



BANK OF ENGLAND

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International policy spillovers at the zero lower bound

Alex Haberis and Anna Lipińska

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International policy spillovers at the zero lower bound

Alex Haberis⁽¹⁾ and Anna Lipińska⁽²⁾

Abstract

In this paper, we consider how monetary policy in a large, foreign economy affects optimal monetary policy in a small open economy ('home') in response to a large global demand shock that pushes both economies to the zero lower bound (ZLB) on nominal interest rates. We show that the inability of foreign monetary policy to stabilise the foreign economy at the ZLB creates a spillover that affects how well the home policymaker is able to stabilise its own economy. We show that more stimulatory foreign policy worsens the home policymaker's trade-off between stabilising inflation and the output gap when home and foreign goods are close substitutes. This reflects the fact that looser foreign policy leads to a relatively more appreciated home real exchange rate, which induces large expenditure switching away from home goods when goods are highly substitutable — just at a time (at the ZLB) when home policy is trying to boost demand for home goods. When goods are not close substitutes the home policymaker's ability to stabilise the economy benefits from more stimulatory foreign policy.

Key words: Small open economy, policy trade-offs, trade structure, zero lower bound.

JEL classification: E58, F41, F42.

(1) Bank of England. Email: alex.haberis@bankofengland.co.uk

(2) Federal Reserve Board. Email: anna.lipinska@frb.gov

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Publications Group, Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 7601 4030 Fax +44 (0)20 7601 3298 email mapublications@bankofengland.co.uk

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Summary

In this paper, we are interested in how the policy of other central banks affects policy in a small open economy in the face of a large global demand shock that leads central banks internationally to cut rates to the zero lower bound (ZLB). Our interest in this issue comes from the policy response to the financial crisis that started in 2007/08. This hit many economies at the same time, leading to large declines in output during what has become known as the ‘Great Recession’. In response, central banks around the world cut policy rates to (close to) zero to offset the deflationary pressure associated with the collapse in demand.

The ZLB creates an interesting set of challenges for monetary policy. This is because the conventional options available to policymakers to stimulate the economy – further rate cuts – are not available. Past academic work has shown that this can lead to trade-offs for policymakers in terms of stabilising inflation and output. In the current conjuncture, with the crisis having led many of the world’s major central banks to cut policy rates to (or close to) the ZLB, the international dimension of these challenges is of particular interest – for instance, in terms of how policy overseas might create spillovers into the policy problem at home, which is the focus of this paper.

In practice, however, it is worth noting, that central banks greeted these challenges during the crisis with ‘unconventional’ quantitative policy action. In this paper, we do not look at unconventional monetary policy measures taken at the ZLB.

Instead, we adopt a more stylised framework for looking at monetary policy strategy, in line with previous research on monetary policy at the ZLB. We adopt this approach in part for its analytical convenience and in part because it allows us to couch our findings in terms of other work. In this framework, policy may be set either under ‘discretion’ or under ‘commitment’. Discretionary policy involves the policymaker taking the action in a given period that gives the best outcome in terms of stabilising inflation and output in *that* period. When following policy under commitment, the policymaker commits to the course of action for all periods that achieves the best stabilisation performance over time.



This is more powerful, because, if it is possible, policy can operate more effectively on expectations about the future, which under discretion are constrained by the belief that policymakers will choose short-sighted policies (the famous ‘time consistency’ problem). In general, at the zero bound, commitment policy allows the policymaker to provide greater stimulus to the economy, which leads to improved stabilisation of inflation and output relative to a purely discretionary policy.

To analyse the issue, we use a model in which there are two countries: a large economy (which we refer to as ‘foreign’) and a small open economy (which we refer to as ‘home’). The foreign economy is large in the sense it is not affected by developments in the home economy, although developments in the foreign economy can affect the home economy.

In our results, we find that in response to a large global demand shock, when foreign policy follows a commitment strategy, this reduces the home policymaker’s ability to stabilise the home economy when home and foreign goods are close substitutes. This is because looser monetary policy in the foreign economy means the home real exchange rate is relatively appreciated compared to when the foreign policymaker sets policy under discretion. When there is a high degree of substitutability between goods, a stronger home real exchange rate induces large expenditure-switching effects away from home goods. This effect outweighs the impact on the demand for home goods from the higher level of foreign aggregate demand resulting from the looser stance of foreign monetary policy. Because our model is based on microeconomic foundations, we are able to work out how foreign policy affects social welfare at home. When goods are highly substitutable, home welfare is higher when foreign policy is set under discretion compared to commitment. In contrast, when goods are not close substitutes across countries, the opposite holds.



1 Introduction

How does monetary policy in a large, foreign economy affect optimal monetary policy in a small open economy in response to large global demand shocks that push both economies to the zero lower bound (ZLB) on nominal interest rates? Our interest in this question is motivated by the recent financial crisis and the monetary policy response to it. The crisis simultaneously hit many economies around the world, leading to large declines in output during what has become known as the ‘Great Recession’. In response, central banks internationally cut policy rates to (close to) zero in order to offset the deflationary pressure associated with the collapse in demand (as well as adopting other ‘unconventional’ quantitative measures).¹ This state of affairs – the majority of the world’s major economies’ being at the zero bound simultaneously – is unprecedented in recent times, and it naturally raises questions about how policies may interact at the ZLB.²

In this paper, we consider these issues through the lens of an open economy New Keynesian model along the lines of Gali and Monacelli (2005) and De Paoli (2009). The model has two economies: a small open economy (which we refer to as ‘home’) and a large, closed economy (which we refer to as ‘foreign’). The foreign economy is large in the sense that it is unaffected by developments in the home economy. Given its openness, the home economy is affected by developments in the foreign economy. To analyse monetary policy at the zero bound, we consider the case of a large demand shock that hits both economies simultaneously and causes their respective natural rates of interest to fall well below zero – in a sense this is a crude characterisation of the shock associated with Great Recession.³

The main conclusion of our analysis is that while welfare in the home economy is always higher when monetary policy follows a commitment strategy – it pays to be credible, consistent with the literature on optimal policy – the inability of foreign monetary policy

¹In this paper, we do not specifically consider the quantitative measures that have formed a large part of central banks’ response to the crisis.

²For popular policy writing on the topic see, eg the *Financial Times* article in October 2010 on ‘Global Implications of QE2’ by Gavyn Davies or the *Economist* article in November 2010 ‘It goes to the Fed’s motive’.

³A similar approach has been undertaken by Levin, Lopez-Salido, Nelson and Yun (2010).



to stabilise the foreign economy at the zero bound creates a spillover that affects how well the home policymaker is able to stabilise its own economy, even under commitment. This spillover can have a material impact on the welfare in the home economy.

The policy spillover arises because at the zero bound, policy is unable to stabilise perfectly the output gap and inflation. We show it is possible to isolate the impact of this spillover by writing the home economy's *IS* and Phillips curves as functions of the foreign output gap. Therefore, for the type of shock we consider, in the absence of the zero bound (or if the shocks were not large enough to push the natural rate below zero), when foreign policy is set optimally, the foreign output gap (and inflation) would be zero at all times and the spillover would not arise.

To analyse how the foreign policy spillover affects the home economy, we compare the home responses for alternative foreign policies that differ in the amount of stimulus they can deliver to the foreign economy. To do so, we compare the cases of foreign policy under commitment and discretion. Past work in a closed economy context has shown that, at the zero bound, commitment policy is able to effectively borrow stimulus from the future to stabilise the economy today Jung, Teranishi and Watanabe (2005). We characterise how these different foreign policies affect optimal policy for the home central bank, assuming home policy minimises welfare-based losses, and considering the cases of home policy under commitment and discretion.

We show that when foreign policy is more stimulatory (ie when the policymaker follows a commitment strategy), this reduces the home policymaker's ability to stabilise the home economy when home and foreign goods are substitutes. This is because looser monetary policy in the foreign economy means the home real exchange rate is relatively appreciated compared to when the foreign policymaker sets policy under discretion. When there is a high degree of substitutability between goods, a stronger home real exchange rate induces large expenditure-switching effects away from home goods. This effect outweighs the impact on the demand for home goods from the higher level of foreign aggregate demand resulting from the looser stance of foreign monetary policy. That is, the commitment policy of the foreign central bank produces a 'beggar-thy-neighbour' effect, widely discussed in the literature on international



economics, on the home economy. This result contrasts with analysis of competitive devaluations in Corsetti and Pesenti (2001). They find that monetary expansion in one country results in welfare gains for its trading partners (when goods are strong substitutes). However, in our case, the fall in demand for home goods associated with the expenditure-switching effect adds to the shortfall in demand that results from home policymaker's inability to loosen sufficiently in the short term on account of the ZLB. In response, the home policymaker keeps policy looser for longer. When goods are not close substitutes, the opposite holds.

We find that the *beggar-thy-neighbour* effect on the home economy from foreign commitment policy when goods are close substitutes results in greater losses for the home economy irrespective of whether home policy follows commitment or discretion. By contrast, when home and foreign goods are complements for home consumers the results go the other way. The home economy is better off when foreign monetary policy is looser because the boost the home economy gets from the higher level of foreign aggregate demand dominates the expenditure-switching away from home goods induced by the stronger home real exchange rate associated with looser monetary foreign policy.

The literature on monetary policy at the ZLB has concentrated mainly on closed economies.⁴ The main finding of these papers is that, while discretionary policy is very costly, optimal commitment, which involves keeping interest rates at the ZLB for longer (than the duration of the shock), can improve macroeconomic stability. A recent paper by Levin *et al* (2010) argues that the costs under commitment policy can also be big for a sizable shock, such as the one associated with the recent crisis and 'Great Recession'. This means that international spillovers coming from the inability of monetary policy to stabilise the economy will be big, even under commitment. In our paper, we exploit this issue by adopting the size of the shock studied in Levin *et al* (2010).

As far as policy in open economies is concerned, Svensson (2001, 2003), Coenen and Wieland (2003), and Nakajima (2008) study the zero interest rate policy in an environment where a single country hits the ZLB. Fujiwara, Sudo and Teranishi (2010) analyses the optimal co-ordination policy in a two-country world faced with the global

⁴See Adam and Bili (2006, 2007), Eggertsson and Woodford (2003), Jung *et al* (2005), Nakov (2008).

shock that leads both countries to the ZLB. The authors find that the nature of co-ordination policy depends on substitutability of traded goods, since this parameter determines the size of international spillovers. The key difference between the work of Fujiwara *et al* (2010) and our analysis is that Fujiwara *et al* (2010) consider policy co-ordination, whereas we study unco-ordinated policies. In addition, they consider a two-country model, while we focus on a small open economy (ie the limiting case of a two-country model). Related to this, a recent Brookings Report on Rethinking Central Banking (Eichengreen *et al* (2011)) argues that monetary spillovers at the ZLB should be internalised in a co-ordinated global monetary policy. Bodenstein, Erceg and Guerrieri (2009) and Erceg and Linde (2010) study the effects of foreign shocks in an open economy when it is at the ZLB. Both papers find, that in this situation, the effects of foreign shocks are usually amplified. This is because, at the ZLB, monetary policy is constrained and cannot provide the necessary stimulus to its economy. Interestingly, Bodenstein *et al* (2009) also show that the spillover effects of foreign shocks do not seem to be much affected by foreign monetary policy. They argue that, although the ZLB makes foreign output fall by more in response to a negative shock, it also reduces the associated home appreciation. Thus the ultimate effect on home output is little changed when compared to the case of no ZLB. This result, as Bodenstein *et al* (2009) acknowledges, however, depends on the assumed trade price elasticity. In our framework, as alluded to above, we show how foreign policy can alter the nature of home policy at the zero bound and the losses suffered, and how these depend on the trade elasticity.

Finally, our paper is also related to Lipinska, Spange and Tanaka (2011), which studies international policy spillovers in case of global cost-push shocks. Lipinska *et al* (2011) shows that, in this case, policy trade-offs in a small open economy depend on foreign policy actions precisely because the cost-push shock introduces trade-offs for policymakers. Our paper shows more generally that foreign policy spillovers emerge in situations when foreign monetary policy is not able to stabilise its economy, which means it faces a policy trade-off between stabilising inflation and the output gap or is constrained by the ZLB because of the size of the shock.

The paper is organised as follows. In Section 2 we outline the model we use to conduct



the analysis; in Section 3 we explain the nature of international spillovers; in Section 4 we derive optimal policy under discretion and commitment in a small open economy under the ZLB; in Section 5 we analyse international spillovers at the ZLB under our benchmark case of home and foreign goods that are substitutes; in Section 6 we discuss alternative modelling assumptions that could have an impact on the nature of international spillovers; Section 7 concludes.

2 Model

In this section, we describe the model we use to conduct the analysis and its calibration.

2.1 *Small open economy model*

The analysis is conducted in a standard small open economy New Keynesian model along the lines of Gali and Monacelli (2005) and De Paoli (2009). The relative simplicity of the model is an advantage in that it means that the spillover effects due to the presence of the ZLB will be more transparent. In the model, there are two countries: ‘home’ (indexed by H) and ‘foreign’ (indexed by F). Representative households in each country supply labour to monopolistically competitive firms producing differentiated goods, and consume goods produced in both the home and foreign economies. Wages are assumed to be fully flexible, but prices are assumed to be sticky as in Calvo (1983). We adopt the approach of De Paoli (2009), which first solves for the equilibrium of the two-country model, and then takes the limit of the size of the home economy to zero. As a result, the home economy becomes a small open economy, whereas the foreign economy behaves like a closed economy: although developments in foreign variables affect the home economy, the opposite is not true. This is because the share of home goods in consumption basket of foreign households is infinitesimal. The model is in the class of cashless-limit economies, see eg Woodford (2003).

We assume there are two kinds of shocks in both economies: country-specific preference and productivity shocks.

Policymakers in both economies conduct welfare-based optimal policies. They can



conduct their policies under discretion or commitment.

2.1.1 Foreign economy

The foreign economy we consider is the same as the one used to analyse optimal policy at the ZLB in a closed economy setting Jung *et al* (2005); Levin *et al* (2010). The non-policy block of the model is represented by two equations: an *IS* curve and a New Keynesian Phillips Curve (*NKPC*), which are both derived from the optimising behaviour of households and firms:

$$\widehat{x}_{F,t}^W = \widehat{x}_{F,t+1}^W - \frac{1}{\rho}(i_{F,t} - \widehat{\pi}_{F,t+1} - r_{F,t}^n), \quad (1)$$

$$\widehat{\pi}_{F,t} = k(\rho + \eta)\widehat{x}_{F,t}^W + \beta\widehat{\pi}_{F,t+1}, \quad (2)$$

where $\frac{1}{\rho}$ is the interest rate elasticity of real aggregate demand, k is the slope of the *NKPC* and η is the inverse of elasticity of labour supply; $\widehat{\pi}_{F,t}$ is the foreign inflation rate, $i_{F,t}$ is the foreign short term nominal interest rate, $r_{F,t}^n$ is the foreign natural interest rate.⁵ The variable $\widehat{x}_{F,t}^W$ is the foreign welfare relevant output gap and is defined by $\widehat{x}_{F,t}^W \equiv \widehat{Y}_{F,t} - \widehat{Y}_{F,t}^T$, where $\widehat{Y}_{F,t}^T$ is the policymaker's welfare relevant target level of output (which is equal to the efficient level of output). Since we assume that there are no mark-up shocks and the steady state is efficient, the efficient level output will be equal to the level of flexible price output; that is, $\widehat{Y}_{F,t}^f = \widehat{Y}_{F,t}^T$ Benigno and Woodford (2005). Aside from the ZLB, the only distortion in the foreign economy arises from the existence of sticky prices. As a result, the welfare relevant and flexible price output gaps will coincide.

It can be shown that the foreign natural interest rate depends on demand shocks and productivity shocks:

⁵The natural rate is defined as the real rate in the flexible price equilibrium or equivalently the real rate consistent with zero inflation.

$$r_{F,t}^n = \frac{\rho\eta}{\rho + \eta} (\Delta \widehat{A}_{F,t+1} - \Delta \widehat{B}_{F,t+1}) + \frac{1 - \beta}{\beta}, \quad (3)$$

where β is the discount factor and $\frac{1-\beta}{\beta}$ is the real interest in the steady state.

2.1.2 Home economy

The home economy can also be represented by an *IS* curve and an *NKPC*:

$$\begin{aligned} \widehat{x}_{H,t}^W &= \widehat{x}_{H,t+1}^W - \frac{1 - \lambda}{\rho_\lambda} (i_{H,t} - \widehat{\pi}_{H,t+1}^{PPI} - r_{H,t}^{n,PPI}) \\ &\quad + \left(\frac{\rho(1 - \lambda) - \rho_\lambda}{\rho_\lambda} \right) \Delta \widehat{x}_{F,t+1}^W + \Delta \widehat{\zeta}_{H,t+1}^Y \end{aligned} \quad (4)$$

$$\begin{aligned} \widehat{\pi}_{H,t}^{PPI} &= \beta \widehat{\pi}_{H,t+1}^{PPI} + k \left(\left(\frac{\eta(1 - \lambda) + \rho_\lambda}{1 - \lambda} \right) \widehat{x}_{H,t}^W + \left(\frac{\rho(1 - \lambda) - \rho_\lambda}{1 - \lambda} \right) \widehat{x}_{F,t}^W \right) \\ &\quad + k \left(\frac{\eta(1 - \lambda) + \rho_\lambda}{1 - \lambda} \right) \widehat{\zeta}_{H,t}^Y \end{aligned} \quad (5)$$

where λ is the degree of openness of home economy,⁶ θ is the intratemporal elasticity of substitution between home and foreign goods, $\widehat{\pi}_{H,t}^{PPI}$ is the (PPI) inflation rate, $i_{H,t}$ is the home short term nominal interest rate and $r_{H,t}^{n,PPI}$ is the home natural real interest rate, defined in terms of PPI.⁷ The variable $\widehat{x}_{H,t}^W$ is the home welfare-relevant output gap, which is the difference between the actual level of output, $\widehat{Y}_{H,t}$, and its welfare relevant target (equivalently, efficient) level, $\widehat{Y}_{H,t}^T$. The variable $\widehat{\zeta}_{H,t}^Y \equiv \widehat{Y}_{H,t}^T - \widehat{Y}_{H,t}^f$ denotes the difference between the levels of efficient and flexible price output. As with the closed economy, we set the steady state mark-up to a level consistent with an efficient steady

⁶Similarly to De Paoli (2009) this parameter also governs the degree of home bias in Home economy, ie the level of home bias is given by $(1 - \lambda)$.

⁷The natural real interest rate in terms of CPI inflation is given by $r_{H,t}^n = i_t - \widehat{\pi}_{t+1}$, where CPI inflation is defined as

$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{\lambda}{1-\lambda} \Delta \widehat{R}\widehat{S}_t$ and $\Delta \widehat{R}\widehat{S}_t$ is a change in the real exchange rate. Therefore, the natural real rate in terms of PPI inflation is $r_{H,t}^{n,PPI} = i_t - \widehat{\pi}_{t+1} + \frac{\lambda}{1-\lambda} \Delta \widehat{R}\widehat{S}_{t+1}$.

state (that is, $\bar{\mu} = \frac{1}{1-\lambda}$, as shown in De Paoli (2009)). Furthermore, we assume that there are no mark-up shocks in the home economy.

However, in the home economy, even if there are no cost-push shocks and the steady state is undistorted, it will not necessarily be the case that the target level of output will be equal to its flexible price counterpart (De Paoli (2009)). This is because, in the small open economy, in addition to the distortion introduced by sticky prices, there is an external distortion that leads to inefficient fluctuations in the terms of trade - referred to as the ‘terms of trade externality’. This externality arises when home and foreign goods are not perfect substitutes for home consumers. When this is the case, a social planner in the home economy may be able to take advantage of a degree of monopoly power in the supply of home goods on world markets to improve home welfare. In general, this external distortion generates endogenous fluctuations in the variable $\widehat{\zeta}_{H,t}^Y$. These fluctuations rise to a trade-off for the home policymaker between stabilising prices and stabilising the welfare relevant output gap; that is, pure price stability is no longer optimal for the small open economy’s policymaker (De Paoli (2009)).

In the home economy, inflation and the output gap additionally depend on developments in the foreign output gap. This dependence is governed by parameter $\rho_\lambda \equiv \frac{\rho(1-\lambda)}{(\rho\theta-1)\lambda(2-\lambda)+1}$. When $\rho\theta = 1$ or if $\lambda = 0$, $\rho_\lambda = \rho(1-\lambda)$. This implies that the foreign output gap terms disappear from the equations (4) and (5). When $\rho\theta = 1$ it will also be the case that the term $\widehat{\zeta}_{H,t}^Y$ is always equal to zero. Therefore, for $\rho\theta = 1$, equations (4) and (5) collapse to those for the closed economy.

The home natural interest rate ($r_{H,t}^{n, PPI}$) depends on both home shocks and foreign shocks; it can be shown to depend on the foreign natural interest rate and differences between home and foreign demand and supply shocks (this captures movements of the real exchange rate in the flexible price equilibrium):

$$r_{H,t}^{n, PPI} = r_{F,t}^n + \frac{\rho_\lambda \eta}{(\rho_\lambda + \eta(1-\lambda))} (\Delta (\widehat{A}_{H,t+1} - \widehat{A}_{F,t+1}) - (1-\lambda) \Delta (\widehat{B}_{H,t+1} - \widehat{B}_{F,t+1})) . \quad (6)$$

Table A: Model parameters

Parameter	Value
Intertemporal elasticity of substitution (ρ^{-1})	1
Intratemporal elasticity of substitution (θ)	3
Frisch elasticity of labour supply (η^{-1})	0.47^{-1}
Degree of openness (λ)	0.5
Subjective discount factor (β)	0.99
Elasticity of substitution across the differentiated products (σ)	10
Probability of not being able to reset price (α)	0.66
$k = (1 - \alpha\beta) (1 - \alpha) / \alpha (1 + \sigma \eta)$	
$k^* = (1 - \alpha^*\beta) (1 - \alpha^*) / \alpha^* (1 + \sigma \eta)$	

Furthermore, if home and foreign shocks are perfectly correlated then domestic and foreign natural interest rate are equalised and $r_{H,t}^{n, PPI} = r_{F,t}^n$.⁸

The values of model parameters are presented in Table A and follow closely De Paoli (2009).

3 Nature of spillovers

3.1 How do foreign developments affect the home economy?

In this paper we consider the effects of a global shock, ie a negative shock that hits both the home and foreign economies in the same way: $r_{H,t}^{n, PPI} = r_{F,t}^n$. Developments in the foreign economy can affect the home economy through two channels: first, the home economy is affected by a real spillover; second, the home economy is affected by a spillover from foreign monetary policy.

The real spillover comes from the fact that the natural real interest rate in the home economy will be affected by shocks in the foreign economy, reflecting the assumption that a proportion of total demand for home goods and services is from foreign consumers and that home consumers consume both home and foreign goods.

The policy spillover arises for two reasons. First, due to the foreign output gap's

⁸When home and foreign shocks are perfectly correlated the real exchange rate does not change in the flexible price equilibrium.

influence over the home output gap and inflation, which is evident from the home economy's *IS* curve and *NKPC*. To the extent that in our model the foreign central bank determines the size and time path of the foreign output gap, this influence reflects foreign policy. Second, if foreign policy fails to bring the level of output in the foreign economy in line with its flexible price counterpart, this will generate fluctuations in the variable $\widehat{\zeta}_{H,t}^Y$, even for global shocks. In practice, however, we will show that the latter component of the policy spillover is small. Neither of the components of the spillover channel would be present in response to efficient shocks if foreign policy were unconstrained by the zero bound (eg, if the shocks were small) and set optimally (so that the output gap remained closed at all times).

When the shocks are symmetric shocks and small (or equivalently if the zero bound does not represent a constraint on policy), monetary policy in both economies would optimally adjust nominal interest rates in line with the change in the natural rate, thereby stabilising the output gap and inflation. In this case, neither component of the foreign policy spillover would be present. And because the shock is global, the real exchange rate would not need to adjust. Therefore, the efficient and flexible price allocations in the home economy would coincide. This means that optimal policy at home would simply involve adjusting the nominal rate in line with the natural rate. This is consistent with Benigno and Benigno (2006), who show that there are no gains from co-ordination in response to symmetric shocks when policy is set optimally. However, if the shock occurs only in the foreign economy, this would affect both the flexible price and efficient levels of output in the home economy such that the two output levels diverged, even if foreign policy perfectly stabilised the foreign output gap. From the home Phillips curve, it is clear that in this case, a home policy response would be warranted, and that, in general, it would not be possible to stabilise home inflation and the welfare relevant output gap at the same time.

3.2 *What is the nature of the policy spillover?*

In this section, we discuss the nature of the policy spillover. Our focus is on the component of the spillover due to the impact of the foreign output gap on the home output gap and inflation, given its greater significance to our results. The nature of the

spillover depends on the substitutability of home and foreign goods for home households (ie, if $\rho\theta > 1$ (< 1), home and foreign goods are substitutes (complements) in the utility and $\rho(1 - \lambda) > \rho_\lambda$ ($\rho(1 - \lambda) < \rho_\lambda$)).

When goods are substitutes (complements), home inflation is increasing (decreasing) in the foreign output gap. These differences arise because foreign variables affect home real marginal costs in two opposing ways. Real marginal cost in the home economy will depend on the real wage demanded by households in units of home production and home productivity. The influence of foreign variables on home real marginal cost therefore results from their impact on the real wage, given that home productivity is determined by home technology.

Considering the effects of a foreign monetary contraction sheds light on the opposing effects. In response to the contraction, foreign output falls, leading to a negative foreign output gap, and the home real exchange rate depreciates. On the one hand, the fall in foreign output, for a given level of home output (and hence consumption of home goods), reduces home consumption of foreign goods. This reduces home consumption overall, thereby raising the marginal utility of consumption. Given labour demand, in order to restore the ratio of the marginal utilities of consumption and leisure, households reduce the amount of time spent as leisure - that is, they increase their labour supply. This pushes down real wages and hence marginal costs. On the other hand, the real depreciation reduces the value of home production in units of home consumption, leading households to supply less labour. This pushes up real marginal costs. For substitutes (complements), the real exchange rate adjustment is smaller (bigger) so the negative foreign output gap has a negative (positive) effect on home real marginal costs and hence inflation.

The home output gap is decreasing (increasing) in the foreign output gap for substitutes (complements). Again, the intuition can be understood by considering the effects of a foreign monetary contraction. The decrease in foreign demand directly decreases demand for home output – referred to as the aggregate demand effect by Corsetti and Pesenti (2001). But the associated real depreciation induces expenditure-switching that raises demand for home output – the expenditure-switching effect. For substitutes

(complements), the expenditure-switching effect dominates (is dominated by) the aggregate demand effect, and overall demand for home output rises (falls), leading to a positive (negative) home output gap.

4 Optimal monetary policy in a small open economy

In this section, we outline the problem facing the monetary policy makers in the home and foreign economies and characterise the solutions for optimal policy under discretion and commitment in the small open economy.

4.1 Objective of monetary policy

The objective functions of the home and foreign central banks can be derived from the utility functions of households in their respective economies. In the foreign economy, the central bank's loss function can be expressed as:

$$L_{F,t_0} = \frac{1}{2} \bar{C}^{1-\rho} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\omega_y^F (\hat{x}_{F,t}^W)^2 + \omega_\pi^F (\hat{\pi}_{F,t})^2 \right] + \text{so tip} \quad (7)$$

Given the efficiency of the steady state and the absence of mark-up shocks, sticky prices represent the sole source of distortions in the foreign economy. These give rise to the welfare losses that the policy maker aims to minimise.

Following De Paoli (2009), the loss function for the policy maker in the small open economy can be expressed as:

$$L_{H,t_0} = \frac{1}{2} \bar{C}^{1-\rho} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\omega_y^H (\hat{x}_{H,t}^W)^2 + \omega_\pi^H (\hat{\pi}_{H,t}^{PPI})^2 \right] + \text{so tip} \quad (8)$$

The weights the central bank assigns to the welfare relevant output and PPI inflation (ω_y^H and ω_π^H , respectively) are functions of the structural parameters of the model.⁹

⁹The expression for the loss function in De Paoli (2009) includes the welfare relevant real exchange rate gap, illustrating the

As discussed above, given our assumptions about the efficiency of the steady state and the absence of mark-up shocks, the home economy is affected by two distortions: sticky prices and the terms of trade externality. Optimal policy in the home economy, therefore, aims to minimise the influence of these two distortions.

In our set-up, nominal interest rates cannot be negative - that is, there is a ZLB:

$$i_t^j \geq 0. \quad (9)$$

Furthermore, the central banks are assumed to be able to adopt perfectly credible policies. They are also assumed not to have access to quantitative measures such as asset purchases when nominal interest rates are zero.

In what follows, we will show that the path of inflation and output gap determined by the optimal policy in a small open economy differs from the path of these variables in the closed economy in three ways. First, the path of inflation and output gap depends on the degree of openness and substitutability of home and foreign goods. Second, it also depends on the path of the foreign output gap. Third, the home central bank's objectives are different insofar as the efficient level of output differs from the flexible price output level.

4.2 *Optimal policy under discretion*

4.2.1 *Optimisation*

The central bank in the home economy minimises (8) with respect to the economy's structural equations ((4) and (5)) and the non-negativity constraint on nominal interest rates (9). Under discretion, the central bank re-optimises each period. The optimisation can be represented by the following Lagrangian:

fact that the terms of trade externality leads the policy maker to optimally reduce inefficient fluctuations in the real exchange rate. Our formulation is equivalent, although we have eliminated the real exchange rate gap term. This implies that the target output level will differ in our formulation compared to if we had written the problem in terms of the real exchange rate gap also. The optimal paths for nominal interest rates, inflation and output remain unchanged, however.

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2} \bar{C}^{1-\rho} E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\omega_y^H (\hat{x}_{H,t}^W)^2 + \omega_\pi^H (\hat{\pi}_{H,t}^{PPI})^2] + \text{soitip} \right. & (10) \\
& + 2\phi_{1,t} \left[-\Delta \hat{x}_{H,t+1}^W - \Delta (\hat{Y}_{H,t}^T - \hat{Y}_{H,t}^f) - \frac{\rho(1-\lambda) - \rho_\lambda}{\rho_\lambda} \Delta \hat{x}_{F,t+1} \right. \\
& \left. \left. + \frac{1-\lambda}{\rho_\lambda} (i_{H,t} - \hat{\pi}_{H,t+1}^{PPI} - r_t^{n,PPI}) \right] \right. \\
& + 2\phi_{2,t} [\hat{\pi}_{H,t}^{PPI} - \beta \hat{\pi}_{H,t+1}^{PPI} \\
& - k \left(\left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right) \hat{x}_{H,t}^W + \left(\frac{\rho(1-\lambda) - \rho_\lambda}{1-\lambda} \right) \hat{x}_{F,t} \right) \\
& \left. \left. - k \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right) \hat{\zeta}_{H,t}^Y \right] \right\}
\end{aligned}$$

The first-order conditions with respect to $\hat{\pi}_{H,t}^H$, $\hat{x}_{H,t}^W$ and $i_{H,t}$ are as follows:

$$\omega_y^H \hat{x}_{H,t}^W + \phi_{1,t} - k \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right) \phi_{2,t} = 0 \quad (11)$$

$$\omega_\pi^H \hat{\pi}_{H,t}^{PPI} + \phi_{2,t} = 0 \quad (12)$$

$$\hat{i}_{H,t} \phi_{1,t} = 0 \quad (13)$$

$$\hat{i}_{H,t} \geq 0 \quad (14)$$

$$\phi_{1,t} \geq 0 \quad (15)$$

where $\phi_{1,t}$ and $\phi_{2,t}$ are the Lagrange multipliers on the constraints. The form of the first-order conditions (FOCs) is broadly the same as for a closed economy. Equation (13) and inequalities (14) and (15) are the Kuhn-Tucker conditions for the non-negativity constraint on the nominal interest rate. When the nominal interest rate is zero, from (14) and (15), it must be the case that $\phi_{1,t}$ is strictly positive (that is, (9) is binding). Similarly, when the nominal interest rate is positive, $\phi_{1,t}$ will be equal to zero.

4.2.2 Dynamic path

The dynamic path for the endogenous variables in the home economy is characterised by two phases.¹⁰ In the first phase, the nominal interest rate is equal to zero (the non-negativity constraint binds, $\phi_{1,t} > 0$). Since the shock to the natural rate is assumed to dissipate over time, it will gradually converge to its steady-state value. As a result, the endogenous variables also converge to their (interior solution) steady-state values.¹¹ In the case of the nominal interest rate, this is strictly greater than zero; therefore, at some point the non-negativity constraint on the nominal interest rate will cease to bind and it will become positive. The final period for which the non-negativity constraint on the nominal interest rate is binding we denote by T^d (that is, $i_{H,T^d} = 0$ and $i_{H,T^d+1} > 0$). From the *IS* curve and *NKPC*, the dynamics of the welfare relevant output gap and inflation in the first phase are governed by the following difference equation:

$$\mathbf{z}_{H,t+1} = \mathbf{A}\mathbf{z}_{H,t} - \mathbf{a}r_{H,t}^{n, PPI} - \mathbf{B}_0\mathbf{v}_{t+1} - \mathbf{B}_1\mathbf{v}_t$$

where $\mathbf{z}_{H,t} = \begin{bmatrix} \widehat{\pi}_{H,t}^{PPI} & \widehat{x}_{H,t}^W \end{bmatrix}'$ and $\mathbf{v}_t = \begin{bmatrix} \widehat{\zeta}_{H,t}^Y & \widehat{x}_{F,t} \end{bmatrix}'$. This can be solved forward to give a unique bounded solution for the home welfare relevant output gap and inflation:

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} \mathbf{a}r_{H,k}^{n, PPI} + \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_0\mathbf{v}_{k+1} + \mathbf{B}_1\mathbf{v}_k) + \mathbf{A}^{-(T^d-t+1)} \mathbf{z}_{H,T^d+1}$$

This differs from the solution for the closed economy in several ways. First, the coefficient matrices \mathbf{A} and \mathbf{a} will be different to their closed economy counterparts insofar as the parameters of the home economy depend on the degree of openness and substitutability of home and foreign goods. Second, the paths of the home variables depend on current and future values of the flexible price and efficient levels of home output and the foreign output gap (that is, the elements of the vector \mathbf{v}_t). Third, it will not necessarily be the case that $\mathbf{z}_{H,T^d+1} = 0$, in contrast to the equivalent case for the

¹⁰Details are given in Appendix A.

¹¹The steady state under discretion is described in Appendix A.

closed economy: even once the natural rate has become positive, it may not necessarily be optimal to set $i_{H,t} = r_{H,t}^{n, PPI}$.

In the second phase, $t = T^d + 1, \dots$, the nominal interest rate is positive and the Lagrange multiplier on the non-negativity constraint is zero $\phi_{1,T^d+1} = \phi_{1,T^d+2} = \dots = 0$. Using this fact, first-order conditions (11) and (12) and the economy's structural equations (4) and (5), it is possible to obtain a unique bounded solution for the home output and inflation, which is given by:

$$\mathbf{z}_{H,t} = \begin{bmatrix} 1 \\ -\frac{\omega_{\pi}^H}{\omega_y^H} k \left(\frac{\eta(1-\lambda) + \rho_{\lambda}}{1-\lambda} \right) \end{bmatrix} \frac{1}{\beta} \sum_{k=t}^{\infty} \Psi_1^{-(k-t+1)} \mathbf{d}'_1 v_k,$$

$$\text{where } \Psi_1 = \frac{\omega_y^H \beta}{\omega_y^H + \omega_{\pi}^H k^2 \left(\frac{\eta(1-\lambda) + \rho_{\lambda}}{1-\lambda} \right)^2}, \mathbf{d}_1 = \left[k \left(\frac{\eta(1-\lambda) + \rho_{\lambda}}{1-\lambda} \right) \quad k \left(\frac{(\rho(1-\lambda) - \rho_{\lambda})}{1-\lambda} \right) \right]'$$

4.3 Optimal policy under commitment

4.3.1 Optimisation

Under commitment, the home central bank's optimisation problem can be represented by the same Lagrangian (10) as for discretion. But in contrast to the case of discretionary policy, under commitment the policy maker is assumed to be able to choose the entire paths of inflation and the output gap to minimise its loss. The first-order conditions to the policy maker's problem with respect to $\widehat{\pi}_{H,t}^{PPI}$, $\widehat{x}_{H,t}^W$ and $i_{H,t}$ are as follows:

$$\omega_y^H \widehat{x}_{H,t}^W + \phi_{1,t} - \beta^{-1} \phi_{1,t-1} - k \left(\frac{\eta(1-\lambda) + \rho_{\lambda}}{1-\lambda} \right) \phi_{2,t} = 0 \quad (16)$$

$$\omega_{\pi}^H \widehat{\pi}_{H,t}^{PPI} - \frac{1-\lambda}{\rho_{\lambda}} \beta^{-1} \phi_{1,t-1} + \phi_{2,t} - \phi_{2,t-1} = 0 \quad (17)$$

$$\widehat{i}_t \phi_{1,t} = 0 \quad (18)$$

$$\widehat{i}_t \geq 0 \quad (19)$$

$$\phi_{1,t} \geq 0 \quad (20)$$

where $\phi_{1,t}$ and $\phi_{2,t}$ are the Lagrange multipliers on the constraints. The implications of the Kuhn-Tucker conditions (18), (19) and (20) for whether the nominal interest rate is positive or not are similar to those for discretionary policy.

4.3.2 Dynamic path

As in the closed economy case studied by Jung *et al* (2005), the dynamic path for the home economy is characterised by three distinct phases.¹²

In the first phase, the nominal interest rate is zero. Given that the system converges back to its steady state as the effects of the shock dissipate, the nominal interest rate will eventually be increased from zero.¹³ The final period in the first phase is denoted by T^c . After substituting for $i_{H,t} = 0$, the *IS* curve (4) and *NKPC* (5) give rise to a difference equation for $t = 1, \dots, T^c$ of the form:

$$\mathbf{z}_{H,t+1} = \mathbf{A}\mathbf{z}_{H,t} - \mathbf{a}r_{H,t}^{n, PPI} - \mathbf{B}_1\mathbf{v}_{t+1} - \mathbf{B}_2\mathbf{v}_t \quad (21)$$

By solving this forward, and using the FOCs (17) and (16), we obtain the following equations for the path of the endogenous variables in the home economy and Lagrange multipliers $\phi_t = \begin{bmatrix} \phi_{1,t} & \phi_{2,t} \end{bmatrix}'$ up to, and including, period T^c :

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} \mathbf{a}r_{H,k}^{n, PPI} + \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_1\mathbf{v}_{k+1} + \mathbf{B}_2\mathbf{v}_k) + \mathbf{A}^{-(T^c-t+1)} \mathbf{z}_{H,T^c+1} \quad (22)$$

$$\phi_t = \mathbf{C}\phi_{t-1} - \mathbf{D}_1\mathbf{z}_{H,t} + \mathbf{D}_2\mathbf{v}_t \quad (23)$$

¹²More details on the solution are given in the Appendix A.

¹³The steady state under commitment is described in Appendix A.

The form of these equations differs from the closed economy case insofar as the paths for the home welfare relevant output gap and inflation also depend on the paths of the gap between the efficient and flexible price levels of output and the foreign output gap. In addition, the elements of the coefficient matrices will differ from their closed economy counterparts since the parameters of the home economy depend on the home economy's degree of openness and substitutability of home and foreign goods.

The equations show that during the phase up to, and including, period T^c , the endogenous variables in the home economy depend on current and future values of the home natural rate, a terminal condition for the zero interest rate policy phase, \mathbf{z}_{H,T^c+1} (referred to by Levin *et al* (2010) as the 'forward guidance vector'), and current and future values of the foreign output gap. As discussed by Levin *et al* (2010), the forward guidance vector pins down the rational expectations equilibrium for the economy in the first phase.

The second phase occurs at period $T^c + 1$ and is distinguished from the first and third phases since in (16) $\phi_{1,t} = \phi_{1,T^c+1} = 0$, but $\phi_{1,t-1} = \phi_{1,T^c} > 0$. This phase effectively acts as a bridge between the other two phases: the first phase depends on the outcome in the second phase since this is when the value of forward guidance vector is determined. In turn, the forward guidance vector and ϕ_{2,T^c+1} depend on the values of the endogenous variables in period $T^c + 2$, which is the initial period of the final phase:

$$\begin{bmatrix} \mathbf{z}_{H,T^c+1} \\ \phi_{2,T^c+1} \end{bmatrix} = \mathbf{F}^{-1} \mathbf{B} \mathbf{z}_{H,T^c+2} + \mathbf{F}^{-1} \mathbf{H} \phi_{T^c} + \mathbf{F}^{-1} \mathbf{K} \nu_{T^c+1}. \quad (24)$$

These equations for phase 2 (obtained by substituting $\phi_{1,T^c+1} = 0$ into the first-order conditions to the policy problem, (17) and (16), and using (5) also differ from those for the closed economy only to the extent that they include the foreign output gap and the parameters depend on the home economy's degree of openness and substitutability of home and foreign goods.

In the final phase ($t = T^c + 2, \dots$), it will be the case that $\phi_{1,T^c+1} = \phi_{1,T^c+2} = \dots = 0$.

Using this fact, (17) and (16), and the structural equations of the economy, (5) and (4), we obtain a unique bounded solution given by:

$$\begin{bmatrix} \widehat{\pi}_{H,t}^{PPI} \\ \widehat{Y}_{H,t}^{Wgap} \\ \phi_{2,t} \end{bmatrix} = \begin{bmatrix} -\frac{\gamma_{12}}{\gamma_{11}} \\ \frac{k}{\omega_y^H} \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \lambda_2 \\ \lambda_2 \end{bmatrix} \phi_{2,t-1} + \begin{bmatrix} C_{1,t} \\ C_{2,t} \\ C_{3,t} \end{bmatrix} \quad (25)$$

where λ_2 is a real eigenvalue of an associated matrix and $\mathbf{c}_t = \begin{bmatrix} C_{1,t} & C_{2,t} & C_{3,t} \end{bmatrix}'$ is a function of current and future values of the foreign output gap and future values of the difference between the levels of efficient and flexible price output in the home economy. As before, this differs from the closed economy solution because the home output gap and inflation depend on developments in the foreign economy, the difference between the policy maker's target level of output and its flexible price level, the degree of openness and substitutability of home and foreign goods.

4.4 Model solution

To solve for the optimal path of the endogenous variables under discretion (commitment) it is necessary to determine the value of T^d (T^c). Following Jung *et al* (2005), we apply an algorithm that computes the path for $\phi_{1,t}$ for an initially high value for T^d (T^c), and then reduces T^d (T^c) by one and re-computes the path for $\phi_{1,t}$ until $\phi_{1,T^d} > 0$ and $\phi_{1,T^d+1} = 0$ ($\phi_{1,T^c} > 0$ and $\phi_{1,T^c+1} = 0$).

5 Results

In this section, we present the simulation results for discretionary and commitment policies in the home and foreign economy in response to a large adverse shock to the natural real rate of interest.

5.1 Policy experiment

We consider the impact of a large negative demand shock at period 0 that causes the natural rate to become negative in that period, following Jung *et al* (2005).¹⁴ We focus on the effects of a simultaneous negative demand shock in both the home and foreign economies (that is, a global shock). The shock is calibrated to match the ‘Great Recession’ shock considered by Levin *et al* (2010) and involves an 8 percentage point in annual terms fall in the home and foreign natural real rates relative to steady state on impact. The effect of the shock gradually dissipates from period 1 onwards deterministically, slowly returning the natural rate to its steady-state level with the persistence parameter equal to 0.85. As in Jung *et al* (2005), this assumption allows us to focus on the optimal path for nominal interest rates in response to the shock in a perfect foresight environment.¹⁵ In this setting, after the shock has occurred agents know that no further shocks will hit. They are therefore able to foresee perfectly the future paths of the natural rate and, consequently, all endogenous variables.

5.2 Benchmark case: home and foreign goods are substitutes

Our benchmark case is characterised by home and foreign goods that are substitutes for home consumers. Ultimately, whether home and foreign goods are substitutes or complements, and whether this changes depending on the horizon considered, is an empirical question that remains somewhat unresolved. That said, the literature survey of Obstfeld and Rogoff (2000b) finds an intratemporal elasticity of substitution between home and foreign goods that is quite high (in the neighbourhood of 5 or 6), consistent with them being substitutes. Given this finding, we focus on the case of substitutes in explaining the impact of the foreign policy spillover on the home economy. However, we also present findings under the alternative assumption that home and foreign goods are complements.

¹⁴Note that our results would not change if we considered a productivity shock.

¹⁵We chose perfect foresight given the main interest of our paper, ie international spillovers at the ZLB. In our opinion, stochastic environment would not change qualitative results of our paper.

5.2.1 Policies under discretion

We consider optimal policy under discretion for the home central bank in response to the global shock when the foreign central bank is also assumed to be following optimal policy with discretion. We find that the responses in the home and foreign economies are the same, reflecting the symmetry of the shock and the fact that policy is set under discretion in both economies: nominal rates are held at zero until the natural real rate is positive; thereafter, nominal rates are set equal to the natural real rate.

Chart 1 shows the responses of the home and foreign economies to the shock. It shows nominal and natural real interest rates (first panel), inflation (second panel), the output gap (third panel), short term and natural real interest rates (fourth panel), and the home real exchange rate (fifth panel).

The responses of the home and foreign economies are the same qualitatively as those in Jung *et al* (2005); quantitatively, the responses in Chart 1 are much larger compared to those presented in Jung *et al* (2005), reflecting the much bigger shock we consider. Nominal interest rates are cut to zero and held at that level until the natural real rate becomes positive. Thereafter, it is optimal each period for the central bank to set the nominal rate equal to the natural rate, stabilising the output gap and inflation. While policy rates are at the zero bound, real interest rates are well above the natural rate: nominal rates cannot be cut on account of the zero bound, and inflation expectations cannot be influenced due to the discretionary nature of policy. The monetary policy stance in both economies, therefore, is tight. As a result, a (very) wide negative output gap opens up, and there is a period of deflation.

How does the foreign policy spillover affect the home economy? To assess this we can consider the responses of the home economy assuming that foreign policy is unconstrained by the zero bound. In this case, the foreign real rate tracks the fall in the natural rate (ie the black dotted line), so that $\widehat{x}_{F,t} = 0$ and $\widehat{\zeta}_{H,t}^Y = 0$. The home responses consistent with this are shown by the green (dash-dotted) lines in the panels in Chart 1.¹⁶

¹⁶In this Chart, and all subsequent Charts, inflation for the home economy refers to PPI inflation, and the home real interest rate is in terms of CPI inflation.

The home economy experiences a much wider negative output gap and a bigger fall in inflation in the absence of the foreign policy spillover. In this case, there is a large appreciation of the home real exchange. This is because the stance of home monetary policy is much tighter than it is in the foreign economy: home real rates are significantly higher. The real appreciation gives rise to substantial expenditure-switching effects away from home goods, pushing down on home demand. Although the direct impact of the appreciation pushes up inflation, overall the fall in demand means that inflation falls by more compared to when there is a spillover from foreign policy.

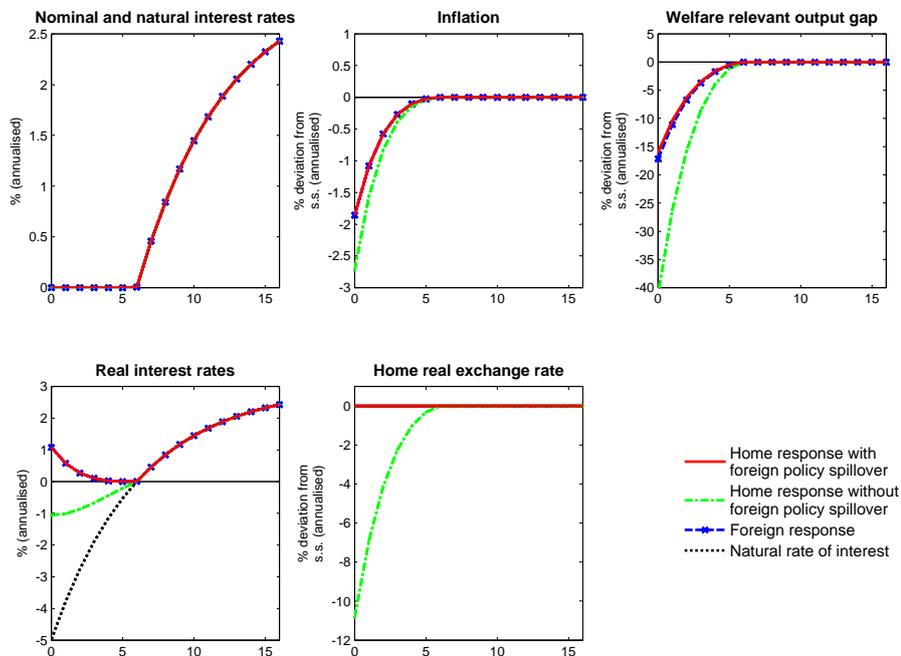
When foreign policy is constrained – and the home economy is affected by the policy spillover – the foreign economy effectively experiences a monetary contraction. Other things equal, this would induce the home real exchange rate to depreciate (as discussed above). However, other things are not equal, and the pressure on home real exchange rate (to depreciate) from the foreign effective monetary contraction is offset by the pressure on the home real exchange (to appreciate) from the effective monetary contraction at home. The offset is exact, leaving the home real exchange rate unchanged. The opposing forces cancel out on account of the symmetric nature of the shock and the fact that, when the policy makers in both economies follow a discretionary strategy, they are equally constrained in their ability to tackle the recession – neither has sufficient tools to stimulate their respective economies.

It is possible to shed light on the dynamics of home demand (as shown in Chart 1) by considering the impact of these opposing effects. When foreign policy is constrained, the resulting negative foreign output gap puts upward pressure on home demand: other things equal, it would be consistent with a depreciation of the home real exchange rate that would induce expenditure-switching towards home goods, boosting home demand. The extent of the boost to home demand from the foreign policy spillover is broadly consistent with the gap between the red (solid) and green (dash-dotted) lines. However, the negative impact on home demand from the effective contraction of home monetary policy offsets this boost, so that the home output gap is negative overall.

The difference between the red (solid) and green (dash-dotted) lines in Chart 1 shows that the home economy benefits from the foreign central bank's inability to stabilise the



Chart 1: Home and foreign response to a global shock for optimal policy under discretion



foreign output gap and inflation at the zero bound. If instead the foreign central bank were not facing the ZLB, the home economy would suffer real appreciation that would further reduce its output gap and cause stronger deflation. This result is evident in the realised welfare losses of the home economy (see Table B).¹⁷ The loss of the home economy under discretion is around three times smaller when the foreign economy also faces the ZLB constraint.

5.2.2 Policies under commitment

We consider optimal policy under commitment for the home central bank in response to the global shock when the foreign central bank also follows commitment policy. In this case, nominal rates in both economies are held at the zero bound for several periods after

¹⁷The losses are measured in terms of permanent shifts in the steady state consumption between given policy and the policy unconstrained by the ZLB. We calculated welfare losses as in Benigno and Lopez-Salido (2006).

Table B: Losses when home and foreign goods are substitutes (% of steady state consumption).

	Home losses	Foreign losses
Home policy set under discretion		
Foreign policy set with commitment	1.9	1
Foreign policy set with discretion	1.7	6
Foreign policy unconstrained by ZLB	4.6	0
Home policy set under commitment		
Foreign policy set with commitment	0.5	1
Foreign policy set with discretion	0.02	6
Foreign policy unconstrained by ZLB	1.2	0

the natural real rate has become positive (as in Jung *et al* (2005)). The spillover from foreign policy does not lengthen home economy's stay at the ZLB compared to what it would be if the foreign policy was not constrained by the ZLB. However, the spillover induces a more gradual tightening of home policy after leaving the ZLB. Under commitment, the responses of the home and foreign economy are no longer symmetric. This results from the fact that commitment policy represents a way of stimulating the economy at the zero bound – ie the policy makers have more tools at their disposal than under discretion. The asymmetry arises because, in using those tools, the home central bank needs to take account of what the foreign policy maker does, whereas the foreign policy maker – because the foreign economy is effectively closed – does not need to take account of the actions of the home policy maker. Chart 2 plots the responses to the shock of macroeconomic variables in the home and foreign economies under commitment policy.

As with discretionary policy, the responses of the foreign economy are the same qualitatively as those in Jung *et al* (2005). The foreign economy's responses under commitment are also much larger compared to those presented in Jung *et al* (2005), consistent with the larger shock we consider. In contrast to policy under discretion, nominal interest rates are held at zero until a number of periods after the natural rate has turned positive - that is, there is policy inertia. The policy maker avoids deflation today effectively by 'borrowing' stimulus from the future. By creating positive inflationary expectations, the central bank reduces the size of the negative output gap relative to

discretionary policy.

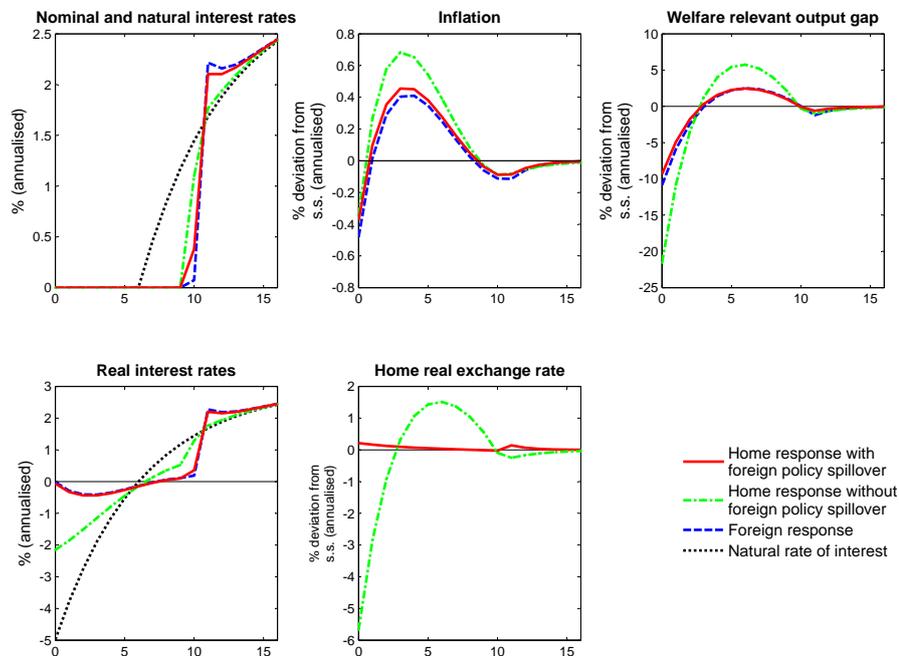
Optimal monetary policy in the home economy in response to the global shock involves a more gradual return of nominal rates to the natural real rate compared to in the foreign economy. This difference arises for two reasons. First, the home economy's openness alters the propagation of the demand shock. Second, the home economy is affected by the spillover from foreign policy. This spillover arises on account of the foreign output gap's influence on the home output gap and inflation. It also reflects the fact that when foreign policy is constrained by the zero bound it drives a wedge between the efficient and flexible price levels of output in the home economy, even though the shock we consider is symmetric.

In Chart 2, the green (dash-dotted) lines show the home economy's response to the shock excluding the effects of the foreign policy spillover (we have again assumed that foreign policy is unconstrained for these simulations). They show that the optimal response of home monetary policy to the effects of the demand shock would be similar to that in the foreign economy: cut nominal rate to zero and hold it there for several periods after the natural rate has become positive. Compared to the foreign economy, however, the home nominal rate would be tightened faster - there would be less policy inertia (the green (dashed-dotted) lines compared to the blue (dashed) in Chart 2). This is because, in the home economy, where the openness of the economy means the real exchange rate is an additional margin of adjustment, the central bank's policy inertia induces a real depreciation, after an initial appreciation. Since goods are assumed to be substitutes, the depreciation induces large expenditure-switching effects that boost home demand. Inflation is also increased. This provides sufficient stimulus to the home economy to allow policy to be tightened earlier than in the foreign economy.

The marginal impact of the foreign policy spillover can be inferred by comparing the red and green lines in Chart 2, where the red lines show the home response to both the demand shock and the foreign policy spillover. Home policy is still tightened faster than foreign policy, reflecting the boost from the depreciation of the home currency. But compared to the response to the effects of the demand shock alone, home policy is now tightened more gradually after leaving zero.



Chart 2: Home and foreign response to a global shock for optimal policy under commitment



As discussed above, the policy spillover operates through the foreign output gap's influence on home inflation and the home output gap, as well as through the wedge it creates between the flexible price and efficient levels of output in the home economy. In this model, for plausible calibrations, the impact of the latter is small (see Chart 6, that compares *ad hoc* and optimal policies, in Appendix B). Therefore, the effect of foreign policy on the home economy largely reflects the impact of foreign policy on the foreign output gap.

When the foreign central bank sets policy with inertia, this stimulates a positive foreign output gap. Under the assumption that goods are substitutes for home consumers, a positive foreign output gap would tend to weigh on home demand (and the home output gap), reflecting the dominance of the expenditure-switching effect over the aggregate demand effect. Therefore, the foreign central bank's policy inertia, by generating a

positive foreign output gap, acts to weigh on home demand, with the peak impact occurring broadly around the time the effects of the natural rate shock are beginning to wane. Absent this spillover, we have seen that home policy would be tightened faster. But when it is present, policy is tightened more gradually in order to provide sufficient offsetting stimulus to the home economy.

Another way to view the effects of foreign policy spillover is via the dynamics of the real exchange rate. In particular, when the foreign policy maker sets policy with inertia, it reduces the extent of the stimulatory real depreciation that home policy inertia is able to generate. This is because when foreign policy is unconstrained, the foreign real interest rate tracks the path of the natural real interest rate. Given home policy is constrained by the zero bound (the green lines), the reduction in the real interest rate the home central bank is able to generate is smaller, which is consistent with a sizable initial real appreciation. To the extent home policy inertia at the ZLB induces a reduction in the real interest rate to a level below the natural real interest rate after around seven to ten periods, this gives rise to a real exchange rate depreciation. Real depreciation stimulates home demand and leads to a sizable positive home output gap. However, when foreign policy is also constrained, the foreign central bank's policy inertia also induces a fall in the foreign real interest rate below the natural rate after around seven to ten periods also. This fall in the foreign real rate reduces the extent of the real depreciation that home policy inertia is able to generate. Therefore, home policy needs to be looser for longer in the presence of the foreign policy spillover.

As in the case of policy under discretion, under commitment, the home economy benefits when policy in the foreign economy is constrained by the ZLB. This benefit is evident in the lower losses the home economy suffers when the foreign policy spillover is present (Table B). When the foreign central bank can set its policy so that the foreign real rate tracks the natural rate and at the same time the home central bank is constrained by the zero bound, there are large and undesirable swings in the home real exchange rate. These fluctuations in home real exchange rate in turn give rise to large and undesirable swings in the home welfare relevant output gap and inflation.



5.2.3 Foreign policy design

In this section, we examine how differences in foreign policy affect the home economy for given home policies. In particular, we consider the home response when foreign policy follows commitment compared to when foreign policy follows discretion.

Chart 3: Home responses to a global shock for home policy under commitment

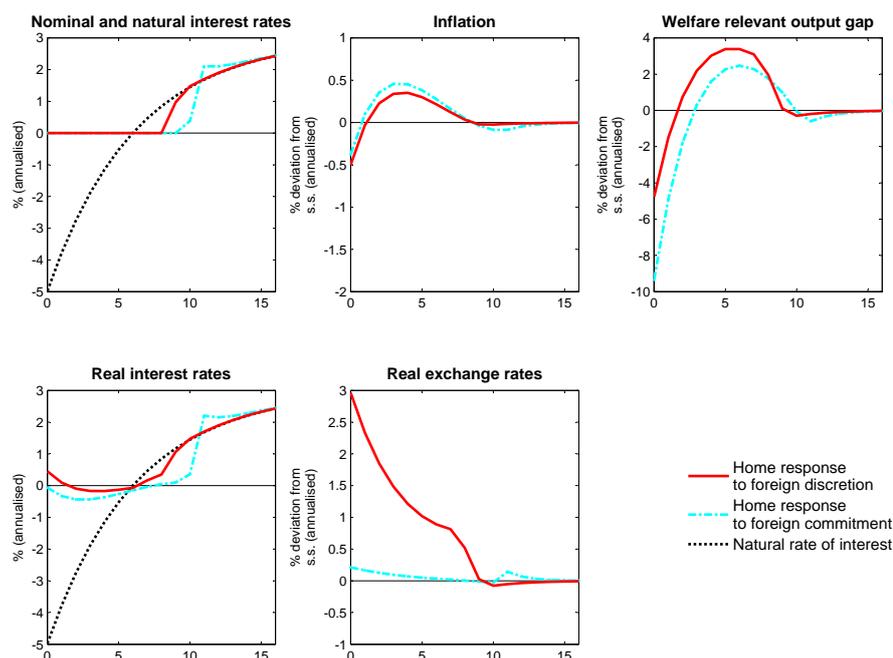


Chart 3 shows the home economy's response to the shock when home policy is set under commitment for the foreign policy maker under commitment and discretion. When home policy is set under commitment, there is less policy inertia when the foreign central bank sets discretionary policy compared to the case of foreign commitment policy. In the case of foreign discretionary policy, the home commitment policy induces a depreciation of the home real exchange rate, boosting home demand. Although the larger fall in foreign demand when foreign policy is discretionary gives rise to a larger aggregate demand effect, this is offset by the expenditure-switching effect. Due to the boost to home demand from the real depreciation, there is less need for stimulus through

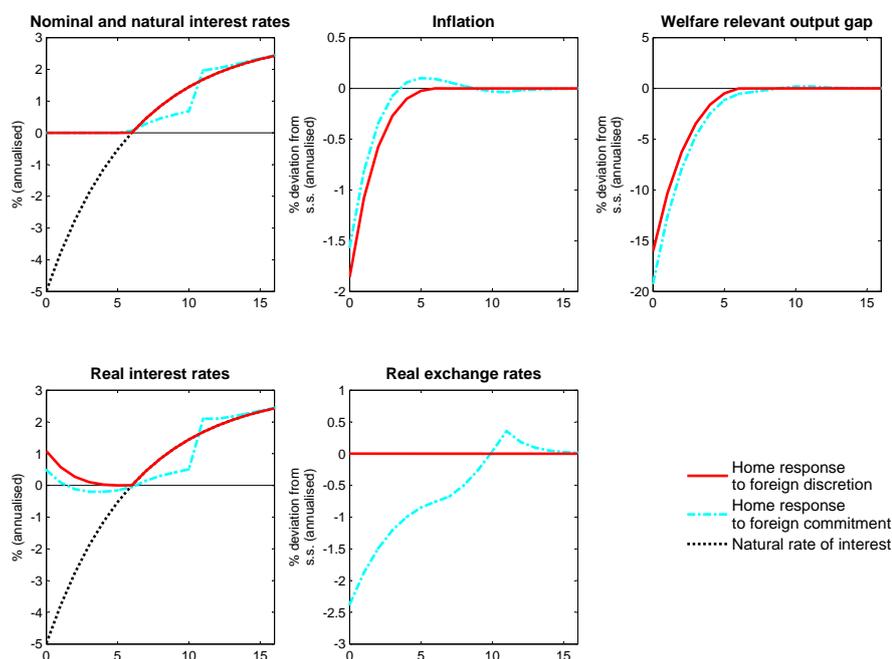
policy inertia by the home central bank. Nevertheless, nominal rates remain at the ZLB until after the natural rate has become positive, reflecting the deflationary impact of the negative demand shock and the real depreciation. In the case of foreign commitment policy, the foreign central bank's policy inertia reduces the extent of the real depreciation of the home exchange rate. Therefore, home policy needs to generate stimulus through greater inertia when foreign policy is set under commitment compared to the case of discretionary foreign policy.

The home economy's losses are larger the less constrained the foreign policy maker (Table B). That is, home losses are largest when foreign policy is completely unconstrained by the zero bound and smallest when the foreign policy maker is only able to follow discretionary policy. When the foreign policy is less constrained, this reduces the extent of the stimulatory real exchange rate depreciation home policy inertia is able to induce.

We can interpret our results also in light of the Svensson (2003)'s proposal of escaping from the liquidity trap through exchange rate depreciation. We show that if the shock is global then the ability of the home central bank to escape from the liquidity trap depends on foreign monetary policy design. Home commitment (which resembles Svensson (2003)'s proposal of an explicit central-bank commitment to a higher future price level) is very successful and brings very small losses only when foreign policy acts under discretion (see Table B).

When home policy is set under discretion, although policy is tightened earlier in the situation of foreign commitment (compared with foreign discretion), the tightening thereafter is much more gradual (Chart 4). The home central bank tightens more gradually when foreign policy is set under commitment because the foreign central bank's policy inertia induces an appreciation of the home real exchange rate. This home appreciation gives rise to an expenditure-switching effect that reduces home demand, which is absent when foreign policy is also discretionary. Under assumption of home and foreign goods being substitutes, this effect dominates the aggregate demand effect. As a result, overall demand for home output declines, giving rise to a wider home output gap. Furthermore, a boost in foreign output gap coming from the foreign commitment puts

Chart 4: Home responses to a global shock for home policy under discretion



upward pressure on home inflation. In sum, foreign commitment policy raises home inflation and widens the home output gap. Faced with this trade-off, home inflation overshoots its steady state level in order to reduce the costs in terms of real activity from the negative output gap.

Under discretion, as with commitment, losses in the home economy are smaller when the foreign economy is relatively more constrained (Table B). Again, this result reflects the extent to which home policy is able to influence the real exchange rate when the zero bound represents a constraint on policy. In addition, it is clear from Table B that home losses are greater when the home policy maker follows discretionary policy, consistent with usual findings in the literature.

Table C: Losses when home and foreign goods are complements (% of steady state consumption).

	Home losses	Foreign losses
Home policy set under discretion		
Foreign policy set with commitment	23.3	1
Foreign policy set with discretion	24.6	6
Foreign policy unconstrained by ZLB	18.9	0
Home policy set under commitment		
Foreign policy set with commitment	8.95	1
Foreign policy set with discretion	10.6	6
Foreign policy unconstrained by ZLB	17.3	0

5.3 Home and foreign goods are complements

When home and foreign goods are complements for home consumers, our findings are reversed: home losses are smaller when the foreign policy maker is able to commit. This finding is clear from Table C, which shows the losses assuming the intratemporal elasticity of substitution between home and foreign goods (θ) is 0.5. This finding arises from the difference in the impact of a foreign monetary contraction under this assumption: the aggregate demand effect dominates the expenditure-switching effect. Although the home economy still benefits from the expenditure-switching towards its goods induced by the real depreciation associated with a period of relatively tight foreign monetary policy, this is outweighed by the cost in terms of lower overall demand it suffers due to the lower aggregate demand in the foreign economy. Therefore, a foreign commitment strategy, by providing the foreign economy with greater stimulus, in turn provides the home economy with a bigger boost. This leads to smaller losses compared to when foreign policy is set with discretion.

6 Discussion

For the purposes of clarity of our analysis we consider a highly stylised model. There are several important alternative modelling assumptions that can have an effect on the nature of international spillovers. These are: imperfect pass-through, limited risk-sharing among countries and domestic frictions in the labour market, such as sticky wages. In

this section we will briefly outline how these alternative features of the model might affect our results.

First, our model is built under assumption that there is a full pass-through from exchange rate movements, given our assumption of producer currency pricing. This implies that when home and foreign goods are substitutes that the expenditure-switching effect resulting from the foreign output gap dynamics is strong and outweighs the aggregate demand effect. However if one takes into account that the pass-through from exchange rate movements to consumer prices may be limited – if we were to assume that there was local currency pricing¹⁸ – the expenditure-switching effect is reduced. As a result, the sign of international spillovers can be reversed. Empirically, there is evidence on some limits to the pass-through in the short run, contrary to the model of producer currency pricing. In the long run, however, pass-through is almost complete. But, at the same time, there is an empirical evidence indicating strong correlation between the exchange rate and terms of trade, which is not the case in models with local currency pricing.¹⁹ These findings point to short-comings in the way pricing decisions are typically built into models like the one we have considered in this paper.

Second, our model assumes that there is a full risk-sharing among countries. This risk-sharing implies that countries can run trade imbalances and finance an increase in their consumption by borrowing from abroad. However, if one assumes that the degree of financial integration between countries is smaller, then the trade balances will be kept closer to zero via the adjustment of quantities produced. As a result, the size of international spillovers will be reduced.

Finally, our model assumes a frictionless labour market. This assumption is important in driving the effects of international spillovers on home inflation. Note that changes in the foreign output gap affect home inflation via a change in home labour supply which in turn affects the real marginal cost. However, if one assumed instead that wages were sticky, then the effect of changes in home labour supply on the real marginal cost would be much more limited. And thus home inflation would be little changed. Moreover, an

¹⁸See eg Bacchetta and van Wincoop (2000).

¹⁹See e.g. Obstfeld and Rogoff (2000a).

additional friction in the labour market will make it harder for the central bank under commitment to engineer inflation expectations in order to boost the economy.

7 Conclusion

In this paper we show that in response to a global shock that pushes the natural rate into negative territory, the inability of monetary policy in a large foreign economy to stabilise the output gap and inflation at the zero lower bound creates a spillover for a small open economy. The resultant negative output gap in the foreign economy creates inefficient fluctuations in the home output gap and inflation. This finding - that international spillovers at the ZLB have an inefficient component - has not previously been explored in the literature. The spillover from foreign policy can alter optimal monetary policy in the home economy and affect the welfare losses agents in the home economy suffer as a result of the shock. The way the spillover affects the home economy depends on the home economy's structure - whether home and foreign goods are substitutes or complements for home consumers. The size of the spillover will depend on policy design in the foreign economy - whether foreign monetary policy is set under commitment or discretion.

The existence of policy spillover at the ZLB in our stylised model suggests that there may be gains from international co-ordination of monetary policy at the ZLB, as in the case of inefficient shocks such as mark-up shocks (Benigno and Benigno (2006)). Thus, it would be interesting to investigate this issue further by comparing fully optimal co-ordinated and unco-ordinated policies of big open economies at the ZLB.



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Appendix A: Optimal policy

In this section, we provide greater detail on the solutions for optimal policy under discretion and commitment in the small open economy.

A.1 Discretion

A.1.1 Steady state

In the home economy's steady state, inflation, the welfare relevant and flexible price output and real exchange rate gaps, the nominal interest rate, the natural real rate and the Lagrange multipliers are equal to their long-run values (indexed by a subscript ∞):

$\widehat{\pi}_{H,t}^{PPI} = \widehat{\pi}_{H,t+1}^{PPI} = \widehat{\pi}_{H,\infty}^{PPI}$; $\widehat{x}_{H,t}^W = \widehat{x}_{H,t+1}^W = \widehat{x}_{H,\infty}^W$; $\widehat{x}_{H,t} = \widehat{x}_{H,t+1} = \widehat{x}_{H,\infty}$;
 $\widehat{RS}_{H,t}^W = \widehat{RS}_{H,t+1}^W = \widehat{RS}_{H,\infty}^W$; $i_{H,t} = i_{H,\infty}$; $r_{H,t}^n = r_{H,\infty}^n$; $\phi_{1,t} = \phi_{1,\infty}$; $\phi_{2,t} = \phi_{2,\infty}$. Under the assumption that the steady state is efficient, $\widehat{Y}_{H,\infty}^T = \widehat{Y}_{H,\infty}^f$ which implies $\widehat{x}_{H,\infty}^W = \widehat{x}_{H,\infty}$. Jung *et al* (2005) show that for the closed economy the interior solution for the steady state is first best for optimal policy under discretion and unique for optimal policy under commitment and discretion; this implies that $\widehat{x}_{F,t} = \widehat{x}_{F,t+1} = \widehat{x}_{F,\infty} = 0$.

Therefore, using this fact, it is possible to derive the steady state for the small open economy, for which there are also interior and corner solutions. The interior solution is for the case when the nominal interest rate is positive and is given by:

$$\widehat{\pi}_{H,\infty}^{PPI} = 0; \widehat{x}_{H,\infty} = 0; \widehat{x}_{H,\infty}^W = 0; i_{H,\infty} = r_{H,\infty}^{n,PPI}; \phi_{1,\infty} = 0; \phi_{2,\infty} = 0. \quad (\mathbf{A-1})$$

The corner solution occurs when nominal interest rates are equal to zero and is given by:

$$\widehat{x}_{H,\infty}^{PPI} = -\frac{(1-\beta)(1-\lambda)}{k(\eta(1-\lambda) + \rho_\lambda)} r_{H,\infty}^{n,PPI},$$

$$\widehat{\pi}_{H,\infty}^{PPI} = -r_{H,\infty}^{n,PPI} < 0, \quad (\mathbf{A-2})$$



$$\phi_{2,\infty} = r_{H,\infty}^{n,PPI},$$

$$\phi_{1,\infty} = \frac{\rho(1-\lambda)}{\rho_\lambda} \left(\omega_H \frac{(1-\beta)(1-\lambda)}{k(\eta(1-\lambda)+\rho_\lambda)} + k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{(1-\lambda)} \right) \right) r_{H,\infty}^{n,PPI}.$$

Following similar arguments to those in Jung *et al* (2005), the interior solution is the first best outcome - it is superior to the corner solution in terms of the central bank's preferences.

A.1.2 Dynamic path

For the phase when the nominal interest rate is zero, $t = 1, \dots, T^d$, the *NKPC* (5), the *IS* curve (4) (after substituting for $i_{H,t} = 0$) and the first-order conditions from the policy maker's optimisation (11) and (12) characterise the optimal path for the endogenous variables. This is given by:

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} \mathbf{a} r_{H,k}^{n,PPI} + \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_1 v_{k+1} + \mathbf{B}_2 v_k) + \mathbf{A}^{-(T^d-t+1)} \mathbf{z}_{H,T^d+1}$$

$$\text{where } \mathbf{A} \equiv \begin{bmatrix} \beta^{-1} & -\beta^{-1}k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \\ -\beta^{-1}\rho_\lambda^{-1}(1-\lambda) & 1 + \beta^{-1}\rho_\lambda^{-1}k(\eta(1-\lambda)+\rho_\lambda) \end{bmatrix}, \mathbf{a} \equiv \begin{bmatrix} 0 \\ \frac{1-\lambda}{\rho_\lambda} \end{bmatrix},$$

$$\mathbf{B}_1 \equiv \begin{bmatrix} 0 & 0 \\ 1 & \frac{\rho(1-\lambda)-\rho_\lambda}{\rho_\lambda} \end{bmatrix},$$

$$\mathbf{B}_2 \equiv \begin{bmatrix} \frac{1}{\beta}k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) & \frac{1}{\beta}k \left(\frac{\rho(1-\lambda)-\rho_\lambda}{1-\lambda} \right) \\ -\frac{1}{\beta\rho_\lambda}k(\eta(1-\lambda)+\rho_\lambda) - 1 & -\frac{1}{\beta}k \left(\frac{\rho(1-\lambda)-\rho_\lambda}{\rho_\lambda} \right) - \frac{\rho(1-\lambda)-\rho_\lambda}{\rho_\lambda} \end{bmatrix}.$$

For $t = T^d + 1, \dots$ the solution is given by:

$$\mathbf{z}_{H,t} = \begin{bmatrix} 1 \\ -\frac{\omega_\pi}{\omega_y}k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \end{bmatrix} \frac{1}{\beta} \sum_{k=t}^{\infty} \Psi_1^{-(k-t+1)} \mathbf{d}'_1 \omega_k,$$

$$\text{where } \mathbf{d}_1 = \begin{bmatrix} k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) & k \left(\frac{\rho(1-\lambda)-\rho_\lambda}{1-\lambda} \right) \end{bmatrix}', \Psi_1 = \frac{\omega_y \beta}{\omega_y + \omega_\pi k^2 \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right)^2}.$$



A.2 Commitment

A.2.1 Steady state

There is a unique steady state for policy with commitment, given by the interior solution:

$$\widehat{\pi}_{H,\infty}^{PPI} = 0; \widehat{x}_{H,\infty} = 0; \widehat{x}_{H,\infty}^W = 0; i_{H,\infty} = r_{H,\infty}^{n,PPI}; \gamma_{1,\infty} = 0; \gamma_{2,\infty} = 0. \quad (\text{A-3})$$

In the corner solution, which occurs when the economy is at the zero lower bound, it will be the case that:

$$\phi_{1,\infty} = -\beta\rho r_{H,\infty}^{n,PPI} < 0.$$

Since this violates the Kuhn-Tucker condition **(19)**, there cannot be a corner steady-state solution. This is also the case for a closed economy.

A.2.2 Dynamic path

In the first phase, $t = 1, \dots, T^c$, the *NKPC*, the *IS* curve (after substituting for $i_{H,t} = 0$) and the first-order conditions from the policy maker's optimisation characterise the optimal path for the endogenous variables, which is given by:

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} \mathbf{a} r_{H,k}^{n,PPI} + \mathbf{A}^{-(T^c-t+1)} \mathbf{z}_{H,T^c+1} + \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_1 \nu_{k+1} + \mathbf{B}_2 \nu_k), \quad (\text{A-4})$$

$$\phi_t = \mathbf{C}\phi_{t-1} - \mathbf{D}_1 \mathbf{z}_{H,t}$$

$$\text{where } \mathbf{C} \equiv \begin{bmatrix} \frac{1}{\beta} + k \frac{1}{\beta\rho\lambda} (\eta(1-\lambda) + \rho\lambda) & k \left(\frac{\eta(1-\lambda) + \rho\lambda}{1-\lambda} \right) \\ \frac{1-\lambda}{\beta\rho\lambda} & 1 \end{bmatrix},$$

$$\mathbf{D}_1 \equiv \begin{bmatrix} k \left(\frac{\eta(1-\lambda) + \rho\lambda}{1-\lambda} \right) \omega_\pi & \omega_y \\ \omega_\pi & 0 \end{bmatrix}.$$



The path for the variables up to and including period T^c depends on the value of \mathbf{z}_{H,T^c+1} . To solve for this we use the fact that in period $T^c + 1$ the non-negativity constraint on nominal interest rates no longer binds: $\phi_{1,t} = 0$. Substituting this into the first-order conditions for the policy problem and using the *NKPC* gives the following equation for \mathbf{z}_{H,T^c+1} and ϕ_{2,T^c+1} :

$$\begin{bmatrix} \mathbf{z}_{H,T^c+1} \\ \phi_{2,T^c+1} \end{bmatrix} = \mathbf{F}^{-1} \mathbf{G} \mathbf{z}_{H,T^c+2} + \mathbf{F}^{-1} \mathbf{H} \phi_{T^c} + \mathbf{F}^{-1} \mathbf{K} v_{T^c+1}, \quad (\text{A-5})$$

$$\text{where } \mathbf{F} \equiv \begin{bmatrix} 1 & -k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) & 0 \\ \omega_\pi & 0 & 1 \\ 0 & \omega_y & -k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \end{bmatrix},$$

$$\mathbf{G} \equiv \begin{bmatrix} \beta & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{H} \equiv \begin{bmatrix} 0 & 0 \\ \frac{1-\lambda}{\rho_\lambda} \beta^{-1} & 1 \\ \beta^{-1} & 0 \end{bmatrix},$$

$$\mathbf{K} \equiv \begin{bmatrix} k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) & k \left(\frac{\rho(1-\lambda)-\rho_\lambda}{1-\lambda} \right) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

To solve this we draw on the solution for the endogenous variables from $T^c + 2$ onwards. Substituting $\phi_{1,T^c+1} = \phi_{1,T^c+2} = \dots = 0$. into the first-order conditions for the policy problem and using the *NKPC* gives a system of difference equations governing the behaviour of $\widehat{\pi}_{H,t+1}$ and $\phi_{2,t}$ for $t = T^c + 2$ of the form:

$$\begin{bmatrix} \widehat{\pi}_{H,t+1}^{PPI} \\ \phi_{2,t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{\pi}_{H,t}^{PPI} \\ \phi_{2,t-1} \end{bmatrix} + \begin{bmatrix} -\frac{1}{\beta} \mathbf{c}'_1 v_t \\ 0 \end{bmatrix},$$

where $\mathbf{M} \equiv \begin{bmatrix} \frac{1}{\beta} + \frac{k^2}{\beta \omega_y} \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right)^2 \omega_\pi & -\frac{k^2}{\beta \omega_y} \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right)^2 \\ -\omega_\pi & 1 \end{bmatrix}$ and

$$\mathbf{c}_1 = \left[k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \quad \frac{k(\rho(1-\lambda)-\rho_\lambda)}{1-\lambda} \right]'$$

The unique bounded solution to this difference equation is given by:

$$\begin{bmatrix} \widehat{\pi}_{H,T^{C+2}}^{PPI} \\ \widehat{Y}_{H,T^{C+2}}^{Wgap} \\ \phi_{2,T^{C+2}} \end{bmatrix} = \begin{bmatrix} -\frac{\gamma_{12}}{\gamma_{11}} \\ \frac{k}{\omega_y} \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \lambda_2 \\ \lambda_2 \end{bmatrix} \phi_{2,T^{C+1}} + \begin{bmatrix} C_{1,T^{C+2}} \\ C_{2,T^{C+2}} \\ C_{3,T^{C+2}} \end{bmatrix} \quad (\mathbf{A-6})$$

where $C_{1,t} = \frac{1}{\beta} \sum_{k=t}^{\infty} \lambda_1^{-(k+1-t)} \mathbf{c}'_1 v_k$,

$$C_{2,t} = -\frac{k}{\omega_y} \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \psi_{21}^{inv} \left(\psi_{22}^{inv} - \frac{\psi_{21}^{inv} \psi_{12}^{inv}}{\psi_{11}^{inv}} \right)^{-1} \frac{1}{\beta} \left[\mathbf{c}'_1 v_t + \sum_{k=t+1}^{\infty} \lambda_1^{-(k-t)} \mathbf{c}'_1 v_k - \lambda_2 \sum_{k=t}^{\infty} \lambda_1^{-(k+1-t)} \mathbf{c}'_1 v_k \right]$$

$$C_{3,t} = -\psi_{21}^{inv} \left(\psi_{22}^{inv} - \frac{\psi_{21}^{inv} \psi_{12}^{inv}}{\psi_{11}^{inv}} \right)^{-1} \frac{1}{\beta} \left[\mathbf{c}'_1 v_t + \sum_{k=t+1}^{\infty} \lambda_1^{-(k-t)} \mathbf{c}'_1 v_k - \lambda_2 \sum_{k=t}^{\infty} \lambda_1^{-(k+1-t)} \mathbf{c}'_1 v_k \right],$$

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \mathbf{\Gamma} \mathbf{M} \mathbf{\Gamma}^{-1}, \quad \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} = \mathbf{\Gamma}.$$

Appendix B: Comparison of charts

These Charts show simulations equivalent to Charts 1 and 2 under the assumption that the home central bank minimises an ad hoc loss function in which the welfare relevant output gap is replaced with the flexible price output gap. The weight on the output gap in the ad hoc loss function is set equal to weight on the welfare relevant output gap in the micro-founded loss function. The small difference between the responses under the different assumptions indicates that the role of the wedge between the efficient and flexible price levels of output created by the foreign policy spillover is small in practice.

Chart 5: Home responses to a global shock for optimal policy under discretion

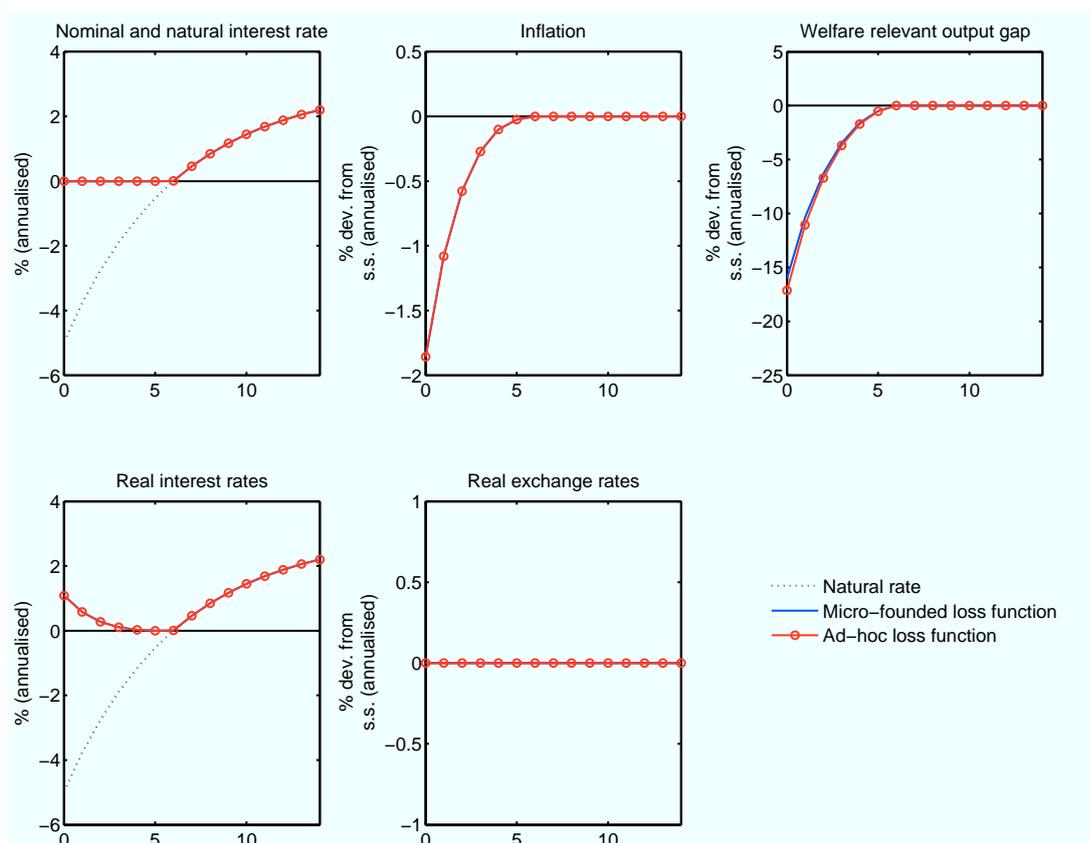


Chart 6: Home responses to a global shock for optimal policy under commitment

