

Key words: Energy prices.

JEL classification: E27, E37.

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Summary

The world price of energy has risen dramatically in recent years. This rise has been persistent. Energy has an important role in all economies, affecting both demand and supply, in ways that depend upon energy intensity and the degree to which an economy produces energy as a raw material. For economies that are significant net producers or net consumers of oil and natural gas, persistent price rises can imply potentially large wealth effects in the absence of full international risk-sharing. The United Kingdom is an interesting case as it represents an economy on the transition path from being broadly self-sufficient in energy to being one that is a significant net importer. Thus in this paper, we analyse the implications of permanent energy price shocks for the UK economy.

To analyse the impact of such shocks we build a dynamic general equilibrium model. This approach allows us to articulate theoretically the wide variety of channels through which energy prices might affect demand and supply by making a careful analysis of how shocks propagate through the economy, a process that inevitably takes time. The calibration process we use involves the careful choice of critical parameters that allow us to match key properties of the UK data. On the supply side, we model how primary energy inputs such as oil and natural gas are used to produce final energy goods such as petrol and electricity and gas distribution. We also model the way that final energy goods enter the production process of non-energy goods. We allow for the direct use of energy in the production process and for energy prices to influence the utilisation of the capital stock. On the demand side we model the substitution in household consumption between final energy goods and non-energy goods. To calibrate the model we construct a UK data set using the National Accounts Input-Output Supply and Use Tables. This allows us to gauge the quantitative importance of the different channels.

We examine how the various channels in the model contribute to the overall response to a permanent energy price shock. We show the quantitative sensitivity of inflation and output responses to the following key assumptions and judgements:

- (i) the degree of nominal rigidity in price and wage-setting;
- (ii) the monetary policy response both domestic and overseas;
- (iii) the assumption about self-sufficiency and its impact on the real exchange rate and import prices;
- (iv) the degree of real wage resistance and the impact on the labour market; and
- (v) the impact on the level and utilisation of the capital stock.

We show that the impact of higher energy prices depends significantly on the monetary policy response to higher energy prices, both here and abroad. When policy does not fully accommodate the shock the degree of nominal wage rigidity is important in determining the extent to which the indirect effects of higher energy prices are able to offset the direct effects of higher petrol and utility prices on inflation. Indeed negative effects on inflation from higher energy prices are possible if these offsetting indirect effects are not synchronous with the direct effects. The degree of self-sufficiency in energy is also important as it leads to significantly different effects on consumption and the real exchange rate. On the supply side, we find that the

effects on potential supply are not likely to be large unless there is significant real wage resistance and higher energy costs affect the utilisation of the capital stock.

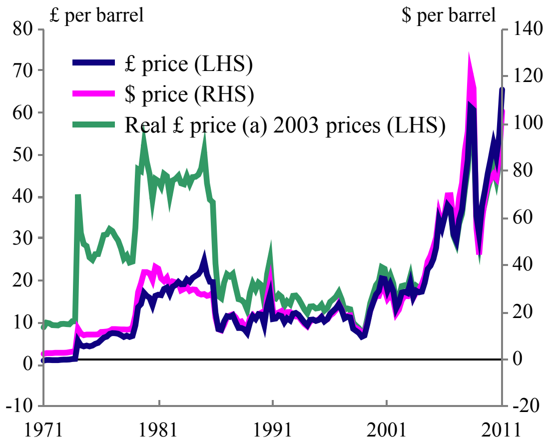
Our model only explores the effects of permanent shocks in a theoretical model. In a companion paper, the model is estimated on actual data to see how well it describes the UK experience.

1 Introduction

This paper examines the impact of higher energy prices on the UK economy. There have been large shifts in the price of crude oil and natural gas since the beginning of 2004 summarised in Charts 1 and 2. Crude oil prices rose from around \$30 per barrel at the end of 2003 to a quarterly average of over \$100 by the end of 2010. As a result the real price of oil faced by the UK economy has risen to a similar extent as the increase in the 1970s. UK wholesale natural gas prices have exhibited similar sharp swings over the past few years although the timing of the movements were by no means synchronous with those of the oil price, reflecting various idiosyncrasies in the UK gas market. These changes in primary energy costs have led to equally significant movements in *final* energy goods prices such as the prices of petrol and 'utilities' output (the electricity and gas distributed to firms and households). These prices have increased by around 60%-100% since the end of 2003 (Chart 3). Chart 4 also shows that oil futures prices have tended to move in line with spot prices. So, to the extent that futures prices are indicative of expectations, it suggests movements in the spot price of oil and gas are normally expected to persist for some time by markets.

In this paper, we outline the properties of one of the models used at the Bank of England to analyse the implications of these large energy price shocks for the UK economy. The UK can be characterised as a small open economy that is also a current producer of primary energy sources such as crude oil and natural gas. However, the UK economy's production of oil and gas is expected to decline markedly over the next 20 years or so, possibly by as much as three quarters. This gradual decline in natural resource extraction is likely to have particular implications for the response of domestic consumption and the real exchange rate to a *permanent* change in energy prices and hence have implications for monetary policy. Our approach is to build an open economy dynamic general equilibrium (DGE) model that includes a variety of standard channels through which energy prices affect both the demand and the supply side of the economy but also facilitates analysis of the implications of declining energy production. We calibrate the model to UK data and examine how the various channels in the model contribute to the overall response of the economy to permanent energy price shocks of a similar magnitude to those observed in the data. The paper sits between two other companion papers. First a descriptive analysis of the key channels of the model using aggregate demand and supply diagrams can be found in Barwell *et al* (2007). A second companion paper (Millard (2011)) uses Bayesian techniques to estimate a linearised stochastic version of the model. This paper models the stochastic process underlying energy prices and looks at how recent shocks to energy prices have contributed recent movements in output and inflation.

Chart 1: Oil prices



(a) Sterling oil prices deflated by the consumption deflator.

Chart 2: Wholesale gas prices

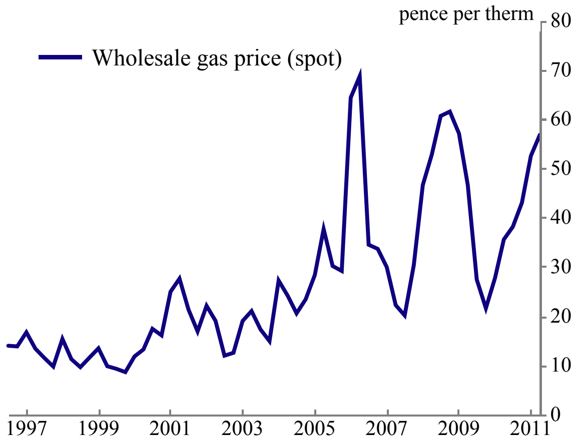


Chart 3: Petrol and utility prices

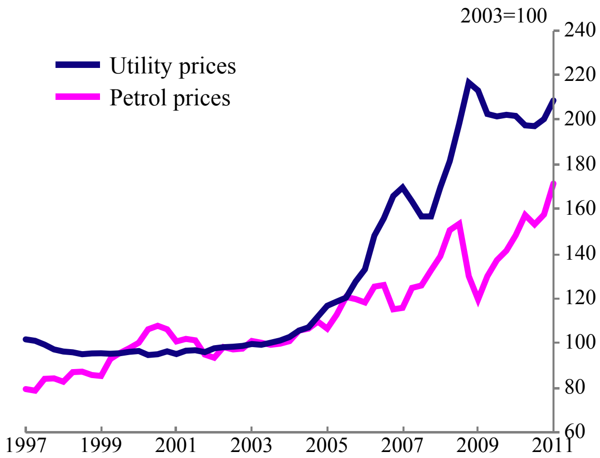
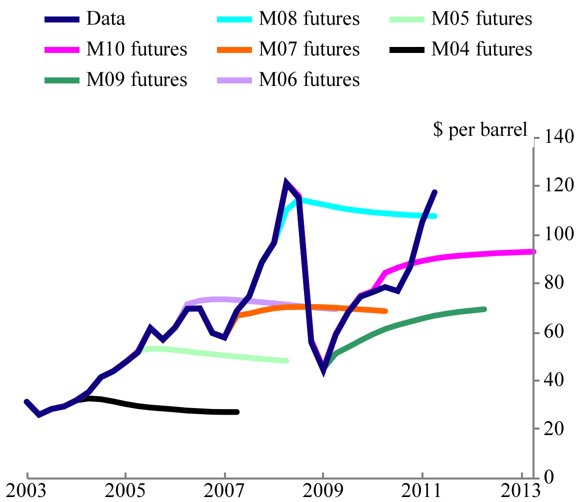


Chart 4: Oil price futures



Note: The chart shows the oil price futures curve in May of each year taken from successive *Bank of England Inflation Reports*.

The plan of the paper is as follows. In Section 2 we provide an overview of the model and how it relates to the various macroeconomic models in the literature that incorporate an energy sector such as Hamilton (1983), Bruno and Sachs (1985), Rotemberg and Woodford (1996), Finn (2000) and Blanchard and Gali (2007). In Section 3 we calibrate the key parameters of the model using a set of model-consistent data derived from the UK National Accounts Supply and Use Tables. In Section 4 we consider the responses of a simplified version of the model to a permanent energy price increase. This allows us to highlight some of the basic channels and mechanisms in the model and allows a comparison with various papers in the literature. In Section 5 we then examine the sensitivity of the model to different supply-side assumptions and different monetary policy reactions. We also examine the case in which energy production declines over time so that the UK economy has to respond to higher energy prices as it moves along a transition path from being self-sufficient in energy to being a significant net importer.

2 The model

In this section, we give a brief overview of the structure of the model. A detailed derivation and list of the model equations is presented in the appendix. Aside from the treatment of energy prices, the structure of the model follows very closely that of the New Keynesian model Harrison and Oomen (2010) (HO, henceforth) have estimated for the United Kingdom. Overall our modelling strategy has two key elements:

- First, the model is designed to be capable of analysing some important features of the recent developments in energy prices noted earlier. The energy price increases we have observed since 2004 appear to contain a sizable permanent component at least in expectation. So we need to be able to examine the effects of permanent energy price shocks on the steady state of the model as well as along the United Kingdom's transition path to becoming a net importer of energy. We also choose to separate the analysis of crude oil and natural gas. As noted earlier crude oil prices have behaved differently to wholesale gas prices in the recent past. This is important for an economy like the United Kingdom given natural gas represents a significant proportion of the economy’s total inputs of primary energy. The future production profiles for crude oil and natural gas are also different. So we allow for separate channels for these two energy prices in our model. We also allow for variable speeds of pass-through from oil and wholesale gas price changes into final energy goods prices. In particular the data suggest petrol prices move more rapidly in response to oil prices than utility prices respond to wholesale gas costs.

- The second element of our approach is to include a wide range of potential channels through which energy price shocks may affect the economy. On the supply side, we model the production of energy – both petrol and ‘utilities’ (electricity and natural gas distribution) – and the way that energy enters the production processes of other ‘non-energy’ goods and services. In these processes we allow for the direct use of energy in production and for energy prices to influence the utilisation of the capital stock. We also allow for sticky wage and price-setting in the non-energy sector as well as real wage resistance by workers. On the demand side we model household consumption of final energy goods (petrol and utilities) separately from other goods and services. This disaggregated approach allows us to consider the implications of different degrees of substitutability between energy and non-energy goods in both production and consumption.

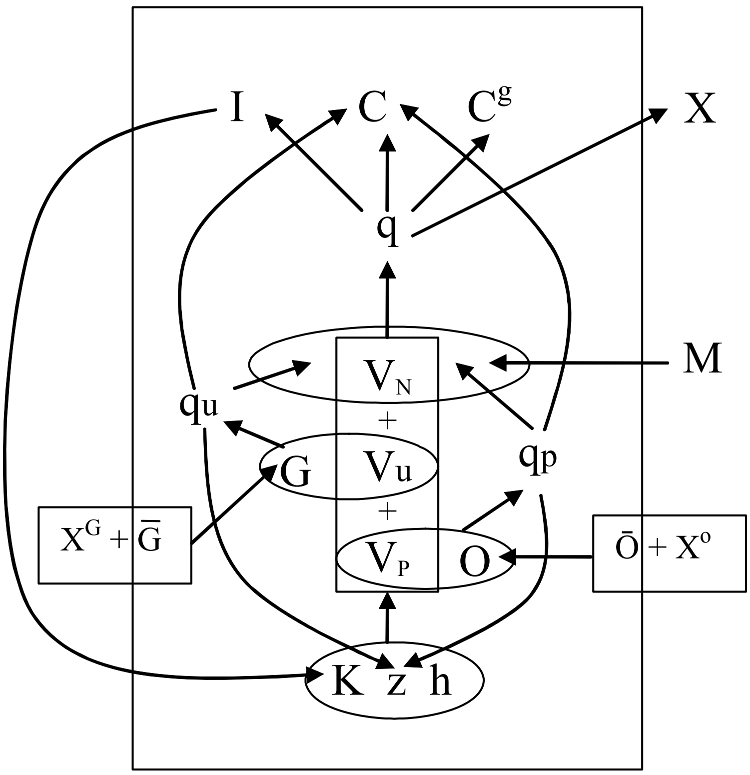
2.1 An overview of the model structure

The key features of the model’s structure can be represented in a simple diagram (Figure 1).

The diagram focuses on the supply side and product markets and does not, for simplicity, show the flow of labour from households to firms. Neither does it depict the flow of payments for products and factors. The large rectangle in the centre of the diagram represents the UK economy and the space outside that rectangle represents the rest of the world.

The diagram shows that physical capital (K), utilised at rate (z) and hours of labour (h) are combined to produce value added (V). Value added is allocated to three sectors: the petrol sector (V_p); the utilities sector (V_u); and the non-energy sector (V_N). Value added is combined with other inputs to produce different goods. The petrol sector combines value added (V_p) and oil (O) to produce petrol (q_p). The quantity of oil used in UK petrol production is the sum of the United Kingdom's endowment of oil ($\overline{O}$) and net trade in oil with the rest of the world (X^o). Similarly, the utilities sector combines value added (V_u) and natural gas (G) to produce utilities (q_u) and the quantity of gas used in production is given by the endowment ($\overline{G}$) and net trade with the rest of the world (X^g).

Figure 1: A diagram of the key linkages in the model



Both petrol and utilities are combined with intermediate imports (M) and value added (V^N) to produce final non-energy output (q). Final non-energy output is sold to households for investment (I), to government (C^g) and to the rest of the world as exports (X). The household's overall consumption bundle (C) combines final non-energy goods (C^n), petrol (C^p) and utilities (C^u). The figure also shows how investment cumulates into the capital stock and that a particular choice of capital utilisation (z) requires 'inputs' of petrol and utilities.

The schematic representation of the model given here simply maps out (some of) the flows of spending in the model, without explaining how they are determined. The next subsection describes how these choices are modelled as the optimal outcomes of constrained maximisation problems by the agents that populate the model.

2.2 Households

Households act to maximise their expected discounted stream of utility, which is defined over consumption, hours worked and real money balances. Households also own the physical capital stock, which they rent to firms. As well as physical capital, households have access to domestic and foreign (nominal, government) bonds, which they can use to borrow and save.
The dynamics of the household's consumption choice are influenced by the assumption of (external) habit formation, so that the current utility of each household depends on the level of aggregate consumption in the previous period. As well as choosing the overall level of consumption, the household must choose each period how to allocate expenditure between the categories of consumption that make up the consumption bundle. Specifically, the overall consumption bundle consists of an aggregate of final (domestically produced) non-energy goods (c^n) and energy goods (c^e):

$$c = f^c\left(c^n, c^e\right)$$

where consumption of final energy goods is defined in terms of consumption of petrol (c^p) and utilities
(c^u):

$$c^e = f^{ce}\left(c^u, c^p\right)$$

and we assume that the bundling functions f^c and f^{ce} are constant elasticity of substitution (CES) functions with elasticities σ^e and σ^p respectively.

Households' labour supply behaviour is formulated in terms of a choice of a nominal wage. We therefore follow Erceg, Henderson and Levin (2000) and assume that each household supplies a differentiated labour service (over which it has monopolistic power) to firms. Households set (nominal) wages in staggered contracts on the understanding that they supply whatever level of labour that firms demand at that wage rate. In particular, a randomly selected fraction of households can renegotiate their wage contracts in each period. The wage rates of households that do not reset their wages are increased by a weighted average of steady-state inflation and the aggregate wage inflation rate observed in the previous period. This assumption gives rise to nominal inertia in wage-setting.

Households rent capital services to firms. Capital services are measured as the product of the physical capital stock in existence at the start of the period multiplied by the capital utilisation rate (z):

$$k_t^s \equiv z_t k_{t-1}$$

so households must choose both how much physical capital to accumulate as well as the rate at which it should be utilised. These choices are linked by the assumption that the depreciation rate of capital is affected by the rate of utilisation. Specifically, we assume that the depreciation rate of capital is given by

$$\delta + f^z(z)$$

where $f^z(z)$ is a monotonically increasing function. Adjustment of the physical capital stock is also subject to quadratic adjustment costs that depend in part on the difference between the change in the capital stock and the change in the aggregate capital stock observed in the previous period, which gives rise to inertia in investment spending. The choice of utilisation rate is also influenced by energy prices. Specifically, we follow Finn (2000) in assuming that the household must purchase e_z units of energy according to the following relationship:[1]

$$\frac{e_{z,t}}{k_{t-1}} = \frac{\chi_e}{\gamma_e} z_t^{\gamma_e}$$

which can be interpreted as a demand curve for the energy required to operate the capital stock. The amount of energy per unit of capital stock is positively related to the capital utilisation rate: using the capital stock more intensively requires more energy. This setup means that an increase in the price of energy creates an incentive to utilise capital less intensively.

Households' choices of domestic and foreign (nominal, government) bonds give rise to no-arbitrage conditions, including an uncovered interest parity condition linking the expected exchange rate change to the differential between domestic and overseas interest rates. This condition also contains a term capturing the effect of an assumption that trade in foreign bonds incurs quadratic costs. These costs are defined relative to a steady-state net foreign asset position and are a common feature of small open economy models: they ensure that the model returns to a unique steady-state net foreign asset position following a transitory shock (see Schmitt-Grohe and Uribe (2003)).

The household's intertemporal budget constraint describes how the household accumulates physical capital and financial assets from its net income flow. The household earns income from supplying labour and capital services to firms plus dividend payments received by firms including (net) proceeds from the sale of oil and gas on world markets. These proceeds are generated from the (costless) operation of a domestic oil well and gas field, that produce exogenous (and potentially time-varying) flows of oil and gas. Household expenditures are allocated to consumption, taxes, adjustment costs and the cost of servicing capital.

2.3 Firms

Production is divided into a number of sectors that produce value added, final outputs and intermediate products. We consider each in turn.
We assume that value added (V) is produced by combining domestic capital and labour:

$$V = f^v\left(h, k^s\right)$$

where h and k^s represent total hours and capital services rented from households. We

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[1]We assume $\chi_e \geq 0$ and $\gamma_e > 1$.

assume that the function f^v is CES with elasticity σ_v and that this sector is perfectly competitive. This means that factor demands are implied by profit maximisation and the price of value added can be derived from the zero-profit condition.

Final non-energy output (q) is produced by a population of imperfectly competitive firms operating a production function:

$$q_t = f^q(B, E)$$

that combines a bundle B, defined below, of value added and imports with energy inputs (E). Again, we assume that the production function is CES (with elasticity σ_q). The bundle of value added (V^N) and intermediate imports (M^N) used by non-energy firms is:

$$B_t = f^B\left(V^N, M^N\right)$$

where we assume that f^B is Cobb-Douglas. This choice is motivated by the roughly constant ratio of total nominal imports to nominal value added in the United Kingdom, despite a clear trend in the relative price of imports. The energy input is defined as a function of petrol and utilities:

$$e_t = f^e\left(I_t^p, I_t^u\right)$$

where I^p and I^u denote intermediate inputs of petrol and utilities and f^e is assumed to be a Leontief aggregator.

Firms that produce final non-energy output maximise the discounted flow of dividends – net of quadratic costs of adjusting prices – subject to the demand curve for their output and the production functions described above. The firm's choice variables are their factor demands and the price for their final output. The demand curve for final non-energy output has a constant elasticity form[2] and the price adjustment costs depend in part on lagged non-energy price inflation, thus incorporating a degree of inertia into non-energy inflation.

Finally, we turn to the production of energy goods. We assume that utilities and petrol are both produced using value added and intermediate inputs of primary energy sources (natural gas and oil respectively):

$$q_t^u = f^u\left(I^g, V^u\right)$$

and

$$q_t^p = f^p\left(I^o, V^p\right)$$

where I^g and I^o denote intermediate inputs of natural gas and oil and f^u and f^p are

[2]Stemming from the assumption that consumers and government have (identical) Dixit-Stiglitz preferences over the varieties of non-energy final output produced by the population of firms.

assumed to be Leontief aggregators. We assume that firms producing petrol and gas are monopolistically competitive and face price adjustment costs of the same form as in the non-energy sector. They set prices and factor demands to maximise the discounted flow of dividends, net of these adjustment costs.

2.4 Fiscal and monetary policy

For simplicity the government is assumed to choose procurement (c^g) according to a simple autoregressive rule and issues money and nominal one-period bonds. The spending and net money and bond issuance is financed by tax revenue. Tax revenue consists of: value added taxes levied (at an exogenous rate) on non-energy output and petrol; a special value added tax applied only to spending on utilities; petrol duties and a lump-sum tax levied on households. The lump-sum tax is the fiscal instrument and moves to satisfy the government's budget constraint (and for simplicity we analyse the case in which the government issues no debt). The distortionary taxes are included to facilitate the calibration of energy shares in output as well as to assist when comparing simulations from the model with patterns in the data.

We assume that the monetary authority sets the nominal interest rate according to a simple monetary reaction function responding to deviations of annual consumer price inflation from target and a measure of the output gap. The measure of the output gap is defined in terms of the deviation of non-energy valued added from the level that would prevail under flexible prices.[3]

2.5 Rest of the world and international trade

The model features trade in intermediate goods, final output and natural resources (oil and natural gas). All non-energy imports are assumed to be used as intermediate goods in production of final non-energy output, as described above. Import prices are assumed to be priced as a mark-up over the exogenous world import price (measured in domestic currency) and are subject to Calvo price adjustment costs as in HO. To finance net expenditures on imported intermediates and energy, the domestic economy sells final non-energy output abroad. We assume that there is a downward-sloping export demand function for domestically produced non-energy output, as in HO. This corresponds to the assumption that the domestic economy's final non-energy output can be treated as a distinct 'brand' among the set of varieties of goods demanded by agents in the rest of the world.

We assume that there is an infinitely elastic supply of oil (gas) available from the world market at exogenous world *relative* prices. This reflects the assumption that these prices are determined on world markets and are unaffected by developments in the domestic economy. The prices of oil and gas in domestic currency are given by the law of one price. Net trade in oil (gas) is the difference between the exogenous endowment of oil (gas) and the quantity used in the production of petrol (utilities).[4]

[3] The policy rule therefore requires an estimate of the 'flex-price' level of output. We solve for the flex-price allocations simultaneously with the sticky-price version of the model by augmenting it with flex-price analogues of each equation.

[4] So a key simplification of the model is that we assume that oil and natural gas extraction rates are exogenous. A more realistic approach would be to model the extraction rates as the choice variable of 'natural resource extractors'

The simplifying assumption of an infinitely elastic supply of oil and natural gas on the world market amounts to assuming that global reserves of oil and gas last forever, which is clearly a false assumption. However, we regard this assumption as a convenient representation of the fact that world reserves of natural resources are expected to last for much longer than UK reserves.

3 Parameterising the model

To parameterise the model we adopt a hybrid approach. To calibrate the key energy sector parameters we construct a set of model-consistent data derived from the UK National Accounts Input-Output Supply and Use Tables (I-O SUTs). These data are used to pin down the key production and expenditure shares of the model in steady state and allows us to back out the key production function and consumption bundle parameters. For the remaining macroeconomic parameters we take a more eclectic approach and use estimated parameters from previous UK and US studies. The aim of the model is not exclusively a data-fitting exercise but to elucidate the impact of oil prices under different, but quantitatively plausible, assumptions about key macroeconomic parameters.

3.1 Macroeconomic parameters for the non-energy sector

Our main source of information on macroeconomic parameters is the model of HO (*op cit*). This model is estimated with Bayesian techniques and has basically the same structure as our model with the exception of the energy price channels. So we can use it to calibrate some of the standard macroeconomic and non-energy sector parameters that are common to both models. These are listed in Table 1. But we also make some specific parameter settings that are different to HO largely to allow for the fact that our parameters should apply to the non-energy private sector rather than the economy as a whole:

- We choose a slightly different level of β from HO so that the model generates a steady-state real interest rate of around 3% in annual terms.

- In the production function, we take the elasticity of substitution between capital and labour in non-energy value added from HO (equal to 0.5). But conditional on this the other production function parameters are set to achieve a labour share of 0.75 in steady state (rather than 0.7 as in HO), matching the share of private sector non-energy value added in the recent data.

- Conditional on the production function parameters, the elasticity of product demand in the non-energy sector is then set at 20 to match the capital-output ratio in the data, using the firm's first-order condition for the demand for capital in steady state.

- Given that we are modelling private sector wage-setting, the elasticity of demand for

as in Gray (1914). Then, given an exogenous stock of oil and natural gas, extractors would choose the optimal extraction rate based on the expected path for energy prices and extraction costs. In future work, we plan to extend the model in this way.

differentiated labour was set somewhat lower than HO at 3.89 implying a wage mark-up of around 1/3. This is close to many of the estimates used in the empirical literature.

- The parameters governing the degree of nominal wage and price rigidities were set so that the average wage and price contract is set for a year, so that there is a 0.25 probability of changing prices and wages in the baseline case, although we will look closely at the impact of relaxing this assumption.

Table 1: Macroeconomic parameters

β	0.9925		σ^c	0.66
δ	0.013		ψ^{hab}	0.69
χ^{bf}	0.01		σ^h	0.43
σ^w	3.89		χ^k	201
η^x	1.50		ϵ^k	0.50
κ^x	0.24		ϕ^z	0.56
ψ^{pm}	0.40		ϵ^w	0.58
ϵ^m	0.17		θ_p	1.5
ψ^w	0.25		θ_{RG}	0.8
σ^v	0.5		ε	20
χ^p	247			

3.2 Shares of energy in consumption and production

To calibrate some of the key parameters in the production and consumption bundles we use shares of energy in consumption and production in 2003 from the I-O SUTs and other consistent national accounts data. The approach here is fairly standard. We use the steady-state solution of the model to back out a set of parameter values that are consistent with the key factor and expenditure shares found in the data. So, for example, we know that in 2003 oil and gas inputs accounted for around 1.6% of total final expenditure at market prices in the economy. The model would suggest this share is pinned down by a number of parameters, one of which is α_q. We constrain the parameter α_q to ensure the energy input share in expenditure holds by inverting the steady-state solution of the model to ensure the share in the model matches its 2003 value in the data, conditional on the values chosen or calibrated for other parameters. We pick 2003 as the basis for the steady-state calibration as this is the point at which both oil and gas prices started to pick up significantly and we are interested in analysing the long-run impact of that step change in energy prices, should it prove permanent. Although the shares of energy in production and consumption have changed since this date, using the model to try and track how these shares have changed over time is a useful way of calibrating the various elasticities of substitution in the model as we discuss below. But it means that to analyse the impact of a permanent increase in energy prices at a date subsequent to 2003 means accounting for any changes in shares that have occurred in the interim period. This can be tracked in later I-O SUT tables. Table 2 summarises the parameters that are jointly pinned down by these production and consumption shares with a brief description of how the parameter relates to a key share in the model and the data.

Table 2: Parameter values relating to energy side assumptions based on 2003 I_O SUTs

Parameter	Value	Calibration target	Description
α_B	0.33	$\frac{p^m m^n}{q+p^u C^u+p^p C^p+p^p X^p}=0.25$	Share of imports in non-extraction final expenditure at market prices
α_q	0.989	$\frac{p^o i^o+p^g i^g}{q+p^u C^u+p^p C^p+p^p X^p}=0.016$	Share of energy inputs in final expenditure at market prices
C^g	0.008	$\frac{C^g}{q+p^u C^u+p^p C^p+p^p X^p}=0.1$	Share of government procurement in final expenditure at market prices
d^p	2.71	$d^p / p^p = 0.617$	Share of duty in final petrol price
$\bar{G}$	0.001	$\frac{p^g \bar{G}}{p^v V+p^o \bar{O}+p^g \bar{G}}=0.0126$	Gross gas production as a share of total private sector value added
$\bar{O}$	0.001	$\frac{p^o \bar{O}}{p^v V+p^o \bar{O}+p^g \bar{G}}=0.0174$	Gross oil production as a share of total private sector value added
ϕ^v	0.949	$\frac{wh}{p^v V}=0.75$	Share of labour in non-extraction value added
ψ^e	0.003	$\frac{p^u C^u}{p^c C}=0.0215$	Share of gas and electricity utilities in consumption (excl rents)
ψ^n	0.27	$\frac{p^p i^p}{p^u i^u}=1.36$	Ratio of petrol to utilities inputs used in production
ψ^p	0.298	$\frac{p^p C^p}{p^c C}=0.03$	Share of petrol in consumption
ψ^{qp}	0.248	$\frac{p^o i^o}{p^{pb} q^p}=0.734$	Share of oil in petrol production
ψ^u	0.756	$\frac{p^g i^g}{p^{ub} q^u}=0.287$	Share of natural gas in utility output
ψ^{xp}		$\frac{x^p}{q^p+m^p}=0.137$	Petrol exported as a ratio of petrol consumed

3.3 *Elasticities of substitution and product demand in the energy sector*

To back out the key elasticities of substitution in the production and consumption bundles we make use of the relationship between relative prices and relative volumes implied by the CES relationship. For example the elasticity of substitution in consumption between petrol and utilities can be backed out from the following relationship if we know the change in relative volumes and the change in relative prices.

$$\frac{1-\psi^p}{\psi^p}\left\{\frac{1-\phi^p}{\phi^p}\right\}^{1-\frac{1}{\sigma_p}}\left\{\frac{c_t^u}{c_t^p}\right\}^{-\frac{1}{\sigma_p}}=\frac{p_t^u}{p_t^p}$$

So to calibrate the various elasticities of substitution we proceed by inputting the change in volumes and relative prices over the period 2003-08 into the log difference of equations, such as the one above, and jointly back out the elasticities of substitution that match the data over this period (Table 3).

For the elasticities of product demand in the petrol and utilities sector we assume a value of 20 the same as in the non-energy sector. There may be good reasons for assuming a lower elasticity for the utilities sector. This is because utility prices in the data appear to respond to natural gas prices by proportionately more in the long run than would be suggested by just the share of gas inputs in utility companies costs in the I-O SUT data and assuming a mark-up of 5%. This may be because other costs in the sector are correlated with gas prices – we do not model coal prices for example. One way of mimicking a larger proportional response to gas prices would be to set a lower elasticity of product demand for the utilities sector. That would ensure a bigger proportionate mark-up on natural gas costs in the utility sector. That in turn would imply the price of utilities responds to a greater degree to a given percentage change in wholesale gas prices as the supernormal profit element of utility prices rises proportionately with gas costs. So when using the model in practice the elasticity of demand in the utilities may need to be changed if changes in natural gas prices appear to have a different effect on utility prices than suggested by the I-O SUT shares.

For the nominal rigidity parameters in the petrol and utilities sectors, we assume that petrol prices are relatively flexible given they appear to respond rapidly to oil prices in the data. We assume that the probability that a petrol company can change its price is 4/5 in a given quarter. For utility companies, we assume a more sluggish pass-through of natural gas costs and assume that on average they change prices once a year which is largely in line with how the large utilities appear to operate. Finally to calibrate the use of energy in facilitating the utilisation of the capital stock we use the parameters from Finn (2000).

Table 3: Parameter values for key elasticities of substitution and elasticities of product demand

σ^p	0.1	Elasticity of substitution between petrol and utilities in the energy bundle
σ^e	0.4	Elasticity of substitution between energy and non-energy goods in consumption bundle
σ^q	0.15	Elasticity of substitution between energy
ε^p	20	Elasticity of product demand within the petrol sector
ε^u	≤ 20	Elasticity of product demand within the utilities sector
χ^{pp}	5.9	Nominal rigidities in petrol pricing
χ^u	247	Nominal rigidities in utility pricing
χ^e	0.01	Usage of energy in utilisation of capital stock
γ_e	1.66	Usage of energy in utilisation of capital stock

3.4 Fiscal parameters

The fiscal parameters are taken directly from the data. The rate of VAT on petrol τ^p is assumed to be the standard 17.5% rate that companies faced in 2003, and on utilities τ^u to be the reduced rate of 5%. The effective VAT rate on other goods and services τ^v is assumed to be 10%, calculated by residual given the total amount of VAT collected in 2003. It is less than 17.5% reflecting the fact that VAT is not levied on some non-energy goods. The share of duties in the retail petrol price (shown in Table 2) was obtained from the Department of Trade and Industry (now the UK Department for Business, Innovation and Skills).

The model assumes that duties and indirect taxes paid on retail petrol also apply to the petrol used as a production input by the non-energy sector (ie both consumers and produces face the same petrol prices). But this implies a total direct and indirect share of petrol in final expenditure that is somewhat higher than implied by the I-O SUTs. So the indirect effect of higher oil prices on the non-energy producing sector may be overstated by this assumption. An alternative calibration to that in Table 2 would be to use the parameter α_q to impose the post-tax share of petrol in final expenditure rather than the primary energy share[5], particularly when using the model to analyse the impact of a known petrol or utility price impact. This might however also require other parameter changes to be consistent with primary energy shares in the I-O SUTs. For example the model could be modified to have a different effective indirect tax rate for industry that matched the share in the I-O SUTs.

3.5 Other parameters

The final set of parameters are those that were chosen in order to normalise certain variables at particular (arbitrary) values in the steady state. Most of these parameters are set by inverting the model to find the parameter values required to satisfy the desired normalisation and are given in Table 4. Others are simply chosen for numerical purposes to ensure the solution of a steady state given these normalisations.

Table 4: Parameter values for key elasticities of substitution and product demand

	Parameter value	Normalisation
κ^c	21.58	$p^c = 1$
κ^h	1.07	$V = 1$
κ^{mon}	0.04	$mon/V = 0.3$
κ^x	0.044	$s = 1$
χ^z	0.0351	$z = 1$
p^{mf}	1	$p^m = 1$
p^{of}	1	$p^o = 1$
p^{gf}	1	$p^g = 1$
y^f	1	
R^f	1.0126	$R^f = R^g$
Π^{ss}	1.005	
Π^{fss}	1	$\Pi^{fss} = \Pi^{ss}$
A	66.83	

[5] That is use α_q to fix the share: $\frac{p^p i^p + p^u i^u}{q + p^u C^u + p^p C^p + p^p X^p}$

rather than: $\frac{p^o i^o + p^g i^g}{q + p^u C^u + p^p C^p + p^p X^p}$

4 A simplified version of the model

In this section we examine impulse responses to a permanent increase in oil and gas prices, based on the estimated/calibrated parameters described above, but also applying a set of simplifying assumptions. These assumptions are designed to represent a 'textbook' case in which some of the long-run effects of an energy price rise captured by our model are absent. Using the simplified version helps to focus on the core mechanisms of the model, in particular the interplay between monetary policy and nominal rigidities in determining the impact of energy price shocks. But it also facilitates a comparison with some of the key results of other models (eg Hamilton (1983), Bruno and Sachs (1985) and Rotemberg and Woodford (1996)) that often make similar simplifications. We then show the implications of moving to our richer baseline model specification in Section 5.

Our simplifying assumptions are as follows:

1) Monetary policy follows the simple Taylor rule specified earlier. Overseas inflation and monetary policy are exogenous in the model and are assumed to remain unchanged following the oil and price increase. So the monetary policy response is assumed to be unilateral.

2) The United Kingdom is assumed to be self-sufficient in oil and gas initially (which was broadly true of 2003) with all dividends from domestic oil production ultimately flowing back to UK households.

3) We assume domestic factor supplies are inelastic in the long run. Labour supply is assumed to be inelastic with respect to the real consumption wage in the long run. So there is no long-run 'real wage resistance' to a permanent energy price shock in the baseline case. We introduce a shift into labour supply via the (κ_h) term that keeps employment unchanged in steady state following an energy price shock. The capital stock and its utilisation rate are also assumed to be fixed (we exogenise the investment relationship and the capital utilisation condition), so the flow of capital services is assumed unchanged.

Given these simplifying assumptions Charts 5-6 show a set of impulse responses to a permanent 100% unanticipated increase in world real oil and gas prices (p^{of} and p^{gf}) occurring in 2003. The responses are shown over a five-year period.

4.1 Short and medium-term effects on inflation and other nominal variables

Chart 5A shows that under the simplified baseline assumptions the annual rate of CPI inflation rises in response to the shock with a peak response of around 0.7 percentage points after around 3-4 quarters. Petrol prices respond quickly given the low degree of nominal rigidity we have calibrated, utility prices respond more slowly and take somewhat longer to have their full effect on the price level. Inflation in the non-energy sector falls with a peak negative impact occurring at around 6 quarters.

Other charts show why inflation in the non-energy sector falls in response to the energy price effect. Firms in this sector initially face both higher petrol and electricity costs and lower nominal expenditure on their products (Charts 5D and 6E). Initially higher energy prices raise firms' costs, but because prices are sticky firms pass them on gradually. They also expect a monetary policy response to the upward pressure on CPI inflation. The reaction function prescribes increases in nominal interest rates. Our baseline specification of the reaction function implies that total nominal demand does not expand sufficiently to accommodate the oil and gas price shock (output falls below the flex-price level). This is compounded by the fact that the low elasticity of substitution between energy and non-energy goods in the consumption bundle means that the share of total consumption expenditure on non-energy goods falls (Chart 6G). As a result total nominal expenditure on non-energy output falls in response to the shock (Chart 6E). Given sticky wages and prices, this fall in nominal demand will lead to a fall in real non-energy final output and employment. The resulting negative output gap and the associated rise in unemployment (Chart 5E) push down on wage inflation. Lower import price inflation also alleviates the pressure on firms' costs. In part this is because the rise in interest differentials caused by the unilateral monetary policy response leads to an appreciation of the effective exchange rate. But there is also a permanent effect on the real exchange rate that will be discussed in more detail below.

The combined fall in wage and import price inflation is broadly sufficient to more than offset the impact of higher energy input costs on non-energy sector inflation (Chart 5D), but is not sufficient for overall CPI inflation to remain unchanged. So the extent to which other prices move to offset the energy price rises is only partial. Chart 6C shows what happens to the level of various prices in the model. Higher relative oil and gas prices drive a wedge between the various relative prices in the model. Ultimately monetary policy determines the consumer price level response and other nominal prices adjust according to the pattern of required relative price movements. The overall CPI price level rises but nominal wages and non-energy final goods prices fall.

It is useful to note what happens to the value added deflator in the non-extraction sector based on the measure inclusive of firms' supernormal mark-up (which is how value added is defined in the national accounts).[6] The value added deflator can be interpreted as the price of firms' final output net of their energy and import costs or, alternatively, as an intermediate price reflecting firms' wages costs, the competitive cost of capital and the mark-up. Initially the price of value added in the non-extraction sector falls, because higher energy costs are not immediately passed through to final consumer and other goods prices. So average profit margins are initially compressed (Chart 5D). As import prices fall and firms restore their profit margins the value added deflator recovers somewhat. But the downward pressure on earnings growth from lower output and employment means that it remains permanently lower than its initial pre-shock level.

[6]In terms of the model the non-extraction value added deflator inclusive of company mark-ups is defined as

$$p^v = (p^b q - p^m m^n - p^p i^p - p^u i^u + p^{ub} q^u - p^g i^g + p^{pb} q^p - p^o i^o)/V$$

Chart 5: Impact of a permanent 100% increase in energy prices (2003 energy shares)

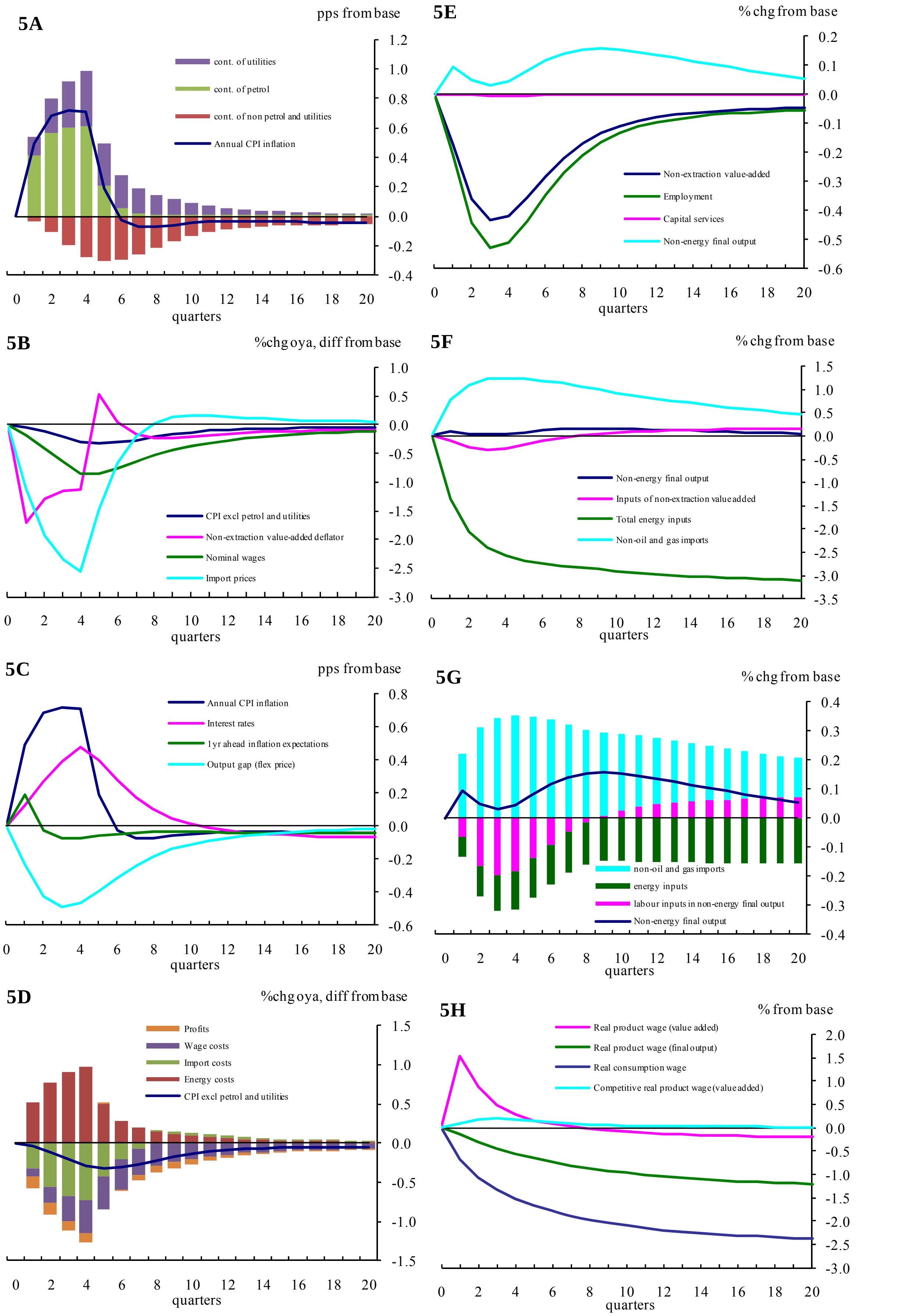
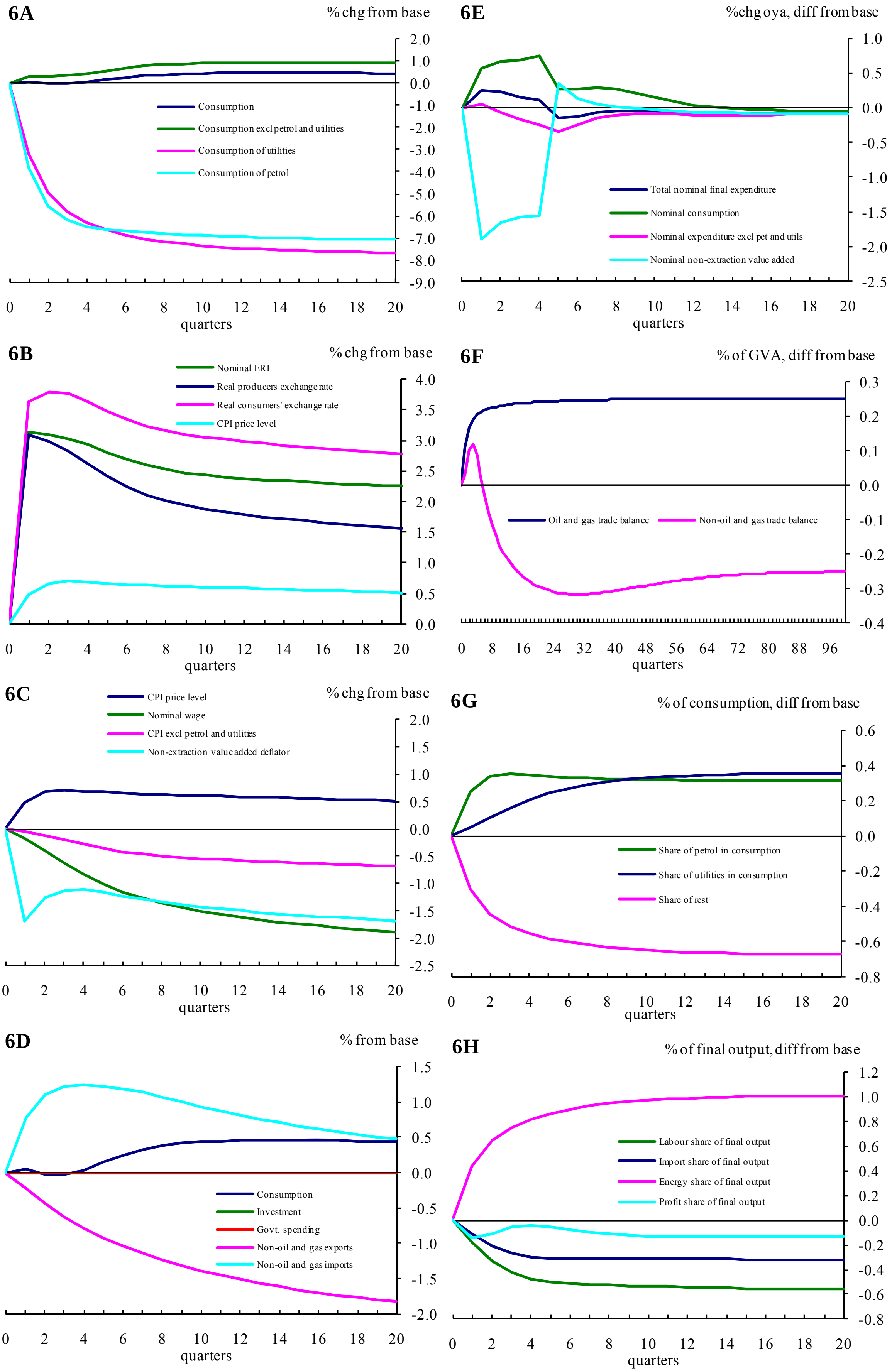


Chart 6: Impact of a permanent 100% increase in energy prices (2003 energy shares)



4.2 *Long-term impact on real factor prices and output*

Higher real energy prices lower the marginal product of other factors of production. Given that in the simplified version of the model labour supply is fixed, the fall in the marginal product of labour must lead to a lower real wage. As Rotemberg and Woodford (1996) stress, it is important to be clear that this is the *real wage defined in terms of final output or consumption goods* that adjusts rather than the real wage expressed in terms of value added. Higher energy prices drive a wedge between the two concepts of the real wage as can be seen in Chart 5H. In perfect competition (with no mark-ups) the real product wage in value added terms will be unchanged in the long run if the capital stock is unchanged and labour supply is inelastic.[7] But the presence of mark-ups and a low elasticity of substitution between energy and other factors imply that the real product wage in value added terms (w/p^v) will fall a little by around 0.2%. As Rotemberg and Woodford (1996) show this is because firms mark up proportionately on all their costs including energy. This implies that the share of supernormal profits in value added will rise slightly following an energy price shock and as a result the real product wage will be lower. But, as shown in Chart 5H, the major adjustment is made by the real consumption wage. The required fall in real consumption wages from a 100% increase in oil and gas prices (and assuming 2003 energy shares) is around 2.5%.

Chart 6H shows what happens to factor shares of non-energy final output. The energy share of final output increases at the expense of the other three factors given the low elasticity of substitution between energy and other inputs. The total profit share declines in the long run mainly as a result of the fall in the normal profit share. Given the assumption of isoelastic demand for varieties of final non-energy goods, the supernormal profit share remains unchanged in the long run.

The long-run impact on consumption and the real exchange rate in the baseline case largely relies on the assumption of self-sufficiency in energy. If the UK economy is self-sufficient then, other things equal, a rise in energy prices should lead to little change in households' real incomes or the equilibrium real exchange rate. This is because households should gain from higher dividends from oil production by exactly an amount that compensates them for a decline in their real labour incomes. But energy is substitutable with other goods and factors. This implies that the United Kingdom's consumption of energy will fall following a rise in energy prices and the energy trade balance will improve (see Chart 6F). In turn this requires an appreciation of the real exchange rate so that the non-energy trade balance falls sufficiently in the long run to keep the current account of the balance of payments in equilibrium.[8] The appreciation of the real exchange rate is also sufficient to allow a small increase in aggregate consumption given the resources freed up by the fall in exports.

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[7] See the response of the 'competitive' real wage, w/p^{vc} using a measure of the value added deflator that does not include the firms' mark-up.

[8] The model assumes that the long-run net foreign asset position of the economy is determined exogenously. But it seems likely that it would in actual fact respond to a permanent energy price shock. Determining the extent and direction of the response, however, would require much more detailed modelling of the composition of UK asset holdings.

In the long run, the potential supply of non-energy final output is little changed and potential value added is relatively unchanged given the simplifying assumption that labour supply is fixed in the long run. Chart 5G shows that the fall in energy inputs lowers output directly, but this is relatively small (-0.15%) given the low degree of substitutability between energy inputs and other factors and its relatively low share of output. The small direct impact of lower energy inputs on output is in line with the observations of Hamilton (1983) and Rotemberg and Woodford (1996) who all note the small direct effect of energy prices on output holding other factor inputs fixed. Offsetting the fall in energy inputs is a positive contribution from higher inputs of imported goods given that the rise in the exchange rate has made them cheaper. Inputs of value added used by the non-energy sector also rise slightly. This reflects the re-allocation of the unchanged stocks of capital and labour towards the non-energy sector given the fall in the consumption and output of the petrol and utilities sector.

5 Variations on the simplified version

In this section we consider how the model responses differ as we relax or change some of the underlying assumptions in the simple baseline case.

5.1 Sensitivity to nominal rigidities

Chart 7 shows the impact on the baseline case when we change the degree of wage and price stickiness in the non-energy sector. In the case where prices are assumed to be more flexible and wages are assumed to be stickier, the effect of the energy price shock on CPI inflation is larger. In the charts we show an extreme case where we increase the probability of changing prices to 0.5 a quarter rather than the 0.25 in the baseline case and make wages almost completely sticky. In this case firms pass on higher energy costs more quickly. For a given level of nominal demand, this means the fall in output must be greater as a result. But there is little downward offset from wage costs despite the contraction in output and lower employment that is generated as a result.

Alternatively when wages and prices are both assumed to be more flexible, the impact on the real economy is smaller. In this case, prices still rise more quickly than the simplified baseline version analysed in Section 4. But as nominal wages now react more quickly to the falls in employment they rapidly offset the higher energy costs. The impact on overall CPI inflation now cycles around zero with the falls in non-energy price inflation occurring later than the pickup in petrol and utility prices. Chart 7 shows the case where we increase the probability of changing prices and wages to 0.5 a quarter (rather than the 0.25 assumed under our baseline calibration). This demonstrates that negative effects on inflation from higher energy prices are possible if the offsetting indirect effects are not synchronous with the direct effects.

Chart 7: Inflation response under nominal different rigidities

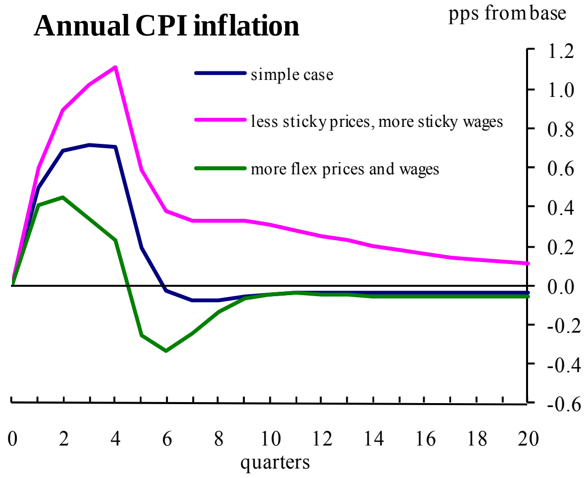
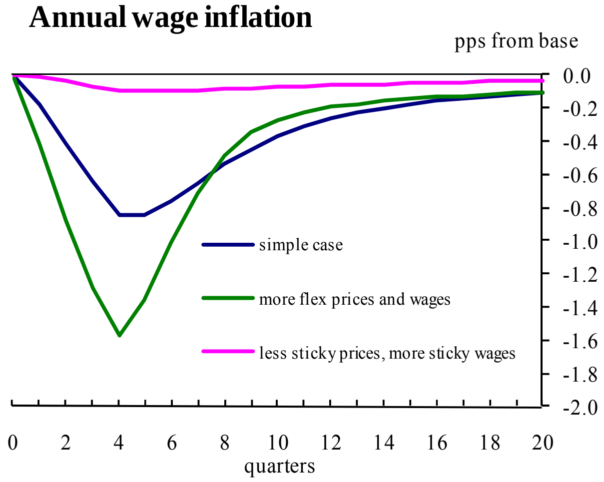


Chart 8: Wage inflation response under different nominal rigidities



5.2 Changing the domestic policy response and the interaction with nominal rigidities

Charts 9 and 10 shows what happens to CPI inflation when we change the assumed policy response. Instead of using a simple Taylor rule, the authorities might attempt to 'accommodate' the energy price shock by looking through the 'first round' or direct impacts of higher energy prices and attempt to stabilise some measure of 'underlying' inflation. In Chart 9 we replace the policy rule with a rule that calls for the complete stabilisation of wage inflation in every period. In this model that is almost identical to the case where the policymaker attempts to keep value added and employment at their flex-price levels, ie there is little or no output gap (Chart 10). In this case the impact on overall inflation is larger as nominal demand is now allowed to expand sufficiently to pay for the higher energy costs faced by each sector while maintaining the wage bill and profits. The required fall in the real consumption wage from higher energy prices is now met entirely through higher prices in the long run when policy is accommodating.

Chart 9 also shows an alternative case where policy attempts to completely stabilise nominal demand growth (specifically nominal consumption growth) in each quarter. In this case we get the opposite effect to accommodation and most of the adjustment in real wages is met through a downward adjustment in nominal wages. This in turn requires a more negative output gap than in the simple case (Chart 10). Charts 9 and 10 also show the more extreme case where the authorities try and stabilise total CPI inflation completely in every period. Unsurprisingly this requires a large initial increase in the output gap in order to generate enough downward adjustment in nominal wages (from the 25% of households who are able to adjust their wage demands in the first period) so that the effect of higher energy costs is completely offset. Note that if prices and wages are fully flexible, nominal demand targeting would be enough to stabilise CPI inflation in each quarter because non-energy inflation would fall synchronously to completely offset the impact of higher energy prices on petrol and utility prices.

Chart 9: Inflation impact under different policy responses

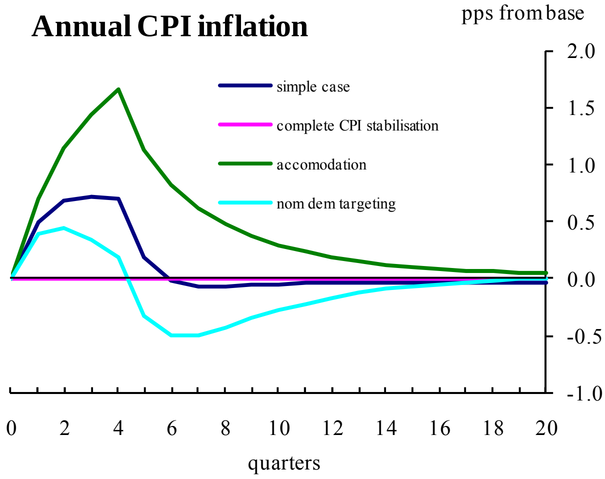
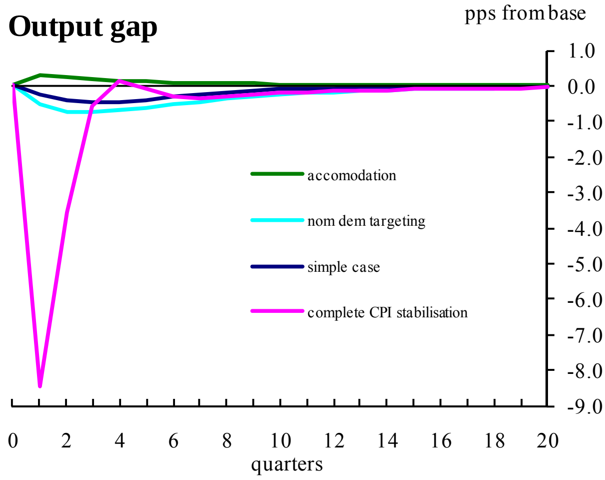


Chart 10: Output gap impact under different policy responses



Overall this analysis underlines the importance of the monetary policy reaction in determining the dynamic behaviour of the model.

5.3 Changing the overseas policy response

In the simplified case the monetary policy reaction to higher energy prices was assumed to be unilateral. But overseas prices and interest rates (and other variables) are also likely to change in response to a rise in global energy prices. This reaction is likely to have a significant bearing on the extent to which exchange rates and import prices respond to the shock. To show the sensitivity of an overseas policy response we make the extreme assumption that overseas interest rates and inflation move in exactly the same way as UK interest rates and inflation. This implies adding the equations to the model so that foreign interest rates and inflation become endogenous variables. Chart 11 shows that this modification implies a larger inflation response. This is because the real exchange no longer overshoots its long-run level in contrast to the simplified model, given that interest rate and inflation differentials remain unchanged (Chart 12). This implies that import price inflation falls less than in the simplified version analysed earlier.

Chart 11: Inflation impact with overseas policy response

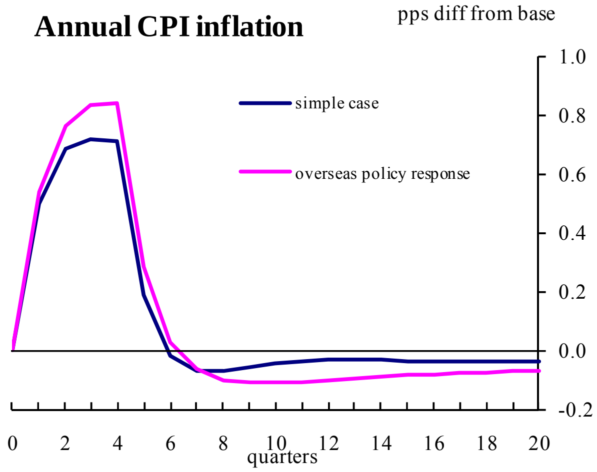
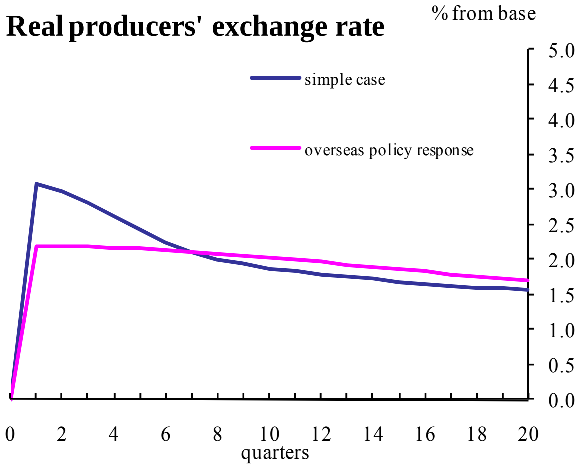


Chart 12: Real exchange rate impact with overseas policy response



5.4 Sensitivity to self-sufficiency: consumption and the real exchange rate

In this section we relax the assumption of self-sufficiency in energy. As is well known, the United Kingdom is predicted to become a significant net importer of energy over the next 10-20 years due to a fall in extraction of oil and natural gas in the North Sea. In Chart 13 we show the sensitivity of the impact of higher energy prices to two different assumptions about the energy deficit in steady state. The first assumes a primary energy deficit of 1% of non-oil value added, (approximately the size deficit towards the end of 2010); the second shows the impact of a 2% deficit which is approximately what the United Kingdom will face in 2020 if production falls in the most pessimistic case.[9]

Chart 13: Inflation impact, energy deficit case

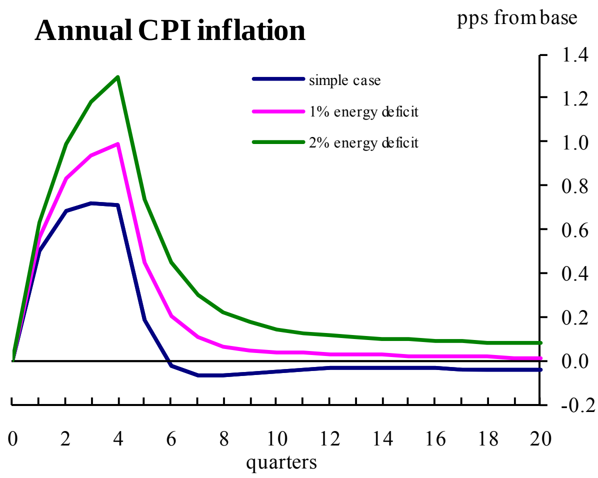
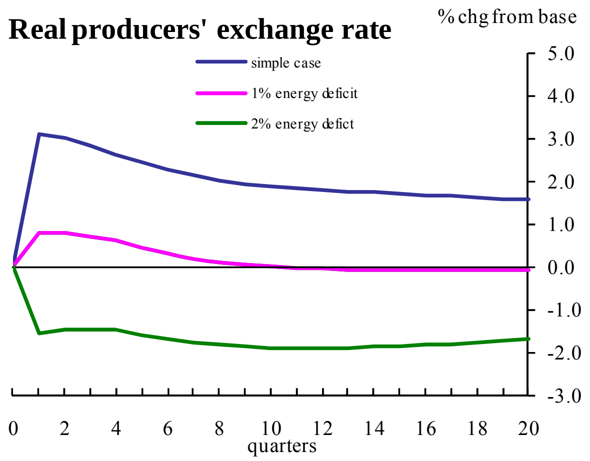


Chart 14: Real exchange rate impact, energy deficit case



[9]See the latest UK offshore operators annual report for 2010 at www.oilandgasuk.co.uk/cmsfiles/modules/publications/pdfs/EC021.pdf.

Chart 15: Real consumption wages, energy deficit case

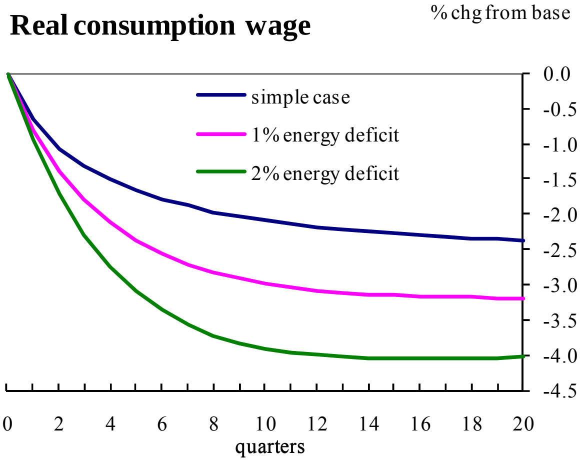
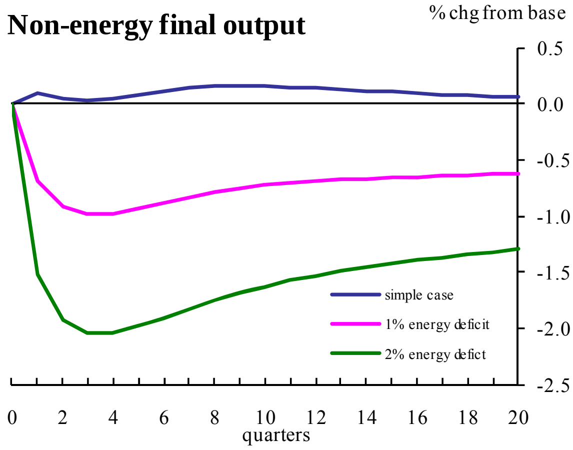


Chart 16: Final output, energy deficit case



The impact on aggregate household consumption from assuming an initial deficit is now negative. This is because the negative effect on the present value of real labour income now exceeds the positive effect from a higher present value of future dividends from oil production. The real exchange rate no longer appreciates and depreciates significantly in the 2% deficit case (Chart 14). This is because with an energy deficit, higher energy prices will ultimately have a larger effect on the value of oil and gas imports than they do on exports. This projected increase in the energy trade deficit exceeds the decline resulting from the fall in domestic use of oil and gas inputs. This means that non-energy exports now need to rise in equilibrium to pay for the worsening of the energy trade deficit. And the real exchange rate must depreciate to ensure this adjustment takes place. This means that the impact on import prices is positive and means that the impact on CPI inflation is larger than the simple case. A further implication is that the required fall in the real consumption wage is larger given that nominal import prices now rise rather than fall in response to higher energy prices (Chart 15). And the impact on final output is now negative as both energy and imported inputs fall in the long run (Chart 16).

Note the results are driven by the fact that, in the model, UK consumers are assumed to hold shares only in the domestic oil well and natural gas field. Indeed, the model assumes that they own all of the shares in the domestic oil and gas well, so there is no foreign ownership of energy production and implicitly any government tax receipts from energy production are passed back to consumers. The theory of international risk-sharing would suggest UK consumers can hedge themselves against the falls in real income arising from higher energy prices by holding shares in overseas energy production (see Bodenstein, Erceg and Guerrieri (2007)). This means that when energy prices rise, UK consumers are compensated by dividends from claims on overseas as well as domestic energy production. And the worsening of the energy trade balance is offset by higher net dividend payments from overseas so the current account of the balance of payments remains unchanged. In other words, it is ownership rather than the production of energy that matters. Most private energy companies are large multinationals operating in a variety of regions of the world. So holding shares in these companies necessarily implies ownership of claims on both domestic and overseas energy production. For example 44% of shares in BP are held by UK residents and institutions,[10] but most of BP's operations are overseas suggesting that UK residents earn dividends on overseas oil production. This means that UK consumers are implicitly hedged against oil price increases to some degree, so these simulations may exaggerate the impact of an energy trade deficit on the UK economy's response to an energy price shock.

5.5 Sensitivity to real wage resistance

The issue of real wage resistance has often been cited as a reason why higher energy prices can have effects on real output over and above its direct effect on energy inputs in the production function (see eg Blanchard and Gali (2007)). But there is considerable debate about how long lasting these effects might be. We implement real wage resistance in our labour market framework as follows.

(i) We alter the indexation rule used by the proportion of workers who do not reset their wage optimally in a given period. In our baseline calibration, the indexation rule places equal weight on the inflation target and lagged (quarterly) nominal wage inflation. To generate real consumption wage rigidity we replace the indexation rule with one which indexes wages to a four-quarter moving average of CPI inflation (ie approximately annual inflation expressed as a quarterly rate). This is similar to the adjustment made to introduce real wage rigidity in Blanchard and Gali (2007).

(ii) In the long run we no longer assume labour supply is inelastic as in the simple case of Section 4 so employment will move in the long run according to the (Frisch) labour supply elasticity of 0.43 estimated by HO.

Chart 17: Wage inflation response under real wage resistance

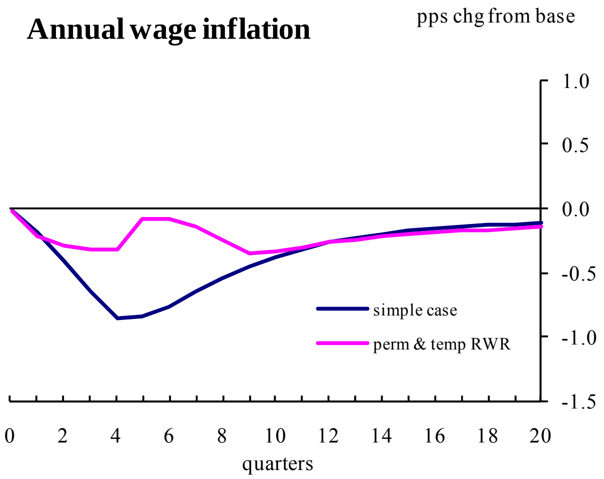
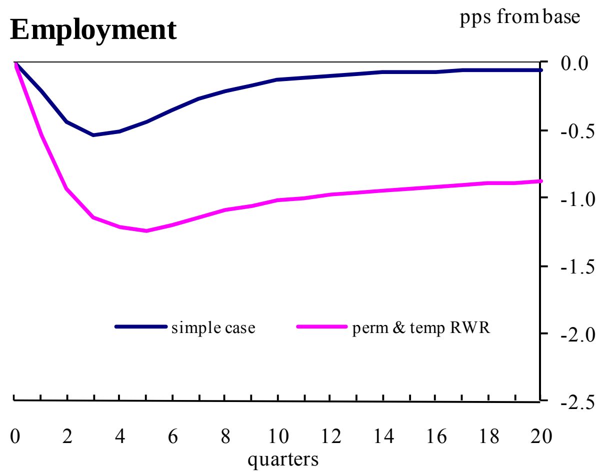


Chart 18: Employment response under real wage resistance



[10]See BP operating information 2006-10.

Charts 17 and 18 show the impact on earnings and employment of assuming the real wage resistance implied by our two assumptions. It leads to a more muted response of earnings growth, given that the indexation rule feeds off increased CPI inflation. This leads to a larger short-term fall in output and employment and a larger total impact on CPI inflation. Because we assume that the real wage resistance is permanent and that the lower real consumption wage leads to a fall in labour supply, this leads to an impact on employment of around 0.9% in the long run, and a much larger fall in potential supply than in the simplified case.

5.6 Sensitivity to capital utilisation

Finally we consider allowing the level and utilisation of the capital stock to adjust to the energy price shock. Higher energy prices and lower energy inputs lead to a fall in the marginal product of capital (in final output terms) that lowers the desire of firms to rent capital from households. This leads to a fall in investment spending and the capital stock falls by just under 0.8% in the long run. This has a small effect on the required adjustment of the real product wage (around 0.2%) and on potential supply.

A larger impact on potential supply can be generated if we make the utilisation of the capital stock dependent on energy, following Finn (2000) and using the parameter settings discussed earlier (setting the parameter χ_e to a positive value). So energy prices will now have an effect on capital services via the utilisation and depreciation rate of capital. In this case the impact on capital services is much more pronounced leading to a long-run fall of around 4% (Chart 20). In crude terms this permanent fall in utilisation would be consistent with a scrapping story in a vintage capital model. If energy prices are permanently higher it is too costly to utilise the entire capital stock and some capital is never used again. The larger fall in potential supply implies that oil prices will have a more pronounced medium-term impact on inflation than in our simple case (Chart 19).

Chart 19: Inflation impact with capital response

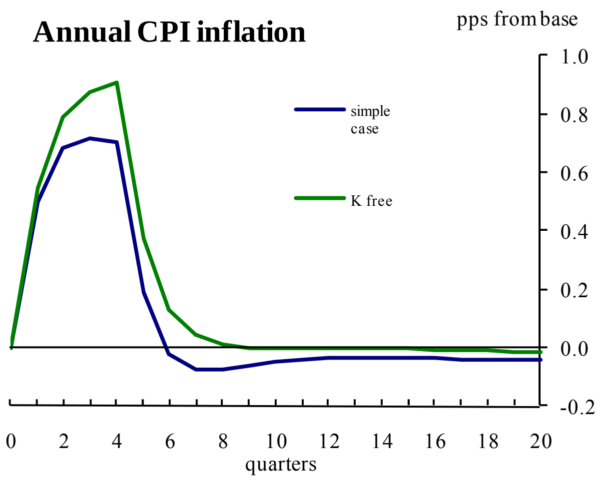
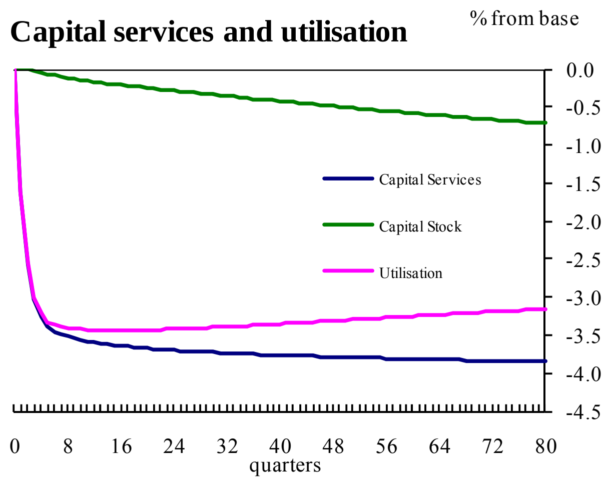


Chart 20: Capital response to energy price shock



6 Conclusion

In this paper we have outlined the key properties of a model that can be used to examine the impact of higher energy prices on the UK economy. We have calibrated it to UK data and examined how the various channels we identify contribute to the responses to permanent energy price shocks of a similar magnitude to those observed in the data. The impact of higher energy prices depends significantly on the monetary policy response to higher energy prices, both here and abroad. The degree of self-sufficiency in energy also makes a large difference via the effects on consumption and the real exchange rate. On the supply side, significant effects are possible through the effects of real consumption-wage rigidity on employment and the higher cost of energy on the degree of capital utilisation. In a companion paper, Millard (2011) uses an estimated stochastic version of the model to examine the contribution of energy prices to inflation and output in the recent data.

Appendices

Appendix A – Model derivation

The model is based on the DSGE model developed by Harrison and Oomen (2010) – HO henceforth – though it is extended to incorporate a range of energy price effects. This appendix provides the derivation of the model. Most of the explanation is given for features that are an extension of the HO model.

Households

Following HO, there is a continuum of households of unit mass. Each household, indexed by $j \in (0,1)$, maximises a utility function defined over consumption (c), hours worked (h) and real money balances (MON / P^c). Their budget constraint describes how end of period holdings of nominal government debt (BG), nominal foreign bonds (BF), capital (k) and money (MON) are given by their start of period holdings, plus net income. Net income consists of earnings from supplying labour (at wage W) and capital services ($k_{t-1}z_t$, rented at rate W^K) to firms plus dividend payments (DV) from firms less expenditures on consumption (c), taxes (τ) adjustment costs (discussed below) and the cost of servicing capital. Capital depreciates at a rate of

$$\delta + \frac{\chi^z}{1+\phi^z}\left[z_t(j)^{1+\phi_z} - z_{ss}^{1+\phi_z}\right]$$

where z is the utilisation rate and z_{ss} denotes its steady-state level. The nominal price of domestic output is P, the nominal price of consumption is P^c and the nominal exchange rate (units of domestic currency per unit of foreign currency) is S.

In contrast to HO, we assume that the domestic economy costlessly operates an oil well and a gas field that produce exogenous (and potentially time-varying) flows of oil and gas denoted $\overline{O}_t$ and $\overline{G}_t$, respectively. Proceeds from the sale of oil and gas on world markets (sold at nominal prices $P_t p_t^o$ and $P_t p_t^g$ respectively) are distributed lump sum to consumers. We also assume that the capital utilisation decision depends on the price of energy, following Finn (2000). Specifically we assume that the household must purchase e_z units of energy according to the following relationship:

$$\frac{e_{z,t}(j)}{k_{t-1}(j)} = \frac{\chi_e}{\gamma_e} z_t(j)^{\gamma_e}$$

for $\chi_e \geq 0$ and $\gamma_e > 1$. We can think of equation energy for utilisation demand as a demand curve for energy. The amount of energy per unit of capital stock is positively related to the capital utilisation rate: using the capital stock more intensively requires more energy. So an increase in the price of energy creates an incentive to utilise capital less intensively.
The maximisation problem is therefore given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\left[1 - \frac{1}{\sigma^c}\right]^{-1} \left[\frac{c_t(j)}{c_{t-1}^{\psi^{hab}}}\right]^{1-\frac{1}{\sigma^c}} - \frac{\sigma^h (\kappa^h)^{-\frac{1}{\sigma^h}}}{\sigma^h + 1} h_{t+r}(j)^{\frac{\sigma^h+1}{\sigma^h}} + \frac{(\kappa^{mon})^{\frac{1}{\sigma^c}}}{1-\frac{1}{\sigma^c}} \left[\frac{MON_t(j)}{P_t^c}\right]^{1-\frac{1}{\sigma^c}} \right]$$

subject to

$$
\begin{gathered}
BG_t(j) + MON_t(j) + \tfrac{BF_t(j)}{S_t} + P_t\tfrac{\chi^{bf}}{2}\Big[\tfrac{BF_t(j)}{P_tS_t} - nfa^{ss}\Big]^2 + P_tk_t(j) - MON_{t-1}(j) \\
-R^g_{t-1}BG_{t-1}(j) - R^f_{t-1}\tfrac{BF_{t-1}(j)}{S_t} - P_t\Big[1 - \Big(\delta + \tfrac{\chi^z}{1+\phi^z}\Big[z_t(j)^{1+\phi_z} - z_{ss}^{1+\phi_z}\Big]\Big)\Big]k_{t-1}(j) \\
+P_t\tfrac{\chi^k}{2k_{t-1}}\Big[k_t(j) - (k_{t-1}/k_{t-2})^{\epsilon^k}k_{t-1}(j)\Big]^2 - W^K_tz_t(j)k_{t-1}(j) - W_t(j)h_t(j) - P_tp^o_t\bar{O}_t \\
-P_tp^g_t\bar{G}_t + P^c_tc_t(j) + P_tp^e_te_{z,t}(j) - DV_t + P_t\tau_t = 0
\end{gathered}
$$

As noted above, the budget constraint contains a number of adjustment costs. As in HO, we include costs of adjusting foreign bond holdings:

$$\frac{\chi^{bf}}{2}\left[\frac{BF_t(j)}{P_tS_t} - nfa^{ss}\right]^2$$

to ensure that the net foreign asset position of the economy is pinned down. We also include quadratic capital adjustment costs of the same form as HO:

$$\frac{\chi^k}{2}\frac{\left[k_t(j) - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\epsilon^k}k_{t-1}(j)\right]^2}{k_{t-1}}$$

The first-order conditions for consumption, money, government bonds, foreign bonds, capital and utilisation are:

$$\left[\frac{c_t(j)}{c_{t-1}^{\psi^{hab}}}\right]^{-\frac{1}{\sigma^c}}\frac{1}{c_{t-1}^{\psi^{hab}}} - P^c_t\lambda_t = 0$$

$$(P^c_t)^{\frac{1}{\sigma^c}-1}\left[\frac{MON_t(j)}{\kappa^{mon}}\right]^{-\frac{1}{\sigma^c}} - \lambda_t + \beta E_t\lambda_{t+1} = 0$$

$$-\lambda_t + \beta R^g_tE_t\lambda_{t+1} = 0$$

$$-\frac{\lambda_t}{S_t} - \chi^{bf}\frac{\lambda_t}{S_t}\left[\frac{BF_t(j)}{P_tS_t} - nfa^{ss}\right] + \beta R^f_tE_t\frac{\lambda_{t+1}}{S_{t+1}} = 0$$

$$
\begin{aligned}
&\lambda_tP_t\left[1 + \chi^k\left(\frac{k_t(j)}{k_{t-1}} - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\epsilon^k}\frac{k_{t-1}(j)}{k_{t-1}}\right)\right] \\
&= \beta E_t\lambda_{t+1}P_{t+1}\left[\begin{array}{c}
1 - \left(\delta + \frac{\chi^z}{1+\phi^z}\left[z_{t+1}(j)^{1+\phi_z} - z_{ss}^{1+\phi_z}\right]\right) \\
+\chi^k\left(\frac{k_{t+1}(j)}{k_t} - \left(\frac{k_t}{k_{t-1}}\right)^{\epsilon^k}\frac{k_t(j)}{k_t}\right)\left(\frac{k_t}{k_{t-1}}\right)^{\epsilon^k} \\
+\frac{W^k_{t+1}}{P_{t+1}}z_{t+1}(j) - p^e_{t+1}\frac{\chi_e}{\gamma_e}z_{t+1}(j)^{\gamma_e}
\end{array}\right]
\end{aligned}
$$

$$W^K_tk_{t-1}(j) - \chi^zz_t(j)^{\phi^z}P_tk_{t-1}(j) - P_tp^e_t\chi_ek_{t-1}(j)z_t(j)^{\gamma_e-1} = 0$$

where λ is the Lagrange multiplier on the budget constraint.

Following HO we assume that each household j supplies a differentiated labour input to the production sector. The labour index, which aggregates over all available labour type, has the CES form:

$$h_t \equiv \left[\int_0^1 h_t(j)^{\frac{\sigma^w - 1}{\sigma^w}} dj \right]^{\frac{\sigma^w}{\sigma^w - 1}},$$

where the parameter $\sigma^w > 1$ is the elasticity of substitution among labour services. The aggregate nominal wage index associated with the minimum cost needed to hire a unit of the composite labour given each household's wage rate is given by:

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\sigma^w} dj \right]^{1-\sigma^w}.$$

Hence, each household i faces a downward sloping demand curve for its own labour.

$$h_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\sigma^w} h_t.$$

Households set (nominal) wages in staggered contracts. In particular, a randomly selected fraction (ψ^w) of households can renegotiate their wage contracts in each period. Whenever a household j has not reset its contract wage since period t, then its wage rate in period $t+r$ is adjusted by an indexation factor, $\xi^w_{t,t+r}$. That is

$$W_{t,t+r}(j) = \xi^w_{t,t+r} \widetilde{W}_t(j) \ .$$

Here, $\widetilde{W}_{t,}(j)$ denotes the optimal wage rate set in period t, and $W_{t,t+r}(j)$ denotes the wage rate in period $t+r$ faced by a household that has set its wage rate at time t. Following HO, the indexation factor is

$$\xi^w_{t,t+r} = \begin{cases} 1 & \text{if } r = 0 \\ (\Pi^{ss})^{1-\epsilon^w} \left(\frac{W_{t+r-1}}{W_{t+r-2}} \right)^{\epsilon^w} \xi^w_{t,t+r-1} & \text{if } r \geq 1. \end{cases}$$

This expression implies that if a household who has set wages in period t does not receive a signal to update its wage at time $t+r$ its wage rate is increased in proportion with the weighted average of the steady-state rate of (gross) price inflation Π^{ss} and the lagged (gross) nominal wage inflation. The parameter $0 < \varepsilon^w < 1$ is the weight attached to the latter. In any period t in which household j is able to reset its contract wage, $W_t(j)$, it aims to maximise the following expression:

$$E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^w)^r \left\{ \Lambda_{t+r} \xi^w_{t,t+r} W_t(j) h_{t+r}(j) - (\kappa^h)^{-\frac{1}{\sigma^h}} \frac{\sigma^h}{\sigma^h + 1} [h_{t+r}(j)]^{\frac{\sigma^h+1}{\sigma^h}} \right\},$$

with first-order condition:

$$0 = E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r \Lambda_{t+r} \xi^w_{t,t+r} (1-\sigma^w) \left(\frac{\xi^w_{t,t+r} \widetilde{W}_t(j)}{W_{t+r}} \right)^{-\sigma^w} h_{t+r}$$

$$+ E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r (\kappa^h)^{-\frac{1}{\sigma^h}} \sigma^w \left[\left(\frac{\xi^w_{t,t+r} \widetilde{W}_t(j)}{W_{t+r}} \right)^{-\sigma^w} h_{t+r} \right]^{\frac{\sigma^h+1}{\sigma^h}} \widetilde{W}_t(j)^{-1}.$$

which can be written in terms of the optimal real wage rate $\widetilde{w}_t \equiv \widetilde{W}_t / P_t$ as:

$$\widetilde{w}_t^{\,1+\frac{\sigma^w}{\sigma^h}} = w_t^{\frac{\sigma^w}{\sigma^h}} \frac{(\kappa^h)^{-\frac{1}{\sigma^h}} \sigma^w}{\sigma^w - 1} \frac{\Xi^w_t}{\Theta^w_t},$$

where

$$\Xi^w_t = h_t^{\frac{\sigma^h+1}{\sigma^h}} + \beta(1-\psi^w) E_t \left[\frac{\xi^w_t w_t}{w_{t+1} \Pi_{t+1}} \right]^{-\sigma^w(\frac{\sigma^h+1}{\sigma^h})} \Xi^w_{t+1}$$

where

$$\xi^w_t = (\Pi^{ss})^{1-\epsilon^w} \left(\frac{W_t}{W_{t-1}} \right)^{\epsilon^w}$$

$$= (\Pi^{ss})^{1-\epsilon^w} \left(\frac{w_t}{w_{t-1}} \Pi_t \right)^{\epsilon^w}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the rate of (gross) output price inflation in period t, and

$$\Theta^w_t = \Lambda_t h_t + \beta(1-\psi^w) E_t \Pi^{-1}_{t+1} \xi^w_t \left[\frac{\xi^w_t w_t}{\Pi_{t+1} w_{t+1}} \right]^{-\sigma^w} \Theta^w_{t+1}.$$

The nominal wage index in period t satisfies:

$$W_t^{1-\sigma^w} \equiv \int_0^1 W_t(j)^{1-\sigma^w} dj$$

$$= (1-\psi^w)(\xi^w_{t-1} W_{t-1})^{1-\sigma^w} + \psi^w \widetilde{W}_t^{1-\sigma^w}.$$

The real wage index can then be written as:

$$w_t^{1-\sigma^w} = (1-\psi^w) \left(\xi^w_{t-1,t} \frac{w_{t-1}}{\Pi_t} \right)^{1-\sigma^w} + \psi^w \widetilde{w}_t^{\,1-\sigma^w}.$$

Unlike HO, we assume that the final consumption bundle consists of a CES aggregate of (domestically produced) non-energy goods (c^n) and energy (c^e):

$$c_t \equiv \kappa^c \left[(1-\psi^e)\{(1-\phi^e) c^n_t\}^{1-\frac{1}{\sigma^e}} + \psi^e \{\phi^e c^e_t\}^{1-\frac{1}{\sigma^e}} \right]^{\frac{\sigma^e}{\sigma^e - 1}}$$

where consumption of energy is defined in terms of consumption of petrol (c^p) and utilities (c^u):

$$c^e_t \equiv \kappa^e \left[(1-\psi^p)\{(1-\phi^p) c^u_t\}^{1-\frac{1}{\sigma^p}} + \psi^p \{\phi^p c^p_t\}^{1-\frac{1}{\sigma^p}} \right]^{\frac{\sigma^p}{\sigma^p - 1}}$$

Nominal expenditure on consumption is simply

$$P_t^c c_t = P_t c_t^n + P_t p_t^u c_t^u + P_t p_t^p c_t^p$$

Optimal consumption choices imply that the relative demands for consumption goods solve the following problem:

$$\max \kappa^c \left[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \psi^e \{c_t^e\}^{1-\frac{1}{\sigma^e}} \right]^{\frac{\sigma^e}{\sigma^e-1}} - \nu[P_t c_t^n + P_t p_t^u c_t^u + P_t p_t^p c_t^p - Z]$$

which has the following first-order conditions:

$$0 = \kappa^c \left[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \psi^e \{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}} \right]^{\frac{\sigma^e}{\sigma^e-1}-1} (1-\psi^e)\{(1-\phi^e)\}^{1-\frac{1}{\sigma^e}} \{c_t^n\}^{-\frac{1}{\sigma^e}} - \nu P_t$$

$$\begin{aligned} 0 = {} & \kappa^c \left[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \psi^e \{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}} \right]^{\frac{\sigma^e}{\sigma^e-1}-1} \psi^e \{\phi^e\}^{1-\frac{1}{\sigma^e}} \{c_t^e\}^{-\frac{1}{\sigma^e}} \\ & \times \kappa^e \left[(1-\psi^p)\{(1-\phi^p)c_t^u\}^{1-\frac{1}{\sigma_p}} + \psi^p \{\phi^p c_t^p\}^{1-\frac{1}{\sigma_p}} \right]^{\frac{\sigma_p}{\sigma_p-1}-1} (1-\psi^p)\{(1-\phi^p)\}^{1-\frac{1}{\sigma_p}} \{c_t^u\}^{-\frac{1}{\sigma_p}} \\ & - \nu P_t p_t^u \end{aligned}$$

$$\begin{aligned} 0 = {} & \kappa^c \left[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \psi^e \{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}} \right]^{\frac{\sigma^e}{\sigma^e-1}-1} \psi^e \{\phi^e\}^{1-\frac{1}{\sigma^e}} \{c_t^e\}^{-\frac{1}{\sigma^e}} \\ & \times \kappa^e \left[(1-\psi^p)\{(1-\phi^p)c_t^u\}^{1-\frac{1}{\sigma_p}} + \psi^p \{\phi^p c_t^p\}^{1-\frac{1}{\sigma_p}} \right]^{\frac{\sigma_p}{\sigma_p-1}-1} \psi^p \{\phi^p\}^{1-\frac{1}{\sigma_p}} \{c_t^p\}^{-\frac{1}{\sigma_p}} \\ & - \nu P_t p_t^p \end{aligned}$$

which can be represented as

$$\frac{\left(\frac{1-\psi^e}{\psi^e}\right)\left\{\frac{1-\phi^e}{\phi^e}\right\}^{1-\frac{1}{\sigma^e}} \left\{\frac{c_t^n}{c_t^e}\right\}^{-\frac{1}{\sigma^e}} \left\{\frac{c_t^e}{c_t^u}\right\}^{-\frac{1}{\sigma_p}}}{[\kappa^e]^{1-\frac{1}{\sigma_p}} (1-\psi^p)\{(1-\phi^p)\}^{1-\frac{1}{\sigma_p}}} = \frac{1}{p_t^u}$$

and

$$\frac{1-\psi^p}{\psi^p}\left\{\frac{1-\phi^p}{\phi^p}\right\}^{1-\frac{1}{\sigma_p}} \left\{\frac{c_t^u}{c_t^p}\right\}^{-\frac{1}{\sigma_p}} = \frac{p_t^u}{p_t^p}$$

Firms

We assume that value added is produced by combining domestic capital and labour using a CES production function:

$$V_t = \kappa^v \left[(1-\alpha_v)\{(1-\phi_v)h_t\}^{1-\frac{1}{\sigma^v}} + \alpha_v \{\phi_v K_t^s\}^{1-\frac{1}{\sigma^v}} \right]^{\frac{\sigma^v}{\sigma^v-1}}$$

where h is total hours and K^s represents capital services rented from households. This sector is perfectly competitive so that factor demands are implied by profit maximisation:

$$\max P_t p_t^{vc} \kappa^v \left[(1-\alpha_v)\{(1-\phi_v)h_t\}^{1-\frac{1}{\sigma^v}} + \alpha_v \{\phi_v K_t^s\}^{1-\frac{1}{\sigma^v}} \right]^{\frac{\sigma^v}{\sigma^v-1}} - P_t w_t h_t - P_t w_t^k K_t^s$$

giving:

$$(1-\alpha_v)\{\kappa^v(1-\phi_v)\}^{1-\frac{1}{\sigma v}}\left\{\frac{V_t}{h_t}\right\}^{\frac{1}{\sigma v}} = \frac{w_t}{p_t^{vc}}$$

and

$$\alpha_v\{\kappa^v\phi_v\}^{1-\frac{1}{\sigma v}}\left\{\frac{V_t}{K_t^s}\right\}^{\frac{1}{\sigma v}} = \frac{w_t^k}{p_t^{vc}}$$

where p^{vc} denotes the perfectly competitive price of value added, which can be derived from the zero-profit condition:

$$p_t^{vc} = w_t\frac{h_t}{V_t} + w_t^k\frac{K_t^s}{V_t}$$

Final non-energy output is produced by firms operating the following production function:

$$q_t = A_t\Big[(1-\alpha_q)\{(1-\phi_q)B_t\}^{1-\frac{1}{\sigma q}} + \alpha_q\{\phi_q E_t\}^{1-\frac{1}{\sigma q}}\Big]^{\frac{\sigma q}{\sigma q-1}}$$

where q is *final* output, consisting of a bundle (B ,defined below) that combines value added and imports and 'energy' (E). We denote exogenous productivity by A.

The bundle of value added (V^N) and imports (M^N) is a Cobb-Douglas aggregator:

$$B_t = \kappa^B(V_t^N)^{1-\alpha_B}(M_t^N)^{\alpha_B}$$

The energy input is a Leontief bundle of petrol and utilities:

$$e_t = \kappa_N \min\left\{\frac{I_t^p}{\psi_N}, \frac{I_t^u}{1-\psi_N}\right\}$$

where I^p and I^u denote intermediate inputs of petrol and utilities. Efficient use of energy inputs implies the following fixed-proportion factor demand conditions:

$$I_t^p = \psi_N e_t$$
$$I_t^u = (1-\psi_N)e_t$$

Nominal dividends are defined as follows:

$$DV_t^q = P_t^b q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N - P_t p_t^p I_t^p - P_t p_t^u I_t^u$$

which says that dividends are the difference between the value of output sold (at basic price, P_t^b) and purchases of value added, petrol and utilities (at market prices). Since petrol and utilities are used in fixed proportions to form the energy input, we can write the dividend flow as:

$$DV_t^q = P_t^b q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N - P_t[\psi_N p_t^p + (1-\psi_N)p_t^u]e_t$$

and treat energy as a single input with price $P_t\left[\psi_N p_t^p + (1-\psi_N)p_t^u\right]$.

Firms maximise the discounted flow of dividends net of the costs of adjusting prices:

$$E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\begin{array}{c} P_t^b(k) \left(\frac{P_t^b(k)}{P_t^b} \right)^{-\eta} q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N \\ -P_t [\psi_N (1+\tau_t^{vat}) p_t^p + (1-\psi_N)(1+\tau_t^u) p_t^u] E_t - \frac{\chi^p}{2} \left(\frac{\frac{P_t^b(k)}{P_{t-1}^b(k)}}{(\Pi^{ss})^{1-\epsilon} \left(\frac{P_{t-1}^b}{P_{t-2}^b} \right)^{\epsilon}} - 1 \right)^2 P_t^b q_t \\ -LM_t \left[\left(\frac{P_t^b(k)}{P_t^b} \right)^{-\eta} q_t - A_t \left[(1-\alpha_q)\{(1-\phi_q) B_t\}^{1-\frac{1}{\sigma^q}} + \alpha_q \{\phi_q e_t\}^{1-\frac{1}{\sigma^q}} \right]^{\frac{\sigma^q}{\sigma^q - 1}} \right] \end{array} \right]$$

subject to (value added) and (materials aggregator). This can be represented as:

$$E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\begin{array}{c} P_t^b(k) \left(\frac{P_t^b(k)}{P_t^b} \right)^{-\eta} q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N \\ -P_t [\psi_N p_t^p + (1-\psi_N) p_t^u] e_t - \frac{\chi^p}{2} (\xi_t^p(k))^2 P_t^b q_t \\ -LM_t \left[\left(\frac{P_t^b(k)}{P_t^b} \right)^{-\eta} q_t - A_t \left[(1-\alpha_q)\{(1-\phi_q) B_t\}^{1-\frac{1}{\sigma^q}} + \alpha_q \{\phi_q e_t\}^{1-\frac{1}{\sigma^q}} \right]^{\frac{\sigma^q}{\sigma^q - 1}} \right] \end{array} \right]$$

where

$$\xi_t^p(k) = \frac{\frac{P_t^b(k)}{P_{t-1}^b(k)}}{(\Pi^{ss})^{1-\epsilon} \left(\frac{P_{t-1}^b}{P_{t-2}^b} \right)^{\epsilon}} - 1$$

summarises the adjustment cost for prices. The adjustment costs depend on the rate at which firm k adjusts its price ($P^b(k)$) relative to a weighted average of trend inflation and lagged aggregate price inflation. This formulation has very similar effects to the assumptions about wage stickiness described above. The first-order conditions are:

$$\frac{P_t p_t^{vc}}{p_t^b} = LM_t A_t^{1-\frac{1}{\sigma^q}} (1-\alpha_q)(1-\phi_q)^{1-\frac{1}{\sigma^q}} \left[\frac{q_t}{B_t} \right]^{\frac{1}{\sigma^q}} (1-\alpha_B) \frac{B_t}{V_t^N}$$

$$\frac{P_t p_t^m}{p_t^b} = LM_t A_t^{1-\frac{1}{\sigma^q}} (1-\alpha_q)(1-\phi_q)^{1-\frac{1}{\sigma^q}} \left[\frac{q_t}{B_t} \right]^{\frac{1}{\sigma^q}} \alpha_B \frac{B_t}{M_t^N}$$

$$P_t \frac{\psi_N p_t^p + (1-\psi_N) p_t^u}{p_t^b} = LM_t A_t^{1-\frac{1}{\sigma^q}} \alpha_q \phi_q^{1-\frac{1}{\sigma^q}} \left[\frac{q_t}{e_t} \right]^{\frac{1}{\sigma^q}}$$

$$
\begin{aligned}
&(1-\eta)\left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta} q_t - \chi^p \xi_t^p(k) P_t^b q_t \frac{\xi_t^p(k)+1}{P_t^b(k)} \\
&+ LM_t \eta \left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta} \frac{q_t}{P_t^b(k)} \\
&= -\beta \chi^p E_t \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^p(k) P_{t+1}^b q_{t+1} \frac{\xi_{t+1}^p(k)+1}{P_t^b(k)}
\end{aligned}
$$

Finally, we turn to the production of energy goods. We assume that utility output (petrol) is produced according to a Leontief combination of value added and gas (oil):

$$q_t^u = \min\left\{\frac{I_t^g}{1-\psi^u}, \frac{V_t^u}{\psi^u}\right\}$$

and

$$q_t^p = \min\left\{\frac{I_t^o}{1-\psi^{qp}}, \frac{V_t^p}{\psi^{qp}}\right\}$$

The factor demands are then simply linear functions of total production:

$$
\begin{aligned}
I_t^g &= (1-\psi^u) q_t^u \\
V_t^u &= \psi^u q_t^u \\
I_t^o &= (1-\psi^{qp}) q_t^p \\
V_t^p &= \psi^{qp} q_t^p
\end{aligned}
$$

Nominal dividends from utilities production are given by:

$$DV_t^u = P_t^{ub} q_t^u - P_t p_t^{vc} V_t^u - P_t p_t^g I_t^g$$

which, given the factor demands can be written as:

$$DV_t^u = \left[P_t^{ub} - \psi^u P_t p_t^{vc} - (1-\psi^u) P_t p_t^g\right] q_t^u$$

We assume monopolistic competition so that the demand schedule for utilities is:

$$q_t^u(k) = \left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} q_t^u$$

and that utilities producers maximise the discounted flow of dividends subject to price adjustment costs:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\left[P_t^{ub}(k) - \psi^u P_t p_t^{vc} - (1-\psi^u) P_t p_t^g\right] \left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} q_t^u - \frac{\chi^u}{2} (\xi_t^u(k))^2 P_t^{ub} q_t^u \right]$$

where

$$\xi_t^u(k) = \frac{\frac{P_t^{ub}(k)}{P_{t-1}^{ub}(k)}}{(\Pi^{ss})^{1-\epsilon_u}\left(\frac{P_{t-1}^{ub}}{P_{t-2}^{ub}}\right)^{\epsilon_u}} - 1$$

summarises the price adjustment costs as before.

The first-order condition for pricing is then:

$$\begin{aligned}(1-\eta_u)\left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} q_t^u - \chi^u \xi_t^u(k) P_t^{ub} q_t^u \frac{\xi_t^u(k)+1}{P_t^{ub}(k)} \\ + [\psi^u P_t p_t^{vc} + (1-\psi^u) P_t p_t^g]\eta_u \left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} \frac{q_t^u}{P_t^{ub}(k)} \\ = -\beta\chi^u E_t \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^u(k) P_{t+1}^{ub} q_{t+1}^u \frac{\xi_{t+1}^u(k)+1}{P_t^{ub}(k)}\end{aligned}$$

Nominal dividends from petrol production are given by:

$$DV_t^p = \left[P_t^{pb} - \psi^{qp} P_t p_t^{vc} - (1-\psi^{qp}) P_t p_t^o\right] q_t^p$$

which is analogous to the expression for dividends from utilities and again the price earned from petrol production, P_t^{pb} is measured at basic prices. More details on the taxation treatment of petrol will be given below.

Again assuming monopolistic competition so that the demand schedule for utilities is:

$$q_t^p(k) = \left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p} q_t^p$$

and that utilities producers maximise the discounted flow of dividends subject to price adjustment costs

$$\max E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\left[P_t^{pb}(k) - \psi^{qp} P_t p_t^{vc} - (1-\psi^{qp}) P_t p_t^o\right] \left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p} q_t^p - \frac{\chi^{pp}}{2} (\xi_t^{pp}(k))^2 P_t q_t^p \right]$$

where

$$\xi_t^{pp}(k) = \frac{\frac{P_t^{pb}(k)}{P_{t-1}^{pb}(k)}}{(\Pi^{ss})^{1-\epsilon_{pp}}\left(\frac{P_{t-1}^{pb}}{P_{t-2}^{pb}}\right)^{\epsilon_{pp}}} - 1$$

summarises the price adjustment costs as before. The first-order condition for pricing is then:

$$
\begin{aligned}
&(1-\eta_p)\left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p} q_t^p - \chi^{pp}\xi_t^{pp}(k)P_t^{pb}q_t^p\frac{\xi_t^{pp}(k)+1}{P_t^{pb}(k)} \\
&+[\psi^{qp}P_t p_t^{vc} + (1-\psi^{qp})P_t p_t^o]\eta_p\left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p}\frac{q_t^p}{P_t^{pb}(k)} \\
&= -\beta\chi^{pp}E_t\frac{\lambda_{t+1}}{\lambda_t}\xi_{t+1}^{pp}(k)P_{t+1}^{pb}q_{t+1}^p\frac{\xi_{t+1}^{pp}(k)+1}{P_t^{pb}(k)}
\end{aligned}
$$

Domestic production of oil and gas are given exogenously by $\overline{O}_t$ and $\overline{G}_t$ respectively.

Rest of the world and exogeneity assumptions

We assume that there is a downward-sloping export demand function for domestically produced goods, as in HO. So the demand for domestic non-energy exports is given by:

$$
x_t^n = \left(x_{t-1}^n\right)^{\psi_x}\left[\kappa_x\left(\frac{S_t P_t^f}{P_t}\right)^{-\eta_x} y_t^f\right]^{1-\psi_x}
$$

where ψ_x captures an assumption that foreign preferences exhibit a form of 'habit formation' similar to that assumed for domestic agents.

We assume that there is an infinitely elastic supply of oil (gas) available from the world market at a world *relative* price p^{of} (p^{gf}). The prices of oil and gas in domestic currency are given by the law of one price:

$$
P_t^o = \frac{p_t^{of}P_t^f}{S_t}
$$
$$
P_t^g = \frac{p_t^{gf}P_t^f}{S_t}
$$

Import prices are assumed to be priced as a mark-up over the world import price (measured in domestic currency) and are subject to Calvo price adjustment costs as in HO. We summarise the import pricing equation as:

$$
\begin{aligned}
\ln\left(\frac{p_t^m\Pi_t}{p_{t-1}^m}\right) &= \frac{\beta}{1+\beta\varepsilon^{pm}}E_t\ln\left(\frac{p_{t+1}^m\Pi_{t+1}}{p_t^m}\right) \\
&+\frac{\varepsilon^{pm}}{1+\beta\varepsilon^{pm}}\ln\left(\frac{p_{t-1}^m\Pi_{t-1}}{p_{t-2}^m}\right)+\frac{\left[1-\beta\left(1-\psi^{pm}\right)\right]\psi^{pm}}{\left(1+\beta\varepsilon^{pm}\right)\left(1-\psi^{pm}\right)}\ln\left(\frac{p_t^{mf}}{s_t p_t^m}\right)
\end{aligned}
$$

Fiscal and monetary policy

The government's nominal budget constraint is satisfied each period:

$$BG_t + MON_t = R^g_{t-1} BG_{t-1} + MON_{t-1} + P_t c^g_t - P_t \tau_t$$
$$-P_t \tau_v \left[p^b_t q_t + p^{pb}_t q^p_t \right] - (1+\tau_v) P_t d^p_t q^p_t - \tau_u P_t p^{ub}_t q^u_t$$

with procurement (c^g) exogenous, lump-sum taxes (τ) move to satisfy a balanced budget process for government debt:

$$BG_t = 0$$

Tax revenue includes value added tax on final output which means that the price of output is given by:

$$P_t = (1+\tau_v) P^b_t$$

The revenue from tax on utilities reflects the fact that utilities are taxed at a different rate τ_u :

$$P_t p^u_t = (1+\tau_u) P_t p^{ub}_t$$

The tax revenue from petrol sales includes duties (d^p) as well as value added taxes so that the market price for petrol is:

$$P_t p^p_t = (1+\tau_v) P_t \left(p^{pb}_t + d^p_t \right)$$

The baseline monetary reaction function says that nominal interest rates respond to deviations of annual consumer price inflation from target:

$$\frac{R^g_t}{R^{gss}} = \left(\frac{R^g_{t-1}}{R^{gss}} \right)^{\theta^{rg}} \left(\frac{\left[P^c_t / P^c_{t-4} \right]^{0.25}}{\Pi^{ss}} \right)^{\theta^{pdot}\left(1-\theta^{rg}\right)}$$

The baseline reaction function does not include a response to an output gap measure, because the precise definition of the output gap may be important.

Aggregation, market clearing and the resource constraint

Total dividends received by households are given by:

$$DV_t = DV^q_t + DV^u_t + DV^p_t$$

Market clearing for value added requires:

$$V_t = V^n_t + V^p_t + V^u_t$$

Market clearing for petrol requires:

$$q^p_t = c^p_t + I^p_t + \psi_N e_{z,t} + X^p_t - M^p_t$$

where we account for the demand for petrol by households to facilitate capital utilisation. We do so under the assumption that the energy bundle used by households is the same Leontief bundle used by non-energy producing firms. Net trade in petrol is assumed to be zero (X_p=M_p).

Again accounting for household demand for utilities to facilitate capital utilisation implies that market clearing for utilities requires:

$$q_t^u = c_t^u + I_t^u + (1-\psi_N)e_{z,t}$$

Total demand for oil can be sourced from the domestic well and net trade:

$$I_t^o = \overline{O}_t - X_t^o$$

where X_t^o measures net trade (and therefore can be negative).

Similarly, total demand for gas satisfies:

$$I_t^g = \overline{G}_t - X_t^g$$

Non-oil final production satisfies demand:

$$q_t = c_t^g + c_t^n + k_t - \left(1 - \delta - \frac{\chi^z}{1+\phi^z}\left[z_t^{1+\phi_z} - 1\right]\right)k_{t-1} + x_t^n$$

Substituting the government budget constraint, the expression for dividends (DV) and the market clearing conditions into the household budget constraint delivers an aggregate resource constraint describing how the net foreign assets of the economy evolve:

$$\begin{aligned} \tfrac{bf_t}{s_t} &= \left(R_{t-1}^f\right)\tfrac{bf_{t-1}}{\Pi_t^f s_t} + x_t^n - p_t^m M_t^N + p_t^g X_t^g + p_t^o X_t^o \\ &- \tfrac{\chi^{bf}}{2}\left[\tfrac{bf_t}{s_t} - nfa^{ss}\right]^2 - \tfrac{\chi^k}{2k_{t-1}}\left[k_t - \left(k_{t-1}/k_{t-2}\right)^{\varepsilon^k} k_{t-1}\right]^2 \end{aligned}$$

Appendix B – Data construction

We have constructed a set of model-consistent data derived from the UK National Accounts to provide the basis for calibrating the energy sector in the model. In most cases we are able to construct data that are broadly equivalent to the concepts that we model, except for some areas where some small but unavoidable double counting is involved. First we focus on estimates of expenditure, output and value added in nominal terms. These are the most important as they are used to set the key parameters in the model that match production and expenditure shares in the data. To simplify notation and to make it easier to compare with model equations we report all nominal value relationships using prices *relative* to the numeraire (the price of final non-energy output).

Total final expenditure (including indirect taxes) on finished goods in the model ($ndem$) is given by the expenditure on non-energy final output q, and the final consumption expenditure on petrol C^p and utilities C^u and the amount of petrol that is exported X^p (little utility sector output is sold overseas):

$$ndem = q + p^u C^u + p^p C^p + p^p X^p$$

Final expenditure on non-energy final output is in turn given by the sum of consumption expenditure on non-energy finished goods, C^n, investment spending Inv, government procurement C^g and non-energy exports X^n.

$$q = C^n + C^g + Inv + X^n$$

We equate the model measure of total final expenditure ($ndem$) with an appropriate measure that can be constructed from the National Accounts - calculated as gross final expenditure at market prices less government spending on labour and capital depreciation (government value added), less consumption of housing services, less exports of oil and gas.[11] Estimates of consumption spending on petrol and utilities (C^p and C^u) both in value and (chain-weighted) volume terms are available from the ONS publication *Consumer Trends*. Subtracting these elements from total consumption expenditure yields the equivalent of model variable C^n in value (nominal) terms. Investment spending (Inv), government procurement (C^g) and non-energy exports can all be derived from the National Accounts.

To move from such measures of final expenditure to a measure of non-extraction value added, which corresponds to the model variable *V*, we first need to deduct indirect taxes. This leads to a measure of final output that corresponds to the sum of the various factor incomes (including the income earned on imported inputs by the overseas sector). In terms of UK data this means moving from a measure evaluated at market prices to one evaluated at basic prices.[12] We then

[11]For further discussion of similar measures, see Churm *et al* (2006). This definition implies that C^g represents government purchases of privately produced goods and services and that total consumption expenditure ($C^n + p^p C^p + p^u C^u$) excludes actual and imputed rental expenditure by households.

[12]An alternative would be to use the factor cost measure depending on whether all indirect taxes are taken out. We choose to construct our data for the value of final output using a basic price valuation as this is the preferred

need to deduct imports and inputs of energy from each of the components of final demand. A further difficulty relates to the treatment of the mark-up when matching value added in the model to that in the data. There is a model measure of nominal value added based on the 'competitive' price of value added ($p^{vc}V$). But in practice the data for nominal profits and value added will include any supernormal element or mark-up. So the appropriate model concept for nominal value added that maps into the data in the National Accounts is given by:

$$p^{v}V = \underbrace{(p^{b}q - p^{m}m^{n} - p^{p}i^{p} - p^{u}i^{u})}_{p^{vn}V^{n}} + \underbrace{(p^{ub}q^{u} - p^{g}i^{g})}_{p^{vu}V^{u}} + \underbrace{(p^{pb}q^{p} - p^{o}i^{o})}_{p^{vp}V^{p}}$$

As the equation makes clear the model also requires us to break down total value added into that used by the petrol production sector (V^{p}), the utilities' sector (V^{u}) and the non-energy sector (V^{n}), by deducting the appropriate inputs from each component final output. Achieving an equivalent break down in the data is important as this will allow us to set the key production function parameters for petrol, utilities and non-energy finished goods.
The main source of these data are the annual Input-Output Supply and Use Tables (I-O SUTs for shorts). To proceed, some simplifying assumptions are required:

1) The model assumes that the domestic petrol and utilities sector use no non-extraction imports in their production. So the portion of non-extraction value added used by these sectors is assumed to correspond to final output less inputs of oil and gas. We assume the same definition in our data set.[13]

2) All non-oil and gas extraction imports ($p^{m}m^{n}$) are assumed to be used in the production of final output (q). Note that the model does not allow for 'direct' imports (imports purchased directly from abroad that do not go through any sort of production process in the United Kingdom). This may not be so extreme an assumption given that pretty much every imported good has to go through a retail or wholesale distribution network of some kind. But it does mean that the measure of final output aggregates across a range of goods some of which may have relatively little domestic value added content.

3) Data for intermediate inputs of petrol and utilities at purchasers prices ($p^{p}i^{p}$ and $p^{u}i^{u}$) used by the non-energy sector can be derived from Table 3 of the I-O SUTs. This involves taking the final output of the petrol and utilities sector (appropriately adjusted for the 'own use' of its output) and subtracting the amount purchased directly by the household sector for consumption. In the case of petrol we also take account of exports and imports. In the model some of the intermediate inputs of petrol and utilities are used directly by the household to facilitate capital utilisation if $\chi_{e} > 0$ and $e_{z} > 0$. And net trade in petrol is assumed to be zero at all times. The model identities for petrol and utility production are given by:

valuation basis for output and value added in the UK National Accounts. This means removing the ONS basic price adjustment from our final expenditure data.

[13] In part this is because the annual I-O SUTs do not provide the breakdown of inputs used by the petrol and utilities into imported and domestic inputs. But the more detailed 1995-based I-O analytical tables suggest this is a reasonable assumption.

$$
\begin{aligned}
q_t^p &= c_t^p + I_t^p + \psi_N e_{z,t} + X^p - M^p \\
q_t^u &= c_t^u + I_t^u + (1-\psi_N) e_{z,t} \\
X^p &= \psi^{xp} Q^p \\
M^p &= X^p
\end{aligned}
$$

4) Value added used by the petrol and utilities sector can also be derived straightforwardly from the I-O SUTs as given by the formula above. This involves taking gross output at basic prices for each sector adjusted for any own use of output (for example some electricity is used in gas distribution) and then removing raw inputs of crude oil and natural gas (model variables $p^g i^g$ and $p^o i^o$). The value added that remains is then the sum of profits and labour compensation attributable to the petrol and utilities sector.

Finally non-extraction value added itself is composed of labour income and the sum of normal and supernormal profits.

$$p^v V^n = w.h + wk.k + \frac{1}{\eta} p^v V^n$$

Data on private sector labour compensation is taken from the Bank of England Quarterly Model.[14] This implicitly assumes that no labour is required to extract crude oil and natural gas which is not an unreasonable approximation given the small amount of labour used in that sector.

Turning to the data for the production and income generated by the extraction sector: data on the value of the United Kingdom's output of crude oil and natural gas can be obtained from the I-O SUTs. This allows us to calibrate a value for the model's total revenue from oil and gas production $p^o \overline{O} + p^g \overline{G}$. This implicitly defines total private sector value added:

$$p^v V^n + p^o \overline{O} + p^g \overline{G}$$

although in practice the oil and gas sector do use a small amount of non-extraction value added and imports. So there is a small element of double counting involved in the above identity unless one were to make oil and gas production a function of capital and labour.
The split between oil and gas production is not given in the I-O SUTs but it can be inferred by examining the industries that purchase output from these sectors. Most oil and gas inputs are purchased by just two broad sectors, the refining industry and the utilities (comprising the electricity and natural gas distribution sectors). Crude oil is not used in significant amounts for power generation in the United Kingdom, so we assume the share of oil and gas inputs purchased by the refining sector represents the share of crude oil output in total oil and gas extraction output. Given the inputs of natural gas and oil used by the energy producing sector ($p^g i^g$ and $p^o i^o$) this defines the trade balance in and natural gas.

[14] See Harrison *et al* (2005).

$$I_t^o - \overline{O}_t = X_t^o$$
$$I_t^g - \overline{G}_t = X_t^g$$

As is shown in the main paper the expected value of this deficit in steady state will be crucial in determining the wealth effects of permanent energy price changes.

Annual values of the key nominal expenditure and output components for 2003 derived from the I-O SUTs under the assumptions discussed above are shown in Table B1 below. Where the calculations involve combining various I-O SUT numbers, we show how the underlying figures have been combined so that they can be traced back to the values in appropriate tables. We use these nominal expenditure components to pin down various parameters in Section 3 of the main paper.

Table B1: Key expenditure and output values from the 2003 I-O SUTs

Expenditure component	Value in 2003 in £mn	Description
$q + p^u C^u + p^p C^p + p^p X^p$	1250618	Final expenditure on goods and services
$p^u C^u$	14930	Consumption expenditure on utilities
$p^p C^p$	21420	Consumption expenditure on petrol
$p^p X^p$	6889	Exports of petrol
$p^v V$	763808	Non-extraction value added
$p^{vu} V^u$	32268+12254-9894-1420-2212-1579-4331-4126	Value added used by utilities sector
$p^{vp} V^p$	15927-921-11010	Value added used by petrol sector
$p^m m^n$	306300	Non-extraction imports
$p^p i^p$	50132-921-6899-21420	Value of petrol inputs
$p^u i^u$	29866+14359-9894-1420-2212-1579-7542-6215	Value of utility inputs
$p^{pb} q^p$	15927-921	Value of petrol production at basic prices
$p^{pu} q^u$	32268+12254-9894-1420-2212-1579	Value of utility production at basic prices
$p^o i^o$	11010	Value of crude oil inputs used in refining
$p^g i^g$	4331+4126	Value of gas inputs used by utilities
$p^o \bar{O} + p^g \bar{G}$	25210-1452	Value of extraction sector production
$w.h$	573609	Private sector compensations

Appendix C – Final equation listing

For completeness, we list the model equation listing here, where we have rearranged equations (often using definitions) and imposed a symmetric equilibrium across agents.

$$\left[\frac{c_t}{c_{t-1}^{\psi^{hab}}}\right]^{-\frac{1}{\sigma^c}} \frac{1}{c_{t-1}^{\psi^{hab}}} - p_t^c \Lambda_t = 0$$

$$\frac{1}{p_t^c}\left[\frac{mon_t}{\kappa^{mon}}\right]^{-\frac{1}{\sigma^c}} - \Lambda_t + \beta E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} = 0$$

$$-\Lambda_t + \beta R_t^g E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} = 0$$

$$-\frac{\Lambda_t}{s_t} - \chi^{bf}\frac{\Lambda_t}{s_t}\left[\frac{bf_t}{s_t} - nfa^{ss}\right] + \beta\left(R_t^f\right)E_t \frac{\Lambda_{t+1}}{s_{t+1}\Pi_{t+1}^f} = 0$$

$$1 + \chi^k\left(\frac{k_t}{k_{t-1}} - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\epsilon^k}\right) = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[\begin{array}{c} 1 - \left(\delta + \frac{\chi^z}{1+\phi^z}\left[z_{t+1}^{1+\phi_z} - z_{ss}^{1+\phi_z}\right]\right) \\ +\chi^k\left(\frac{k_{t+1}}{k_t} - \left(\frac{k_t}{k_{t-1}}\right)^{\epsilon^k}\right)\left(\frac{k_t}{k_{t-1}}\right)^{\epsilon^k} \\ +w_{t+1}^k z_{t+1} - p_{t+1}^e \frac{\chi_e}{\gamma_e} z_{t+1}^{\gamma_e} \end{array}\right]$$

$$w_t^K k_{t-1} - \chi^z z_t^{\phi^z} k_{t-1} - \chi_e p_t^e k_{t-1} z_t^{\gamma_e - 1} = 0$$

$$\widetilde{w}_t^{1+\frac{\sigma^w}{\sigma^h}} = w_t^{\frac{\sigma^w}{\sigma^h}} \frac{(\kappa^h)^{-\frac{1}{\sigma^h}}\sigma^w}{\sigma^w - 1}\frac{\Xi_t^w}{\Theta_t^w},$$

$$\Xi_t^w = h_t^{\frac{\sigma^h+1}{\sigma^h}} + \beta(1-\psi^w)E_t\left[\frac{\xi_t^w w_t}{w_{t+1}\Pi_{t+1}}\right]^{-\sigma^w\left(\frac{\sigma^h+1}{\sigma^h}\right)}\Xi_{t+1}^w$$

$$\xi_t^w = (\Pi^{ss})^{1-\epsilon^w}\left(\frac{w_t}{w_{t-1}}\Pi_t\right)^{\epsilon^w}$$

$$\Theta_t^w = \Lambda_t h_t + \beta(1-\psi^w)E_t \Pi_{t+1}^{-1}\xi_t^w\left[\frac{\xi_t^w w_t}{\Pi_{t+1} w_{t+1}}\right]^{-\sigma^w}\Theta_{t+1}^w.$$

$$w_t^{1-\sigma^w} = (1-\psi^w)\left(\xi_{t-1}^w \frac{w_{t-1}}{\Pi_t}\right)^{1-\sigma^w} + \psi^w \widetilde{w}_t^{1-\sigma^w}.$$

$$c_t = \kappa^c \Big[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \psi^e \{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}} \Big]^{\frac{\sigma^e}{\sigma^e-1}}$$

$$c_t^e = \kappa^e \Big[(1-\psi^p)\{(1-\phi^p)c_t^u\}^{1-\frac{1}{\sigma_p}} + \psi^p \{\phi^p c_t^p\}^{1-\frac{1}{\sigma_p}} \Big]^{\frac{\sigma_p}{\sigma_p-1}}$$

$$p_t^c c_t = c_t^n + p_t^u c_t^u + p_t^p c_t^p$$

$$\frac{\left(\frac{1-\psi^e}{\psi^e}\right)\left\{\frac{1-\phi^e}{\phi^e}\right\}^{1-\frac{1}{\sigma^e}}\left\{\frac{c_t^n}{c_t^e}\right\}^{-\frac{1}{\sigma^e}}\left\{\frac{c_t^e}{c_t^u}\right\}^{-\frac{1}{\sigma_p}}}{[\kappa^e]^{1-\frac{1}{\sigma_p}}(1-\psi^p)\{(1-\phi^p)\}^{1-\frac{1}{\sigma_p}}} = \frac{1}{p_t^u}$$

$$\frac{1-\psi^p}{\psi^p}\left\{\frac{1-\phi^p}{\phi^p}\right\}^{1-\frac{1}{\sigma_p}}\left\{\frac{c_t^u}{c_t^p}\right\}^{-\frac{1}{\sigma_p}} = \frac{p_t^u}{p_t^p}$$

$$V_t = \kappa^v \Big[(1-\alpha_v)\{(1-\phi_v)h_t\}^{1-\frac{1}{\sigma^v}} + \alpha_v \{\phi_v z_t k_{t-1}\}^{1-\frac{1}{\sigma^v}} \Big]^{\frac{\sigma^v}{\sigma^v-1}}$$

$$(1-\alpha_v)\{\kappa^v(1-\phi_v)\}^{1-\frac{1}{\sigma^v}}\left\{\frac{V_t}{h_t}\right\}^{\frac{1}{\sigma^v}} = \frac{w_t}{p_t^{vc}}$$

$$\alpha_v\{\kappa^v\phi_v\}^{1-\frac{1}{\sigma^v}}\left\{\frac{V_t}{z_t k_{t-1}}\right\}^{\frac{1}{\sigma^v}} = \frac{w_t^k}{p_t^{vc}}$$

$$q_t = A_t \Big[(1-\alpha_q)\{(1-\phi_q)B_t\}^{1-\frac{1}{\sigma^q}} + \alpha_q \{\phi_q e_t\}^{1-\frac{1}{\sigma^q}} \Big]^{\frac{\sigma^q}{\sigma^q-1}}$$

$$B_t = \kappa^B (V_t^N)^{1-\alpha_B}(M_t^N)^{\alpha_B}$$

$$I_t^p = \psi_N e_t$$

$$I_t^u = (1-\psi_N)e_t$$

$$dv_t^q = p_t^b q_t - p_t^{vc}V_t^N - p_t^m M_t^N - [\psi_N p_t^p + (1-\psi_N)p_t^u]e_t$$

$$\xi_t^p = \frac{p_t^b \Pi_t / p_{t-1}^b}{(\Pi^{ss})^{1-\epsilon}\left(p_{t-1}^b \Pi_{t-1}/p_{t-2}^b\right)^{\varepsilon}} - 1$$

$$\frac{p_t^{vc}}{p_t^b} = \mu_t A_t^{1-\frac{1}{\sigma^q}}(1-\alpha_q)(1-\phi_q)^{1-\frac{1}{\sigma^q}}\left[\frac{q_t}{B_t}\right]^{\frac{1}{\sigma^q}}(1-\alpha_B)\frac{B_t}{V_t^N}$$

$$\frac{p_t^m}{p_t^b} = \mu_t A_t^{1-\frac{1}{\sigma^q}}(1-\alpha_q)(1-\phi_q)^{1-\frac{1}{\sigma^q}}\left[\frac{q_t}{B_t}\right]^{\frac{1}{\sigma^q}}\alpha_B\frac{B_t}{M_t^N}$$

$$\frac{\psi_N p_t^p + (1-\psi_N)p_t^u}{p_t^b} = \mu_t A_t^{1-\frac{1}{\sigma^q}}\alpha_q\phi_q^{1-\frac{1}{\sigma^q}}\left[\frac{q_t}{e_t}\right]^{\frac{1}{\sigma^q}}$$

$$(1-\eta) - \chi^p\xi_t^p(\xi_t^p+1) + \eta\mu_t = -\beta\chi^p E_t\frac{p_{t+1}^b q_{t+1}\Lambda_{t+1}}{p_t^b q_t\Lambda_t}\xi_{t+1}^p(\xi_{t+1}^p+1)$$

$$I_t^g = (1-\psi^u)q_t^u$$

$$V_t^u = \psi^u q_t^u$$

$$I_t^o = (1-\psi^{qp})q_t^p$$

$$V_t^p = \psi^{qp}q_t^p$$

$$dv_t^u = \left[p_t^{ub} - \psi^u p_t^{vc} - (1-\psi^u)p_t^g\right]q_t^u$$

$$\xi_t^u = \frac{p_t^{ub}\Pi_t/p_{t-1}^{ub}}{(\Pi^{ss})^{1-\epsilon_u}\left(p_{t-1}^{ub}\Pi_{t-1}/p_{t-2}^{ub}\right)^{\epsilon_u}} - 1$$

$$(1-\eta_u) - \chi^u\xi_t^u(\xi_t^u+1) + \frac{\psi^u p_t^{vc} + (1-\psi^u)p_t^g}{p_t^{ub}}\eta_u = -\beta\chi^u E_t\frac{\Lambda_{t+1}}{\Lambda_t}\frac{p_{t+1}^{ub}q_{t+1}^u}{p_t^{ub}q_t^u}\xi_{t+1}^u(\xi_{t+1}^u+1)$$

$$dv_t^p = \left[p_t^{pb} - \psi^{qp}p_t^{vc} - (1-\psi^{qp})p_t^o\right]q_t^p$$

$$\xi_t^{pp} = \frac{p_t^{pb}\Pi_t/p_{t-1}^{pb}}{(\Pi^{ss})^{1-\epsilon_{pp}}\left(p_{t-1}^{pb}\Pi_{t-1}/p_{t-2}^{pb}\right)^{\epsilon_{pp}}} - 1$$

$$(1-\eta_p) - \chi^{pp}\xi_t^{pp}(\xi_t^{pp}+1) + \frac{\psi^{qp}p_t^{vc} + (1-\psi^{qp})p_t^o}{p_t^{pb}}\eta_p$$

$$= -\beta\chi^{pp}E_t\frac{\Lambda_{t+1}}{\Lambda_t}\frac{p_{t+1}^{pb}q_{t+1}^p}{p_t^{pb}q_t^p}\xi_{t+1}^{pp}(\xi_{t+1}^{pp}+1)$$

$$1 = (1+\tau_v)p_t^b$$

$$p_t^u = (1+\tau_u)p_t^{ub}$$

$$p_t^p = (1+\tau_v)\left(p_t^{pb} + d_t^p\right)$$

$$p_t^e = \psi_N p_t^p + (1-\psi_N)p_t^u$$

$$x_t^n = (x_{t-1}^n)^{\psi_x}\left[\kappa_x s_t^{-\eta_x}y_t^f\right]^{1-\psi_x}$$

$$p_t^o = \frac{p_t^{of}}{s_t}$$

$$p_t^g = \frac{p_t^{gf}}{s_t}$$

$$\ln\left(\frac{p_t^m\Pi_t}{p_{t-1}^m}\right) = \frac{\beta}{1+\beta\epsilon^{pm}}E_t\ln\left(\frac{p_{t+1}^m\Pi_{t+1}}{p_t^m}\right) + \frac{\epsilon^{pm}}{1+\beta\epsilon^{pm}}\ln\left(\frac{p_{t-1}^m\Pi_{t-1}}{p_{t-2}^m}\right) + \frac{[1-\beta(1-\psi^{pm})]\psi^{pm}}{(1+\beta\epsilon^{pm})(1-\psi^{pm})}\ln\left(\frac{p_t^{mf}}{s_t p_t^m}\right)$$

$$\frac{R_t^g}{R^{gss}} = \left(\frac{R_{t-1}^g}{R^{gss}}\right)^{\theta^{rg}}\left(\frac{\left[p_t^c\Pi_t\Pi_{t-1}\Pi_{t-2}\Pi_{t-3}/p_{t-4}^c\right]^{0.25}}{\Pi^{ss}}\right)^{\theta^{pdot}\left(1-\theta^{rg}\right)}$$

$$dv_t = dv_t^q + dv_t^u + dv_t^p$$

$$V_t = V_t^n + V_t^p + V_t^u$$

$$q_t^p = c_t^p + I_t^p + \psi_N e_{z,t} + X^p - M^p$$

$$q_t^u = c_t^u + I_t^u + (1 - \psi_N) e_{z,t}$$

$$M^p = X^p$$

$$X^p = \psi^{xp} Q^p$$

$$\frac{e_{z,t}}{k_{t-1}} = \frac{\chi_e}{\gamma_e} z_t^{\gamma_e}$$

$$I_t^o = \bar{O}_t - X_t^o$$

$$I_t^g = \bar{G}_t - X_t^g$$

$$q_t = c_t^g + c_t^n + k_t - \left(1 - \delta - \frac{\chi^z}{1+\phi_z} \left[z_t^{1+\phi_z} - z_{ss}^{1+\phi_z} \right] \right) k_{t-1} + x_t^n$$

$$\frac{bf_t}{s_t} = R_{t-1}^f \frac{bf_{t-1}}{\Pi_t^f s_t} + x_t^n - p_t^m M_t^N + p_t^g X_t^g + p_t^o X_t^o$$

$$-\frac{\chi^{bf}}{2} \left[\frac{bf_t}{s_t} - nfa^{ss} \right]^2 - \frac{\chi^k}{2k_{t-1}} \left[k_t - (k_{t-1}/k_{t-2})^{\epsilon^k} k_{t-1} \right]^2$$

References

Barwell, R, Thomas, R S J and Turnbull, K (2007) 'The macroeconomic impact of higher energy prices on the UK economy', *Bank of England Quarterly Bulletin*, Vol. 47, no. 4, pages 522-32.

Blanchard, O and Gali, J (2007), 'The macroeconomic effects of oil shocks: why are the 2000s so different from the 1970s?', *NBER Working Paper no.13368*.

Bodenstein, M, Erceg, C J and Guerrieri, L (2007), 'Oil shocks and external adjustment'**,** *International Finance Discussion Paper no. 897*, Board of Governors of the Federal Reserve System.

Bruno, M and Sachs, J D (1985), *Economics of worldwide stagflation*, Harvard University Press.

Churm, R, Srinivasan, S, Thomas, R, Mahajan, S, Maitland-Smith, F and Tily, G (2006), 'Measuring market sector activity in the United Kingdom', *Bank of England Quarterly Bulletin* Vol. 46, No. 4, pages 404-14.

Erceg, C J, Henderson, D W and Levin, A T (2000), 'Optimal monetary policy with staggered wage and price contracts'*, Journal of Monetary Economics*, Vol. 46(2), pages 281-313.

Finn, M G (2000), 'Perfect competition and the effects of energy price increases on economic activity', *Journal of Money, Credit and Banking*, Vol. 32, pages 400-16.

Gray, L C (1914) 'Rent under the assumption of exhaustibility' *Quarterly Journal of Economics,* Vol. 28, pages 466-89.

Hamilton, J D (1983), 'Oil and the macroeconomy since World War II', *Journal of Political Economy*, Vol. 91, pages 228-48

Harrison, R, Nikolov, K, Quinn, M, Ramsay, G, Scott, A and Thomas, R (2005), *The Bank of England Quarterly Model*, London: Bank of England.

Harrison, R and Oomen, O (2010), 'Evaluating and estimating a DSGE model for the United Kingdom', *Bank of England Working Paper no. 380.*

Millard, S (2011), 'An estimated DSGE model of energy, costs and inflation in the United Kingdom', *Bank of England Working Paper no. 432*.

Rotemberg, J and Woodford, M (1996), 'Imperfect competition and the effects of energy price increases on economic activity', *Journal of Money, Credit and Banking*, Vol. 28, pages 549-77.

Schmitt-Grohe, S and Uribe M (2003), 'Closing small open economy models', *Journal of International Economics,* Vol. 61, pages 163-85.