



BANK OF ENGLAND

Working Paper No. 450

Forecasting UK GDP growth, inflation and interest rates under structural change: a comparison of models with time-varying parameters

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Konstantinos Theodoridis

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Alina Barnett,⁽¹⁾ Haroon Mumtaz⁽²⁾ and Konstantinos Theodoridis⁽³⁾

Abstract

Evidence from a large and growing empirical literature strongly suggests that there have been changes in inflation and output dynamics in the United Kingdom. This is largely based on a class of econometric models that allow for time-variation in coefficients and volatilities of shocks. While these have been used extensively to study evolving dynamics and for structural analysis, there is little evidence on their usefulness in forecasting UK output growth, inflation and the short-term interest rate. This paper attempts to fill this gap by comparing the performance of a wide variety of time-varying parameter models in forecasting output growth, inflation and a short rate. We find that allowing for time-varying parameters can lead to large and statistically significant gains in forecast accuracy.

Key words: Time-varying parameters, stochastic volatility, VAR, FAVAR, forecasting, Bayesian estimation.

JEL classification: C32, E37, E47.

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Summary

In recent years, a number of papers have applied econometric models that allow for changes in model parameters. In general, this literature has examined and investigated how the properties of key macroeconomic variables have changed over the last three decades. So the underlying econometric models in these studies have therefore been used in a descriptive role.

The aim of this paper, instead, is to consider if these sophisticated models can offer gains in a forecasting context - specifically, GDP growth, CPI inflation and the short-term interest rate relative to simpler econometric models that assume fixed parameters. We consider 24 forecasting models that differ along two dimensions. First, they model the time-variation in parameters in different ways and allow for either gradual or abrupt shifts. Second, some of the models incorporate more economic information than others and include a larger number of explanatory variables in an efficient manner while still allowing for time-varying parameters.

We estimate these models at every quarter from 1976 Q1 to 2007 Q4. At each point in time we use the estimates of each model to forecast GDP, CPI inflation and the short-term interest rate. We then construct the average squared deviation of these forecasts from the observed value relative to forecasts from a simple benchmark model.

A comparison of this statistic across the 24 forecasting models indicates that allowing for time-varying parameters can lead to gains in forecasting. In particular, models that incorporate a gradual change in parameters and also include a large set of explanatory variables do particularly well as far as the inflation forecast is concerned recording gains (over the benchmark) which are significant from a statistical point of view. Models that include this extra information also appear to be useful in forecasting interest rates. Models that incorporate more abrupt changes in parameters can do well when forecasting GDP growth. This feature also appears to surface during the financial crisis of 2008-09 when this type of parameter variation proves helpful in predicting the large contraction in GDP growth.



1 Introduction

A large and growing literature has proposed and applied a number of empirical models that incorporate the possibility of structural shifts in the model parameters. The series of papers by Tom Sargent and co-authors on the evolving dynamics of US inflation is a often cited example of this literature. In particular, Cogley and Sargent (2002), Cogley and Sargent (2005) and Cogley, Primiceri and Sargent (2008) use time-varying parameter VARs (TVP-VAR) to explore the possibility of shifts in inflation dynamics, with Benati (2007) applying this methodology to model the temporal shifts in UK macroeconomic dynamics. In contrast, Sims and Zha (2006), model changing US macroeconomic dynamics using a regime-switching VAR (see Groen and Mumtaz (2008) for an application to the United Kingdom). Balke (2000) highlights potential non-linearities in output and inflation dynamics and use threshold VAR (TVAR) models to explore non-linear dynamics in output and inflation. Recent papers have estimated time-varying factor augmented VAR (TVP-FAVAR) models in order to incorporate more information into the empirical model. For example, Baumeister, Liu and Mumtaz (2010) argue that incorporating a large information set can be important when modelling changes in the monetary transmission mechanism and use a TVP-FAVAR to estimate the evolving response to US monetary policy shocks.

Most of this literature has focused on describing the evolution in macroeconomic dynamics. In contrast, research on the forecasting ability of these models has been more limited in number and scope. D'Agostino, Gambetti and Giannone (2011) focus on TVP-VARs and show that they provide more accurate forecasts of US inflation and unemployment when compared to fixed-coefficient VARs. In a recent contribution, Eickmeier, Lemke and Marcellino (2011) present a comparison of the forecasting performance of the TVP-FAVAR with its fixed-coefficient counterpart and AR models with time-varying parameters for US data over the 1995-2007 period. The authors show that there are some gains (in terms of forecasting performance) from allowing time-variation in model parameters and exploiting a large information set.

The aim of this paper is to extend the forecast comparison exercise in D'Agostino *et al*



(2011) and Eickmeier *et al* (2011) along two dimensions. First, our paper compares the forecast performance of a much wider range of models with time-varying parameters. In particular, we compare the forecasting performance of (a range of) regime-switching models, TVP-VARs, TVP-FAVARs, TVARs, smooth transition VARs (ST-VARs), the unobserved component model with stochastic volatility proposed by Stock and Watson (2007), rolling VARs and recursive VARs. The forecast comparison is carried out recursively over the period 1976 Q1 to 2007 Q4 and thus covers a longer period than Eickmeier *et al* (2011). Second, while previous papers have largely focused on the United States, we work with UK data and try to establish of these time-varying parameter models are useful for forecasting UK inflation, GDP growth and the short-term interest rate. This is a policy relevant question as the United Kingdom has experienced large changes in the dynamics of key macro variables over the last three decades. In addition, the recent financial crisis has been associated with large movements in inflation and output growth again highlighting the possibility of structural change. Note also that our analysis has a different focus than the analysis in Eklund, Kapetanios and Price (2010) and Clark and McCracken (2009). While these papers largely focus on forecasting performance under structural change in a Monte Carlo setting our exercise is a direct application to UK data using time-varying parameter models that are currently popular in empirical work.¹

The forecast comparison exercise brings out the following main results:

- On average, the TVP-VAR model delivers the most accurate forecasts for GDP growth at the one-year forecast horizon, with a root mean squared error (RMSE) 6% lower than an AR(1) model. The TVAR model also performs well, especially over the post-1992 period.
- Models with time-varying parameters lead to a substantial improvement in inflation forecasts. At the one-year horizon, the TVP-FAVAR model has an average RMSE 23% lower than an AR(1) model. A similar forecasting performance is delivered by

¹Faust and Wright (2011) compare the performance of a large number of models in forecasting US inflation. Their focus, however, is not exclusively on models with time-varying parameters.

the TVP-VAR model and Stock and Watson's unobserved component model, where the latter delivers the most accurate forecasts over the post-1992 period.

- Over the recent financial crisis, models that allow for regime-switching and non-linear dynamics appear to be more successful in matching the profile of inflation and GDP growth than specifications that allow for parameter drift.

The paper is organised as follows. Section 2 provides details on the data used in this study and describes the real time out of sample forecasting exercise. Section 3 describes the main forecasting models used in this study. Section 4 describes the main results in detail.

2 Data and forecasting methodology

2.1 Data

Our main data set consists of quarterly annualised real GDP growth, quarterly annualised inflation and the three-month Treasury bill rate. Quarterly data on these variables is available from 1955 Q1 to 2010 Q4.

The GDP growth series is constructed using real-time data on GDP obtained from the Office for National Statistics. Vintages of GDP data covering our sample period are available 1976 Q1 onwards and these are used in our forecasting exercise as described below. GDP growth is defined as 400 times the log difference of GDP.

The inflation series is based on the seasonally adjusted harmonised index of consumer prices spliced with the retail prices index excluding mortgage payments. This data is obtained from the Bank of England database. Inflation is calculated as 400 times the log difference of this price index. The three-month Treasury bill rate is obtained from Global Financial Data.



Root mean squared error

In particular, we use root mean squared error (RMSE) calculated as

$$RMSE = \sqrt{\sum_{t=T+1}^{T+h} \frac{(\hat{Z}_t - Z_t)^2}{h}} \quad (1)$$

where $T + 1, T + 2, \dots, T + h$ denotes the forecast horizon, \hat{Z}_t denotes the forecast, while Z_t denotes actual data. For GDP growth, the forecast error $\hat{Z}_t - Z_t$ is calculated using the latest available vintage. We estimate the RMSE for $h = 1, 4, 8$ and 12 quarters.

In order to compare the performance of the different forecasting models we use the RMSE of each model relative to a benchmark model: an AR(1) regression estimated via OLS recursively over each subsample.

Diebold-Mariano statistic

To test formally whether the predictive accuracy delivered by the non-linear models considered in this study is superior to that obtained using the AR(1) regression estimated via OLS recursively over each subsample, we use the statistic developed by Diebold and Mariano (1995).² The accuracy of each forecast is measured by using the squared error loss function $-L(\hat{Z}_t^i, Z_t) = (\hat{Z}_t^i - Z_t)^2$ where $t = T + 1, \dots, T + R$ and R is the length of the forecast evaluation sample. Under the null hypothesis the expected forecast loss of using one model instead of the other is the same

$$H_0 : E [L(\hat{Z}_t^i, Z_t)] = E [L(\hat{Z}_t^{AR}, Z_t)] \quad (2)$$

This can be tested as a t -statistic, namely

$$\left| \frac{\sqrt{R} \frac{1}{R} \sum_{t=T+1}^{T+R} d_t}{\hat{\sigma}_d} \right| > 1.96 \quad (3)$$

where $d_t = (\hat{Z}_t^i - Z_t)^2 - (\hat{Z}_t^{iAR} - Z_t)^2$ and $\hat{\sigma}_d^2$ is the heteroskedasticity and autocorrelation consistent variance estimator developed by Newey and West (1987).

²Note the Diebold-Mariano (DM) statistic is calculated for the entire sample, not for each point in time as the RMSE is derived. Furthermore, the DM statistic will coincide with the RMSE only if the forecasting horizon equals one. Finally, there could be cases when the DM test is unable to distinguish between models even when there are quite large reductions in RMSE.

The information that the h -step ahead forecast error follows a moving average process of order $h - 1$ is used to decide about the bandwidth of the kernel.³

Trace statistic

In addition we calculate the trace of the forecast error covariance matrix – Ω – to assess the multivariate performance of the competing models. Consider the singular valued decomposition of $\Omega = V\Lambda V'$ where V is the matrix of eigenvectors and Λ is the diagonal matrix with eigenvalues in descending order. The eigenvalues are the variances of the principal components and the trace of Ω equals the sum of their eigenvalues. Based on these observations Adolfson, Linde and Villani (2007) argue that the trace will, to a large extent, be determined by the forecasting performance of the least predictable dimensions (largest eigenvalues). It should be mentioned that this statistic has its limitations. For instance, Clements and Hendry (1995) point out that the model ranking based on this statistic is affected by linear transformations of the forecasting variables. However, this is not the case in our exercise since all variables are expressed in percentage terms.

3 Forecasting models

In this section we provide a description of the forecasting models used in this study. Note that the forecasts that the Monetary Policy Committee publishes in the *Inflation Report* are their best collective judgement of future developments given particular interest rate paths and are not based on any particular formal model. Naturally, they are informed by the insights from many different models, including models that recognise the existence of structural change such as those examined here.

Following D'Agostino *et al* (2011) (and the convention in a large number of papers using VARs with time-varying parameters – see for example Cogley and Sargent

³Given that these models are non-linear and their parameters are functions of time (not just a sequence that converges to a fixed point) and that we have calculated the DM statistic for the entire sample (not just for each data release, $T \rightarrow \infty$), we can perhaps make the assumption that these models are non-nested and standard asymptotic theory can be applied. The same is true for the AR(1).

(2002), Primiceri (2005) and Cogley and Sargent (2005)) we use a lag length of two in all models considered below.

3.1 Regime-switching VAR

To account for the possibility of structural shifts, we model inflation, output and interest rate dynamics using a regime-switching VAR of the following form

$$Z_t = c_{S_t} + \sum_{j=1}^K B_{S_t} Z_{t-j} + \Omega_{H_t}^{1/2} \epsilon_t \quad (4)$$

where Z_t is a $T \times 3$ data matrix that contains GDP growth, inflation and the interest rate. B_S and Ω_h are regime-dependent autoregressive coefficients and reduced-form variance-covariance matrices. The VAR model allows for M breaks at unknown dates, as in Chib (1998), these are modelled via the latent state variable S_t for the VAR coefficients and H_t for the error covariance matrix. In our most general regime-switching model, the state variables S and H are assumed to evolve independently with their transition governed by a first-order Markov chains with $M + 1$ regimes with restricted transition probabilities $p_{ij} = p(S_t = j | S_{t-1} = i)$ and $q_{ij} = p(H_t = j | H_{t-1} = i)$. The transition probability matrices are defined as

$$p_{ij}, q_{ij} > 0 \text{ if } i = j \quad (5)$$

$$p_{ij}, q_{ij} > 0 \text{ if } j = i + 1$$

$$p_{MM}, q_{MM} = 1$$

$$p_{ij}, q_{ij} = 0 \text{ otherwise.}$$

For example, if $M = 4$ the transition matrices are defined as

$$\tilde{P} = \begin{pmatrix} p_{11} & 0 & 0 & 0 \\ 1 - p_{11} & p_{22} & 0 & 0 \\ 0 & 1 - p_{22} & p_{33} & 0 \\ 0 & 0 & 1 - p_{33} & 1 \end{pmatrix}, \tilde{Q} = \begin{pmatrix} q_{11} & 0 & 0 & 0 \\ 1 - q_{11} & q_{22} & 0 & 0 \\ 0 & 1 - q_{22} & q_{33} & 0 \\ 0 & 0 & 1 - q_{33} & 1 \end{pmatrix}$$

Equations (4) and (5) define a Markov-switching VAR with non-recurrent states where transitions are allowed in a sequential manner. For example, to move from regime 1 to regime 3, the process has to visit regime 2. Similarly, transitions to past regimes are not allowed. However, this structure is not necessarily more restrictive than a standard Markov-switching model, but simply implies that any new regimes are given a new

label, rather than being explicitly linked to past states (as in a standard Markov-switching model). This formulation implies that the regimes are identified by assumption and no ‘label switching’ problem exists when implementing the Gibbs sampler. This feature offers a clear computational advantage (relative to regime-switching VAR with unrestricted transition probabilities) by removing the need for regime normalisation which can be computationally challenging as the number of regimes become larger.

We estimate three versions of this regime-switching model: (1) The general switching model as set out in equation (4) which allows for independent breaks in the VAR coefficients and error covariance. (2) A version of the regime-switching VAR where the breaks in VAR coefficients and the covariance matrix are restricted to occur jointly and (3) A version of the regime-switching VAR where only breaks in the VAR coefficients are allowed. Specification (2) is estimated to gauge if allowing for different timing in variance and coefficient breaks offers any advantage in terms of forecasting performance. Specification (3) which does not include volatility breaks is included in order to shed light on the role played by heteroscedasticity. In each case, we allow for up to three breaks or four regimes.

Versions of this regime-switching model have been used in a number of recent studies to describe the changing dynamics of key macroeconomic time series. For example Sims and Zha (2006) argue that a model that incorporates regime-switching dynamics provides a good description of the evolution of monetary policy and inflation dynamics in the United States. Groen and Mumtaz (2008) provide a similar analysis for the United Kingdom and show that a regime-switching VAR is useful for describing the change in inflation persistence. It may also be argued that allowing for discrete shifts in coefficients and error variances is especially appropriate given the current crisis and its associated impact on macroeconomic variables.

The models are estimated using a Gibbs sampling algorithm. The prior distributions and conditional posteriors are described in the appendix. Note that we employ a normal inverse Wishart prior on the VAR parameters in each regime. However, as described in the appendix the tightness parameters are set to large values rendering the



prior distributions non-informative.

3.2 Time-varying VAR

In a recent paper, D'Agostino *et al* (2011) show that a VAR with time-varying parameters and stochastic volatility performs well in forecasting US macroeconomic data. In addition, a voluminous literature has used the time-varying VAR model to investigate the possibility of a temporal shift in UK and US inflation dynamics. Prominent examples of papers that employ this model for the United States include Cogley and Sargent (2002), Cogley and Sargent (2005) and Cogley *et al* (2008). Benati (2007) and Mumtaz and Sunder-Plassmann (2010) use the time-varying VAR model to capture the time-varying dynamics of UK macroeconomic and financial time series. Relative to the regime-switching model, the time-varying VAR incorporates a more flexible specification for time-varying parameters. In particular, it allows independent time-variation in each VAR equation.

We use a general version of this model as a forecasting model for UK GDP growth, inflation and interest rates. In particular we employ the following specification:

$$Z_t = c_t + \sum_{j=1}^K B_j Z_{t-j} + \Omega_t^{1/2} \varepsilon_t \quad (6)$$

where the VAR coefficients $\Phi_t = \{c_t, B_t\}$ evolve as random walks

$$\Phi_t = \Phi_{t-1} + \eta_t$$

As in Cogley and Sargent (2005), the covariance matrix of the innovations v_t is factored as

$$\text{VAR}(v_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})'. \quad (7)$$

The time-varying matrices H_t and A_t are defined as:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 \\ 0 & h_{2,t} & 0 \\ 0 & 0 & h_{3,t} \end{bmatrix} \quad A_t \equiv \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix} \quad (8)$$

with the $h_{i,t}$ evolving as geometric random walks,

$$\ln h_{i,t} = \ln h_{i,t-1} + \tilde{v}_t.$$

Following Primiceri (2005), we postulate the non-zero and non-one elements of the matrix A_t to evolve as driftless random walks,

$$\alpha_t = \alpha_{t-1} + \tau_t, \quad (9)$$

In our first specification we consider a version of this TVP-VAR estimated recently by Cogley *et al* (2008) and Baumeister and Benati (2010). These authors generalise the specification adopted in Cogley and Sargent (2005) by allowing for stochastic volatility in η_t . In particular, we model $\text{var}(\eta_t) = Q_t$ as

$$Q_t = \tilde{A}^{-1} \tilde{H}_t (\tilde{A}^{-1})' \quad (10)$$

where \tilde{A} is a lower triangular matrix, while $\tilde{H}_t = \text{diag}(\tilde{h}_{1t}, \dots, \tilde{h}_{Mt})$ with $M = N \times (N \times L + 1)$ and \tilde{h}_{jt} evolving as

$$\ln \tilde{h}_{jt} = \ln \tilde{h}_{jt-1} + \tilde{u}_t \quad (11)$$

As discussed in Baumeister and Benati (2010), one advantage of this extended specification is that it allows for the possibility that the degree of time-variation varies over the sample. For example, it accounts for the possibility that the VAR coefficients change faster during crisis periods while the degree of parameter drift is smaller over tranquil periods. We consider two further restricted specifications. First, following D'Agostino *et al* (2011), we estimate a time-varying VAR with a constant degree of parameter drift – ie $Q_t = Q$. Second, to explore the role played by heteroscedasticity, we estimate a TVP-VAR with $\Omega_t = \Omega$ as in Cogley and Sargent (2002).

The models are estimated using a Gibbs sampling algorithm. The priors distributions and conditional posteriors are described in the appendix. We point out two aspects: first, the prior for Q (and for \tilde{H}_0 in the specification which allows for a time-varying Q) is set using a pre-sample of $T_0 = 40$ quarters. In particular, let Q_{OLS} denote the OLS estimate of the coefficient covariance matrix using the training sample. When Q is

time-invariant, its prior distribution is assumed to be inverse Wishart with a scale matrix given by $\bar{Q} = Q_{OLS} \times T_0 \times k$ where the scalar $k = 3.5e - 04$ as in Cogley and Sargent (2005). The prior degrees of freedom are set equal to $T_0 = 40$ the length of the training sample. When Q is time-varying, a prior distribution is required for the initial values of \tilde{H}_t . The mean of this log normal prior is set as the log of $diag(A_{OLS}Q_{OLS}A'_{OLS}) \times T_0 \times k$ where A_{OLS} is the inverse of the Choleski decomposition of Q_{OLS} . The variance of the prior distribution is set to 1. On a log scale this represents an agnostic prior about the initial value of \tilde{H}_t .

3.3 Time-varying factor augmented VAR

Recent empirical work on the evolving monetary transmission mechanism has employed time-varying factor augmented VAR (TVP-FAVAR) models as a way of incorporating additional information into the empirical specification (see for example Baumeister *et al* (2010) and Eickmeier *et al* (2011) and the references therein). As shown in Eickmeier *et al* (2011), these models therefore offer a convenient way to combine a large information set and time-varying dynamics. As shown by Eickmeier *et al* (2011) for the United States, the time-varying FAVAR delivers superior forecasting performance than its fixed-coefficient counterpart and time-varying models that do not include information from a large data set. Following Eickmeier *et al* (2011) we estimate the following TVP-FAVAR model

$$\begin{aligned} X_t &= \beta F_t + e_{it} & (12) \\ F_t &= c_t + \sum_{j=1}^K B_j F_{t-j} + \Omega_t^{1/2} \epsilon_t \end{aligned}$$

where $X_t = [x_{it}, z_t]$. x_{it} is a $T \times M$ matrix of macroeconomic and financial variables and z_t is the variable we are interested in forecasting. That is z_t is either GDP growth, inflation or the three-month Treasury bill rate. The matrix F_t contains the K latent factors that summarise the information in the panel x_{it} and z_t . That is $z_t = [f_{1t}, \dots, f_{Kt}, z_t]$. β denotes the factor-loading matrix, while e_{it} represents the idiosyncratic component. We allow for first-order serial correlation in e_{it} with $e_{it} = \rho_i e_{it-1} + v_{it}$. More details on the observation equation can be found in Baumeister *et al* (2010). The dynamics of F_t are described by a time-varying VAR model with stochastic volatility. The coefficients of this transition equation evolve as random walks (the shock on the random walk has a

fixed variance). The variance of the shocks is specified as in equation (7). In our application we fix the number of latent factors to three for computational reasons. In particular, with a larger number of endogenous variables it becomes increasingly difficult to keep the transition equation of the model stable at each point in time. Given that the model needs to be estimated recursively (over 100 recursive data samples) computational efficiency is vital in our exercise.

The model is estimated using a Gibbs sampling algorithm. This algorithm is an extended version of the sampler used for the time-varying VAR and is described in Appendix D. Note that the priors for the hyperparameters of the transition equation are set as described in the previous section. Appendix H describes the data set x_{it} used for the forecasting exercise. In short, x_{it} contains 43 variables that represent data on real activity, inflation, money supply, interest rates and exchange rates. This data set is chosen as it is consistently available over the sample period used in our forecasting exercise.

We also consider two restricted versions of the model in (12). First, we fix $\Omega_t = \Omega$ and only allow time-variation in the coefficients of the transition equation. Second, we estimate a fixed-coefficient FAVAR. These restricted models are estimated to gauge the role played by time-varying parameters (in addition to the impact of the larger information set) in driving any change in forecasting performance.

3.4 Unobserved component model with stochastic volatility

In a recent contribution Stock and Watson (2007) show that a univariate unobserved component (UC) model with stochastic volatility performs well in forecasting US inflation. Following Stock and Watson (2007) we consider this model as a possible alternative specification to forecast UK data. The UC model with stochastic volatility

is given by:

$$\tilde{Z}_t = \beta_t + \sqrt{\sigma_t} \epsilon_t$$

$$\beta_t = \beta_{t-1} + \sqrt{\omega_t} v_t$$

$$\ln \sigma_t = \ln \sigma_{t-1} + e_{1t}, \text{var}(e_{1t}) = g_1$$

$$\ln \omega_t = \ln \omega_{t-1} + e_{2t}, \text{var}(e_{2t}) = g_2$$

where \tilde{Z}_t contains data on GDP growth, inflation or the three-month Treasury bill yield. The model is estimated using a MCMC algorithm which is described in Appendix E.

3.5 Threshold and smooth transition VAR models

Threshold and smooth transition VAR models allow for different VAR parameters in different regimes. The switching mechanism in this case is intuitive and simple, making these models very attractive and, consequently, popular. In addition (unlike regime-switching and time-varying parameter models), the time-variation in the parameters is linked explicitly to a threshold variable. In other words, parameters are allowed to be different in expansions and recessions, periods of high or low inflation and periods of high and low interest rates. While regime-switching and time-varying models can account for this possibility, the parameter change in these models is governed by a more general process.

These models can be expressed as follows:

$$Z_t = c_{\tilde{S}_t} + \sum_{j=1}^K B_{j,\tilde{S}_t} + \Omega_{\tilde{S}_t}^{1/2} \epsilon_t \quad (13)$$

However, the state variable \tilde{S}_t is constructed differently now. In the threshold case \tilde{S}_t is a discrete variable that takes values 1 or 0 according to the following rule

$$\tilde{S}_t = \begin{cases} 1 & \text{if } Z_{i,t-d} \leq c_1 \\ 1 & \text{if } c_1 < Z_{i,t-d} \leq c_2 \\ \vdots & \\ 1 & \text{if } c_{M-2} < Z_{i,t-d} \leq c_{M-1} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

where $Z_{i,t}$ is the variable i of the Z vector, M is the number of regimes and $c = (c_1, \dots, c_{M-1})'$ is the vector of threshold values. In our exercise M has been set

equal to two and d equal to one, implying that

$$\tilde{S}_t = \begin{cases} 1 & \text{if } Z_{i,t-1} \leq c \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

In the smooth transition case \tilde{S}_t is a continuous variable given by

$$\tilde{S}_t = \begin{cases} \frac{1}{1+\exp(-\gamma(Z_{i,t-1}-c))} \\ 1 - \frac{1}{1+\exp(-\gamma(Z_{i,t-1}-c))} \end{cases} \quad (16)$$

If we assume conjugate priors for the VAR parameters then conditional on γ and c the posterior distribution of the VAR coefficient vector is the conditional normal Wishart distribution. Unfortunately, the posterior distribution of γ and c conditional the VAR parameter vector is unknown, meaning that we have to employ both the Gibbs and Metropolis-Hasting samplers to derive the full posterior distribution of the entire estimated parameter vector (Chen and Lee (1995), Chen (1998) and Lopes and Salazar (2006)).

3.6 Rolling and recursive VAR model

Our final two forecasting models are based on the following VAR

$$Z_t = c + \sum_{j=1}^K BZ_{t-j} + \Omega^{1/2}\epsilon_t \quad (17)$$

where Z_t is a $T \times 3$ data matrix that contains GDP growth, inflation and the interest rate. The recursive VAR is estimated recursively starting in 1976 Q1 until the end of the sample period. The rolling VAR model uses a ten-year rolling window to estimate the model parameters. From an applied point of view, the main virtue of these models is the fact that they are simple to estimate. A finding that these models forecast well relative to the more sophisticated alternatives would therefore have practical importance.

3.7 Bayesian model averaging

We also consider if the average forecast from our 24 forecasting models can improve upon the individual forecasts presented above. In particular, we combine the forecasts

using Bayesian model averaging (BMA):

$$\hat{Z}_{t,BMA} = \sum_{m=1}^{24} \hat{Z}_{t,m} P(Z_t \setminus m) \quad (18)$$

where $\hat{Z}_{t,BMA}$ denotes the BMA forecast at time t , $\hat{Z}_{t,m}$ denotes the forecast from model m and $P(Z_t \setminus m)$ is the marginal likelihood.

Calculation of the marginal likelihood in equation (18) is the key task when estimating $\hat{Z}_{t,BMA}$. Following Chib (1995) we consider the following representation for the log marginal likelihood:

$$\ln P(Z_t \setminus m) = \ln F(Z_t \setminus \hat{\Xi}, m) + \ln p(\hat{\Xi}) - \ln G(\hat{\Xi} \setminus Z_t) \quad (19)$$

where $\ln F(Z_t \setminus \Xi, m)$ is the log likelihood, $\ln p(\hat{\Xi})$ is the log prior density and $\ln G(\hat{\Xi} \setminus Z_t)$ is the log posterior density with all three terms evaluated at the posterior mean for the model parameters $\hat{\Xi}$. The prior density $\ln p(\hat{\Xi})$ is easy to evaluate. Similarly, the log likelihood of the models we consider can be evaluated either directly or via non-linear filters. The final term $\ln G(\hat{\Xi} \setminus Z_t)$ requires more work. Following Chib (1995) and Chib and Jeliazkov (2001) we proceed by factorising $\ln G(\hat{\Xi} \setminus Z_t)$ into conditional and marginal densities of various parameter blocks and using additional and Gibbs and Metropolis runs to approximate these densities. Details are provided in the appendix.

In Table B we present the estimated log marginal likelihoods (calculated over the full estimation sample) for the forecasting models. The TVP-FAVAR model with homoscedastic shocks has the largest marginal likelihood followed by the TVP-FAVAR model that allows for time-varying coefficients and heteroscedastic shocks. Within the TVP-VAR models, the homoscedastic version performs the best in terms of the marginal likelihood with little support for the generalised TVP-VAR that allows for a time-varying Q matrix. It is also interesting to note that the threshold and STAR models fit the data better than the regime-switching models implying that the type of non-linear dynamics built into these models are important for UK data. Finally, the rolling and recursive VARs fit the data well with a marginal likelihood substantially higher than the time-varying VARs. This suggests that for our data set this simple form

of parameter variation inherent in these models is preferred to stochastic parameter drift.

4 Results

The left-hand side of Tables C to E present the estimated RMSE for each model at the one, four, eight and twelve quarter horizons. The colours indicate how better (green) or worse (blue) these different forecasting models do compared to an AR(1) model. The right-hand side offers an alternative evaluation of the forecasting performance in terms of the Diebold and Mariano (1995) test. As for the RMSE, green indicates that these models outperform the AR(1), blue the opposite and white that these models' forecasts are statistically indistinguishable from each other. We evaluate the forecast performance over the full sample, 1976 Q1 to 2007 Q4 and over the period 1993 Q1 to 2007 Q4. Results for this latter subsample are presented separately as recent studies have highlighted a decline in predictability over the great moderation period (see for example Benati and Surico (2008)). It is, therefore, interesting to check if the forecasting models described above can outperform the AR(1) model over this subsample. The final table presents a multivariate measure of forecast accuracy, namely the trace of the forecast error covariance matrix.

4.1 Overall forecast performance

4.1.1 GDP growth

Consider the RMSE for GDP growth. Over the full sample, 1976-2007, the TVP-VAR model outperforms the other forecasting models at all forecasting horizons. The largest reduction in RMSE relative to the AR(1) model occurs at the one and four-quarter forecast horizon with the the TVP-VAR model's performance close to the AR model at longer horizons. However the DM statistic is not able to distinguish between the forecasting performance of an AR(1) model and that of a TVP-VAR (standard) (and hence white cells on the right-hand side of Table C). Note also that allowing for heteroscedastic shocks in the TVP model improves forecasting performance – the homoscedastic TVP-VAR has a larger RMSE at all forecast horizons. The TVAR and

ST-VAR model with inflation as the threshold variable also performs well with a relative RMSE close to the TVP-VAR at the one and four-quarter horizons. In addition, the UC model performs well at the one-quarter horizon. Note that this is the only model which shows a statistically significant improvement over the AR(1) benchmark according to the Diebold and Mariano (1995) test.

Over the great moderation period, the TVP-VAR model is the best-performing model at the four-quarter forecast horizon according to a RMSE criterion. However, the TVAR (with inflation as the threshold variable) outperforms the TVP-VAR at one-quarter horizon, while the FAVAR model delivers the most accurate GDP forecasts at the eight and twelve-quarter horizons, albeit with a forecast accuracy close to the AR(1) model.

Chart 1 explores the evolution of the RMSE of these models over the forecasting period. The chart plots the smoothed relative RMSE for the TVP-VAR, TVAR and ST-VAR models at the four-quarter horizon over the forecasting sample. In the pre-2000 period, the performance of three models is quite similar. The performance of all three models deteriorates after 2000. Note, however, that this deterioration is largest for the TVP-VAR and smallest for the TVAR model.

4.1.2 Inflation

At the one-quarter forecast horizon, the TVP-VAR model, the UC model and the TVP-FAVAR model deliver, on average, the most accurate forecasts for inflation. These models have a RMSE 12% to 15% lower than the AR(1) model. This improvement over the AR(1) benchmark is starker at the four-quarter horizon. At the four-quarter horizon, the TVP-FAVAR model has the lowest relative RMSE on average over the full forecast sample – an improvement of 23% over the AR model. The TVP-FAVAR with homoscedastic shocks delivers a very similar result to its heteroscedastic counterpart. Note, however, that the inflation forecasts from the fixed-coefficient FAVAR model are substantially less accurate. These results suggest that the extra information included in the factor model *and* the presence of time-varying coefficients leads to an improvement in the accuracy of inflation forecasts. At the four-quarter horizon, the BMA forecast also does well with a relative RMSE close to the TVP-FAVAR models.

It is also worth noting that the Diebold and Mariano (1995) test indicates the TVP-FAVAR models are the only specifications (at the four-quarter horizon and over the full sample) with a significant improvement in forecast performance relative to the benchmark model. The TVP-FAVAR models are the best performers at the eight and the twelve-quarter horizon.

Over the great moderation period, Stock and Watson's unobserved component model leads to the most accurate inflation forecasts at the one, four and eight-quarter horizon. For example, at the four-quarter horizon, this model leads to an average RMSE which is 54% lower than the AR(1) benchmark. Over this subsample a number of other forecasting models also stand out. For example, at the one-quarter horizon, the three regime-switching VAR (with time-invariant covariance) and the rolling VAR model have the lowest RMSE after the UC model. At the four-quarter horizon, the TVP-FAVAR, the TVP-VAR and the BMA procedure also deliver forecasts almost as accurate as the UC model. The Diebold and Mariano (1995) test provides strong evidence that over this subsample these forecasting models provide significantly more accurate forecasts than the benchmark.

In Chart 2 we examine the evolution of the relative RMSE (at the four-quarter horizon) of the best-performing inflation forecasting models. It is interesting to note that there is a distinct change in relative RMSEs after the early 1990s, with the inflation-targeting period characterised by a substantially improved performance by the four time-varying parameter models relative to the benchmark. Note that several studies have documented a change in UK inflation dynamics at this juncture (see for example Benati (2007)). Our results suggest that models with evolving parameters were able to adapt to this change better than recursively estimated fixed-coefficient models. During the pre-1990 period, the TVP-FAVAR model generally has the lowest relative RMSE, especially during the late 1970s and the early 1980s when some of the other forecasting models in Chart 2 appeared to be inaccurate relative to the benchmark.

Overall, our results for inflation point to the role played by time-varying parameters in delivering accurate inflation forecasts.



4.1.3 Short-term interest rates

Stock and Watson's unobserved component model has the lowest relative RMSE (on average over the period 1976-2007) at the one and four-quarter horizon in forecasting the short-term interest rate. The FAVAR model also performs well at the four-quarter horizon. At longer forecast horizons, the TVAR model (with inflation as the threshold variable) produces the lowest relative RMSE on average. Note that when considering the entire forecast period, the gains from these models relative to the benchmark are modest.

In contrast, when considering the great moderation period the difference in the performance of some of the forecasting models and the AR(1) benchmark are larger. At the one and four-quarter horizons, the TVP-VAR model produces the most accurate interest rate forecasts leading to a 20% reduction in RMSE relative to the AR(1) model (with a significant Diebold and Mariano (1995) test statistic). The TVAR model is the best-performing model at longer horizons over this period.

4.2 Model-specific results

In this subsection, we consider forecasting performance across different specifications of the estimated models. Consider, first, the regime-switching models. It is immediately clear that allowing for independent regime shifts in the VAR coefficients and error covariance matrix, generally leads to a deterioration in forecasting accuracy (as measured by the trace of the forecast error covariance matrix). In fact, within the estimated regime-switching models, the best forecasting performance (ie lowest trace) appears to be delivered by the specification that imposes a common regime variable for the VAR coefficients and the covariance matrix or only allows the coefficients to switch.

From a forecast accuracy point of view, the general TVP-VAR proposed in Cogley *et al* (2008) and Baumeister and Benati (2010) (that allows for a time-varying Q matrix) displays the least favourable performance within the TVP-VARs considered in this study. The best performance on the basis of the trace of the forecast error covariance

matrix and over all forecast horizons is delivered by the standard TVP-VAR that allows for stochastic volatility and the homoscedastic TVP-VAR.

According to the trace criteria, the TVP-FAVAR with homoscedastic shocks delivers the best forecast performance (within the FAVAR models) at the four, eight and twelve quarter horizons. Note, also that at these forecasting horizons (and over the full forecast sample) this model performs better than all the other competing models. This again brings out the influence of time-varying parameters and a large information set on forecast performance.

The TVAR model that uses the lag of inflation as the threshold variable consistently delivers more accurate forecasts of all three variables at the four, eight and twelve-quarter forecasts horizon, pointing to the importance of regime switches driven by lagged inflation. The estimates for ST-VAR model suggest a similar result, albeit the difference across the STAR models is less stark.

The performance of the recursive and rolling VAR models is quite similar to each other. The rolling VAR performs slightly better than the recursive VAR model at the one, eight and twelve quarter horizons, with the recursive VAR delivering a lower trace at the four-quarter horizon.

Finally, the BMA procedure does well on the basis of the trace at the four-quarter horizon with an estimated trace close to the best-performing model. At longer forecast horizons, the BMA procedure produces the most accurate forecast in terms of the trace.

4.3 Forecast performance and the recent financial crisis

In this subsection we consider how the forecasts from our models perform when considering the period 2008 Q1 to 2010 Q4, a period over which the financial crisis intensified. In particular, we consider how the forecasting models perform given the information set at 2007 Q4. Chart 3 plots the one step and four step ahead recursive forecasts for these variables from the 24 forecasting models alongside the actual realised values over these quarters.

Consider the one step ahead GDP forecast in the top-left panel. One striking feature of this panel is that most models predicted zero or positive growth over the second half of 2008 and 2009 when actual growth was strongly negative. In 2008 Q4, actual annualised GDP growth was -3.6%. The three STAR models were the only specifications to predict negative GDP growth with forecasts of around -1%. Note that these models predicted a growth rate in 2008 Q4 which was much lower than the realised value. In 2009 Q1, the prediction of these three STAR models of a GDP growth rate of -7% was fairly close to the actual value, with the forecasts from these models overshooting in the next quarter. By 2009 Q3, a number of other forecasting models (RSVAR* (three and four regimes), RSVAR** (three and four regimes), TVP-FAVAR and FAVAR, UC, TVAR (inflation) and the rolling VAR) were also predicting negative growth. Overall, the STAR models delivered plausible (but volatile) one-step forecasts over the large contraction in GDP growth in 2008 and 2009. The top-right panel of the chart shows that a similar interpretation can be placed on the four step ahead GDP forecasts.

The middle panel of Chart 3 shows the one and four step ahead forecasts for inflation. In 2008 Q4, annualised quarterly inflation was 5.5%. It then dropped to 0.8% in the next quarter before reaching a trough at 0.1%. Inflation then rose to around 4% by mid-2010. Most of the forecasting models failed to predict (at the one-quarter horizon), the large drop in inflation between 2008 Q4 and 2009 Q1. The exceptions are the general regime-switching VAR with two regimes and the three STAR models. These four specifications predict a fall in inflation (over this quarter) from 3% to 10%, while the other forecasting models essentially indicate no change. In contrast, the majority of the forecasting models predict a large fall in inflation between 2009 Q1 and 2009 Q2, while actual inflation remained fairly stable. Over the second half of 2009 and 2010, the gentle increase in inflation is matched by the profile of most one step ahead forecasts. At the four-quarter forecast horizon, most forecasting models predicted higher-than-actual inflation over 2008 and the first half of 2009, and underpredicted inflation over the second half of 2009 and 2010 Q1.

As with the one step ahead inflation forecast, the forecasting models have a hard time in matching the sharp fall in the short-term interest rate in the last quarter of 2008



(bottom-left panel). However, after 2009 Q2, the models predict interest rates close to zero. The bottom-right panel shows that the four-quarter forecasts are quite far away from the true realised values, possibly reflecting the atheoretical nature of the forecasting models.

5 Conclusions

This paper investigates the performance of a variety of models with time-varying parameters in forecasting UK GDP growth, inflation and the short-term interest rate. Overall, different models perform better than others on occasions, therefore it is most appropriate to consider a suite of models when forecasting – and that is what policymakers typically do, including the MPC.

More specifically, the TVP-VAR and the TVAR model provide forecasts for GDP growth with a lower average RMSE than an AR(1) model. The TVAR model also appears to perform better in matching the GDP growth profile over the recent financial crisis.

Models with time-varying parameters lead to a large improvement in inflation forecasting performance over the AR(1) benchmark. In particular, the TVP-FAVAR model, the TVP-VAR and the UC model have a RMSE that is substantially lower than the benchmark model.

Stock and Watson's unobserved component model, the FAVAR, the TVP-VAR and the TVAR stand out when considering the interest rate forecast. But, in general, it appears that the models considered in this study are less successful at forecasting interest rates than GDP growth and inflation.

Across the three variables, the TVP-FAVAR model stands out, delivering the most accurate forecasts at the four-quarter horizon on average over the sample. This highlights the role played by a large information set and time-varying parameters in delivering forecast accuracy.

Appendix A: Tables and charts

Chart 1: RMSE error at the four-quarter horizon of TVP-VAR, TVAR and ST-VAR models in forecasting GDP growth (relative to an AR(1) model)

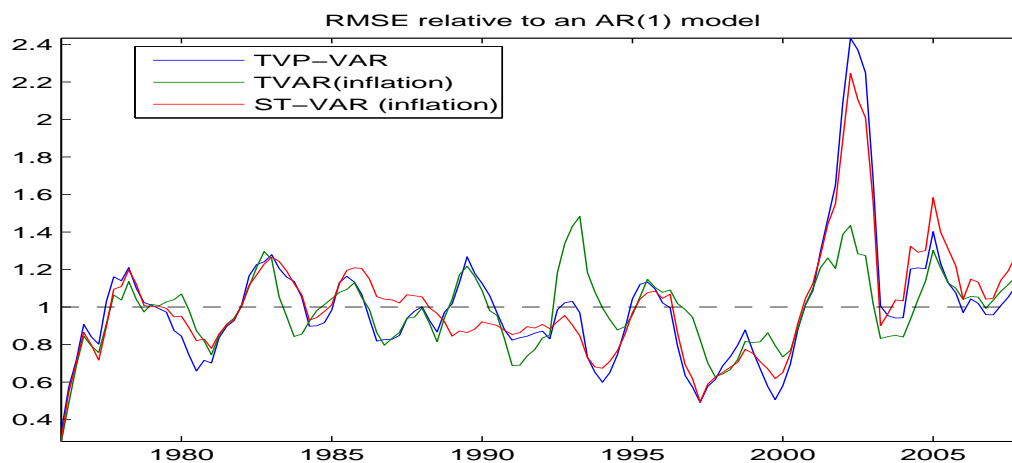


Chart 2: RMSE error at the four-quarter horizon of TVP-FAVAR, TVP-VAR, rolling VAR and UC models in forecasting inflation (relative to an AR(1) model)

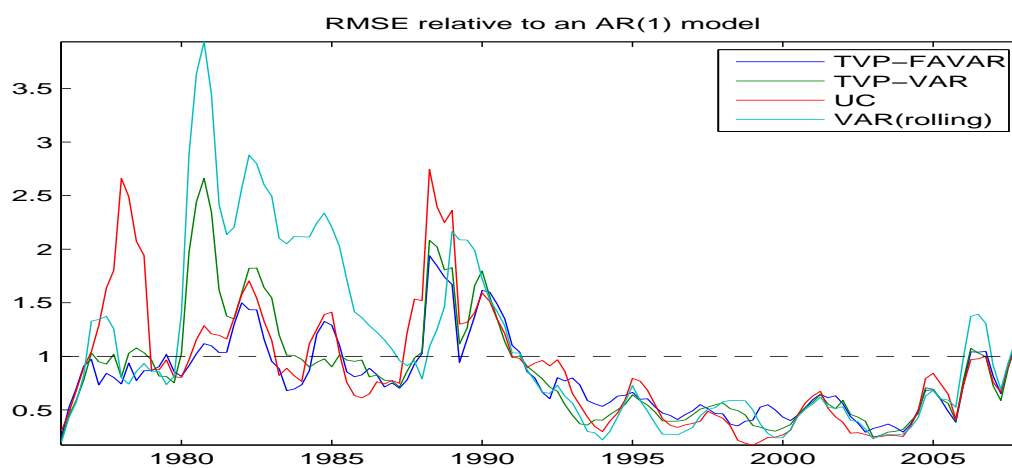


Chart 3: Forecasts and actual data 2008 Q1 to 2010 Q4

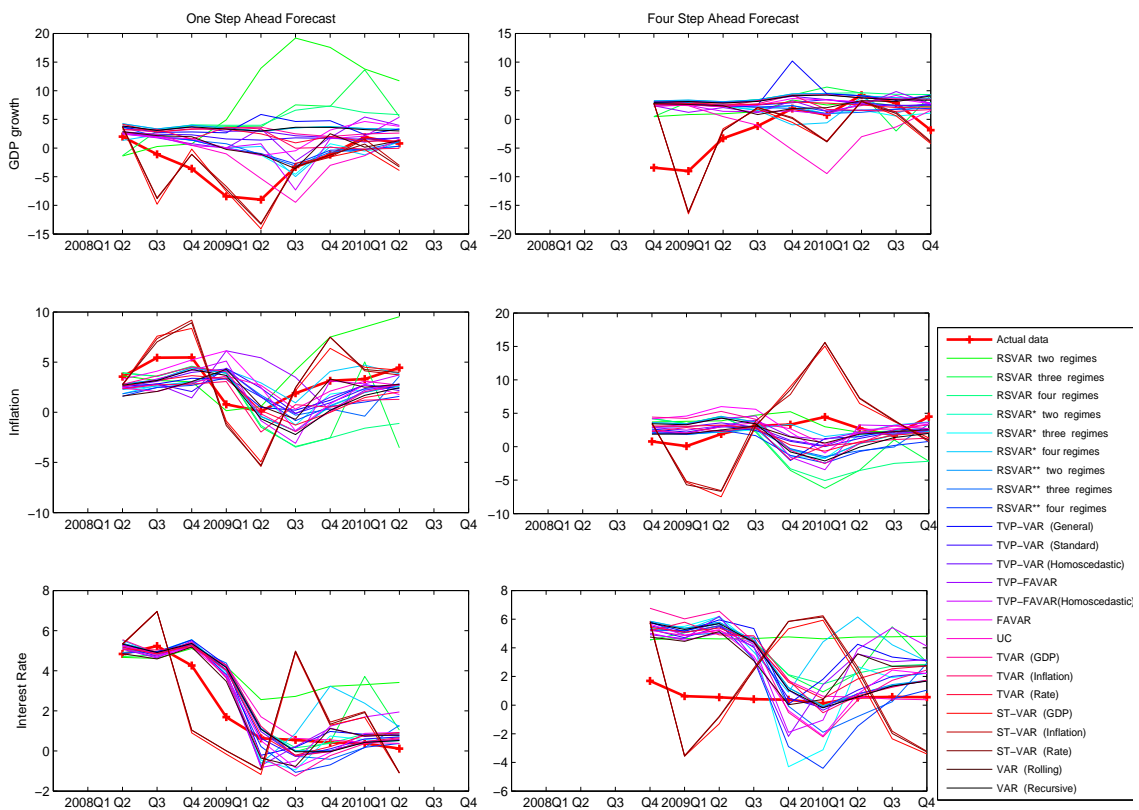


Table A: Notes for tables

Acronym	Explanation
RMSE	Denotes the root mean squared error of each model relative to an AR(1) model
DM	Denotes the Diebold-Mariano forecast evaluation statistic relative to an AR(1) model
probV	Is the probability value of DM statistic under the null that both models produce equal accurate forecasts, with a * by the DM value indicating rejection of null at 5% significance level
RSVARs	Markov-switching VARs with independent switching in the coefficients and the covariance
RSVARs*	Markov-switching VARs with joint switching in the coefficients and the covariance
RSVARs**	Markov-switching VARs with only the coefficients switching
TVP-VAR (General)	Time Varying VAR with time varying degree of parameter drift
TVP-VAR (Standard)	Time-varying VAR with constant degree of parameter drift
TVP-VAR (Homoscedastic)	Time-varying VAR with constant variance-covariance matrix of the VAR residuals
TVP-FAVAR	Time-varying factor augmented VAR model
TVP-FAVAR (Homoscedastic)	Time-varying factor augmented VAR model with constant variance-covariance matrix of the VAR residuals
FAVAR	Factor augmented VAR model
UC	Unobserved component model
TVAR (X)	Threshold VAR model using X as the threshold variable
ST-VAR (X)	Smooth transition VAR model using X as the threshold variable
VAR (Rolling)	VAR estimated with a rolling constant window
VAR (Recursive)	VAR estimated recursively
BMA	Denotes the Bayesian weighted forecasting model

Table B: Log marginal likelihood for the forecasting models over the full estimation sample

	Log marginal likelihood
RSVAR two regimes	-915.428
RSVAR three regimes	-970.105
RSVAR four regimes	-1028.07
RSVAR* two regimes	-986.545
RSVAR* three regimes	-984.634
RSVAR* four regimes	-1063.59
RSVAR** two regimes	-978.358
RSVAR** three regimes	-1054.53
RSVAR** four regimes	-1112.36
TVP-VAR (General)	-3663.45
TVP-VAR (Standard)	-2352.16
TVP-VAR (Homoscedastic)	-1151.24
TVP-FAVAR	527.5055
TVP-FAVAR (Homoscedastic)	5673.594
FAVAR	-56.5631
UC	-202.318
TVAR (GDP)	-423.984
TVAR (Inflation)	-414.781
TVAR (Rate)	-363.855
ST-VAR (GDP)	-381.474
ST-VAR (Inflation)	-265.502
ST-VAR (Rate)	-291.512
VAR (Rolling)	-24.1139
VAR (Recursive)	116.9715

Table C

RMSE of each model relative to an AR(1) model for GDP

Models	1976-2007				1992-2007				Models	1976-2007				1992-2007			
	1 Q	4 Q	8 Q	12 Q	1 Q	4 Q	8 Q	12 Q		1 Q	4 Q	8 Q	12 Q	1 Q	4 Q	8 Q	12 Q
RSVAR two regimes	2.80	1.70	1.42	1.32	3.95	2.03	1.60	1.44	6.10	2.70	0.76	0.89	6.75	1.80	0.26	0.21	
RSVAR three regimes	1.53	1.16	1.07	1.04	1.47	1.06	1.01	1.01	2.50	-0.60	-0.20	1.65	2.67	-1.13	-0.98	1.26	
RSVAR four regimes	1.70	1.24	1.13	1.08	1.40	1.08	1.04	1.03	3.02	-0.80	-0.43	0.95	2.39	0.21	1.11	0.97	
RSVAR* two regimes	1.03	1.04	1.04	1.05	1.38	1.13	1.08	1.06	0.78	0.85	2.40	0.83	1.73	0.73	1.38	0.93	
RSVAR* three regimes	1.00	1.11	1.12	1.13	1.06	1.03	1.05	1.06	-0.38	1.16	1.74	1.83	0.00	0.69	1.83	1.05	
RSVAR* four regimes	1.03	1.09	1.11	1.13	1.09	0.98	0.99	1.00	0.21	1.09	1.66	1.89	0.07	-0.75	1.57	1.02	
RSVAR** two regimes	1.08	1.04	1.05	1.06	1.35	1.11	1.07	1.06	0.29	0.55	2.24	1.45	1.57	0.73	1.37	0.96	
RSVAR** three regimes	1.02	1.17	1.18	1.18	1.13	1.06	1.08	1.09	-0.02	1.05	1.16	1.27	0.50	0.98	1.78	1.13	
RSVAR** four regimes	1.01	1.16	1.19	1.20	1.04	1.00	1.05	1.06	0.07	1.04	1.21	1.44	-0.28	0.58	1.74	1.14	
TVP-VAR (General)	1.59	1.31	1.24	1.21	1.21	1.05	1.05	1.07	3.73	1.72	2.08	2.41	1.05	0.63	5.25	3.00	
TVP-VAR (Standard)	0.88	0.94	0.97	0.99	1.01	0.94	0.98	0.99	-1.09	-0.25	1.40	1.35	0.70	0.32	1.29	0.96	
TVP-VAR (Homoscedastic)	0.92	0.97	0.98	1.00	1.18	1.01	1.02	1.02	-1.09	-0.25	1.40	1.35	0.70	0.32	1.29	0.96	
TVP-FAVAR	1.00	1.04	1.03	1.03	1.17	1.12	1.09	1.08	0.08	0.76	1.08	0.69	1.47	2.19	1.58	-0.18	
TVP-FAVAR (Homoscedastic)	1.07	1.06	1.05	1.05	1.23	1.17	1.13	1.12	1.12	0.99	1.51	0.80	1.90	1.99	1.05	0.49	
FAVAR	1.02	0.99	0.99	0.99	1.11	1.14	0.97	0.98	0.73	0.03	0.62	-0.43	0.52	-0.80	0.78	0.71	
UC	0.94	1.04	1.06	1.06	1.19	0.95	1.13	1.09	-1.68	3.21	2.38	0.98	1.13	2.73	3.02	0.43	
TVAR (GDP)	1.18	1.05	1.06	1.06	1.01	0.98	1.02	1.03	1.54	0.31	1.48	0.16	-0.37	0.01	1.31	-0.86	
TVAR (Inflation)	0.89	0.96	0.99	1.01	0.99	1.02	1.00	1.02	-0.21	-0.35	0.76	1.39	-0.24	-0.17	0.80	0.74	
TVAR (Rate)	1.01	0.99	1.00	1.02	1.19	0.97	1.02	1.03	0.77	0.17	0.91	1.10	0.66	-0.30	0.89	0.70	
ST-VAR (GDP)	0.96	0.98	0.99	1.00	1.09	0.97	0.98	0.99	0.16	-0.08	0.99	0.69	0.10	-0.11	1.09	0.79	
ST-VAR (Inflation)	0.94	0.97	0.98	1.00	1.09	0.97	0.98	0.99	-0.44	-0.45	1.00	1.30	0.17	-0.03	1.08	0.82	
ST-VAR (Rate)	0.94	0.97	0.98	1.00	1.09	1.01	0.99	1.00	-0.87	-0.27	0.97	0.95	0.14	-0.05	1.13	0.81	
VAR (Rolling)	0.96	0.99	1.03	1.04	1.00	1.02	1.06	1.06	0.24	0.75	2.39	1.33	-0.27	0.71	1.96	0.81	
VAR (Recursive)	0.97	0.98	1.00	1.01	1.19	1.17	1.02	1.02	-0.60	0.00	1.30	1.46	0.78	0.28	1.26	0.88	
BMA	1.10	1.07	1.05	1.05	1.24	0.00	1.13	1.12	1.37	0.83	1.54	0.69	1.90	1.98	0.99	0.50	

Note: RMSE values smaller than 1 and DM values smaller than -1,645 indicate that our forecasting models outperform the AR(1) (green). RMSE values greater than 1 and DM values greater than 1,645 suggest the opposite (blue). If DM is between these two numbers the null of statically equally accurate forecasts cannot be rejected (white).

Table E

RMSE of each model relative to an AR(1) model for the interest rate

Models	1976-2007				1992-2007			
	1Q	4Q	8Q	12Q	1Q	4Q	8Q	12Q
RSVAR two regimes	3.49	2.87	2.03	1.72	1.78	1.46	1.04	0.87
RSVAR three regimes	1.53	1.54	1.37	1.27	0.92	0.85	0.79	0.75
RSVAR four regimes	1.60	1.73	1.55	1.41	0.77	0.81	0.80	0.78
RSVAR** two regimes	1.12	1.04	1.03	1.00	0.91	1.03	1.15	1.20
RSVAR** three regimes	1.41	1.11	1.06	1.00	0.99	1.04	1.05	1.01
RSVAR** four regimes	1.37	1.10	1.04	0.97	0.86	0.97	0.93	0.84
RSVAR** two regimes	1.23	1.09	1.04	1.03	0.87	0.99	1.12	1.17
RSVAR** three regimes	1.47	1.14	1.09	1.06	1.10	1.15	1.20	1.19
RSVAR** four regimes	1.36	1.12	1.07	1.01	0.80	0.91	0.99	0.95
TVP-VAR (General)	1.08	1.22	1.25	1.20	0.84	0.97	1.02	1.01
TVP-VAR (Standard)	1.04	1.05	1.06	1.03	0.75	0.79	0.79	0.72
TVP-VAR (Homoscedastic)	1.08	1.08	1.07	1.04	0.76	0.79	0.81	0.79
TVP-FAVAR	1.08	1.13	1.14	1.11	0.81	0.87	0.87	0.80
TVP-FAVAR (Homoscedastic)	1.03	1.05	1.03	1.01	0.94	0.99	0.90	0.85
FAVAR	0.99	0.97	0.98	0.97	0.80	0.87	0.96	1.01
UC	0.97	0.97	1.00	0.98	0.89	1.23	0.87	0.80
TVAR (GDP)	1.03	1.08	1.18	1.23	1.00	0.80	1.44	1.52
TVAR (Inflation)	1.01	0.98	0.97	0.93	0.80	1.01	0.78	0.71
TVAR (Rate)	1.03	1.04	1.03	0.99	0.91	0.93	0.99	0.93
ST-VAR (GDP)	0.99	1.01	1.03	1.04	0.83	0.91	1.03	1.07
ST-VAR (Inflation)	0.99	1.01	1.03	1.03	0.82	0.92	1.00	1.04
ST-VAR (Rate)	0.99	1.01	1.03	1.03	0.82	1.02	1.01	1.05
VAR (Rolling)	1.14	1.08	1.03	0.97	0.88	0.92	1.07	1.00
VAR (Recursive)	0.99	1.00	1.02	1.02	0.82	0.93	1.01	1.05
BMA	1.05	1.07	1.05	1.02	0.88	0.00	0.86	0.82

Models	1976-2007				1992-2007			
	1Q	4Q	8Q	12Q	1Q	4Q	8Q	12Q
RSVAR two regimes	6.21	4.14	2.66	1.59	3.09	0.73	-1.22	-1.22
RSVAR three regimes	2.67	2.60	1.79	1.22	-0.60	-1.71	-2.25	-2.25
RSVAR four regimes	3.56	2.87	2.31	1.59	-1.47	-1.66	-2.48	-2.48
RSVAR** two regimes	1.72	-0.33	-0.87	-0.79	-0.72	1.73	4.70	4.70
RSVAR** three regimes	1.58	0.54	-0.60	-1.10	0.15	0.65	-0.25	-0.25
RSVAR** four regimes	1.65	0.40	-0.45	-0.84	-1.04	-0.39	-1.63	-1.63
RSVAR** two regimes	1.82	0.58	-0.71	-0.41	-2.15	2.15	5.25	5.25
RSVAR** three regimes	1.63	0.47	-0.41	-0.44	1.52	1.52	1.68	1.68
RSVAR** four regimes	1.52	0.57	-0.24	-0.40	-1.63	-0.54	-0.26	-0.26
TVP-VAR (General)	1.52	2.12	1.67	1.76	-1.10	0.00	-0.44	-0.44
TVP-VAR (Standard)	1.85	1.65	1.03	0.52	-2.11	-2.38	-2.75	-2.75
TVP-VAR (Homoscedastic)	1.85	1.65	1.03	0.52	-2.11	-2.38	-2.75	-2.75
TVP-FAVAR	1.67	1.80	1.55	1.11	-1.02	-1.46	-1.98	-1.98
TVP-FAVAR (Homoscedastic)	0.57	1.04	0.66	0.46	-0.80	-1.22	-2.11	-2.11
FAVAR	0.61	0.22	-1.31	-1.56	-2.26	-2.49	0.16	0.16
UC	-0.82	-0.43	0.35	0.09	-1.63	-1.78	-2.00	-2.00
TVAR (GDP)	0.93	2.12	5.05	4.62	-0.53	4.78	4.98	4.98
TVAR (Inflation)	0.70	0.35	-0.03	-0.62	-2.15	-2.85	-2.51	-2.51
TVAR (Rate)	0.81	0.56	-0.48	-1.06	-1.65	-1.06	-2.08	-2.08
ST-VAR (GDP)	0.38	0.75	1.59	1.42	-2.16	1.13	4.08	4.08
ST-VAR (Inflation)	0.49	0.69	0.72	0.55	-2.44	-1.84	1.46	1.46
ST-VAR (Rate)	0.40	0.69	0.77	0.60	-2.38	-1.48	2.14	2.14
VAR (Rolling)	2.21	0.44	-1.01	-1.80	-1.07	0.43	-0.37	-0.37
VAR (Recursive)	0.13	0.57	0.26	0.01	-2.40	-1.64	3.06	3.06
BMA	2.01	1.54	0.87	-0.12	-0.84	-1.28	-2.23	-2.23

Note: RMSE values smaller than 1 and DM values smaller than -1,645 indicate that our forecasting models outperform the AR(1) (green). RMSE values greater than 1 and DM values greater than 1,645 suggest the opposite (blue). If DM is between these two numbers the null of statically equally accurate forecasts cannot be rejected (white).

Table F

Trace- multivariate forecast evaluation statistic relative to an AR(1) model

<i>Models</i>	1976-2007				1992-2007			
	1 Q	4 Q	8 Q	12 Q	1 Q	4 Q	8 Q	12 Q
RSVAR two regimes	16.07	2.86	1.73	1.32	12.15	1.08	0.43	0.38
RSVAR three regimes	4.21	2.50	1.78	1.52	2.34	0.72	0.68	0.70
RSVAR four regimes	4.40	2.82	1.73	1.53	1.87	0.84	0.75	0.74
RSVAR* two regimes	1.03	1.25	1.38	1.48	1.12	0.62	0.79	0.90
RSVAR* three regimes	1.05	1.25	0.93	0.92	0.73	0.44	0.43	0.40
RSVAR* four regimes	1.03	1.18	0.88	0.82	0.73	0.35	0.34	0.28
RSVAR** two regimes	1.13	1.07	1.14	1.19	1.08	0.60	0.78	0.89
RSVAR** three regimes	1.02	1.49	0.96	0.94	0.82	0.51	0.53	0.53
RSVAR** four regimes	1.05	1.58	0.96	0.90	0.70	0.39	0.42	0.40
TVP-VAR (General)	2.61	1.33	1.23	0.84	0.86	0.43	0.47	0.49
TVP-VAR (Standard)	0.95	0.94	0.93	0.82	0.82	0.42	0.46	0.45
TVP-VAR (Homoscedastic)	0.95	0.94	0.93	0.82	0.82	0.42	0.46	0.45
TVP-FAVAR	0.92	0.90	0.88	0.81	0.87	0.44	0.33	0.29
TVP-FAVAR (Homoscedastic)	1.02	0.77	0.70	0.64	0.93	0.42	0.32	0.30
FAVAR	1.30	0.80	0.77	0.78	0.81	0.52	0.67	0.77
UC	0.90	0.86	0.75	0.66	0.76	0.41	0.30	0.25
TVAR (GDP)	1.28	1.00	1.29	1.39	0.64	0.62	0.81	1.01
TVAR (Inflation)	0.92	0.93	0.94	0.83	0.86	0.46	0.39	0.35
TVAR (Rate)	1.03	1.06	1.13	1.04	0.96	0.59	0.57	0.54
ST-VAR (GDP)	1.01	0.96	1.10	1.10	0.73	0.53	0.69	0.80
ST-VAR (Inflation)	0.99	0.95	1.03	0.98	0.76	0.54	0.66	0.73
ST-VAR (Rate)	0.95	0.99	1.10	1.08	0.75	0.55	0.70	0.79
VAR (Rolling)	0.95	1.09	1.08	1.01	0.68	0.44	0.40	0.35
VAR (Recursive)	0.97	1.00	1.12	1.08	0.84	0.50	0.64	0.73
BMA	1.09	0.87	0.69	0.50	0.93	0.42	0.31	0.29

Note: A trace value smaller than 1 indicates that our forecasting models outperform the AR(1) (green). A trace value greater than 1 indicates the opposite (blue).

Appendix B: Regime-switching VAR

Consider the change-point VAR model

$$Z_t = c_S + \sum_{j=1}^K B_S Z_{t-j} + \Omega_H^{1/2} \epsilon_t \quad (\mathbf{B-1})$$

where $S = 1 \dots M$ follows a M-state Markov chain. The Gibbs sampler cycles through the following steps.

1. Sampling the states S_t :

Given starting values for the VAR parameters and covariances and the transition probabilities, the unobserved state variables S and H are drawn using multi-move Gibbs sampling to draw from the joint conditional density $f(S_t | Z_t, c_s, B_{1,s}, \dots, B_{K,s}, \tilde{P})$ and $f(H_t | Z_t, c_s, B_{1,s}, \dots, B_{K,s}, \tilde{Q})$ Kim and Nelson (1999, Chapter 9) show that the Markov property of the state variable implies that

$$f(\bar{S}_t | Z_t) = f(\bar{S}_T | Z_T) \prod_{t=1}^{T-1} f(\bar{S}_t | \bar{S}_{t+1}, Z_t) \quad (\mathbf{B-2})$$

where $\bar{S}_t = S_t$ or H_t . This density can be simulated in two steps:

(a) Calculating $f(\bar{S}_T | Z_T)$: The Hamilton (1989) filter provides $f(\bar{S}_t | Z_t), t = 1, \dots, T$. The last iteration of the filter provides $f(\bar{S}_T | Z_T)$.

(b) Calculating $f(\bar{S}_t | \bar{S}_{t+1}, Z_t)$: Kim and Nelson (1999, Chapter 9) show that

$$f(\bar{S}_t | \bar{S}_{t+1}, Y_t) \propto f(\bar{S}_{t+1} | \bar{S}_t) f(\bar{S}_t | Y_t) \quad (\mathbf{B-3})$$

where $f(\bar{S}_{t+1} | \bar{S}_t)$ is the transition probability and $f(\bar{S}_t | Z_t)$ is obtained via Hamilton (1989) filter in step a. Kim and Nelson (1999) (page 214) show how to sample \bar{S}_t from **(B-3)**.



2. Sampling $c_S, B_{1,S}, \dots, B_{k,S}$,

Conditional on a draw for S_t and H_t the model in equation **(B-1)** is simply a sequence of Bayesian VAR models with heteroscedasticity (when Ω evolves independently) . Collecting the VAR coefficients for regime $S = J$ into the $(N \times (N \times P + 1)) \times 1$ vector Υ^S , the left-hand side of equation **(B-1)** for regime $S = J$ into the matrix Y_t^S and the right-hand side (ie lags and the intercept terms) of equation **(B-1)** into the matrix X_t^S , we rewrite the VAR in each regime in state-space form

$$\begin{aligned} \text{vec}(Y_t^S) &= (I_N \otimes X_t^S) \Upsilon_t^S + \Omega_H^{1/2} \varepsilon_t^S \\ \Upsilon_t^S &= \Upsilon_{t-1}^S \end{aligned}$$

and use the Carter and Kohn (2004) algorithm to derive the mean and variance of Υ_t^S . As described below, this requires a Kalman filter recursion. The initial state and its covariance for the Kalman filter is specified as the mean and variance of the normal inverse Wishart prior. The prior mean and variance is set using dummy observations:

$$Y_D = \begin{pmatrix} \frac{\text{diag}(\gamma_1 \sigma_1 \dots \gamma_N \sigma_N)}{\tau} \\ 0_{N \times (P-1) \times N} \\ \dots \\ \text{diag}(\sigma_1 \dots \sigma_N) \\ \dots \\ 0_{1 \times N} \end{pmatrix}, \text{ and } X_D = \begin{pmatrix} \frac{J_P \otimes \text{diag}(\sigma_1 \dots \sigma_N)}{\tau} & 0_{NP \times 1} \\ 0_{N \times NP} & 0_{N \times 1} \\ \dots \\ 0_{1 \times NP} & c \end{pmatrix} \quad \text{(B-4)}$$

where $\sigma_1 \dots \sigma_N$ represents standard deviations of the error term of an AR model estimated using each endogenous variable, γ_1 to γ_N denotes the prior mean for the coefficients on the first lag, τ is the tightness of the prior on the VAR coefficients and c is the tightness of the prior on the constant terms. We set $\tau = 10$ and $c = 1/10000$ in our implementation.

3. Sampling Ω_H



Conditional on a draw for S_t and H_t , the error covariance matrix has an inverse Wishart conditional posterior. We calculate the residuals of the VAR model as

$$\hat{\varepsilon}_t = \sum_{S=1}^J (Y_t^S - X_t^S \bar{\Upsilon}^S) \times I(S_t = J) \quad (\mathbf{B-5})$$

where $\bar{\Upsilon}^S$ equals Υ^S reshaped into a $(N \times P + 1) \times N$ matrix (to be conformable with X_t^S) and $I(\cdot)$ is an indicator variable. Note that when $S_t = H_t$ (i.e. the coefficients and the error covariance matrix switch jointly) or $\Omega_H = \Omega$ steps 2 and 3 simplify and standard formulas for the conditional posterior of Bayesian VARs can be used.

4. Sampling \tilde{P} and \tilde{Q} :

The prior for the non-zero elements of the transition probability matrix p_{ij} is of the following form

$$p_{ij}^0 = D(u_{ij})$$

where $D(\cdot)$ denotes the Dirichlet distribution and $u_{ij} = 15$ if $i = j$ and $u_{ij} = 1$ if $i \neq j$. This choice of u_{ij} implies that the regimes are fairly persistent. The posterior distribution is:

$$p_{ij} = D(u_{ij} + \eta_{ij})$$

where η_{ij} denotes the number of times regime i is followed by regime j .

We use 200,000 iterations and discard the first 199,000 as burn-in.

B.1 Calculation of the marginal likelihood

A detailed description of the calculation of the marginal likelihood for change-point models can be found in Bauwens and Rombouts (2012).

Appendix C: Time-varying VAR

Consider the general time-varying VAR model

$$Z_t = c_t + \sum_{j=1}^K B_j Z_{t-j} + \Omega_t^{1/2} \varepsilon_t$$

where

$$\Phi_t = \Phi_{t-1} + \eta_t, \text{var}(\eta_t) = Q_t \quad (\text{C-1})$$

$$\text{VAR}(v_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})'. \quad (\text{C-2})$$

$$Q_t = \tilde{A}^{-1} \tilde{H}_t (\tilde{A}^{-1})' \quad (\text{C-3})$$

where the structure of A_t , H_t and \tilde{A} is described in the text.

C.2 Prior distributions and starting values

The initial conditions for the VAR coefficients ϕ_0 are obtained via an OLS estimate of a fixed-coefficient VAR using the first 40 observations of the sample period. Let \hat{v}^{ols} denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data described above. The prior for the diagonal elements of the VAR covariance matrix (see (8)) is defined as $\ln h_0 \sim N(\ln \mu_0, I_3)$ where μ_0 are the diagonal elements of the Cholesky decomposition of \hat{v}^{ols} . The prior for the off-diagonal elements A_t is $A_0 \sim N(\hat{a}^{ols}, V(\hat{a}^{ols}))$ where \hat{a}^{ols} are the off-diagonal elements of \hat{v}^{ols} , with each row scaled by the corresponding element on the diagonal. $V(\hat{a}^{ols})$ is assumed to be diagonal with the elements set equal to ten times the absolute value of the corresponding element of \hat{a}^{ols} .

Let Q_{OLS} denote the OLS estimate of the coefficient covariance matrix using the training sample. When Q is time-invariant, its prior distribution is assumed to be inverse Wishart with a scale matrix given by $\bar{Q} = Q_{OLS} \times T_0 \times k$ where the scalar

$k = 3.5e - 04$ as in Cogley and Sargent (2005). The prior degrees of freedom are set equal to $T_0 = 40$ the length of the training sample. When Q is time-varying, a prior distribution is required for the initial values of \tilde{H}_t . The mean of this log normal prior is set as the log of $\text{diag}(A_{OLS}Q_{OLS}A'_{OLS}) \times T_0 \times k$ where A_{OLS} is the inverse of the Choleski decomposition of Q_{OLS} . The variance of the prior distribution is set to 1.

The prior distribution for the blocks of S is inverse Wishart: $S_{i,0} \sim IW(\bar{S}_i, K_i)$ where $i = 1..3$ indexes the blocks of S . \bar{S}_i is calibrated using \hat{a}^{ols} . Specifically, \bar{S}_i is a diagonal matrix with the relevant elements of \hat{a}^{ols} multiplied by 10^{-3} . Following Cogley and Sargent (2005) we postulate an inverse-gamma distribution for the elements of G , $\sigma_i^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right)$.

C.3 Simulating the posterior distributions

Time-varying VAR coefficients

The distribution of the time-varying VAR coefficients ϕ_t conditional on all other parameters and hyperparameters is linear and Gaussian: $\phi_t \setminus X_{i,t}, \Xi \sim N\left(\phi_{T \setminus T}, P_{T \setminus T}\right)$ and $\phi_t \setminus \phi_{t+1}, X_{i,t}, \Xi \sim N\left(\phi_{t \setminus t+1, \phi_{t+1}}, P_{t \setminus t+1, \phi_{t+1}}\right)$ where $t = T - 1, ..1$, Ξ denotes a vector that holds all the other VAR parameters and

$\phi_{T \setminus T} = E\left(\phi_T \setminus X_{i,t}, \Xi\right)$, $P_{T \setminus T} = \text{Cov}\left(\phi_T \setminus X_{i,t}, \Xi\right)$, $\phi_{t \setminus t+1, \phi_{t+1}} = E\left(\phi_t \setminus X_{i,t}, \Xi, \phi_{t+1}\right)$ and $P_{t \setminus t+1, \phi_{t+1}} = \text{Cov}\left(\phi_t \setminus X_{i,t}, \Xi, \phi_{t+1}\right)$. As shown by Carter and Kohn (2004) the simulation proceeds as follows. First we use the Kalman filter to draw $\phi_{T \setminus T}$ and $P_{T \setminus T}$ and then proceed backwards in time using $\phi_{t|t+1} = \phi_{t|t} + P_{t|t}P_{t+1|t}^{-1}\left(\phi_{t+1} - \phi_t\right)$ and $\phi_{t|t+1} = \phi_{t|t} - P_{t|t}P_{t+1|t}^{-1}P_{t|t}$.

Elements of H_t

Following Cogley and Sargent (2005), the diagonal elements of the VAR covariance matrix are sampled using the Metropolis Hastings algorithm in Jacquier, Polson and Rossi (2004). Given a draw for ϕ_t the VAR model can be written as $A'_t(\tilde{Z}_t) = u_t$.

Where $\tilde{Z}_t = Z_t - \sum_{l=1}^L \phi_{l,t}Z_{t-l} = v_t$ and $\text{VAR}(u_t) = H_t$. Jacquier *et al* (2004) note that conditional on other VAR parameters, the distribution $h_{it}, i = 1..3$ is given by

$f(h_{it}/h_{it-1}, h_{it+1}, u_{it}) = f(u_{it}/h_{it}) \times f(h_{it}/h_{it-1}) \times f(h_{it+1}/h_{it}) =$
 $h_{it}^{-0.5} \exp\left(\frac{-u_{it}^2}{2h_{it}}\right) \times h_{it}^{-1} \exp\left(\frac{-(\ln h_{it} - \mu)^2}{2\sigma_{h_i}}\right)$ where μ and σ_{h_i} denote the mean and the
 variance of the log-normal density $h_{it}^{-1} \exp\left(\frac{-(\ln h_{it} - \mu)^2}{2\sigma_{h_i}}\right)$. Jacquier *et al* (2004) suggest
 using $h_{it}^{-1} \exp\left(\frac{-(\ln h_{it} - \mu)^2}{2\sigma_{h_i}}\right)$ as the candidate generating density with the acceptance
 probability defined as the ratio of the conditional likelihood $h_{it}^{-0.5} \exp\left(\frac{-u_{it}^2}{2h_{it}}\right)$ at the old
 and the new draw. This algorithm is applied at each period in the sample.

Element of A_t

Given a draw for ϕ_t the VAR model can be written as $A_t'(\tilde{Z}_t) = u_t$ where
 $\tilde{Z}_t = Z_t - \sum_{l=1}^L \phi_{l,t} Z_{t-l} = v_t$ and $VAR(u_t) = H_t$. This is a system of equations with
 time-varying coefficients and given a block diagonal form for $Var(\tau_t)$ the standard
 methods for state-space models described in Carter and Kohn (2004) can be applied.

VAR hyperparameters

Conditional on Z_t , $\phi_{l,t}$, H_t , and A_t , the innovations to $\phi_{l,t}$, H_t , and A_t are observable,
 which allows us to draw the hyperparameters - the elements of Q , S , and the σ_i^2 - from
 their respective distributions. When Q is time-varying, its diagonal elements are drawn
 using the Jacquier *et al* (2004) Metropolis step described above.

We use 50,000 iterations and discard the first 49,000 as burn-in.

C.4 Calculation of the marginal likelihood

Following Chib (1995) the log marginal likelihood is defined as
 $\ln P(Z_t \setminus m) = \ln F(Z_t \setminus \hat{\Xi}, m) + \ln p(\hat{\Xi}) - \ln G(\hat{\Xi} \setminus Z_t)$. We use a particle filter to evaluate
 the log likelihood of the models with stochastic volatility. The log likelihood for the
 model with homoscedastic shocks can be evaluated using the Kalman filter.

For the most general TVP model, the posterior density is defined as

$G(\hat{\Xi} \setminus Z_t) = G(\hat{C}, \hat{M}, \hat{G}, \hat{D})$ where \hat{C} denotes the posterior mean of the non-zeros
 elements of \tilde{A} , \hat{M} denotes the posterior mean of the variance-covariance of τ_t , \hat{G}

denotes the posterior mean of the variance of \tilde{v}_t and \hat{D} is the posterior mean of the variance of \tilde{u}_t (see Section 3.2 for a definition of τ_t, \tilde{v}_t and \tilde{u}_t). We drop the dependence on Z_t for notational simplicity.

The posterior distribution can be factored as

$$G(\hat{C}, \hat{M}, \hat{G}, \hat{D}) = H(\hat{C} \setminus \hat{M}, \hat{G}, \hat{D}) \times H(\hat{M} \setminus \hat{G}, \hat{D}) \times H(\hat{G} \setminus \hat{D}) \times H(\hat{D}) \quad (\text{C-4})$$

Consider each term on the right-hand side of equation (C-4):

The term $H(\hat{C} \setminus \hat{M}, \hat{G}, \hat{D})$ can be expressed as

$$\int H(\hat{C} \setminus \hat{M}, \hat{G}, \hat{D}, \Theta) \times H(\Theta \setminus \hat{M}, \hat{G}, \hat{D}) d\Theta \quad (\text{C-5})$$

where $\Theta = \{\Phi_t, H_t, A_t, \tilde{H}_t\}$ denotes the state variables in the model. Note that $H(\hat{C} \setminus \hat{M}, \hat{G}, \hat{D}, \Theta)$ is a complete conditional and can be approximated using an additional Gibbs run that samples from the conditional densities of the states given the posterior mean of the model parameters ie $\Theta_j \setminus \hat{C}, \hat{M}, \hat{G}, \hat{D}$ and then evaluates the normal density $H(\hat{C} \setminus \hat{M}, \hat{G}, \hat{D}, \Theta_j)$ after a burn-in period. $H(\hat{C} \setminus \hat{M}, \hat{G}, \hat{D})$ is approximated as $\frac{1}{J} \sum_{j=1}^J H(\hat{C} \setminus \hat{M}, \hat{G}, \hat{D}, \Theta_j)$ where J denotes the number of retained Gibbs draws.

Similarly, the term $H(\hat{M} \setminus \hat{G}, \hat{D})$ is approximated via a Gibbs run that samples from the following conditional densities: (1) $M_j \setminus \hat{G}, \hat{D}, C_j, \Theta_j$ (2) $C_j \setminus \hat{G}, \hat{D}, M_j, \Theta_j$ and (3) $\Theta_j \setminus \hat{G}, \hat{D}, M_j, C_j$. After a burn-in period $H(\hat{M} \setminus \hat{G}, \hat{D}) \approx \frac{1}{J} \sum_{j=1}^J H(\hat{M} \setminus \hat{G}, \hat{D}, C_j, \Theta_j)$ where $H(\hat{M} \setminus \hat{G}, \hat{D}, C_j, \Theta_j)$ is the inverse Wishart density.

The term $H(\hat{G} \setminus \hat{D})$ is approximated via a Gibbs run that cycles through the following conditionals: (1) $G_j \setminus C_j, M_j, \hat{D}, \Theta_j$ (2) $C_j \setminus G_j, M_j, \hat{D}, \Theta_j$ (3) $M_j \setminus C_j, G_j, \hat{D}, \Theta_j$ and (4) $\Theta_j \setminus M_j, C_j, G_j, \hat{D}$. After a burn-in period $H(\hat{G} \setminus \hat{D}) \approx \frac{1}{J} \sum_{j=1}^J H(\hat{G} \setminus C_j, M_j, \hat{D}, \Theta_j)$ where $H(G_j \setminus C_j, M_j, \hat{D}, \Theta_j)$ is an inverse gamma density for each element of G .

The term $H(\hat{D})$ is approximated via a Gibbs run that cycles through all the conditionals: (1) $G_j \setminus C_j, M_j, D_j, \Theta_j$ (2) $C_j \setminus G_j, M_j, D_j, \Theta_j$ (3) $M_j \setminus G_j, C_j, D_j, \Theta_j$ (4) $D_j \setminus M_j, G_j, C_j, \Theta_j$ and (5) $\Theta_j \setminus M_j, C_j, G_j, D_j$. After a burn-in period, $H(\hat{D}) \approx \frac{1}{J} \sum_{j=1}^J H(\hat{D} \setminus M_j, G_j, C_j, \Theta_j)$ which is an inverse gamma density for each element of D .

We use 20,000 replications in these additional Gibbs runs discarding the first 15,000 as burn-in.



Appendix D: Time-varying FAVAR model

Our time-varying FAVAR model consists of the following equations

$$\begin{aligned} X_{it} &= \beta F_t + e_{it} & \text{(D-1)} \\ F_{k,t} &= c_t + \sum_{l=1}^2 \phi_{l,t} F_{k,t-l} + v_t \\ e_{it} &= \rho_i e_{it-1} + \varepsilon_{it} \end{aligned}$$

with $F = \{F_t^1, F_t^2, F_t^3\}$, β denotes the factor loading matrix and the coefficients $\tilde{\phi}_{l,t} = \{c_t, \phi_{l,t}\}$ follow a random walk:

$$\tilde{\phi}_{l,t} = \tilde{\phi}_{l,t-1} + \eta_t$$

The covariance matrix of the innovations v_t is factored as

$$\text{VAR}(v_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})' \quad \text{(D-2)}$$

where the time-varying matrices H_t and A_t are given as in the time-varying VAR model:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 \\ 0 & h_{2,t} & 0 \\ 0 & 0 & h_{3,t} \end{bmatrix} \quad A_t \equiv \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix} \quad \text{(D-3)}$$

with the $h_{i,t}$ evolving as geometric random walks

$$\ln h_{i,t} = \ln h_{i,t-1} + v_t.$$

Following Primiceri (2005) we postulate that the non-zero and non-one elements of the matrix A_t evolve as driftless random walks

$$\alpha_t = \alpha_{t-1} + \bar{\tau}_t \quad \text{(D-4)}$$

and we assume the vector $[\varepsilon_t', \eta_t', \bar{\tau}_t', \tilde{v}_t']'$ to be distributed as

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \\ \bar{\tau}_t \\ v_t \end{bmatrix} \sim N(0, V), \text{ with } V = \begin{bmatrix} R_t & 0 & 0 & 0 \\ 0 & Q_F & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \text{ and } G = \text{diag}(\sigma_1^2, \dots, \sigma_K^2). \quad \text{(D-5)}$$

Bernanke, Boivin and Eliasch (2005) show that identification of the FAVAR model given by equations (D-1) requires putting some restrictions on the matrix of factor loadings.

Following their example we assume that the top $J \times J$ block of β_{ik} is an identity matrix. The model is then estimated using a Gibbs sampling algorithm with the conditional prior and posterior distributions described below.

D.5 Prior distributions and starting values

Following Bernanke *et al* (2005) we centre our prior on the factors (and obtain starting values) by using a principal components (PC) estimator applied to each $X_{i,t}$. In order to reflect the uncertainty surrounding the choice of starting values, a large prior covariance of the states ($P_{0/0}$) is assumed.

Starting values for the factor loadings are also obtained from the PC estimator after imposing the above restrictions. The priors on the diagonal elements of R are assumed to be inverse gamma

$$R_{ii} \sim IG(R_0, V_0).$$

where $R_0 = 0.01$ and $V_0 = 1$ denote the prior scale parameter and the prior degrees of freedom respectively. The prior distributions for the parameters of the transition equation are set as described for the time-varying VAR model in Section C.2.

D.6 Simulating the posterior distributions

Factors and factor loadings

This closely follows Bernanke *et al* (2005). Details can also be found in Kim and Nelson (1999).

Factors

The conditional posterior distribution of the factors F_t is linear and Gaussian

$$F_T \setminus X_{i,t}, R_t, \Xi \sim N(F_{T \setminus T}, P_{T \setminus T})$$

$$F_t \setminus F_{t+1}, X_{i,t}, R_t, \Xi \sim N(F_{t \setminus t+1, F_{t+1}}, P_{t \setminus t+1, F_{t+1}})$$

where $t = T - 1, \dots, 1$, the vector Ξ holds all other FAVAR parameters. Carter and Kohn (2004) is used to calculate the mean and variance of these distributions. For details see Kim and Nelson (1999).

Elements of R

Following Bernanke *et al* (2005) R is a diagonal matrix. The diagonal elements R_{ii} are drawn from the following inverse gamma distribution

$$R_{ii} \sim IG(\bar{R}_{ii}, T + V_0)$$

where

$$\bar{R}_{ii} = \hat{\xi}'_i \hat{\xi}_i + R_0$$

where $\hat{\xi}_i$ denoting the residual $X_{it}^* - F_t'^* \beta_k$ where $X_{it}^* = X_{it} - \rho_i X_{it-1}$ and $F_t^* = F_t - \rho_i F_{t-1}$

Factor loadings

The factor loadings are sampled from

$$\beta_i \sim N(\beta^*, M^*)$$

where $\beta^* = \left(\Sigma_0^{-1} + \frac{1}{R_{ii}} F_t'^* F_t^* \right)^{-1} \left(\Sigma_0^{-1} B_0 + \frac{1}{R_{ii}} F_t'^* X_{it}^* \right)$ and $M^* = \left(\Sigma_0^{-1} + \frac{1}{R_{ii}} F_t'^* F_t^* \right)^{-1}$.

Note $B_0 = 0$ and Σ_0 is an identity matrix.

Autocorrelation coefficients

The autocorrelation coefficients ρ_i are sampled from

$$\rho_i \sim N(\rho^*, V^*)$$

where $\rho^* = \left(\Sigma_{\rho 0}^{-1} + \frac{1}{R_{ii}} e'_{t-1} e_{t-1} \right)^{-1} \left(\Sigma_{\rho 0}^{-1} \rho_0 + \frac{1}{R_{ii}} e'_{t-1} e_t \right)$ and $V^* = \left(\Sigma_{\rho 0}^{-1} + \frac{1}{R_{ii}} e'_{t-1} e_{t-1} \right)^{-1}$

with $\rho_0 = 0$ and $\Sigma_{\rho 0} = 1$

Elements of the time-varying VAR (transition equation)

Given an estimate of the factors, the model becomes a VAR with drifting coefficients and covariances and we use the algorithm described in Section C.3 to sample from the conditional posterior distributions.

As estimation of this model is more computationally intensive, we use 10,000 draws of the MCMC algorithm and use the last 1,000 draws for inference.

D.7 Calculation of the marginal likelihood

The log likelihood for the TVP-FAVAR models is evaluated using a particle filter while the Kalman filter is used for the fixed-coefficient FAVAR. The posterior density for the most general model (ie the TVP-FAVAR with stochastic volatility) is defined as $G(\hat{\Xi} \setminus Z_t) = G(\hat{\beta}, \hat{\rho}, \hat{R}, \hat{Q}_F, \hat{M}, \hat{G})$ where \hat{M} denotes the posterior mean of the variance-covariance of $\bar{\tau}_t$, \hat{G} denotes the posterior mean of the variance of v_t and $\hat{\beta}, \hat{\rho}, \hat{R}, \hat{Q}_F$ denote the posterior means of the model parameters described above. We drop the dependence on Z_t for notational simplicity. This posterior distribution can be factored as

$$G(\hat{\beta}, \hat{\rho}, \hat{R}, \hat{Q}_F, \hat{M}, \hat{G}) = \tag{D-6}$$

$$H(\hat{\beta} \setminus \hat{\rho}, \hat{R}, \hat{Q}_F, \hat{M}, \hat{G}) \times H(\hat{\rho} \setminus \hat{R}, \hat{Q}_F, \hat{M}, \hat{G}) \times$$

$$H(\hat{R} \setminus \hat{Q}_F, \hat{M}, \hat{G}) \times H(\hat{Q}_F \setminus \hat{M}, \hat{G}) \times H(\hat{M} \setminus \hat{G}) \times H(\hat{G})$$

where (as in the case of the TVP-VAR) each density on the RHS can be written as a ‘weighted average’ across the state variables $\Theta = \{F_t, \Phi_t, H_t, A_t\}$. For example $H(\hat{\beta} \setminus \hat{\rho}, \hat{R}, \hat{Q}_F, \hat{M}, \hat{G}) = \int H(\hat{\beta} \setminus \hat{\rho}, \hat{R}, \hat{Q}_F, \hat{M}, \hat{G}, \Theta) \times H(\Theta \setminus \hat{\rho}, \hat{R}, \hat{Q}_F, \hat{M}, \hat{G}) d\Theta$. The form of each density on the RHS of equation (D-6) can be approximated using the method in Chib (1995). That is, as in the case of the TVP-VAR model, we use additional Gibbs iterations to approximate each of these densities and integrate over the states. We use 10,000 iterations with a burn-in period of 7,000 iterations.

Appendix E: Unobserved component model with stochastic volatility

Consider the UC model:

$$\tilde{Z}_t = \beta_t + \sqrt{\sigma_t} \varepsilon_t$$

$$\beta_t = \beta_{t-1} + \sqrt{\omega_t} v_t$$

$$\ln \sigma_t = \ln \sigma_{t-1} + e_{1t}, \text{var}(e_{1t}) = g_1$$

$$\ln \omega_t = \ln \omega_{t-1} + e_{2t}, \text{var}(e_{2t}) = g_2$$

E.8 Priors and starting values

The prior for the initial value of the stochastic volatility $\ln \sigma_t$ is defined as $\ln \sigma_0 \sim N(\ln \mu_0, 10)$ where μ_0 is the variance of $\tilde{Z}_{t0} - \beta_{t0}$ where $t0$ denotes the training sample of 40 observations and β_{t0} is an initial estimate of the trend using an HP filter. Similarly $\ln \omega_0 \sim N(\ln \omega_0, 10)$ where $\omega_0 = \Delta \beta_{t0}$. The prior for g_1 and g_2 is inverse gamma with prior scale parameter set equal to 0.01 and 0.0001 respectively with degrees of freedom set equal to one.

E.9 Simulating the posterior distributions

Conditional on a value for g_1 and g_2 the Metropolis algorithm described in Jacquier *et al* (2004) is used to draw σ_t and ω_t . β_t is drawn using the Carter and Kohn (2004) algorithm. Given a draw for σ_t and ω_t , g_1 and g_2 can easily be sampled from the inverse gamma distribution. We use 10,000 draws of the MCMC algorithm and use the last 1,000 draws for inference.

E.10 Calculating the marginal likelihood

The log likelihood function for this model is calculated using a particle filter. The posterior density in the equation for the marginal likelihood is defined as

$G(\hat{\Xi} \setminus Z_t) = G(\hat{g}_1, \hat{g}_2)$ where we have dropped the dependence on Z_t for notational



simplicity. This density can be factored as

$$G(\hat{g}_1, \hat{g}_2) = H(\hat{g}_1 \setminus \hat{g}_2) \times H(\hat{g}_2)$$

where $H(\hat{g}_1 \setminus \hat{g}_2) = \int H(\hat{g}_1 \setminus \hat{g}_2, \Theta) \times H(\Theta \setminus \hat{g}_2) d\Theta$ and

$H(\hat{g}_2) = \int H(\hat{g}_2 \setminus \Theta) \times H(\Theta) d\Theta$ where $\Theta = \{\beta_t, \sigma_t, \omega_t\}$ denotes the state variables in the model. As described above for the TVP-VAR and the TVP-FAVAR models, additional Gibbs runs can be used to approximate these two terms. We use 10,000 iterations in these additional Gibbs samplers and discard the first 7,000 as burn-in.



Appendix F: Threshold and smooth transition VAR models

F.11 Prior distribution

The prior distribution of the VAR parameter vector in each regime

$$\beta_{\tilde{S}_t} \equiv \left(\text{vec}(B_{1,\tilde{S}_t})', \dots, \text{vec}(B_{K,\tilde{S}_t})', \text{vech}(\Omega_{\tilde{S}_t})' \right)'$$

has the same natural conjugate normal Wishart prior distribution. The prior moments of $\beta_{\tilde{S}_t}$ and the tightness hyperparameters around these moments have been set equal to those used in Section C.2. The prior distribution of c is the truncated normal distribution with mean equal to the mean of $Z_{i,t-1}$ and the standard deviation is adjusted to deliver the appropriate acceptance rate (between 25%-40%). The distribution of c is truncated between the 0.15 and 0.85 quantile of the empirical distribution of $Z_{i,t-1}$ to ensure that at least 15% of the observations lie in this regime. Similar to Engemann and Owyang (2010), the prior distribution used for γ is the gamma distribution with both hyperparameters equal to one.

F.12 Posterior estimation

This section briefly describes the steps of the Gibbs and Metropolis-Hasting sampling scheme used to derive the posterior distribution of the entire parameter vector. For more details please consult the studies of Chen and Lee (1995) and Lopes and Salazar (2006)

STEP 1 For c^{j-1} and γ^{j-1} \tilde{S}_t is constructed using (15) for TVAR or (16) for ST-VAR

STEP 2 Given \tilde{S}_t from STEP 1 we derive the OLS version of $\beta_{\tilde{S}_t}$ ($\hat{\beta}_{\tilde{S}_t}$)

STEP 3 $\hat{\beta}_{\tilde{S}_t}$ is combined with the prior moments of $\beta_{\tilde{S}_t}$ to construct its posterior conditional moments, which are used to draw from the normal Wishart distribution

$$(\beta_{\tilde{S}_t}^j)$$



STEP 4 Given $\beta_{s_t}^j$, c^j and γ^j are generated by

$$c^j = c^{j-1} + \sigma_c u_{c,t}$$

$$\gamma^j = \gamma^{j-1} + \sigma_\gamma u_{\gamma,t}$$

STEP 5 If the ratio $\frac{L(Z_t; \beta_{s_t}^j, c^j, \gamma^j) p(\beta_{s_t}^j) p(c^j) p(\gamma^j)}{L(Z_t; \beta_{s_t}^{j-1}, c^{j-1}, \gamma^{j-1}) p(\beta_{s_t}^{j-1}) p(c^{j-1}) p(\gamma^{j-1})}$ is greater than a random variable generated by the uniform over the unit interval then the draw $\beta_{s_t}^j$, c^j and γ^j is accepted – set $c^{j-1} = c^j$ and $\gamma^{j-1} = \gamma^j$ and proceed to STEP 1. Otherwise the draw is discarded – $c^{j-1} = c^{j-1}$ and $\gamma^{j-1} = \gamma^{j-1}$ and proceed to STEP 1.

The values of σ_c and σ_γ have been calibrated to deliver an appropriate acceptance rate.

F.13 Calculating the marginal likelihood

The log likelihood of these models is available in analytical form. The posterior distribution is defined as $G(\hat{\Xi} \setminus Z_t) = G(\hat{B}, \hat{\Omega}, \hat{c}, \hat{\gamma})$. This density can be factored as

$$G(\hat{B}, \hat{\Omega}, \hat{c}, \hat{\gamma}) = G(\hat{B} \setminus \hat{\Omega}, \hat{c}, \hat{\gamma}) \times G(\hat{\Omega} \setminus \hat{c}, \hat{\gamma}) \times G(\hat{c} \setminus \hat{\gamma}) \times G(\setminus \hat{\gamma}) \quad (\mathbf{F-1})$$

The first two terms on the RHS of equation (F-1) can be approximated via extra Gibbs runs while the method in Chib and Jeliazkov (2001) is used to approximate the unknown densities in the last two terms.

Appendix G: Rolling and recursive VARs

We use the natural conjugate prior for the VAR described in equation (B-4) with $\tau = 10$ and $c = 1/10000$. Details on the posterior moments can be found in Banbura, Giannone and Reichlin (2010). An analytical expression for the marginal likelihood can be found in Carriero, Clark and Marcellino (2011).



Appendix H: Data for the FAVAR models

The data set used to estimate the FAVAR models is listed in Table G. Note that when estimating the model to forecast inflation, we include GDP growth and the short-term interest rate in X_{it} (see equation **(D-1)**) along with the variables in Table G. When estimating the model to forecast GDP growth we include inflation and the interest rate in X_{it} along with the variables in Table G. Similarly, GDP growth and inflation are added to the panel when estimating the model to forecast interest rates.



Table G: Data used to estimate FAVAR model. ONS denotes Office for National Statistics. IFS is International Financial Statistics. GFD is Global Financial Data

Variable no	Variable Name	Source	Transformation
1	General government: Final consumption expenditure	ONS	Log Difference
2	ESA95 output index: F: Construction:	ONS	Log Difference
3	Total exports	ONS	Log Difference
4	Total imports	ONS	Log Difference
5	Gross Fixed Capital Formation	ONS	Log Difference
6	IOP: Manufacturing	ONS	Log Difference
7	SA95 output index: Transport storage & communication	ONS	Log Difference
8	SA95 output index: Total	ONS	Log Difference
9	ESA95 output index: Distribution, hotels & catering; repairs	ONS	Log Difference
10	IOP: All production industries	ONS	Log Difference
11	IOP: Electricity, gas and water supply	ONS	Log Difference
12	IOP: Manuf of food, drink & tobacco	ONS	Log Difference
13	IOP: Manuf coke/petroleum prod/nuclear fuels	ONS	Log Difference
14	IOP: Manuf of chemicals & man-made fibres	ONS	Log Difference
15	Consumption	ONS	Log Difference
16	Trade balance	ONS	None
17	RPI total Food	ONS	Log Difference
18	RPI total non-food	ONS	Log Difference
19	RPI total all items other than seasonal food	ONS	Log Difference
20	GDP Deflator	ONS	Log Difference
21	Wages	ONS	Log Difference
22	Import prices	IFS	Log Difference
23	Export prices	IFS	Log Difference
24	M4 deposits	BOE	Log Difference
25	M4 lending	BOE	Log Difference
26	Real Nationwide house prices	Nationwide	Log Difference
27	Dividend yield	GFD	None
28	PE ratio	GFD	None
29	FTSE All-Share index	GFD	Log Difference
30	Pounds US dollar rate	GFD	Log Difference
31	Pounds euro rate	GFD	Log Difference
32	Pounds yen rate	GFD	Log Difference
33	NEER	GFD	Log Difference
34	Pounds Canadian dollar rate	GFD	Log Difference
35	Pounds Australian dollar rate	GFD	Log Difference
36	Corporate bond yield	GFD	None
37	Unemployment Rate	GFD	None
38	5-year govt bond yield	GFD	None
39	10-year govt bond yield	GFD	None
40	20-year govt bond yield	GFD	None
41	Commodity price index	GFD	Log Difference
42	Brent oil price	GFD	Log Difference
43	Industrial production index	GFD	Log Difference
44	United Kingdom composite leading indicators	GFD	Log Difference

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