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The international transmission of volatility shocks: an empirical analysis

Haroon Mumtaz and Konstantinos Theodoridis

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Abstract

This paper proposes an empirical model which can be used to estimate the impact of changes in the volatility of shocks to US real activity on the UK economy. The proposed empirical model is a structural VAR where the volatility of structural shocks is time varying and is allowed to affect the level of endogenous variables. Using this extended SVAR model we estimate that a one standard deviation increase in the volatility of the shock to US real GDP leads to a decline in UK GDP growth of 0.1% and a 0.1% increase in UK CPI inflation. We then use a non-linear small open economy New Keynesian business cycle model calibrated to US/UK economies to investigate what kind of stochastic volatility shocks can deliver such behaviour. We find that shocks that generate marginal cost uncertainty — such as foreign wage mark-up and productivity stochastic volatility shocks — can reproduce the macroeconomic aggregate responses obtained by the empirical model. An increase in uncertainty, associated with foreign demand shocks on the other hand has a negligible impact on the domestic economy.

Key words: Stochastic volatility, Gibbs sampling, DSGE model.

JEL classification: F42, C32.

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Contents

Summary

The recent financial crisis has been characterised by increasingly volatile macroeconomic data in the United States and the United Kingdom. In this paper we devise an empirical model to estimate the impact of this increase in volatility or uncertainty on the UK economy. In particular we examine the impact of an increase in uncertainty associated with US real activity. Uncertainty about growth in large economies has been a key consideration for policymakers in recent years.

The empirical model that we propose is an extension of vector autoregression (VAR) models. VAR models link each variable included in the model to past values of all the variables in the system. The residual associated with each variable is typically assumed to have a constant variance. For example if the model included US GDP growth, the variance of the residual to the relevant equation would be constant. This also implies that in this modelling set-up, the uncertainty associated with each variable (as proxied by the residual variance) is fixed over time. Given recent events, this may not be a good assumption.

Our paper extends this model along two dimensions. First, we allow the residual variance to change over time – in other words we allow for stochastic volatility. Second, we allow this stochastic volatility to enter as an explanatory variable in each equation of the model. We can therefore gauge the effect of volatility on each variable included in the VAR model.

In our empirical application, we include US GDP growth, US CPI inflation, the federal funds rate, UK GDP growth, UK CPI inflation and Bank Rate in the extended VAR model. We then try to estimate the impact of an increase in the stochastic volatility associated with the residual of the US GDP growth equation. We find that if this volatility increases by one standard deviation, UK GDP growth declines by 0.1% and UK CPI inflation increases by 0.1%. The impact of this shock on the US GDP growth and inflation is very similar. The impact is statistically important albeit small in economic terms.

We then employ a theoretical model of the open economy to understand the transmission channel of this shock. Model simulations indicate that it can be interpreted as a sudden change in the volatility associated with shocks to US wages or productivity – ie shocks to US ‘supply’. A sudden increase in the volatilty of these shocks leads to an increase in precautionary savings by consumers who are more uncertain about the future. This leads to a reduction in consumption and subsequently GDP growth in both countries.

Workers try to insure themselves against uncertainty about future wages by demanding higher pay in the current period and this puts upward pressure on inflation.

1 Introduction

A vast body of empirical research has focused on estimating the domestic impact of structural economic shocks originating from the rest of the world. Structural vector autoregressions ((S)VARs), originally proposed by Sims (1980), have featured prominently in this literature as they offer a flexible data-driven approach to modelling the international transmission mechanism. Prominent papers that adopt this approach include VAR studies by Cushman and Zha (1997), Kim (2001) and Scholl and Uhlig (2006) amongst many others.

While the international transmission mechanism of these shocks has been studied deeply, the role played by changes in the volatility of these shocks has been ignored in this literature. Most of the adopted SVAR models in these papers assume homoscedastic shocks. Studies that do allow for time-varying shock volatility (see for example Mumtaz and Sunder-Plassmann (2010)) do not incorporate a direct impact of the shock variance on the endogenous variables. The omission of this transmission channel is a potential problem because of three considerations.

First, a growing number of studies have demonstrated that the volatility of structural shocks such as monetary policy, supply and demand has fluctuated substantially in large industrialised countries like the United States. For example, the estimated volatility of the (US) monetary policy shock in Primiceri (2005) increases by more than 100% during the early 1980s. Second, the recent financial crisis has highlighted the fact that macroeconomic volatility cannot be regarded as a ‘pre-great moderation phenomenon’. Therefore, gauging the potential impact of an increase in shock volatility in the outside world is a relevant concern for policymakers in small open economies such as the United Kingdom. Third, there is a growing body of theoretical work that has identified channels through which changes in volatility can affect the real economy. For example, Bloom (2009) presents simulations from a model where higher uncertainty causes firms to pause their hiring and investment leading to a drop in real activity. Using a non-linear small open economy DSGE model, Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2009) suggest a channel through which changes in real interest rate volatility can affect open economies that use foreign debt to smooth consumption and to hedge against idiosyncratic productivity shocks. As real interest rate volatility increases and as countries are increasingly exposed to variations in marginal utility, they reduce the level of foreign debt by cutting consumption. Investment falls as foreign debt becomes a less attractive hedge for productivity shocks leading to a fall in real activity.

This paper attempts to fill this gap in the literature by using an extended SVAR model to estimate the effect of an increase in *the volatility of shocks to US real activity* on the UK economy. The extensions to the SVAR model proposed in this paper include: (1) allowing for time-varying variance of structural shocks via a stochastic volatility specification and (2) by allowing a dynamic interaction between the level of the endogenous variables in the VAR and this time-varying volatility. This extended VAR model can therefore be used to not only gauge the effect of foreign shocks but also the impact of changes in the volatility of the shock in question.

We focus the paper on the potential impact of the volatility of shocks to US real activity because of the policy relevance of this question. In particular, during the recent recession and financial crisis, uncertainty about growth in the United States (and the euro area) has been a key concern for policymakers. The methodology proposed in this paper is used to quantify the potential impact of this uncertainty.

Using this extended SVAR model we find that a one standard deviation shock to the volatility of US real activity shocks leads to a 0.15% decline in UK quarterly GDP growth and a 0.1% increase in quarterly UK CPI inflation.

We next use an otherwise standard small open economy New Keynesian DSGE model calibrated to US and UK economies to investigate what kind of structural uncertainty shock could generate the dynamic behaviour obtained by the empirical model. The model discussed in Appendix B is a simplified version of the model developed by Adolfson, Laseen, Linde and Villani (2007). Our specification does not consider physical capital dynamics and we use a closed economy model DSGE model to describe the foreign economy (this is a version of Smets and Wouters (2007) model but again without capital). The simulations presented below illustrate that shocks that generate marginal cost uncertainty – supply type volatility shocks such as wage mark-up and productivity stochastic volatility shocks – can reproduce the dynamic paths that we see in the data.

In Chart 1 we summarise the basic transmission mechanism. An innovation to the volatility of foreign supply type shocks leads to an increase in foreign inflation because: (1) workers prefer to set higher current wages as an insurance against the possibility that they may be 'locked in' to a contractual agreement (via the Calvo mechanism) to supply more labour when demand is high without being able to renegotiate wages. (2) Firms prefer to set higher prices in the current period to avoid a similar scenario. (3) Because firms are subject to a working capital constraint, a higher interest rate (via the Taylor rule) puts additional upward pressure on marginal costs. Foreign output, on the other hand, declines because risk-averse agents save more in the face of uncertainty and households supply less labour to match the higher wages. The real exchange rate depreciates (from the domestic country perspective) because the interest rate differential between the foreign and domestic economies is positive and this occurs because domestic inflation rises by less than foreign inflation. Exports from the domestic economy increase as a consequence. However this is not enough to prevent a contraction of domestic output which is driven by a fall in domestic consumption (due to precautionary saving). Domestic CPI inflation increases for exactly the reasons discussed for the foreign economy. However for the domestic economy there is an additional channel: the exchange rate depreciation leads to an increase in import prices and this puts further upward pressure on domestic CPI inflation.

Our model simulations show that innovations to the volatility of demand type shocks lead to impulse responses that are qualitatively similar but are of negligible magnitude. This results from the fact that a level demand shock does not create a trade-off between output and inflation for the policymaker.

The paper is organised as follows: Sections 2 and 3 introduce the SVAR model and discuss the estimation method. The results from the VAR model are presented in Section 4. We introduce the open economy DSGE model in Section 5 and present the model simulation in Section 5.4.

2 Empirical model

We estimate the following VAR model with stochastic volatility:

$$Z_t = c + \sum_{j=1}^{P} \beta_j Z_{t-j} + \sum_{j=0}^{J} \gamma_j \tilde{h}_{t-j} + \Omega_t^{1/2} e_t,\ e_t \sim N(0,1) \quad \textbf{(1)}$$

where

$$\Omega_t = A^{-1} H_t A^{-1\prime} \quad \textbf{(2)}$$

In equation **(1)** Z_t denotes the N macroeconomic variables (US GDP growth, US CPI inflation, the federal funds rate, UK GDP growth, UK CPI inflation and Bank Rate in our application below), while $\tilde{h}_t = [h_{1t}, h_{2t} ... h_{Nt}]$ refers to the log volatility of the structural shocks in the VAR. This latter feature can be seen more clearly by considering our application where $N = 6$. The structure of H_t in equation **(2)** is then given by

$$H_t = \begin{pmatrix} \exp(h_{1t}) & 0 & 0 & 0 & 0 & 0 \\ 0 & \exp(h_{2t}) & 0 & 0 & 0 & 0 \\ 0 & 0 & \exp(h_{3t}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \exp(h_{4t}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \exp(h_{5t}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \exp(h_{6t}) \end{pmatrix} \quad \textbf{(3)}$$

The structure of the A matrix is chosen by the econometrician to model the contemporaneous relationship amongst the reduced-form shocks. We discuss our choice of the structure of the A matrix in section 3.2 below.

The transition equation for the stochastic volatility is given by:

$$\tilde{h}_{i,t} = \theta_i \tilde{h}_{i,t-1} + \eta_{i,t},\ \eta_{i,t} \sim N(0, Q_i),\ E\left(e_t, \eta_{i,t}\right) = 0,\ i = 1, 2..N \quad \textbf{(4)}$$

where Q_i is a diagonal matrix. There are two noteworthy features about the complete system defined by equations **(1)**, **(2)** and **(4)**. First, equation **(1)** allows the volatility of the *structural* shocks $\tilde{h}_t$ to have an impact on the endogenous variables Z_t. [1] Second, note that the structure of the matrix A in equation **(2)** determines the interpretation of structural shocks and hence their volatility H_t. In the 6 variable example above with Z_t containing US GDP growth, US CPI inflation, the federal funds rate, UK GDP growth, UK CPI inflation and Bank Rate (in that order), a lower triangular structure for A_t would imply that one could interpret h_{1t} as the log volatility of the shock to US real activity, where this shock is identified via the assumption that UK shocks have no contemporaneous impact on US real activity – ie US GDP is the most exogenous variable in the system. Alternatively, one may restrict the signs of elements of A^{-1} to identify the shocks via contemporaneous sign restrictions. The ability to place an economic interpretation on some or all of the shocks is important as it allows the model to tackle the analysis of the impact of volatility in a theoretically consistent manner.

Note that equation **(4)** makes the simplifying assumption that the shocks to the volatility equation η_{it} and the observation equation e_t are uncorrelated and Q_i is a diagonal matrix. With these assumption in place,

[1] In our specification the log volatility enters the VAR equations rather than its level. This is primarily because the former specification proved to be substantially more computationally stable than the latter in our experiments. In particular, the level specification is sensitive to the scaling of the variables with the possibility of overflow whenever the scale of the variables is somewhat large.

one can interpret an innovation in η_{it} as a shock to volatility of the structural shock of interest and then calculate the response of h_t and Z_t. On the other hand, if these assumptions are relaxed, further identifying restrictions are required to distinguish amongst the volatility shocks and to separate the innovation to the volatility from the innovation to the level. Note that in this more general scenario (ie with a full covariance matrix amongst the volatility and level innovations), identification of the volatility shocks is substantially more involved. In particular, there is no simple way to assign $h_{i,t}$ to a particular structural shock (as done in the proposed model above) and the researcher has to take a stand on the restrictions to place on the contemporaneous relationships amongst the volatilities. In contrast, the assumptions in equation **(4)** allows the use of standard identification schemes (that apply to the contemporaneous relationships amongst the *level* of the reduced-form shocks rather than their volatility). To retain this ease of intepretation of $h_{i,t}$ we incorporate the assumption of a diagonal Q_i and no correlation amongst e_t and η_{it} in the proposed empirical model.

The time-series model considered in this section can be seen as a simplification of the reduced-from version of a small open economy DSGE model with stochastic volatility – such as the one discussed in Appendix B. To explore this point we start from the studies of Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011b), Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2011a) and Fernández-Villaverde and Rubio-Ramírez (2010) that argue that in order to study the effects of volatility on a DSGE economy we need to approximate agents' decision rule at least up to the third order. This is due to the fact that for any perturbation order less than three, the stochastic volatility components do not enter into the policy function with non-zero coefficients. [2] We express, therefore, the non-linear state-space representation of a third-order approximated DSGE model as follows

$$\mathcal{Y}_t = g\left(\mathcal{X}_{t-1}, \mathcal{Z}_{t-1}, \Sigma_{t-1}, \mathcal{U}_t, \mathcal{E}_t\right) \quad \textbf{(5)}$$

$$\mathcal{X}_t = h\left(\mathcal{X}_{t-1}, \mathcal{Z}_{t-1}, \Sigma_{t-1}, \mathcal{U}_t, \mathcal{E}_t\right) \quad \textbf{(6)}$$

where $\mathcal{Y}_t$ is the vector of non-predetermined variables, $\mathcal{X}_t$ denotes the vector of the endogenous predetermine variables, while $\mathcal{Z}_t$ stands for the vector of the exogenous predetermine variables, the vector of stochastic volatility series – Σ_t – is kept separated from $\mathcal{Z}_t$. $\mathcal{U}_t$ is the vector of the structural shocks that perturbate the economy, while the stochastic volatility shocks are collected in the vector $\mathcal{E}_t$. [3] We eliminate spurious high-order terms by adopting the pruning approach developed by Kim, Kim,

[2]In the second-order approximation case the stochastic volatility shocks affect the state vector indirectly, through their cross-products with their level shocks (see, Fernández-Villaverde *et al* (2011b) and Fernández-Villaverde and Rubio-Ramírez (2010, Section 4.4)).

[3]We have adopted the notation used by Fernández-Villaverde and Rubio-Ramírez (2010)

Schaumburg and Sims (2008).

Our observable vector Z_t consists of elements of $\mathcal{S}_t = \left(\mathcal{Y}_t', \mathcal{X}_t'\right)' = f\left(\mathcal{X}_{t-1}, \mathcal{Z}_{t-1}, \Sigma_{t-1}, \mathcal{U}_t, \mathcal{E}_t\right)$ – $Z_t = J\mathcal{S}_t$, where J is a selection matrix of ones and zeros. It is not hard to see that the following derivatives

$$\frac{\partial Z_t}{\partial \mathcal{E}_t} = J\frac{\partial \mathcal{S}_t}{\partial \mathcal{E}_t} = J\frac{\partial f\left(\mathcal{X}_{t-1}, \mathcal{Z}_{t-1}, \Sigma_{t-1}, \mathcal{U}_t, \mathcal{E}_t\right)}{\partial \mathcal{E}_t} \tag{7}$$

$$\frac{\partial Z_t}{\partial \Sigma_{t-1}} = J\frac{\partial \mathcal{S}_t}{\partial \Sigma_{t-1}} = J\frac{\partial f\left(\mathcal{X}_{t-1}, \mathcal{Z}_{t-1}, \Sigma_{t-1}, \mathcal{U}_t, \mathcal{E}_t\right)}{\partial \Sigma_{t-1}} \tag{8}$$

capture the contemporaneous and the persistence effect – respectively – that the stochastic volatility vector has on the theoretical economy. In our empirical model – expressions **(1)** and **(2)** – these effects are summarised by matrices γ_0 and γ_1, in other words, our non-linear SVAR model captures the third-order non-linear dynamics implied by the model. Clearly, the mapping from the theoretical to empirical model is not one to one, as there are several cross-products implied by **(7)** and **(8)** not included in **(1)**, however, the time-series model appears rich enough to capture the macroeconomic aggregates responses to uncertainty perturbations. [4]

The model proposed above is related to a number of recent contributions. For example, the structure of the stochastic volatility model used above closely resembles the formulations used in time-varying VAR models (see Primiceri (2005)). Our model differs from these studies in that it allows a direct impact of the volatilities on the level of the endogenous variables. The model proposed above can be thought of as a multivariate extension of the stochastic volatility in mean model proposed in Koopman and Uspensky (2000) and applied in Berument, Yalcin and Yildirim (2009), Kwiatkowski (2010) and Lemoine and Mougin (2010). In addition, our model has similarities with the stochastic volatility models with leverage studied in Asai and McAleer (2009). However, unlike these contributions, the model proposed above is formulated with the aim of characterising the dynamic effects of volatility of *structural* shocks.

[4] In a promising work Aruoba, Bocola and Schorfheide (2011) propose a new class of non-linear time-series model that can be used to evaluate DSGE models that have been solved using second-order perturbations methods. The authors, however, identify the difficulties of working with the multivariate version of these models – all the DSGE evaluation exercises are carried out using univariate non-linear models.

3 Estimation

3.1 The Gibbs sampling algorithm

The non-linear state-space model consisting of the observation equation **(1)** and transition equation **(4)** is estimated using a Gibbs sampling algorithm. The appendix presents details of the priors and the conditional posterior distributions while a summary of the algorithm is presented below.

The Gibbs sampling algorithm proceeds in the following steps:

1. Conditional on a draw for the stochastic volatility $\tilde{h}_t$, and the matrix A, equation **(1)** represents a VAR model where the disturbances are heteroscedastic and correlated with the regressors $\tilde{h}_t$ (if the contemporaneous value of $\tilde{h}_t$ is included on the RHS of equation **(1)**). However, conditional on $\tilde{h}_t$, and the matrix A, the form of heteroscedasticity and this correlation is known and can be removed via a simple GLS transformation of the model. Once this is done, standard results for conditional posterior distribution of the VAR coefficients apply to $\Gamma = [c, \beta, \gamma]$.
2. Conditional on a draw for $\tilde{h}_t$ and Γ, the elements of the matrix A can be drawn using a series of linear regression models amongst the elements of the residual matrix $v_{it} = \Omega_t^{1/2} e_{it}$ as shown in Cogley and Sargent (2005). Conditional on $\tilde{h}_t$, the autoregressive parameters θ_i and variances Q_i can be drawn using standard results for linear regressions.
3. Conditional on Γ, A, θ_i and Q_i, the stochastic volatilities are simulated using a date by date independence Metropolis step as described in Jacquier, Polson and Rossi (2004) (see also Carlin, Polson and Stoffer (1992)).

We use 100,000 replications and base our inference on the last 10,000 replications. The recursive means of the retained draws (see appendix) show little fluctuation providing support for convergence of the algorithm.

3.2 Model specification and the identification of the shock to US real activity

In our application, the vector of endogenous variables Z_t contains quarterly data on US GDP growth, US CPI inflation, the federal funds rate, UK GDP growth, UK CPI inflation and Bank Rate over the period

1975Q1 to 2011Q3. [5] We employ the following benchmark VAR specification:

$$Z_t = c + \sum_{j=1}^{2} \beta_j Z_{t-j} + \sum_{j=0}^{1} \gamma_j \tilde{h}_{t-j} + \Omega_t^{1/2} e_t \tag{9}$$

The structure of the prior on β and the γ matrices (described in the appendix) incorporates a small open economy assumption for the United Kingdom. In particular, we incorporate the prior belief that the lagged UK variables and stochastic volatilities have a negligible impact on the United States.

The lag length of the endogenous variables is set at two reflecting convention in studies employing similar VAR models to quarterly data (see for example Cogley and Sargent (2005) and Primiceri (2005)). In our benchmark model, the contemporaneous and the lagged value of $\tilde{h}_{it}$ is allowed to affect Z_t. Given that we employ quarterly data, we allow the possibility of an impact of $\tilde{h}_t$ within a three-month period. We show in the sensitivity analysis below that the benchmark results are not affected if longer lags of volatility are included in the mean equations.

In order to identify the US demand/real activity shock we consider the following recursive structure for $\tilde{A} = A^{-1}$

$$\tilde{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \tilde{a}_{2,1} & 1 & 0 & 0 & 0 & 0 \\ \tilde{a}_{3,1} & \tilde{a}_{3,2} & 1 & 0 & 0 & 0 \\ \tilde{a}_{4,1} & \tilde{a}_{4,2} & \tilde{a}_{4,3} & 1 & 0 & 0 \\ \tilde{a}_{5,1} & \tilde{a}_{5,2} & \tilde{a}_{5,3} & \tilde{a}_{5,4} & 1 & 0 \\ \tilde{a}_{6,1} & \tilde{a}_{6,2} & \tilde{a}_{6,3} & \tilde{a}_{6,4} & \tilde{a}_{6,5} & 1 \end{pmatrix} \tag{10}$$

Given the ordering of the endogenous variables (as US GDP growth, US CPI inflation, the federal funds rate, UK GDP growth, UK CPI inflation and Bank Rate) this structure for $\tilde{A}$ implies that the first shock is identified as an exogenous increase in US real activity where we are agnostic about the source of the shock. We use the non-linear DSGE model in Section 5 to analyse the direction of the response to an increase in the volatility of this shock and consider alternative sources of the shock within the model.

[5] The US data is obtained from the FRED database. The FRED codes are as follows: (1) Real GDP: GDPC96 (2) CPI: CPIAUCSL (3) Three-month treasury bill rate: FEDFUNDS. UK real GDP is obtained from the Office for National Statistics (code ABMI). UK CPI and Bank Rate is obtained from the Bank of England database.

4 Results

4.1 Estimated volatility

The first column in the top row of Chart 2 presents the estimated volatility of the shock to US real activity. The volatility of this shock is highest in the pre-1985 period reaching its peak during the late 1970s. The post-1985 period contains smaller increases at the time of the first Gulf war during the early 1990s, the recession of 2000 and then towards the end of the sample coinciding with the recent financial crisis. The profile for the volatility of the shock to the US inflation equation is similar with the highest variance concentrated in the pre-1985 sample. One noticeable feature, however, is the substantial increase in the volatility of this shock during the recent crisis. The evolution of the volatility of the federal funds rate shock is very similar to the estimate in Benati and Mumtaz (2007), with large increases during the great inflation of the mid-1970s and then during Paul Volcker's experiment of targeting non-borrowed reserves at the end of the 1970s. Note that the great moderation period – starting from the mid-1980s – was associated, on the whole, with less volatile policy shocks. An exceptions to this stability is the recent recession.

The top fourth panel of the chart shows the estimated volatility for the shock to the UK GDP growth equation. The volatility of this shock is high during most of the 1970s. It reached a peak in 1975 before declining towards the end of this decade. The volatility increased again in 1980, before declining and settling at a lower level over most of the remaining sample period. Note, however, that the recent recession saw an increase in this volatility. The profile of the volatility of the shock to the inflation equation is similar to the one depicted in Benati (2008). The volatility of this shock was high during the mid-1970s. This volatility rose again during the early 1980s before settling at a low level. The volatility was high over the period coinciding with the ERM crisis and then declined and remained stable over the great moderation. The recent crisis, however, saw a sharp increase in the variance of this shock. The volatility of the Bank Rate shock was high from the mid-1970s to the early 1990s with the estimated profile close to that reported in Benati (2008). While the volatility of this shock has been relatively low over the post-1992 period, the recent crisis saw an increase in this volatility.

In Table A we show the estimated posterior distribution of the parameters of the transition equation. The estimated variance of the US real activity volatility shock Q_1 is estimated to be lower than the variance of the US inflation and the federal funds rate volatility shocks respectively. The estimates for θ_i suggest that the estimated stochastic volatility of the shocks in the VAR is highly persistent.

4.2 Impulse response to US real activity volatility shocks

Chart 3 plots the impulse response to a one standard deviation increase in the variance of the US real activity shock where the real activity shock is identified using the recursive structure in equation **(10)**. A one standard deviation shock increases the log volatility of the real activity shock by around 20%. It is worth noting that the increase in the log variance of this shock over the crisis period (2008Q1 to 2009Q1) was approximately 60%. Quarterly US GDP growth falls by 0.05% at the two to three-year horizon. The response of US CPI inflation is positive with this variable increasing by 0.1%. The response of the federal funds rate is more persistent. The federal funds rate increases by 0.2%, but the estimated error bands include a zero response. The impact of this shock on UK GDP growth is negative with growth falling by about 0.1%. UK CPI inflation increases by 0.1% two years after the shock, with the response persisting for about 10 quarters. Bank Rate increases in response to this shock, but the response is estimated imprecisely.

In summary, these results indicate that an innovation to the volatility of the US real activity shock results in a fall in UK GDP growth and an increase in UK CPI inflation. In the section below, we show that these results are robust to various changes in the VAR specification and alternative assumptions about the identification of the US real activity shock.

4.2.1 Sensitivity analysis

In order to test the robustness of the results to our choice of endogenous variables we estimate an expanded VAR system that includes three additional variables for the United Kingdom. In particular, we include UK broad money (M4) growth, the growth rate of the dollar to the pound exchange rate and the growth rate of the FTSE All-Share return index. We include the latter two variables to account for the external sector and domestic asset prices, respectively. Money growth is included to account for the period of monetary targetting during the 1980s. The estimated impulse responses from this expanded system are shown in Chart 4. The key results in this model are similar to the benchmark case. In particular, the increase in the volatility of the US real activity shock leads to a fall in UK GDP growth and an increase in UK CPI inflation. Bank Rate is also estimated to decline. The volatility shock results in a fall in the growth of the FTSE, with a decline of about 0.4%. M4 growth is negative two quarters after the shock with a decline of 0.1%. There is evidence of an exchange rate depreciation after this shock, with exchange rate growth about -0.5%. However, after 5 quarters, the exchange rate is estimated to appreciate, with a growth rate of 0.5%. As in the benchmark case, US CPI inflation increases as a result of this shock and GDP growth declines.

The left panel of Chart 5 shows the impulse response to the real activity volatility shock from a version of the VAR model that allows the contemporaneous value and two lags of $\tilde{h}_t$ to affect the endogenous variables. The estimated impact of the shock is very similar to the benchmark. In particular, GDP growth declines in the United States and the United Kingdom while inflation increases. Similarly, the middle panel of Chart 5 shows that when a measure of de-trended GDP is used instead of GDP growth the direction of the impulse responses are relatively unchanged. The response of HP-filtered GDP in the United States and the United Kingdom is persistent with a decline of 0.2% at the one-year horizon. The third panel of this chart shows impulse responses from a version of the model where US GDP is the second variable in the recursive ordering implicit in equation **(10)**. The main results using this alternative ordering are similar to the benchmark case. In particular, GDP growth in the United States and the United Kingdom declines while inflation is estimated to increase in response to the volatility shock. The final column of the chart shows impulse responses from a version of the model where US GDP is the third variable in the recursive ordering. The responses of GDP growth and inflation in the two countries are very similar to the benchmark case. Note, however, that the response of the federal funds rate differs in this specification.

To summarise, this sensitivity analysis supports the idea that an increase in US real activity shock variance leads to a decline in UK GDP growth and an increase in UK inflation

5 Explaining the results: a non-linear DSGE model

5.1 The model

We use a small open economy DSGE model with stochastic volatility to study the theoretical plausibility of the results obtained from the SVAR model. The structural model used in this study is a simplified version of the model developed by Adolfson *et al* (2007). Our specification does not contain capital dynamics and, instead of assuming exogenous driving processes for all foreign variables (Justiniano and

Preston (2010), Adolfson *et al* (2007)), we explicitly model the foreign economy (Bauerle and Menz (2008)). [6] The use of a closed economy non-linear DSGE model to describe the foreign economy (similar to Smets and Wouters (2007) but we abstract again from capital dynamics) is a necessity in our case. The reason is that we want to study how uncertainty shocks originated aboard are transmitted to domestic economy and this cannot be captured from a set of linearised equations accurately.

The model is described in detail in the appendix. Here we provide a summary of the key features. Following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) both – domestic and foreign – economies are characterised by a number of nominal and real frictions such as sticky prices, sticky wages, working capital channel – firms borrow money from a financial intermediate to finance part of their wage bill – and habit persistence. Moreover, both economies are populated by a continuum of infinitely lived households that consume and supply labour. Domestic and foreign households have monopoly power over their labour and they set wages based on Calvo type staggered pricing contracts and backward indexation rules. Domestic households can invest in either domestic or foreign one-period bonds, while foreign agents are restricted from holding domestic bonds. The interest rates on both domestic foreign bonds are subject to risk-premia, which are functions of the net foreign asset position and lagged deviations from the uncovered interest rate parity (UIP).

On the supply side of the economy, there is a continuum of monopolistically competitive firms producing a variety of domestic goods used for the production of the final good, which can be either consumed domestically by agents or the government or – in the domestic economy – be exported. There is also a monopolistically competitive retail sector, which consists of firms that buy a homogenous good in the world market that it is turned into a differentiated consumption good. All sectors are assumed to follow a Calvo staggered pricing schemes and backward indexation rules.

Monetary authorities in both countries adjust their instruments – short-term interest rates – based on a Taylor-type policy rule. Additionally, we assume that domestic authorities responds to real exchange variations to reduce the stabilisation trade-off between inflation and output caused by incomplete pass-through (see Gali and Monacelli (2005)).

[6]We abstract from capital to make our model parsimonious and computationally tractable. Adding capital leads to a significant increase in the dimension of the state vector and increases the computational costs considerably. Note that, in principle an additional asset such as capital could help agents to smooth consumption more efficiently potentially implying smaller effects (than those reported below) due to uncertainty shocks.

5.2 Calibration

We use the estimates reported in Table A to calibrate the evolution of the SV processes in the model. To be precise, the first row is used for the US preference and government spending SV – demand – processes, the second line for the parameters that describe the evolution of US domestic price mark-ups, wage mark-ups, labour supply and productivity SV – supply – processes. Finally, we use the third line to calibrate the uncertainty of the non-systematic part of the foreign Taylor rule. Similar to Fernández-Villaverde and Rubio-Ramírez (2010) we set for both countries both the inverse of intertemporal substitutionlog utility and the Frisch elasticity parameters equal to 2 ($\sigma = \sigma^* = \varphi = \varphi^* = 2$), and the Frisch elasticity equal to 2. Following Adolfson *et al* (2007) we assume 5% steady-state wages mark-ups ($\lambda_w = \lambda_w^* = 1.05$). We set the steady-state value of labour supply shock $\psi = \psi^*$ equal to 7.5, implying that the steady-state value of hours worked is approximately 30%. The value of $\beta = \beta^*$ (0.99) pins down the steady-state value of the real interest rate (4%, annually). For simplicity, we assume zero inflation in both countries. The values of elasticity of substitution between domestic and imported goods (η), between domestic and foreign exports (η_f) and the weight on import consumption (a) are taken from Harrison and Oomen (2010), however, these values are very similar to those used by Adolfson *et al* (2007). All domestic Calvo and indexation parameters – coefficients of the domestic, import, export and wages Phillips curves – are those estimated by Adolfson *et al* (2007), while the foreign ones by Smets and Wouters (2007). The steady-state mark-ups of domestic producers in both countries is 17% ($\lambda_d = \lambda_d^* = 1.17$), while importers have more monopoly power when they set prices $\lambda_m = 1.62$ (see Adolfson *et al*, 2007). We set λ_x equal to 1.05. The coefficients of the risk premium function $\chi_a = 0.14$ and $\chi_s = 1.25$ are used by Adolfson *et al* (2007) and Christiano, Trabandt and Walentin (2007), respectively. As it is explained in Christiano *et al* (2007) a value of greater than one is required for χ_s to reconcile the negative reduced-form regression coefficient of the UIP that we see in the data. We use Taylor coefficient estimates taken from Smets and Wouters (2007) for both countries $\left(\phi_\pi = \phi_{\pi^*} = 2.03, \phi_y = \phi_{y^*} = 0.3\right)$ and similar to Vukotic (2007) we set ϕ_q equal to 0.13. Finally, for most of the level foreign shock autoregressive coefficients we use the estimates of Smets and Wouters (2007).

5.3 Solution

The model is solved using third-order perturbation methods (see Judd (1998)) since for any order below three stochastic volatility shocks that we are interested in do not enter into the decision rule as independent components. One difficulty of using these higher-order solution techniques is that paths simulated by the approximated policy function often explode. As it is explained by Kim *et al* (2008) regular perturbation approximations are polynomials that have multiple steady state and could yield unbounded solutions. In other words, this approximation is valid only locally and along the simulation path we may enter into a region where its validity is not preserved anymore.

To avoid this problem Kim *et al* (2008) suggest to 'prune' all those terms that have an order that is higher than the approximation order, while Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2012) show how this logic can be applied to any order. Although there are studies that question the legitimacy of this approach (see Haan and Wind (2010)), it has by now been widely accepted as the only reliable way to get the solution of n – where $n > 1$ – order approximated DSGE model.

Finally, we follow the procedure described in the online technical appendix of Fernández-Villaverde *et al* (2011b) to generate the responses of model variables to stochastic volatility shocks. To be precise, we simulate the model for 4,000 periods and we use the last 2,000 observations to calculate the ergodic mean of the state vector. Starting from the ergodic mean we perturbate the system with a x times standard-deviation stochastic volatility shock that rises the uncertainty of the level of the shocks by 100% and we report the impulse responses as percentage deviations from their ergodic mean. Similar to Fernández-Villaverde *et al*, we also check whether the generalised impulse responses (seeKoop, Pesaran and Potter (1996))are different from those constructed using the procedure above and we found no significant discrepancies.

5.4 Description of the exercises and summary of the results

Supply stochastic volatility shocks

The first exercise is to see how agents in both economies respond to *foreign* uncertainty supply shocks. [7] Stochastic volatility shocks increase the uncertainty of level supply shocks by 20%. Chart 6 illustrates agents' optimal responses to i) wage mark-up and productivity uncertainty shocks – red solid line, ii) wage mark-up, productivity and labour supply uncertainty shocks – black dashed-dotted line, and iii) to wage mark-up, productivity, labour supply and price mark-up uncertainty shocks.

[7] All the shocks that hit the domestic economy are switched off. This choice has been made in order to meet Matlab's memory requirements.

There is no doubt that these shock have a significant effect on the economy, output falls persistently below its steady state by almost 0.4% and it does not recover even 10 years after the shock. Inflation rises by almost 0.4% above the target and stays elevated for more than 40 quarters. Another interesting observation is that the maximum impact of the shock takes place three years later. The effects on the domestic economy are of similar order domestic GDP contracts by almost 0.1% when home inflation rises by 0.1%. Recession and 'stagflation' are also the key features of the SVAR responses, making the supply stochastic volatility shocks a candidate structural interpretation of the empirical evidence.

Consider the transmission mechanism underlying these results. Note that foreign agents respond to higher uncertainty by decreasing consumption and increasing savings as they are risk-averse. As explained in Basu and Bundick (2011) (see also Chart 12) in the absence of labour market frictions, lower consumption increases labour supply – the marginal utility of consumption λ_t rises – and leads to lower wages. In our case, households have monopoly power when they set their wages and they respond to higher uncertainty by actually increasing the wedge (equilibrium actual mark-ups) [8] between the wage and the marginal rate of substitution. Households act in this way in order to hedge themselves against future variation in labour demand. To be more explicit, let us say that an uncertainty shock hits the economy and a household signs a Calvo contract today. The houeshold knows that if in the future the economy 'improves', then with a constant probability ξ_w it will not be able to reset its wage and in order to honour its contract it has to supply as much labour as is demanded by the firm at higher disutility cost. In order to avoid this situation, it increases its wage wedge today. As it can be seen from Chart 13 the labour supply schedule in this case moves to the left, which leads to higher wages and, consequently, to a larger negative effect on output. [9] Foreign producers respond to higher uncertainty in a similar manner, meaning that they increase their price wedge or endogenous mark-up in order to insure themselves against higher demand in the future. The logic is the same: if a firm that signs a Calvo contract sets the price too low today and there is a positive demand shock in the future then they end up selling a higher amount of goods with a loss in order to honour its contract. This is due to the fact that with a constant probability a fraction of firms may not be able to reset prices optimally. Higher price wedges and higher marginal cost due to higher wages imply higher inflation and the foreign policymaker responds by increasing the short-term interest rate to bring inflation back to its target.

Domestic households and firms act in a similar way when they form decisions about wages and prices,

[8] We use the term 'wedge' instead of equilibrium actual mark-ups to avoid confusion with the exogenous/desired markup processes.

[9] Firms also have monopoly power in our setup and they respond to higher marginal cost with lower demanded labour in order to preserve their profits.

respectively. They use mark-ups as an insurance device to avoid working or producing more when a good shock hits the economy as they may not be able to optimally reset wages and prices, respectively, due to Calvo contracts.

As explained in Benigno, Benigno and Nistico (2011), households in home economy have an additional device to insure themselves against higher uncertainty – foreign bonds. The authors argue that an asset can be considered as a good hedging device against uncertainty if it pays well when money is needed. In other words, if the expected nominal exchange rate depreciates, then investing in foreign bonds is a good strategy since it offers a better return when it is actually required. Home economy households can accumulate foreign bonds through trade surplus, however, given that the foreign demand is suppressed during this period, positive net trade can be achieved only through a weaker exchange rate. As it has been discussed earlier, households and firms will respond to higher supply shock uncertainty by biasing upwards their wage and price decisions and this pushes inflation and nominal interest rate higher. However, the home interest rate rises by less than the foreign policy rate and this causes the exchange rate to depreciate. This happens because rebalancing takes place in the domestic economy. Domestic economy agents do not substitute expensive foreign consumption with cheap domestic one but they prefer to go through a significant de-leveraging.

Another interesting feature that we observe from Chart 6 is that uncertainty related with labour supply and price mark-up shocks does not cause as much damage to the economy as wage mark-up and productivity stochastic volatility shocks. The reason is that the latter two shocks affect the marginal cost directly causing inflation to increase significantly. The monetary authority (that is devoted to stabilising prices) tightens policy and this reduces demand further.

Demand stochastic volatility shocks

Chart 7 illustrates what happens to both – foreign and domestic – economies when the uncertainty of US discount factor – β – and government spending shocks rise by 20%. Interestingly, demand stochastic volatility shocks do not seem to have a significant impact on the economy when compared to supply uncertainty shock that we studied earlier. The impact of these shocks is about two or three orders of magnitude smaller than the supply uncertainty shocks.

This difference arises because of the following reason: a demand level shock moves output and inflation in the same direction and as a consequence the policymaker can easily offset the impact of this shock. For instance, a negative demand shock that lowers output also decreases inflation. Economic agents are less concerned about uncertainty regarding this shock as they are aware that monetary authorities will follow an expansionary policy to restore both output and inflation. Note that supply level shocks move output and inflation in different directions. Higher uncertainty regarding the realisation of supply shocks implies that agents are less certain about the "support" they can receive from the monetary authority (in the face of negative shocks) and as a consequence the precautionary savings mechanisms (discussed in the context of supply uncertainty shocks above) are employed more intensively.

5.5 Key model characteristics

The simulations reported above represent fairly strong evidence in favour of the hypothesis that the real activity uncertainty shocks identified in our VAR model are consistent with uncertainty about supply type shocks. In this subsection we explore this result further. In particular, we aim to identify those key model characteristics that are responsible for the main findings of our simulations. In the following exercises we conduct the same experiment as considered in the benchmark case: the foreign economy is hit by a set of stochastic volatility shocks that increase the uncertainty of the level of foreign productivity, wage mark-up, labour supply and price mark-up shocks by 100%. We call the responses derived by the model that has been used so far benchmark and they are compared with those obtained when selected features of the model are switched off.

Switching off the working capital friction: $\phi_f = \phi_{f^*} = 0$

When a fraction of the wage bill must be financed in advance, then firms must borrow and this links the short-term interest rate with firms' marginal cost. The studies of Christiano *et al* (2005), Adolfson *et al* (2007) and Christiano *et al* (2007) present time-series evidence in favour of this mechanism. We saw earlier that supply stochastic volatility shock lowers output and increases inflation. Since the policymaker responds to higher inflation with higher interest rates, these frictions amplify the effects of the shock as it magnifies the negative trade-off between inflation and output and, consequently, limits monetary authorities' ability to offset the consequences of the shocks. In Chart 8 we switch off this friction in order to assess its contribution to our simulations.

From Chart 8 it is clear that this friction enlarges the adverse effects of the shock, however, this does not seem to be the main driver. [10]

Switching off the labour market frictions: $\xi_w = \xi_{w^*} = 0.05$ *and* $\kappa_w = \kappa_{w^*} = 0$

We next investigate the role played by labour market frictions. As it is was mentioned earlier, households' monopoly power over their wages offsets some of the labour supply effect caused by precautionary savings. Chart 12 illustrates that, absent labour market frictions, an increase in uncertainty moves the labour supply schedule to the right, implying lower wages, more hours and this moderates some of the negative impact of the shock. However, from Chart 13 we see that the labour supply effect is mitigated – or even reversed depending on model's calibration – when households can set their wages. Under higher wages and with sticky prices, firms are going to cut hours back by more, relatively to the flexible wages case.

Chart 9 suggests that absent labour market frictions the effects of supply uncertainty shocks are very small. In this case the real wage is set as a mark-up λ_w over the marginal rate of substitution, implying that the labour supply needs to adjust in order for this relationship to hold. This seems to mitigate the importance of the marginal cost uncertainty channel discussed earlier. Workers now respond to higher uncertainty by supplying more labour and this put a downward pressure on wages and, consequently, on the marginal cost. Flexible wages reduce (or even eliminates) the trade-off faced by monetary authorities (since they are now happy to loosen policy as inflation does not rise) and this makes agents less concerned about supply stochastic volatility shocks.

Standard UIP: $\chi_s = 0$

We now study how the second term of the risk premium function – $\chi_s \left(\frac{rr_{t-1}}{rr^*_{t-1}} \frac{q_{t-1}}{q_t} - 1 \right)$ – influences the results. This terms allows the exchange rate to deviate persistently from the UIP. This is a well known stylised fact (see, Eichenbaum and Evans (1995)) that cannot be reproduced by the DSGE model easily (see, Christiano, Trabandt and Walentin (2011)). By setting χ_s equal to 0, the risk premium function collapses to a more standard format (see Adolfson *et al* (2007)).

Chart 10 shows how agents' behaviour changes when this additional risk channel is switched off. The

[10] Fernández-Villaverde *et al* (2011b, online Appendix) arrive at similar conclusion.

foreign agents' responses are not affected by this change – as in a small open economy model home agents' actions do not alter foreign economy measures.

The blue dashed line in Chart 10 displays the response to a foreign supply uncertainty shock under the assumption that $\chi_s = 0$. Domestic inflation is now lower mainly because wage and price equilibrium mark-ups are lower, however, the story remains the broadly unchanged. The domestic economy goes through a re-balancing phase, where households reduce – both domestic and imported – consumption and increase savings in order to decrease their exposure costly labour and financial income variations.

Similar to working capital channel the effects of uncertainty supply shocks are weaker when the standard UIP is employed, however, these differences are small suggesting that this friction cannot be the main driver of the results.

Stochastic volatility shock persistence

From Chart 2 it is apparent that uncertainty – for all shocks and both countries – is characterised by long cycles. In other words, when uncertainty is high it remains elevated for a prolonged period and vice versa and this seems consistent with the persistence estimates displayed in Table A. The question that we ask in this exercise is: what are the effects of uncertainty supply shocks when they are expected to last only for a short period of time? We do this by setting the persistence parameter equal to 0.7. Chart 11 illustrates the results from this exercise and these are quite revealing. Despite the fact that the economy is subject to the same set of frictions as in the benchmark case the consequence of uncertainty supply shocks are negligible when the disturbances are expected to die out quickly.

In summary, the sensitivity analysis reveals that the degree of persistence of supply uncertainty shocks combined with labour frictions in the DSGE model are important with regards to matching the VAR evidence on the sign and size of impulse responses to real activity volatility shocks.

6 Conclusion

This paper investigates the international transmission of US real activity uncertainty shocks to the UK economy. To do this, we develop an open economy structural VAR model that allows the volatility of US real activity shocks to be timevarying and to have an impact on the endogenous variables. Shocks to US real activity shock uncertainty in the VAR model result in an increase in US and UK inflation and a fall in US and UK output growth.

We then use a non-linear open economy DSGE model to try and distinguish between different structural uncertainty shocks that are consistent with these empirical results. In particular, we consider an increase in foreign supply shock uncertainty and foreign demand shock uncertainty, respectively. We find that the sign and magnitude of the VAR responses are consistent with the supply uncertainty shocks. These shocks lead to higher inflation in both economies through higher marginal cost and to lower output due to precautionary savings.

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Appendix A: The Gibbs sampling algorithm

Prior distributions and starting values

VAR coefficients

We use a training sample of 40 observations to derive the initial conditions for the VAR coefficients Γ_0 (to be used in the Kalman filter as described below). In particular, the values for Γ_0 are obtained via an OLS estimate of equation **(1)** using an initial estimate of the stochastic volatility (using data over the traning sample). The covariance around these initial conditions P_0 is set to a diagonal matrix with diagonal elements equal to 1. This initial estimate of stochastic volatility estimate is estimated as $\left(\Delta Z_t^0\right)^2$ where Z_t^0 denotes the data matrix over the training sample. This measure of volatility is added as exogenous regressors to a VAR in Z_t^0 in order to provide a rough guess for initial conditions for the VAR coefficients.

Elements of H_t

Let $\hat{\upsilon}^{ols}$ denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data described above. The prior for $\tilde{h}_t$ at $t = 0$ is defined as $\ln h_0 \sim N(\ln \mu_0, I_6)$ where μ_0 are the diagonal elements of the Cholesky decomposition of $\hat{\upsilon}^{ols}$.

Elements of A

The prior for the off-diagonal elements A is $A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$ where $\hat{a}^{ols}$ are the off-diagonal elements of $\hat{\upsilon}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$.

Parameters of the transition equation

Following Cogley and Sargent (2005) we postulate an inverse-gamma distribution for the elements of $Q_i \sim IG\left(\frac{Q_{i0}}{2}, \frac{1}{2}\right)$ where $Q_{i0} = 0.01$. The prior for θ_i is given as $N\left(\theta_{i,0}, 1\right)$ where $\theta_{i,0} = 0.8$.

Simulating the posterior distributions

VAR coefficients

The distribution of the VAR coefficients Γ conditional on all other parameters Ξ and the stochastic volatility $\tilde{h}_t$ is linear and Gaussian: $\Gamma \backslash Z_t, \tilde{h}_t, \Xi \sim N\left(\Gamma_{T \backslash T}, P_{T \backslash T}\right)$ where $\Gamma_{T \backslash T} = E\left(\Gamma_T \backslash Z_t, \tilde{h}_t, \Xi\right)$, $P_{T \backslash T} = Cov\left(\Gamma_T \backslash Z_t, \tilde{h}_t, \Xi\right)$. Following Carter and Kohn (2004) we use the Kalman filter to estimate $\Gamma_{T \backslash T}$ and $P_{T \backslash T}$ where we account for the fact that the covariance matrix of the VAR residuals changes through time and that the contemporaneous value of $\tilde{h}_t$ on the RHS of the VAR induces a correlation with the residual term. Note that since we condition on $\tilde{h}_t$ and A, the form of the heteroscedasticity and correlation is known. The final iteration of the Kalman filter at time T delivers $\Gamma_{T \backslash T}$ and $P_{T \backslash T}$.[11] This application of the Carter and Kohn (2004) algorithm to this heteroscedastic VAR model is equivalent to a GLS transformation of the model.

Element of A_t

Given a draw for Γ and $\tilde{h}_t$ the VAR model can be written as $A'\left(\tilde{Z}_t\right) = e_t$ where $\tilde{Z}_t = Z_t - c + \sum_{j=1}^{P} \beta_j Z_{it-j} + \sum_{j=0}^{J} \gamma_j \tilde{h}_{it-j} = v_t$ and $VAR\left(e_t\right) = H_t$. This is a system of linear equations with known form of heteroscedasticity. The conditional distributions for a linear regression apply to this system after a simple GLS transformation to make the errors homoscedastic. More details on this step can be found in Cogley and Sargent (2005).

Elements of H_t

Conditional on the VAR coefficients and the parameters of the transition equation, the model has a multivariate non-linear state-space representation. Carlin *et al* (1992) show that the conditional distribution of the state variables in a general state-space model can be written as the product of three terms:

$$\tilde{h}_t \backslash Z_t, \Xi \propto f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1} \backslash \tilde{h}_t\right) \times f\left(Z_t \backslash \tilde{h}_t, \Xi\right) \qquad \textbf{(A-1)}$$

where Ξ denotes all other parameters. In the context of stochastic volatility models, Jacquier *et al* (2004) show that this density is a product of log normal densities for $\bar{h}_t$ and $\bar{h}_{t+1}$ and a normal density for Z_t

[11] The Kalman filter is initialised using the initial conditions (Γ_0, P_0) described above.

where $\bar{h}_t = \exp\left(\tilde{h}_t\right)$ .Carlin *et al* (1992) derive the general form of the mean and variance of the underlying normal density for $f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \propto f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1} \backslash \tilde{h}_t\right)$ and show that this is given as

$$f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \sim N\left(B_{2t} b_{2t}, B_{2t}\right) \qquad \textbf{(A-2)}$$

where $B_{2t}^{-1} = Q^{-1} + F'Q^{-1}F$ and $b_{2t} = \tilde{h}_{t-1}F'Q^{-1} + \tilde{h}_{t+1}Q^{-1}F$. Note that due to the non-linearity of the observation equation of the model an analytical expression for the complete conditional $\tilde{h}_t \backslash Z_t, \Xi$ is unavailable and a metropolis step is required.

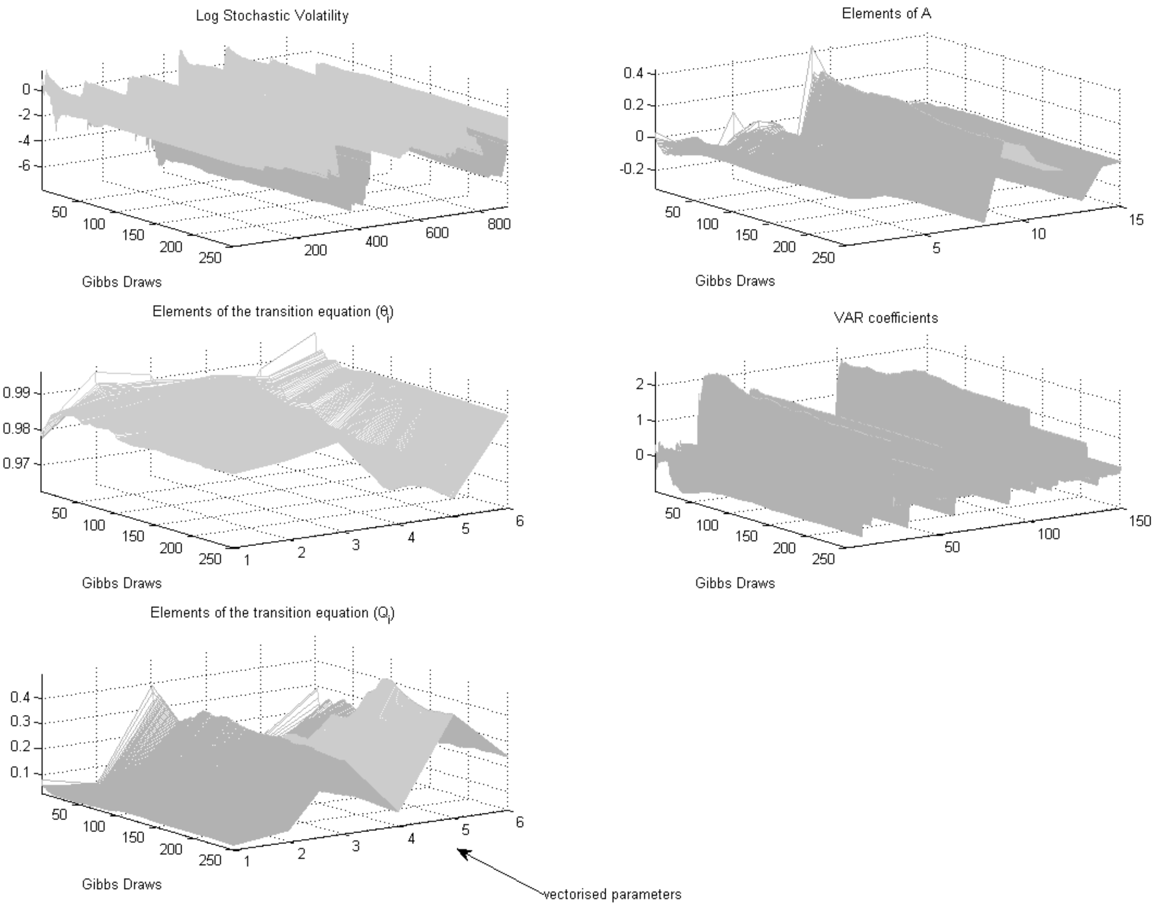
Following Jacquier *et al* (2004) we draw from **(A-1)** using a date-by-date independence metropolis step using the density in **(A-2)** as the candidate generating density. This choice imples that the acceptance probability is given by the ratio of the conditional likelihood $f\left(Z_t \backslash \tilde{h}_t, \Xi\right)$ at the old and the new draw. In order to take endpoints into account, the algorithm is modified slightly for the initial condition and the last observation. Details of these changes can be found in Jacquier *et al* (2004).

Parameters of the transition equation

Conditional on a draw for $\tilde{h}_t$ the transition equation **(4)** is a simply a sequence of linear regressions and the standard normal and inverse Gamma conditional posteriors apply.

Convergence

The MCMC algorithm is applied using 100,000 iterations discarding the first 90,000 as burn-in. The chart below plots recursive means calculated using intervals of 20 draws for the retained draws of the main VAR parameters. The show little fluctuations providing evidence for convergence of the algorithm.


Log Stochastic Volatility
0
-2
-4
-6
50
100
150
200
250
200
400
600
800
Gibbs Draws
Elements of A
0.4
0.2
0
-0.2
50
100
150
200
250
5
10
15
Gibbs Draws
Elements of the transition equation (θi)
0.99
0.98
0.97
50
100
150
200
250
1
2
3
4
5
6
Gibbs Draws
VAR coefficients
2
1
0
50
100
150
200
250
50
100
150
Gibbs Draws
Elements of the transition equation (Qi)
0.4
0.3
0.2
0.1
50
100
150
200
250
1
2
3
4
5
6
Gibbs Draws
vectorised parameters

Appendix B: Domestic economy

The description of all structural parameters used in this model and their values are provided by Table B.

Firms

Three types of firms are operated in the domestic economy. The intermediate monopolistically competitive domestic firms use labour supplied by households to produce a differentiated good that is sold to a final good producer who employs a continuum of these differentiated goods in her constant elasticity of substitution – CES – production to deliver the final good. The monopolistically competitive importing firms use a costless technology and turn a homogenous good – bought in the world market – into a differentiated good, which is then sold to the domestic consumers. The exporting monopolistically competitive firms use similar ‘brand naming’ technology and transform the domestic final good into a differentiated product that is sold to foreign households.

Domestic Firms

This sector consists of three firms, the ‘labour packer’ who hires labour from households and transforms it into a homogenous input good – h_t^d, a continuum of monopolistically competitive firms that buys h_t^d and produces an intermediate $y_{i,t}$ and the final good producer who combines all these intermediate products into a single good consumed by households. The final good producer’s CES production function is given by

$$y_t^d = \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_{d,t}}} di\right]^{\lambda_{d,t}} \qquad \textbf{(B-1)}$$

where

$$\lambda_{d,t} = \left(1-\rho_{\lambda_d}\right)\lambda_d + \rho_{\lambda_d}\lambda_{d,t-1} + \sigma_{\lambda_d} e^{\tilde{\sigma}_{\lambda_d,t}} \omega_{\lambda_d,t} \qquad \textbf{(B-2)}$$

$$\tilde{\sigma}_{\lambda_d,t} = \rho_{\tilde{\sigma}_{\lambda_d}}\tilde{\sigma}_{\lambda_d,t-1} + \sigma_{\tilde{\sigma}_{\lambda_d}}\omega_{\tilde{\sigma}_{\lambda_d},t} \qquad \textbf{(B-3)}$$

denotes the time-varying conditional heteroscedastic mark-up in the domestic good market. The final good producer’s demand curve for $y_{i,t}$ arises from the profit minimisation problem –

$\max_{y_{i,t}} \left\{ p_t \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_{d,t}}} di \right]^{\lambda_{d,t}} - p_{i,t} y_{i,t} \right\}$

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} y_t^d \tag{B-4}$$

The final good price index is obtained by combining **(B-1)** and **(B-4)**

$$p_t = \left[\int_0^1 p_{i,t}^{\frac{1}{1-\lambda_{d,t}}} di \right]^{1-\lambda_{d,t}} \tag{B-5}$$

Intermediate good producers use the following production function

$$y_{i,t} = z_t h_{i,t}^d \tag{B-6}$$

where

$$z_t = \left(1-\rho_z\right) z + \rho_z z_{t-1} + \sigma_z e^{\tilde{\sigma}_{z,t}} \omega_{z,t} \tag{B-7}$$

$$\tilde{\sigma}_{z,t} = \rho_{\tilde{\sigma}_z} \tilde{\sigma}_{z,t-1} + \sigma_{\tilde{\sigma}_z} \omega_{\tilde{\sigma}_{z,t}} \tag{B-8}$$

is a stationary exogenous conditional heteroscedastic technological process and $h_{i,t}^d$ is the amount of homogeneous labour rented by the firm i^{ih}. The intermediate firm select $h_{i,t}^d$ in order to minimise its production cost

$$\min_{h_{i,t}^d} \tilde{\omega}_t r_t^w h_{i,t}^d + mc_t p_t \left[y_{i,t} - z_t h_{i,t}^d \right] \tag{B-9}$$

Similar to Adolfson *et al* (2007) we assume that a fraction of intermediate firms' wage bill has to be financed in advance

$$r_t^w \equiv \phi_f r_{t-1} + 1 \tag{B-10}$$

where is r_{t-1} is the gross nominal interest rate. It is not hard to see that absent to this working capital constraint – $\phi_f = 0$ – **(B-9)** collapses to standard firms' minimisation problem. The real marginal cost for the intermediate firms is given by the first order condition of **(B-9)** with respect to $H_{i,t}$ is

$$mc_t = \frac{\omega_t r_t^w}{z_t} \tag{B-11}$$

where $\omega_t \equiv \frac{\tilde{\omega}_t}{p_t}$ is the real wage.

A fraction – $\left(1-\xi_d\right)$ – of intermediate firms receive a random signal and they are allowed to optimally reset their prices – $p_{i,t}^{new}$. The proportion – ξ_d – of firms that cannot reoptimise prices will set p_t based on backward-looking rule

$$p_t = \pi_{t-1}^{\kappa_d} p_{t-1} \tag{B-12}$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation and κ_d is the indexation parameter. The pricing problem of firm i is then

$$\max_{p_{i,t}^{new}} E_t \sum_{j=0}^{\infty} \left(\beta\xi_d\right)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \pi_{t+s-1}^{\kappa_d} \frac{p_{i,t}^{new}}{p_{t+j}} - mc_{t+j} \right) y_{i,t+j} \right\} \qquad \textbf{(B-13)}$$

subject to

$$y_{i,t+j} = \left(\prod_{s=1}^{j} \pi_{t+s-1}^{\kappa_d} \frac{p_{i,t}^{new}}{p_{t+j}} \right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} y_{t+j}^d \qquad \textbf{(B-14)}$$

The first-order condition is expressed as system of difference equations

$$f_{1,t} = \lambda_t mc_t y_t^d + \beta\xi_d E_t \left(\frac{\pi_t^{\kappa_d}}{\pi_{t+1}} \right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} f_{1,t+1} \qquad \textbf{(B-15)}$$

$$f_{2,t} = \lambda_t \bar{\pi}_t y_t^d + \beta\xi_d E_t \left(\frac{\pi_t^{\kappa_d}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_{d,t}-1}} \left(\frac{\bar{\pi}_t}{\bar{\pi}_{t+1}} \right) f_{2,t+1} \qquad \textbf{(B-16)}$$

$$0 = \lambda_{d,t} f_{1,t} - f_{2,t} \qquad \textbf{(B-17)}$$

$$1 = \xi_d \left(\frac{\pi_{t-1}^{\kappa_d}}{\pi_t} \right)^{-\frac{1}{\lambda_{d,t}-1}} + \left(1-\xi_d\right) \bar{\pi}_t^{-\frac{1}{\lambda_{d,t}-1}} \qquad \textbf{(B-18)}$$

where $\bar{\pi}_t \equiv \frac{p_t^{new}}{p_t}$.

Importing firms

The import sector consists of a continuum of monopolistically competitive firms that buy a homogenous good in the world market at price p_t^*. These firms have access to a costless technology and transform the homogenous good into a differentiated product – $c_{i,t}^m$ – consumed by domestic households. Similar to Justiniano and Preston (2010) and Adolfson *et al* (2007) we assume local currency in order to allow for incomplete exchange rate pass-through to the import prices. To be precise, the importing firms follow the Calvo price-setting scheme, meaning that a fraction – $1-\xi_m$ – of them is allowed to reset their price optimally – $p_{m,t}^{new}$ – only when they receive a random price change signal, while those firms that missed this signal can only index their prices by past inflation – $p_{m,t} = \pi_{m,t-1}^{\kappa_m} p_{m,t-1}$. The pricing problem of the firm becomes

$$\max_{p_{i,t}^{m,new}} E_t \sum_{j=0}^{\infty} \left(\beta\xi_m\right)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^m\right)^{\kappa_m} \frac{p_{i,t}^{m,new}}{p_{t+j}^m} - mc_t^m \right) c_{i,t}^m \right\} \qquad \textbf{(B-19)}$$

where $mc_t^m \equiv \frac{s_t p_t^*}{p_t^m}$ is the real marginal cost of the importing firm and s_t is the nominal exchange rate.

The final import good is a composite of a continuum of these differentiated imported good and it is given by the following CES production function

$$c_t^m = \left[\int_0^1 \left(c_{i,t}^m\right)^{\frac{1}{\lambda_{m,t}}} di\right]^{\lambda_{m,t}} \quad \textbf{(B-20)}$$

where

$$\lambda_{m,t} = \left(1-\rho_{\lambda_m}\right)\lambda_m + \rho_{\lambda_m}\lambda_{m,t-1} + \sigma_{\lambda dm} e^{\tilde{\sigma}_{\lambda m,t}} \omega_{\lambda_m,t} \quad \textbf{(B-21)}$$

$$\tilde{\sigma}_{\lambda_m,t} = \rho_{\tilde{\sigma}_{\lambda m}} \tilde{\sigma}_{\lambda_m,t-1} + \sigma_{\tilde{\sigma}_{\lambda m}} \omega_{\tilde{\sigma}_{\lambda m},t} \quad \textbf{(B-22)}$$

is the time-varying conditional heteroscedastic mark-up in the import good market. Taking p_t^m and $p_{i,t}^m$ as given the final import good producer's demand curve for $C_{i,t}^m$ can be derived form the profit minimisation problem – $\max_{c_{i,t}^m} \left\{ p_t^m \left[\int_0^1 \left(c_{i,t}^m\right)^{\frac{1}{\lambda_{m,t}}} di\right]^{\lambda_{m,t}} - p_{i,t}^m c_{i,t}^m \right\}$

$$c_{i,t}^m = \left(\frac{p_{i,t}^m}{p_t^m}\right)^{-\frac{\lambda_{m,t}}{\lambda_{m,t}-1}} c_t^m \quad \textbf{(B-23)}$$

Finally, total amount of imported goods is obtained by integrating out over all differentiated imported goods

$$C_t^m = \int_0^1 c_{i,t}^m di \quad \textbf{(B-24)}$$

$p_{i,t}^{m,new}$ is derived by maximising **(B-19)** subject to

$$c_{i,t+j}^m = \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^m\right)^{\kappa_m} \frac{p_{i.t}^{m,new}}{p_{t+j}^m}\right)^{-\frac{\lambda_{m,t}}{\lambda_{m,t-1}}} c_{t+j}^m \quad \textbf{(B-25)}$$

and the first-order condition – expressed as a system of first-order difference equations – is

$$g_{1,t} = \lambda_t m c_t^m c_{t+j}^m + \beta \xi_m E_t \left(\frac{\left(\pi_t^m\right)^{\kappa_m}}{\pi_{t+1}^m}\right)^{-\frac{\lambda_{m,t}}{\lambda_{m,t}-1}} g_{1,t+1} \quad \textbf{(B-26)}$$

$$g_{2,t} = \lambda_t \bar{\pi}_t^m c_{t+j}^m + \beta \xi_m E_t \left(\frac{\left(\pi_t^m\right)^{\kappa_m}}{\pi_{t+1}^m}\right)^{-\frac{1}{\lambda_{m,t}-1}} \left(\frac{\bar{\pi}_t^m}{\bar{\pi}_{t+1}^m}\right) g_{2,t+1} \quad \textbf{(B-27)}$$

$$0 = \lambda_{m,t} g_{1,t} - g_{2,t} \quad \textbf{(B-28)}$$

$$1 = \xi_m \left(\frac{\left(\pi_{t-1}^m\right)^{\kappa_m}}{\pi_t^m}\right)^{-\frac{1}{\lambda_{m,t}-1}} + \left(1-\xi_m\right)\left(\bar{\pi}_t^m\right)^{-\frac{1}{\lambda_{m,t}-1}} \quad \textbf{(B-29)}$$

where $\bar{\pi}_t^m = \frac{p_t^{m,new}}{p_t^m}$.

Exporting firms

Again there is a continuum of exporting firms indexed by i on the unit interval. Each firm i buys a homogenous final domestic good in the domestic market and differentiates it by using costless banding technology. They next sell the differentiated goods to the rest of the world. Foreign households' demand scedule is given by

$$c_{i,t}^{x} = \left(\frac{p_{i,t}^{x}}{p_{t}^{x}}\right)^{-\frac{\lambda_{m,t}}{\lambda_{m,t}-1}} c_{t}^{x} \tag{B-30}$$

where $\frac{\lambda_{m,t}}{\lambda_{m,t}-1}$ denotes the time-varying elasticity of substitution between differentiated exporting goods, where

$$\lambda_{x,t} = \left(1-\rho_{\lambda_x}\right)\lambda_x + \rho_{\lambda_x}\lambda_{x,t-1} + \sigma_{\lambda_x} e^{\tilde{\sigma}_{\lambda_x,t}}\omega_{\lambda_x,t} \tag{B-31}$$

$$\tilde{\sigma}_{\lambda_x,t} = \rho_{\tilde{\sigma}_{\lambda_x}}\tilde{\sigma}_{\lambda_x,t-1} + \sigma_{\tilde{\sigma}_{\lambda_x}}\omega_{\tilde{\sigma}_{\lambda_x},t} \tag{B-32}$$

The exported price index is

$$p_{t}^{x} = \left[\int_{0}^{1}\left(p_{i,t}^{x}\right)^{\frac{1}{1-\lambda_{d,t}}} di\right]^{1-\lambda_{d,t}} \tag{B-33}$$

The total amount of exported goods is obtained by integrating over all goods

$$C_{t}^{x} = \int_{0}^{1} c_{i,t}^{x} di \tag{B-34}$$

Similar to Gali and Monacelli (2005) we assume that the domestic economy is small relative to the economy and its contribution to the aggregate foreign demand is negligible – see also Justiniano and Preston (2010) and Adolfson *et al* (2007). Motivated by the same studies we further assume that the foreign demand for the domestic consumption – c_t^x – is given by

$$c_{t}^{x} = \varsigma\left(\frac{p_{t}^{x}}{p_{t}^{*}}\right)^{-\eta_f} y_{t}^{*} \tag{B-35}$$

where y_t^* and p_t^* is the foreign demand and price level, respectively. Under Calvo pricing contract and indexation $p_{i,t}^{x,new}$ is derived by maximising

$$\max_{p_{i,t}^{x,new}} E_t \sum_{j=0}^{\infty}\left(\beta\xi_x\right)^j \frac{\lambda_{t+j}}{\lambda_t}\left\{\left(\prod_{s=1}^{j}\left(\pi_{t+s-1}^{x}\right)^{\kappa_m}\frac{p_{i,t}^{x,new}}{p_{t+j}^{x}} - mc_{t}^{x}\right)c_{i,t}^{x}\right\} \tag{B-36}$$

subject to

$$c_{i,t+j}^{x} = \left(\prod_{s=1}^{j}\left(\pi_{t+s-1}^{*}\right)^{\kappa_m}\frac{p_{i.t}^{x,new}}{p_{t+j}^{x}}\right)^{-\frac{\lambda_{x,t}}{\lambda_{x,t-1}}} c_{t+j}^{x} \tag{B-37}$$

where $mc_t^x = \frac{p_t}{s_t p_t^x}$. The first-order condition can be expressed as

$$u_{1,t} = \lambda_t mc_t^x c_{t+j}^x + \beta \xi_x E_t \left(\frac{\left(\pi_t^*\right)^{\kappa_x}}{\pi_{t+1}^x} \right)^{-\frac{\lambda_{x,t}}{\lambda_{x,t-1}}} u_{1,t+1} \tag{B-38}$$

$$u_{2,t} = \lambda_t \bar{\pi}_t^x c_{t+j}^x + \beta \xi_x E_t \left(\frac{\left(\pi_t^*\right)^{\kappa_x}}{\pi_{t+1}^x} \right)^{-\frac{1}{\lambda_{x,t-1}}} \left(\frac{\bar{\pi}_t^x}{\bar{\pi}_{t+1}^x} \right) u_{2,t+1} \tag{B-39}$$

$$0 = \lambda_{x,t} u_{1,t} - u_{2,t} \tag{B-40}$$

$$1 = \xi_x \left(\frac{\left(\pi_{t-1}^*\right)^{\kappa_x}}{\pi_t^x} \right)^{-\frac{1}{\lambda_{x,t-1}}} + \left(1 - \xi_x\right) \left(\bar{\pi}_t^x\right)^{-\frac{1}{\lambda_{x,t-1}}} \tag{B-41}$$

where $\bar{\pi}_t^x = \frac{p_t^{x,new}}{p_t^x}$.

Households

The domestic economy is populated by a continuum of households that attain utility from consumption – $c_{\kappa,t+j}$ – and leisure – $-h_{\kappa,t+j}$. Household's preferences are separable

$$E_t \sum_{j=0}^{\infty} \beta^j d_{t+j} \left\{ \frac{\left(c_{\kappa,t+j} - bc_{\kappa,t+j-1}\right)^{1-\sigma_c}}{1-\sigma_c} - \psi_{t+j} \frac{h_{\kappa,t+j}^{1+\varphi}}{1+\varphi} d\kappa \right\} \tag{B-42}$$

where d_t and ψ_t is a conditional heteroscedastic discount factor and labour supply shock, respectively

$$d_t = \left(1 - \rho_d\right) d + \rho_d d_{t-1} + \sigma_d e^{\tilde{\sigma}_{d,t}} \omega_{d,t} \tag{B-43}$$

$$\tilde{\sigma}_{d,t} = \rho_{\tilde{\sigma}_d} \tilde{\sigma}_{d,t-1} + \sigma_{\tilde{\sigma}_d} \omega_{\tilde{\sigma}_d,t} \tag{B-44}$$

$$\psi_t = \left(1 - \rho_\psi\right) \psi + \rho_\psi \psi_{t-1} + \sigma_\psi e^{\tilde{\sigma}_{\psi,t}} \omega_{\psi,t} \tag{B-45}$$

$$\tilde{\sigma}_{\psi,t} = \rho_{\tilde{\sigma}_\psi} \tilde{\sigma}_{\psi,t-1} + \sigma_{\tilde{\sigma}_\psi} \omega_{\tilde{\sigma}_\psi,t} \tag{B-46}$$

β is the discount factor, φ the inverse of the Frisch elasticity, σ_c the inverse of intertemporal elasticity of substitution and b the habit formation parameter.

Aggregate consumption is function of domestically produced – $c_{\kappa,t}^d$ – and imported good – $c_{\kappa,t}^m$

$$c_{\kappa,t} = \left[(1-\alpha)^{\frac{1}{\eta_c}} \left(c_{\kappa,t}^d\right)^{\frac{\eta_c-1}{\eta_c}} + \alpha^{\frac{1}{\eta_c}} \left(c_{\kappa,t}^m\right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}} \tag{B-47}$$

The elasticity of substitution between domestic and foreign goods in given by η_c and α measures the 'trade openness'. The maximisation of **(B-47)** subject to the budget constraint $p_t^c c_{\kappa,t} = p_t c_{\kappa,t}^d + p_t^m c_{\kappa,t}^m$

delivers the following demand functions

$$c^d_{\kappa,t} = (1-\alpha)\,\tilde{p}_t^{-\eta_c} c_{\kappa,t} \qquad \textbf{(B-48)}$$

$$c^m_{\kappa,t} = \alpha\left(\tilde{p}^m_t\right)^{-\eta_c} c_{\kappa,t} \qquad \textbf{(B-49)}$$

where $\tilde{p}_t \equiv \frac{p_t}{p^c_t}$ and $\tilde{p}^m_t \equiv \frac{p^m_t}{p^c_t}$ is the relative domestic consumption and import price, respectively. Plugging **(B-48)** and **(B-49)** into the budget constraint we obtain the definition of the consumer price index – CPI

$$\left(p^c_t\right)^{1-\eta_c} = (1-\alpha)\,p_t^{1-\eta_c} + \alpha\left(p^m_t\right)^{1-\eta_c} \qquad \textbf{(B-50)}$$

alternatively

$$1 = (1-\alpha)\,\tilde{p}_t^{1-\eta_c} + \alpha\left(\tilde{p}^m_t\right)^{1-\eta_c} \qquad \textbf{(B-51)}$$

Another interesting relation that links the CPI inflation – $\pi^c_t = \frac{p^c_t}{p^c_{t-1}}$ – with the home produced inflation – π_t, the imported inflation – π^m_t – and the terms of trade is the following one

$$\left(\frac{\pi^c_t}{\tilde{p}_{t-1}}\right)^{1-\eta_c} = (1-\alpha)\,\pi_t^{1-\eta_c} + \alpha\left(\pi^m_t T_{t-1}\right)^{1-\eta_c} \qquad \textbf{(B-52)}$$

In order to avoid keeping track of the entire distribution of households' wealth we make the assumption that households have access to complete financial markets and they can insurance themselves against idiosyncratic risks through the purchase of the appropriate portfolio of securities. This assumption, which is standard in this framework, gives rise to the following common real budget constraint

$$b_{\kappa,t} + \frac{p^c_t}{p_t}c_{\kappa,t} = \frac{r^h_{t-1}}{\pi_t}b_{\kappa,t-1} + \omega_{\kappa,t}h_{\kappa,t} + F_t + \omega_{\kappa,t}h_{\kappa,t} + TR_t \qquad \textbf{(B-53)}$$

The household κ uses its labour income – $\frac{\omega_{\kappa,t}}{\tilde{p}_t}h_{\kappa,t}$, gross interest rate debt payments – $\frac{r_{t-1}}{\pi_t}b_{\kappa,t-1}$, where $b_{\kappa,t} = b^d_{\kappa,t} + \frac{s_t b^*_{\kappa,t}}{p_t}$ is the sum of domestic and foreign government bonds, government transfers – TR_t – and profits – F_t – to finance consumption and new purchases of financial assets – $b_{\kappa,t} + s_t b^*_{\kappa,t} + p^c_t c_{\kappa,t}$. The household maximises **(B-42)** with respect to $c_{\kappa,t}$ and $b_{\kappa,t}$ subject to **(B-53)**

$$\frac{d_t}{\left(c_{\kappa,t} - bc_{\kappa,t-1}\right)^{\sigma_c}} - E_t\frac{\beta b d_{t+1}}{\left(c_{\kappa,t+1} - hc_{\kappa,t}\right)^{\sigma_c}} = \frac{\lambda_{\kappa,t}}{\tilde{p}_t} \qquad \textbf{(B-54)}$$

$$\lambda_{\kappa,t} = \beta E_t\left\{\lambda_{\kappa,t+1}\frac{r^h_t}{\pi^c_{t+1}}\frac{\pi^c_{t+1}}{\pi_{t+1}}\right\} \qquad \textbf{(B-55)}$$

Financial intermediary

The financial intermediary firm issues bonds to households paying a gross interest rate r_t^h. The firm then purchases a portfolio of foreign bonds (paying gross interest r_t^*) and domestic government issued bonds (paying interest r_t). For simplicity we assume that all government bonds issued are purchased by this firm. The interest rates received by this firm depend on the level of bonds the firm issues. The firm's maximisation problem

$$\max_{b_{\kappa,t}^d,\alpha_{\kappa,t}} \frac{r_t^h}{E_t\pi_{t+1}^c} b_{\kappa,t} - \frac{r_t}{E_t\pi_{t+1}^c} b_{\kappa,t}^d \Phi_{R^h}(a_t) - \frac{r_t^*}{E_t\pi_{t+1}^c} s_{t+1} \frac{b_{\kappa,t}^*}{p_t} \Phi_{UIP}\left(a_t, \frac{q_t}{q_{t-1}}, \frac{rr_{t-1}}{rr_{t-1}^*}, \tilde{\phi}_t\right)$$

or

$$\max_{b_{\kappa,t}^d,\alpha_{\kappa,t}} \frac{r_t^h}{E_t\pi_{t+1}^c} b_{\kappa,t} - \frac{r_t}{E_t\pi_{t+1}^c} b_{\kappa,t}^d \Phi(a_t) - \frac{r_t^*}{E_t\pi_{t+1}^*} \frac{E_t q_{t+1}}{q_t} \alpha_{\kappa,t} \Phi\left(a_t, \frac{q_t}{q_{t-1}}, \frac{rr_{t-1}}{rr_{t-1}^*}, \tilde{\phi}_t\right) \qquad \textbf{(B-56)}$$

subject to

$$b_{\kappa,t} = b_{\kappa,t}^d + \frac{s_t b_t^*}{p_t} = b_{\kappa,t}^d + \alpha_{\kappa,t} \qquad \textbf{(B-57)}$$

Following Adolfson *et al* (2007) and Christiano *et al* (2011) the net foreign asset position is defined as $a_t \equiv \frac{s_t b_t^*}{p_t}$, while $rr_t = \frac{r_t}{E_t\pi_{t+1}^c}$ and $rr_t^* = \frac{r_t^*}{E_t\pi_{t+1}^*}$ denote the domestic and foreign real interest rate, respectively and $q_t = \frac{s_t p_t^*}{p_t^c}$ is the real exchange rate. Furthermore, in order to ensure that the model is stationary we link the risk premium charged on r_t a function of the net foreign asset position **(**Schmitt-Grohe and Uribe (2003)**)**

$$\Phi_{R^h}(a_t) \equiv e^{-\chi_a a_t} \qquad \textbf{(B-58)}$$

$$\Phi_{UIP}\left(a_t, \frac{q_t}{q_{t-1}}, \frac{rr_{t-1}}{rr_{t-1}^*}, \tilde{\phi}_t\right) \equiv e^{-\chi_a a_t - \chi_s\left(\frac{rr_{t-1}}{rr_{t-1}^*}\frac{q_{t-1}}{q_t}-1\right)-\tilde{\phi}_t} \qquad \textbf{(B-59)}$$

The second expression is less familiar as the risk premium charged to foreign interest rates becomes a function of lag deviations from UIP. This formulation, which can be viewed as a – reduced-form – financial friction, introduces additional dynamics to the UIP equation that seem consistent with empirical evidence (see the discussion in Christiano *et al* (2011)). Finally, $\tilde{\phi}_t$ is a conditional heteroscedastic foreign risk premium shock

$$\tilde{\phi}_t = \left(1-\rho_{\tilde{\phi}}\right)\tilde{\phi} + \rho_{\tilde{\phi}}\tilde{\phi}_{t-1} + \sigma_{\tilde{\phi}} e^{\tilde{\sigma}_{\tilde{\phi},t}} \omega_{\tilde{\phi},t} \qquad \textbf{(B-60)}$$

$$\tilde{\sigma}_{\tilde{\phi},t} = \rho_{\tilde{\sigma}_{\tilde{\phi}}} \tilde{\sigma}_{\tilde{\phi},t-1} + \sigma_{\tilde{\sigma}_{\tilde{\phi}}} \omega_{\tilde{\sigma}_{\tilde{\phi}},t} \qquad \textbf{(B-61)}$$

The first-order conditions are

$$\frac{r_t^h}{E_t\pi_{t+1}^c} = \frac{r_t}{E_t\pi_{t+1}^c}\Phi\left(a_t\right) \quad \textbf{(B-62)}$$

$$\frac{r_t^h}{E_t\pi_{t+1}^c} = \frac{r_t^*}{E_t\pi_{t+1}^*}\frac{E_tq_{t+1}}{q_t}\Phi\left(a_t, \frac{q_t}{q_{t-1}}, \frac{rr_{t-1}}{rr_{t-1}^*}, \tilde{\phi}_t\right) \quad \textbf{(B-63)}$$

The first equation suggests that the real interest paid to households is function of the real interest adjusted for a premium charged when households save less or borrow more. The UIP can be obtained by combining the two equations

$$\frac{r_t}{E_t\pi_{t+1}^c}e^{-\chi_a a_t} = \frac{r_t^*}{E_t\pi_{t+1}^*}\frac{E_tq_{t+1}}{q_t}e^{-\chi_a a_t-\chi_s\left(\frac{rr_{t-1}}{rr_{t-1}^*}\frac{q_{t-1}}{q_t}-1\right)-\tilde{\phi}_t}$$

$$\frac{r_t}{E_t\pi_{t+1}^c} = \frac{r_t^*}{E_t\pi_{t+1}^*}\frac{E_tq_{t+1}}{q_t}e^{-\chi_s\left(\frac{rr_{t-1}}{rr_{t-1}^*}\frac{q_{t-1}}{q_t}-1\right)-\tilde{\phi}_t} \quad \textbf{(B-64)}$$

Wages

We follow Erceg, Henderson and Levin (2000) and assume that each monopolistically competitive household supplies a differentiated labour service to the production section. They set their nominal wage and supply any amount of labour demanded by the firms at that wage rate. For convenience, we assume that there exist a representative firm that combines households' labour inputs into a homogenous input hood - h_t^d - using a CES production function

$$h_t^d = \left[\int_0^1 h_{\kappa,t}^{\frac{1}{\lambda_{w,t}}}d\kappa\right]^{\lambda_{w,t}} \quad \textbf{(B-65)}$$

where $\lambda_{w,t}$ is the conditional heteroscedastic time-varying wage mark-up

$$\lambda_{w,t} = \left(1-\rho_{\lambda_w}\right)\lambda_w + \rho_{\lambda_w}\lambda_{w,t-1} + \sigma_{\lambda_w}e^{\tilde{\sigma}_{\lambda_w,t}}\omega_{\lambda_w,t} \quad \textbf{(B-66)}$$

$$\tilde{\sigma}_{\lambda_w,t} = \rho_{\tilde{\sigma}_{\lambda_w}}\tilde{\sigma}_{\lambda_w,t-1} + \sigma_{\tilde{\sigma}_{\lambda_w}}\omega_{\tilde{\sigma}_{\lambda w},t} \quad \textbf{(B-67)}$$

Taking w_t and $w_{\kappa,t}$ as given the aggregator's demand for the labour hours of household κ results its profit maximisation $\max_{h_{\kappa,t}}\left\{w_t\left[\int_0^1 h_{\kappa,t}^{\frac{1}{\lambda_{w,t}}}d\kappa\right]^{\lambda_{w,t}} - w_{\kappa,t}h_{\kappa,t}\right\}$

$$h_{\kappa,t} = \left(\frac{w_{\kappa,t}}{w_t}\right)^{-\frac{\lambda_{w,t}}{\lambda_{w,t}-1}}h_t^d \quad \textbf{(B-68)}$$

The aggregate wage arise from the profit condition and the demand curve

$$w_t = \left[\int_0^1 w_{\kappa,t}^{\frac{1}{1-\lambda_{w,t}}}d\kappa\right]^{1-\lambda_{w,t}} \quad \textbf{(B-69)}$$

In each period, a function – $1 - \xi_w$ – of households receive a random signal and they are allowed to reset wages optimally – w_t^{new}. All other households can only partially index their wages by past inflation. The problem of setting wages can be described as follows

$$\max_{w_t^{new}} E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left\{ -d_{t+j} \psi_{t+j} \frac{h_{\kappa,t+j}^{1+\varphi}}{1+\varphi} + \lambda_{t+j} \prod_{s=1}^{j} \frac{\left(\pi_{t+s-1}^c\right)^{\kappa_w}}{\pi_{t+s}^c} w_{\kappa,t} h_{\kappa,t} \right\} \quad \textbf{(B-70)}$$

subject to

$$h_{\kappa,t} = \left(\prod_{s=1}^{j} \frac{\left(\pi_{t+s-1}^c\right)^{\kappa w}}{\pi_{t+s}^c} \frac{w_{\kappa,t}}{w_t} \right)^{-\frac{\lambda_{w,t}}{\lambda_{w,t-1}}} h_t^d \quad \textbf{(B-71)}$$

The first order is summarised by the following recursive equations

$$v_{1,t} = \frac{1}{\lambda_{w,t}} \left(w_t^{new}\right)^{\frac{1}{1-\lambda_{w,t}}} \lambda_t w_t^{\frac{\lambda_{w,t}}{\lambda_{w,t-1}}} h_t^d + \beta \xi_w E_t \left(\frac{\left(\pi_t^c\right)^{\kappa_w}}{\pi_{t+1}^c} \right)^{\frac{1}{1-\lambda_{w,t}}} \left(\frac{w_{t+1}^{new}}{w_t^{new}} \right)^{\frac{1}{\lambda_{w,t-1}}} v_{1,t+1} \quad \textbf{(B-72)}$$

$$v_{1,t} = d_t \psi_t \left(\frac{w_t}{w_t^{new}} \right)^{\frac{(1+\varphi)\lambda_{w,t}}{\lambda_{w,t-1}}} \left(h_t^d\right)^{1+\varphi} + \beta \xi_w E_t \left(\frac{\left(\pi_t^c\right)^{\kappa_w}}{\pi_{t+1}^c} \right)^{\frac{(1+\varphi)\lambda_{w,t}}{1-\lambda_{w,t}}} \left(\frac{w_{t+1}^{new}}{w_t^{new}} \right)^{\frac{(1+\varphi)\lambda_{w,t}}{\lambda_{w,t-1}}} v_{1,t+1} \quad \textbf{(B-73)}$$

$$w_t^{\frac{1}{1-\lambda_{w,t}}} = \xi_w \left(\frac{\left(\pi_{t-1}^c\right)^{\kappa_w}}{\pi_t^c} \right)^{\frac{1}{1-\lambda_{w,t}}} w_{t-1}^{\frac{1}{1-\lambda_{w,t}}} + (1-\xi_w) \left(w_t^{new}\right)^{\frac{1}{1-\lambda_{w,t}}} \quad \textbf{(B-74)}$$

Monetary policy

The monetary authority sets its instrument short-term interest rate according to a Taylor rule

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\phi_R} \left(\pi_t^c\right)^{(1-\phi_R)\phi_\pi} \left(\frac{y_t}{y}\right)^{(1-\phi_R)\phi_y} \left(\frac{q_t}{q}\right)^{(1-\phi_R)\phi_q} e^{\sigma_R e^{\tilde{\sigma}_{R,t}} \omega_{R,t}} \quad \textbf{(B-75)}$$

The variance of the monetary policy shock evolves over time according to an AR(1) process

$$\tilde{\sigma}_{R,t} = \rho_{\tilde{\sigma}_R} \tilde{\sigma}_{R,t-1} + \sigma_{\tilde{\sigma}_{R,t}} \omega_{\tilde{\sigma}_R,t} \quad \textbf{(B-76)}$$

In other words, the policymaker adjusts the nominal interest rate in response to its lag value, to inflation deviations from the target – $\pi = 1$ – and to output deviations from the long-run equilibrium – y.

Market clearing conditions

The resource constraint implies that

$$y_t^d = c_t^d + c_t^x + g_t \quad \textbf{(B-77)}$$

Substituting **(B-48)** into **(B-77)** we obtain

$$(1-\alpha)\,\tilde{p}_t^{-\eta_c} c_t + c_t^x + g_t = \frac{z_t h_t^d}{\upsilon_t^p} \tag{B-78}$$

where $\upsilon_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} d_i$ is the price dispersion term and it is given by

$$\upsilon_t^p = \xi_d \left(\frac{\pi_{t-1}^{\kappa_d}}{\pi_t}\right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} \upsilon_{t-1}^p + \left(1-\xi_d\right)\bar{\pi}_t^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} \tag{B-79}$$

The evolution of the net foreign assets position is obtained from the aggregate household's budget constraint and using the definition of profits

$$F_t = p_t^c C_t - p_t^m C_t^m - p_t C_t^d + p_t Y_t - w_t h_t + p_t^m C_t^m - s_t p_t^* + s_t p_t^x C_t^x - p_t C_t^x \tag{B-80}$$

$$a_t = \frac{s_t p_t^x}{p_t} C_t^x - \frac{s_t p_t^*}{p_t} C_t^m + \frac{r_{t-1}^*}{\pi_t^*} \Phi\left(a_{t-1}, \tilde{\phi}_{t-1}\right) \frac{q_t}{q_{t-1}} \frac{\pi_t^c}{\pi_t} a_{t-1} \tag{B-81}$$

$$a_t = \frac{q_t}{\tilde{p}_t}\left(\tilde{p}_t^x C_t^x - C_t^m\right) + \frac{r_{t-1}^*}{\pi_t^*} \Phi\left(a_{t-1}, \tilde{\phi}_{t-1}\right) \frac{q_t}{q_{t-1}} \frac{\pi_t^c}{\pi_t} a_{t-1} \tag{B-82}$$

$$h_t^d = \frac{h_t}{\upsilon_t^w} \tag{B-83}$$

where $\upsilon_t^w = \int_0^1 \left(\frac{w_{i,t}}{w_t}\right)^{-\frac{\lambda_{w,t}}{\lambda_{w,t}-1}} d_i$ is the wage dispersion term and its evolution is described

$$\upsilon_t^w = \xi_w \left(\frac{\left(\pi_{t-1}^c\right)^{\kappa_w}}{\pi_t^c}\right)^{\frac{\lambda_{w,t}}{1-\lambda_{w,t}}} \left(\frac{w_{t-1}}{w_t}\right)^{\frac{\lambda_{w,t}}{1-\lambda_{w,t}}} \upsilon_{t-1}^w + (1-\xi_w)\left(\frac{w_t^{new}}{w_t}\right)^{\frac{\lambda_{w,t}}{1-\lambda_{w,t}}} \tag{B-84}$$

The marketing clearing condition for total imports is

$$C_t^m = c_t^m \upsilon_t^m \tag{B-85}$$

where $\upsilon_t^m = \int_0^1 \left(\frac{p_{i,t}^m}{p_t^m}\right)^{-\frac{\lambda_{m,t}}{\lambda_{m,t}-1}} d_i$ is the import price dispersion term with its law of motion

$$\upsilon_t^m = \xi_m \left(\frac{\left(\pi_{t-1}^m\right)^{\kappa_m}}{\pi_t^m}\right)^{-\frac{\lambda_{m,t}}{\lambda_{m,t}-1}} \upsilon_{t-1}^m + \left(1-\xi_m\right)\left(\bar{\pi}_t^m\right)^{-\frac{\lambda_{m,t}}{\lambda_{m,t}-1}} \tag{B-86}$$

Finally, he marketing clearing condition for total export is given by

$$C_t^x = c_t^x \upsilon_t^x \tag{B-87}$$

where $\upsilon_t^x = \int_0^1 \left(\frac{p_{i,t}^x}{p_t^x}\right)^{-\frac{\lambda_{x,t}}{\lambda_{x,t}-1}} d_i$ is the export price dispersion term with

$$\upsilon_t^x = \xi_x \left(\frac{\left(\pi_{t-1}^*\right)^{\kappa_x}}{\pi_t^x}\right)^{-\frac{1}{\lambda_{x,t}-1}} \upsilon_{t-1}^x + \left(1-\xi_x\right)\left(\bar{\pi}_t^x\right)^{-\frac{1}{\lambda_{x,t}-1}} \tag{B-88}$$

Appendix C: Foreign economy

The foreign agents' decision problems are very similar to those discussed in the previous section. To avoid repeating ourselves and to save some space we just list in this section the first-order conditions required for the solution of the model. We keep the same notation with the domestic economy and we add a star symbol – $*$ – to separate the foreign from the domestic variables.

Supply

The production of the intermediate good production is given by

$$y_{i,t}^{*} = z_t^{*} h_{i,t}^{d,*} \tag{C-1}$$

where – similar to the domestic economy – the productivity shock is conditionally heteroscedastic

$$\begin{aligned} z_t^{*} &= \left(1-\rho_{z^*}\right) z^{*} + \rho_{z^*} z_{t-1}^{*} + \sigma_{z^*} e^{\tilde{\sigma}_{z^*,t}} \omega_{z^*,t} && \text{(C-2)} \\ \tilde{\sigma}_{z^*,t} &= \rho_{\tilde{\sigma}_{z^*}} \tilde{\sigma}_{z^*,t-1} + \sigma_{\tilde{\sigma}_{z^*}} \omega_{\tilde{\sigma}_{z^*},t} && \text{(C-3)} \end{aligned}$$

A fraction – ϕ_f^{*} – of intermediate firms' wage bill has to be financed prior to the production

$$r_t^{w,*} \equiv \phi_f^{*} r_{t-1}^{*} + 1 \tag{C-4}$$

The marginal cost is given

$$mc_t^{*} = \frac{w_t^{*} r_t^{w,*}}{z_t^{*}} \tag{C-5}$$

while the following equations describe firm's pricing first-order conditions

$$\begin{aligned} f_{1,t}^{*} &= \lambda_t^{*} mc_t^{*} y_t^{d,*} + \beta \xi_d^{*} E_t \left(\frac{\left(\pi_t^{c,*}\right)^{\kappa_d}}{\pi_{t+1}^{c,*}} \right)^{-\frac{\lambda_{d,t}^{*}}{\lambda_{d,t}^{*}-1}} f_{1,t+1}^{*} && \text{(C-6)} \\ f_{2,t}^{*} &= \lambda_t^{*} \bar{\pi}_t^{*} y_t^{d,*} + \beta \xi_d^{*} E_t \left(\frac{\left(\pi_t^{c,*}\right)^{\kappa_d}}{\pi_{t+1}^{c,*}} \right)^{-\frac{1}{\lambda_{d,t}^{*}-1}} \left(\frac{\bar{\pi}_t^{*}}{\bar{\pi}_{t+1}^{*}} \right) f_{2,t+1}^{*} && \text{(C-7)} \\ 0 &= \lambda_{d,t}^{*} f_{1,t}^{*} - f_{2,t}^{*} && \text{(C-8)} \\ 1 &= \xi_d^{*} \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_d}}{\pi_t^{c,*}} \right)^{-\frac{1}{\lambda_{d,t}^{*}-1}} + \left(1-\xi_d\right) \left(\bar{\pi}_t^{*}\right)^{-\frac{1}{\lambda_{d,t}^{*}-1}} && \text{(C-9)} \end{aligned}$$

where $\bar{\pi}_t^* \equiv \frac{p_t^{new,*}}{p_t^*}$.

Households

Domestic household's utility maximisation first-order conditions with respect to $C_t^*$and B_t^* subject to its budget constraint the consumption Euler condition

$$\frac{d_t^*}{\left(c_t^* - bc_{t-1}^*\right)^{\sigma_c}} - E_t \frac{\beta b d_{t+1}^*}{\left(c_{t+1}^* - hc_t^*\right)^{\sigma_c}} = \lambda_t^* \quad \textbf{(C-10)}$$

$$\lambda_t^* = \beta E_t \left\{ \lambda_{t+1}^* \frac{r_t^*}{\pi_{t+1}^{c,*}} \right\} \quad \textbf{(C-11)}$$

Agents' utility function is subject to conditionally heteroscedastic – discount factor and labour supply – shocks

$$d_t^* = \left(1 - \rho_{d^*}\right) d^* + \rho_{d^*} d_{t-1}^* + \sigma_{d^*} e^{\tilde{\sigma}_{d^*,t}} \omega_{d^*,t} \quad \textbf{(C-12)}$$

$$\tilde{\sigma}_{d^*,t} = \rho_{\tilde{\sigma}_{d^*}} \tilde{\sigma}_{d^*,t-1} + \sigma_{\tilde{\sigma}_{d^*}} \omega_{\tilde{\sigma}_{d^*},t} \quad \textbf{(C-13)}$$

$$\psi_t^* = \left(1 - \rho_{\psi^*}\right) \psi^* + \rho_{\psi^*} \psi_{t-1}^* + \sigma_{\psi^*} e^{\tilde{\sigma}_{\psi^*,t}} \omega_{\psi^*,t} \quad \textbf{(C-14)}$$

$$\tilde{\sigma}_{\psi^*,t} = \rho_{\tilde{\sigma}_{\psi^*}} \tilde{\sigma}_{\psi^*,t-1} + \sigma_{\tilde{\sigma}_{\psi^*}} \omega_{\tilde{\sigma}_{\psi^*},t} \quad \textbf{(C-15)}$$

Wages

The evolution of wages in the foreign economy is described by

$$v_{1,t}^{*} = \frac{1}{\lambda_{w,t}^{*}}\left(w_t^{new,*}\right)^{\frac{1}{1-\lambda_{w,t}^{*}}}\lambda_t^{*}\left(w_t^{*}\right)^{\frac{\lambda_{w,t}^{*}}{\lambda_{w,t}^{*}-1}}h_t^{d,*} + \beta\xi_w^{*}E_t\left(\frac{\left(\pi_t^{c,*}\right)^{\kappa_w}}{\pi_{t+1}^{c,*}}\right)^{\frac{1}{1-\lambda_{w,t}^{*}}}\left(\frac{w_{t+1}^{new,*}}{w_t^{new,*}}\right)^{\frac{1}{\lambda_{w,t}^{*}-1}}v_{1,t+1}^{*} \quad \textbf{(C-16)}$$

$$v_{1,t}^{*} = d_t^{*}\psi_t^{*}\left(\frac{w_t^{*}}{w_t^{new,*}}\right)^{\frac{(1+\varphi)\lambda_{w,t}^{*}}{\lambda_{w,t}^{*}-1}}\left(h_t^{d,*}\right)^{1+\varphi} + \beta\xi_w^{*}E_t\left(\frac{\left(\pi_t^{c,*}\right)^{\kappa_w}}{\pi_{t+1}^{c,*}}\right)^{\frac{(1+\varphi)\lambda_{w,t}^{*}}{1-\lambda_{w,t}^{*}}}\left(\frac{w_{t+1}^{new,*}}{w_t^{new,*}}\right)^{\frac{(1+\varphi)\lambda_{w,t}^{*}}{\lambda_{w,t}^{*}-1}}v_{1,t+1}^{*} \quad \textbf{(C-17)}$$

$$\left(w_t^{*}\right)^{\frac{1}{1-\lambda_{w,t}^{*}}} = \xi_w^{*}\left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_w}}{\pi_t^{c,*}}\right)^{\frac{1}{1-\lambda_{w,t}^{*}}}\left(w_{t-1}^{*}\right)^{\frac{1}{1-\lambda_{w,t}^{*}}} + \left(1-\xi_w^{*}\right)\left(w_t^{new,*}\right)^{\frac{1}{1-\lambda_{w,t}^{*}}} \quad \textbf{(C-18)}$$

Monetary policy

Foreign monetary policy authorities follow a similar with the domestic economy Taylor type rule

$$\frac{r_t^{*}}{r} = \left(\frac{r_{t-1}^{*}}{r}\right)^{\phi_{R^*}}\left(\pi_t^{c,*}\right)^{\left(1-\phi_{R^*}\right)\phi_{\pi^*}}\left(\frac{y_t^{d,*}}{y^{*}}\right)^{\left(1-\phi_{R^*}\right)\phi_{Y^*}} e^{\sigma_{R^*}e^{\tilde{\sigma}_{R^*,t}}\omega_{R^*,t}} \quad \textbf{(C-19)}$$

where the variance of the monetary policy shock evolves over time

$$\tilde{\sigma}_{R^*,t} = \rho_{\tilde{\sigma}_{R^*}}\tilde{\sigma}_{R^*,t-1} + \sigma_{\tilde{\sigma}_{R^*,t}}\omega_{\tilde{\sigma}_{R^*},t} \quad \textbf{(C-20)}$$

Market clearing conditions

$$y_t^{d,*} = c_t^{*} + g_t^{*} \quad \textbf{(C-21)}$$

where

$$g_t^{*} = \left(1-\rho_{g^*}\right)g^{*} + \rho_{g^*}g_{t-1}^{*} + \sigma_{g^*}e^{\tilde{\sigma}_{g^*,t}}\omega_{g^*,t} \quad \textbf{(C-22)}$$

$$\tilde{\sigma}_{g^*,t} = \rho_{\tilde{\sigma}_{g^*}}\tilde{\sigma}_{g^*,t-1} + \sigma_{\tilde{\sigma}_{g^*}}\omega_{\tilde{\sigma}_{g^*},t} \quad \textbf{(C-23)}$$

$$y_t^{d,*} = \frac{z_t^* h_t^{d,*}}{\upsilon_t^{p,*}} \tag{C-24}$$

where $\upsilon_t^{p,*}$ is the price dispresion term

$$\upsilon_t^{p,*} = \xi_d^* \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_d}}{\pi_t^{c,*}} \right)^{-\frac{\lambda_{d,t}^*}{\lambda_{d,t}^*-1}} \upsilon_{t-1}^{p,*} + \left(1 - \xi_d\right) \left(\bar{\pi}_t^*\right)^{-\frac{\lambda_{d,t}^*}{\lambda_{d,t}^*-1}} \tag{C-25}$$

$$h_t^{d,*} = \frac{h_t^*}{\upsilon_t^{w,*}} \tag{C-26}$$

where $\upsilon_t^{w,*}$ is the wage dispersion term

$$\upsilon_t^{w,*} = \xi_w^* \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_w}}{\pi_t^{c,*}} \right)^{\frac{\lambda_{w,t}^*}{1-\lambda_{w,t}^*}} \left(\frac{w_{t-1}^*}{w_t^*} \right)^{\frac{\lambda_{w,t}^*}{1-\lambda_{w,t}^*}} + \left(1 - \xi_w^*\right) \left(\frac{w_t^{new,*}}{w_t^*} \right)^{\frac{\lambda_{w,t}^*}{1-\lambda_{w,t}^*}} \tag{C-27}$$

Appendix D: Steady states

Domestic economy

We assume that

$$\pi^c = \pi = \pi^m = q = 1 \tag{D-1}$$

This implies that

$$r = \frac{1}{\beta} \tag{D-2}$$

Under the assumption that $A = 0$

$$r^* = r \tag{D-3}$$

To pin down the value of $\frac{P_t^c}{P_t}$ we use **(B-50)**

$$\begin{aligned}
\left(\frac{p^c}{p}\right)^{1-\eta_c} &= (1-\alpha) + \alpha\left(\frac{p^m}{p}\right)^{1-\eta_c} \\
\left(\frac{1}{\tilde{p}}\right)^{1-\eta_c} &= (1-\alpha) + \alpha\left(\tilde{p}^m\right)^{1-\eta_c}
\end{aligned}$$

We know also the importing firm's flexible price decision

$$\begin{aligned}
mc^m &= \frac{sp^*}{p^m} = \frac{sp^*}{p^c}\frac{p^c}{p^m} = q\frac{1}{\tilde{p}^m} \\
\tilde{p}^m &= \lambda_m
\end{aligned}$$

From the definition of the CPI

$$\begin{aligned}
1 &= (1-\alpha)\,\tilde{p}^{1-\eta_c} + \alpha\left(\tilde{p}^m\right)^{1-\eta_c} \\
1 &= (1-\alpha)\,\tilde{p}^{1-\eta_c} + \alpha\lambda_m^{1-\eta_c} \\
\tilde{p} &= \left(\frac{1-\alpha\lambda_m^{1-\eta_c}}{1-\alpha}\right)^{\frac{1}{1-\eta_c}}
\end{aligned}$$

The export price equals the foreign price level and we assume that the law of one price holds for the exporters

$$mc^x = \frac{p}{sp^x} = \frac{p^c}{sp^*}\frac{p^*}{p^c}\frac{p}{p^x}$$
$$\frac{1}{\lambda_x} = q\frac{\tilde{p}}{\tilde{p}^x}$$
$$\tilde{p}^x = \lambda_x \tilde{p}$$

The terms of trade

$$T = \frac{\tilde{p}^m}{\tilde{p}} \quad \textbf{(D-4)}$$

From **(B-10)** we know the steady-state value of R^H

$$r^w = \phi_f r + 1 \quad \textbf{(D-5)}$$

The long-run value of the real marginal cost is

$$mc = \frac{1}{\lambda_d} \quad \textbf{(D-6)}$$

From **(B-11)** we know that

$$\frac{1}{\lambda_d} = \frac{wr^w}{z}$$
$$w = \frac{z}{\lambda_d r^w} \quad \textbf{(D-7)}$$

From (**B-54**)

$$\frac{(1-\beta b)\,d}{((1-b)\,c)^{\sigma_c}} = \frac{\lambda}{\tilde{p}}$$
$$\lambda = \frac{1}{c^{\sigma_c}}\frac{\tilde{p}\,(1-\beta b)\,d}{(1-b)^{\sigma_c}} \quad \textbf{(D-8)}$$

From the wage pricing equation we know that

$$0 = w\frac{\lambda}{\lambda_w} - d\psi\left(h^d\right)^{\varphi}$$
$$h^d = \left[\frac{w\frac{\lambda}{\lambda_w}}{d\psi}\right]^{\frac{1}{\varphi}} \quad \textbf{(D-9)}$$

From **(B-48)**, **(B-49)** and **(B-77)** we have

$$(1-\alpha)\,\tilde{p}^{-\eta_c}c + c^x + g = \frac{zh^d}{v^p} \quad \textbf{(D-10)}$$

The condition $a = 0$ implies

$$\begin{aligned} C^x - C^m &= c^x v^x - c^m v^m = 0 \\ c^x &= \frac{v^m}{v^x} c^m \end{aligned} \tag{D-11}$$

Then

$$\begin{aligned}
(1-\alpha)\, \tilde{p}^{-\eta_c} c + \frac{v^m}{v^x} \alpha \left(\tilde{p}^m\right)^{-\eta_c} c + g &= \frac{z h^d}{v^p} \\
\left((1-\alpha)\, \tilde{p}^{-\eta_c} + \frac{v^m}{v^x} \alpha \left(\tilde{p}^m\right)^{-\eta_c} \right) c &= \frac{z h^d}{v^p} - g
\end{aligned}$$

$$\begin{aligned}
\left((1-\alpha)\, \tilde{p}^{-\eta_c} + \frac{v^m}{v^x} \alpha \left(\tilde{p}^m\right)^{-\eta_c} \right) c &= \frac{z}{v^p} \left[\frac{w \frac{\lambda}{\lambda_w}}{d \psi} \right]^{\frac{1}{\varphi}} - g \\
\left((1-\alpha)\, \tilde{p}^{-\eta_c} + \frac{v^m}{v^x} \alpha \left(\tilde{p}^m\right)^{-\eta_c} \right) c &= \frac{z}{v^p} \left[\frac{\frac{w}{\lambda_w}}{d \psi} \frac{1}{c^{\sigma_c}} \frac{\tilde{p}\,(1-\beta b)\, d}{(1-b)^{\sigma_c}} \right]^{\frac{1}{\varphi}} - g \\
\left((1-\alpha)\, \tilde{p}^{-\eta_c} + \frac{v^m}{v^x} \alpha \left(\tilde{p}^m\right)^{-\eta_c} \right) c &= \frac{z}{v^p} c^{-\frac{\sigma_c}{\varphi}} \left[\frac{\frac{w}{\lambda_w}}{d \psi} \frac{\tilde{p}\,(1-\beta b)\, d}{(1-b)^{\sigma_c}} \right]^{\frac{1}{\varphi}} - g \\
\left((1-\alpha)\, \tilde{p}^{-\eta_c} + \frac{v^m}{v^x} \alpha \left(\tilde{p}^m\right)^{-\eta_c} \right) &= \frac{z}{v^p} c^{-\frac{\sigma_c}{\varphi}-1} \left[\frac{\frac{w}{\lambda_w}}{d \psi} \frac{\tilde{p}\,(1-\beta b)\, d}{(1-b)^{\sigma_c}} \right]^{\frac{1}{\varphi}} - \frac{g}{c}
\end{aligned}$$

$$c = \left(\frac{\left((1-\alpha)\, \tilde{p}^{-\eta_c} + \frac{v^m}{v^x} \alpha \left(\tilde{p}^m\right)^{-\eta_c} \right) + \frac{g}{c}}{\frac{z}{v^p} \left[\frac{\frac{w}{\lambda_w}}{d\psi} \frac{\tilde{p}(1-\beta b) d}{(1-b)^{\sigma_c}} \right]^{\frac{1}{\varphi}}} \right)^{-\frac{\varphi}{\varphi+\sigma_c}} \tag{D-12}$$

From **(B-73)** we get

$$v_1 = \frac{d \psi h^{1+\varphi}}{1-\beta \xi_w} \tag{D-13}$$

From **(B-16)** and **(B-17)** we get

$$f_1 = \frac{\lambda m c y}{1-\beta \xi_d} \tag{D-14}$$

$$f_2 = \frac{\lambda y}{1-\beta \xi_d} \tag{D-15}$$

Finally, from **(B-26)** and **(B-27)** we get

$$g_1 = \frac{\lambda m c^m c^m}{1-\beta \xi_m} \tag{D-16}$$

$$g_2 = \frac{\lambda c^m}{1-\beta \xi_m} \tag{D-17}$$

Foreign economy

$$\pi^{c,*} = 1 \tag{D-18}$$

$$r^{w,*} = \phi_f^* r^* + 1 \tag{D-19}$$

$$mc^* = \frac{1}{\lambda_d^*} \tag{D-20}$$

$$w^* = \frac{z^*}{\lambda_d^* r^{w,*}} \tag{D-21}$$

$$\lambda^* = \frac{1}{(c^*)^{\sigma_c}} \frac{(1-\beta b)\, d}{(1-b)^{\sigma_c}} \tag{D-22}$$

$$h^* = \left[\frac{w \frac{\lambda}{\lambda_w}}{d\psi} \right]^{\frac{1}{\varphi}} \tag{D-23}$$

$$\begin{aligned}
c^* + g^* &= \frac{z^* h^{d,*}}{\upsilon^{p,*}} \\
c^* + g^* &= \frac{z^*}{\upsilon^{p,*}} \left[\frac{\frac{w}{\lambda_w}}{d\psi} \frac{1}{(c^*)^{\sigma_c}} \frac{(1-\beta b)\, d}{(1-b)^{\sigma_c}} \right]^{\frac{1}{\varphi}} \\
c^* + g^* &= \frac{z^*}{\upsilon^{p,*}} (c^*)^{-\frac{\sigma_c}{\varphi}} \left[\frac{\frac{w}{\lambda_w}}{d\psi} \frac{(1-\beta b)\, d}{(1-b)^{\sigma_c}} \right]^{\frac{1}{\varphi}}
\end{aligned}$$

$$c^* = \left(\frac{1 + \frac{g^*}{c^*}}{\frac{z^*}{\upsilon^{p,*}} \left[\frac{\frac{w}{\lambda_w}}{d\psi} \frac{(1-\beta b) d}{(1-b)^{\sigma_c}} \right]^{\frac{1}{\varphi}}} \right)^{-\frac{\varphi}{\varphi+\sigma_c}} \tag{D-24}$$

Tables

Table A: Posterior estimates

Parameter	Posterior Estimate	Parameter	Posterior Estimate
θ_1	0.9859 (0.97, 0.99)	Q_1	0.0389 (0.02, 0.07)
θ_2	0.9865 (0.97, 0.99)	Q_2	0.0649 (0.04, 0.10)
θ_3	0.9896 (0.98, 0.99)	Q_3	0.2926 (0.19, 0.44)
θ_4	0.9862 (0.97, 0.99)	Q_4	0.0655 (0.03, 0.13)
θ_5	0.9738 (0.94, 0.99)	Q_5	0.3321 (0.11, 0.76)
θ_6	0.9902 (0.98, 0.99)	Q_6	0.2197 (0.13, 0.35)

Table B: DSGE Model Parameters

Parameters	Description	Value	Source
	Domestic Economy		
α	Weight on import consumption	0.25	Harrison and Oomen (2010)
σ	Inverse of intertemporal substitution	2.00	Fernández-Villaverde *et al* (2011a)
φ	Inverse of labour supply elasticity	2.00	Fernández-Villaverde *et al* (2011a)
ξ_d	Calvo probability: domestic producer prices	0.88	Adolfson *et al* (2007)
ξ_m	Calvo probability: importing firm prices	0.46	Adolfson *et al* (2007)
η	EoS between domestic and imported goods	1.77	Harrison and Oomen (2010)
h	Habit formation	0.69	Adolfson *et al* (2007)
κ_d	Indexation: domestic producer prices	0.21	Adolfson *et al* (2007)
κ_m	Indexation: import firm prices	0.16	Adolfson *et al* (2007)
ϕ_R	Interest rate smoothing: monetary policy rule	0.89	Smets and Wouters (2007)
ϕ_π	Inflation reaction: monetary policy rule	2.03	Smets and Wouters (2007)
ϕ_y	Output reaction: monetary policy rule	0.30	Smets and Wouters (2007)
ϕ_q	Output reaction: monetary policy rule	0.13	Vukotic (2007)
η_f	EoS between domestic and foreign exports	1.46	Harrison and Oomen (2010)
χ_a	Risk premium: NFA	0.05	Adolfson *et al* (2007)
χ_s	Risk premium: nominal exchange rate	0.75	Assumption
ξ_w	Calvo probability: wages	0.70	Adolfson *et al* (2007)
κ_w	Indexation: wages	0.52	Adolfson *et al* (2007)
β	Time discount factor	0.99	Calibrated
λ_d	Domestic firms mark-up: steady state	1.17	Adolfson *et al* (2007)
λ_m	Import firms mark-up: steady state	1.62	Adolfson *et al* (2007)
λ_x	Export firms mark-up: steady state	1.05	Assumption
ϕ_f	Fraction of wages financed in advance	1.00	Christiano *et al* (2011)
λ_w	Wages mark-up: steady state	1.05	Adolfson *et al* (2007)
ξ_x	Calvo probability: domestic producer prices	0.64	Adolfson *et al* (2007)
κ_x	Indexation: export firm prices	0.14	Adolfson *et al* (2007)
	Foreign Economy		
$\rho_{\lambda_{d^*}}$	Persistence: domestic prices mark-up shock	0.95	Smets and Wouters (2007)
$\rho_{\lambda_{w^*}}$	Persistence: wages mark-up shock	0.96	Smets and Wouters (2007)
ρ_{ψ^*}	Persistence: labour supply shock	0.96	Assumption
ρ_{z^*}	Persistence: productivity shock	0.96	Smets and Wouters (2007)
ρ_{d^*}	Persistence: preference shock	0.95	Assumption
ρ_{g^*}	Persistence: government spending shock	0.95	Fernández-Villaverde *et al* (2011a)
ξ_{d^*}	Calvo probability: domestic producer prices	0.55	Smets and Wouters (2007)
ξ_{w^*}	Calvo probability: wages	0.70	Smets and Wouters (2007)
κ_{d^*}	Indexation: domestic producer prices	0.24	Smets and Wouters (2007)
κ_{w^*}	Indexation: wages	0.58	Smets and Wouters (2007)

Charts

Chart 1: Flow diagram describing the international transmission of supply uncertainty shocks in the DSGE model.

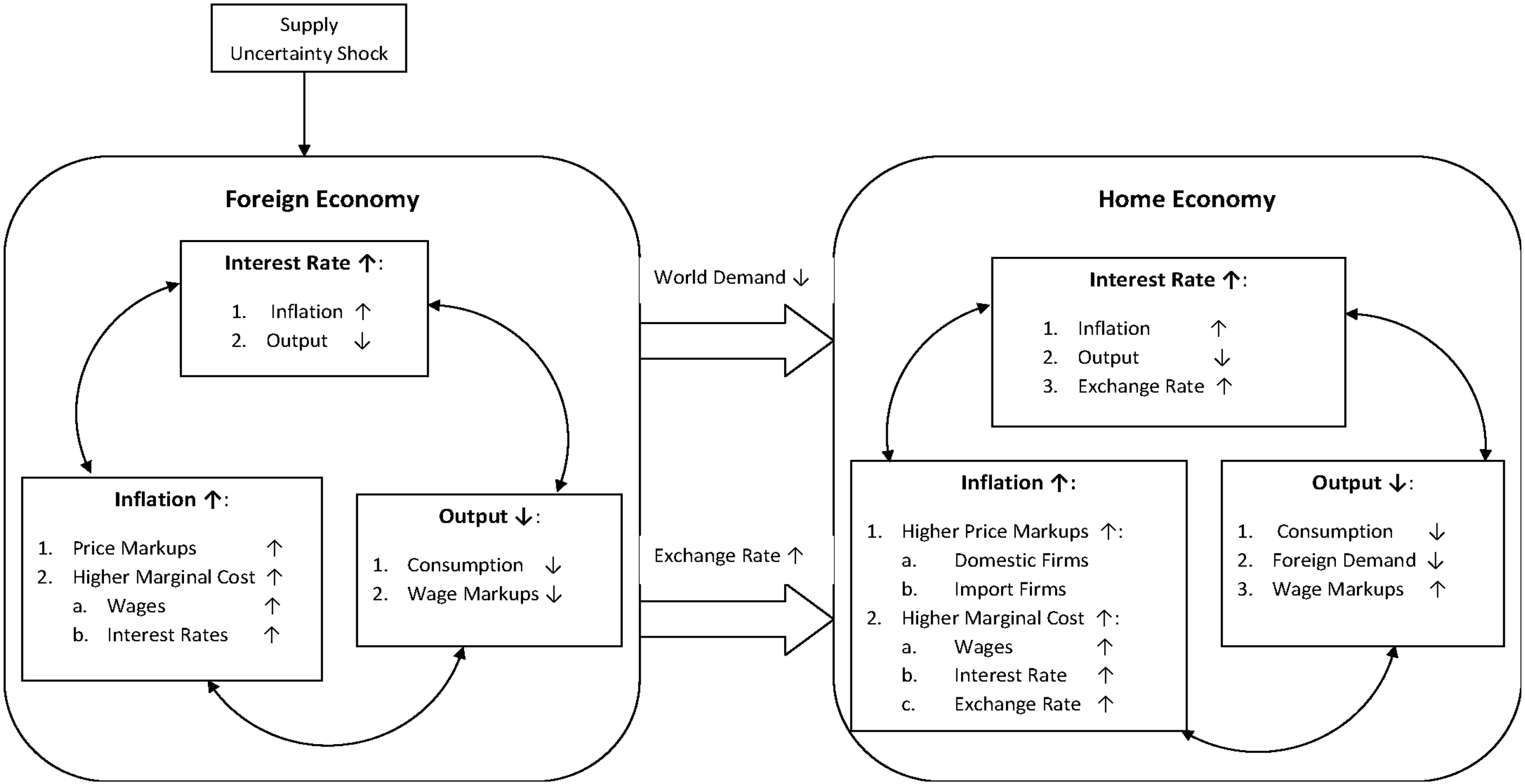


Chart 2: Estimated standard deviation of structural shocks

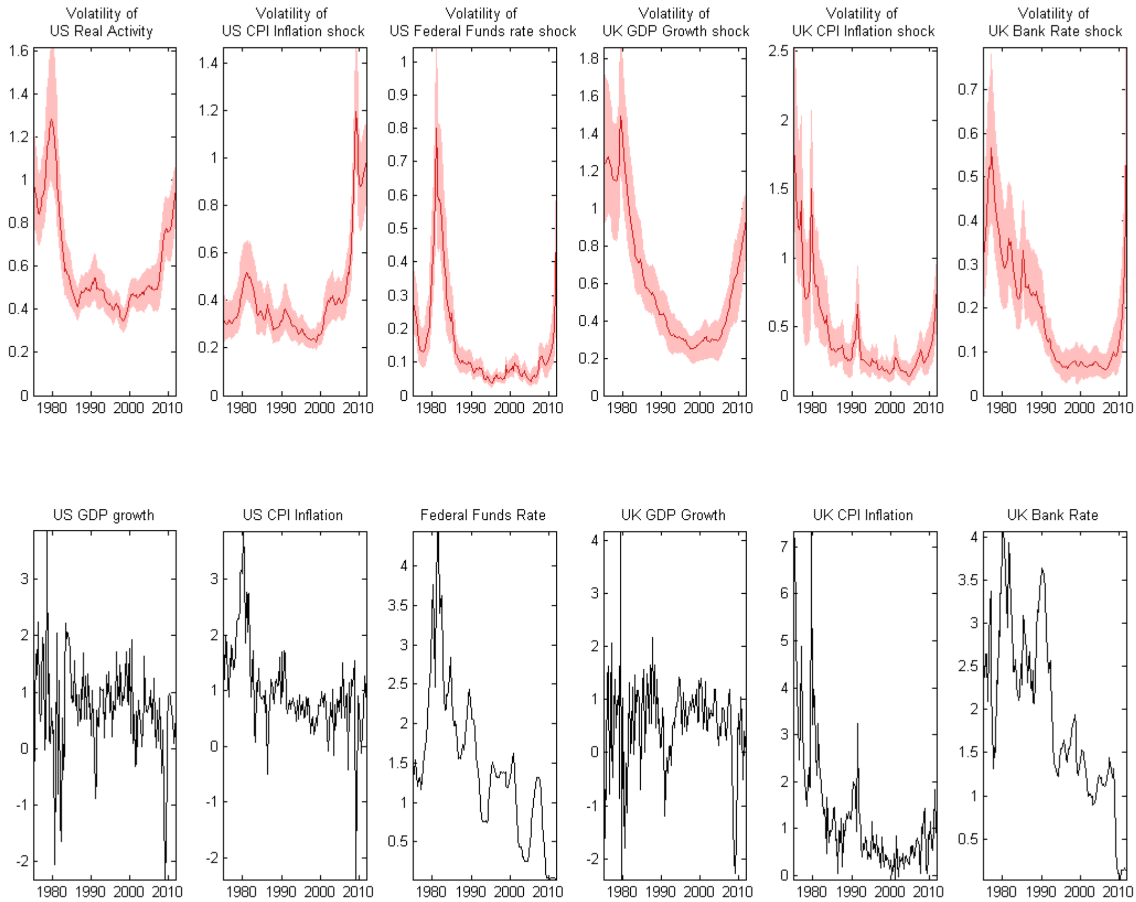


Chart 3: The response to a 100% increase in the volatility of the shock to US real activity. Median responses (solid line) and 68% error bands (shaded area).

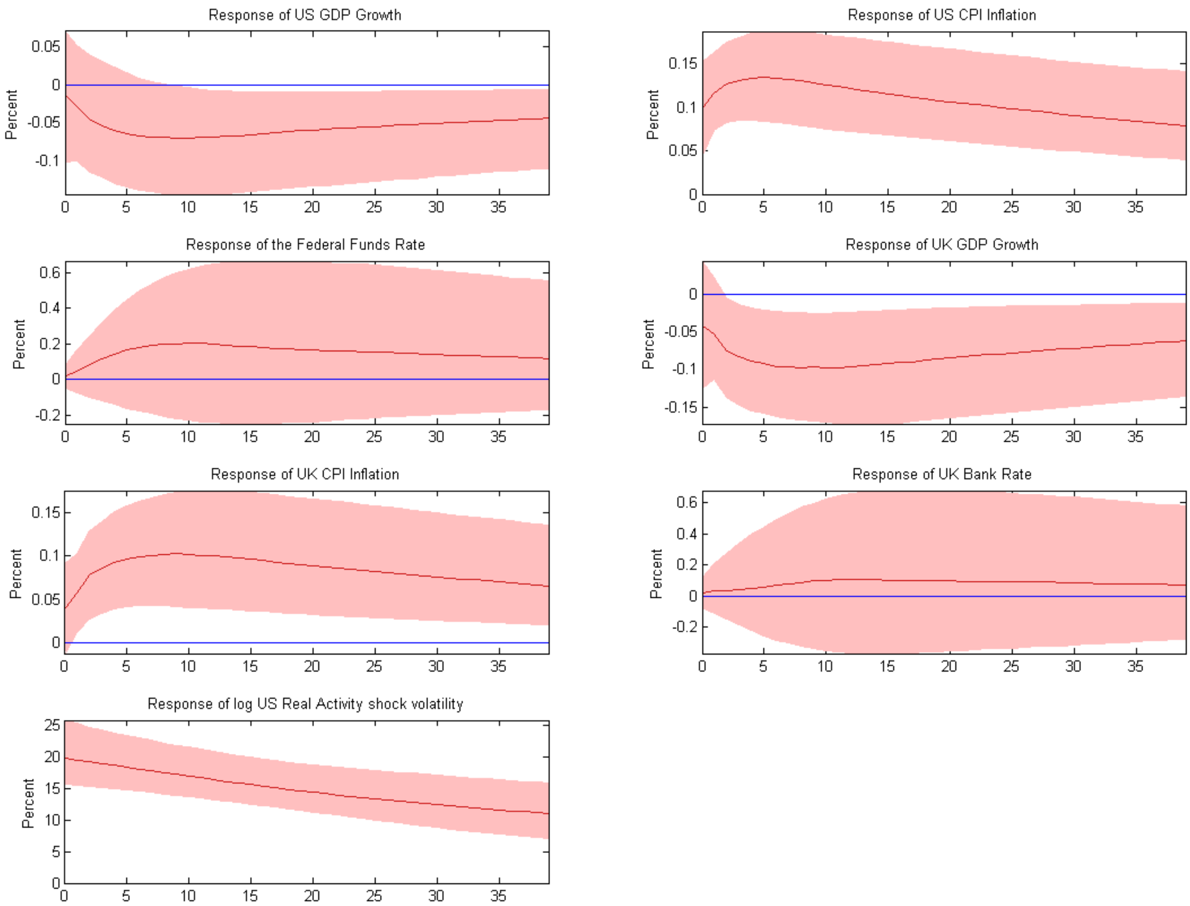


Chart 4: The response to a 100% increase in the volatility of the shock to US real activity in an expanded VAR system. Median responses (solid line) and 68% error bands (shaded area).

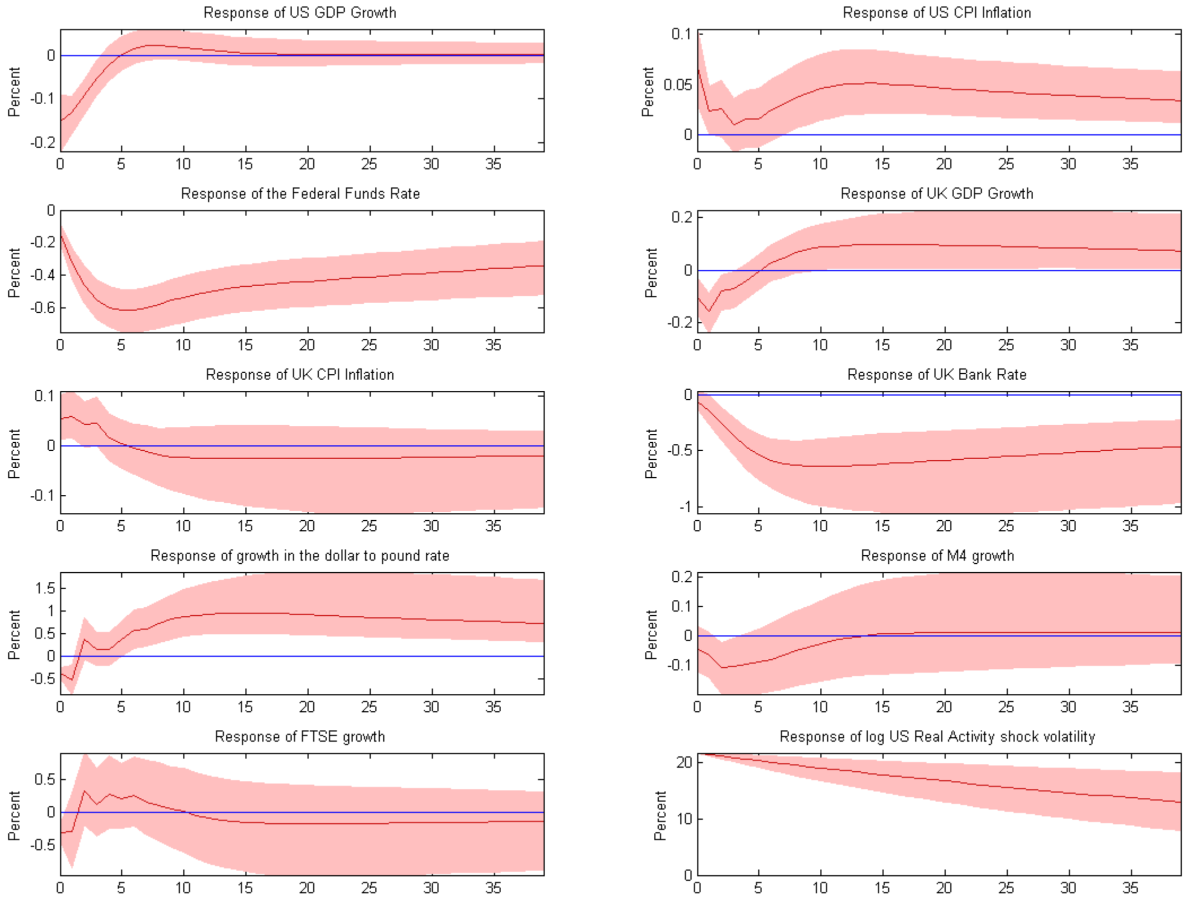


Chart 5: The response to a 100% increase in the volatility of the shock to US real activity. Further sensitivity analysis. Median responses (solid line) and 68% error bands (shaded area).

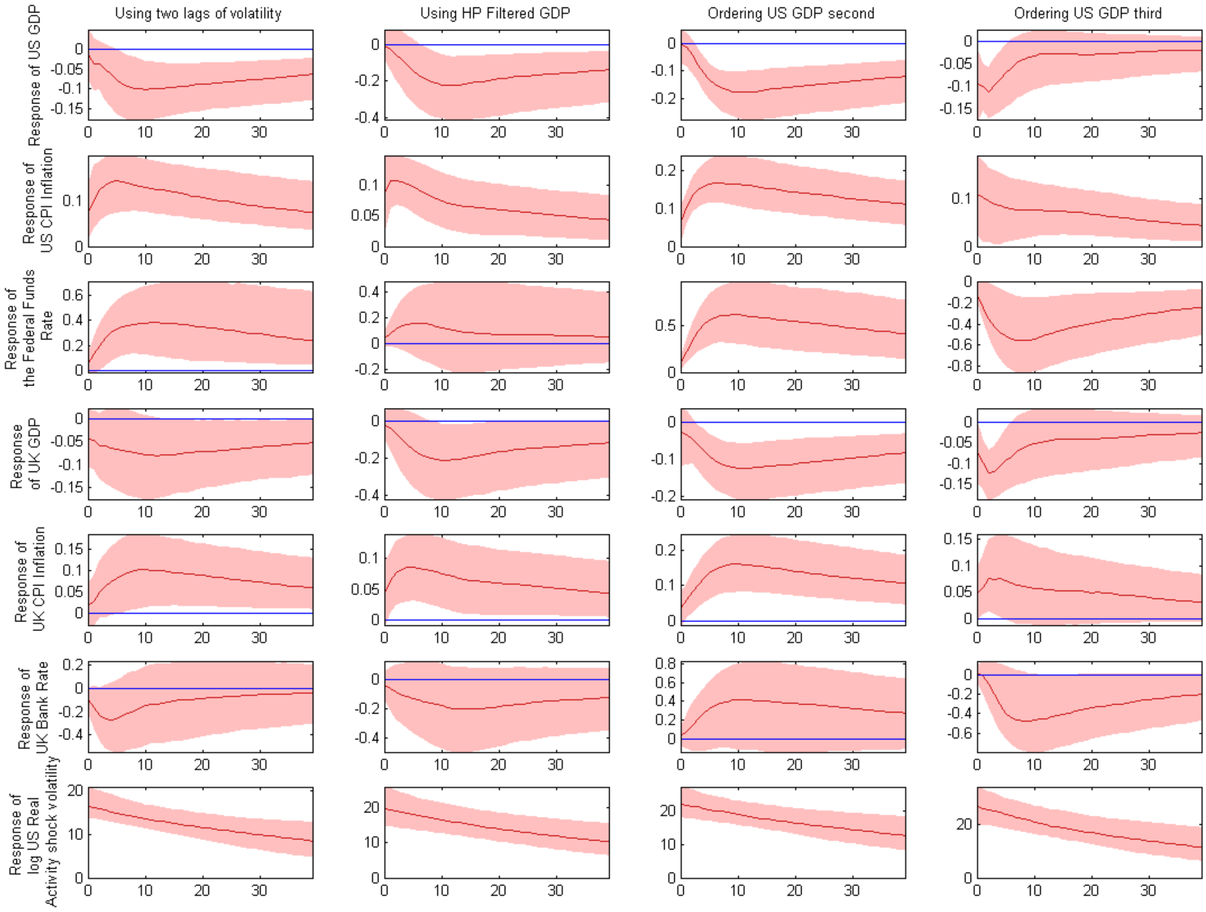


Chart 6: Uncertainty shocks increase supply shock uncertainty by 20%. Nominal Interest rates and inflation rates are reported in annual terms. PrMp = price mark-up, LbSp = labour supply, WgMp = wage mark-up, Prod = productivity, Unc = uncertainty shock, Nom-ExR-G = home defined nominal exchange rate growth, H = home economy, F = foreign economy.

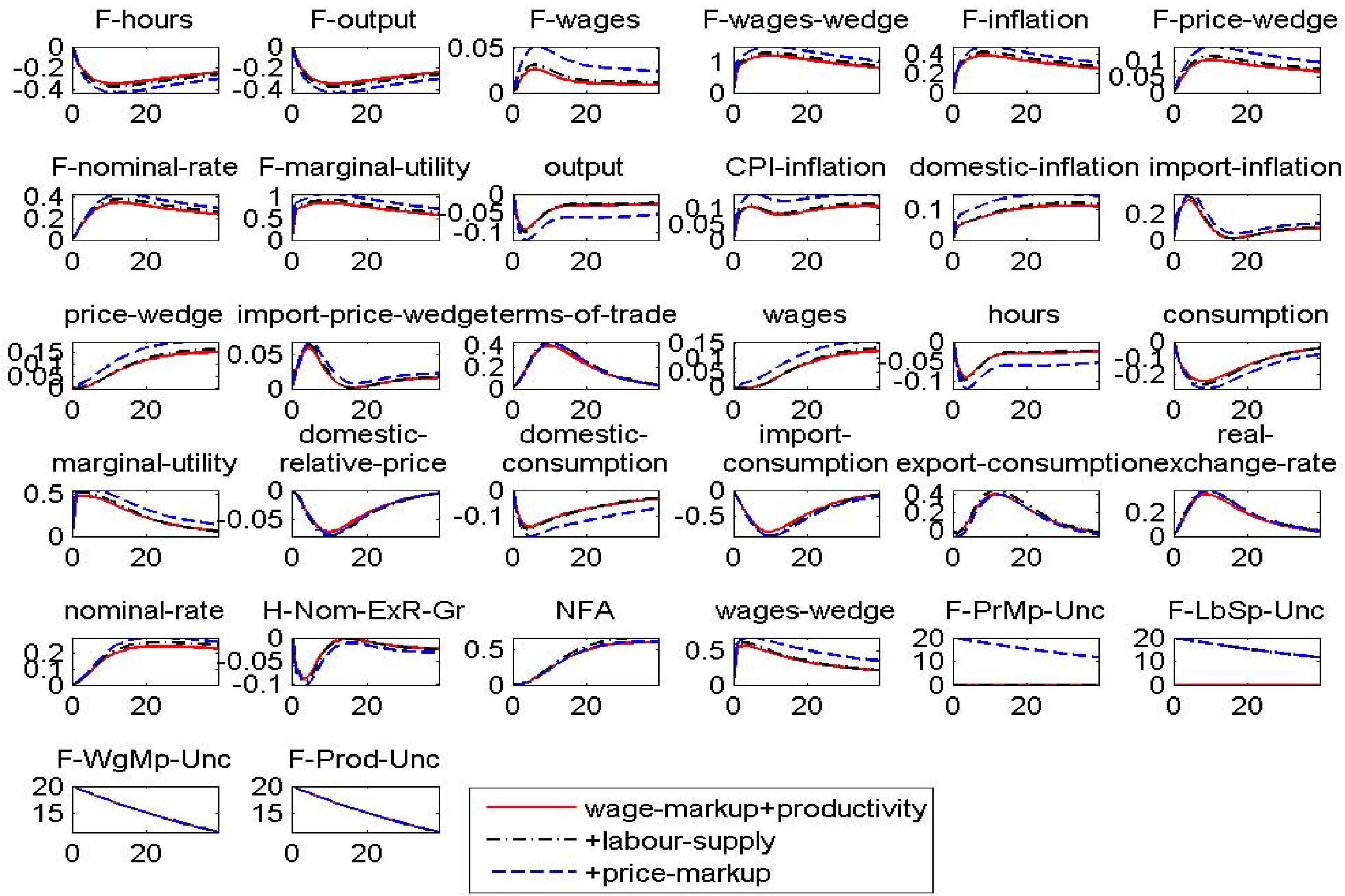


Chart 7: Uncertainty shocks increase demand shock uncertainty by 20%. Nominal interest rates and inflation rates are reported in annual terms. Dm = discount factor (β), G = government spending, Unc = uncertainty shock, Nom-ExR-G = home defined nominal exchange rate growth, H = home economy, F = foreign economy.

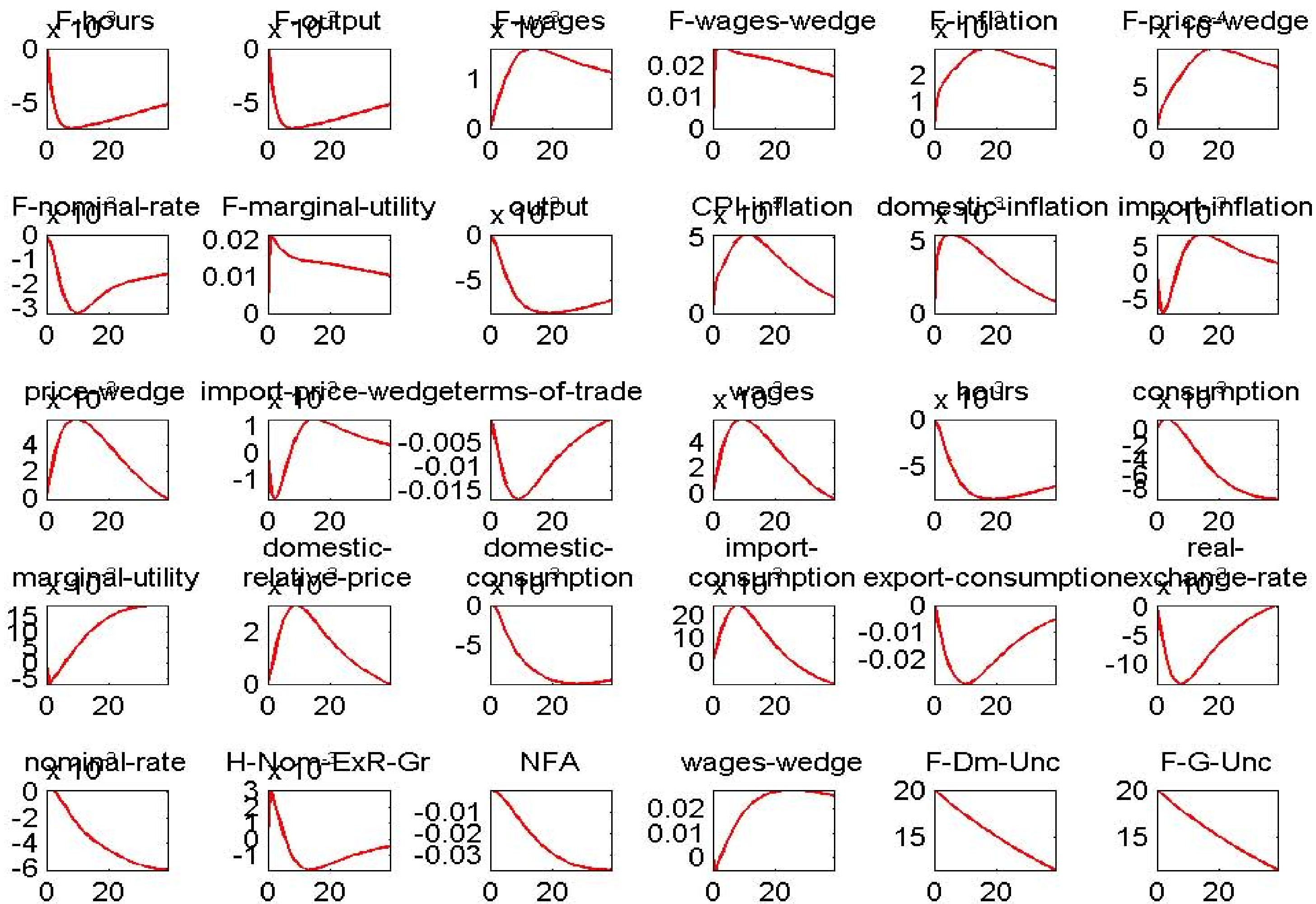


Chart 8: Working capital sensitivity exercise. Uncertainty shocks rise supply shock uncertainty by 20%. Nominal interest rates and inflation rates are reported in annual terms. PrMp = price mark-up, LbSp = labour supply, WgMp = wage mark-up, Prod = productivity, Unc = uncertainty shock and UK-Nom-ExR-G = UK defined nominal exchange rate growth.

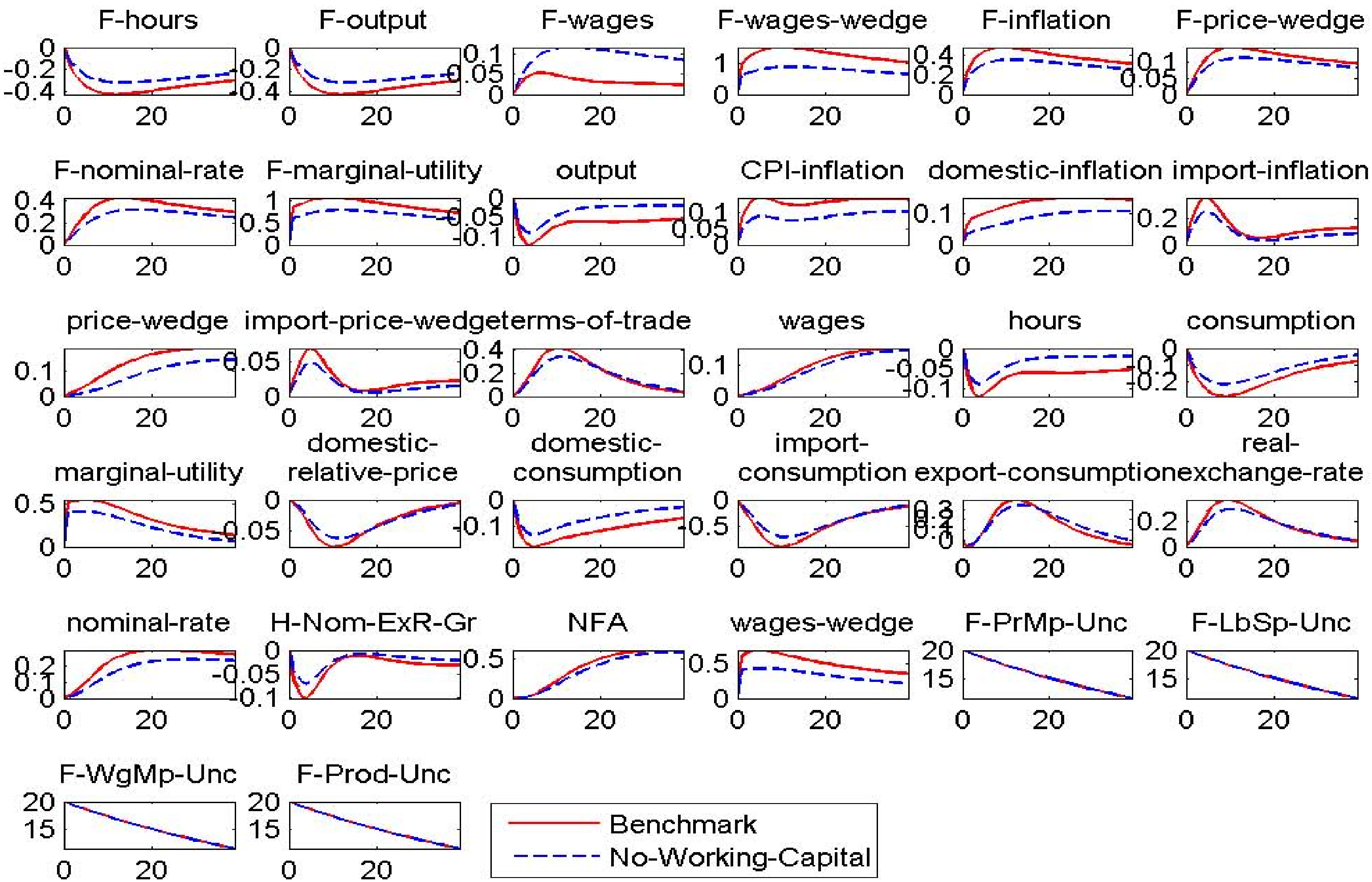


Chart 9: Labour market frictions sensitivity exercise. Uncertainty shocks increase supply shock uncertainty by 20%**. Nominal interest rates and inflation rates are reported in annual terms. PrMp = price mark-up, LbSp = labour supply, WgMp = wage mark-up, Prod = productivity, Unc = uncertainty shock, Nom-ExR-G = home defined nominal exchange rate growth, H = home economy, F = foreign economy.**

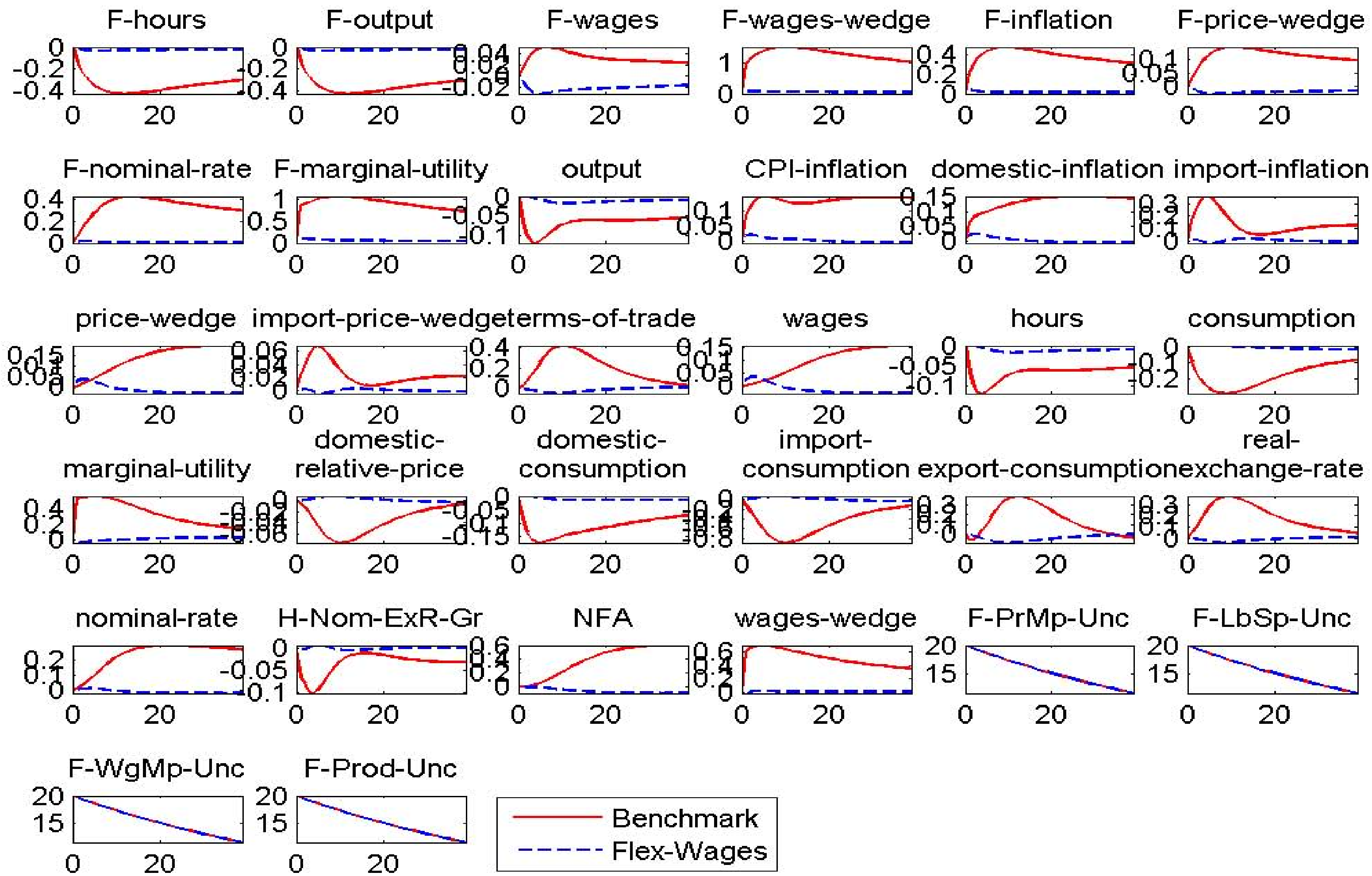


Chart 10: Risk premium function sensitivity exercise. Uncertainty shocks increase supply shock uncertainty by 20%**. Nominal interest rates and inflation rates are reported in annual terms. PrMp = price mark-up, LbSp = labour supply, WgMp = wage mark-up, Prod = productivity, Unc = uncertainty shock, Nom-ExR-G = home defined nominal exchange rate growth, H = home economy, F = foreign economy.**

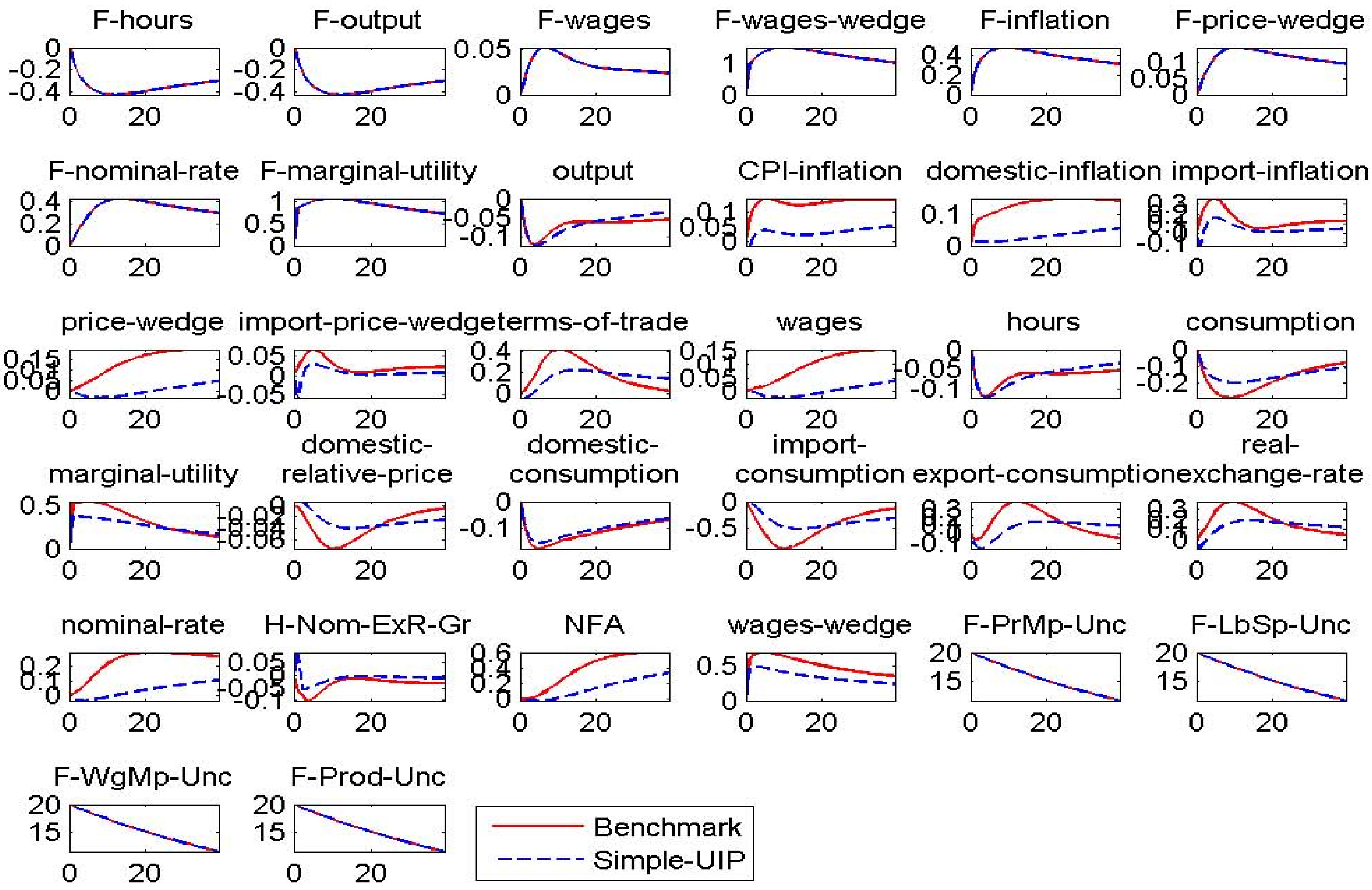


Chart 11: Persistence sensitivity exercise. Uncertainty shocks increase supply shock uncertainty by 20%. Nominal interest rates and inflation rates are reported in annual terms. PrMp = price mark-up, LbSp = labour supply, WgMp = wage markup, Prod = productivity, Unc = uncertainty shock, Nom-ExR-G = home defined nominal exchange rate growth, H = home economy, F = foreign economy.

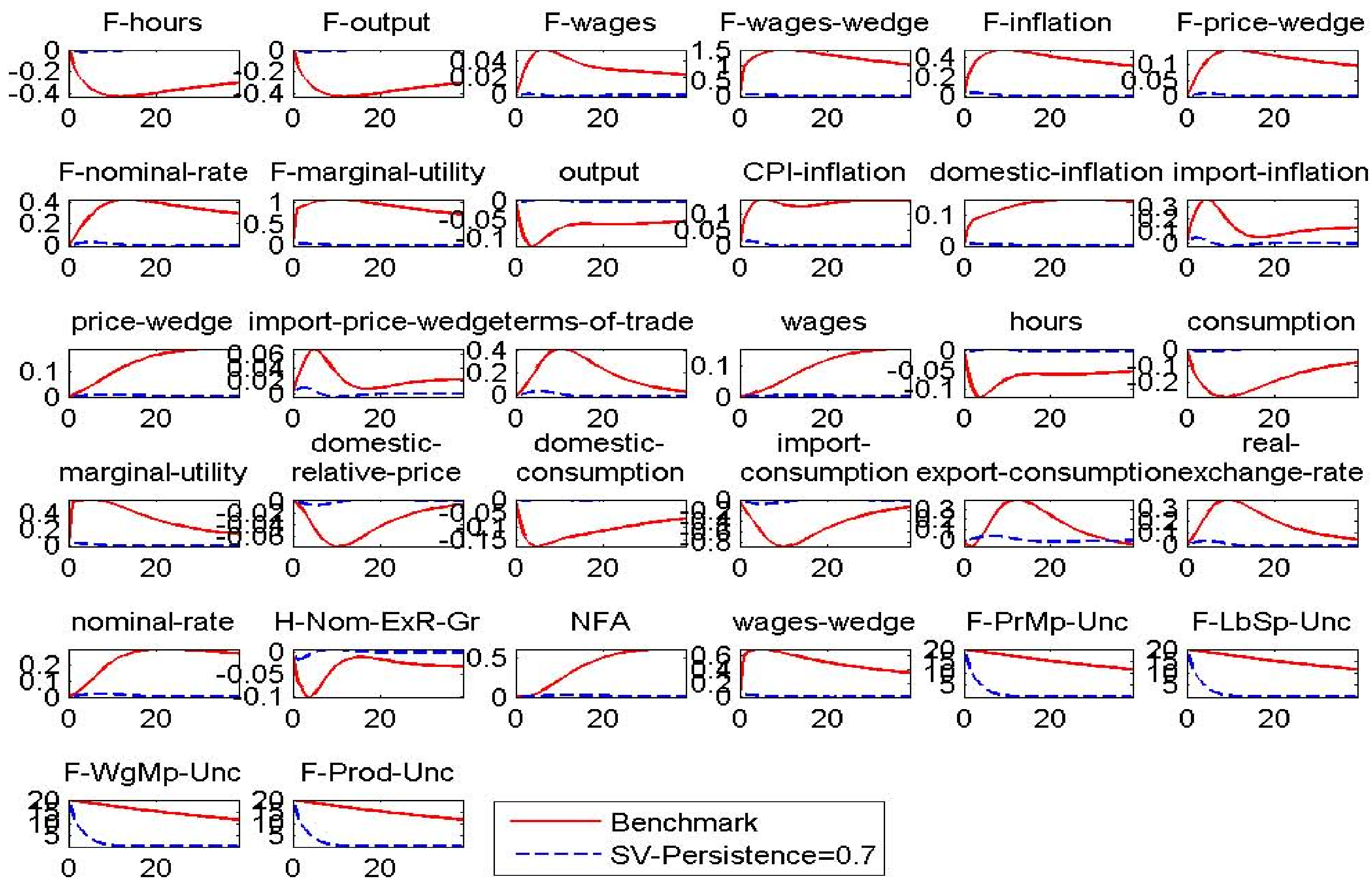


Chart 12: Sticky prices model

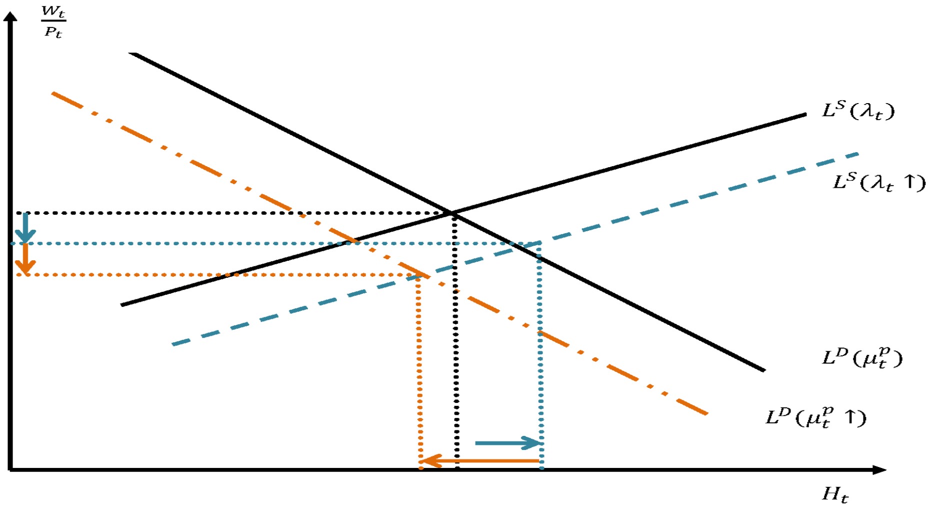


Chart 13: Sticky prices and wages model