BANK OF ENGLAND

Working Paper No. 480
Central counterparties and the topology of clearing networks

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August 2013

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Abstract

Given a network of client-clearer relationships, we define central clearing as a function transforming bilateral trading exposures into centrally cleared exposures. By using numerical simulations, we study how this function is affected by the network's topology, focusing on the exposures of the central counterparty. By assuming that margin requirements are a linear function of exposures, we also draw conclusions as to how the network topology affects aggregate margin requirements.

Key words: Central counterparty, CCP, clearing, settlement, network analysis.

JEL classification: E58, G01, G18.

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The views expressed here are those of the authors; they do not necessarily reflect the views of their parent institutions. Most of the practical knowledge about central counterparties behind this paper comes from Matthew Dive, who patiently took us through many intricacies of clearing. Edwin Latter and Anne Wetherilt provided useful feedback which helped to improve this paper. We are extremely grateful to Darrel Duffie and Rob Bliss for encouragement and for insightful discussion. We owe thanks to Thor Koeppl and to Katzeteru Tao, who discussed earlier versions of this paper. We also thank Simon Debbage, Claire Halsall and Matthew Vital for suggestions. Kimmo Soramäki gratefully acknowledges a grant from Säästöpankkien Tutkimussäätiö. This paper was finalised on 15 July 2013.

The Bank of England's working paper series is externally refereed.

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ISSN 1749-9135 (on-line)

Summary

The 2008-09 financial crisis prompted reforms in important parts of the financial infrastructure. Central counterparties (CCPs) are playing a major role in this reform, especially for over-the-counter (OTC) derivatives. Notably, the G20 leaders agreed in Pittsburgh in September 2009 that "All standardised over-the-counter derivative contracts should be traded on exchanges or electronic trading platforms, where appropriate, and be cleared through central counterparties end-2012 at the latest". Since 2009, a substantial amount of progress has been made in defining new standards and implementing infrastructure reforms.

The main function of CCPs is to novate contracts between trading parties, becoming the 'seller to every buyer, and buyer to every seller'. By so doing, CCPs concentrate counterparty credit risk on themselves, sitting at the vertex of what can be seen as clearing networks.

In the simplest, theoretical case, a clearing network comprises the CCP at the vertex and, directly linked to this, a number of general clearing members (GCMs). Almost invariably though, the clearing network is more articulated, as some GCMs may in turn work as clearing agents for other entities (be these banks or market participants in general), and so forth in a sequence of tiers.

What are the consequences of such stratification? More generally: how does the *topology* of a clearing network affect the systemic risk-reduction role of central clearing? This paper develops a stylised model to look into this question.

The topology of a clearing network will have an effect both on credit exposures and market participants' liquidity needs as margin calls are issued by the CCP in order to manage its exposures, and possibly also by GCMs when clearing for second-tier entities.

To analyse these issues we proceed as follows. First, we lay out a stylised but general model of central clearing. Then, we look at how *initial* bilateral *exposures* are transformed by the network into centrally *cleared exposures*, which in turn generate *liquidity demands*. The model allows exposures and liquidity demands for any network topology to be computed. We can look at the effects on exposures and liquidity demands arising in different topologies.

The model is highly simplified, flattening out fine but important detail of how, for example, exposures may be netted across the network, or how margins may be computed. Moreover, the model takes initial bilateral exposures as exogenous random variables, mechanically turning them into cleared exposures without including any behavioural component.

However, because of its simplicity, this work sheds some light on the properties of clearing networks. Its results give insights into the effects of tiering and concentration[1] on the systemic risk-reduction role of central clearing. Tiering appears to increase some key risks faced by the CCP. For example, it increases the likelihood of large exposures, and makes them more

[1] 'Concentration' here refers to the way second-tier members are distributed across GCMs.

unpredictable. CCP exposures are on average smaller in concentrated systems, while extreme exposures become less frequent. The effects on margin needs are, interestingly, non-monotonic but, unfortunately, less clear-cut as they crucially depend on details of the margining methodology; in particular, on whether 're-hypothecation' is allowed or not.

1 Introduction

1.1 Market infrastructures

The 2008-2009 financial crisis prompted reforms in important parts of the financial market infrastructure. This is not because the infrastructures themselves experienced failures (on the contrary, they held up remarkably well), but because infrastructures could have mitigated some of the problems that fuelled the crisis, among which were lack of information and lack of trust[1]. Central banks and regulators world-wide are now encouraging greater use of infrastructures, and enhanced risk management within those infrastructures, to help avoid future crises.

As part of this effort, in September 2009 the G20 leaders agreed in Pittsburgh that all standardised over-the-counter derivative contracts should be traded on exchanges or electronic trading platforms, where appropriate, and be cleared through CCPs by the end of 2012. New standards have been set by CPSS-IOSCO in their April 2012 "Principles for Financial Market Infrastructures". In February 2012, IOSCO published recommendations on requirements for mandatory central clearing.[2] The Financial Stability Board has published three progress reports on implementation of OTC derivatives markets reforms (April and October 2011, and June 2012)[3], while the Committee on the Global Financial System, in its November 2011 report[4] considers alternative clearing configurations and their merits from a policy perspective.

CCPs are entities whose main function is to novate contracts between the trading parties, becoming the 'seller to every buyer, and buyer to every seller.' By doing so, CCPs relieve their clients of direct counterparty risk (other than to the CCP itself), which they themselves manage eg by calling margins and collecting default fund contributions.

The widespread view is that central clearing can reduce systemic risk, because exposures are then concentrated and dealt with by dedicated infrastructures, which can offer rigorous and transparent risk management. Yet, the choice of CCP *vs* non-CCP is only part of the issue, as CCP-cleared systems can still be organized in very different architectures. In one extreme model, all market participants directly connect to the CCP. In another, the CCP clears for a limited number of institutions, which in turn clear for other participants, and so on in a hierarchy of tiers. This paper argues that the question 'which network should there be around a CCP?' is as relevant as the original question 'should a CCP be introduced?'

Table 1 gives an idea of how varied clearing networks can be. At one extreme there is EuroCCP, which appears highly tiered, ie few General Clearing Members (GCM)

[1] "Small lessons from a big crisis", speech by Andrew G Haldane, 8 May 2009.
[2] "Requirements for mandatory clearing", Technical Committee Of The International Organization Of Securities Commissions, February 2012.
[3] "OTC derivatives markets reforms", FSF (2010, 2011).
[4] "The macrofinancial implications of alternative configurations for access to central counterparties in OTC derivatives markets", CGFS Papers No 46, BIS (2011).

serve a large number of indirect participants. At the other extreme, LCH RepoClear appears as much flatter, with a ratio of indirect participants to GCMs of about two.

However, tiering does not fully characterize a network, as two systems with the same number of members and same degree of tiering may still be very different. In one, the majority of indirect members may clear at a few large GCMs –we will call this a 'concentrated' network. In the other, the indirect members could be evenly distributed across the GCMs.

	GCMs	Indirect members	Ratio indirect members / GCMs
Euro CCP	25	> 550	> 22
LCH EquityClear LSE	37	> 530	> 14
Eurex	58	235	4.1
LCH SwapClear	22	80	3.6
LCH RepoClear	42	80	1.9

Source: CCPs' websites

Table 1: Examples of network structures

This paper focuses on these two dimensions of clearing networks: tiering and concentration, looking at how these features affect some aspects of functioning of the network itself, as specified below.

The stability of the whole network depends on the survival of its central node, the CCP; hence, the main focus of the analysis will be on the *CCP exposures*. The other issue we look into is the total *liquidity* required by the network. CCP clearing is based on margin calls, issued by the CCP in order to manage its exposures[5]. The amount of margin is thus a measure of the resources absorbed by the network, and will be the second focus of the analysis.

The paper has a simple structure. First, we build a very stylized but general model of a clearing network. Then, using Monte Carlo simulations, we look at how *initial exposures* are transformed by the network into *cleared exposures*, in turn generating *liquidity demands*. The model enables exposures and liquidity demands to be computed for any network topology. We can look at the effects on exposures and liquidity demands arising in different topologies.

We model initial exposures as random variables, so the resulting cleared exposures and liquidity demands are also random. The respective distributions are found to vary greatly across networks. By looking at the overall shape of these distributions (as opposed to simple statistics such as average values) we gain insights into several aspects of the 'risks' arising in different topologies.

Our findings suggest that tiering increases some key risks faced by the CCP. In particular, it increases the likelihood of very large exposures and makes them more unpredictable – mainly because a tiered network has larger participants, with possibly very unbalanced positions. Exposures faced by a CCP are (on average)

[5] Margin calls may also be issued by GCMs, on the second tier entities they clear for.

smaller in concentrated systems. Moreover, extreme exposures become less likely. The results on margin needs are more complex and non-monotonic. In addition, they crucially depend on whether “re-hypothecation” is allowed or not.

1.2 Related literature

Recent studies have looked at networks in financial infrastructures with theoretical models, or have applied network analysis to financial-infrastructure data. These include analyses of payment systems (Becher et al. 2008, Soramäki et al., 2007, Bech and Atalay, 2008) and studies of interbank lending markets (Akram and Christophersen 2010, Boss et al. 2004, Bech and Atalay 2008, Iori et al. 2008, Wetherilt et al. 2010).

Research on clearing arrangements, in particular from a network theory perspective, is relatively scant. Jackson and Manning (2007) show that, if market participants differ in their credit quality and the CCP cannot tailor margins to individual default probabilities, then tiered arrangements will be favoured by high credit quality agents. On the other hand, Moser (2002) shows that there are reasons why participants with a high credit reputation may favour non-tiered systems: broadening first-tier participation may increase market depth and liquidity. Duffie and Zhu (2011) show that, under specific assumptions on the network of original exposures, central clearing may be less netting efficient than bilateral clearing[6]. Pirrong (2009) considers the effects of CCP netting on the distribution of losses following a default.

This paper connects mostly to Jackson and Manning (2007) and to Duffie and Zhu (2011) in its modelling approach. Its main innovations are: a) providing a model of clearing which can encompass a large variety clearing networks; b) offering the first systematic study of the statistical properties of exposures and margin needs across clearing network topologies.

By focusing on exposures and margins, this paper leaves aside other aspects of clearing, most notably the issue of defaults. We do model (in a very stylized manner) margin requirements, but do not explain how margins are used by the CCP, nor the consequences of member defaults, should margins be insufficient. We also disregard several other issues of CCP risk management: membership criteria, loss-allocation rules, and other default procedures.

The paper proceeds as follows: Section 2 describes our model, Section 3 presents the results, and Section 4 provides the conclusions.

[6] Netting is ‘efficient’ in the sense that it reduces expected exposures –as indeed exposures of different sign are compensated.

2 Model

2.1 Original bilateral exposures

We consider a single contract, traded on a market with N agents or 'counterparties'. We do not specify the details of the contract. All we say is that it produces exposures between participants, summarized in a $N \times N$ matrix T.

T_{ij} represents the nominal position of trader i against trader j. For example, $T_{12} = x$ means 'trader 1 is long x contracts against trader 2'. We assume that trades in T are already the result of bilateral netting, ie that T is negative-symmetric about its main diagonal: $T_{ij} = -T_{ji}$ for all i, j.

Whenever $T_{ij} > 0$, i and j are both exposed to each other. So e.g. $T_{ij} < 0$ should *not* be read as 'i has a negative exposure (possibly is not exposed) to j' but instead as '*i is exposed to j, j is exposed to i, and the size of these exposures is given by the absolute value of* T_{ij}'. Both parties are exposed to the risk of the other party not fulfilling its obligation.

2.2 Clearing networks

The N counterparties, plus the CCP which has no exposures deriving from proprietary trading (and thus does not figure in matrix T), are arranged in a directed acyclic network, where the link $x \rightarrow y$ means that x clears through y. We consider networks with three types of nodes:

a) one <u>CCP</u>, with no 'parent' but with one or more 'children' of type b);

b) a number V (between 2 and N) of <u>GCMs</u>, which are 'children' of the CCP and, possibly, 'parents' of one or more nodes of type c);

c) $N - \mathrm{V}$ <u>clients</u>, each with exactly one GCM as a 'parent', and no 'children'. Note that we do not allow clients to clear via multiple GCMs.

Given any node i, we denote its parent by $C(i)$, standing for i's clearer. The network is thus fully described by the $N \times 1$ vector C.

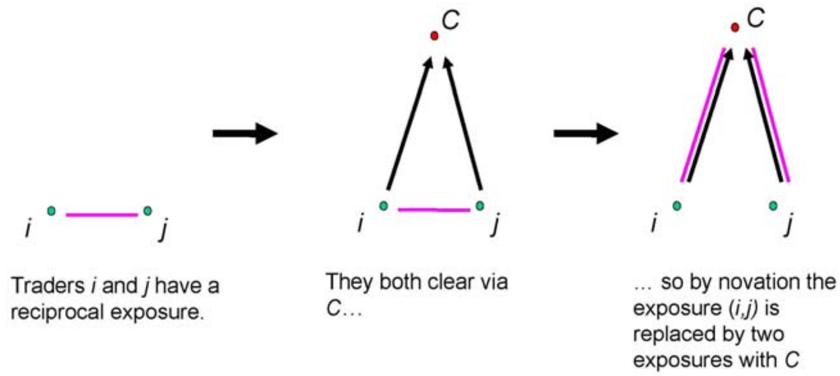
In the reminder of the paper, we refer to both CCPs and GCMs as 'clearers': the CCP is the clearer for the GCMs, which in turn clear for the 'clients'.

2.3 Clearing by means of novation

Clearing, carried out by the CCP and by the GCMs, transforms trading exposures into 'cleared' exposures. The basic step of this process, which we will call 'clearing' instead of 'central clearing' for brevity, is novation:

Novation: *replacement of an exposure of size x between i and j ≠ C(i) by a pair of offsetting exposures of size x: one between i and C(i), the other between C(i) and j, so that the net market position of each involved party is left unchanged.*

Figure 1: Novation



As Figure 1 shows, novation re-arranges exposures between non-adjacent nodes (link *i*,*j*) into exposures along network links (links *i*,*C* and *j*,*C*).

Central clearing consists in repeatedly applying novation, until exposures are only between participants and their clearers[7]. A formal description of this process is the following (with running index *s* = 0,1...):

$$
\begin{aligned}
&T(0) = T \\
&T_{ij}(s+1) = 0 \quad \mathit{for\ any\ pair}\ (i,j){:}\ j\ \neq C(i) \\
&T_{iC(j)}(s+1) = T_{iC(i)}(s) + T_{ij}(s)\ \mathit{for\ the\ above}\ (i,j) \\
&T_{jC(i)}(s+1) = T_{jC(i)}(s) + T_{ji}(s)\ \ \mathit{for\ the\ above}\ (i,j)
\end{aligned}
\tag{1}
$$

It is easy to see that this process ends after a finite number of steps, say *s**, and that its end point $T(s = s^*)$ does not depend on the precise sequence of chosen pairs (*i*,*j*), while of course it depends on the network (vector) *C.*

Denoting the end point of the clearing process by T^*, we define the ***clearing function*** $\Gamma^C\colon \mathbb{R}^{2N} \to \mathbb{R}^{2N}$ as follows:

$$\Gamma^C(T) \equiv T^*(T, C).$$

So, given a network *C* and exposure matrix *T*, the clearing function gives a new matrix of cleared exposures T^*.

This model of clearing, where all exposures are added together, reflects the accounting practice of ***account pooling***, whereby a clearer pools (i.e. sums) exposures from its own proprietary trading with positions generated by its clients. Alternative accounting methods exist, and will in future be mandatory in some jurisdictions, but we focus on account pooling for its simplicity[8].

---

[7] Note that, applying novation sequentially, exposures may be created which will be novated at a later stage. This happens when novation is applied to a T_{ij} such that *C*(*i*) ≠ *C*(*j*). In this case, an exposure between *j* and *C*(*i*) is created, which is between non-adjacent participants because *C*(*i*) ≠ *C*(*i*).

[8] Under account segregation, clearers keep separate proprietary and client trades, to clear them separately with the CCP. Under the European Market Infrastructure Regulation (EMIR), client positions will have to be segregated from house positions. They may be individually segregated or held in "omnibus" client accounts. Where the client has an individually segregated account there will be no netting of trades with those of other clients of the GCM (i.e. no "internalisation" of trades). Modelling segregated accounts is cumbersome, but conceptually simple: one needs to keep track of several *T*s, one for proprietary, the other(s) for client exposures.

For a given GCM k we define k's 'group' (i.e. pool of clients) in the obvious way:

$$g_k = k \cup \{i: k = C(i)\}.$$

Assuming account pooling, we can define k's "***internalized trades***", or those that do not result in a net exposure between the CCP and GCM k, as

$$I_k = \{T_{ij}: (i,j) \in g_k \times g_k\} \tag{2}$$

i.e. trades within k's group. Similar, k's "***non-internalized trades***" are defined as:

$$NI_k = \{T_{ij}: i \in g_k \text{ and } j \notin g_k\}.$$

Recalling that cleared exposures are only between participants and their respective clearers, a moment of reflection reveals that:

$$\Gamma^C(T) = T_{ij}{}^* = \begin{cases} \sum_{T_{\cdot,\cdot} \in NI_j} T_{\cdot,\cdot} & for\ i = CCP\ and\ j \in \Gamma \\ \sum_k T_{jk} & for\ i \in \Gamma\ and\ j = C^{-1}(i) \\ 0 & otherwise \end{cases} \tag{3}$$

That is:

i) the cleared exposure between the CCP and a GCM j is given by the sum of j's non-internalized trades (first line),
ii) that between a GCM i and a client j is given by j's trades (second line);
iii) all other exposures are zero (third line).

2.4 Margins

Clearers (CCP and GCMs) collect margins from the parties they clear for, in order to manage counterparty credit risk. Margins are typically of two types: initial and variation margins. Initial margin is collected to protect against future price movements. Variation margins involve marking contracts to current market prices and settling any variation. This ensures that initial margin continues to provide a desired level of insurance against future price movements.

For simplicity we concentrate on initial margin, disregarding default fund and variation margin.

Margins are paid by each institution to its clearer. We assume that long and short positions are equally risky[9], so clearers collect margins in equal measure for long and short exposures. We also assume that margins depend linearly on the size, or absolute value, of exposures. This is equivalent to saying that each contract attracts the same margin, and that long and short contracts are 'netted'. The **margin due** by i to $C(i)$ is therefore:

[9] As eg. if prices are equally likely to move in each direction.

$$m(i, C(i)) = m_i = \kappa \left| T_{iC(i)}{}^* \right| \tag{4}$$

where κ is a constant, set equal to 1 for simplicity. In reality, GCMs may adjust margin calls on the basis of perceived creditworthiness of their clients. For simplicity, we assume that the same margining rule *m*(.) is adopted by all participants.

It should be noted that, in our networks, clients only pay margins, as they do not novate contracts. GCMs, on the other hand, pay margins (to the CCP) and receive them (from their clients). Finally, the CCP only receives margins, being the ultimate clearer.

The above implies that margins flow through the network from bottom up. Note that, due to internalization (see Section 2.3), a GCM may end up retaining part of its clients' margins. Indeed, in our model the GCM collects margins for all client trades, but will need to clear at the CCP only the non-internalized ones; hence it will pay margin only on these latter.

Margin needs can be defined under two assumptions[10]:

1. Re-hypothecation. GCMs are allowed to, and choose to, 'recycle' the margins received from clients to pay margin to the CCP. In this case, the margin need of a participant is the net of margin due *minus* margin received;
2. No re-hypothecation. This is the case if GCMs are not allowed to reuse margin received from clients to pay margin to the CCP. Here, the margin need of a participant equals the margin due[11].

More formally, the **margin need** of *i* is:

$$\mu_i = \begin{cases} max\{0, m_i - \Sigma_{k \in g_i} m_k\} & rehypothecation \\ m_i & no\ rehypothecation. \end{cases}$$

where g_i is the group of clients of GCM *i*. Note that re-hypothecation is unrelated to both account pooling and internalization (Section 2.3). These two determine the size of the cleared exposures; re-hypothecation instead determines how the corresponding margins calls can be met.

We define **systemic margin need** as the value of margin payments required by the network as a whole:

$$\mu R = \sum_{i \in \Gamma} max\{m_i, \Sigma_{k \in g_i \setminus i} m_k\} \quad under\ rehypothecation \tag{5}$$

$$\mu NR = \sum_{i \in N} \mu(i) = \sum_{i \in N} m(i) \qquad under\ no\ rehypothecation \tag{5'}$$

[10] In reality, re-hypothecation practices may differ across GCMs. To simplify, we assume that either it is allowed and done across the whole network, or it is not allowed at all.
[11] Note: trades can still be 'internalized' and not result in a net exposure to the CCP.

Under re-hypothecation, the amount of margin needed is the sum of the margin needs by each group as a whole. For group g_i, this equals the margin need of the GCM *i* plus the margin needs of its clients, that is $max\{0, m_i - \Sigma_{k \in g_i \setminus i} m_k\} + \Sigma_{k \in g_i \setminus i} \mu_k = max\{m_i, \Sigma_{k \in g_i \setminus i} m_k\}$. Without re-hypothecation, it is simply the sum of individual margin needs.

Summing up: margin needs depend on margins due *m*(.). Margins due depend on T^* (eq. (4)), and in turn T^* depends on *C* and on *T* (eq. (3)). We thus have:

$$\mu R = \mu R^C(T): \ \ \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}$$
$$\mu NR = \mu NR^C(T): \ \ \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}.$$

So, a clearing network *C* defines (two) margin need functions. Each of them maps bilateral trading exposures (*T*s) into single numbers.

3 Analysis of the model

Sections 2.3 and 2.4 led to the definition of the following:

a) the clearing function $\Gamma^C(T)$;
b) the margin need functions $\mu R^C(T)$ and $\mu NR^C(T)$.

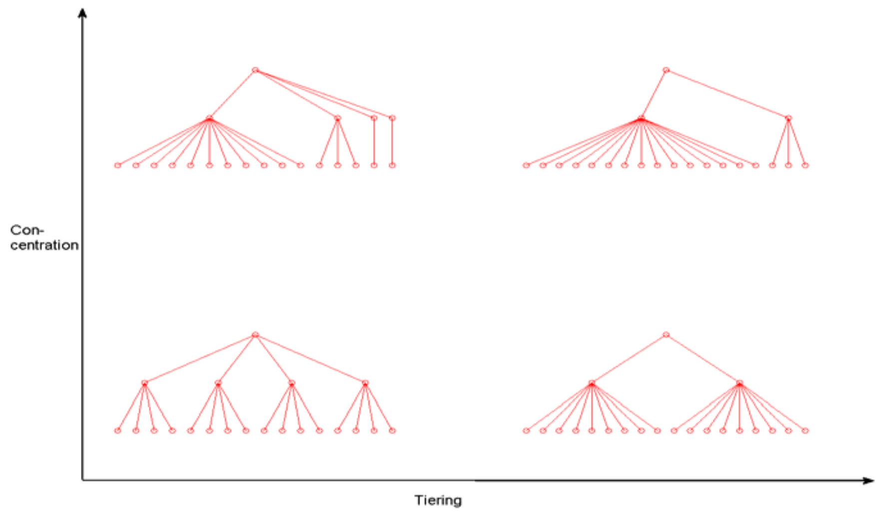
Our aim is to study the properties of a) and b), as the network *C* changes. We do so using Monte Carlo simulations. We fix the number of participants *N* and proceed in four steps:

I. randomly create many trading matrixes *T*s (daily positions)
II. construct all possible clearing networks Cs (call *Ξ* the set of such networks);
III. compute $\Gamma^C(T)$, $\mu R^C(T)$, $\mu NR^C(T)$ for each $C \in \Xi$ and *T*;
IV. compute statistics of these functions, and relate them to the network properties of the corresponding *C*s*.*

To perform step IV, we classify networks along two dimensions: tiering and concentration, as illustrated in Figure 2.

The two networks on the left display lower tiering than the two on the right, as they have four participants connected to the CCP instead of two. The two networks at the bottom feature less concentration than the two on the top, as clients there are equally split among GCMs.

Figure 2: Clearing networks



Formally, we define the tiering level simply as the number of clients:

$$Tiering = N - 1 - \mathrm{V},$$

where $\mathrm{V} = \|\Gamma\|$ is the number of GCMs.

Concentration is measured by a Gini coefficient. That is, we introduce the following notation[12]

$$s_i = \frac{|g_i - 1|}{N - 1 - \mathrm{V}} \quad \text{and} \quad S_k = \sum_{i=1}^{k} s_i \, , k = 1..\mathrm{V}\,.$$

Then, we label GCMs by size (so that $s_i < s_j$ for $i < j$). Finally, we define:

$$Concentration = \frac{2}{V-1} \sum_{k=1}^{V} \left(\frac{k}{V} - S_k \right),$$

which varies between 0 (when all s_i are equal ie concentration is minimal) and 1 (when all s_i are zero, except for one).

Unfortunately the mapping $f(C) = (tiering(C), concentration(C))$ is not an injective function[13], i.e. different networks may feature the same tiering and concentration levels. However, any two networks with identical tiering and concentration turn out to produce extremely similar exposures and margin needs, allowing us to approximate our simulations by average values across these 'duplicate' networks.

[12] In words: s_i is the share of clients cleared by *i*, and S_k is the cumulated share up to GCM *k*.

[13] A function *f* is injective if, whenever x≠y, *f*(x) ≠ *f*(y).

Summing up, we identify each network with a pair of tiering/concentration 'co-ordinates', and we present our results in a tiering-concentration space.

4 Results

Given the key role of the CCP, we concentrate on its exposures after novation has taken place, looking at its:

1. total expected exposure (expected across realizations of *T*),
2. single expected exposure (the above, averaged across GCMs),
3. extreme exposures (largest across GCMs and across realizations of *T*).

The first two indicators tell us about 'average' risks. The third is instead most relevant if the CCP has to ensure against extreme events.

As Eq. (3) reveals, a client's exposure is unaffected by the network topology, as it only depends on the client's own trades[14].

We then consider margin needs, looking at:

1. expected margin needs (expected across different realizations of *T*),
2. extreme margin needs (95th percentile across *T*s)

We consider in the base case a network consisting of 21 nodes (N = 21), i.e. 20 counterparties and one CCP. Such N is large enough to produce a large variety of clearing networks, while at the same time keeping their number manageable. With $N = 21$ we have precisely 626 topologically different tree networks, with a number of GCMs between 2 and 20, and concentration levels in [0,1][15]. For each network we obtain statistics of cleared exposures and margin requirements, thus obtaining about 626 data points in the tiering-concentration space[16]. Further details on simulations (in particular, on the generation of the random *T*s) are in the Appendix. Appendix IV, in particular, presents some robustness checks on the effect of network size.

Exposures by GCMs are summarized in the Appendix.

4.1.1 CCP's total expected exposure

The total expected exposure of the CCP is defined as:

$$CTE = \mathrm{E}\left[\sum_{k\in\Gamma}\left|T_{CCP,k}{}^{*}\right|\right] \tag{6}$$

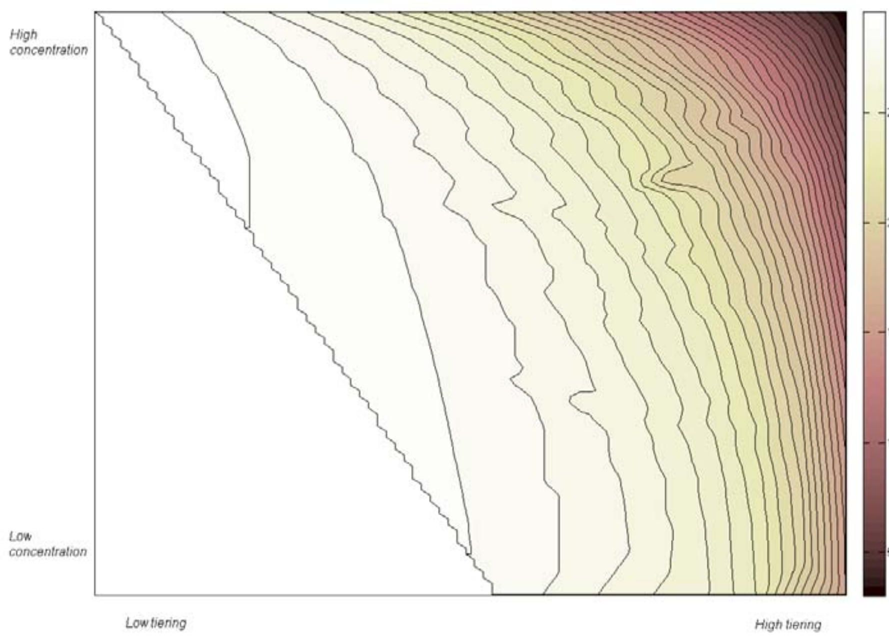
[14] It should be however noted that in practice, through contractual clauses, client exposure may actually be to CCP. This model abstracts from this and, coherently with the abstract definition of novation given above, assumes that exposures arise only between a party and its clearer agent.
[15] Note that low-tiering networks can only display a limited range of concentration levels: with e.g. 18 GCMs, there are only 2 clients, which cannot be uniformly distributed across the GCMs. For 2 to 20 GCMs, there are respectively 10, 33, 64, 84, 90, 82, 70, 54, 42, 30, 22, 15, 11, 7, 5, 3, 2, 1, 1 networks.
[16] As noted above, not all 626 different networks feature a unique tiering/concentration combination. So, we actually obtain fewer (i.e. 579) 'data points'.

where expectation is taken over realizations of T. Our first result is:

- *RESULT 1: tiering and concentration decrease CTE.*

This is illustrated by Figure 3. The contour plot shows *CTE* as a function of tiering (horizontal axis) and concentration (vertical axis). Darker shades correspond to lower *CTE*. The figure is ‘blank’ in the bottom left portion, because low-tiering networks can only display a very limited range of concentration levels[17].

Figure 3: CCP’s total expected exposure



The reason behind Result 1 is that that tiering and concentration both increase ‘internalization’ and ‘netting’. The formal argument is presented in the Appendix.

4.1.2 CCP’s single expected exposure

We now consider the expected exposure of the CCP against a single counterparty, averaged across realizations of Ts (days) and across GCMs. That is, the exposure of the previous section divided by the number of GCMs:

$$CSE = \frac{CTE}{\mathrm{V}} \tag{7}$$

We have the following:

- *RESULT 2: tiering increases and concentration decreases CSE.*

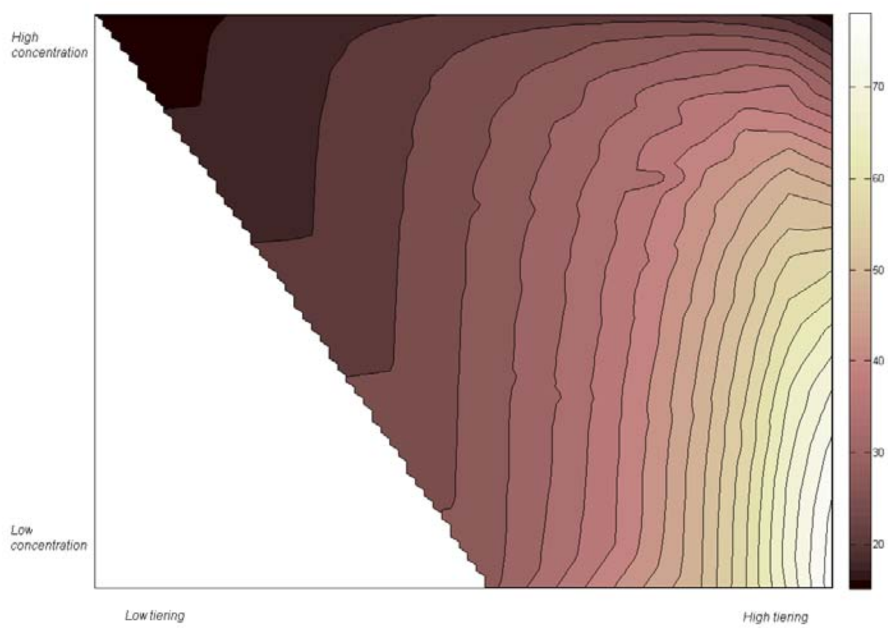
This is illustrated by Figure 4, where again darker shades correspond to lower *CSE*, and the figure is ‘blank’ in the bottom left portion.

[17] Eg, if there are only 2 clients and several GCMs, we can only observe two concentration levels: either i) both clients are assigned to the same GCM or ii) they are assigned to two GCPs.

The second part of *RESULT 2* immediately follows from *RESULT 1*: concentration reduces *CTE*, so it clearly reduces $CSE = CTE/NC$ (note that V, the number of GCMs, is fixed when varying concentration).

The effect of tiering on CSE is less trivial. A tiered network has *few* and hence *large* GCMs, capable of creating large exposures, instead of many small ones as a non-tiered one. However, large GCMs also internalize more trades, so it is not entirely trivial that the total effect on *CSE* is positive. A more precise argument is again given in the Appendix.

Figure 4: CCP's single expected exposure



4.1.3 CCP's extreme exposures

Sections 4.1.1 and 4.1.2 looked at *average* exposures, but more significant risks may reside in *extreme* exposures. We consider 'extreme' exposures in two senses: extreme across realizations of *T* and across GCMs.

Largest exposure across GCMs

The largest exposure of the CCP is defined as

$$CME = \max_{k \in \Gamma} \{T_{CCP,k}{}^{*}\} \tag{8}$$

An analytical expression for *CME* seems very difficult to reach. Indeed, *CME* is the last order statistic of a vector of non-independent, different random variables[18].

[18] To see non-independence, recall that a CCP-cleared trade comprises two symmetric legs with two different GCMs. These legs make CCP exposures correlated. Also, GCMs may have a different number of clients. So, their exposures with the CCP are random draws from different distributions.

Intuition, as often for order statistics, does not go far. For example, the previous section showed that concentration increases the single *average* CCP exposure. So, it is tempting to conclude that also the single *maximum* exposure should rise with concentration. However, this turns out to be a fallacy: as shown later, *CME* actually falls with concentration, both in expected value and in other statistics. Intuitive explanations on the effects of tiering are equally riddled with pitfalls.

So, we just report the results of our Monte Carlo simulations, without attempting any 'explanation' or formal proof.

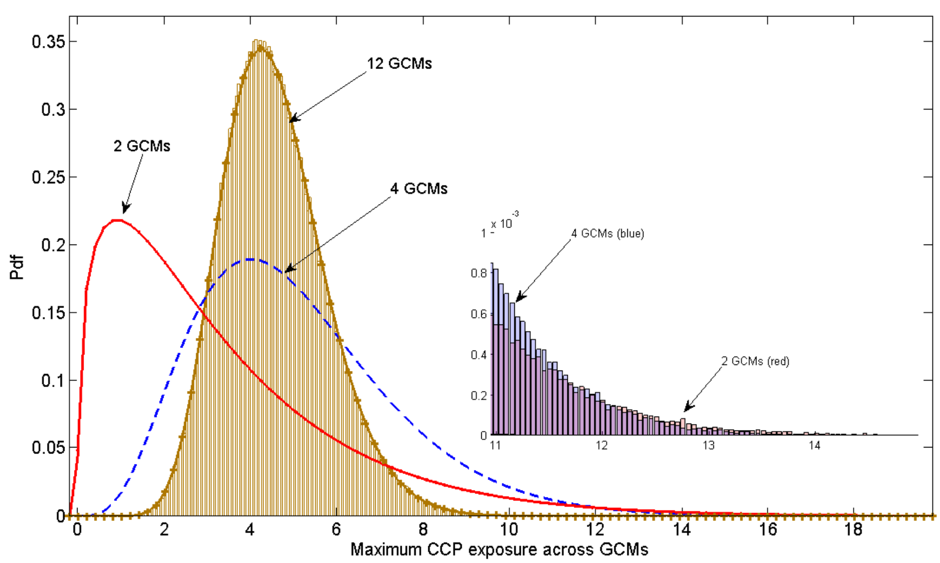
Figure 5 shows the distribution of *CME* for three levels of tiering[19]. The inset shows the detail of the right tail (with red and blue histograms corresponding to red and blue lines in the main picture, ie to 2 and 4 GCMs). We do not have analytical results for the shape of such distribution but, interestingly, the Gamma-distribution yields an extremely good fit.

The content of Figure 5 is summarized as

- *RESULT 3: tiering i) decreases CME's expected value, ii) increases CME's variance, iii) increases the likelihood of extreme CMEs[20].*

In other words, de-tiering on average *increases* the CCP's largest exposure. However, it makes it more predictable, and reduces the chances of it being very large.

Figure 5: CCP largest exposure (tiering)

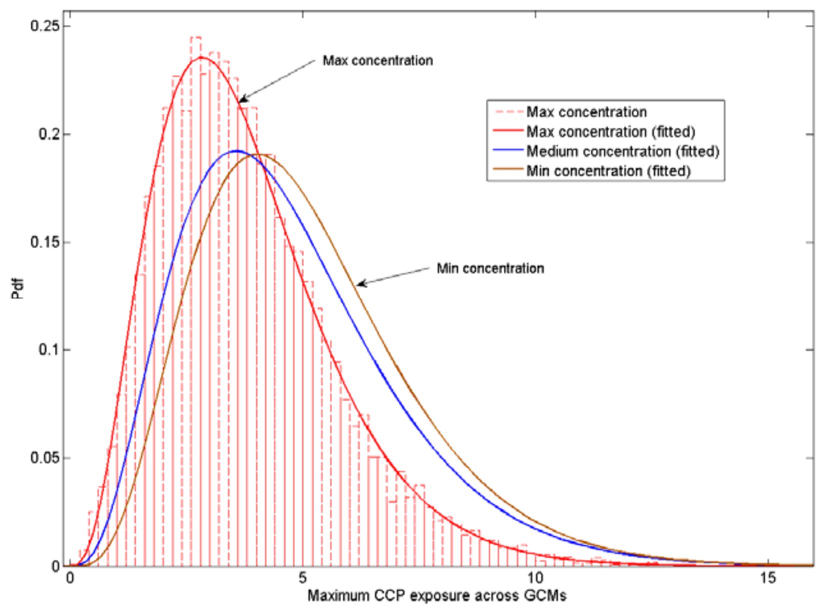


[19] For ease of computation, we obtained Figure 5 for N = 13. The concentration level is kept constant to zero, so each GCM has 0, 2 or 5 clients, depending on whether there are 12, 4 or 2 GCMs. Our results are robust to changes in N (see Appendix).

[20] For this latest fact, an intuitive explanation does seem possible. Suppose the original exposures come from a distribution with finite support. Then, the maximum CCP-GCM exposure is finite and is an increasing function of the size of the GCM.

Figure 6 looks at the effects of concentration. Here, the probability density functions correspond to three systems with four GCMs each, and clients distributed as follows: maximum concentration (8, 0, 0, 0), medium concentration (5, 2, 1, 0) and minimum concentration (2, 2, 2, 2). Concentration appears to decrease the expected value of the maximum exposure, to reduce its variance[21], and to makes the right tail of the distribution thinner.

Figure 6: CCP largest exposure (concentration)



We summarize the content of Figure 6 as:

- *RESULT 4: concentration i) decreases CME's average, ii) decreases CME's variance, and reduces the likelihood of extreme CMEs.*

In synthesis, concentration reduces the risks stemming from the CCP largest exposure across GCMs. This result may be driven by netting which increases as concentration rises (as more business is cleared at large GCMs)

Extreme-event exposures

Sections 4.1.1 and 4.1.2 studied *average* CCP exposures. To investigate extreme market events, we look at the whole *distribution* of CCP's exposures, concentrating on its tail.

[21] This is not very visible from the pictures.

As we have one such pdf for each tiering and concentration combination, we cannot graphically illustrate them all. Figure 7 thus plots the pdf of CTE (defined in Eq. (6)) for a given concentration level, and four different tiering levels[22].

Figure 7: Total CCP exposure (*CTE*) – distribution across realizations of T (days)

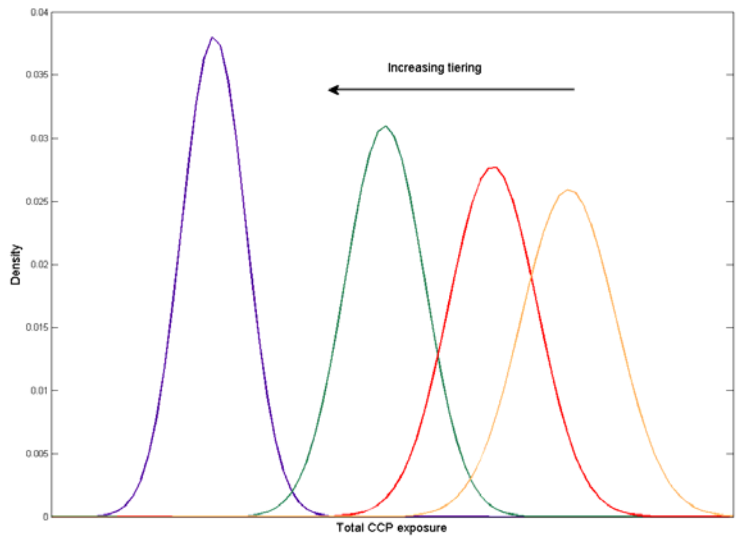


Figure 7 reveals that tiering reduces expected total CCP exposure, and may be summarized as follows

- *RESULT 5: tiering reduces the likelihood of large total CCP exposures. It also reduces the variance of the total CCP exposure.*

Figure 7 also confirms a finding of Figure 3, i.e. that average CTE falls with higher tiering. Similar results arise when keeping tiering constant while increasing concentration: concentration reduces expected total exposure for the CCP.

4.2 Margins

We now move from exposures to margin needs. For both cases (ie with and without re-hypothecation), we consider margin needs under average and extreme market circumstances.

These results may shed light on whether (and which) clearing networks may produce liquidity stresses on their participants.

[22] In Figure 7 concentration equals to zero, ie all GCMs have the same size.

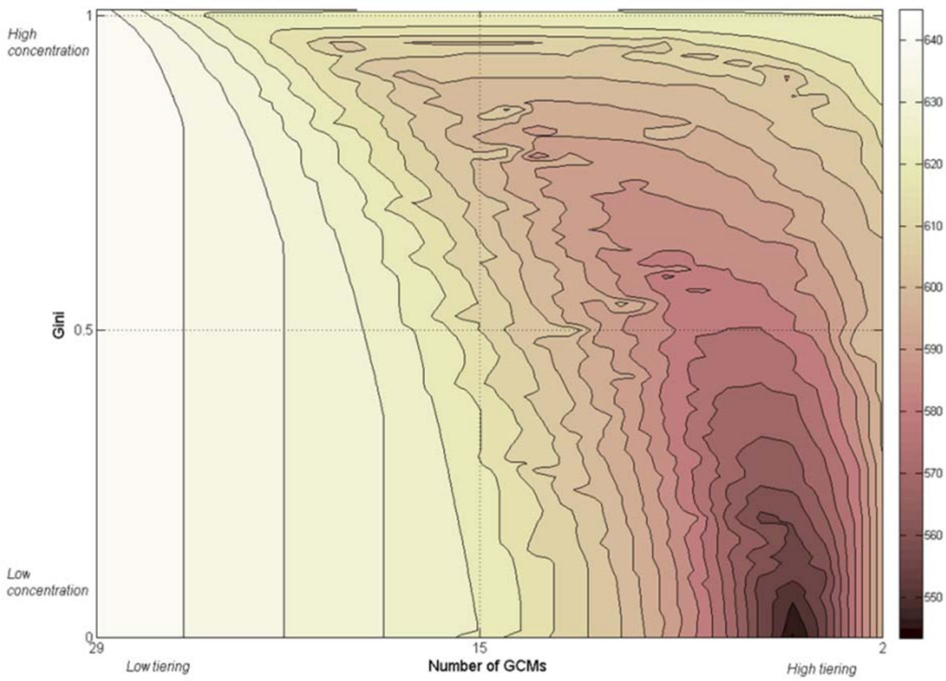
4.2.1 Expected margin needs - re-hypothecation

Here we study the function μR (Eq. 5). Figure 8 is a contour plot of $\mathrm{E}[\mu R]$ against tiering and concentration – where the expected value is taken as the average across realisations of *T* (). The content of Figure 8 is summarized as follows:

- *Tiering first decreases, then increases margin needs*.

The effects of concentration are less clear-cut but, in most of the cases, margin needs increase with it.

Figure 8: Expected margin needs - re-hypothecation



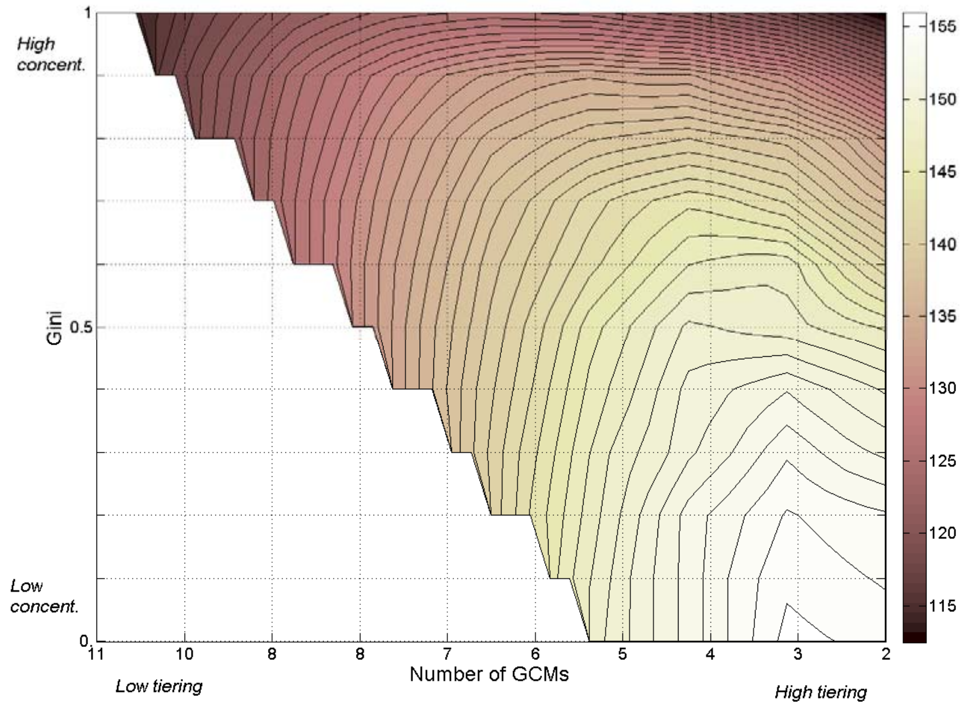
A heuristic explanation of this pattern is given in the Appendix.

4.2.2 Expected margin needs – no re-hypothecation

Margin needs in this case equal the sum of all margins paid in the network, i.e. the sum of all cleared exposures T^* (Eq. 5'). Figure 9 plots $\mathrm{E}[\mu NR]$, where the expected value is taken across realizations of *T*, against tiering and concentration. Its content is reassumed as:

- *Without re-hypothecation, average margin needs depend non-monotonically on tiering and decrease with concentration.*

Figure 9: Expected margin needs – no re-hypothecation



Without giving rigorous proof, the Appendix explains the pattern of Figure 9.

Predictably, margin needs are higher when re-hypothecation is not allowed. The size of this saving depends on the model's parameters[23]. However, such savings are higher when tiering is high, and when concentration is low. Indeed, in these cases it turns out that large GCMs receive margin payments in excess of what they pay to the CCP. This entails a 'waste' of margins in terms of recycling, as part of the margins paid by clients are held by the GCMs and not recycled.

Our main results on average margin needs are summarized as follows:

- *RESULT 6: the effects of tiering and concentration on margin needs can be different depending on whether re-hypothecation is allowed or not. If re-hypothecation is not allowed, concentration may help save on margin needs. On the other hand, re-hypothecation is most powerful as a margin-saving device when tiering is high and concentration is low.*

4.2.3 Extreme margin needs

Having looked at 'average events' we now consider margin needs in extreme circumstances. Figure 10 shows how tiering and concentration affect the 95th percentile of margin needs, when re-hypothecation is not allowed.

[23] In particular, on *T*'s distribution.

Figure 10: Margin needs (extreme events)

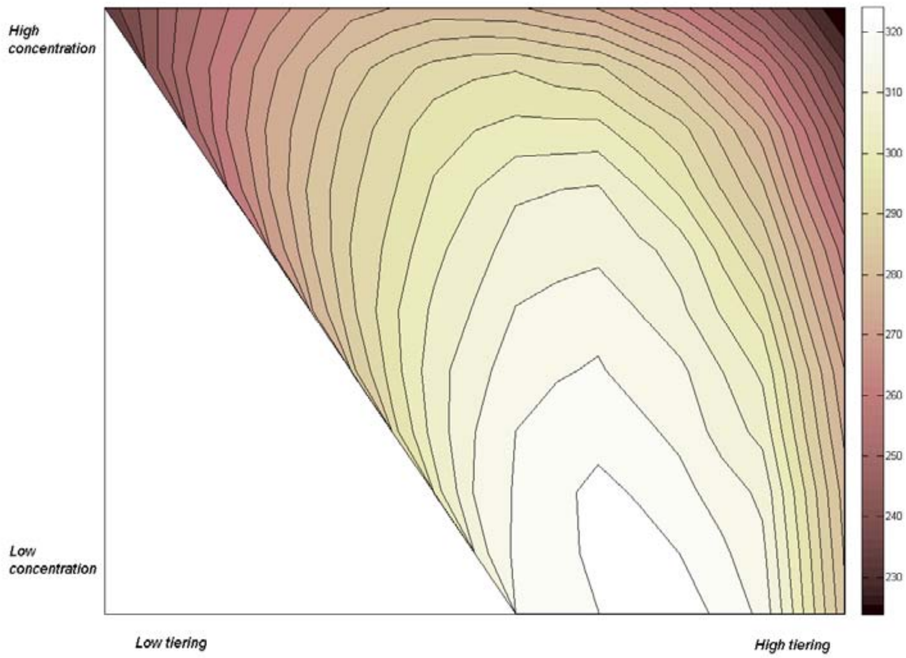


Figure 10 is very similar to Figure 9. This is probably not surprising, as both figures are about the same random variable: Figure 10 represents the 95% quantile, Figure 9 the average.

In our model, margin needs without re-hypothecation equal the sum of all exposures in the network. So Figure 10 also shows the total amount of resources needed, if each *i* is required to provide margin covering 100% of its exposures in 95% of the realizations of *T*. Figure 10 shows:

- *RESULT 7:*
  - *i) systems with low concentration may face particularly large initial margin needs;*
  - *ii) in systems with an intermediate level of concentration, initial margin needs depend non-monotonically with tiering: as this increases, they first increase and then decrease.*

5 Conclusions

This paper models clearing systems as networks whose function is to transform bilateral exposures. By looking at the mechanics of such networks, we study how their topology affects the resulting exposures between CCPs and their members, and the margin requirements for both members and their clients.

Our results on CCP **exposures** are summarized in the following table.

CCP exposure against…	Dependence on…	
	…tiering	…concentration
… all GCMs (total)		
▪ Average	*negative*	*negative*
▪ Likelihood of extreme exposures	*negative*	*negative*
… average GCM	*positive*	*negative*
… GCM with largest exposures		
▪ Average over realizations	*negative*	*negative*
▪ Likelihood of extreme exposures	*positive*	*negative*

Table 2: Relationship between exposures and network characteristics

Our results suggest that tiering has a complex effect on the risks faced by the CCP. On one hand, higher tiering *reduces* the CCP's total exposure (its expected value, variance, and likelihood of extreme realizations). On the other hand, higher tiering *increases* the CCP's expected single exposure towards the average GCM.

The 2012 CPSS-IOSCO Principles for Financial Market Infrastructures require CCPs to "maintain (…) financial resources sufficient to cover (…) the default of the participant and its affiliates that would (…) cause the largest aggregate credit exposure (…) in extreme but plausible market conditions"[24].

We find that higher tiering reduces such largest exposure on average, but increases the probability of it being very large. Moreover, tiering increases the variance of the CCP's largest exposure, making it less predictable.

The effects of concentration on the CCP are even more clear-cut. Concentration decreases risks for the CCP in all exposure-related aspects: it decreases the average single exposure, the total exposure, the largest expected exposure, the variance of such exposures and the skewness of their distribution (the right tail). This suggests

[24] This cover is elevated to the *two* largest participants for a CCP "involved in activities with a more-complex risk profile or that is systemically important in multiple jurisdictions".

that, for a given total volume being cleared, a CCP faces reduced risks (in terms of a simple measure of counterparty credit exposure) when this volume is *unequally* divided by GCMs, i.e. when some GCMs are 'large', and others are 'small'. This is because more risk is 'internalized' in the GCM as more offsetting client positions are netted out of exposures between GCM and CCP.

This paper also offer insights into the costs of central clearing, measured by the **amount of margin** (i.e. collateral) needed for it, which is important from both an 'efficiency' and a 'risk' perspective. High margin requirements can put excessive strain on participants; similarly, if a network can operate safely with lower margin needs, the freed up collateral can be employed elsewhere.

Our results on margins are summarized in the following table:

	Tiering	**Concentration**
Systemic margins requirements		
Re-hypothecation	*neg / pos*	*positive*
No re-hypothecation	*pos / neg*	*negative*

Table 3: Sign of the relationship between margin needs and network characteristics

The relationship between the network topology and margin needs turns out to depend crucially on whether re-hypothecation is allowed or not. An obvious result is that a network requires more margins without re-hypothecation. Less obvious findings are that the relationship between tiering and margins is non-monotonic, and it is inverted when re-hypothecation is introduced. In a network with re-hypothecation, concentration increases margin requirements, while tiering first decreases, and then increases them. The opposite is true if re-hypothecation is not allowed. Concentration reduces margin requirements while tiering first increases margin requirements, and then decreases them.

In our last experiment, we assume that each participant is required to hold enough liquid margin resources to cover extreme distributions of trades[25], and look at how this prudential buffer depends on tiering and concentration. Concentration is found to have a beneficial effect, decreasing the size of the required buffer. Tiering has a non-monotonic effect: the needed buffer is lowest for extreme tiering levels, and reaches a maximum for intermediate levels.

All our results depend on the assumptions made. A key assumption is the single account hypothesis. In the model each GCM holds a unique account at the CCP, where proprietary and client trades are pooled together.

A second important assumption is the statistical independence of pre-clearing exposures. This could be easily removed in our simulations, but it would be difficult to decide *how* this assumption should be removed. If traders are not ex-ante

[25] Ie to cover for 100% of its exposures, in 95% of the days (realizations of T).

identical, then the way trades are distributed among GCMs would be expected to influence the results. In other words, tiering and concentration would be insufficient to describe a clearing network, and we could not express our results in terms of these two simple dimensions.

A third important assumption is that margins are a simple (linear) function of exposures.

Last but not least, we have assumed that bilateral exposures do not depend on the clearing network (C). In reality this may not be true. For example: a clearing network which is very liquidity intensive due to its particular topology could discourage trading in the first place (as clearing would be expensive). In turn, this could reduce the pre-clearing bilateral exposures.

Appendix

I Generation of random Ts

We draw our instances of *T* in the simplest way: all $T_{ij}: i \neq j$ are iid random draws from a Normal(0,1).

This seems a reasonable assumption for exposures in a centrally cleared market, where traders do not distinguish between counterparties, and indeed, trading platforms ensure anonymity and 'fungibility' of contracts.

This assumption is more questionable for an OTC market, where preferential trading partnerships may emerge. These would give rise to e.g. sparse matrices *T*, or at least some asymmetry in the trading exposures T_{ij}.

II GCM Exposures

GCMs can differ in the number of their clients. So, for given tiering and concentration levels, it is impossible to represent in a compact way the cleared exposures of *all* GCMs. However, cleared exposures of a single GCM have a simple analytical form.

<u>II.1 Cleared exposures towards the CCP</u>

Consider a GCM *i*, with a group of *S* members. It is easy to see that $\|NI_i\| = NS - S^2$. Hence, *i*'s exposure to the CCP is the sum of $NS - S^2 = \sigma$ normal random variables, that is:

$$T_{i,CCP}{}^{*} \sim N(0, \sigma).$$

It can immediately be verified that the expected *absolute value* of $T_{i,CCP}$ scales with $\sigma = NS - S^2$. So, it first increases, then decreases in S.

Intuitively, the reason is the following. At first, acquiring clients creates more non-internalized exposures, so *i*'s exposure to the CCP grows. But after a while internalization kicks in; $\|NI_i\|$ starts to fall, and so does *i*'s exposure.

<u>II.2 Cleared exposures towards clients</u>

Consider a GCM *i.* Its cleared exposure to a client *j* is a random variable

$$T_{i,client}{}^{*} \sim N(0, N-1)$$

as, indeed, it is the sum of all the exposures inherited from *i*.

More difficult to determine is *i*'s largest exposure across clients. Formally, if *i* has *G* clients, its largest exposure is the 1st order statistic of a vector of *G* normally distributed random variables, which may be correlated. Indeed, if both *j* and *k* clear at *i*, the exposures (*i*,*j*) and (*i*,*k*) are sums of normal r.v.s with a term in common, albeit of opposite sign, i.e. the exposure (*j*,*k*).

Such correlation makes it very difficult to analytically compute the distribution of the maximum exposure. Monte Carlo simulations however show that, as a GCM acquires clients, the *expected value* of its largest exposure grows, but the *variance* of such exposure falls. That is: large GCMs are exposed to larger but more predictable risks.

III Proofs and explanations of some results

To simplify notation, given a set of exposures *A*, we adopt the following shorthand:

$$\sum_{T_{rs}\in A} T_{rs} = \Sigma T_A \qquad \text{(A0)}$$

III.1 Proof of RESULT 1: Tiering and concentration decrease CTETo prove that higher tiering decreases *CTE*, consider a network *C* with non-internalized payments NI. Substituting (3) into (6), and recalling the shorthand defined in (A0) we obtain:

$$CTE(C) = E\left[\sum_{k\in\Gamma^c} abs(\Sigma NI_k)\right] == E\left[abs(\Sigma NI_r) + \sum_{t\neq i,k} abs(\Sigma NI_k)\right] \qquad \text{(A1)}$$

Now **decrease** tiering by 'promoting' a client $i \in g_r$ as a GCM. The non-internalized exposures in the new network are now $NI'_i = \{T_{ij}\}_{j=1..N}$, $NI'_r = NI_r \backslash \{T_{ij}\}_{j\notin g_r}$ and $NI'_t = NI_t$ for $t \neq i,k$ and so:

$$CTE(C') = E\left[abs(\Sigma NI'_i) + abs(\Sigma NI'_r) + \sum_{t\neq i,k} abs(\Sigma NI'_t)\right] \qquad \text{(A2)}$$

The last term is common to both (A1) and (A2), so let's compare the different terms

$A = abs(\Sigma NI_r)$ and $B = abs(\Sigma NI'_i) + abs(\Sigma NI'_r)$. The following diagram makes clear that the sums in *B* contain, together, all the terms in *A*, plus some others (to be precise, those $T_{i,j}: j \in g_r$). Thus, because $abs(x) + abs(y) \geq abs(x+y)$ we have $B \geq A$. From which $CTE(C') \geq CTE(C)$.

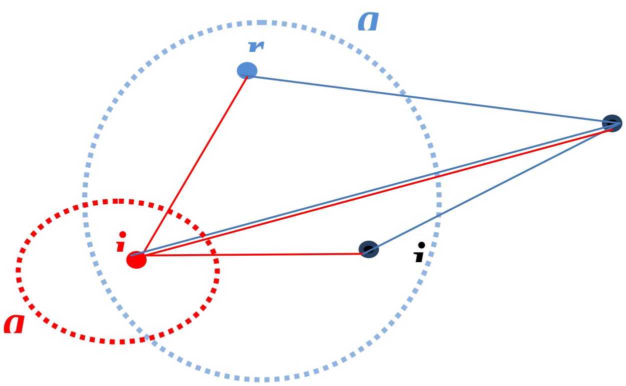


Figure 11 – Non-internalized trades when *i* becomes a GCM

Let us now prove that concentration decreases *CTE*. Start from a network *C* and **increase** concentration by moving a client *i* from a correspondent *k* to a correspondent q with an equal or larger group of clients. It is obvious that $|NI'| \leq |NI|$ ie the number of exposures contributing to $CTE(C')$ is no larger than the number of those contributing to $CTE(C)$. Hence, as expectation is taken, $CTE(C') \leq CTE(C)$. ■

III.2 Explanation of RESULT 2: Tiering increases CSE

To simplify, we instead show that tiering increases the ratio NI/V, ie non-internalized trades per GCM. To see this, consider then a network where the *N* participants (ignore the CCP) are equally split across two GCMs. Then, $NI = s(N) = (N/2)^2$. Increase the number of GCMs to four, splitting each group in two equal-sized groups. This gives $s(N/2) = (N/4)^2$ new cross-group exposures which, summed to the previous ones, gives a total of $2(N/4)^2$ non-internalized exposures. Repeating this process produces a geometric series whose kth term is $\mathrm{N}^2/2^{\mathrm{k+1}}$. So NI increases with V, but at a decreasing rate; hence NI/V falls with V.

III.3 Explanation of Figure 8: With re-hypothecation, tiering first decreases, then increases systemic margin needs

To intuitively see this, recall that

$$\mu R = \sum_{i \in \Gamma} max\{m_i, \Sigma_{k \in g_i \setminus i} m_k\} \tag{A3}$$

When tiering is low, groups are small. Hence for each correspondent *i* we should expect $m_i > \Sigma_{k \in g_i \setminus i} m_k$ so $\mu R = \sum_{i \in \Gamma} m_i$. As tiering decreases, members move out of the top tier, so the summation in μR loses terms and total margin needs fall. In the meantime, some GCMs gain clients so, at certain point, $m_i = \Sigma_{k \in g_i \setminus i} m_k$ for some GCM *i*. At that point, as new clients join *i*, $max\{m_i, \Sigma_{k \in g_i \setminus i} m_k\}$ start to grow. That is, a term of the summation in μR starts to grow, increasing μR.

III.4 Explanation of Figure 9: Without re-hypothecation, margin needs a) decrease with concentration, and b) depend non-monotonically on tiering.

The reason behind a) is the following. The margin needs of clients are unaffected by *C*. Hence, to assess the effect of concentration on margins we only need to consider the effect of concentration on the margins paid by the GCMs. These equal (a multiple of) the total exposure of the CCP. Figure 3 and the discussion therein showed that the CCP total exposure decreases with concentration. Hence, so does the aggregate GCMs margin need, and the systemic margin need.

The reason behind b) is the following. As tiering increases, the total margins paid by the GCMs fall (see previous paragraph). On the other hand, the bottom tier becomes larger, so more margins are paid by the bottom tier to the GCMs. So, there are two countervailing effects on total margin requirements. It is not surprising that the tiering-margin relationship is non-monotonic, as one effect or the other prevails at opposite extremes of tiering.

IV Robustness check

Most numerical results are derived simulating a model with N=21. Such size allows a wide range of tiering and concentration levels, while at the same time keeping the model numerically approachable.

Replicating the analysis for various Ns suggests that our results are robust to changes in N. The following pictures illustrate large exposures for larger systems (N=51 and N=101). The substance of our results is clearly unaffected. The same has been verified for the results on margins and average exposures.

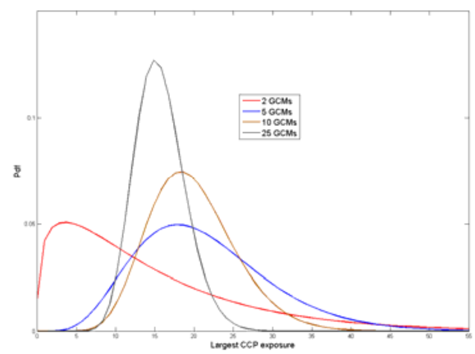


Figure 12 - Largest CCP exposure - 50 members (compare with Fig. 5)

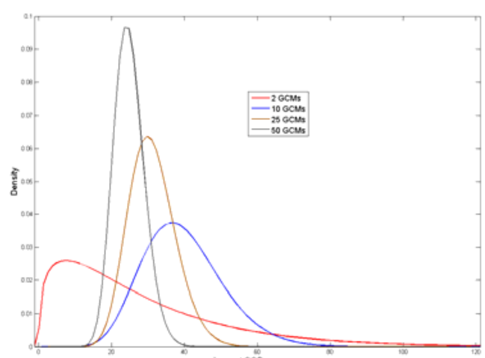


Figure 13 - Largest CCP exposure - 100 members (compare with previous)

Song et al. (2012), in a work still preliminary when this paper was written, give the following analytical expression for the CME's limiting distribution:

$$\lim_{L\to\infty} p(CME \geq L) \sim e^{-\beta L^2}$$

where

$$\beta = 2/(N^2 - 4|g^*|)$$

and $|g^*|$ is the largest GCM's group size. This result states that the probability of observing very large CMEs decays faster, when the 'top' GCM is larger. If confirmed, this would actually imply that extremely large CMEs are less likely in highly tiered systems. In Fig. 5 and 12, this would mean that the red curves eventually fall below all other curves. We could not observe this reversal in none of the systems we simulated; we estimate that, if this reversal really does take place, it can only be happening at events whose probability is smaller than 10−7 (for CMEs happening say 1 day every 30′000 years). Thus we conclude that, if our Result 3 admits exceptions, these are relevant only from a theoretical point of view.

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