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The business cycle implications of banks' maturity transformation

Martin M Andreasen, Marcelo Ferman and Pawel Zabczyk

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The business cycle implications of banks' maturity transformation

Martin M Andreasen,⁽¹⁾ Marcelo Ferman⁽²⁾ and Pawel Zabczyk⁽³⁾

Abstract

This paper develops a DSGE model in which banks use short-term deposits to provide firms with long-term credit. The demand for long-term credit arises because firms borrow in order to finance their capital stock which they only adjust at infrequent intervals. We show within a real business cycle framework that maturity transformation in the banking sector in general attenuates the output response to a technological shock. Implications of long-term nominal contracts are also examined in a New Keynesian version of the model, where we find that maturity transformation reduces the real effects of a monetary policy shock.

Key words: Banks, DSGE model, financial frictions, firm heterogeneity, maturity transformation.

JEL classification: E32, E44, E22, G21.

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Summary

Economists, including those at central banks, have a keen interest in understanding the impact of different types of disturbances and tracing how they work through the economy. Such analyses are often conducted using dynamic stochastic general equilibrium (DSGE) models. These models use theory to describe how all the actors in the economy behave, and how they interact over time to produce an economy-wide outcome. The word ‘stochastic’ indicates that there is a fundamental uncertainty pervading the economy, with different types of random ‘shocks’ affecting the dynamics of prices and quantities.

The recent economic crisis highlighted the importance of financial factors in the propagation of economic disturbances. While some analyses, most notably the well-known studies by Kiyotaki and Moore and Bernanke, Gertler and Gilchrist have studied the role of financial frictions, they did so without explicitly modelling the behaviour of the banking sector. A growing number of papers has therefore incorporated this sector into general equilibrium models. With a few exceptions, however, this literature abstracts from a key aspect of banks’ behaviour - ie, the fact that banks fund themselves using short-term deposits while providing long-term credit. This so-called ‘maturity transformation’ has the potential to affect the propagation of stochastic shocks, and the aim of this paper is to propose a DSGE model which helps to clarify how.

A general equilibrium approach is essential for our analysis, because we are interested not only in explaining how long-term credit affects the economy but also in the important feedback effects from the rest of the economy to banks and their credit supply. There are, however, several technical difficulties which mean that maturity transformation based on long-term credit has not been widely studied in a DSGE set up. The framework we propose overcomes these difficulties and remains conveniently tractable. We assume, in particular, that firms need credit to purchase their capital stock and that they change their level of capital at random intervals - meaning they require financing for longer periods of time.

Importantly, we show that this set up, by itself, has no implications for shock propagation. This means that the aggregate effects of maturity transformation we obtain are *not* a trivial implication of the infrequent capital adjustment assumption. It is only when we introduce banks, which use



accumulated wealth and short-term deposits from the household sector to provide longer-term credit to firms, that maturity transformation starts playing a role.

We illustrate the quantitative implications of maturity transformation in two standard types of DSGE models – one in which firms can adjust their prices instantly, and one in which they can only reset them at infrequent intervals. We focus on stochastic shocks affecting productivity and nominal interest rates. Our analysis highlights the existence of a *credit maturity attenuator effect*, meaning that the response of output to both types of shocks decreases with higher degrees of maturity transformation.

A positive unexpected change in firm productivity has a smaller effect on output because banks' revenues respond less to the shock. In particular, many loans will have been granted prior to the shock, and cannot be adjusted quickly. This smaller increase in banks' net worth means that the increase in the amount of credit they can supply will also be smaller, constraining the increase in output – relative to the case of no maturity mismatch and no long-term lending.

In a model in which firms cannot adjust their prices instantly, increasing the degree of maturity transformation also attenuates the fall in output following an unexpected increase in interest rates. This can be explained by three main channels. First, the resultant fall in production lowers the price of capital. As above, changes in the price of capital have weaker effects on banks' revenues for higher degrees of maturity transformation, and this reduces the fall in output following the disturbance. Second, the shock generates a fall in inflation and raises the *ex-post* real interest rate on loans. The aggregate value of loans falls by less in the presence of maturity transformation (due to the first channel) and the higher *ex-post* real rate therefore has a larger positive effect on banks' balance sheets and output than without long-term loans. Finally, the smaller reduction in output (and income) following the shock implies that households' deposits fall by less with maturity transformation. Banks are therefore able to provide more credit and this reduces the contraction in output.



1 Introduction

The seminal contributions by Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Bernanke, Gertler and Gilchrist (1999) show how financial frictions augment the propagation of shocks in otherwise standard real business cycle (RBC) models.¹ This well-known financial accelerator effect is derived without explicitly modelling the behaviour of the banking sector and a growing literature has therefore incorporated this sector into a general equilibrium framework.² With a few exceptions, banks are assumed to receive one-period deposits which are instantaneously passed on to firms as one-period credit. Hence, most of the papers in this literature do not address a key aspect of banks' behaviour, namely the transformation of short-term deposits into long-term credit.

The aim of this paper is to examine how banks' maturity transformation affects business cycle dynamics. Our main contribution is to show how maturity transformation in the banking sector can be introduced in otherwise standard dynamic stochastic general equilibrium (DSGE) models, including the models by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). We then illustrate the quantitative implications of maturity transformation, first in a simple RBC model with long-term real contracts and subsequently in a New Keynesian model with long-term nominal contracts.

Some implications of maturity transformation have been studied outside a general equilibrium framework. For instance, Flannery and James (1984), Vourougou (1990), and Akella and Greenbaum (1992) document that asset prices of banks with a large maturity mismatch on their balance sheets react more to unanticipated interest rate changes than asset prices of banks with a small maturity mismatch. Additionally, the papers by Gambacorta and Mistrulli (2004) and Van den Heuvel (2006) argue that banks' maturity transformation also affects the transmission mechanism of a monetary policy shock. In our context, however, a general equilibrium framework is necessary because we are interested not only in explaining how long-term credit affects the economy but also in the important feedback effects from the rest of the economy to banks and their credit supply.

¹See also Berger and Udell (1992); Peek and Rosengren (2000); Hoggarth, Reis and Saporta (2002); Dell'Ariccia, Detragiache and Rajan (2008); Chari, Christiano and Kehoe (2008); Campello, Graham and Harvey (2009) for a discussion of the real impact of financial shocks.

²See for instance Chen (2001), Aikman and Paustian (2006), Goodfriend and McCallum (2007), Teranishi (2008), Gertler and Karadi (2009), Gertler and Kiyotaki (2009), and Gerali, Neri, Sessa and Signoretti (2009).



Maturity transformation based on long-term credit has to our knowledge not been studied in a general equilibrium setting, although long-term financial contracts have been examined by Gertler (1992) and Smith and Wang (2006).³ This may partly be explained by the fact that introducing long-term credit and maturity transformation in a general equilibrium framework is quite challenging for at least three reasons. First, one needs to explain why firms demand long-term credit. Second, banks' portfolios of outstanding loans are difficult to keep track of in the presence of long-term credit. Finally, and related to the second point, model aggregation is often very difficult or simply infeasible when banks provide long-term credit.

The framework we propose overcomes these three difficulties and remains conveniently tractable. Our novel assumption is to consider the case where firms face a constant probability α_k of being unable to adjust their capital stock in every period. The capital level of firms which cannot adjust their capital stock is assumed to slowly depreciate over time. This set up generates a demand for long-term credit when we impose the standard assumption that firms borrow in order to finance their capital stock. That is, firms require a given amount of credit for potentially many periods, because they may be unable to adjust their capital levels for many periods in the future.

Interestingly, our set up with infrequent capital adjustments implies heterogeneity at the firm level. In particular, the firm-level dynamics of capital in our model is in line with the main stylised fact which the literature on non-convex investment adjustment costs aims to explain, ie that firms usually invest in a lumpy fashion (Caballero and Engel (1999); Cooper and Haltiwanger (2006)). However, we show for a wide class of DSGE models without a banking sector that the dynamics of prices and *aggregate* variables are unchanged relative to the case where firms adjust capital in every period. This result relies on firms having a Cobb-Douglas production function, as the scale of each firm then becomes irrelevant for all prices and aggregate quantities. We refer to this result as the '*irrelevance of infrequent capital adjustments*'. This is a very important result because it shows that the constraint we impose on firms' ability to adjust capital does not affect the aggregate properties of many existing DSGE models. Crucially, the aggregate effects of maturity transformation we obtain in a model with a banking sector are *not* a trivial implication of the infrequent capital adjustment assumption.

³The paper by Gertler and Karadi (2009) implicitly allows for maturity transformation by letting banks receive one-period deposits and invest in firms' equity, which have infinite duration.

Our next step is to introduce a banking sector into the model. We specify the behaviour of banks along the lines suggested by Gertler and Karadi (2009) and Gertler and Kiyotaki (2009). That is, banks receive short-term deposits from the household sector and face an agency problem in the relationship with households. Differently from Gertler and Karadi (2009) and Gertler and Kiyotaki (2009), banks' assets consist in our case of long-term credit contracts supplied to firms. As we match the life of the credit contracts to the number of periods the firm does not adjust capital, the average life of banks' assets in the economy as a whole is $\mathcal{D} \equiv 1/(1 - \alpha_k)$. When $\alpha_k > 0$, this implies that banks face a maturity transformation problem because they use short-term deposits and accumulated wealth to provide long-term credit. The standard case of no maturity transformation in the banking sector is thus recovered when $\alpha_k = 0$.

We first illustrate the quantitative implications of maturity transformation in a simple RBC model with long-term real contracts following a positive technological shock. Our analysis shows the existence of a *credit maturity attenuator effect*, meaning that the response of output to this shock is weaker the higher the degree of maturity transformation. The intuition for this result is as follows. The positive technological shock increases the demand for capital and its price. In the model without maturity transformation, the entire portfolio of loans in banks' balance sheets is instantly reset to reflect the higher price of capital. This means that firms now need to borrow more to finance the same amount of productive capital. Banks provide the extra funds to firms and consequently benefit from higher revenues. With maturity transformation, on the other hand, only a fraction of all loans in banks' balance sheets is instantly reset, creating a smaller increase in banks' revenues. As a result, the increase in banks' net worth and consequently in output are weaker the higher the degree of maturity transformation.

Our second illustration studies the quantitative implications of maturity transformation in a New Keynesian model with nominal financial contracts. In the case of long-term lending, the distinction between nominal and real contracts is especially interesting because long-term inflation expectations directly affect firms' decisions. Here, we focus on how maturity transformation affects the monetary transmission mechanism.

We find that increasing the degree of maturity transformation attenuates the fall in output following a contractionary monetary policy shock. This result can be explained by three main channels. First, the fall in real activity lowers the price of capital. As before, changes in the price



of capital have weaker effects on banks' revenues for higher degrees of maturity transformation, and this reduces the fall in output following the monetary contraction. Second, there is a debt-deflation mechanism that interacts with the channel just described. The monetary contraction generates a fall in inflation and raises the *ex-post* real interest rate on loans. The aggregate value of loans falls by less in the presence of maturity transformation (due to the first channel) and the higher *ex-post* real rate therefore has a larger positive effect on banks' balance sheets and output than without long-term loans. Finally, the smaller reduction in output (and income) following the shock implies that households' deposits fall by less with maturity transformation. Banks are therefore able to provide more credit and this reduces the contraction in output.

The remainder of the paper is structured as follows. Section 2 extends the simple RBC model with infrequent capital adjustments and analyses the implications of this assumption. This model is extended in Section 3 with a banking sector performing maturity transformation based on real financial contracts. The following section explores how maturity transformation and long-term nominal contracts affect the monetary transmission mechanism within a New Keynesian model. Concluding comments are provided in Section 5.

2 A standard RBC model with infrequent capital adjustments

The aim of this section is to describe how a standard real business cycle (RBC) model can be extended to incorporate the idea that firms do not optimally choose capital in every period. We show that this extension does not affect the dynamics of any prices and aggregate variables in the model. This result holds under weak assumptions and generalises to a wide class of DSGE models. We proceed as follows. Sections 2.1 to 2.3 describe how we modify the standard RBC model. The implications of this assumption are then analysed in Section 2.4.

2.1 Households

Consider a representative household which consumes c_t , provides labour h_t , and accumulates capital k_t^s . The contingency plans for c_t , h_t , and i_t are determined by maximising

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{(c_{t+j} - b c_{t+j-1})^{1-\phi_0}}{1 - \phi_0} - \phi_2 \frac{h_{t+j}^{1+\phi_1}}{1 + \phi_1} \right) \quad (1)$$

subject to

$$c_t + i_t = h_t w_t + r_t^k k_t^s \quad (2)$$

$$k_{t+1}^s = (1 - \delta) k_t^s + i_t \left[1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] \quad (3)$$

and the usual no-Ponzi game condition. The left-hand side of equation (2) lists expenditures on consumption and investment i_t , while the right-hand side lists the sources of income. We let w_t denote the real wage and r_t^k be the real rental rate of capital. As in Christiano *et al* (2005), the household's preferences are assumed to display internal habits with intensity parameter b . The capital depreciation is determined by δ , while the capital accumulation equation includes quadratic adjustment costs as in Christiano *et al* (2005).

2.2 Firms

We assume a continuum of firms indexed by $i \in [0, 1]$ and owned by the household. Profit in each period is given by the difference between firms' output and costs, where the latter are composed of capital rental fees $r_t^k k_{i,t}$ and the wage bill $w_t h_{i,t}$. Both costs are paid at the end of the period. We assume that output is produced from capital and labour according to a standard Cobb-Douglas production function

$$y_{i,t} = a_t k_{i,t}^\theta h_{i,t}^{1-\theta}. \quad (4)$$

The aggregate level of productivity a_t is assumed to evolve according to

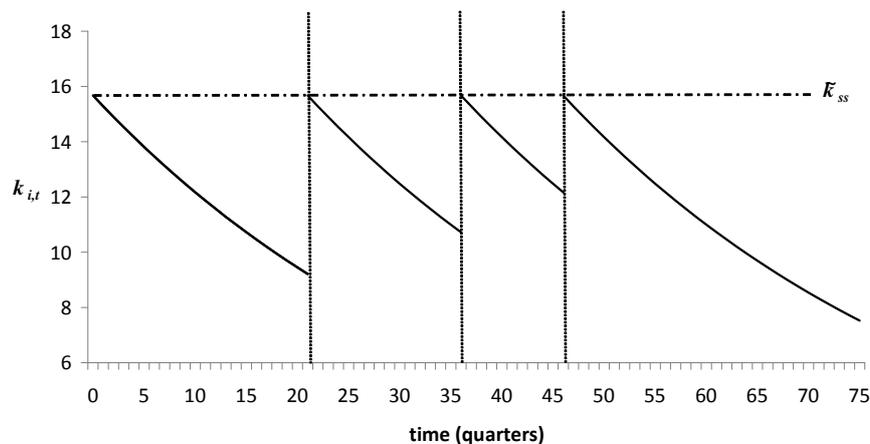
$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a, \quad (5)$$

where $\varepsilon_t^a \sim \mathcal{NID}(0, \sigma_a^2)$ and $\rho_a \in (-1, 1)$.

The model has so far been completely standard. We now depart from the typical RBC set up by assuming that firms can only choose their optimal capital level with probability $1 - \alpha_k$ in every period. The probability $\alpha_k \in [0, 1)$ is assumed to be the same for all firms and across time. Capital for firms which cannot reoptimise is assumed to depreciate by the rate δ over time. All firms, however, are allowed to choose labour in every period as in the standard RBC model.

One way to rationalise the restriction we impose on firms' ability to adjust capital is as follows. The decision of a firm to purchase a new machine or to set up a new plant usually involves large fixed costs. These could be costs related to gathering information, decision-making, and training

Figure 1: Infrequent capital adjustments - dynamics at the firm level



Notes: Bold lines represent the capital of the considered firm. Vertical lines mark the periods in which the firm is allowed to reoptimize capital. The dotted horizontal line represents the steady-state level.

the workforce. We do not attempt to model the exact nature of these costs and how firms choose which period to adjust capital, but our set up still captures the main macroeconomic implications of firms' infrequent changes in capital.

To see how this assumption affects the level of capital for the i 'th firm, consider the example displayed in Figure 1 for an economy in steady state. The downward-sloping lines denote the capital level for the i 'th firm over time. The dashed horizontal line represents the optimal choice of capital for firms that are able to optimise (\tilde{k}_{ss}), whereas vertical lines mark the periods in which the firm is allowed to reoptimize capital. In this example, the firm is not allowed to reoptimize capital from period zero until the first vertical line and simply sees its capital depreciate. Once the vertical line is reached the firm adjusts its capital stock and chooses \tilde{k}_{ss} . In the following periods capital depreciates again until the firm is allowed to adjust capital once more. Note that the vertical lines are not equidistant, reflecting our assumption of random capital adjustment dates.

It is important to note that the dynamics of capital at the firm level implied by our assumption is in line with the key finding in the empirical literature on non-convex investment adjustment costs (Caballero and Engel (1999); Cooper and Haltiwanger (2006)). This literature uses micro data to document that firms usually invest in a lumpy fashion, ie there are many periods of investment inaction followed by spikes in the level of investment and capital.

Our assumption on firms' ability to adjust their capital level implies that there are two groups of firms in every period : i) a fraction $1 - \alpha_k$ which potentially change their capital level and ii) the remaining fraction α_k which produce using the depreciated capital chosen in the past. All reoptimising firms choose the same level of capital due to absence of cross-sectional heterogeneity. We denote this capital level by \tilde{k}_t . By the same token, all firms that produce in period t using capital chosen in period $t - m$ also set the same level of labour which we denote by $\tilde{h}_{t|t-m}$ for $m = \{1, 2, \dots\}$.⁴ Hence, firms adjusting capital in period t solve the problem

$$\max_{\tilde{k}} \mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[a_{t+j} ((1 - \delta)^j \tilde{k}_t)^\theta \tilde{h}_{t+j|t}^{1-\theta} - r_{t+j}^k (1 - \delta)^j \tilde{k}_t - w_{t+j} \tilde{h}_{t+j|t} \right]. \quad (6)$$

We see that firms account for the fact that they might not adjust capital for potentially many periods. Note that capital depreciates while the firm does not adjust its capital level, and the amount of capital available in period $t + j$ for a firm that last optimised in period t is $(1 - \delta)^j \tilde{k}_t$.

The first-order condition for the choice of capital \tilde{k}_t is given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} \theta (1 - \delta)^{j\theta} \tilde{k}_t^{\theta-1} \tilde{h}_{t+j|t}^{1-\theta} - r_{t+j}^k (1 - \delta)^j \right) = 0. \quad (7)$$

If $\alpha_k > 0$, the optimal choice of capital now depends on the discounted value of all future expected marginal products of capital and rental rates. Note also that the discount factor between periods t and $t + j$ incorporates α_k^j which is the probability that the firm cannot adjust its level of capital after j periods. If $\alpha_k = 0$, equation (7) reduces to the standard case where the firm sets capital such that its marginal product equates the rental rate.

The first-order condition for labour is given by

$$h_{i,t} = \left(\frac{w_t}{a_t (1 - \theta)} \right)^{-\frac{1}{\theta}} k_{i,t} \text{ for } i \in [0, 1]. \quad (8)$$

Here, we do not need to distinguish between optimising and non-optimising firms because all firms are allowed to optimally set their labour demand each period. It is important to note that the capital-labour ratio only depends on aggregate variables and is therefore identical for all firms.

⁴A similar notation for capital implies $\tilde{k}_{t|t-m} \equiv \tilde{k}_{t-m} (1 - \delta)^m$.

2.3 Market clearing and aggregation

In equilibrium, the aggregate supply of capital must equal the capital demand of all firms, ie

$$k_t^s = \int_0^1 k_{i,t} di. \quad (9)$$

A fraction of $1 - \alpha_k$ firms choose \tilde{k}_t in period t . The capital demand among non-reoptimising firms is equal to the aggregate capital in period $t - 1$ rescaled by α_k and adjusted for depreciation. This is because all firms face the same probability of being allowed to adjust capital. Market clearing in the rental market for capital is therefore given by

$$k_t^s = (1 - \alpha_k) \tilde{k}_t + \alpha_k (1 - \delta) k_{t-1}^s. \quad (10)$$

Note that $k_t^s = \tilde{k}_t$ when $\alpha_k = 0$ and all firms are allowed to adjust their capital level in every period.

Market clearing in the labour market implies

$$h_t = \int_0^1 h_{i,t} di, \quad (11)$$

and (8) therefore gives

$$h_t = \left(\frac{w_t}{a_t (1 - \theta)} \right)^{-\frac{1}{\theta}} k_t^s. \quad (12)$$

Finally, the goods market clears when

$$y_t \equiv \int_0^1 y_{i,t} di = c_t + i_t. \quad (13)$$

2.4 Implications of infrequent capital adjustments

The parameter α_k determines the fraction of firms reoptimising capital in a given period, or equivalently the average numbers of periods that the i 'th firm operates without adjusting its capital level. It is therefore natural to expect that different values of α_k result in different business cycle implications for prices and aggregate variables in the model. For instance, large values of α_k imply that adjusting firms are more forward looking compared to the case where α_k is small, and this could potentially give rise to different dynamics for prices and aggregate variables. This simple intuition turns out *not* to be correct: different values of α_k actually gives exactly the same aggregate model dynamics.⁵ We summarise this result in Proposition 1.

⁵Note that the implications of infrequent capital adjustments differ substantially from the well-known real effects of staggered *nominal price* contracts when specified following Calvo (1983).

Proposition 1 The parameter α_k has no impact on the law of motions for $c_t, i_t, h_t, w_t, r_t^k, k_t^s$, and a_t .

Proof. The model consists of eight variables $c_t, i_t, h_t, w_t, r_t^k, k_t^s, a_t, \tilde{k}_t$ and eight equations. The parameter α_k only enters in (7) and (10). The dynamics of k_t^s follows from \tilde{k}_t and the system can therefore be reduced to seven equations in seven variables $c_t, i_t, h_t, w_t, r_t^k, \tilde{k}_t, a_t$. Note also that (12) implies $\tilde{k}_t^{\theta-1} \tilde{h}_{t+j|t}^{1-\theta} = \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1-\theta}{\theta}}$ which allow us to simplify the algebra. To prove the proposition, we need to show that the first-order condition for capital when $\alpha_k = 0$ is equivalent to the first-order condition for capital when $\alpha_k > 0$, ie

$$\forall t : a_t \theta \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} = r_t^k \Leftrightarrow$$

$$\forall t : E_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} (1-\delta)^j \left(a_{t+j} \theta \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} - r_{t+j}^k \right) = 0.$$

To show \Rightarrow we observe that $a_t \theta \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} = r_t^k$ implies that each of the elements in the infinite sum is equal to zero and so is the conditional expectations. To prove \Leftarrow we first lead the infinite sum by one period and multiply the expression by $\alpha_k \beta (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} > 0$. This gives

$$E_{t+1} \left[\sum_{i=1}^{+\infty} \alpha_k^i \beta^i \frac{\lambda_{t+i}}{\lambda_t} (1-\delta)^i \left(a_{t+i} \theta \left(\frac{w_{t+i}}{a_{t+i}(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} - r_{t+i}^k \right) \right] = 0$$

and by the law of iterated expectations

$$E_t \left[\sum_{i=1}^{+\infty} \alpha_k^i \beta^i \frac{\lambda_{t+i}}{\lambda_t} (1-\delta)^i \left(a_{t+i} \theta \left(\frac{w_{t+i}}{a_{t+i}(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} - r_{t+i}^k \right) \right] = 0. \quad (14)$$

Another way to express the infinite sum is by

$$E_t \left[a_t \theta \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} - r_t^k \right] +$$

$$E_t \left[\sum_{j=1}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} (1-\delta)^j \left(a_{t+j} \theta \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} - r_{t+j}^k \right) \right] = 0$$

Using (14), this expression reduces to

$$a_t \theta \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} = r_t^k$$

as required.

The intuition behind this irrelevance proposition is simple. When the capital supply is predetermined, it does not matter if a fraction of firms cannot change their capital level because

the other firms have to demand the remaining amount of capital to ensure equilibrium in the capital market. The fact that the capital-labour ratio is the same across firms further implies that aggregate labour demand is similar to the case where all firms can adjust capital. The aggregate output produced by firms is also unaffected due to the presence of constant returns to scale in the production function. The result in Proposition 1 is thus similar to the well-known result from microeconomics for a market in perfect competition and constant returns to scale, where only the aggregate production level can be determined but not the production level of the individual firms.

There are at least two interesting implications of the infrequent capital adjustments at the firm level. First, the distortion on firms' ability to change their capital level does not break the relation from the standard RBC model, where the marginal product of capital equals its rental price. In other words, the induced distortion in the capital market does not lead to any inefficiencies because the remaining part of the economy is sufficiently flexible to compensate for the imposed friction.

Second, the infrequent capital adjustments give rise to firm heterogeneity. There will be firms which have not adjusted their capital levels for a long time and hence have small capital levels due to the effect of depreciation. These firms will therefore produce a small amount of output and will also have a low labour demand due to (8). Similarly, there will also be firms which have recently adjusted their capital levels and therefore produce relatively high quantities and have high labour demands. This firm heterogeneity relates to the literature on firm-specific capital as in Sveen and Weinke (2005), Woodford (2005), among others.

When proving Proposition 1 we only used two assumptions from our RBC model, besides a predetermined capital supply. Hence, the irrelevance result for α_k holds for all DSGE models with these two properties. We state this observation in Corollary 1.

Corollary 1 Proposition 1 holds for any DSGE model with the following two properties:

1. The capital labour ratio is identical for all firms
2. The parameter α_k only enters into the equilibrium conditions for capital



Examples of DSGE models with these properties are models with sticky prices, sticky wages, monopolistic competition, habits, to name just a few. The three most obvious ways to break the irrelevance of the infrequent capital adjustments can be inferred from (8). That is, if firms i) do not have a Cobb-Douglas production function, ii) face firm-specific productivity shocks, or iii) face different wage levels due to imperfections in the labour market.

Another way to break the irrelevance of infrequent capital adjustments is to make α_k affect the remaining part of the economy. We will in the next section show how this can be accomplished by introducing a banking sector into the model.

3 An RBC model with banks and maturity transformation

This section incorporates a banking sector into the RBC model developed above. Here, we impose the standard assumption that firms need to borrow prior to financing their desired level of capital. This requirement combined with infrequent capital adjustments generate a demand for long-term credit at the firm level. Banks use one-period deposits from households and accumulated wealth (ie net worth) to meet this demand. As a result, banks face a maturity transformation problem because they use short-term deposits to provide long-term credit.

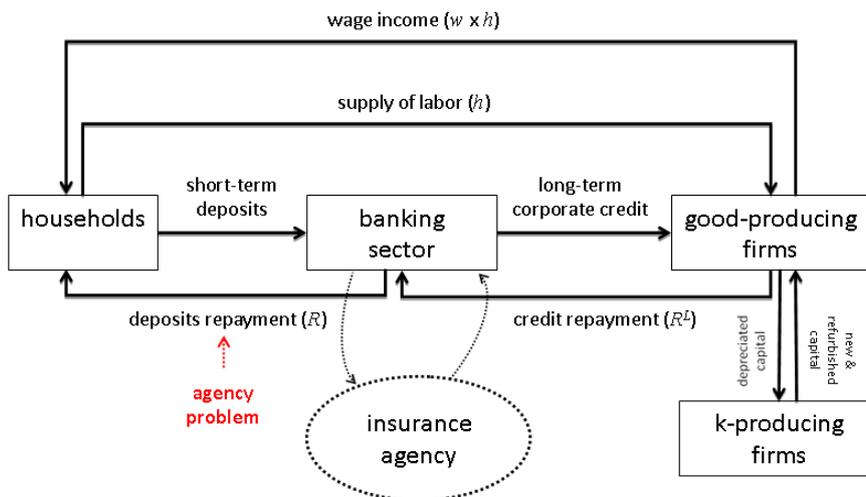
Having outlined the novel feature of our model, we now turn to the details. The economy is assumed to have four agents: i) households, ii) banks, iii) good-producing firms, and iv) capital-producing firms. The latter type of firms are standard in the literature and introduced to facilitate the aggregation (see for instance Bernanke *et al* (1999)).

The interactions between the four types of agents are displayed in Figure 2.⁶ Households supply labour to the good-producing firms and make short-term deposits in banks. Banks then use these deposits together with their own wealth to provide long-term credit to good-producing firms. The good-producing firms hire labour and use credit to obtain capital from the capital-producers. The latter firms simply repair the depreciated capital and build new capital which they provide to good-producing firms.

We proceed as follows. Sections 3.1 and 3.2 revisit the problems for the households and

⁶For simplicity, Figure 2 does not show profit flows going from firms and banks to households.

Figure 2: RBC model with banks and maturity transformation



good-producing firms when banks are present. Sections 3.3 and 3.4 are devoted to the behaviour of banks and the capital-producing firm, respectively. Market clearing conditions and the model calibration are discussed in Section 3.5. We then study the quantitative implication of maturity transformation following a technology shock in Section 3.6.

3.1 Households

Each household is inhabited by workers and bankers. Workers provide labour h_t to good-producing firms and in exchange receive labour income $w_t h_t$. Each banker manages a bank and accumulates wealth that is eventually transferred to his respective household. It is assumed that a banker becomes a worker with probability α_b in each period, and only in this event is the wealth of the banker transferred to the household. Each household postpones consumption from periods t to $t + 1$ by holding short-term deposits in banks.⁷ Deposits b_t made in period t are repaid in the beginning of period $t + 1$ at the gross deposit rate R_t .

The households’ preferences are as in Section 2.1. The lifetime utility function is maximised with respect to c_t , b_t , and h_t subject to

$$c_t + b_t = h_t w_t + R_{t-1} b_{t-1} + T_t. \tag{15}$$

⁷As in Gertler and Karadi (2009), it is assumed that a household is only allowed to deposit savings in banks owned by bankers from a different household. Additionally, it is assumed that within a household there is perfect consumption insurance.

Here, T_t denotes the net transfers of profits from firms and banks. Note that the households are not allowed to accumulate capital, as in the previous model, but are forced to postpone consumption through deposits in banks.

3.2 *Good-producing firms*

We impose the requirement on good-producing firms that they need credit to finance their capital stock. With infrequent capital adjustments these firms therefore demand long-term credit which we assume is provided by banks.

It is convenient in this set up to match the number of periods a firm cannot adjust capital to the duration of its financial contract with the bank. That is, the financial contract lasts for all periods where the firm cannot adjust its capital level, and a new contract is signed whenever the firm is allowed to adjust capital. Since the latter event happens with probability $1 - \alpha_k$ in each period, the exact maturity of a contract is not known *ex-ante*. The average maturity of all existing contracts, however, is known and given by $\mathcal{D} = 1 / (1 - \alpha_k)$.

The specific obligations in the financial contract are as follows. A contract signed in period t specifies the amount of capital \tilde{k}_t that the good-producing firm wants to finance for as long as it cannot reoptimise capital. As in Section 2.2, capital depreciates over time, meaning that after j periods the firm only needs funds for $(1 - \delta)^j \tilde{k}_t p_t^k$ units of capital. Here, p_t^k denotes the real price of capital. The bank provides credit to finance the rental of capital throughout the contract at a constant (net) interest rate $r_t^L + \delta$. The first component of the loan rate r_t^L reflects the fact that firms need external finance, whereas the second component δ refers to the depreciation cost associated with capital usage. It should be emphasised that we do not consider informational asymmetries between banks and the firm, implying that the firm cannot deviate from the signed contract or renegotiate it as considered in Hart and Moore (1998).

As in the standard RBC model, good-producing firms also hire labour which is combined with capital in a Cobb-Douglas production function. We continue to assume that the wage bill is paid after production takes place, implying that demand for credit is uniquely associated with firms' capital level.



The assumptions above are summarised in the expression for $profit_{t+j|t}$, ie the profit in $t + j$ for a firm that entered a financial contract in period t :

$$profit_{t+j|t} = \underbrace{a_{t+j} [(1 - \delta)^j \tilde{k}_t]^\theta h_{t+j|t}^{1-\theta}}_{\text{production revenue}} - \underbrace{w_{t+j} \tilde{h}_{t+j|t}}_{\text{wage bill}} - \underbrace{(r_t^L + \delta) p_t^k [(1 - \delta)^j \tilde{k}_t]}_{\text{capital rental bill}}. \quad (16)$$

Note that all future cash flow between the firm and the bank are determined with certainty for the duration of the contract. That is, the firm needs to fund \tilde{k}_t units of capital based on a fixed price p_t^k , which is done at the fixed loan rate r_t^L .

The good-producing firm determines capital and labour by maximising the net present value of future profits. Using the households' stochastic discount factor, the first-order condition for the optimal level of capital \tilde{k}_t is given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\theta a_{t+j} (1 - \delta)^{j\theta} (\tilde{k}_t)^{\theta-1} h_{t+j|t}^{1-\theta} - (r_t^L + \delta) p_t^k (1 - \delta)^j \right] = 0. \quad (17)$$

The price for financing one unit of capital throughout the contract is thus constant and given by $(r_t^L + \delta) p_t^k$. The first-order condition for the optimal choice of labour is exactly as in the standard RBC model, ie as in (8).

3.3 The banking sector

We incorporate banks following the approach suggested by Gertler and Kiyotaki (2009) and Gertler and Karadi (2009). Their specification has two key elements. The first is an agency problem that characterises the interaction between households and banks and limits banks' leverage. This in turn limits the amount of credit provided by banks to the good-producing firms. The agency problem only constrains banks' supply of credit as long as banks cannot accumulate sufficient wealth to be independent of deposits from households. The second key element is therefore to assume that bankers retire with probability α_b in each period, and when doing so, transfer wealth back to their respective households. The retired bankers are assumed to be replaced by new bankers with a sufficiently low initial wealth to make the aggregate wealth of the banking sector bounded.⁸

Although our model is very similar to the model by Gertler and Karadi (2009), the existence of long-term financial contracts complicates the aggregation. This is because new bankers must

⁸Note that their second assumption generates heterogeneity in the banking sector and there does not exist a representative bank.

inherit the outstanding long-term contracts from the retired bankers, but the new bankers may not be able to do so with a low initial wealth. We want to maintain the assumption of bankers having to retire with probability α_b , because this justifies the transfer of wealth from the banking sector to the households and in turn to consumption. Our solution is to introduce an insurance agency financed by a proportional tax on banks' profit. When a banker retires, the role of this agency is to create a new bank with an identical asset and liability structure and effectively guarantee the outstanding contracts of the old bank. This agency therefore ensures the existence of a representative bank and that the wealth of this bank is bounded with an appropriately calibrated tax rate.

We next describe the balance sheet of the representative bank in Section 3.3.1 and present the agency problem in Section 3.3.2.

3.3.1 Banks' balance sheets

As mentioned earlier, the representative bank uses accumulated wealth n_t and short-term deposits from households b_t to provide credit to good-producing firms. This implies the following identity for the bank's balance sheet

$$len_t \equiv n_t + b_t, \quad (18)$$

where len_t represents the amount of lending.

The net wealth generated by the bank in period t is given by

$$n_{t+1} = (1 - \tau) [rev_t - R_t b_t], \quad (19)$$

where τ is the proportional tax rate and rev_t denotes revenue from lending to good-producing firms. The term $R_t b_t$ constitutes the value of deposits repaid to consumers. Combining the last two equations gives the following law of motion for the bank's net wealth

$$n_{t+1} = (1 - \tau) [rev_t - R_t len_t + R_t n_t]. \quad (20)$$

The imposed structure for firms' inability to adjust capital implies simple expressions for len_t



and rev_t . Starting with the total amount of lending in period t , we have

$$\begin{aligned}
 len_t &\equiv \int_0^1 p_{i,t}^k k_{i,t} di & (21) \\
 &= \underbrace{(1 - \alpha_k) p_t^k \tilde{k}_t}_{\text{adjust in period } t} + \underbrace{(1 - \alpha_k) \alpha_k (1 - \delta) p_{t-1}^k \tilde{k}_{t-1}}_{\text{adjust in period } t-1} + \dots \\
 &= (1 - \alpha_k) \sum_{j=0}^{\infty} ((1 - \delta) \alpha_k)^j p_{t-j}^k \tilde{k}_{t-j}
 \end{aligned}$$

where simple recursions are easily derived. Similarly, for the total revenue we have

$$rev_t = (1 - \alpha_k) \sum_{j=0}^{\infty} ((1 - \delta) \alpha_k)^j R_{t-j}^L p_{t-j}^k \tilde{k}_{t-j}. \quad (22)$$

Here, $R_t^L \equiv 1 + r_t^L$ is the gross loan rate. The intuition for these equations is as follows. A fraction $(1 - \alpha_k)$ of the bank's lending and revenue in period t relates to credit provided to adjusting firms in the same period. Likewise, a fraction $(1 - \alpha_k) \alpha_k (1 - \delta)$ of lending and revenue relates to credit provided to firms that last adjusted capital in period $t - 1$, and so on. For all contracts, the loans made j periods in the past are repaid at the rate R_{t-j}^L . Thus, a large values of α_k makes the bank's balance sheet less exposed to changes in R_t^L compared to small values of α_k . The most important thing to notice, however, is that α_k affects the bank's lending and revenue and thereby its balance sheet, implying that the irrelevance theorem of infrequent capital adjustments in Section 2.4 does not hold for this model.

3.3.2 The agency problem

As in Gertler and Karadi (2009), we assume that bankers can divert a fraction Λ of their deposits and wealth at the beginning of the period, and transfer this amount of money back to their corresponding households. The cost for bankers of diverting is that depositors can force them into bankruptcy and recover the remaining fraction $1 - \Lambda$ of assets. Bankers therefore choose to divert whenever the benefit from diverting, ie Λlen_t , is greater than the value associated with staying in business as a banker, ie V_t . This gives the following incentive constraint

$$\underbrace{V_t}_{\substack{\text{banker's loss} \\ \text{from diverting}}} \geq \underbrace{\Lambda len_t}_{\substack{\text{banker's gain} \\ \text{from diverting}}} \quad (23)$$

for households to have deposits in banks. The continuation value V_t of a bank is given by

$$V_t = \mathbf{E}_t \sum_{j=0}^{+\infty} (1 - \alpha_b) \alpha_b^j \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t} n_{t+j+1}. \quad (24)$$

This expression reflects the idea that bankers attempt to maximise their expected wealth at the point of retirement where they transfer n_t to their respective household. Note that the discount factor in (24) is adjusted by $(1 - \alpha_b) \alpha_b^j$ to reflect the fact that retirement itself is stochastic and therefore could happen with positive probability in any period.

We assume that lending to the good-producing firms is profitable for banks. This implies that banks lend up to the limit allowed by the incentive constraint, which therefore is assumed to hold with equality. Consequently, the amount of credit provided by the representative bank is limited by its accumulated wealth through the relation

$$len_t = (lev_t) n_t \quad (25)$$

where

$$lev_t \equiv \frac{x_{2,t}}{\frac{\Lambda}{1-\tau} - x_{1,t}} \quad (26)$$

is the bank's leverage ratio. The two control variables $x_{1,t}$ and $x_{2,t}$ follow simple recursions derived in Appendix B.1.

3.4 Capital-producing firms

A capital-producing firm is assumed to control the aggregate supply of capital. This firm takes depreciated capital from all good-producing firms and invests in new capital before sending the 'refurbished' capital back to these firms. The decisions by the capital-producing firm are closely related to the financial contract provided by the representative bank. This is because the capital-producing firm trades capital at individual prices with each of the good-producing firms. That is, throughout a given financial contract, capital is traded at the price when this contract was signed. For instance, if a contract was signed in period $t - 4$, then the capital-producing firm trades capital with this particular firm at the price p_{t-4}^k throughout the contract. That is, when the good-producing firm enters a financial contract, it obtains the right to borrow at the constant rate r_t^L based on the current value of its capital stock p_t^k . By doing this we ensure that within each financial contract the cash flows between banks and good-producing firms are known with certainty.⁹

⁹Another way to justify this assumption is to consider the bank and the capital-producing firm as a joint entity.

More specifically, the net present value of profit for the capital-producing firm is given by

$$profit_t^k = \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} [v_{t+j} - v_{t+j}(1 - \delta) - i_{t+j}]. \quad (27)$$

Here, v_t is a value aggregate given by

$$v_t \equiv (1 - \alpha_k) \sum_{j=0}^{+\infty} \alpha_k^j p_{t-j}^k (1 - \delta)^j \tilde{k}_{t-j}, \quad (28)$$

or equivalently

$$v_t = (1 - \alpha_k) p_t^k \tilde{k}_t + \alpha_k (1 - \delta) v_{t-1}. \quad (29)$$

According to (27), the capital-producing firm obtains depreciated capital from good-producing firms $v_t(1 - \delta)$ and allocates resources to investments i_t . The output from this production process is an upgraded capital stock, which is sent to the good-producing firms resulting in revenue v_t .

When maximising profits, the firm is constrained by the evolution of \tilde{k}_t , ie

$$k_t = (1 - \alpha_k) \tilde{k}_t + \alpha_k (1 - \delta) k_{t-1}, \quad (30)$$

and the law of motion for aggregate capital:

$$k_{t+1} = (1 - \delta)k_t + i_t \left[1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right]. \quad (31)$$

The optimisation of (27) is described in Appendix B.2. An important point to note is that the lagrange multiplier for (31), ie q_t , is the standard Tobin's Q and indicates a marginal change in profit following a marginal change in the next period capital k_{t+1} . On the other hand, the price of capital p_t^k denotes the marginal change in profit for a marginal change in current capital k_t .

3.5 Market clearing and calibration

Market clearing conditions in the capital, labour, and good markets are similar to those derived in Section 2.3, and technology evolves according to the AR(1) process in (5).¹⁰

The model is calibrated to the post-war US economy in Table A. We chose standard values for the discount factor $\beta = 0.9926$, the capital share $\theta = 0.36$, the coefficient of relative risk-aversion $\phi_0 = 1$, and the rate of depreciation $\delta = 0.025$. In line with the estimates in

¹⁰The complete list of equations in the model is shown in Appendix B.3.

Table A: Baseline calibration

β	0.9926	Λ	0.2
b	0.65	α_b	0.972
ϕ_0	1	τ	0.017
ϕ_1	1/3	κ	2.5
θ	0.36	δ	0.025
α_k	<i>free</i>	ρ_a	0.90
		σ_a	0.7%

Christiano *et al* (2005), we set the intensity of habits to $b = 0.65$ and investment adjustment costs to $\kappa = 2.5$. The inverse Frisch elasticity of the labour supply ϕ_1 is set to 1/3. This is slightly below the value estimated in Smets and Wouters (2007) but preferred to account for the fact that there are no wage rigidities in our model. The parameters affecting the evolution of technological shocks are set to $\rho_a = 0.90$ and $\sigma_a = 0.007$.

There are three parameters that directly affect the behaviour of banks: i) the fraction of banks' assets that can be diverted Λ , ii) the probability that a banker retires α_b , and iii) the tax rate on banks' wealth τ . We calibrate these parameters to generate an external financing premium of 100 annualised basis points and a steady-state leverage ratio of 4 in the banking sector as in Gertler and Karadi (2009).¹¹ The value of α_k determines the average duration of financial contracts and is left as a free parameter to explore the implications of maturity transformation. Finally, we compute the model solution by a standard log-linear approximation.¹²

3.6 Implications of maturity transformation: a shock to technology

Figure 3 shows impulse response functions to a positive technological shock. In each graph, the continuous line shows the model with banks and no maturity transformation, ie in case the average duration of contracts in the economy, \mathcal{D} , is set equal to 1. The dashed lines, on the other hand, correspond to two different calibrations of the model with maturity transformation – $\mathcal{D} = 4$ and $\mathcal{D} = 12$.

¹¹ Simple algebra shows that the steady-state level of the external financing premium implied by our model does not depend on α_k .

¹² All versions of the model are implemented in Dynare. Codes are available on request.

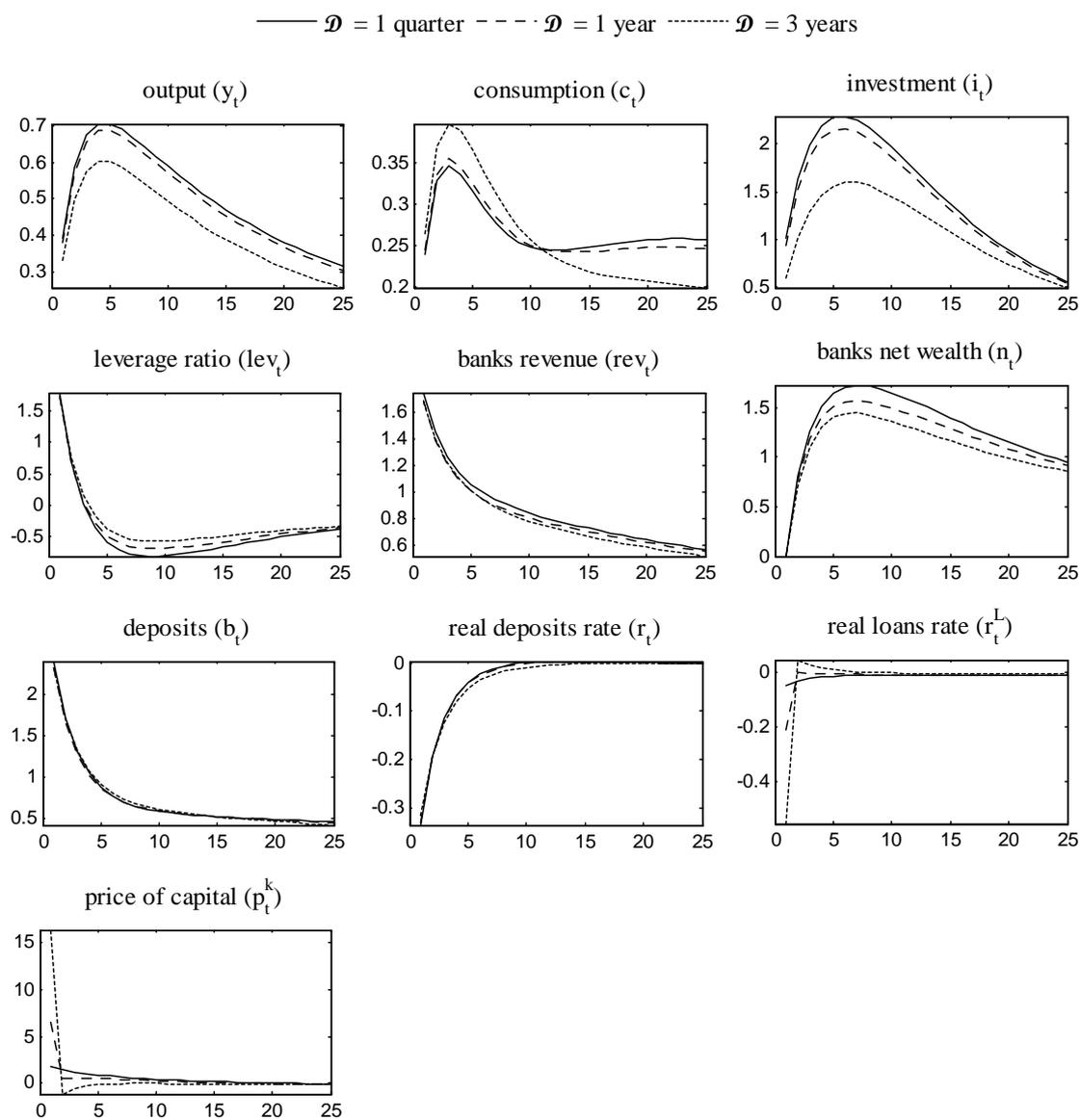
We start by analysing the model without maturity transformation. As in standard RBC models, the shock generates an increase in consumption, investment, and output. Households become temporarily richer and therefore raise their deposits b_t while r_t falls. With a higher level of deposits, banks increase their supply of credit, resulting in a fall in the loan rate r_t^L . Firms demand more capital and therefore its price p_t^k increases. This means that they now need to borrow more in order to finance each unit of capital, and firms therefore increase their demand for credit. These combined effects generate an increase in banks' net worth as shown in Figure 3. As banks' financial position is strengthened following the shock, restrictions to credit provision are relaxed and banks' leverage ratio increases. We therefore obtain a financial accelerator effect in the sense of Bernanke *et al* (1999).

The business cycle implications of maturity transformation can be considered by comparing the full and dashed lines in Figure 3. We see that increasing the average duration of loans to $\mathcal{D} = 4$ and $\mathcal{D} = 12$ generates weaker responses in output following the shock. Accordingly, our model predicts a *credit maturity attenuator effect*. To understand why, consider banks' balance sheet equations (20) to (22). The presence of maturity transformation ($\alpha_k > 0$) implies that only a fraction of all loans is reset to reflect a higher price of capital p_t^k following the shock. The remaining fraction of contracts was signed in the past and does not respond to changes p_t^k . Consequently, good-producing firms increase their demand for credit by a smaller amount the higher the degree of maturity transformation. Banks' revenues and net worth therefore increase by less, which in turn results in a weaker response of output to the shock.

Interestingly, in our general equilibrium set up, the effects of different degrees of maturity transformation are felt not only in the relation between banks and good-producing firms, but also in the behaviour of all agents in the economy. Capital producers, for example, know that higher degrees of maturity transformation are associated with weaker increases in the demand for capital after the shock. They therefore raise investment by less compared to the case without maturity transformation, resulting in more room for households' consumption to increase. Over time, however, the smaller increase in investment affects households' income and, consequently, consumption goes back to the steady state faster the higher the degree of maturity transformation.



Figure 3: Impulse responses to a positive technological shock



Notes: Impulse response to a one standard deviation positive shock to technology. In each graph the vertical axis measures percentage deviation from the deterministic steady state of the respective variable, whereas the horizontal axis measures quarters after the shock hits.

4 A New Keynesian model: nominal financial contracts

The analysis has so far focused on long-term financial contracts set in real terms, ie with inflation protection. Such insurance against inflation is often not available in reality and most lending is therefore conducted based on nominal contracts. The distinction between nominal and real contracts is especially interesting in our set up, because long-term inflation expectations here have a larger impact on firms' decisions compared to one-period nominal contracts as considered in Christiano, Motto and Rostagno (2003) and Christiano, Motto and Rostagno (2007). The aim of this section is therefore to extend the model presented in Section 3 to nominal contracts and study how maturity transformation affects the monetary transmission mechanism in an otherwise standard New Keynesian model.

We proceed as follows. Sections 4.1 and 4.2 revisit the problems for the good-producing firms and banks, respectively, when we have long-term nominal contracts. To introduce price stickiness into the model, Section 4.3 follows Gertler and Karadi (2009) and adds retail firms to the economy. Monetary policy and market clearing conditions are outlined in Section 4.4. Section 4.5 then studies the quantitative implications of maturity transformation following a monetary policy shock.

4.1 Good-producing firms

The basic set up for the good-producing firms is similar to the one presented in Section 3.2, except firms now need to borrow based on the nominal price of their capital stock when signing the contract. To see the implications of this assumption, let P_t denote the nominal price level of aggregate output (defined below) and let P_t^k be the nominal price of capital. The expression for real profit in period $t + j$ for a firm that entered a contract in period t is then

$$profit_{t+j|t} = \underbrace{\frac{P_{t+j}^{int}}{P_{t+j}} a_{t+j} [(1 - \delta)^j \tilde{k}_t]^\theta h_{t+j|t}^{1-\theta}}_{\text{production revenue}} - \underbrace{w_{t+j} \tilde{h}_{t+j|t}}_{\text{wage bill}} - \underbrace{(r_t^L + \delta) (1 - \delta)^j \tilde{k}_t \frac{P_t^k}{P_{t+j}}}_{\text{capital rental bill}}, \quad (32)$$

where P_t^{int} is the nominal price of the good produced by the firm. That is, the firm borrows $\tilde{k}_t P_t^k$ units of *cash* throughout the contract, and the interest rate on this loan $r_t^{L,nom}$ is now expressed in nominal terms. Importantly, changes in the price level P_t affects the *real* value of the loan and



thereby its implied real interest rate. This effect is easily seen by rewriting the firm's profit as

$$\begin{aligned} profit_{t+j|t} &= p_{t+j}^{int} a_{t+j} (1 - \delta)^{\theta j} (\tilde{k}_t)^\theta h_{t+j|t}^{1-\theta} - w_t h_{t+j|t} \\ &\quad - (r_t^{L,nom} + \delta) \left(\prod_{i=1}^j \pi_{t+i} \right)^{-1} p_t^k \tilde{k}_t (1 - \delta)^j, \end{aligned} \quad (33)$$

where we define the real prices $p_t^{int} \equiv P_t^{int} / P_t$ and $p_t^k \equiv P_t^k / P_t$. Moreover, $\pi_t \equiv P_t / P_{t-1}$ denotes the gross inflation rate. Hence, higher inflation during the contract erodes the real value of the loan and hence lowers its real interest rate $(r_t^{L,nom} + \delta) \left(\prod_{i=1}^j \pi_{t+i} \right)^{-1}$, and *vice versa* for lower inflation. The firm and the bank are aware of this effect when signing the contract, and $r_t^{L,nom}$ therefore accounts for long-term inflation expectations.

As in Section 3.2, the good-producing firm determines capital and labour by maximising the net present value of future profits. Applying the households' stochastic discount factor, the first-order condition for the optimal level of capital \tilde{k}_t is now

$$\mathbf{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(p_{t+j}^{int} \theta a_{t+j} (1 - \delta)^{\theta j} (\tilde{k}_t)^{\theta-1} h_{t+j|t}^{1-\theta} - \frac{(r_t^{L,nom} + \delta)}{\prod_{i=1}^j \pi_{t+i}} p_t^k (1 - \delta)^j \right) = 0. \quad (34)$$

The first-order condition for labour remains unchanged as in equation (8).

4.2 The banking sector

The behaviour of the representative bank is similar to the case with real contracts. However, the fact that contracts are set in nominal terms introduces a debt-deflation channel following Fisher (1933). We briefly describe how this effect operates via banks' balance sheet within our model.

Redoing the arguments in Section 3.3 for nominal variables imply that

$$N_{t+1} = (1 - \tau) [REV_t - R_t^{nom} LEN_t + R_t^{nom} N_t], \quad (35)$$

where N_t is *nominal* net worth, REV_t is *nominal* revenue, and LEN_t is *nominal* lending.

Re-expressing this equation in real terms implies

$$n_{t+1} = (1 - \tau) \left[\frac{rev_t}{\pi_{t+1}} - R_t^{nom} \frac{len_t}{\pi_{t+1}} + R_t^{nom} \frac{n_t}{\pi_{t+1}} \right] \quad (36)$$

where $rev_t \equiv REV_t / P_t$, $len_t \equiv LEN_t / P_t$ and $n_t \equiv N_t / P_t$. The important difference compared to the corresponding equation based on real contracts in (20) is the correction for inflation.

Hence, a reduction in inflation increases the real value of banks' net worth from the previous

period n_t/π_{t+1} and their revenue rev_t/π_{t+1} . The real value of deposits len_t/π_{t+1} also increase, but the combined effect is likely to be positive, in so far as banks are running a surplus in period t .

This effect from inflation introduces a debt-deflation mechanism whereby fundamental macroeconomic shocks affect real activity. The channel operates in the following way. Unpredictable macro shocks may move inflation temporarily away from what was expected when contracts were signed, resulting in changes in the *ex-post* real revenue of long-term loans. This in turn affects banks net worth and therefore also the supply of credit.

The remaining equations for the banking sector are as in Section 3.3, given appropriate corrections for inflation (see Appendix C.1).

4.3 Retail firms

The final output in the economy is assumed to be a CES composite produced from differentiated retail goods, ie

$$y_t = \left[\int_0^1 y_{f,t}^{\frac{\eta-1}{\eta}} df \right]^{\frac{\eta}{\eta-1}}, \quad (37)$$

where $\eta > 1$ and $y_{f,t}$ is the product from retail firm f . Cost minimisation implies the standard demand function

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\eta} y_t, \quad (38)$$

where $P_{f,t}$ is the price of the retail good from firm f . The aggregate price level is thus given $P_t = \left[\int_0^1 P_{f,t}^{1-\eta} df \right]^{\frac{1}{1-\eta}}$.

The role of the individual retail firms is to re-package the good from the good-producing firms using a linear production technology. Nominal rigidity is introduced based on a Calvo-style formulation, where only a fraction $1 - \alpha_p$ of retail firms can reset their prices every period. This price is denoted by P_t^* . The remaining fraction α_p of retail firms simply let $P_{f,t} = P_{f,t-1}$. Accordingly, the problem for retail firms adjusting prices in period t is given by

$$\max_{P_t^*} \mathbf{E}_t \sum_{i=0}^{\infty} (\alpha_p \beta)^i \frac{\lambda_{t+i}}{\lambda_t} \left[\frac{P_t^*}{P_{t+i}} - P_{t+i}^{int} \right] y_{f,t+i} \quad (39)$$

subject to (38).



4.4 Monetary policy and market clearing conditions

Monetary policy is specified by a standard Taylor-rule

$$r_t^{nom} = \rho r_{t-1}^{nom} + (1 - \rho) \left(r_{ss}^{nom} + \phi_\pi \log \left(\frac{\pi_t}{\pi_{ss}} \right) + \phi_y \log \left(\frac{y_t}{y_{ss}} \right) \right) + \varepsilon_t^r \quad (40)$$

where $R_t^{nom} \equiv 1 + r_t^{nom}$ and $\varepsilon_t^r \sim \mathcal{NID}(0, \sigma_r^2)$. That is, central bank aims to close the inflation and output gaps, while potentially smoothing changes in the policy rate.

The market clearing conditions are standard and stated in Appendix C.1.

4.5 Implications of maturity transformation: a monetary policy shock

This section examines effects of maturity transformation following a positive monetary policy shock, ie an exogenous increase in r_t^{nom} . The real part of the model is calibrated as in Table A. The parameters associated to the nominal frictions are calibrated as follows. Inflation in the steady state is assumed to be zero, while we let $\alpha_p = 0.75$ so that retail firms on average change their prices once every year. The value of η is set to 6, consistent with a 20% price mark up as implied by the benchmark estimate in Christiano *et al* (2005). Finally, the coefficients in the Taylor rule are taken from the post-1984 estimates in Justiniano and Primiceri (2008), ie $\rho = 0.84$, $\phi_\pi = 2.37$, and $\phi_y = 0.02$. Figure 4 displays the impulse response functions to a monetary policy shock of 25 basis points (equivalent to an annualised 100 basis points shock). As before, the continuous line represents the model without maturity transformation ($\mathcal{D} = 1$), whereas dashed lines refer to different calibrations of maturity transformation with $\mathcal{D} = 4$ and $\mathcal{D} = 12$.

Starting with the simpler model where $\mathcal{D} = 1$, the policy shock generates an increase in the implied real deposit rate (r_t^{nom} increases and π_t decreases) which results in the familiar contraction in consumption, investment, output, and inflation. The reduction in inflation increases the real value of banks' nominal assets and banks are therefore better off on impact. However, the fall in the demand for capital and the associated fall in p_t^k reduces banks' real revenues, lowering their net worth from the second period onwards.¹³ The positive comovement between net worth and output generates a financial accelerator effect as in Bernanke *et al* (1999).

¹³Note in equation (36) that on impact movements in n_t following any shock are only a result of the change in inflation. Changes in rev_t , len_t and R_t^{nom} can only affect banks' net worth from the second period and onwards.

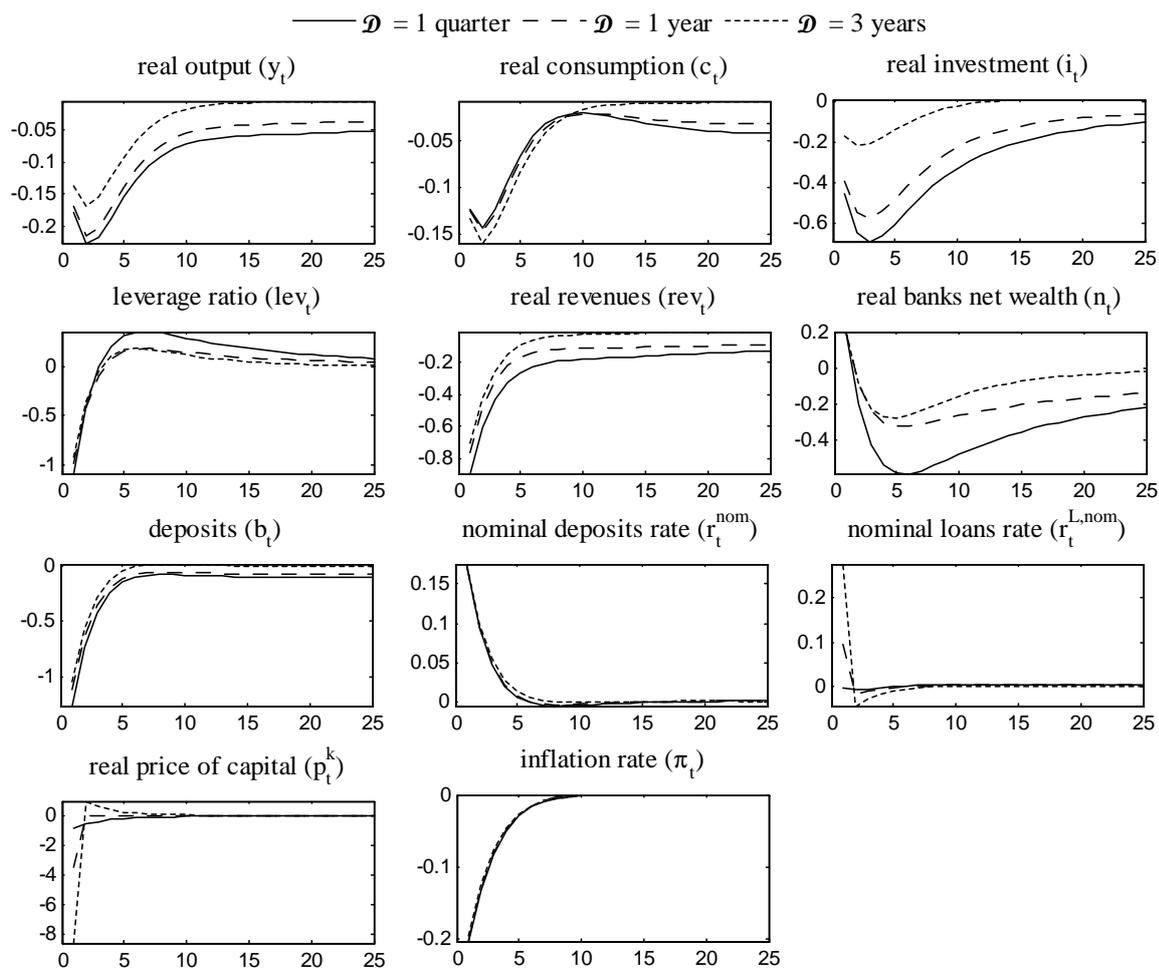
We next study how maturity transformation affects the monetary transmission mechanism. Our model predicts that the fall in output is weaker the higher the degree of maturity transformation. In other words, we also obtain a *credit maturity attenuator effect* in the case of a monetary policy shock. Notably, this is in contrast to the ‘bank capital channel’ analysed in a somewhat different set up eg by Van den Heuvel (2006). According to this theory, the presence of maturity mismatches in banks’ balance sheets implies that only a small fraction of loans can be quickly adjusted following a monetary policy shock, whereas deposits are almost entirely adjusted on impact. This means that an increase in the policy rate would have a negative impact on banks’ profits and consequently on the supply of credit, potentially exacerbating the real effects of the shock. To explain the difference between this theory and our result, we focus on how maturity transformation affects banks’ net worth within our model. Here, we emphasise three general equilibrium effects, which do not feature so prominently in the analysis focusing on the bank capital channel.

First, in the model without maturity transformation the fall in the price of capital p_t^k implies a reduction in the value of *all* loans, and banks therefore see a fall in their revenues. However, with maturity transformation only a fraction $1 - \alpha_k$ of loans are reset every period to reflect the fall in p_t^k . Accordingly, banks revenues do not fall as much the higher the degree of maturity transformation.

A second general equilibrium effect occurs as a result of the debt-deflation channel discussed in Section 4.2. The reduction in inflation following the shock raises the *ex-post* real interest rates paid by the good-producing firms. The aggregate value of loans fall by less in the presence maturity transformation (due to the first channel) and the higher *ex-post* real rate therefore has a larger positive effect on banks’ balance sheets and output than without long-term loans.

The third general equilibrium effect is as follows. With maturity transformation, the smaller reduction in banks’ net worth n_t implies that output (and income) does not fall as much as in the case without long-term contracts. Hence, the decline in households’ deposits is smaller, and banks are able to provide more credit to good-producing firms. As a result, this effect also reduces the contraction in output following the shock.

Figure 4: Impulse responses to a positive monetary policy shock



Notes: Impulse response to a 25 basis points positive monetary policy shock. In each graph the vertical axis measures percentage deviation from the deterministic steady state of the respective variable, whereas the horizontal axis measures quarters after the shock hits.

5 Conclusion

This paper shows how to introduce a banking sector with maturity transformation into an otherwise standard DSGE model. Our novel assumption is to consider the case where firms face a constant probability of being unable to reset their capital level in every period. We first show that this restriction on firms' ability to adjust capital does not affect prices and aggregate quantities in a wide range of DSGE models. Importantly, the considered friction generates a demand for long-term credit when we impose the standard requirement that firms borrow when financing their capital stock. As a result, banks face a maturity transformation problem because they use short-term deposits and accumulated wealth to fund the provision of long-term credit. Within an RBC model featuring long-term contracts and banks, we then analyse the quantitative implications of maturity transformation following a positive technological shock. Our model suggests that the responses of the model economy to this shock are in general weaker the higher the degree of maturity transformation in the banking sector.

The final part of our paper studies implications of maturity transformation when financial contracts are set in nominal terms. We therefore extend the considered RBC model with sticky prices, long-term nominal contracts, and a central bank. Effects of maturity transformation within the banking sector are then analysed following a positive monetary policy shock. We once again conclude that responses in the economy in general are weaker the higher the degree of maturity transformation in the banking sector.

Our way of incorporating maturity transformation is only a first step in analysing this topic in a dynamic stochastic general equilibrium set up. Interesting extensions could introduce extra financing options for firms, possibly by breaking the match between the duration of firms' exposure and their financial contract. This would also have the potential to create a time-varying maturity transformation problem within the banking sector. Studying higher-order effects and the impact of risk on banks' behaviour would also make for interesting extensions.

Appendix A: A standard RBC model with infrequent capital adjustments

A.1 Households

The representative household's problem can be summarised by the following lagrangian:

$$\begin{aligned} \mathcal{L} = & \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{(c_{t+j} - b c_{t+j-1})^{1-\phi_0}}{1-\phi_0} - \phi_2 \frac{h_{t+j}^{1+\phi_1}}{1+\phi_1} \right) + \\ & \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \lambda_{t+j} [h_{t+j} w_{t+j} + R_{t+j}^k k_{t+j} - c_{t+j} - i_{t+j}] + \\ & \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j q_{t+j} \lambda_{t+j} \left[(1-\delta) k_{t+j}^s + i_{t+j} \left[1 - S \left(\frac{i_{t+j}}{i_{t-1+j}} \right) \right] - k_{t+1+j}^s \right], \end{aligned}$$

where λ_t is the lagrange multiplier associated with the budget constraint. The first-order conditions are:

i **Consumption, c_t :**

$$\lambda_t = \mathbf{E}_t \left[\frac{1}{(c_t - b c_{t-1})^{\phi_0}} - \frac{\beta b}{(c_{t+1} - b c_t)^{\phi_0}} \right]$$

ii **Labour, h_t :**

$$\phi_2 h_t^{\phi_1} = \lambda_t w_t$$

iii **Physical capital stock, k_{t+1}^s :**

$$1 = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{R_{t+1}^k + q_{t+1} (1-\delta)}{q_t} \right) \right]$$

iv **Investment, i_t :**

$$q_t = \frac{1 - \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left(\frac{i_{t+1}}{i_t} \right)^2 S' \left(\frac{i_{t+1}}{i_t} \right) \right]}{\left[1 - S \left(\frac{i_t}{i_{t-1}} \right) - \frac{i_t}{i_{t-1}} S' \left(\frac{i_t}{i_{t-1}} \right) \right]}$$

A.2 Firms

The profit of firm i in period $t + j$ is

$$a_t k_{i,t+j}^\theta h_{i,t+j}^{1-\theta} - R_{t+j}^k k_{i,t+j} - w_{t+j} h_{i,t+j},$$



and the firm seeks to maximise its expected discounted value of profits given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} (a_{t+j} k_{i,t+j}^\theta h_{i,t+j}^{1-\theta} - R_{t+j}^k k_{i,t+j} - w_{t+j} h_{i,t+j}).$$

This problem is divided in two steps. We first derive the i^{th} firm's demand of labour, which takes the standard form since labour is optimally chosen in every period. In the second step, we derive the optimal value of capital $\tilde{k}_{i,t}$ for firms that are able to adjust capital in period t . Note that a firm adjusting capital in period t faces a probability α_k^j of not being able to reoptimise after j periods in the future and hence have $(1 - \delta)^j \tilde{k}_{i,t}$ units of capital in period $t + j$.

i Labour, h_t :

In every period $t + j$, for $j = 0, 1, 2, \dots$, all firms are allowed to adjust their labour demand. Hence, we can ignore the dynamic dimension of the firm's problem which implies

$$h_{i,t+j} = \left(\frac{w_{t+j}}{a_{t+j} (1 - \theta)} \right)^{-\frac{1}{\theta}} k_{i,t+j}.$$

The period $t + j$ demand for labour for a firm that last reoptimised in period t , $\tilde{h}_{i,t+j|t}$, is given by

$$\tilde{h}_{i,t+j|t} = \left(\frac{w_{t+j}}{a_{t+j} (1 - \theta)} \right)^{-\frac{1}{\theta}} (1 - \delta)^j \tilde{k}_{i,t}$$

ii Capital, \tilde{k}_t :

A firm adjusting capital in period t chooses $\tilde{k}_{i,t}$ to maximise the present discounted value of profits. This firm therefore solves

$$\max_{\tilde{k}_{i,t}} \mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} ((1 - \delta)^j \tilde{k}_{i,t})^\theta \tilde{h}_{i,t+j|t}^{1-\theta} - R_{t+j}^k (1 - \delta)^j \tilde{k}_{i,t} - w_{t+j} h_{i,t+j|t} \right)$$

Using the demand for labour derived above, the optimality condition associated with the capital choice for firm i can be written as

$$\mathbf{E}_t \sum_{l=0}^{+\infty} (\alpha_k \beta (1 - \delta))^l \frac{\lambda_{t+l}}{\lambda_t} \left(a_{t+l} \theta \left(\frac{w_{t+l}}{a_{t+l} (1 - \theta)} \right)^{-\frac{1-\theta}{\theta}} - R_{t+l}^k \right) = 0$$

Appendix B: An RBC model with banks and maturity transformation

B.1 Recursions for $x_{1,t}$ and $x_{2,t}$

The expected discounted value of bank equity V_t can be expressed as

$$\begin{aligned}
 V_t &= \mathbf{E}_t \sum_{i=0}^{\infty} (1 - \alpha_b) \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} (1 - \tau) [rev_{t+i} - R_{t+i} len_{t+i} + R_{t+i} n_{t+i}] \\
 &= (1 - \tau) \left\{ len_t \left(\mathbf{E}_t \sum_{i=0}^{\infty} (1 - \alpha_b) \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \left[\frac{rev_{t+i}}{len_t} - R_{t+i} \frac{len_{t+i}}{len_t} \right] \right) \right. \\
 &\quad \left. + n_t \left(\mathbf{E}_t \sum_{i=0}^{\infty} (1 - \alpha_b) \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \frac{R_{t+i} n_{t+i}}{n_t} \right) \right\} \\
 &= (1 - \tau) [len_t x_{1,t} + n_t x_{2,t}]
 \end{aligned}$$

where we have defined

$$\begin{aligned}
 x_{1,t} &\equiv \mathbf{E}_t \sum_{i=0}^{\infty} (1 - \alpha_b) \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \left[\frac{rev_{t+i}}{len_t} - R_{t+i} \frac{len_{t+i}}{len_t} \right] \\
 x_{2,t} &\equiv \mathbf{E}_t \sum_{i=0}^{\infty} (1 - \alpha_b) \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \frac{R_{t+i} n_{t+i}}{n_t}
 \end{aligned}$$

Straightforward algebra then implies the following recursions:

$$\begin{aligned}
 x_{1,t} &= E_t (1 - \alpha_b) \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{rev_t}{len_t} - R_t \right] + \mathbf{E}_t \left[\alpha_b \beta x_{1,t+1} \frac{len_{t+1}}{len_t} \frac{\lambda_{t+1}}{\lambda_t} \right] \\
 x_{2,t} &= (1 - \alpha_b) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] R_t + \mathbf{E}_t \left[x_{2,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \right]
 \end{aligned}$$

B.2 First-order conditions for the capital-producing firm

To simplify the optimisation, we isolate \tilde{k}_t from (30) and substitute it into (29). Hence, we need to optimise (27) with respect to v_t , k_t , and i_t subject to (30) and (31). The lagrange function then

reads:

$$\begin{aligned}\mathcal{L} &= \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} [\delta v_{t+j} - i_{t+j}] \\ &+ \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} u_{1,t+j} [(k_{t+j} - \alpha_k (1 - \delta) k_{t-1+j}) p_{t+j}^k + \alpha_k (1 - \delta) v_{t-1+j} - v_{t+j}] \\ &+ \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} q_{t+j} \left[(1 - \delta) k_{t+j} + i_{t+j} \left[1 - \frac{\kappa}{2} \left(\frac{i_{t+j}}{i_{t+j-1}} - 1 \right)^2 \right] - k_{t+j+1} \right]\end{aligned}$$

The first-order conditions are:

i. **The value aggregate** v_t :

$$u_{1,t} = \delta + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} \alpha_k (1 - \delta) \right]$$

ii. **Capital** k_t :

$$q_t + \mathbf{E}_t \left[\beta^2 \frac{\lambda_{t+2}}{\lambda_t} u_{1,t+2} \alpha_k (1 - \delta) p_{t+2}^k \right] = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} p_{t+1}^k \right] + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} (1 - \delta) \right].$$

iii. **Investment** i_t :

$$1 = q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right) + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{i_{t+1}^2}{i_t^2} \right]$$

Notice that q_t is the standard Tobin's Q, ie indicating the marginal change in profit of a marginal change in k_{t+1} . On the other hand, p_t^k is the marginal change in profit of a marginal change in k_t .

B.3 Model summary

Household:

- 1) $\lambda_t = \mathbf{E}_t [(c_t - bc_{t-1})^{-\sigma_c} - \beta b (c_{t+1} - bc_t)^{-\sigma_c}]$
- 2) $1 = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} R_t \right]$
- 3) $\phi_2 h_t^{\phi_1} = \lambda_t w_t$

Good-producing firms:

- 4) $h_t = \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1}{\theta}} k_t$
- 5) $z_{1,t} = (r_t^L + \delta) p_t^k z_{2,t}$
- 6) $z_{1,t} = \theta a_t \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} z_{1,t+1} ((1-\delta) \alpha_k \beta) \right]$
- 7) $z_{2,t} = 1 + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} ((1-\delta) \beta \alpha_k) z_{2,t+1} \right]$
- 8) $k_t = (1 - \alpha_k) \tilde{k}_t + \alpha_k (1 - \delta) k_{t-1}$

Banking sector:

- 9) $n_{t+1} = (1 - \tau) [rev_t - R_t len_t + R_t n_t]$
- 10) $rev_t = (1 - \alpha_k) R_t^L p_t^k \tilde{k}_t + (1 - \delta) \alpha_k rev_{t-1}$
- 11) $len_t = (1 - \alpha_k) p_t^k \tilde{k}_t + (1 - \delta) \alpha_k len_{t-1}$
- 12) $lev_t \equiv \frac{len_t}{n_t} = \frac{x_{2,t}}{\frac{\Lambda}{1-\tau} - x_{1,t}}$
- 13) $V_t = (1 - \tau) [len_t x_{1,t} + n_t x_{2,t}]$
- 14) $x_{1,t} = \mathbf{E}_t (1 - \alpha_b) \beta^1 \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{rev_t}{len_t} - R_t \right] + \mathbf{E}_t \left[\alpha_b \beta x_{1,t+1} \frac{len_{t+1}}{len_t} \frac{\lambda_{t+1}}{\lambda_t} \right]$
- 15) $x_{2,t} = (1 - \alpha_b) \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] R_t + \mathbf{E}_t \left[x_{2,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \right]$

Capital-producing firm:

- 16) $k_{t+1} = (1 - \delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right]$
- 17) $u_{1,t} = \delta + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} \alpha_k (1 - \delta) \right]$
- 18) $1 = q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right) + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{i_{t+1}^2}{i_t^2} \right]$
- 19) $q_t + \mathbf{E}_t \left[\beta^2 \frac{\lambda_{t+2}}{\lambda_t} u_{1,t+2} \alpha_k (1 - \delta) p_{t+2}^k \right] = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} p_{t+1}^k \right] + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} (1 - \delta) \right]$
- 20) $v_t = (1 - \alpha_k) \tilde{k}_t p_t^k + \alpha_k (1 - \delta) v_{t-1}$

Market clearing conditions:

- 21) $y_t = a_t k_t^\theta h_t^{1-\theta}$
- 22) $y_t = c_t + i_t$

Exogenous processes:

- 23) $\log a_t = \rho_a \log a_{t-1} + \varepsilon_t^a$

Appendix C: The New Keynesian model with banks and maturity transformation

C.1 Model summary

Household:

- 1) $\lambda_t = \mathbf{E}_t [(c_t - bc_{t-1})^{-\sigma_c} - \beta b (c_{t+1} - bc_t)^{-\sigma_c}]$
- 2) $1 = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^{nom}}{\pi_{t+1}} \right]$
- 3) $\phi_2 h_t^{\phi_1} = \lambda_t w_t$

Intermediate goods-producing firms:

- 4) $h_t = \left(\frac{w_t}{p_t^{int} a_t (1-\theta)} \right)^{-\frac{1}{\theta}} k_t$
- 5) $z_{1,t} = (r_t^{L,nom} + \delta) p_t^k z_{2,t}$
- 6) $z_{1,t} = p_t^{int} \theta a_t \left(\frac{w_t}{p_t^{int} a_t (1-\theta)} \right)^{-\frac{1-\theta}{\theta}} + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} z_{1,t+1} ((1-\delta) \alpha_k \beta) \right]$
- 7) $z_{2,t} = 1 + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} ((1-\delta) \beta \alpha_k) z_{2,t+1} \right]$
- 8) $k_t = (1 - \alpha_k) \tilde{k}_t + \alpha_k (1 - \delta) k_{t-1}$

Financial intermediaries:

- 9) $n_{t+1} = (1 - \tau) \pi_{t+1}^{-1} [rev_t - R_t^{nom} len_t + R_t^{nom} n_t]$
- 10) $rev_t = (1 - \alpha_k) R_t^{L,nom} p_t^k \tilde{k}_t + (1 - \delta) \alpha_k rev_{t-1} \pi_t^{-1}$
- 11) $len_t = (1 - \alpha_k) p_t^k \tilde{k}_t + (1 - \delta) \alpha_k len_{t-1} \pi_t^{-1}$
- 12) $lev_t \equiv \frac{len_t}{n_t}$
- 13) $lev_t = \frac{x_{2,t}}{\frac{\Lambda}{1-\tau} - x_{1,t}}$
- 14) $x_{1,t} = \mathbf{E}_t (1 - \alpha_b) \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1} \left[\frac{rev_t}{len_t} - R_t^{nom} \right] + \mathbf{E}_t \left[\alpha_b \beta x_{1,t+1} \frac{len_{t+1}}{len_t} \frac{\lambda_{t+1}}{\lambda_t} \right]$
- 15) $x_{2,t} = (1 - \alpha_b) \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1} \right] R_t^{nom} + \mathbf{E}_t \left[x_{2,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \right]$

Capital-producing firms:

- 16) $k_{t+1} = (1 - \delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right]$
- 17) $u_{1,t} = \delta + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1} u_{1,t+1} \alpha_k (1 - \delta) \right]$
- 18) $1 = q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right) + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{i_{t+1}^2}{i_t^2} \right]$
- 19) $q_t + \mathbf{E}_t \left[\beta^2 \frac{\lambda_{t+2}}{\lambda_t} u_{1,t+2} \alpha_k (1 - \delta) p_{t+2}^k \right] = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} p_{t+1}^k \right] + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} (1 - \delta) \right]$

Retail firms:

- 20) $\frac{P_t^*}{P_t} = \frac{num_t}{den_t}$
- 21) $num_t = \mu p_t^{int} y_t + \mathbf{E}_t \left[\alpha_p \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^\eta num_{t+1} \right]$
- 22) $den_t = y_t + \mathbf{E}_t \left[\alpha_p \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\eta-1} den_{t+1} \right]$
- 23) $\pi_t = \left[(1 - \alpha_p) \left(\frac{P_t^*}{P_t} \pi_t \right)^{1-\eta} + \alpha_p \right]^{\frac{1}{1-\eta}}$



Market clearing conditions:

$$24) \quad y_t^{int} = a_t k_t^\theta h_t^{1-\theta}$$

$$25) \quad y_t = \Delta_t^{-1} y_t^{int}$$

$$26) \quad \Delta_t = (1 - \alpha_p) \left(\frac{p_t^*}{p_t} \right)^{-\eta} + \alpha_p (\pi_t)^\eta \Delta_{t-1}$$

$$27) \quad y_t = c_t + i_t$$

$$28) \quad r_t^{nom} = \rho r_{t-1}^{nom} + (1 - \rho) [r_{ss}^{nom} + \phi_\pi \pi_t + \phi_y (\log y_t - \log y_{ss})] + \varepsilon_t^r$$

Exogenous processes:

$$29) \quad \log a_t = \rho_a \log a_{t-1} + \varepsilon_t^a$$



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