



BANK OF ENGLAND

# Working Paper No. 458

## A network model of financial system resilience

Kartik Anand, Prasanna Gai, Sujit Kapadia,  
Simon Brennan and Matthew Willison

July 2012

Working papers describe research in progress by the author(s) and are published to elicit comments and to further debate. Any views expressed are solely those of the author(s) and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee or Financial Policy Committee.



BANK OF ENGLAND

## Working Paper No. 458

# A network model of financial system resilience

Kartik Anand,<sup>(1)</sup> Prasanna Gai,<sup>(2)</sup> Sujit Kapadia,<sup>(3)</sup> Simon Brennan<sup>(5)</sup> and Matthew Willison<sup>(6)</sup>

### Abstract

We examine the role of macroeconomic fluctuations, asset market liquidity, and network structure in determining contagion and aggregate losses in a stylised financial system. Systemic instability is explored in a financial network comprising three distinct, but interconnected, sets of agents — domestic banks, overseas banks, and firms. Calibrating the model to advanced country banking sector data, this preliminary model generates broadly sensible aggregate loss distributions which are bimodal in nature. We demonstrate how systemic crises may occur and analyse how our results are influenced by fire-sale externalities and the feedback effects from curtailed lending in the macroeconomy. We also illustrate the resilience of our model financial system to stress scenarios with sharply rising corporate default rates and falling asset prices.

**Key words:** Contagion, financial crises, network models, systemic risk.

**JEL classification:** C63, G01, G17, G21.

---

(1) Technische Universität Berlin. Email: kartik.anand@tu-berlin.de

(2) Department of Economics, University of Auckland and National University of Singapore, Risk Management Institute. Email: p.gai@auckland.ac.nz

(3) Bank of England. Email: sujit.kapadia@bankofengland.co.uk

(4) Bank of England. Email: simon.brennan@bankofengland.co.uk

(5) Bank of England. Email: matthew.willison@bankofengland.co.uk

The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England. The paper is forthcoming in the *Journal of Economic Behavior and Organization*. We are grateful to David Aikman, Dilek Bulbul, Andy Haldane, Simon Hall, Reimer Kuhn, Myron Kwast, Paul Ormerod, Joe Pearlman, Gabriel Sterne and Martin Summer for helpful comments and suggestions for improvements. The comments of participants at the GRETA Associati CREDIT 2009 conference on Credit Risk, Financial Crises, and the Macroeconomy (Venice, 24–25 September 2008), the Royal Economic Society Annual Conference (Guildford, 20–22 April 2009), the Money, Macro and Finance Research Group Conference on Financial Stability and Transmission of Credit Risk (London, 21 May 2009), the Bank for International Settlements workshop on Challenges in Banking Research (Basel, 28–29 May 2009), the Econometric Society European Congress (Barcelona, 23–27 August 2009), the European Central Bank workshop on Recent Advances in Modelling Systemic Risk Using Network Analysis (Frankfurt am Main, 5 October 2009), conference on Post-crisis Financial Sector Performance (Bangor, 23 June 2011), conference on Convergence, Interconnectedness and Crises: Insurance and Banking (Philadelphia, 8–10 December 2011), and seminar participants at the Bank of England and Office of Financial Research (US Department of the Treasury) are also gratefully acknowledged. This paper was finalised on 1 June 2012.

The Bank of England's working paper series is externally refereed.

Information on the Bank's working paper series can be found at [www.bankofengland.co.uk/publications/Pages/workingpapers/default.aspx](http://www.bankofengland.co.uk/publications/Pages/workingpapers/default.aspx)

Publications Group, Bank of England, Threadneedle Street, London, EC2R 8AH  
Telephone +44 (0)20 7601 4030 Fax +44 (0)20 7601 3298 email [mapublications@bankofengland.co.uk](mailto:mapublications@bankofengland.co.uk)

## Contents

|   |    |
|---|----|
| Summary   | 3  |
| 1 Introduction  | 5  |
| 2 A stylised financial system and the transmission of shocks    | 8  |
| 3 Model calibration   | 12 |
| 3.1 Structure of balance sheets                                 | 13 |
| 3.2 Distribution of exposure sizes between banks                | 14 |
| 3.3 Connections between banks                                   | 16 |
| 3.4 Distribution of loans to firms and equity holdings          | 17 |
| 3.5 Corporate default probabilities                             | 18 |
| 3.6 Additional parameters                                       | 19 |
| 4 Credit events, aggregate losses, and feedback effects         | 20 |
| 4.1 The baseline aggregate loss distribution                    | 20 |
| 4.2 Macroeconomic shocks  | 21 |
| 4.3 Feedback effects  | 23 |
| 4.4 A more realistic setting - heterogeneity of capital buffers | 25 |
| 5 ‘Stress testing’  | 26 |
| 6 Conclusion  | 27 |
| A A statistical model of the financial system                   | 29 |
| Financial relationships between different types of agent        | 29 |
| Financial relationships between agents of the same type         | 30 |
| Bank balance sheets   | 31 |
| Crisis dynamics   | 32 |
| B Principle of maximum entropy                                  | 36 |
| References  | 38 |



## Summary

The complex and opaque nature of modern financial systems poses a considerable challenge for the analysis of systemic resilience. An intricate web of claims and obligations links households and firms to a wide variety of financial institutions such as banks, insurance companies, and investment firms. The rapid development of securitisation and credit derivative markets has also made exposures between agents more difficult to assess and monitor in the absence of trade repositories. The global financial crisis illustrates how intertwined the financial network has become, while also making clear the potential for widespread losses and instability.

Recent efforts by central banks to measure and assess systemic risk have emphasised the important role played by network effects, fire-sale externalities, and funding liquidity risk in financial stability. A general insight is that these factors generate ‘fat tails’ in the distribution of aggregate losses for the banking system. That is, the financial system may incur very large losses with small probabilities.

Central bank studies typically rely on highly detailed, and relatively static, balance sheet data to establish precise linkages between banks in the domestic financial system and to derive banking system losses. This can be constraining when true linkages are not known (such as with credit risk transfer or off balance sheet activity) or when shocks strike financial institutions external to the core banking system. The pre-defined balance sheet interlinkages in these models also preclude analysis of how network structure matters for system resilience. The crisis has emphasised how network linkages and interactions between financial institutions are critical to understanding systemic risk. And the growing importance of ‘stress-testing’ exercises in the policy debate about financial stability points to the need for analyses that help overcome such limitations.

In this paper, we set out a general framework to gauge systemic risk in circumstances when data about the reach of financial exposures are limited and shocks are international in nature. We present a statistical model of a financial system involving a diverse set of financial agents, namely domestic banks, overseas banks, and firms, which are linked together by their claims on each other. We calibrate the model to advanced country banking sector data to illustrate how macroeconomic fluctuations, asset market liquidity and network structure interact to determine



aggregate credit losses and contagion. Although the calibration is deliberately broad brush so as to emphasise the qualitative nature of the results, we obtain plausible loss distributions and can quantify, within the context of our model, the size of the macroeconomic or financial sector shock that may be necessary for system-wide failure to occur.

The model highlights how shocks are propagated through the direct interlinkages of claims and obligations among (and between) domestic banks and overseas banks. But it also shows how defaults across the network are amplified by asset fire sales and curtailed lending in the macroeconomy as ‘credit crunch’ effects take hold in the event of distress. In addition, we illustrate how greater heterogeneity of bank balance sheets leads to more realistic outcomes, characterised by the failure of some – but not all – banks in extreme scenarios. We also demonstrate how the model can be used to ‘stress test’ the banking system. The results obtained are entirely illustrative and only intended to demonstrate the usefulness of the framework.

## 1 Introduction

The complex and opaque nature of modern financial systems poses a considerable challenge for the analysis of systemic resilience. An intricate web of claims and obligations links households and firms to a wide variety of financial institutions such as banks, insurance companies, and investment firms. The rapid development of securitisation and credit derivative markets has also made exposures between agents difficult to assess and monitor in the absence of trade repositories. The global financial crisis of 2007–08 illustrates how intertwined the financial network has become, while also making clear the potential for widespread losses and instability.

Recent efforts by central banks to measure and assess systemic risk have emphasised the important role played by network effects, fire-sale externalities, and funding liquidity risk in financial stability.<sup>1</sup> A general insight, highlighted by Alessandri, Gai, Kapadia, Mora and Puhf (2009), is that these factors generate ‘*fat tails*’ in the distribution of aggregate losses for the banking system. This is consistent with recent analytical work which suggests that financial systems, like other complex networks, have ‘tipping points’ and display a ‘*robust-yet-fragile*’ tendency – with sharp discontinuities emerging following some unexpected shocks, with other shocks resulting in benign effects (May, Levin and Sugihara (2008); May and Haldane (2011); Gai and Kapadia (2010); Gai, Haldane and Kapadia (2011)).

Central bank studies typically rely on highly detailed, and relatively static, balance sheet data to establish precise linkages between banks in the domestic financial system and to derive banking system losses. This can be constraining when true linkages are not known (such as with credit risk transfer or off balance sheet activity) or when shocks strike financial institutions external to the core banking system. The pre-defined balance sheet interlinkages in these models also preclude analysis of how network structure matters for system resilience. The crisis has emphasised how network linkages and interactions between financial institutions are critical to understanding systemic risk. And the growing importance of ‘stress-testing’ exercises in the policy debate about financial stability points to the need for analyses that help overcome such limitations.

---

<sup>1</sup> See, for example, Elsinger, Lehar and Summer (2006) for the Austrian banking system, Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne and Willison (2009) for the United Kingdom, and Gauthier, Lehar and Souissi (2010) for Canada. Foglia (2009) provides a detailed overview of systemic risk assessment models being developed by central banks. Cifuentes, Ferrucci and Shin (2005) and Morris and Shin (2009) offer analytical accounts of fire-sale effects and funding liquidity risk.

In this paper, we set out a general framework to gauge systemic risk in circumstances when data about the reach of financial exposures is limited and shocks are international in nature. We present a statistical model of a financial system involving a diverse set of financial agents, namely domestic banks, overseas banks and firms,<sup>2</sup> which are linked together by their claims on each other. We calibrate the model to advanced country banking sector data to illustrate how macroeconomic fluctuations, asset market liquidity and network structure interact to determine aggregate credit losses and contagion. Although the calibration is deliberately broad-brush so as to emphasise the qualitative nature of the results, we obtain plausible fat-tailed (bimodal) aggregate loss distributions and can quantify, within the context of our model, the size of the macroeconomic or financial sector shock that may be necessary for system-wide failure to occur.

The model highlights how shocks are propagated through the direct interlinkages of claims and obligations among (and between) domestic banks and overseas banks. But it also shows how defaults across the network are amplified by asset fire sales and curtailed lending in the macroeconomy as ‘credit crunch’ effects take hold in the event of distress. In addition, we illustrate how greater heterogeneity of bank balance sheets leads to more realistic outcomes, characterised by the failure of some – but not all – banks in extreme scenarios.

We also demonstrate how the model can be used to ‘stress test’ the banking system. We draw on current best practice in stress testing to examine the consequences for bank failure in our model. The results obtained are entirely illustrative and only intended to demonstrate the usefulness of the framework. Specifically, we consider a scenario in which a default rate on corporate exposures of around 4.5% is accompanied by a 20% fall in equity prices as a result of fire sales. Faced with such stress, and with the simplifying assumption of 100% loss given default (LGD), approximately one quarter of our model banking system is pushed into default. The assumption of 100% LGD suggests this estimate sets a conservative upper bound on bank failures.

Our analysis complements recent work that draws on techniques from network science and statistical physics to study credit contagion and model credit risk losses in banks’ portfolios (Giesecke and Weber (2004, 2006); Horst (2007); Hatchett and Kuhn (2009); Gai and Kapadia (2010); May and Arinaminpathy (2010)). But the networks in these models typically involve

---

<sup>2</sup>Our notion of overseas banks specifically refers to banks that are headquartered in foreign countries rather than necessarily transnational. For exposition, we choose to interpret the domestic banks as the largest banks in the United Kingdom and the overseas ones as overseas banks with cross-border exposures to the United Kingdom. Moreover, we focus on banks for ease of exposition. Our approach could easily be extended to capture other intermediaries.

homogeneous agents – a firm or a bank – and do not capture the twin effects of macroeconomic and fire-sale feedbacks.

Our analysis also relates to the literature which seeks to obtain analytical valuation results for complex portfolio credit derivatives by considering default correlation and credit contagion among firms in a dynamic setting (see Errais, Giesecke and Goldberg (2010); Longstaff and Rajan (2008)). In contrast to these papers, clearly specified bank balance sheets are central to our approach, with bilateral linkages precisely defined with reference to these. And our differing modelling strategy, which focuses on the transmission of contagion along these links, reflects the greater structure embedded in our network set-up.

In choosing to model the complexity of a heterogeneous financial system with feedback effects, we eschew formal optimising behaviour and strategic interactions by the agents in our financial network. Instead, we allow for plausible ‘rules of thumb’ that permit banks to curtail lending and dispose assets in a fire sale. The size and structure of financial linkages is kept constant as default cascades develop. While this assumption may be defensible in the midst of a rapidly developing crisis, it is clearly at odds with recent work on financial networks (Leitner (2005); Castiglionesi and Navarro (2007)) that builds upon the seminal contribution of Allen and Gale (2000).<sup>3</sup> The stylised nature of these models means, however, that they cannot be used for systemic risk assessment. So our paper should be viewed as a very preliminary first step towards an integrated model of systemic risk that both takes complexity seriously and incorporates rules of thumb that go some way towards capturing plausible behavioural responses.

The paper proceeds as follows. Section 2 provides an informal discussion of our approach and explains how shocks are propagated and amplified in a stylised financial system comprising diverse agents. Section 3 describes model calibration and discusses a novel approach to deriving the distributions of balance sheet exposures in environments when data are limited. Section 4 presents the baseline aggregate loss distribution obtained from stochastic simulations and considers how liquidity risk and macroeconomic feedbacks might affect system stability. Section 5 presents an example of how the model can be used for a banking system stress test. A final section concludes. Formal details of the model and a description of how distributions of exposures are obtained from maximum entropy techniques are presented in the appendices.

---

<sup>3</sup>See Allen and Babus (2008) for a survey of network-theoretic approaches to modelling financial issues. Jackson (2008) provides detailed discussion of strategic behaviour on networks.



## 2 A stylised financial system and the transmission of shocks

The financial system in our model can be viewed as a core-periphery structure with three interconnected layers – domestic banks, overseas banks and firms. While we do not model lending to households in this structure, the framework could easily be augmented to include a role for them in a similar way to the treatment of firms.

A network of core *domestic banks* sits at the centre of the system. A distinguishing feature of this group is that each bank interacts with all other banks, ie the (sub) network of domestic banks is complete. This structure reflects the importance of core banks within money markets in national financial structures.<sup>4</sup>

Beyond this group of core banks lies a group of *overseas banks*, ie banks operating and headquartered in other countries and peripheral to the core domestic financial system. Unlike domestic banks, the (sub-) network of overseas banks is incomplete and exhibits a ‘*small-world*’ property – each overseas bank interacts with institutions in its immediate vicinity and only interacts with more distant institutions with some probability. The sparseness of the links between overseas banks relative to the complete network of domestic banks reflects the much greater diversity of activities and institutions in this sector, both in terms of activity and location. It is also consistent with recent evidence from von Peter (2007) on the financial linkages between international banking centres.

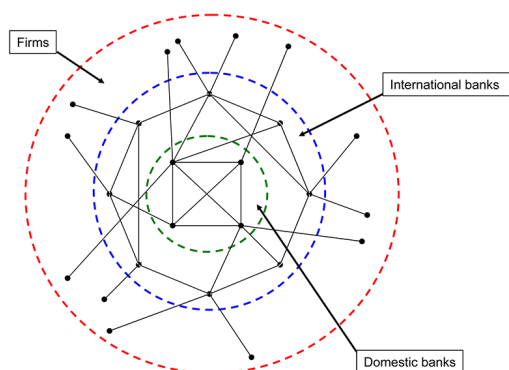
The outer-most layer of the financial system is comprised of *firms* in the economy. Firms are assumed not to lend to each other and do not own shares in one another. They are, thus, not connected to each other in any way. This assumption is made for tractability. Firms are, however, assigned an exogenous credit rating (investment or speculative grade), are subject to common aggregate economy-wide shocks, and exposed to the risk of restrictions in bank credit. The performance of different firms across the economy is therefore correlated following a shock to the financial system.

Although the three layers of the financial system are distinct, each group is linked to the others. Domestic and overseas banks can lend to, and borrow from, each other. They are also able to lend to, and own shares (direct investments) in, firms. The financial relationships across layers are

---

<sup>4</sup>Our data on lending between UK banks corroborates this assumption.

**Chart 1: The stylised financial system where the filled circles - nodes - represent banks/firms, and the links between nodes depict credit or equity relationships. There are three distinct layers: (i) a core of domestic banks; (ii) a peripheral layers of overseas banks; and (iii) an outer layer of firms**



modelled as random graphs. In other words, entities belonging to different layers are linked to each other with a given probability. These probabilities are independent between pairs of entities, and the random links mean banks can differ in terms of lending to, and equity holdings in, firms.

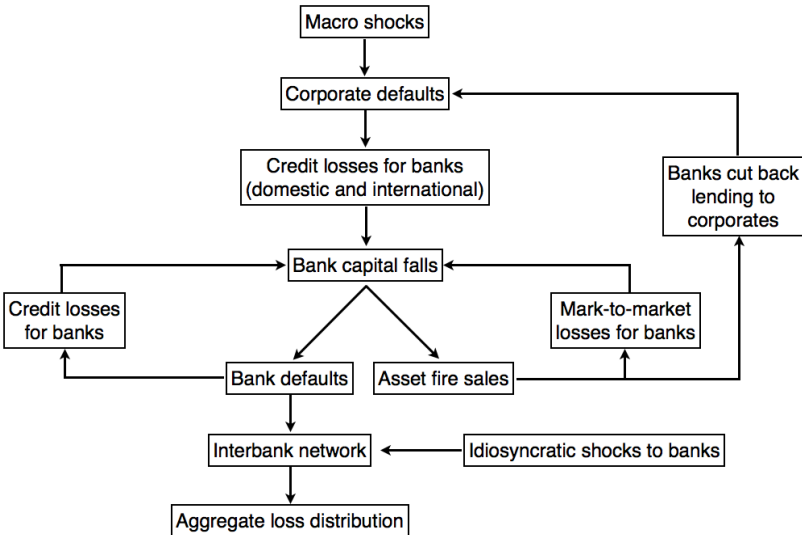
Chart 1 illustrates the financial system. Our use of random graph techniques to model the interlinkages between different types of agent can be viewed as a metaphor for the opacity and reach of modern financial instruments. Policymakers frequently highlight the way in which financial innovation has enabled financial intermediaries to ‘slice and dice’ credit risks to the peripheries of the financial system (Bank of England (2007); Trichet (2008)). The value of instruments such as credit derivatives and their related exposures are difficult to monitor as a result, justifying the probabilistic treatment adopted here.

As Chart 1 shows, banks and firms are represented as nodes in a network. Although not illustrated, links between nodes reflect credit or equity relationships and the network is directed, with incoming links representing assets (ie monies owed to an entity by a counterparty or shares in the case of banks’ relationships with firms) and outgoing links representing liabilities. Chart 2 presents the typical balance sheet of a bank in our model financial system. Total assets comprise loans to firms, loans to other banks (domestic and overseas), shares in firms, and government securities. The liability side of the balance sheet includes customer deposits, interbank borrowing, and the bank’s capital buffer. Our balance sheet structure is sufficiently simple to be tractable while including enough granularity to be interesting.

**Chart 2: Typical balance sheet of a bank in the financial system**

|                              |                               |
|------------------------------|-------------------------------|
| Government bonds             | Customer deposits             |
| Equity against firms         |                               |
| Loans against firms          | Liabilities to overseas banks |
| Loans against overseas banks | Liabilities to domestic banks |
| Loans against domestic banks | Capital buffer                |

**Chart 3: Structure of the model**



Appendix A provides a formal presentation of the model. It specifies banks' balance sheets and shows how our assumptions about connectivity allow the financial system described above to be cast in convenient matrix form. It also specifies how shocks give rise to crisis dynamics and contagion.

An informal sense of the mapping from shocks to systemic risk can, however, be readily gleaned from Chart 3. Macroeconomic disturbances can trigger firm defaults, leading to credit losses and losses on holdings of firm equities at some banks. These shocks can trigger the default of a financial institution and generate a default cascade among banks that are directly linked.<sup>5</sup> But as the losses at an individual bank mount, approaching a critical fraction of capital, it is also likely to take defensive action to try to protect itself from failure. Specifically, it is likely to sell equities once in distress and cut back on its lending to firms.<sup>6</sup> The fire sale of equities and resultant price decline gives rise to mark-to-market losses, forcing other banks to write down the value of their assets and potentially enter into their own fire sales and tighten their own lending to firms. Meanwhile restrictions in credit increase the probability of default of firms, magnifying the initial shock. Direct contagion is, thus, reinforced by fire sales and macroeconomic feedback effects.

We assume that the banking sector is the sole provider of credit for firms who do not have direct access to credit markets or any other channel of credit. Our focus and assumption is a stylised attempt to capture macroeconomic feedback loops from the financial system to the real economy. Moreover, one may expect our assumption to be qualitatively true during periods of financial crisis, where due to a confluence of high capital search costs and a hoarding of liquidity by all institutions, ie a credit crunch, the probability of default for firms increases, which exacerbates the financial crisis.

In the mechanistic setting adopted here, banks follow rules of thumb when confronted with distress. Although plausible, these rules have no micro-foundations. But they can be viewed as being consistent with rational, optimising and myopic behaviour. Facing a highly uncertain recovery rate and timing of economic recovery in the midst of crisis, banks are likely to assume a worst-case scenario and be willing to pull credit lines.<sup>7</sup> These channels are subsumed with other

---

<sup>5</sup>Alternatively, a financial institution may fail for idiosyncratic reasons without there being a macroeconomic shock.

<sup>6</sup>Our approach does not model the dynamic restructuring of balance sheets. In other words, the actual transfer of equity from one bank to another as a result of the fire sale is absent. We motivate this stylised assumption by qualifying our fire sale as an *anticipated* fire sale. Once a bank's capital falls below the critical threshold, all other market participants will anticipate that the bank will perform a fire sale in the near future. It is this anticipation that results in the fall of equity prices.

<sup>7</sup>For simplicity we do not assume an explicit link between a fall in equity prices and the default probabilities of firms.

factors that collectively raise the credit risks of firms.

### 3 Model calibration

We attempt to characterise the state of a modern financial system prior to the onset of a financial crisis. Although we draw upon United Kingdom data for much of our calibration, our choice of parameters is intended to be purely illustrative and does not purport to quantify systemic risk in the United Kingdom. Our intention, instead, is to showcase how the model can usefully generate plausible measures of systemic risk and clarify the interplay between macro-financial shocks, market liquidity, and network structure within a financial system. Since some of the exposure data are confidential in nature, we describe qualitatively how we calibrate some of the key statistics necessary for our maximum entropy procedure. These exposure statistics are, however, reported in detail where the data are publicly available.

The network consists of 17 domestic banks, 240 overseas banks, and 50,000 firms. Seventeen domestic-owned banks accounted for 95% of banking assets in the United Kingdom at end-2007, while three quarters of foreign exchange turnover during 2004 was accounted for by some 240 non-UK banks located in 20 countries according to the Bank for International Settlements (2008) *Consolidated Banking Statistics Report*, which publishes aggregate statistics on cross-border loans and explicitly excludes non-bank financial entities such as insurance companies and hedge funds.<sup>8</sup> As we discuss later, these data allows us to estimate interbank linkages and hence explore interbank contagion across international borders. Our choice of the number of firms is based on the UK Department for Business, Innovation and Skills' press release on *Small and Medium Enterprise Statistics*. At the beginning of 2008 they recorded approximately 33,000 firms in the United Kingdom with 50 employees or more. Our choice of 50,000 firms is broadly illustrative of this statistic. Clearly, the number of overseas banks and firms can be much larger, so our choice simply indicates the situation facing an economy with a highly developed and integrated financial sector.

Given the paucity of data about exposures between banks internationally, and between firms and overseas banks, we rely on deriving distributions of exposure sizes and the number of links between the three types of agent from a limited data set. We use quarterly time-series data on

---

<sup>8</sup> While the BIS report includes data from 29 countries, we focus on the 20 largest countries in terms of foreign exchange turnover against UK banks.

balance sheets over a four-year horizon (2004-07) to fit least biased distributions. These establish the financial connections of our network. Appendix B shows how empirical constraints observed in the data are accounted for in selecting a least biased distribution that also maximises information content.

A novel feature of the calibration is our use of the principle of maximum entropy to estimate the distributions of exposures between different participants. Specifically, we use the principle of maximum entropy to approximate the empirical distribution of exposures. The entropy, which is a function of the probability distribution, is a measure of the *predictability* of exposures. When the entropy is large, there is greater uncertainty on our current state of knowledge and it is harder to predict typical exposure values. In this case, the distribution of exposures is broad. On the other hand, when the entropy is small, the distribution is sharply peaked around a small range of exposures, thereby improving the predictability. The principle of maximum entropy postulates that subject to known constraints (knowledge of the first few moments from the empirical distribution, for example), the probability distribution that best represents our current knowledge and that is least biased is the one with maximal entropy. Importantly, the principle does not require the modeller to make prior assumptions on the shape of the probability distribution. We implement our maximum entropy procedure using the algorithm provided by Mohammad-Djafari (1991).

Our approach is distinct from the maximum entropy methods used in central bank analyses of interbank networks (see for example Elsinger *et al* (2006); Upper (2011)). These studies estimate realisations of exposure matrices whose entropy is as close as possible to a reference matrix. The entropy here is a function of the exposure matrices themselves, which have been suitably re-scaled to satisfy properties of probability distributions.

### **3.1 Structure of balance sheets**

We use end-2007 published accounts data for the 17 UK banks and corresponding data for overseas banks reported in BankScope to characterise the balance sheets used in the model. The average total asset size for UK and overseas banks are £400 billion and £150 billion respectively. Specifically, the mean for the overseas banks is calculated by aggregating equally over all banks. By virtue of the large sample size and heterogeneity across countries, we obtain a lower mean for the overseas banks. Furthermore, the larger mean for UK banks captures the relatively dense

concentration of the domestic banking sector, which includes some of the largest global banks. For UK banks, equities, loans to firms and interbank assets (the sum of claims against both overseas and other UK banks) made up 10%, 80% and 10% of assets respectively, on average. The data from Bankscope suggest a similar picture for overseas banks' balance sheets, so we adopt the same composition for these balance sheets as well.

### **3.2 Distribution of exposure sizes between banks**

We calibrate the distribution of interbank loans between domestic (UK) banks using confidential quarterly data on regulatory large exposures for 17 banks between 2004-07. The empirical mean, standard deviation and skewness for the bilateral claims between our domestic banks were calculated and form the constraints in calculating the maximum entropy probability density function (PDF). Chart 4 plots the maximum entropy PDF (solid line) against the data (circles) on a Y-logarithmic scale.<sup>9</sup> For comparison, we also plot a fitted log normal distribution (dashed line). Both the maximum entropy and log-normal distributions fit the empirical distribution fairly well.

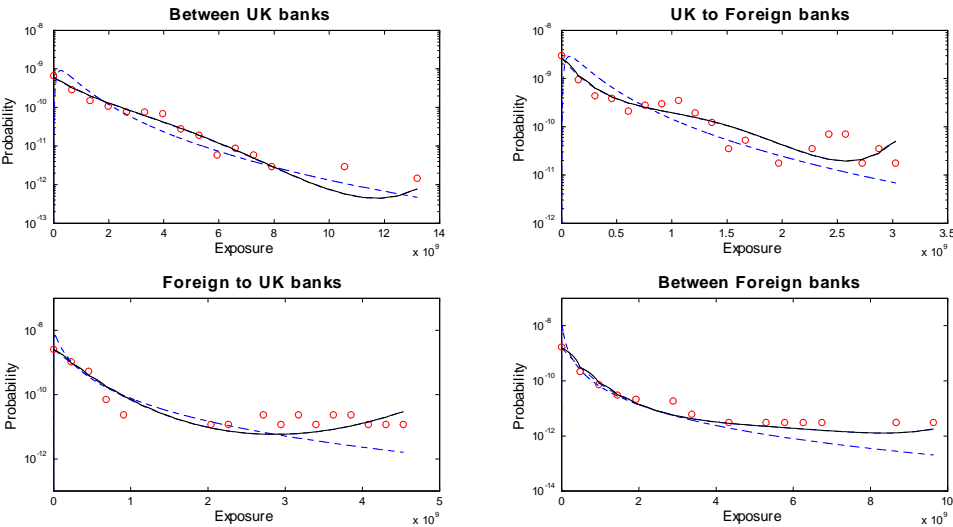
To establish the distribution of loan sizes between domestic and overseas banks, we suppose that the 240 banks originate from the 20 most financially advanced countries that accounted for three quarters of foreign exchange turnover during 2004 against UK banks, and for which data are readily available in the Bank for International Settlements (2008), *Consolidated Banking Statistics Report*. We use this information to establish the sterling claims of UK banks on other countries' banking systems and *vice versa*. We assume that all claims from overseas banks are channeled uniformly through the 17 core domestic banks and the 12 banks in each foreign country. We approximate the individual bank-to-bank claims by dividing the aggregated claims of all domestic banks by the number of UK banks (17) and the number of overseas banks per country (12).

Empirical means, standard deviations and skewness statistics were also calculated for claims held by the domestic banks against overseas banks. For those held by overseas banks against domestic banks, the mean, standard deviation and skewness were £0.28 billion, £0.67 billion, and 4.56. Chart 4 plots the maximum entropy and fitted log-normal PDFs of loans sizes between overseas

---

<sup>9</sup>The PDF for the actual data was obtained by binning the bilateral exposures and normalising the weight attributed to each bin. The circles in Chart 4 correspond to bin centres.

**Chart 4: Empirical distributions (circles) for the interbank asset sizes on a Y-logarithmic axis, ie the probability has a logarithmic scale. We plot against the empirical distribution both the maximum-entropy distribution (solid line) and the fitted log-normal distribution (dashed line).**





and domestic banks. There is again reasonable agreement between the actual and fitted PDFs, although the maximum-entropy PDF seems to capture the fat-tailed nature of the distribution somewhat better.

The overseas banks in our system, of course, also lend to each other as well as to banks within their own jurisdictions. Data on interbank lending within foreign banking systems is not available, however. So we suppose that each overseas bank lends to ten of its local counterparties<sup>10</sup> and that lending between these banks follows the same statistics as interbank lending within the United Kingdom.

In order to calibrate the distribution of exposure sizes between overseas banks in different countries, we make use of cross-border claims data from the BIS. The mean, standard deviation, and skewness of each exposure is £0.25 billion, £0.81 billion, and 6.84 respectively. The fitted maximum entropy distribution in Chart 4 provides a reasonable description of the data, including the fatness in the tail.

### ***3.3 Connections between banks***

In addition to exposure sizes, we also need to establish the number of links between banks to construct the financial network. To obtain the maximum entropy distribution for the number of links that a bank has against other banks, we use the results of Bianconi (2009) that for uncorrelated networks, the maximum entropy distribution for the number of links is a Poisson distribution. Uncorrelated networks are those where the degrees of nodes are not correlated. To argue that our financial network may be modelled by an uncorrelated network we assume that domestic banks are owned by domestic shareholders only. If, on the other hand, domestic banks were owned by foreign shareholders, this would lead to bias in the structure of links between domestic and overseas banks. This assumption holds for our selection of domestic banks. And since we do not know the identities of the other overseas banks, our assumption serves as a null hypothesis for the structure of linkages.

To construct this distribution for our financial system, we need the average number of links between agents – banks or firms – of type X against those of type Y, denoted  $\langle c^{XY} \rangle$ . This implies

---

<sup>10</sup>As made clear below, to utilise the ‘small-world’ network algorithm of Watts and Strogatz (1998) the number of local counterparts for each overseas bank must be even.

that the probability that any link is present between two banks is  $p^{XY} = \frac{\langle c^{XY} \rangle}{N^Y}$ , where  $N^Y$  is the total number of banks of type Y. So the problem reduces to estimating the average number of links between the various types of banks.

We take the domestic banking network as being completely connected. The average number of connections for claims held by domestic banks against overseas banks is obtained by taking the average total interbank assets of a domestic bank, subtracting the average total assets held against other domestic banks, and dividing by the average size of an exposure between a domestic and overseas bank. This suggests that each domestic bank is exposed to 52 overseas banks.

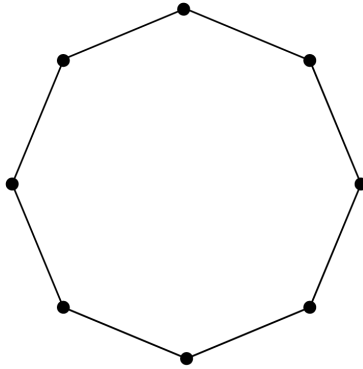
To establish connections between overseas banks, we assume that each overseas bank is connected, on average, to four domestic banks. In this case, the fraction of domestic banks that each overseas bank has loans with (4/17) is roughly equal to the fraction of overseas banks that each domestic bank has loans against (52/240). Each overseas bank lends to ten other banks in its own country. We model the network of all overseas banks as a ‘small-world’ network, where each overseas bank is linked to those in immediate proximity (banks in the same country) and has occasional ‘long-range’ connections to banks in other countries. This means that the number of ‘immediate-neighbour’ connections (between overseas banks in the same country) is  $2\kappa = 10$ . We obtain the average number of ‘long range’ connections by taking average total interbank assets (10% of £150 billion), subtracting the average assets held against domestic banks ( $4 \times$  £0.28 billion) and those held against other banks in the same country, and dividing this quantity by the average size of an exposure between overseas banks in different countries (£0.25 billion). This gives approximately seven ‘long-range’ connections. Defining the ‘long-range’ wiring probability as  $p$ , the average degree for each node is  $2\kappa(1 + p)$ , implying that  $p = 0.7$ .

As Chart 5 illustrates, we arrange the nodes of overseas banks in a ring, connecting each to its immediate (local) neighbours, and then randomly (with probability  $p$ ) allowing an overseas bank to form connections with another bank overseas that is chosen from a uniform distribution over all overseas banks. This procedure is iterated over all overseas banks.

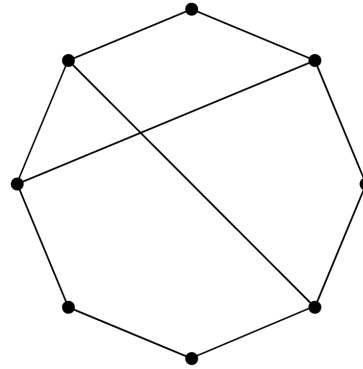
### ***3.4 Distribution of loans to firms and equity holdings***

In the absence of data on individual bank lending to firms, we use our breakdown of banks’ balance sheets to suppose that each loan and equity holding is, on average, £100 million and £10

**Chart 5: The small-world nature of cross-border financial interlinkages. Starting with a regular lattice where each bank is connected to its two nearest-neighbour banks (one on either side), we add ‘long-range’ links at random (with probability  $p$ ) between banks to get the small-world network.**



Regular network



Small world network

million, respectively for domestic banks and £12 million and £1.2 million for overseas banks. The data on average balance sheet size and contributions from loans and equities allow us to infer the connections between domestic (D) and overseas banks (I) and firms (F). These are  $\langle c^{DF} \rangle = 3200$  and  $\langle c^{IF} \rangle = 3200$  for loans, and  $\langle d^{DF} \rangle = 4000$  and  $\langle d^{IF} \rangle = 4000$  for equities.

### 3.5 Corporate default probabilities

Our calibration of corporate sector default probabilities is based on a study of US investment and speculative grade firms by Schuermann and Hanson (2004). They use credit rating data from Standard and Poor’s over the period 1981-2002 to establish Gaussian density functions for annual default probabilities in each grade. We base our default probabilities upon these parameterised density functions. Specifically, we treat the default probability in investment (A) grade category as having a mean and standard deviation of  $\langle PD^{IG} \rangle = 8.65 \times 10^{-5}$  and  $\sigma^{IG} = 2 \times 10^{-5}$ , respectively. The non-investment grade (BB) category has a mean and standard deviation of  $\langle PD^{NIG} \rangle = 6.3 \times 10^{-3}$  and  $\sigma^{NIG} = 6.1 \times 10^{-4}$ , respectively. The proportion of firms that are investment grade (speculative grade) within the system is 0.7 (0.3). We take the firm LGD to be 35%.

The probabilities of default for all firms are also influenced by an common economy-wide shock

$\mu_G$ . The larger  $\mu_G$ , indicating a worsening of the macroeconomic outlook, the higher are the probabilities of default for all firms, irrespective of their rating grade. We do not calibrate  $\mu_G$ , instead we use it as a variable in order to explore how large a macroeconomic shock needs to be in order to ‘tip’ the financial system into a systemic crisis.

### 3.6 Additional parameters

Our model also makes use of some additional parameters that are critical in determining the extent of feedback effects following a shock to the financial network.<sup>11</sup> Specifically,

- Ratio of capital to assets (leverage ratio),  $\pi$  : we initially set banks’ (unweighted) capital buffers to be a uniform  $\pi = 4\%$  of total assets, a figure drawn from the 2005 published accounts of a range of large overseas banks. In Section 4.4, we relax the assumption of uniform capital buffers allowing them to vary in the range 4% – 24%.
- Trigger rule for fire sales,  $\alpha$  : once a bank’s losses from the combined effects of corporate defaults, mark to market losses on its equities, and interbank losses amount to 50% of its initial capital buffer, the bank will decide to put its own tradable assets – equity – up for sale. This trigger level of 50% amounts to setting the parameter  $\alpha = 0.5$ .
- Liquidity discount parameter,  $\lambda$  : we set  $\lambda = 0.7$  to reflect the fact that once 10 % of equity is put up for sale, the equity price  $q(t)$  will fall by 7%. Our parameterisation of the price impact of a fire sale is, to a large extent, arbitrary since evidence on the price impact of fire sales is scarce. Mitchell, Pedersen and Pulvino (2007) analyse fire sales of US convertible bonds by hedge funds in 2005, and suggest that price discounts were around 3% when some 5% of the market was sold. This would correspond to a value of  $\lambda = 0.57$  in our model. Given this estimate is based upon a period of relatively low stress in the financial system, we adopt a value for  $\lambda$  consistent with a more significant price impact.
- Macroeconomic feedback parameter,  $\psi$  : since the literature has not yet settled on an estimate of the macroeconomic feedback, we (somewhat arbitrarily) set  $\psi = 6.25 \times 10^{-5}$  as a working hypothesis<sup>12</sup>. This implies, fairly plausibly, that if a bank reduces the volume of credit it issues to firms by 20% this will increase the probability of default equally for all firms that have pre-existing loans against the bank, irrespective of whether the bank is domestic or overseas.

---

<sup>11</sup>A complete mathematical account for how these various parameters influence a bank’s state is provided in Appendix A.

<sup>12</sup>Our results are robust to changes in the macroeconomic feedback and liquidity discount parameters. For example, setting  $\lambda = 0.7 \pm 0.1$  and  $\psi = (6.25 \pm 2.00) * 10^{-5}$  does not change our qualitative findings.

Specifically, we set  $\psi = 0.2 / \left( \frac{(c^{DF}) + (c^{IF})}{2} \right)$ . Thus, if all banks have more loans, on average, the impact of one bank tightening its credit conditions is mapped into a smaller rise in the probability of default of firms. The trigger rule for a bank to tighten lending conditions is identical to that for fire sales and is governed by the ratio  $\alpha$  of losses to the capital buffer.

#### 4 Credit events, aggregate losses, and feedback effects

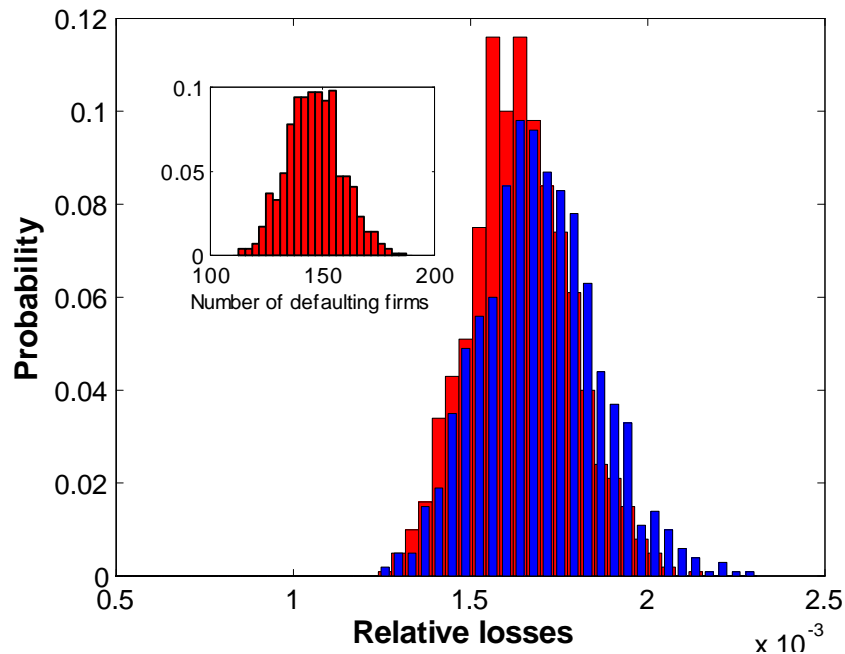
We now present a plausible aggregate loss distribution for the calibrated financial system and evaluate its response to adverse credit shocks. Standard models of systemic risk do not consider the complexity implied by cross-border financial linkages and are typically limited in their characterisation of the feedback effects from asset fire sales and tightening credit conditions in the macroeconomy. The extent to which these factors combine to generate fat tails in the aggregate loss distribution is important to the assessment of financial system resilience. Our stark feedback assumptions are made for tractability and to highlight our findings, but can be readily relaxed without affecting the spirit of the results. Given both this and the broad-brushed nature of the more general calibration, the results presented below should be taken as purely illustrative rather than as a precise measure of systemic risk in the financial system.

##### 4.1 *The baseline aggregate loss distribution*

We perform a series of stochastic simulations to obtain an aggregate loss distribution under a set of baseline assumptions that asset fire sales have no price impact ( $\lambda = 0$ ), there are no macroeconomic feedback effects ( $\psi = 0$ ), nor an aggregate macroeconomic shock to firms ( $\mu_G = 0$ ). For each simulation, we generate balance sheets where exposures are drawn from the connectivity and asset size distributions described in Section 3. Next, through a series of Bernoulli trials, we set some of the firms to default. These defaults are registered on banks' balance sheets, possibly prompting defaults, fire sales or credit crunch feedbacks. Once the shock has run its course through the network and asset prices have adjusted, we measure the fraction of failed banks and the balance sheets of all banks in the system. We perform 1,000 such simulations and create a distribution for financial system assets lost.

The inset of Chart 6 presents the normalised histogram for the number of defaulting firms from the initial credit shock. The number of failures from subjecting firms to a shock is characterised

**Chart 6: Distribution of losses relative to system assets for the entire banking network (red) and the domestic banking network (blue) for the baseline scenario. In the insert we show the corresponding normalised histogram for the number of defaulted firms.**



by a Bernoulli distribution function (see description in Appendix A). The typical defaulting firm is small in relation to the overall financial system – the average size of a defaulting firm is 0.003% of total system assets. In the baseline, 150 firms default on average in each simulation.

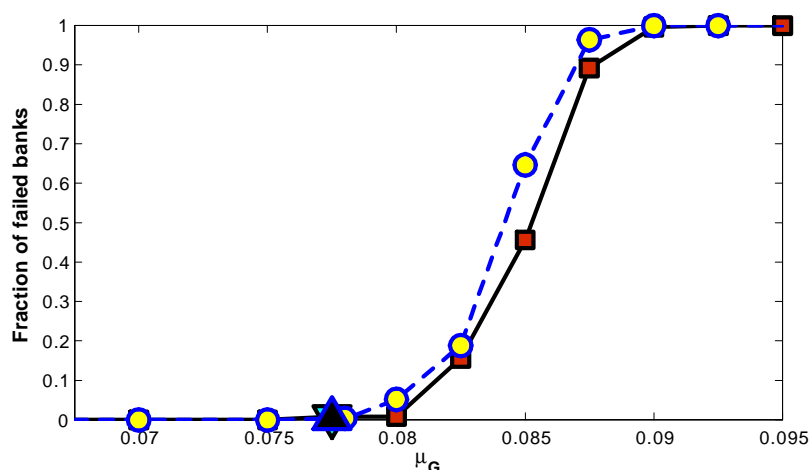
Chart 6 presents the aggregate loss distribution for the financial system, as a fraction of total banking system assets, for the domestic bank network and the overall financial system. The idiosyncratic defaults on the scale described above have very little impact on system resilience – there are no bank defaults. Average system losses as a result of the idiosyncratic shocks are some 0.17% of the domestic banking system and 0.16% of the overall system. In other words, both domestic and overseas banks are similarly affected following idiosyncratic corporate failures.

#### 4.2 *Macroeconomic shocks*

Aggregate macroeconomic shocks to the financial system have an adverse effect on firms and enter the model via the parameter,  $\mu_G$ , increases in which drive higher levels of firm default.

We begin by attempting to identify the scale of firm default, absent any feedback effects from fire

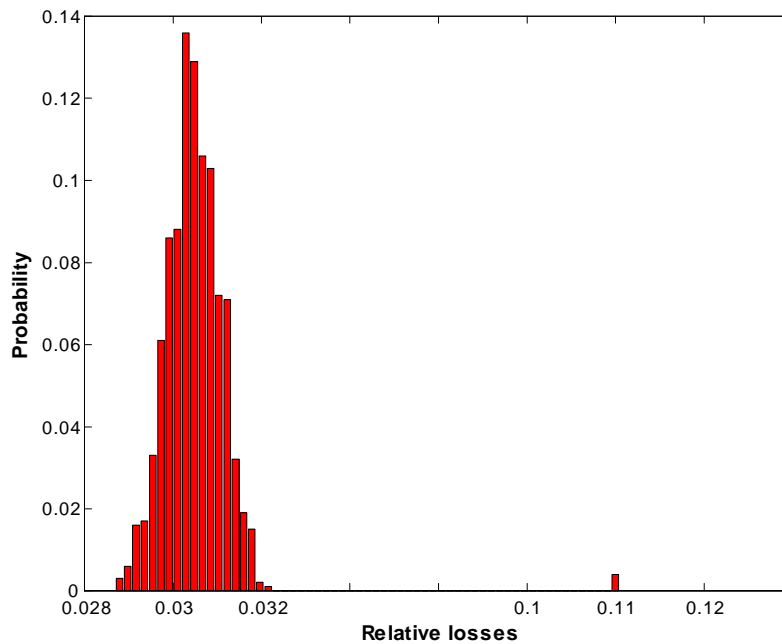
**Chart 7: The average fraction of failed banks as a function of the aggregate macroeconomic shock,  $\mu_G$ . The initial instance of total system breakdown is indicated by the downward-facing triangle. The dashed line represents the case with  $\psi = 6.25 \times 10^{-5}$  and  $\lambda = 0$ . The upward-facing triangle indicates the corresponding first instance of total breakdown.**



sales or a credit crunch, that triggers the first instances of complete financial system failures (ie the failure of all domestic and overseas banks). The results reported for each  $\mu_G$  are compiled from performing 500 draws of the shock and letting them run their course through the system. The initial instance of system-wide failure occurs at  $\mu_G^c = 0.078$ . This point is depicted in Chart 7 by a downward-facing triangle, and is associated with the default of 2,700 firms on average. For  $\mu_G < \mu_G^c$ , we only observe cases where *at most* one overseas bank fails. At  $\mu_G^c$ , by contrast, we observe that there are no bank failures 99.5% of the time, one bank failing 0.1% of the time, and in the remainder 0.4% of cases the entire system fails. In these instances of complete network failure, the initial macroeconomic shock reduces the capital buffer for banks holding loans and equity against the defaulting firms, triggering the direct failure of a few banks. Interbank linkages then lead to direct contagion, as the similarity among banks in their ability to absorb shocks leads to a starkly bi-polar result in which *all* banks fail once widespread contagion has broken out. At the critical value  $\mu_G^c$ , the loss distribution becomes bimodal for the first time.

Chart 7 also shows the average fraction of failed banks (solid line with squares) as a function of  $\mu_G$ . Due to the assumed homogeneity in banks' ability to withstand shocks, each square represents the probability the financial system will collapse for a given level of macroeconomic shock. As  $\mu_G$  approaches 0.09, the probability of system failure accelerates towards unity. There is an inflection point associated with  $\mu_G \approx 0.085$ . Here, the probability mass is equally

**Chart 8: Loss distributions for the stressed aggregate macroeconomic shock scenario where  $\mu = 0.078$ .**



distributed between the two modes of the aggregate loss distribution, representing the ‘*phase transition*’ or ‘*tipping point*’ of the financial system. For macroeconomic shocks above this level, the financial system will always collapse.

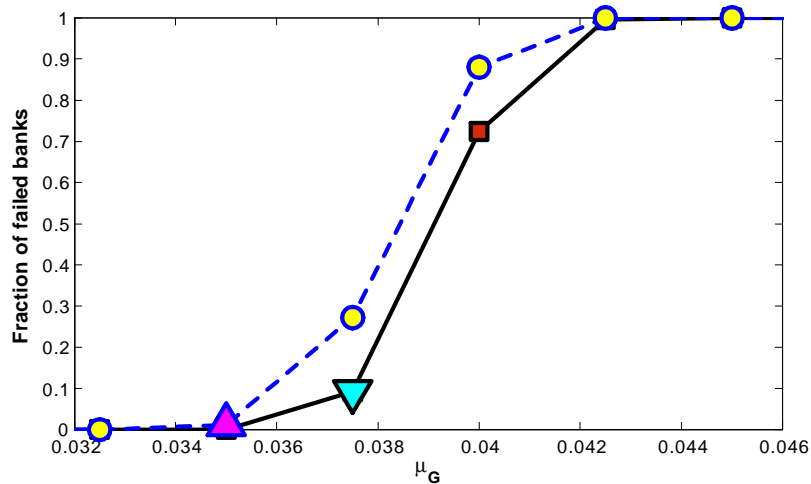
Chart 8 plots the aggregate loss distribution for the entire banking system for the stressed scenario where  $\mu_G = 0.078$ . As can be seen, the losses under the adverse scenario are an order of magnitude greater than those in the baseline and the distribution is bimodal. The probability mass is concentrated around: (a) small losses of around 3% of system assets; and (b) a few extreme instances where around 11% of system assets are lost. In these extreme cases, the entire financial system collapses.

### **4.3 Feedback effects**

We now investigate the effects of asset fire sales and the withdrawal of bank lending to firms on the aggregate loss distribution. When banks are in distress and losses mount in excess of a trigger threshold,  $\alpha$ , of their capital buffer, they sell their holdings of equities and simultaneously tighten their lending to firms. The withdrawal of credit from remaining firms increases their probability of default. As further credit losses mount, the feedback effects of reduced bank lending amplify



**Chart 9: The average fraction of failed banks as a function of  $\mu_G$ . The black line represents the case where  $\lambda = 0.7$  and  $\psi = 0$ . The downward-facing triangle indicates the first instance of total system breakdown. The dashed blue line is for  $\lambda = 0.7$  and  $\psi = 6.25 \times 10^{-5}$ . The upward-facing triangle indicates the first instance of total breakdown for these parameters.**



the losses to banks and, together with the mark-to-market effects of fire sales, contribute to further financial instability.

We initially focus on the pure macroeconomic feedback effect of a credit crunch and abstract away from the possibility of any distress fire sales. So that  $\lambda = 0$  and  $\psi = 6.25 \times 10^{-5}$ . In this case we find no shift in the tipping point  $\mu_G^c = 0.078$ . However, as Chart 7 indicates, the average fraction of failed banks is higher, as indicated by an upward shift of the curve in Chart 7. In the case of a pure fire-sale effect, ie  $\psi = 0$  and  $\lambda = 0.7$ , Chart 9 shows the minimum critical quantum of credit risk necessary to instigate system collapse is brought forward sharply to  $\mu_G^c = 0.037$ . The average fraction of failed banks again shows the probability of system-wide failure.

Allowing for the possibility that banks both tighten credit conditions and engage in fire sales when in distress, further brings forward the first instance of system failure. Our calibration suggests that  $\mu_G^c = 0.031$  in this case and, as Chart 9 shows, the probability of system failure is greater for all values of  $\mu_G$ . The intuition is straightforward. In the first round, the tightening of credit by banks pushes further firms into bankruptcy, amplifying the extent of credit losses among banks. The ensuing distress of some banks leads to further fire sales and a second round of credit tightening that further raises the probability of firm default. The cycle only terminates

once the entire banking system fails.<sup>13</sup>

Under this calibration of our model, macroeconomic feedbacks are less substantial than fire sales. But both are difficult to calibrate meaningfully. While our fire-sale calibration is based on Mitchell *et al* (2007), research on the appropriate calibration of macroeconomic feedbacks is limited. While it may well be that this feedback is more substantial than we assume, the combined effects of both the fire-sale and macroeconomic feedbacks appear plausible. A thorough calibration of these mechanisms is beyond the scope of this paper and is an avenue for future analysis.

#### **4.4 A more realistic setting - heterogeneity of capital buffers**

Our depiction of financial fragility has been extremely stark – a change in the size of a credit shock around a critical value determines whether the entire network collapses or not. More realistically, one might expect situations in which intermediate outcomes obtain, in which only some banks fail but the rest of the system continues to function.

We therefore relax the assumption that all banks have the same capital buffer, and allow it to vary from institution to institution. The capital-asset ratio,  $\pi$ , is now drawn from a uniform distribution with support  $[0.04, 0.24]$ , more representative of the variation in buffers seen in practice in some countries.

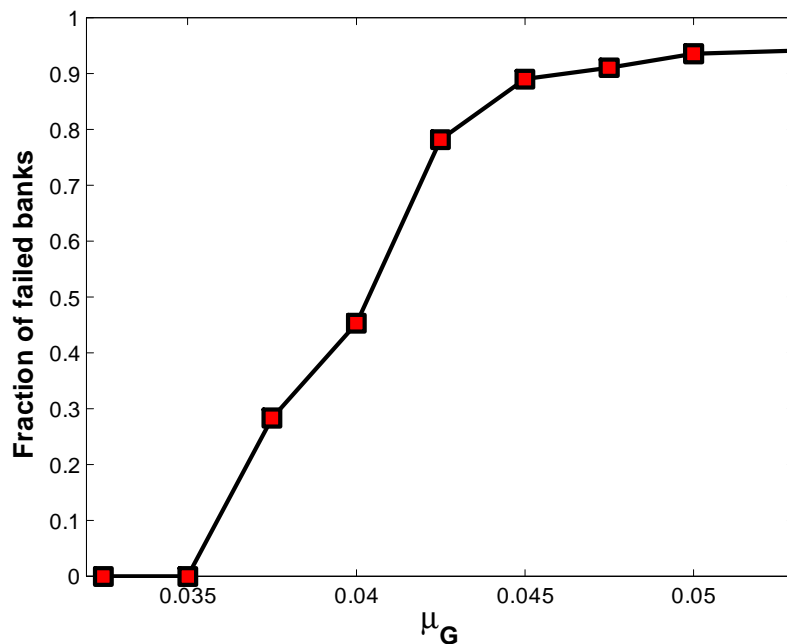
Chart 10 depicts a much richer set of results. We note, for example, that for  $\mu_G = 0.0375$ , on average 2.8% of banks default. The banks that fail are both foreign and domestic. In particular instances of the simulations, 219 banks collapsed (85% of the the total system), while the few remaining, by virtue of higher capital buffers did not. As  $\mu_G$  increases to 0.0475 a similar conclusion is drawn. On average 91% (or 234) of banks in the system collapse. Once again, a few banks are found to be sufficiently well capitalised to survive the shock and feedback effects.

The highest default rates between 1920 and 2006 were, 1.7% for investment grade – ratings class A – firms and 11.1% for speculative grade – ratings class Ba – firms (Moody's (2007)). These higher default rates were witnessed, in particular, during the Great Depression, which was a

---

<sup>13</sup>Our results are robust to changes in the macroeconomic feedback and liquidity discount parameters. For example, setting  $\lambda = 0.7 \pm 0.1$  and  $\psi = (6.25 \pm 2.00) \times 10^{-5}$  does not change our qualitative findings.

**Chart 10: The average fraction of failed banks as a function of  $\mu_G$ , where  $\lambda = 0.7$ ,  $\psi = 6.25 \times 10^{-5}$  and  $\pi$  is random and uniformly distributed in the interval  $[0.4, 0.24]$ .**



period of widespread bank failures. In our model, these figures correspond to a probability of default for the average of 4.5%, which is generated by taking  $\mu_G = 0.045$ . With liquidity and macro-feedback effects switched on and allowing banks to have heterogenous levels of capital in our model, we find that with  $\mu_G = 0.045$  a significant fraction (89%) of the entire financial system collapses, on average.

## 5 ‘Stress testing’

In the wake of the global financial crisis, policymakers have subjected banking systems to ‘stress tests’. While we cannot hope to do full justice to such exercises, our model is well versed to provide a caricature of tests that include both macroeconomic feedback and asset fire sales. Moreover, our use of a random graph structure sidesteps the challenge faced by regulators of assessing the true scale of network connections due to complex financial products.

Specifically, we may ask how well domestic banks absorb firm exposure loss rates of around 4.5% ( $\mu_G = 0.044$ ) and a 35% haircut of equity exposures stemming from a 20% equity price fall ( $\lambda = 0.37$ ). Under this scenario we find that there are no bank failures in 67% of the simulated instances. However, in the remaining 33% of cases we have, on average, 200 (78%) overseas and

domestic banks failing. This implies that an overall average of 25% of banks fail. In the instances where no banks default, the losses solely due to firm defaults amount to 1.8% of total system assets. However, in the remainder of instances, we have, on average, 19% of all assets being lost.

The high average percentage of bank failures may be due to the difficulties of calibrating macroeconomic and fire-sale feedback loops. These elements exacerbate shocks to the banking system, as discussed in Sections 4.2 and 4.3, thereby heightening the fragility of the financial system. Secondly, we take a 100% LGD on interbank exposures. This assumption is stark and intended to be purely illustrative. Actual LGDs are likely to be low.<sup>14</sup> Taking, as we do, a loss rate of 100% amplifies losses due to bank defaults, which further contributes to the degradation of systemic stability. A final exacerbating factor is that firm and banking sector losses are incurred instantaneously in our model, while in real financial systems these would crystallise over a period of time, giving banks time to offset their losses with earnings.

## 6 Conclusion

Modern financial systems are characterised by complex interlinkages and a diverse set of agents. Our paper develops a general framework to gauge system stability in the presence of such linkages and heterogeneity. Calibrating the model using data based on advanced country banking sectors that are largely public, we illustrate how macroeconomic fluctuations, asset market liquidity, and network structure interact to determine aggregate credit losses and contagion. Although our calibration is broad-brush in nature to emphasise the qualitative aspects of the model, the results show how system stability might begin to be quantified in a statistical fashion, particularly when data about the reach of modern financial instruments are limited and shocks are international in nature.

A thorough understanding of both the qualitative and quantitative features of aggregate loss distributions in the banking system is important for policymakers concerned with systemic risk. Our findings indicate that macroeconomic shocks and asset price feedback effects intertwine to generate fat tails in these distributions and show how large-scale financial disruption may be possible. We also show how the heterogeneity of bank balance sheets gives rise to more realistic situations in which some banks fail, but the overall system remains resilient. Higher capital

---

<sup>14</sup>For instance, published results on loss rates (eg James (1991)), report a loss rate of 40% for banks. Relatedly, Altman and Kishore (1996) estimate the recovery rates (100-LGD %) on defaulting bonds of financial institutions between 1978-95 to be about 36%, on average. However, recovery rates vary by type of institution: mortgage banks 68%, finance companies, 46% and commercial banks, 29%.

promotes stability in our model. However, as we abstract away from the cost of capital, our model cannot comment on the welfare implications of such a policy. Nevertheless some recent proposals on appropriate capital levels for globally systemically important banks (eg Basel Committee on Banking Supervision (2011)) are broadly in line with our findings.

The model clearly illustrates how complex financial systems are vulnerable to system-wide breakdown of the type observed during the recent global financial crisis. It can also be used to inform stress-testing exercises. Drawing on the types of scenarios and shocks typically used to gauge financial sector resilience, we find the model generates outcomes that are broadly plausible. In particular, the default rates in the corporate sector necessary to trigger a systemic financial event in the model are comparable to those witnessed during the Great Depression.

Our model imposes several simplifying restrictions on connectivity; principally that they are static and do not evolve over time. Relaxing these restrictions and altering the topology of the network may affect risk-sharing and change the degree to which shocks are dispersed safely across the financial system. A thorough evaluation of changing the network linkages between and among different types of agent is a task we leave for future research. An even greater challenge is to incorporate optimising behavioural responses into this type of network model, while retaining the complexities in its structure.

## A A statistical model of the financial system

### *Financial relationships between different types of agent*

The financial system consists of  $N$  agents who belong to one of three types: (1)  $N^D$  domestic banks, (2)  $N^I$  overseas banks and (3)  $N^F$  firms<sup>15</sup>, where  $N = N^D + N^I + N^F$ .

Each agent is represented by a node on a directed graph and linked to each other through their assets, liabilities, and equity holdings. Specifically, for an agent  $i$ , an incoming link from agent  $j$  represents an asset - either loans or equities - on  $i$ 's balance sheet. Let the value of loans and equities from agent  $i$  to  $j$  be  $A_{ij}, Q_{ij} \in \mathbb{R}^+$ , respectively. Outgoing links represent an agent's liabilities with value  $L_{ij} \in \mathbb{R}^+$ .

Connections between agents of different types are formed randomly. The variables  $c_{ij}, d_{ij} \in \{0, 1\}$  denote whether agent  $i$  holds a loan or equity assets against agent  $j$ . This, we write

$$A_{ij} = c_{ij} S_{ij} \tag{A-1}$$

and

$$Q_{ij} = d_{ij} T_{ij} \tag{A-2}$$

where  $S_{ij}, T_{ij} \in \mathbb{R}^+$  are random variables that describe the extent of the exposure.

The statistics of our random variables are governed by the *type* of the lending and borrowing agents; ie whether one or the other is a domestic or overseas bank or a firm. We define  $\rho_{DI}(S_{ij})$  as the probability density function (PDF) of loans from domestic bank, labelled  $i$ , to the overseas bank, labelled  $j$ . Similarly, we can define the PDF  $\rho_{IF}(T_{ij})$  of equity holdings between the

---

<sup>15</sup>We furthermore denote by  $\aleph^D, \aleph^I$  and  $\aleph^F$  the set of domestic banks, overseas banks and firms, respectively.

overseas bank  $i$  and firm  $j$ . Considering all possible combinations of agent types, and hence lending arrangements, the statistics for sizes of loan and equity holdings is governed by 18 different probability distributions.

For the connectivity coefficients  $c_{ij}$  and  $d_{ij}$  as well we can apply a similar procedure to define  $\tilde{\rho}_{DI}(c_{ij})$  as the probability mass function (PMF) for the presence (or absence) of a loan from domestic bank  $i$  to overseas bank  $j$ . Similarly,  $\tilde{\rho}_{IF}(d_{ij})$  defines the PMF determining the probability with which overseas bank  $i$  holds equity of firm  $j$  in our financial system. The financial relationships between different types of agent can now be given a convenient matrix form. Their interactions are summarised by the matrix

$$F = \left[ \begin{array}{ccc|ccc} \mathbf{A}^{DD} & \mathbf{A}^{DI} & \mathbf{A}^{DF} & \mathbf{Q}^{DD} & \mathbf{Q}^{DI} & \mathbf{Q}^{DF} \\ \mathbf{A}^{ID} & \mathbf{A}^{II} & \mathbf{A}^{IF} & \mathbf{Q}^{ID} & \mathbf{Q}^{II} & \mathbf{Q}^{IF} \\ \mathbf{A}^{FD} & \mathbf{A}^{FI} & \mathbf{A}^{FF} & \mathbf{Q}^{FD} & \mathbf{Q}^{FI} & \mathbf{Q}^{FF} \end{array} \right] \quad (\text{A-3})$$

where  $\mathbf{A}^{XY}$  and  $\mathbf{Q}^{XY}$  are matrices of exposures from type  $X$  agents to other type  $Y$  agents, whose elements are  $c_{ij} S_{ij}$  and  $d_{ij} T_{ij}$ , respectively, with  $i \in \aleph^X$  and  $j \in \aleph^Y$ .

The form used for our exposure PDFs and connectively PMFs are spelt out in the calibration section.

### ***Financial relationships between agents of the same type***

We here assume that all core domestic banks hold assets against every other domestic bank. Hence in matrix  $\mathbf{A}^{DD}$  we have that  $c_{ij} = 1$ . This forms a complete network of the core banks through their lending relationships. Firms do not hold assets or equity against each other or against domestic and overseas banks. Moreover, banks only hold equity of firms. Hence the matrices  $\mathbf{A}^{FD}$ ,  $\mathbf{A}^{FI}$ ,  $\mathbf{A}^{FF}$ ,  $\mathbf{Q}^{DD}$ ,  $\mathbf{Q}^{DI}$ ,  $\mathbf{Q}^{ID}$ ,  $\mathbf{Q}^{II}$ ,  $\mathbf{Q}^{FD}$ ,  $\mathbf{Q}^{FI}$  and  $\mathbf{Q}^{FF}$  are all equal to zero.

Interactions between overseas banks take place on a small-world network. Such networks are characterised by: (i) their clustering coefficient, which reflects the clique-like relationship between a node and its nearest neighbours; (ii) long-range links between ‘distant’ nodes which result in a short average path length (ie a short average number of links between any two nodes).

The connectivity coefficients between overseas banks, ie  $c_{ij}$ , with  $i, j \in \aleph^I$ , are constructed using the algorithm proposed by Watts and Strogatz (1998). Pictorially, we arrange the nodes of overseas banks in a ring and connect each node to its  $2\kappa$  nearest neighbours. Next, starting with the first bank, we add with probability  $p$  a ‘long-range’ link to another bank outside its nearest-neighbourhood. We perform this random draw and ‘long-range’ link addition with probability  $p$  a total of  $2\kappa$  times for the first bank. This procedure is iterated over all overseas banks. The total number of ‘long-range’ links is  $N^I 2\kappa p$ .

Taken together, our assumptions on connectivity lead to a restricted matrix and imply that our financial system can be represented as

$$F' = \left[ \begin{array}{ccc|c} \mathbf{A}^{DD} & \mathbf{A}^{DI} & \mathbf{A}^{DF} & \mathbf{Q}^{DF} \\ \mathbf{A}^{ID} & \mathbf{A}^{II} & \mathbf{A}^{IF} & \mathbf{Q}^{IF} \end{array} \right] \quad (\text{A-4})$$

### **Bank balance sheets**

We now describe the bank balance sheets depicted in Chart 2 formally. The total assets of bank  $i$ , which may be either domestic or overseas, is

$$A_i = \sum_{j \in \aleph_i^D} S_{ij} + \sum_{j \in \aleph_i^I} S_{ij} + \sum_{j \in \aleph_i^F} S_{ij} + \sum_{j \in \mathfrak{S}_i^F} T_{ij} + B_i \quad (\text{A-5})$$

where

$$\aleph_i^X = \{j \in \aleph^X \setminus i \mid c_{ij} = 1\} \quad (\text{A-6})$$

and

$$\mathfrak{S}_i^F = \{j \in \aleph^F \setminus i \mid d_{ij} = 1\} \quad (\text{A-7})$$

and  $B_i$  denotes the level of government bonds. The set  $\aleph_i^X$  denotes the set of institutions  $j$  (type



$X$ ) against whom bank  $i$  holds an asset. Similarly,  $\mathfrak{S}_i^X$  denotes the set of firms  $j$  whose shares bank  $i$  owns. The total liabilities are

$$L_i = \sum_{j \in \tilde{\mathfrak{N}}_i^D} S_{ji} + \sum_{j \in \tilde{\mathfrak{N}}_i^X} S_{ji} + K_i + D_i \quad (\text{A-8})$$

where  $D_i$  denotes external liabilities such as customer deposits and the initial capital buffer  $K_i = \pi A_i$  is a fixed fraction  $\pi \in (0, 1)$  of assets on the balance sheet. As before,  $\tilde{\mathfrak{N}}_i^X = \{j \in \mathfrak{N}^X \setminus i \mid c_{ji} = 1\}$  denotes the set of banks  $j$  (type  $X$ ) to whom bank  $i$  has a liability.

If the initial assets of each bank drawn from the asset distribution exceeds initial liabilities, the liability-side of the balance sheet is ‘topped’ up by customer deposits to ensure that total assets are equal to total liabilities. Conversely, if liabilities exceed assets, the difference is accounted for on the asset side by holdings of government bonds.

### *Crisis dynamics*

In our simulations, a bank will default if its total losses are greater than its capital buffer. We consider a two-state model; ie during each instance of the internal simulation time  $t \in \mathbb{N}$ , bank  $i$  is either solvent ( $v_i(t) = 0$ ) or it has defaulted ( $v_i(t) = 1$ ). Defining the total losses incurred by bank  $i$  as  $\mathcal{L}_{i,tot}(t) \geq 0$ , we obtain the following update rule:

$$v_i(t+1) = \Theta(\mathcal{L}_{i,tot}(t) - K_i) \quad (\text{A-9})$$

where  $\Theta(\cdot)$  is the Heaviside function. In what follows, we specify the various components that contribute to bank losses.

A crisis is instigated by shocks to firms. We model firm default using a Bernoulli model, of the sort widely used in the credit risk literature (Gordy (2000)) and in the risk management industry. Similar to that of banks, we define the state of firm  $\ell$  as being solvent ( $\mu_\ell(t) = 0$ ) or defaulted on its loans to banks ( $\mu_\ell(t) = 1$ ). Furthermore, firms are classified according to their creditworthiness, which is quantified by a probability of default  $PD_\ell(t) \in (0, 1)$ , for firm  $\ell$  at

time  $t$ . All firms fall into one of two categories: (i) investment-grade (IG); or (ii) non-investment/speculative grade (NIG). The probability of default  $PD_\ell^\eta(t)$  for firm  $\ell$  (of grade  $\eta$ ) at time  $t$  is given by

$$PD_\ell^\eta(t) = R_\ell(\eta) + \mu_G + \psi \left( \sum_{i \in W_i^D} \phi_i(t) + \sum_{i \in W_\ell^I} \phi_i(t) \right) \quad (\text{A-10})$$

where  $R_\ell(\eta) \in (0, 1)$  is drawn from the distribution of  $\rho_\eta(PD)$  for firms in grade  $\eta$ . The second term  $\mu_G$  reflects an aggregate economy-wide shock to all firms. The final term in the equation above reflects the macro-feedback loop. The indicator variable  $\phi_i(t)$  denotes whether bank  $i$ 's intent to perform a fire sale ( $\phi_i(t) = 1$ ) or abstain ( $\phi_i(t) = 0$ ) from such drastic action. If bank  $i$  is forced to perform a fire sale, the action is accompanied by the bank cutting back in its lending to firms. This act leaves firms more vulnerable to default, thereby resulting in an increase of their  $PD$  by an amount  $\psi$ . Thus for each bank (domestic and overseas) performing a fire sale, against whom firm  $\ell$  has borrowed (denoted by the set  $W_\ell^X = \{i \in \mathbb{N}^X | c_{i\ell} = 1\}$ , there will be a  $\psi$  increase to the firm  $PD$ .

Firms default according to a series of Bernoulli trials, ie starting with all firms being solvent, at specific time  $t^*$ , each firm  $\ell$  will default independently of others with probability  $PD_\ell^\eta(t^*)$ . These times  $t^*$  occur each time the PDs of firms are incremented by factors of  $\psi$  due to the cutting back of lending by the banks.

Default severs the connections (loans and shares) between banks and firms. We take the firm loan recovery rate to be  $\beta$  and assume that share prices of the defaulted firms drop to zero.<sup>16</sup> Thus, losses from firms for bank  $i$  are

$$\mathcal{L}_{j,c}(t) = \sum_{k \in \mathbb{N}_j^D} S_{jk} \eta_k(t) + \sum_{k \in \mathbb{N}_j^I} S_{jk} \eta_k(t) \quad (\text{A-11})$$

Contagion may also spread indirectly as a result of mark-to-market losses on balance sheets brought on by fire sales of assets by banks in distress. As any individual bank incurs losses, it is

<sup>16</sup>The stylised zero recovery rate assumption simplifies the mathematical structure of the model. And, though we adopt it in our simulations, the framework allows for this assumption to be relaxed in a straightforward manner.

likely to take defensive actions to protect itself from failure. One option, exercised by some institutions since the advent of the current financial crisis, is for the bank to sell assets. Therefore, we allow banks to sell equities when they are in distress (we suppose that debt is completely illiquid and therefore cannot be sold). Specifically, banks engage in fire sales of equities once losses mount above a certain fraction  $\alpha \in (0, 1)$  of their capital, ie

$$\phi_j(t) = \Theta (\mathcal{L}_{j,c}(t) + \mathcal{L}_{j,f}(t) - \alpha K_j) \quad (\text{A-12})$$

Let  $Q(t) \geq 0$  be the equity held by banks participating in a fire sale at time  $t$ , ie

$$Q(t) = \sum_{i \in \mathbb{N}^D} \phi_i(t) \left( \sum_{\ell \in \mathbb{S}^F} T_{i\ell} \right) + \sum_{i \in \mathbb{N}^I} \phi_i(t) \left( \sum_{\ell \in \mathbb{S}^F} T_{i\ell} \right) \quad (\text{A-13})$$

and  $Q > 0$  be the total equity held by all banks. The dynamics of the equity price,  $q(t) \geq 0$ , are determined by a form of ‘cash in the market’ pricing (Allen and Gale (2005); Cifuentes *et al* (2005)), where the price is reducing in the ratio of the equities for sale to the quantity of equities not being sold, a proxy for non-distressed potential buyers. We therefore write

$$q(t+1) = q(t) \left( 1 - \lambda \frac{Q(t)}{Q - Q(t)} \right) \quad (\text{A-14})$$

where  $\lambda \in \mathbb{R}^+$  is a parameter that measures the price impact of a fire sale.<sup>17</sup> If the market is extremely liquid,  $\lambda = 0$  and there is no price impact from asset sales, whereas  $\lambda > 0$  implies that equity prices fall sharply for a given amount of distressed assets on the market.

When the equity price falls, banks incur mark-to-market losses on their equity holdings. Bank  $j$ 's total losses at time  $t$  are thus given by

$$\mathcal{L}_{j,tot}(t) = \mathcal{L}_{j,c}(t) + \mathcal{L}_{j,f} + \sum_{k \in \mathbb{S}^F} T_{jk} (q(0) - q(t)) (1 - \mu_k(t)) \quad (\text{A-15})$$

---

<sup>17</sup>While the prescribed form of equity price captures an acceleration in price fall as more equity is dumped onto the market, we must explicitly demand that negative prices are not allowed. This may be achieved by multiplying the right-hand side of equation (A-14) by  $\Theta(q(t))$ .

where the last term refers to losses incurred due to a fall in equity prices of firms that did not default from the initial shock.

When one bank has defaulted, related counterparty and mark-to-market losses may cause other banks to default. This process continues iteratively, with continually updating counterparty and mark-to-market losses, until no further banks are pushed into default.<sup>18</sup>

---

<sup>18</sup>Eisenberg and Noe (2001) demonstrate that, following an initial default in such a system, a unique vector which clears the obligations of all parties exists.



## B Principle of maximum entropy

Let us define  $P(X)$  to be the probability distribution for the random variable  $X \in \mathbb{N}$ . We shall for the moment concentrate on the case of discrete random variables but the theory may be readily generalised to the case of continuous random variables.

Suppose we can observe and empirically measure the first  $M$  raw moments of the distribution, which we write as

$$\mu_n = \sum_{x \geq 0} x^n P(X = x), \quad n = 0, 1, \dots, M \quad (\mathbf{B-1})$$

The  $n = 0$  case simply reflects that the probability distribution must be normalised, ie  $\mu_0 = 1$ .

Our goal is to find the least biased form for  $P(x)$  that satisfies the constraints given by equation **(B-1)**. The principle of maximum entropy states that the distribution we seek is the one that maximises the information entropy. We can solve for this distribution from the Lagrange function

$$S[P] = - \sum_{x \geq 0} P(x) \log P(x) + \sum_{n=0}^M \lambda_n \left[ \sum_{x \geq 0} x^n P(X = x) \right] \quad (\mathbf{B-2})$$

where  $\lambda_n \in \mathbb{R}$  indicate the Lagrange multipliers that we must solve for. The first term in equation **(B-2)** gives us the information entropy. Our maximal entropy distribution is given by solving  $\partial S[P]/\partial P(x) = 0$ , which yields

$$P(x) = \exp \left( - \left[ \lambda_0 + \lambda_1 x + \lambda_2 x^2 + \dots + \lambda_M x^M \right] \right) \quad (\mathbf{B-3})$$

where

$$\lambda_0 = \log Z$$



$$= \log \sum_{x \geq 0} \exp(-[\lambda_0 + \lambda_1 \langle x \rangle + \lambda_2 \langle x^2 \rangle + \dots + \lambda_M \langle x^M \rangle]) \quad (\text{B-4})$$

enforces the normalisation of the probability distribution and the Lagrange multipliers are given as the solutions to the set of  $M + 1$  equations

$$\mu_n = \int dx x^n \exp\left(-\sum_{m=0}^M \lambda_m x^m\right), \quad n = 0, 1, \dots, M \quad (\text{B-5})$$

Closed form analytical expressions for the distributions are available only when  $M \leq 2$ . For  $M = 0$ , we only specify that the probability distribution must be normalised. This corresponds to Laplace's principle of indifference, which dictates that if we have no prior information to distinguish between different states of a system we must associate equal probability to each state. For  $M = 1$ , we impose that the distribution must be normalised and specify its mean. If the mean is positive then we get an exponential distribution. Finally, when  $M = 2$  and the support for the random variable is the entire real axis, we obtain the Normal distribution function. For higher values of  $M$  there is no closed form analytical expression and we must rely on numerical methods to solve for the distribution. In particular, we follow the method proposed by Mohammad-Djafari (1991) for the estimation.

## References

- Aikman, D, Alessandri, P, Eklund, B, Gai, P, Kapadia, S, Martin, E, Mora, N, Sterne, G and Willison, M (2009)**, ‘Funding liquidity risk in a quantitative model of systemic stability’, *Bank of England Working Paper No.372*.
- Alessandri, P, Gai, P, Kapadia, S, Mora, N and Puh, C (2009)**, ‘Towards a framework for quantifying systemic stability’, *International Journal of Central Banking*, Vol. 5, pages 47–82.
- Allen, F and Babus, A (2008)**, ‘Networks in finance’, in Kleindorder, P and Wind, J (eds), *Network-based strategies and competencies*, Wharton School Publishing, University of Pennsylvania.
- Allen, F and Gale, D (2000)**, ‘Financial contagion’, *Journal of Political Economy*, Vol. 108, pages 1–33.
- Allen, F and Gale, D (2005)**, ‘From cash-in-the-market pricing to financial fragility’, *Journal of the European Economic Association*, Vol. 3, pages 535–46.
- Altman, E L and Kishore, V M (1996)**, ‘Almost everything you wanted to know about recoveries on defaulted bonds’, *Financial Analysts Journal*, Vol. 52, pages 57–64.
- Bank for International Settlements (2008)**, ‘Consolidated banking statistics’, Bank for International Settlements.
- Bank of England (2007)**, *Financial Stability Report*, April.
- Basel Committee on Banking Supervision (2011)**, ‘Global systemically important banks: assessment methodology and the additional loss absorbency requirements’, Bank for International Settlements.
- Bianconi, G (2009)**, ‘The entropy of network ensembles’, *Physical Review E*, Vol. 79, page 036,114.
- Castiglionesi, F and Navarro, N (2007)**, ‘Optimal fragile financial networks’, *Tilburg University Discussion Paper No.2007-100*.
- Cifuentes, R, Ferrucci, G and Shin, H S (2005)**, ‘Liquidity risk and contagion’, *Journal of the European Economic Association*, pages 556–66.
- Eisenberg, L and Noe, T (2001)**, ‘Systemic risk in financial systems’, *Management Science*, Vol. 47, pages 236–49.
- Elsinger, H, Lehar, A and Summer, M (2006)**, ‘Using market information for banking systems’, *International Journal of Central Banking*, Vol. 27, pages 137–65.
- Errais, E, Giesecke, K and Goldberg, L R (2010)**, ‘Affine point processes and portfolio credit risk’, *SIAM Journal on Financial Mathematics*, Vol. 1, pages 642–65.



- Foglia, A (2009)**, ‘Stress testing credit risk: a survey of authorities’ approaches’, *International Journal of Central Banking*, Vol. 5, pages 5–45.
- Gai, P, Haldane, A G and Kapadia, S (2011)**, ‘Complexity, crises and contagion’, *Journal of Monetary Economics*, Vol. 58, pages 415–536.
- Gai, P and Kapadia, S (2010)**, ‘Contagion in financial networks’, *Proceedings of the Royal Society A*, Vol. 466, pages 2,401–23.
- Gauthier, C, Lehar, A and Souissi, M (2010)**, ‘Macroprudential capital requirements and systemic risk’, *Bank of Canada Working Paper No.2010-4*.
- Giesecke, K and Weber, S (2004)**, ‘Cyclical correlations, credit contagion, and portfolio losses’, *Journal of Banking and Finance*, Vol. 28, pages 3,009–36.
- Giesecke, K and Weber, S (2006)**, ‘Credit contagion and aggregate losses’, *Journal of Economic Dynamics and Control*, Vol. 30, pages 741–67.
- Gordy, M (2000)**, ‘A comparative anatomy of credit risk models’, *Journal of Banking and Finance*, Vol. 24, pages 119–49.
- Hatchett, J and Kuhn, R (2009)**, ‘Credit contagion and credit risk’, *Quantitative Finance*, Vol. 9, pages 373–82.
- Horst, U (2007)**, ‘Stochastic cascades, credit contagion and large portfolio losses’, *Journal of Economic Behavior and Organization*, Vol. 63, pages 25–54.
- Jackson, M (2008)**, *Social and economic networks*, Princeton University Press.
- James, C (1991)**, ‘The losses realized in bank failures’, *Journal of Finance*, Vol. 46, pages 1,223–42.
- Leitner, Y (2005)**, ‘Fragile financial networks’, *Journal of Finance*, Vol. 60, pages 2,925–53.
- Longstaff, F A and Rajan, A (2008)**, ‘An empirical analysis of the pricing of collateralized debt obligations’, *Journal of Finance*, Vol. 63, pages 529–63.
- May, R M and Arinaminpathy, N (2010)**, ‘Systemic risk: the dynamics of model banking systems’, *Journal of Royal Society: Interface*, Vol. 7, pages 823–38.
- May, R M and Haldane, A G (2011)**, ‘Systemic risk in banking ecosystems’, *Nature*, Vol. 469, pages 351–55.
- May, R M, Levin, S and Sugihara, G (2008)**, ‘Ecology for bankers’, *Nature*, Vol. 451, pages 893–95.
- Mitchell, M, Pedersen, L H and Pulvino, T (2007)**, ‘Slow moving capital’, *American Economic Review*, Vol. 97, pages 215–20.





**Mohammad-Djafari, A (1991)**, ‘A matlab program to calculate the maximum entropy distributions’, in Grandy, T W (ed), *Maximum entropy and Bayesian methods: MaxEnt proceedings*, Kluwer Academic Publishers, pages 221–33.

**Moody’s (2007)**, ‘Corporate default and recovery rates, 1920-2006’, *Moody’s investors service global credit research*, February.

**Morris, S and Shin, H S (2009)**, ‘Illiquidity component of credit risk’, *mimeo*.

**Schuermann, T and Hanson, S (2004)**, ‘Estimating probabilities of default’, *Federal Reserve Bank of New York Staff Reports No.190*.

**Trichet, J-C (2008)**, ‘The recent turbulence in global financial markets’, keynote address to the New York University Global Economic Policy Forum.

**Upper, C (2011)**, ‘Simulation methods to assess the danger of contagion in interbank markets’, *Journal of Financial Stability*, Vol. 7, pages 111–25.

**von Peter, G (2007)**, ‘International banking centers: a network perspective’, *Bank for International Settlements Quarterly Review*, December, pages 33–45.

**Watts, D and Strogatz, S (1998)**, ‘Collective dynamics of "small world" networks’, *Nature*, Vol. 393, pages 440–42.