Abstract

This paper examines the role of macroprudential capital requirements in preventing inefficient credit booms in a model with reputational externalities. Unprofitable banks have strong incentives to invest in risky assets and generate inefficient credit booms when macroeconomic fundamentals are good in order to signal high ability. We show that across-the-system countercyclical capital requirements that deter credit booms are constrained optimal when fundamentals are within an intermediate range. We also show that when fundamentals are deteriorating, a public announcement of that fact can itself play a powerful role in preventing inefficient credit booms, providing an additional channel through which macroprudential policies can improve outcomes.

Key words: Macroprudential policy, credit booms, bank capital regulation.

JEL classification: G01, G38, E6.
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Summary

This paper considers the role of macroprudential countercyclical capital adequacy regulation in moderating credit cycles in a simple theoretical model. In our model, banks not only care about returns on their investments, but also their reputations. Imperfect information about banks’ abilities and profitability means that they suffer a bigger reputational loss if they fail to make money when macroeconomic fundamentals are good than when they are bad. This is because when fundamentals are good, high-ability banks are more likely to earn high profits, such that markets attribute low profits to the low ability of bank managers. The fear of getting a bad market reputation gives low-ability bank managers the incentive to hide low profits and extend excessive credit in a bid to ‘gamble for reputation’ when fundamentals are good. This generates socially inefficient credit booms which ultimately lead to bank losses.

Our analysis suggests that countercyclical capital adequacy requirements are constrained socially optimal when macroeconomic fundamentals are within an intermediate range. By helping to reduce the incidence of inefficient credit booms, countercyclical capital adequacy requirements help to meet the dual objectives of moderating credit cycles and enhancing banking sector resilience. We are also able to separate two effects of countercyclical capital requirements on banks’ risk-taking incentives, namely (i) the direct effect of raising the cost of risk-taking, and (ii) the indirect effect of making information about the state of macroeconomic fundamentals public. We demonstrate that the latter can have a powerful effect in reducing banks’ risk-taking incentives when fundamentals are rapidly deteriorating.

Our analysis focuses on a particular role for capital adequacy requirements, namely, that of preventing banks from investing in risky projects that have negative net present value. There are other rationales for countercyclical capital adequacy requirements which we have not considered here, including enhancing loss absorbance and avoiding socially costly financial crises. Our analysis also focuses on the role of capital adequacy requirements in preventing inefficient credit booms, and does not examine its potential role in preventing inefficient credit crunches. Examining all these aspects of countercyclical capital requirements in a single framework is left for future research.
1 Introduction

One of the key elements of the Basel III framework is the countercyclical capital buffer. According to the Basel Committee on Banking Supervision (BCBS (2010)), the primary aim of the countercyclical capital buffer regime is to use a capital cushion to achieve the broader macroprudential goal of protecting the banking sector from periods of excess aggregate credit growth that have often been associated with the build-up of system-wide risk. In enhancing the resilience of the banking sector over the credit cycle, the countercyclical capital buffer regime may also help to lean against credit in the build-up phase of the cycle in the first place.

This paper considers the role of macroprudential countercyclical capital adequacy regulation when banks have the incentive to hide losses and undertake inefficient lending in order to safeguard their reputation. In our model, banks not only care about returns on their investment, but also their reputations as being ‘high ability’. Imperfect information over banks’ abilities and profitability means that they suffer a bigger reputational loss if they fail to announce profits when macroeconomic fundamentals are good than when they are bad. This is because when fundamentals are good, high-ability banks are more likely to earn high profits, such that markets attribute low profits to the low ability of bank managers. The fear of getting a bad market reputation gives low-ability bank managers the incentive to hide low profits and extend excessive credit in a bid to ‘gamble for reputation’ when macroeconomic fundamentals are good, thus generating socially inefficient credit booms. We use the global games modelling framework in order to analyse the impact of policy when the reputational effect gives rise to strategic complementarity in banks’ incentive to gamble.\(^1\)

This paper makes two contributions to the literature. First, we characterise optimal macroprudential capital requirement in preventing inefficient credit booms driven by reputational incentives when the regulator cannot observe banks’ type. Our analysis suggests that there is a case for countercyclical capital adequacy requirements because the presence of the reputational effect means that banks’ incentives to gamble are strongest when macroeconomic fundamentals are good. By helping to reduce the incidence of inefficient credit booms, which ultimately lead to bank losses, countercyclical capital adequacy requirements help to meet the dual objectives of

moderating credit cycles and enhancing banking sector resilience.

Higher aggregate capital adequacy requirements could come at a cost, however, if they increase funding costs for all banks, including those high-ability banks that do not have incentives to gamble. This could happen if Modigliani-Miller theorem fails to hold such that increasing the share of equity finance increases the total funding cost for banks – for example, due to the tax advantage associated with debt financing and the presence of deposit insurance which artificially reduces the cost of deposits. If this assumption holds, the regulator faces a trade-off between the need to deter gambling by low-ability banks on the one hand, and the need to avoid imposing high funding cost on high-ability banks on the other hand. We demonstrate that, given this trade-off, countercyclical macroprudential capital adequacy regulation is constrained socially optimal when macroeconomic fundamentals are within an intermediate range.

Our second contribution is to separate two effects of countercyclical capital requirements on banks’ risk-taking incentives, namely (i) the direct effect of raising the cost of risk-taking, and (ii) the indirect effect of making information about the state of macroeconomic fundamentals public. We demonstrate that the latter can have a powerful effect in reducing banks’ risk-taking incentives when fundamentals are rapidly deteriorating.

Our paper is related to a number of existing papers which analyse the impact of strategic interdependence on banks’ risk-taking incentives, including Acharya (2009), and Acharya and Yorulmazer (2008). Our main contribution to this theoretical literature is to model the role of macroprudential capital adequacy regulation explicitly, so that we can characterise optimal countercyclical regulation within a framework that presents a novel reason for cyclicality in risk-taking. In our model, the rationale for countercyclical capital regulation arises because of a procyclical, non-pecuniary externality (reputational concerns) that causes banks’ risk-taking incentives to rise during macroeconomic upswings. In this respect, the underlying distortion we model is close in spirit to that of Rajan (1994) in particular, but see also Gorton and He (2008), Scharfstein and Stein (1990), Froot, Scharfstein and Stein (1992), and Thakor (2006). This rationale is related to but distinct from those articulated by Bianchi (2010) and Lorenzoni (2008), who suggest that countercyclical capital requirements – or higher capital requirements on assets with higher correlation with macroeconomic shocks – could be desirable if private agents’ failure to internalise the pecuniary cost of increasing leverage on \textit{ex post} asset prices and others’
collateral constraints leads to *ex ante* overborrowing. It is also distinct from macroeconomic rationally that emphasise the hedging benefits derived from the issuance of outside equity by banks in general equilibrium, as in Gertler, Kiyotaki and Queralto (2011).

Our theory also offers empirical implications. For instance, our analysis predicts cross-sectional convergence of bank profits during credit booms as low-ability banks’ attempt to hide their low returns in order to mimic the high-ability banks. Similarly, the finding by Drehmann, Borio, Gambacorta, Jiménez and Trucharte (2010) that credit-to-GDP ratio is a good leading indicator of banking crises can be explained by our theory that suggests that inefficient credit booms preceding banking crises are associated with gambling by those banks trying to mimic profitable banks.

This paper is organised as follows. Section 2 provides the most basic set-up of the model, in which banks receive noisy signals about the macroeconomic fundamentals in deciding whether to gamble for reputation or not. The analytical solution in this section helps us to illustrate how capital adequacy requirement affects banks’ incentives to gamble and hence the credit cycles. We also discuss the empirical implications of our analysis. Section 3 explicitly analyses the optimal countercyclical capital adequacy regulation, using a model in which banks receive both public and private signals about macroeconomic fundamentals. Section 4 considers the effect of public announcement of the macroeconomic fundamentals on banks’ risk-taking incentives. Section 5 concludes.

2 The model

We first set-up a simple global games model in which those banks receiving low returns in the interim decide whether to gamble in order to preserve their reputations, based on a private signal they receive about macroeconomic fundamentals. This simple set-up helps to illustrate the impact of the reputational considerations on banks’ incentives to gamble, and how capital adequacy requirement affects these. We will characterise the optimal countercyclical capital adequacy requirement in Section 3. The timing of the game is summarised in Chart 1.
2.1 Set-up

The model consists of three dates, \( t = 0, 1, 2 \), and there is a continuum of \textit{ex ante} identical banks. Each bank invests 1 at \( t = 0 \) in a risky project. A fraction \( k \) of the investment is funded by equity, while fraction \( 1 - k \) is funded by debt. We normalise the cost of debt to zero. The cost of employing equity financing is \( c > 0 \), such that the unit investment costs \( c k \) to fund.\(^2\) The cost of equity is taken as given by the bank, and for the moment, we assume that \( k \) is exogenous. As we will illustrate in the next section, \( k \) can be used as a policy tool to prevent inefficient credit booms.

2.1.1 Initial returns

The structure of pay-offs to the initial investment is summarised in Table A. At \( t = 1 \), banks privately observe the return from an initial investment made in \( t = 0 \). A fraction \( \alpha \) of banks are high ability and observe high returns \( R_H \) with probability \( f(\theta) \), such that in the population as a whole, a fraction \( \alpha f(\theta) \) of banks observe \( R_H \). The remaining fraction of high-ability banks privately observe low returns \( R_L < R_H \). The parameter \( \theta \) indexes macroeconomic fundamentals, which determine the fraction of high-ability banks that observe high returns in the first period. We assume that \( f'(\theta) > 0 \), such that the fraction of high-ability banks receiving high returns increases as macroeconomic fundamentals, \( \theta \), improve. High-ability banks observing \( R_H \) publicly announce these returns, raise new finance, and invest one unit for another period, at cost \( c k \). Banks that have observed \( R_H \) from their \( t = 0 \) investments can be sure that their \( t = 1 \) investments will return \( R_H \) at \( t = 2 \).

A fraction \( 1 - \alpha \) of the banks turn out to be low ability, and receive low returns \( R_L < R_H \) on their initial investments. Of these, a fraction \( 1 - \lambda \) are unable to access subsequent investment opportunities. The remaining fraction of low-ability banks \( (1 - \alpha)\lambda \) together with the ‘unlucky’ fraction \( \alpha [1 - f(\theta)] \) of high-ability banks, all of which privately observe \( R_L < R_H \) at \( t = 1 \), face a further investment choice that we describe next. Hence in this set-up low ability is distinguished from high ability along two dimensions. First, low-ability banks are never able to

\(^2\)The cost of equity \( c > 0 \) could reflect the foregone tax advantage of debt. There could be other deadweight costs associated with equity issuance as opposed to debt issuance where the latter provides a monitoring advantage, eg Calomiris and Kahn (1991).
make high returns on their initial investments. Second, low-ability banks are not always able to access \( t = 1 \) investment opportunities.

\[ \text{2.1.2 Choice at } t = 1 \]

If a bank that received low returns in the interim chooses to announce its true return of \( R_L \), it is unable to raise new finance to invest at \( t = 1 \). But given that interim returns are observed privately, banks observing \( R_L \) can mimic lucky high-ability types by announcing \( R_H \), too. They can then raise new finance at cost \( c_k \), and invest one unit: this investment constitutes ‘gambling for reputation’. In particular, having observed low returns, investing in a subsequent project yields a \( t = 2 \) return of \( 2R_H - R_L \) with probability \( b \in [0, 1] \), such that at \( t = 2 \), total announced profits are \( 2(R_H - c_k) \), which are exactly the same as those of the lucky high-ability banks. But the gamble could fail. With probability \( 1 - b \), banks lose all of their \( t = 1 \) profits, such that they have to announce zero profits in \( t = 2 \). The probability of the \( t = 1 \) gamble being successful is independent of a bank’s ability, whereas the probability of the \( t = 0 \) investment being successful depends on a bank’s ability. Hence we think of the gambling option as a highly risky short-run strategy whose return distribution is invariant to the characteristics of those who execute it. Because following such a strategy requires that the bank raises finance and invests twice over, we also think of this as implying ‘rapid balance sheet expansion’.

Banks that fail to announce a final profit of \( 2(R_H - c_k) \) at \( t = 2 \) suffer reputational damage \( p(\theta, l) \), where \( l \in [0, 1] \) is the proportion of banks that can take the risky gamble having observed initial returns of \( R_L \) and choose to do so. We assume that reputational damage has the following properties: (a) \( \partial p(\theta, l) / \partial \theta > 0 \), so that as fundamentals improve, the reputational cost for announcing low returns increases; and (b) \( \partial p(\theta, l) / \partial l > 0 \), so that as the proportion of banks taking the risky gamble increases, the reputational damage of announcing low returns increases. We discuss this assumption in greater detail below in Section 2.3.

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**Table A: Probabilities of returns at \( t = 1 \) on \( t = 0 \) investment**

<table>
<thead>
<tr>
<th>Initial return</th>
<th>High ability</th>
<th>Low ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_H )</td>
<td>( \alpha f(\theta) )</td>
<td>0</td>
</tr>
<tr>
<td>( R_L )</td>
<td>( \alpha [1 - f(\theta)] )</td>
<td>( (1 - \alpha) )</td>
</tr>
</tbody>
</table>
2.1.3 Private signals

In making a decision about whether to gamble for reputation, banks have to make an assessment of whether other banks will also gamble, as their reputational cost of announcing low returns will depend on what others will do. In making this decision, each bank $i \in [0, 1]$ receives a noisy private signal $x_i$ about fundamentals at $t = 1$:

$$x_i = \theta + \sigma \varepsilon_i, \quad \sigma > 0,$$

where the noise terms are distributed with density $g(.)$ with support on the real line. Given this set-up, a bank’s expected pay-off from gambling at $t = 1$ is:

$$b[2(R_H - ck)] + (1 - b)[-2ck - p(\theta, l)],$$

whereas the pay-off to playing safe is

$$R_L - ck - p(\theta, l).$$

From a social perspective, gambling for reputation is inefficient if

$$b < \frac{R_L + ck}{2R_H}, \quad (1)$$

ie if the gamble is sufficiently risky. We assume condition (1) holds throughout our analysis.

Taken together, the game gives a banker’s marginal pay-off to gambling $\pi(\theta, l)$ as

$$\pi(\theta, l) = b[2R_H + p(\theta, l)] - R_L - ck. \quad (2)$$

Note that in our set-up, reputational considerations generate a source of strategic interdependence between banks’ actions: each banker has a stronger incentive to gamble when they believe that others are doing the same. So the reputational consideration is the friction which induces banks to take the socially inefficient action of gambling for reputation and generates inefficient credit booms: in its absence, banks will never choose to gamble, as $\pi(\theta, l)$ will always be negative by equation (1).

2.2 Equilibrium

We analyse the problem faced by a bank that has observed low initial returns. At this juncture, it has to choose an action {gamble, safe} to maximise its expected pay-off. Suppose that a bank that has received $R_L$ and signal $x_i$ at $t = 1$ uses the following switching strategy:

$$s(\theta) = \{\text{gamble if } x_i \geq \theta^*, \text{ don’t if } x_i < \theta^* \}.$$
Using equation (2) and the results in Morris and Shin (2003), we can prove the following:

**Proposition 1** The unique symmetric switching equilibrium value of fundamentals \( \theta^* \) above which banks co-ordinate on gambling following low initial returns is given implicitly by:

\[
\int_0^1 p(\theta^*, l) dl = \frac{R_L + ck - 2bR_H}{b}.
\]

**Proof.** See Appendix A.1.

Consider a simple case in which \( p(\theta, l) = \theta + l - 1 \). Then \( \theta^* \) is given by

\[
\theta^* = \frac{1}{2} + \frac{R_L + ck}{b} - 2R_H.
\]

Note that the gambling threshold \( \theta^* \) is increasing in \( k \), the capital held by banks. This is very intuitive: a bank has a weaker incentive to gamble if it has to finance a higher proportion of the new lending by costly capital, as it diminishes the expected return from gambling relative to playing safe. Thus, a bank with a higher level of capital tends to play safe even if their private signal points to relatively strong fundamentals. Were the gamble to pay off with a higher probability (ie \( b \) is high), this effect would be mitigated: banks would then choose to gamble even if their private signal suggests fundamentals are low, as they are more likely to be able to avoid a reputational penalty; thus, \( \theta^* \) would fall. Note that our model assumes unlimited liability, so the mechanism via which higher capital reduces risk-taking in our model is different from that in Furlong and Keeley (1989) and Tanaka and Hoggarth (2006), in which banks’ risk-taking incentives arises from the implicit subsidy from (mis-priced) deposit insurance or limited liability.

These results are quite general for \( p(\cdot) \) with the properties we described above. Therefore, we write \( \theta^* = \theta^*(k) \), in which:

\[
\frac{d\theta^*(k)}{dk} > 0,
\]

such that higher bank capital raises the threshold level of the private signal above which banks take the gambling option.
2.3 The reputational effect: an explanation

In our model, a banker’s reputation is assessed by the market, which cannot observe ability or fundamentals. It can be shown that the reputational damage is related to the probability of being low-ability bank conditional on failing to achieve high returns. Consider the following example:

**Example 1** Let the reputational penalty \( p(\theta, l) \) be a monotonic function \( h(\cdot) \) of the probability of being low ability conditional on failing to achieve high returns, \( P(\theta, l) \). The joint probability of being low ability and failing to achieve high returns is

\[
(1 - \alpha) \left[ (1 - \lambda) + \lambda \left\{ (1 - l) + l(1 - b) \right\} \right] = (1 - \alpha) (1 - \lambda lb)
\]

while the unconditional probability of failing to achieve high returns is

\[
(1 - \alpha) (1 - \lambda lb) + \alpha [1 - f(\theta)] \left\{ (1 - l) + l(1 - b) \right\}
\]

\[
= (1 - \alpha) (1 - \lambda lb) + \alpha [1 - f(\theta)] (1 - lb).
\]

Then the probability of being low ability conditional on low returns \( P(\theta, l) \) is given by

\[
P(\theta, l) = \frac{(1 - \alpha) (1 - \lambda lb)}{(1 - \alpha) (1 - \lambda lb) + \alpha [1 - f(\theta)] (1 - lb)},
\]

which satisfies

\[
\frac{\partial P(\theta, l)}{\partial \theta} > 0, \quad \frac{\partial P(\theta, l)}{\partial l} > 0.
\]

Then if \( p(\theta, l) = h[P(\theta, l)] \), by the monotonicity of \( h(\cdot) \) we have that \( \partial p(\theta, l) / \partial \theta > 0 \) and \( \partial p(\theta, l) / \partial l > 0 \).

Heuristically, property (a) \( \partial p(\theta, l) / \partial \theta > 0 \) follows from the observation that as \( \theta \) rises, high-ability types are more likely to receive high initial returns (Table A). In the extreme case where \( f(\theta) = 1 \), all high types always announce high returns; so announcing low returns is a sure signal that ability is low. Property (b) \( \partial p(\theta, l) / \partial l > 0 \) follows from the fact that as the number of gamblers rises, the proportion of banks realising final low returns decreases. But those banks that still manage to post low returns must be more likely to be low-ability types who lack access to the gambling technology. For example, if gambling always paid off, posting low returns would be a sure sign of low ability (though posting high returns would not be a sure sign of high ability). At a high level, property (a) captures Citigroup CEO Chuck Prince’s 2007 remark that ‘as long as the music is playing, you’ve got to get up and dance’, while property (b) captures
Keynes’ famous line about a banker who, ‘when ruined, is ruined in a conventional and orthodox way with his fellows, so that no-one can really blame him’.

There are several reasons why bankers may be averse to admitting to bad results when everyone else is doing well and have the incentive to ‘keep up with the Goldmans’. First, their compensation, promotion and dismissal – as well as their ability to secure another job – may be implicitly or explicitly linked to their performance relative to others in the industry: indeed, a banker’s performance relative to others in the industry is a good signal of their ability when the banking industry is subject to a common shock. Foster and Young (2010), for example, argue that there is no compensation contract that can separate high-ability managers from low-ability managers when managers’ strategies and positions are not transparent. Murphy (1999), updating Gibbons and Murphy (1990), finds that CEO pay in financial services is likely to be evaluated relative to market and industry returns among S&P 500 financial services companies. Explicit relative performance evaluation is used by 57% of the financial services firms in Murphy’s (1999) survey.

Second, policymakers’ inclination to bail out banks when they fail together rather than when they fail in isolation – due to their concerns about systemic risk associated with multiple bank failures – may also give bankers the incentive to avoid failure by gambling when other banks are doing well.

Equally, the function \( p(\theta, l) \) could capture the continuation value of the bank’s operations in a situation where the bank’s future funding costs depend on the market’s perception of ability. In this context, being perceived as low ability would raise future funding costs, reducing profitability, and so the banker’s pay-off. Knowing this, the banker would take steps to avoid the stigma of being perceived negatively by the market.

Our story also relies on imperfect information about fundamentals and ability. Empirically, Slovin, Sushka and Polonchek (1992) find that market participants take individual bank stock issuance as signals of value for other banking firms. In particular, commercial bank equity issues are associated with a significant negative valuation effect of -0.6% on rival commercial banking.

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3Holmstrom (1982) argues that relative performance evaluation is useful if agents face some common uncertainty, such that other agents’ performance reveals information about an agent’s unobservable choices that cannot be inferred from his or her own measured performance.

4See Table 9, page 2,538.

5See, for example, Acharya and Yorulmazer (2008).
firms. Slovin et al (1992) interpret this as evidence that an individual bank’s issuance conveys not just institution-specific information to the market, but industry-wide information regarding fundamentals too. That is, information released by one bank conveys information to the market about industry value, which triggers a re-appraisal of other banks’ market values. Rajan (1994) also finds evidence in favour of cross-bank informational effects. When benchmarking in compensation ties individual incentives to relative performance, these informational externalities generate strong incentives to herd.

2.4 Empirical implications

This simple private signals model has a number of empirical implications. We focus on two. First, reputational incentives drive low-ability banks to gamble when macro fundamentals are sufficiently high. This generates an inefficient credit boom in the model, which is followed by the realisation of large-scale losses. In other words, credit booms should precede crises, and even small changes in fundamentals can have a large impact on the path for credit. Work by Drehmann et al (2010) supports this view, arguing that the ratio of credit to GDP can be a useful indicator of subsequent distress. In Chart 2, we plot the ratio of credit to GDP for the United Kingdom, since 1963. The series have been filtered using a band-pass filter, which isolates variation in the ratio over a particular frequency range. Consistent with Drehmann et al (2010) and Aikman, Haldane and Nelson (2010), we show variation in the ratio of credit to GDP over the 1-20 year frequency range. Shaded regions indicate periods of banking distress, namely, the 1973-75 secondary banking crisis, the 1990-94 small banks crisis, and the recent episode. The chart illustrates that a medium-term build-up in the ratio of credit to GDP has tended to lead crisis periods.

Second, on the microeconomic level, the efforts of low-ability banks to mimic their high-ability counterparts implies a compression in the distribution of announced profits during credit booms.

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6Rajan examines the cross-bank effects resulting from Bank of New England Corp.’s announcement that, prompted by the regulator, it would boost loan loss reserves in response to growing losses in 1989. Banks with headquarters in one state in New England suffered disproportionate cumulative abnormal returns of ~8%. Using data on real estate firms, Rajan argues that the announcement conveyed information to the market about the state of the New England real estate sector in general, rather than conveying only institution-specific information in particular.

7This is equivalent to passing a relatively ‘smooth’ trend through the series. An HP filter with a high value of the smoothing parameter would achieve this. We use a band-pass filter because it allows us to be more precise about the band of the frequency domain over which the filter returns cyclical variation.

8In the early 1990s, the Bank of England provided liquidity support to a few small banks in order to prevent a widespread loss of confidence in the banking system. 25 banks failed or closed during this period. The emergency liquidity assistance provided by the Bank is regarded as having safeguarded the system as a whole, which was vulnerable to a tightening in wholesale markets. See Logan (2000) for discussion.
It is during these periods that standing out from the crowd is most damaging to reputation. Chart 3 plots the cross-sectional dispersion of equity returns for major UK banks and the top 100 UK private non-financial corporations (PNFCs) for 1997-2009. It is striking that the cross-sectional dispersion tended to be lower for banks versus PNFCs for much of the period, despite banks operating at much higher levels of leverage. Further, this compression reached its nadir in the boom years of 2004-07. This phase maps our model, which says that standing out from the crowd is worst for reputation in a boom, to the micro data. A similar story is told in Chart 4, which shows the cross-sectional dispersion in the return on equity (ROE) for major UK banks versus PNFCs.

We turn next to an examination of what policy actions might contribute to mitigating the inefficient credit booms that the model predicts. To do that, we extend our model to include a policymaker explicitly.

3 Capital adequacy regulation

3.1 Game with public and private signals

Let us now consider how a regulator may set $k$, which can be interpreted as the regulatory capital adequacy requirement. To do that, the regulator needs to know the distribution of $\theta$, such that (s)he can estimate what proportion of banks would receive low returns and hence would potentially have incentives to gamble at time $t = 1$. So suppose now that $\theta \sim \mathcal{N}(y, \tau^2)$, and that all agents in the model (including the regulator) observe this distribution. The distribution of fundamentals is therefore a public signal. The regulator sets the capital adequacy requirement, $k^*$, at $t = 0$, which applies to investments made at both $t = 0$ and $t = 1$, so as to maximise social welfare. The rest of the game’s set-up is as before.

We solve the model backwards, first working out banks’ strategies at $t = 1$ given that they now observe a public signal about $\theta \sim \mathcal{N}(y, \tau^2)$ (namely, its distribution) in addition to the private signal, which we now assume follows the process $x_i = \theta + \varepsilon_i$, where $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$. Given these two signals, a bank’s posterior belief of $\theta$ conditional on the two signals will be normal with a mean of:

$$\bar{\theta}_i = \frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2},$$

(4)
and standard deviation

\[ \sqrt{\frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}}, \]

(see DeGroot (1970)). Suppose banks that have received \( R_L \) at \( t = 1 \) use the following switching strategy, gambling when their posterior mean exceeds some threshold \( \theta^* \), and playing safe if not:

\[ s(\bar{\theta}) = \begin{cases} \text{gamble if } \bar{\theta} \geq \theta^*, \text{ don't if } \bar{\theta} < \theta^* \end{cases} \]

(5)

To solve for the equilibrium, consider a simple functional form for bank reputation, \( p(\theta, l) = \theta + l - 1 \) (see example below). Following the solution method used by Morris and Shin (2003), we can prove the following:

**Proposition 2** There exists a unique symmetric switching equilibrium with cut-off \( \theta^* \), where \( \theta^* \) solves the equation:

\[ \theta^*(k, y) = \Phi\left( \sqrt{\gamma} (\theta^*(k, y) - y) \right) + \frac{R_L + ck}{b} - 2R_H, \]

(6)

in which \( \Phi(\cdot) \) is the normal cdf, as long as \( \gamma \equiv (\sigma^2/\tau^4) [(\sigma^2 + \tau^2) / (\sigma^2 + 2\tau^2)] \leq 2\pi \).

**Proof.** See Appendix A.2.

The condition \( \gamma \leq 2\pi \) implies that the unique equilibrium exists only when the public signal is quite noisy relative to the private signal; Morris and Shin (2003) show that when this condition is violated, multiple equilibria can arise. Expression (6) defines banks’ reaction function to the public signal about the fundamental, \( y \), and the capital adequacy requirement, \( k \). It can be shown that, by totally differentiating equation (6),

\[ \frac{d\theta^*}{dk}(k, y) = \frac{c/b}{1 - \phi\left( \sqrt{\gamma \theta^*(k, y) - y} \right) \sqrt{\gamma}} > 0, \]

(7)

and

\[ \frac{d\theta^*}{dy}(k, y) = -\phi\left( \sqrt{\gamma \theta^*(k, y) - y} \right) \sqrt{\gamma} < 0, \]

(8)

in which \( \phi(\cdot) \) is the normal pdf. Equation (7) says that, as before, a higher capital adequacy requirement increases the threshold of the private signal above which banks start gambling, and hence it helps to reduce the incidence of gambling. In addition, equation (8) says that a higher public signal \( y \) reduces the threshold of private signal at which banks start gambling. This is because the higher \( y \), the more likely it is that other banks will also choose to gamble, and since all banks observe \( y \), all banks know this. As such, each bank has an increased incentive to
gamble even if his own private signal is low. Thus, a high public signal makes it more likely that banks will co-ordinate on the gambling equilibrium, all else equal.

**Example 2** (Continued) A linear form for reputational costs can be derived as a first-order Taylor approximation for $P(\theta, l)$. Approximate $P(\theta, l)$ around $\{\hat{\theta}, \hat{l}\}$ by

$$P^{\text{app}}(\theta, l) = \hat{P} + P_\theta(\theta - \hat{\theta}) + P_l(l - \hat{l}),$$

where $\hat{P}$ denotes $P(\theta, l)$ evaluated at $\{\hat{\theta}, \hat{l}\}$, and subscripts denote partial derivatives. Let $h(\cdot)$ be an affine function of $P^{\text{app}}(\theta, l)$, such that $h(P^{\text{app}}) = \varphi_0 + \varphi_1 P^{\text{app}}(\theta, l)$ where $\varphi_j, j = 0, 1$ are constants to be chosen. Then by appropriate choice of the $\varphi_j$s for the desired approximation point $\{\hat{\theta}, \hat{l}\}$, we have

$$p(\theta, l) = \varphi_0 + \varphi_1 \left[ \hat{P} + \hat{P}_\theta(\theta - \hat{\theta}) + \hat{P}_l(l - \hat{l}) \right]$$

which reduces to $p(\theta, l) = -1 + \theta + l$ when

$$\hat{P}_\theta = \hat{P}_0, \quad \varphi_1 = \frac{1}{\hat{P}_0}, \quad \varphi_0 = \varphi_1 \hat{P}_0 \hat{\theta} + \varphi_1 \hat{P}_l \hat{l} - \varphi_1 \hat{P} - 1.$$

For example, $p(\theta, l) = \theta + l - 1$ used above approximates $h[P(\theta, l)]$ around $\{f(\hat{\theta}), \hat{l}\} = \{0.05, 0.5\}$ when $\{\varphi_0, \varphi_1\} = \{-29.80, 124.23\}$ under a baseline calibration in which $\alpha = 0.8, b = 0.09, \lambda = 0.5$.

### 3.2 The optimal capital requirement

We now consider how the policymaker might set the *aggregate* capital requirement which applies system-wide, to all banks. In setting the capital requirement, the policymaker faces the following trade-off. On the one hand, raising the capital requirement deters gambling by those banks that have received low profits in the interim, and thus leans against inefficient investments. On the other hand, it also increases the funding cost for all banks and thus reduces their pay-offs, including for those which have received high profits in the interim and therefore have no incentive to gamble. Capital requirements set too high will also affect lending: beyond a certain point, raising $k$ makes all pay-offs negative, even those of lucky high-ability banks.

To examine the optimal capital requirement, suppose that the policymaker chooses $k$ to maximise social welfare, $S$, consisting of a weighted sum of banks’ expected returns given their reaction
function defined implicitly in equation (6):

$$\max_k S(k, y) \equiv \alpha f(y) \times 2(R_H - ck) + (1 - \alpha)(1 - \lambda) \times (R_L - ck)$$

$$+ [(1 - \alpha)\lambda + \alpha \{1 - f(y)\}] \times \Pi(k, \theta^*)$$,

s.t. \( \theta^* = \theta^*(k, y) \),

where

$$\Pi(k, \theta^*) \equiv \Pr(\text{gambles})[b(2R_H - 2ck) + (1 - b)(-2ck)] + \Pr(\text{safe})(R_L - ck).$$

The function \( \Pi(k, \theta^*) \) is the expected pay-off of unprofitable banks, where \( \Pr(\text{safe}) \) defines the probability of unprofitable banks playing the safe strategy, given the public and private signals about fundamentals and the capital requirement, while \( \Pr(\text{gambles}) \) is defined analogously. We show in the Appendix that:

$$\Pr(\text{safe}) = \theta^*(k, y) - \frac{R_L + ck}{b} + 2R_H.$$

Note that the social welfare function in expression (9) is not a weighted sum of banks’ utility functions. This is because the reputational effect, \( p(\theta, l) \), is a private cost which induces banks to gamble for reputation, so that the policymaker does not place any weight on it. Thus, the policymaker’s objective as formulated in equation (9) can be interpreted as minimising the banks’ expected losses caused by gambling and inefficient credit booms, while avoiding the imposition of excessive funding costs on the entire banking system.

Solving for the policymaker’s first-order condition, the optimal capital requirement \( k^* \) – and hence the regulator’s optimal choice of \( \theta^*(k^*, y) \) – is given by the solution to the following (see Appendix A.4):

$$[(1 - \alpha)\lambda + \alpha \{1 - f(y)\}] \frac{\partial \Pr(\text{safe})}{\partial k} = (\pi^s - \pi^g)$$

$$= c \left(2\alpha f(y) + (1 - \alpha)(1 - \lambda) + (2 - \Pr(\text{safe}))[(1 - \alpha)\lambda + \alpha \{1 - f(y)\}]\right),$$

where \( \pi^g \equiv b(2R_H - 2ck) + (1 - b)(-2ck) \) and \( \pi^s \equiv R_L - ck \) are banks’ net returns from gambling and safe options, respectively, and:

$$\frac{\partial \Pr(\text{safe})}{\partial k} = \frac{d\theta^*(k, y)}{dk} - \frac{c}{b} > 0,$$

$$\pi^s - \pi^g = R_L - 2bR_H + ck > 0.$$

The first-order condition equates the marginal cost of increasing the capital requirement with the marginal benefit. The marginal cost of raising \( k \) is linear in \( c \) for all bank types across all states of
the world. Since finance is raised twice by high-return banks and gamblers, a 2 appears on the right-hand side of the first-order condition, adjusted by the fraction of low-return banks that are expected to play it safe. The marginal benefit of higher capital requirements on the left-hand side of the first-order condition captures the marginal social gain associated with reduced gambling by the low-return banks. In the Appendix, we show that the second-order condition is negative – and an interior solution exists – only when \( \gamma \) is sufficiently close to 2\( \pi \), and \( k^* \), which solves the first-order condition, above gives rise to \( \theta^* (k^*, y) > y \). Otherwise, we will have a corner solution, as we will illustrate later using simulations.

Is the optimal capital adequacy requirement countercyclical? We show that indeed it is, as long as macroeconomic fundamentals are within a certain range:

**Proposition 3** When the public signal about the macroeconomic fundamentals, \( y \), is within a range, \( y \in [\bar{y}, \tilde{y}] \), and the public signal is neither too noisy nor too informative, \( \gamma \in (\gamma, 2\pi] \), the policymaker’s optimal capital requirement \( k^* \) is procyclical, such that \( dk^*/dy > 0 \).

**Proof.** See Appendix A.5.

This is the core result of our paper. We turn next to an explanation of why it arises.

### 3.3 Simulations

We now show our results graphically in order to illustrate the intuition behind them. Chart 5 plots aggregate credit supply (expected at \( t = 0 \) for different values of \( y \)) under our baseline calibration;\(^9\) this illustrates how a higher capital adequacy requirement can mitigate inefficient credit booms. The green dotted line in Chart 5 represents the efficient, ‘no gambling’ level of credit supply, given by \( \alpha f(y) \times 2 + [1 - \alpha f(y)] \times 1 \) as \( \lambda \to 1 \), which rises gently with \( y \). The blue and the red lines show the aggregate credit supply with gambling, given by \( \alpha f(y) \times 2 + [1 - \alpha f(y)] \times \{ \text{Pr(safe) x 1 + [1 - Pr(safe)] x 2} \} \) as \( \lambda \to 1 \), for different levels of capital requirements, \( k = 10\% \) and \( k = 20\% \), respectively. As the blue and the red lines show, banks’ gambling incentives generate inefficient credit booms when fundamentals are high; and a

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\(^9\)We use \( \{ \alpha = 0.8, b = 0.09, c = 0.15, R_L = 1, R_H = 2, \sigma = 0.5, \tau = 0.414, f(z) = (1 + e^{-z})^{-1} \} \) and let \( \lambda \to 1 \). Clearly, the quantitative features of the simulations will depend on the logistic form we have chosen for \( f(.) \). But note that the foregoing theoretical results do not make an assumption about the form for \( f(.) \) other than that it is increasing.
higher capital requirement mitigates inefficient credit booms by increasing the range of fundamentals in which banks choose not to gamble, and by reducing gambling for any given level of fundamentals.

Our analysis points to a particular view of the ‘transmission mechanism’ of capital regulation. The model suggests that risky gambling requires fast balance sheet expansion: low initial return banks must raise funds twice over in order to finance their gambling for reputation. That rapid balance sheet expansion is an indicator of potential future stress in our model is reminiscent of the recent experience. In this context, a capital requirement penalises at the margin low-return banks whose choice to gamble requires them to raise extra funds. Higher capital requirements imply that these marginal funds are more costly as long as \( c > 0 \) (e.g. debt has a tax advantage).

Chart 6 plots the optimal capital adequacy requirement \( k^* \), for a different range of the public signal about the fundamentals, \( y \), under our baseline calibration. As this shows, the optimal capital requirement is zero when \( y \) is below a threshold, but procyclical for an intermediate range of \( y \), and then becomes zero again when \( y \) is above a certain threshold.

To understand why this is the case, note that capital requirements have a non-linear impact on banks’ incentives to gamble, as Chart 7 illustrates. When the capital requirement is low, almost all banks gamble in expectation, whereas when it is high, almost all of them are expected to choose to play safe. In the intermediate range of \( k \), a small increase in capital requirements will lead to a rapid reduction in gambling as banks switch from gambling to playing safe. As \( y \) becomes larger, banks’ incentives to gamble becomes greater, and hence a higher capital requirement is needed to deter gambling.

As a result, the social benefit of increasing \( k \) is non-linear. By contrast, the cost of increasing \( k \) is linear given the opportunity cost of raising capital \( c \). Consequently, the social welfare function (9), is not globally concave, as shown in Chart 8. This is why we have corner solutions for some range of \( y \).

The comparative statics are intuitive, too. For instance, as the cost of raising equity, \( c \), falls, it

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10High-return high-ability banks also raise finance and invest twice over, also expanding their balance sheets ‘rapidly’. But when gambling by low-return banks takes place, the aggregate banking sector balance sheet expands more rapidly than when gambling by low-return banks does not occur.
becomes optimal for the regulator to set a higher capital requirement for any given \( y \) (Chart 9).

Moreover, the optimal capital requirement becomes more strongly countercyclical as \( c \) falls.\(^{11}\)

4 The role of public information

We turn next to the role of public information in our model. We separate out the two effects of countercyclical capital requirements on banks’ risk-taking incentives, namely (i) the direct effect of raising the cost of risk-taking, and (ii) the indirect effect of making information about the state of macroeconomic fundamentals public – for example, via the publication of the central bank financial stability reports. If in our set-up, banks were not to observe \( y \) directly, but were instead to find out \( y \) only because the regulator announces it in order to explain their choice of countercyclical capital requirements (and that the regulator can be trusted to announce the true state of \( y \)), capital adequacy requirements would affect banks’ gambling incentives through two distinct channels. First, higher capital adequacy requirements would increase the cost of gambling directly. Second, information about \( y \) would play a role in co-ordinating banks’ actions between gambling and non-gambling equilibria.

To distinguish these two effects, Chart 10 plots the switching point, \( \theta^* \), in the game where banks only have private information (given by equation (3)), and in the game where they are also given public information about \( y \) (given by equation (6)); all the other parameters, including \( k \), are held constant. Thus, the gap between the two lines gives us the marginal effect of public information on banks’ risk-taking incentives for different values of \( y \). As the chart illustrates, public information has a powerful effect in deterring gambling when \( y \) is low. This suggests that ‘moral suasion’ – ie telling banks to stop taking risks – can potentially act as a powerful deterrence when the fundamentals are deteriorating and the policymakers’ warning is thought to reveal accurate information about fundamentals.

By contrast, telling banks that fundamentals are currently good can have a counterproductive effect of encouraging them to co-ordinate to the gambling equilibrium, when the lack of detailed information about banks’ risk-taking activities prevents policymakers from implementing a targeted policy. So how should policymakers communicate when fundamentals are good? If

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\(^{11}\)Similarly, when the policymaker’s objective is characterised by a concern for low-return banks, which is increasing in parameter \( \delta \), we can show that as \( \delta \) rises – ie the regulator becomes more concerned about the social cost associated with gambling – the optimal capital adequacy requirement becomes more stringent and more strongly countercyclical.
future fundamentals are affected by banks’ current risk-taking decisions\(^\text{12}\) then an effective communication strategy for policymakers might be to highlight the future risks to the banking system created by banks’ current risk-taking. For instance, the public release of stress test results could serve this purpose. Although our static framework does not allow us to model explicitly the impact of future fundamentals on banks’ current risk-taking incentives, banks in the real world make long-term investments which are affected by current as well as future fundamentals, and it is plausible that future fundamentals are endogenous to banks’ current risk-taking, as losses caused by unproductive investments could ultimately lead to a banking crisis and a large output loss. In this sense, publicly announcing the results of stress tests can serve as a macroprudential policy tool in itself to the extent that stress tests ‘look through’ contemporaneous exuberance to reveal underlying fragilities. The macroprudential toolkit can therefore operate both directly on costs (through \(k\)), and indirectly on beliefs, which affect outcomes in a world of imperfect information.

5 Conclusions

This paper contributes to the nascent literature on macroprudential regulation by articulating the trade-off faced by policymakers in setting countercyclical capital adequacy requirements when banks have the incentives to make high-risk, high-return investments in order to maintain their reputations in an imperfect information environment. We show that countercyclical capital adequacy requirements are socially optimal for an intermediate range of fundamentals but not when fundamentals are either very weak or very strong. In the intermediate range, some high-ability banks perform well, so low-ability banks have an incentive to gamble in order to safeguard their reputations – or to ‘keep up with the Goldmans’. Optimal macroprudential policy works against this incentive by raising the cost of gambling through higher capital requirement as fundamentals improve.

When fundamentals are very weak however, few banks make profits and hence unprofitable banks have no incentive to gamble in order to preserve their reputations; thus, there is no need to increase capital adequacy requirements in response to a small improvement in fundamentals. And when fundamentals are very strong, most high-ability banks make profits and hence the

\(^{12}\)Rajan (1994) makes such an assumption, as do Aikman et al (2010). Current lending to impaired borrowers could impair bank capital, constraining intermediaries’ future ability to lend to fund productive investment, leading to declining output, see e.g. Gertler and Karadi (2011) and related models.
unprofitable banks have very strong incentives to gamble in order to avoid being labelled as ‘low ability’; in this case, policymakers cannot deter gambling by the unprofitable banks without also imposing excessively high funding costs on high-ability banks, which have no incentive to gamble. This suggests that, when fundamentals are very strong, the need for policymakers to invest in obtaining detailed information about banks’ balance sheets and their investment strategies is particularly strong.

Our analysis also clarifies the role of central bank communication in deterring gambling via its impact on banks’ beliefs. In particular, we show that a warning by policymakers that the fundamentals are deteriorating can be effective in preventing inefficient credit booms when that warning is seen to reveal the true state of the fundamentals and thus helps to co-ordinate banks’ beliefs to the efficient equilibrium. When fundamentals are good, policymakers may wish to focus on communicating the potential damage to future fundamentals and banks’ profitability caused by their current risk-taking activities – for example by releasing stress test results or regular conjunctural analysis of financial stability issues.

Our analysis focuses on a particular role for capital adequacy requirements, namely, that of preventing banks from investing in risky projects that have negative net present value. There are other rationales for countercyclical capital adequacy requirements which we have not considered here, including enhancing loss absorbance and avoiding socially costly financial crises. Our analysis also focuses on the role of capital adequacy requirements in preventing inefficient credit booms, and does not examine its potential role in preventing inefficient credit crunches. Examining all these aspects of countercyclical capital requirements in a single framework is left for future research.
Appendix A: Proofs

A.1 Proof of Proposition 1

Our model already satisfies two conditions set out in Morris and Shin (2003), whose technology we subsequently employ, namely:

Condition 1: *Action Monotonicity*: By \( \frac{\partial p(\theta, l)}{\partial l} > 0 \), \( \pi(\theta, l) \) is non-decreasing in \( l \);

Condition 2: *State Monotonicity*: By \( \frac{\partial p(\theta, l)}{\partial \theta} > 0 \), \( \pi(\theta, l) \) is non-decreasing in \( \theta \);

and we specify \( p(\theta, l) \) is such that:

Condition 3: *Strict Laplacian State Monotonicity*: there exists a unique \( \theta^* \) solving

\[
\int_{l=0}^{1} \pi(\theta^*, l)dl = 0;
\]

holds. Next, suppose \( p(\cdot) \) implies that

Condition 4: There exist \( \theta \in \mathbb{R}, \tilde{\theta} \in \mathbb{R} \) and \( \varepsilon \in \mathbb{R}_{++}, \) such that (a) \( \pi(\theta, l) \leq -\varepsilon \) for all \( l \) and for \( \theta < \tilde{\theta} \); and (b) \( \pi(\theta, l) > \varepsilon \) for all \( l \) and \( \theta > \tilde{\theta} \).

This condition implies that, for sufficiently low (high) values of fundamentals, choosing the safe (risky) option having observed low returns is a dominant action regardless of the aggregate proportion of banks that do so too. In the intervening interval, the dominant action depends on the proportion of banks that follow that action too. Finally, we require that

Condition 5: *Continuity*: \( \int_{l=0}^{1} g(l)\pi(x, l)dl \) is continuous with respect to signal \( x \) and density \( g(.) \).
Condition 6: *Finite expectations of signals*: \( \int_{-\infty}^{\infty} z f(z) dz \) is well defined.

These six conditions ensure the model complies with the generic formulation of Morris and Shin (2003). We therefore use the following result, taken from their paper:

**Lemma 4** (Morris and Shin (2003), Prop. 2.2): Let \( \theta^* \) be defined by Condition 3. For any \( \delta > 0 \), there exists \( \sigma > 0 \) such that for all \( \sigma < \sigma \), if strategy \( s \) survives iterated deletion of strictly dominated strategies, then \( s(x) = \{ \text{safe} \} \) for all \( x \leq \theta^* - \delta \) and \( s(x) = \{ \text{gamble} \} \) for all \( x \geq \theta^* + \delta \).

(We refer readers to Morris and Shin (2003) for the proof.) In words, this says that the support of fundamentals can be divided into two regions: one, for which \( \theta < \theta^* \), in which banks co-ordinate on choosing the safe option conditional on observing low initial returns. Intuitively, fundamentals are not sufficiently high to cause severe reputational damage to announcing low returns when all other banks do so too. In the second region, in which \( \theta > \theta^* \), high fundamentals imply a large degree of reputational damage to announcing low returns. Hence, all banks co-ordinate on the gambling option to minimise the reputational downside to having made a bad initial investment.

Lemma 1 and equation (2) together prove Proposition 1.

### A.2 Proof of Proposition 2

A banker with private signal \( x_i \) forms an expectation of the proportion of gamblers \( l \) given by

\[
\mathbb{E}[l|x_i] = \mathbb{E} \left[ \int_{j \in [0,1]} 1 \ (\text{banker } j \text{ gambles}|x_i) \ dj \right],
\]

where \( 1 \ (\cdot) \) is the indicator function. Under threshold strategies, this is

\[
\mathbb{E}[l|x_i] = \int_{j \in [0,1]} \mathbb{E} \left[ 1 \ (\bar{\theta}_j \geq \theta^*|x_i) \right] dj.
\]

where \( \bar{\theta}_j \) is \( j \)'s posterior mean. Using the expression for \( j \)'s posterior mean, this is

\[
\int_{j \in [0,1]} \mathbb{E} \left[ 1 \ (\bar{\theta}_j \geq \theta^*|x_i) \right] dj = \int_{j \in [0,1]} \Pr \left[ \left( x_j \geq \frac{\sigma^2 + \tau^2 \theta^* - \sigma^2 y}{\tau^2} \right) |x_i \right] dj
\]

\[
= \int_{j \in [0,1]} \Pr \left[ \left( \theta + \varepsilon_j \geq \frac{\sigma^2 + \tau^2 \theta^* - \sigma^2 y}{\tau^2} \right) |x_i \right] dj.
\]
where we have used the private signal in going from the first line to the second. By independence of the noise across agents, this can be written

\[
\int_{j \in [0,1]} \Pr \left[ \varepsilon_j \geq \frac{\sigma^2}{\tau^2} (\theta^* - y) + (\theta^* - \theta|x_i) \right] dj
= 1 \times \Pr \left[ \varepsilon_j \geq \frac{\sigma^2}{\tau^2} (\theta^* - y) + (\theta^* - \theta|x_i) \right]
= \Pr \left[ (x_j|x_i) \geq \frac{\sigma^2}{\tau^2} (\theta^* - y) + \theta^* \right].
\]

Note that

\[
x_j|x_i = (\theta|x_i) + \varepsilon_j = \mathcal{N} \left( \bar{\theta}, \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2} \right) + \mathcal{N} (0, \sigma^2)
= \mathcal{N} \left( \bar{\theta}, \frac{2\sigma^2 \tau^2 + \sigma^4}{\sigma^2 + \tau^2} \right).
\]

Then we have that

\[
E[l|x_i] = 1 - \Phi \left( \frac{\frac{\sigma^2}{\tau^2} (\theta^* - y) + \theta^* - \bar{\theta}}{\sqrt{\frac{2\sigma^2 \tau^2 + \sigma^4}{\sigma^2 + \tau^2}}} \right).
\]

Note that evaluated at the threshold posterior \( \theta^* = \bar{\theta} \), the expected fraction of gamblers \( l^* \) is then

\[
l^* = 1 - \Phi \left( \sqrt{\gamma} (\theta^* - y) \right)
\]

(A-1)

At the threshold, the expected pay-off to gambling is then

\[
\pi(\theta^*, l^*) = b [2R_H + \theta^* - \Phi (\sqrt{\gamma} (\theta^* - y))] - R_L - ck,
\]

(A-2)

which must equal zero in order for banks to be indifferent between gambling and playing safe. Uniqueness requires that equation (A-2) is strictly increasing in \( \theta^* \). This is the case when

\[
d\pi^* / d\theta^* = b [1 - \sqrt{\gamma} \phi (\sqrt{\gamma} (\theta^* - y))] > 0,
\]

which means the necessary condition for uniqueness is

\[
\frac{1}{\phi (\sqrt{\gamma} (\theta^* - y))} > \sqrt{\gamma}.
\]

Since the normal pdf reaches a maximum at \( 1 / \sqrt{2\pi} \), a sufficient condition is that

\[
\gamma < 2\pi.
\]

Therefore the unique threshold is given by

\[
\theta^* = \Phi (\sqrt{\gamma} (\theta^* - y)) + \frac{R_L + ck}{b} - 2R_H.
\]

(See also Morris and Shin (2003), Prop. 3.1).
A.3 Derivation of Pr(safe)

Given banks’ strategies, the probability of a bank gambling in the symmetric switching equilibrium is given by \( l^* \) in (A-1) when \( \theta^* = \overline{\theta} \). Thus, the probability of a bank which has observed \( R_L \) at \( t = 1 \) choosing to gamble is:

\[
Pr(gamble) = 1 - \Phi(\sqrt{\pi}(\theta^* - y)),
\]

and \( \theta^* \) is given by (6). Rearranging (6) and substituting into the above gives:

\[
Pr(gamble) = 1 - \left[ \theta^*(k, y) - \frac{R_L + ck}{b} + 2R_H \right],
\]

\[
Pr(safe) = \theta^*(k, y) - \frac{R_L + ck}{b} + 2R_H.
\]

A.4 The first and the second-order conditions of the policymaker’s maximisation problem

The policymaker’s first-order condition is given by

\[
\frac{\partial S(k, y)}{\partial k} = -2\alpha f(y)c - (1 - \alpha)(1 - \lambda)c + [(1 - \alpha)\lambda + \alpha \{1 - f(y)\}]\frac{\partial \Pi(k, \theta^*)}{\partial k} = 0, \quad (A-3)
\]

Using

\[
\Pi(k, \theta^*) \equiv [1 - Pr(safe)] [b(2R_H - 2ck) + (1 - b)(-2ck)] + Pr(safe)(R_L - ck)
\]

\[
= 2bR_H - 2ck + Pr(safe)(\pi^s - \pi^g)
\]

\[
= 2bR_H - 2ck + Pr(safe)(R_L - 2bR_H + ck),
\]

where \( \pi^g \equiv b(2R_H - 2ck) + (1 - b)(-2ck) \) and \( \pi^s \equiv R_L - ck \) are banks’ returns from gambling and safe options, respectively, such that \( \pi^s - \pi^g > 0 \). Then

\[
\frac{\partial \Pi(k, \theta^*)}{\partial k} = -2c + c Pr(safe) + \frac{\partial Pr(safe)}{\partial k} (\pi^s - \pi^g). \quad (A-4)
\]

Using this in the first-order condition yields

\[
[(1 - \alpha)\lambda + \alpha \{1 - f(y)\}] \frac{\partial Pr(safe)}{\partial k} (\pi^s - \pi^g)
\]

\[
= c (2\alpha f(y) + (1 - \alpha)(1 - \lambda) + (2 - Pr(safe)) [(1 - \alpha)\lambda + \alpha \{1 - f(y)\}])
\]

in the text.

The second-order condition for a maximum is satisfied if and only if

\[
\frac{\partial^2 S(k, y)}{\partial k^2} = [(1 - \alpha)\lambda + \alpha \{1 - f(y)\}] \frac{\partial^2 \Pi(k, \theta^*)}{\partial k^2} < 0,
\]
for which a sufficient condition is

$$\frac{\partial^2 \Pi(k, \theta^*)}{\partial k^2} = c \frac{\partial \Pr(\text{safe})}{\partial k} + \frac{\partial^2 \Pr(\text{safe})}{\partial k^2} \left( \pi^a - \pi^g \right) + c \frac{\partial \Pr(\text{safe})}{\partial k}$$

$$= 2c \frac{\partial \Pr(\text{safe})}{\partial k} + \frac{\partial^2 \Pr(\text{safe})}{\partial k^2} \left( \pi^a - \pi^g \right) < 0$$

By the definition of $\Pr(\text{safe})$ we have that

$$\frac{\partial \Pr(\text{safe})}{\partial k} = d\theta^* - \frac{c}{b}$$

$$\frac{\partial^2 \Pr(\text{safe})}{\partial k^2} = \frac{d^2 \theta^*}{dk^2}$$

so the sufficient condition becomes

$$\frac{\partial^2 \Pi(k, \theta^*)}{\partial k^2} = 2c \left( \frac{d\theta^*}{dk} - \frac{c}{b} \right) + \frac{d^2 \theta^*}{dk^2} \left( \pi^a - \pi^g \right)$$

$$= 2c \left( \frac{1}{1 - \phi \left( \sqrt{\gamma} (\theta^*(k, y) - y) \right) \sqrt{\gamma}} - 1 \right) \frac{c}{b} + \frac{d^2 \theta^*}{dk^2} \left( \pi^a - \pi^g \right) < 0$$

From (7),

$$\frac{d^2 \theta^*}{dk^2} = \frac{-c/b \times \gamma^{3/2} (\theta^* - y) \times \phi \left( \sqrt{\gamma} (\theta^* - y) \right)}{\left( 1 - \phi \left( \sqrt{\gamma} (\theta^* - y) \right) \sqrt{\gamma} \right)^2} \times \frac{d\theta^*}{dk}$$

which means the sufficient condition reduces to

$$\frac{\partial^2 \Pi(k, \theta^*)}{\partial k^2} = \frac{(c^2/b) \sqrt{\gamma} \phi \left( \sqrt{\gamma} (\theta^* - y) \right)}{1 - \phi \left( \sqrt{\gamma} (\theta^*(k, y) - y) \right) \sqrt{\gamma}} \left[ 2 - \frac{(1/b) \gamma (\theta^* - y) \left( \pi^a - \pi^g \right)}{\left( 1 - \phi \left( \sqrt{\gamma} (\theta^* - y) \right) \sqrt{\gamma} \right)^2} \right] < 0$$

where the first term is positive. If $\gamma = 0$, the whole expression becomes zero; but as $\gamma \rightarrow 2\pi$, $\phi \left( \sqrt{\gamma} (\theta^* - y) \right) \sqrt{\gamma} \rightarrow 1$, such that the denominator of the second term in the square brackets tends to zero. Then as long as $\theta^* > y$, the second-order condition is negative. In other words, as long as $\gamma$ is sufficiently large (ie the public signal is quite precise relative to the private signal), the SOC is satisfied and the policymaker’s optimal choice is to set $k^*$ such that $\theta^*(k^*, y) > y$.

### A.5 Proof of Proposition 3

For there to be a case for countercyclical capital adequacy requirement, ie $dk^*/dy > 0$, it must be the case for the relevant range of $y$ (ie $y < \bar{y}$) that $\partial^2 S(k, y)/\partial k\partial y > 0$. From (A-3),

$$\frac{\partial^2 S(k, y)}{\partial k\partial y} = -\alpha f'(y) 2c - \alpha f'(y) \frac{\partial \Pi(k, \theta^*)}{\partial k} + \left[ (1 - \alpha) \lambda + \alpha \left( 1 - f(y) \right) \right] \frac{\partial^2 \Pi(k, \theta^*)}{\partial k\partial y},$$
where \( f'(y) > 0 \). Evaluated at \( k^* \) given by FOC (A-3), we have

\[
\frac{\partial \Pi(k^*, \theta^*)}{\partial k} = \frac{2\alpha f(y)c}{(1 - \alpha)\lambda + \alpha \{1 - f(y)\}}
\]

So

\[
\frac{\partial^2 S(k, y)}{\partial k \partial y} = -\alpha f'(y)2c - \alpha f'(y)\frac{2\alpha f(y)c}{(1 - \alpha)\lambda + \alpha \{1 - f(y)\}}
\]

\[+\left((1 - \alpha)\lambda + \alpha \{1 - f(y)\}\right)\frac{\partial^2 \Pi(k, \theta^*)}{\partial k \partial y}
\]

Since the first two terms are positive, a necessary and sufficient condition for countercyclical capital adequacy requirement is \( \frac{\partial^2 \Pi(k, \theta^*)}{\partial k \partial y} > 0 \).

From equation (A-4)

\[
\frac{\partial^2 \Pi(k, \theta^*)}{\partial k \partial y} = \frac{\partial \text{Pr(safe)}}{\partial y} + \frac{\partial^2 \text{Pr(safe)}}{\partial k \partial y}(\pi^* - \pi^g),
\]

where

\[
\frac{\partial \text{Pr(safe)}}{\partial y} = \frac{d\theta^*}{dy} = -\phi\left(\sqrt{\gamma}(\theta^* - y)\right)\sqrt{\gamma} < 0,
\]

\[
\frac{\partial^2 \text{Pr(safe)}}{\partial k \partial y} = \frac{d^2\theta^*}{dk dy},
\]

\[
\frac{d\theta^*}{dk} = \frac{c/b}{1 - \phi\left(\sqrt{\gamma}(\theta^* - y)\right)\sqrt{\gamma}} > 0,
\]

\[
\frac{d^2\theta^*}{dk dy} = \frac{-\left(c/b\right)\gamma(\theta^* - y)\phi\left(\sqrt{\gamma}(\theta^* - y)\right)}{(1 - \phi\left(\sqrt{\gamma}(\theta^* - y)\right)\sqrt{\gamma})^2 - \sqrt{\gamma}\left(\frac{d\theta^*}{dy} - 1\right)},
\]

in which

\[
\frac{d\theta^*(k, y)}{dy} - 1 = \frac{-1}{1 - \phi\left(\sqrt{\gamma}(\theta^* - y)\right)\sqrt{\gamma}} < 0.
\]

So

\[
\frac{d\theta^*}{dk dy} = \frac{\left(c/b\right)\gamma(\theta^* - y)\phi\left(\sqrt{\gamma}(\theta^* - y)\right)}{(1 - \phi\left(\sqrt{\gamma}(\theta^* - y)\right)\sqrt{\gamma})^2 - \sqrt{\gamma}\left(\frac{d\theta^*}{dy} - 1\right)}
\]

which is positive iff \( \theta^* - y > 0 \). Use this in \( \frac{\partial^2 \Pi(k, \theta^*)}{\partial k \partial y} \) to give

\[
\frac{\partial^2 \Pi(k, \theta^*)}{\partial k \partial y} = c\frac{\phi(\cdot)\sqrt{\gamma}}{1 - \phi(\cdot)\sqrt{\gamma}}\left\{-1 + \left(1/b\right)\gamma(\theta^* - y)\left(\pi^* - \pi^g\right)\right\},
\]

which is positive iff

\[
\frac{(1/b)\gamma(\theta^* - y)}{(1 - \phi(\cdot)\sqrt{\gamma})^2}\left(\pi^* - \pi^g\right) > 1.
\]

A necessary condition for this is \( \theta^* - y > 0 \). For this, since \( d\theta^*(k, y)/dy - 1 < 0 \), there exists a value of \( y, \bar{y} \), such that \( \theta^* - y > 0 \) for \( y < \bar{y} \). Then as \( \gamma \rightarrow 2\pi, \phi(\cdot)\sqrt{\gamma} \rightarrow 1 \), such that when
there exists some $\gamma, \gamma < 2\pi$, such that, for $\gamma \in (\gamma, 2\pi]$, $\partial^2 \Pi(k, \theta^*)/\partial k \partial y > 0$. The lower bound on the noise ratio, $\gamma$, solves:

$$
\frac{(1/b) \gamma (\theta^* - y)}{1 - \phi \left( \sqrt{\gamma} (\theta^* - y) \right)} (\pi^\theta - \pi^y) = 1.
$$

### A.6 Limited liability

In our analysis, we have abstracted from the distortions caused by limited liability in order to focus on the role of reputational concerns on risk-taking incentives. The standard arguments around limited liability would imply the addition of a further distortion to our model, which would tend to reinforce the proclivity of bankers concerned about their reputations to take excessive risk. If the write-downs suffered by equity holders were shifted to some other agent (eg the government) when risky gambles fail or when low returns are announced, the marginal incentive to gamble in the game becomes

$$
\pi(\theta, l) = b[2R_H + p(\theta, l)] - 2bcL.
$$

When $p(\cdot)$ takes the linear form above, the corresponding limited liability (‘LL’) cut-off in the private signals game becomes

$$
\theta^*_L = \frac{1}{2} + 2ck - 2R_H,
$$

which falls below $\theta^*$ (equation (3)) whenever $b < (R_L/2ck) + (1/2)$. Hence, intuitively, limited liability would enhance incentives to gamble in our model. Increases in the capital requirement would continue to disincentivise gambling, by $\partial \theta^*_L/\partial k > 0$.

In the public signals case the equilibrium condition is given by

$$
\pi(\theta^*, l^*) = b[2R_H + \theta^* - \Phi (\sqrt{\gamma} (\theta^* - y))] - 2bcL
$$

such that

$$
\theta^*_L = \Phi (\sqrt{\gamma} (\theta^*_L - y)) + 2ck - 2R_H.
$$

Capital requirements discourage gambling by

$$
\frac{d\theta^*_L}{dk} = \frac{2c}{1 - \phi \left( \sqrt{\gamma} (\theta^*_L - y) \right)} > 0
$$

13Since then the pay-off to a failed gamble is simply $-p(\theta, l)$ as is the pay-off to announcing low returns when $R_L - ck < 0$. 
If it were the case that $\theta_{LL}^* = \theta^*$ in the baseline case, the threshold would respond more strongly to capital requirements in the limited liability case as long as $2 > 1/b$, or $b > 1/2$. Otherwise the presence of limited liability attenuates the effect of the capital requirement on the threshold.

### A.7 Pecuniary spillovers

We have also abstracted from pecuniary spillovers that may operate through asset values. Such spillovers can generate reductions in measured risk, relaxing value-at-risk constraints, or bring about mark-to-market increases in net worth, both of which can lead to an endogenously generated elevated incentive for balance sheet expansion (see eg Adrian and Shin (2010)). A simple way to include such an effect in our model would be as follows. Suppose that the risky gambles undertaken by low-return banks bid up the collateral values of the (ultimately loss-making) projects in which they invest. In this way, the larger is the proportion of gamblers, the smaller the loss faced by unsuccessful gamblers. Let the pay-off associated with a failed gamble be $\zeta l - 2ck < 0$ (where in our baseline model we set $\zeta = 0$ for all $l$ when a gamble fails).

The parameter $\zeta \geq 0$ measures the extent of the positive pecuniary spillover that arises from gambling. Using this in the private signals model yields a marginal gambling incentive of

$$
\pi(\theta, l) = b \left[ 2R_H + p(\theta, l) + \frac{1 - b}{b} \zeta l \right] - R_L - ck.
$$

In the private signals game, the cut-off then becomes

$$
\theta_g^* = 1 - \left[ 1 + \frac{1 - b}{b} \zeta \right] \frac{1}{2} + \frac{R_L + ck}{b} - 2R_H,
$$

such that $\theta_g^* = \theta^*$ only when $\zeta = 0$. In the presence of pecuniary spillovers, such that $\zeta > 0$, the cut-off falls ($\theta_g^* < \theta^*$), and risk-taking is more likely. Once more, this would reinforce the case for capital regulation, which would lean against the pecuniary effects through $\partial \theta^*_g / \partial k > 0$.

In the public signals case with linear $p(\theta, l)$ the equilibrium condition is:

$$
\pi(\theta^*, l^*) = b \left[ 2R_H + \theta^* - 1 + \left( 1 + \frac{1 - b}{b} \zeta \right) \left( 1 - \Phi(\sqrt{\gamma}(\theta^* - y)) \right) \right] - R_L - ck.
$$

Equilibrium occurs when this expression equals zero, giving

$$
\theta_g^* = -\frac{1 - b}{b} \zeta + \left( 1 + \frac{1 - b}{b} \zeta \right) \Phi(\sqrt{\gamma}(\theta^*_g - y)) + \frac{R_L + ck}{b} - 2R_H.
$$

Capital requirements move the threshold according to

$$
\frac{d\theta^*_g}{dk} = \frac{c/b}{1 - (1 + \frac{1 - b}{b} \zeta) \Phi(\sqrt{\gamma}(\theta^*_g - y))} \sqrt{\gamma}
$$
If it were the case that $\theta^*_g = \theta^*$ in the baseline case, the threshold would respond more strongly to capital requirements in the pecuniary externality case as long as $1 + (1 - b) / b \zeta > 1$. 
Chart 1: The timing and pay-offs of the game
Chart 2: Band-pass filtered ratio of UK credit:GDP, 1963 Q2-2010 Q2. The credit series is M4 lending, which comprises monetary financial institutions’ sterling net lending to private sector. The filter returns cyclical variation in the ratio over the 1-20 year frequency range. Shaded regions indicate periods of distress: 1973 Q4-1975 Q4 (secondary banking crisis); 1990 Q3-1994 Q4 (small banks crisis); 2008 Q3–2010 Q2.
Chart 3: Cross-sectional dispersion of equity returns of major UK banks and top UK 100 PNFCs (by market cap)

Chart 4: Cross-sectional dispersion of ROE of top ten UK banks and top ten UK PNFCs (by market cap)
Chart 5: Aggregate credit supply

Chart 6: Optimal capital adequacy requirement, $k^*$
Chart 7: The impact of capital adequacy requirement on banks’ incentives to gamble

Chart 8: Social welfare function
Chart 9: The effect of lower costs of raising equity on optimal capital adequacy requirements, blue ($c = 15\%$), red dashed ($c = 10\%$)

Chart 10: The role of public information


Aikman, D, Haldane, A and Nelson, B (2010), ‘Curbing the credit cycle’, speech delivered at Columbia University Center on Capitalism and Society Annual Conference.


