



BANK OF ENGLAND

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The pitfalls of speed-limit interest rate rules at the zero lower bound

Charles Brendon,⁽¹⁾ Matthias Paustian⁽²⁾ and Tony Yates⁽³⁾

Abstract

We show that interest rate rules that feed back on the growth rates of target variables (such as output or asset prices) may induce recessions in the presence of a zero lower bound, through purely self-fulfilling dynamics. This pathology is illustrated in a small New Keynesian model with interest rates responding to the growth rate of output, and in a version of a model by Matteo Iacoviello where interest rates respond to the growth rate of house prices and credit. Our results provide a cautionary note, contrasting with previous work which has suggested several desirable properties of speed-limit rules, namely that they are devices enabling the policymaker (i) to side-step uncertainty about natural rates, (ii) to counter booms and busts in asset prices or (iii) to implement optimal commitment policies.

Key words: Speed-limit rules, commitment, zero lower bound, self-fulfilling prophecies.

JEL classification: E52, E58, E61.

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Summary

This paper looks at how well ‘speed-limit’ rules for setting central bank interest rates do at stabilising the economy when we consider the possibility that from time to time interest rates may get trapped at their natural floor of zero (the ‘zero bound’). A more common approach taken by researchers is to study interest rate setting procedures such as Taylor rules, where the interest rate is (for example) raised if inflation exceeds target, or the output gap is positive. A speed-limit rule, by contrast, is a rule where it is decided how far to raise rates based not on the *level*, but the rate of change of some concept like the output gap. Interest in speed-limit rules stemmed from two lines of thought, both of which ignored the zero bound as a factor for policy, (largely because at the time this work was done rates were so far above the zero bound that it was considered highly unlikely they would ever get there). One was that speed-limit rules seemed to provide a way to insulate central banks from policy errors that occurred through mis-measurement of key concepts like the output gap (the difference between actual and potential output); it was easier to measure the rate of change than the level. Another was that speed-limit rules were shown to be a way for central banks to implement what the academic literature has termed ‘optimal commitment policies’. These are policies that stabilise inflation and the output gap (or whatever society cares about) as well as possible, and by using the power of inflation expectations to anchor inflation, through making commitments not to simply think afresh as each period and each new shock to the economy comes along.

Our paper provides a cautionary note to those contemplating speed-limit rules, to weigh against these benefits. We find that there is a chance that rates could end up pinned at the zero bound through self-fulfilling expectations of low inflation, even if there were no fundamental shocks depressing the economy. Normally, in models of rational expectations like ours, if rates followed a Taylor rule with interest rates sufficiently responsive to inflation, and the zero bound were not in play, self-fulfilling recessions would be ruled out. Anyone who contemplated the possibility of future low inflation would recognise that this would itself drive inflation down (through the Phillips curve, the relationship determining inflation which includes a large role for expectations). That alone would prompt a sharp cut in rates, and one that would not be reversed until the output gap that was opened up by the lower inflation was closed. However, under a speed-limit rule, and faced with the zero bound, agents in the economy would correctly surmise that things will be different. First, rates cannot fall so far to begin with to counter the fall in inflation. And second, agents would forecast that after the initial fall in inflation and opening up of the output gap, the central bank would tighten more quickly. This is because it would be concerned to make sure the output gap does not close too quickly (given its concern for the ‘speed limit’). This means people forecast tighter policy tomorrow, which validates the initial forecast of low inflation. Inflation and the output gap fall, and interest rates are pushed to the zero bound, simply because agents in the economy believe it will. This problem of self-fulfilling attacks at the zero bound also afflicts policy rules that involve terms in the rate of change in house prices, in a New Keynesian model modified to include housing.

1 Introduction

Following the financial crisis there has been a rapid expansion in the academic literature investigating the policy implications of the zero lower bound on the nominal interest rate. Simultaneously, a debate has emerged in policy circles about whether monetary policymakers paid sufficient attention to asset-price growth and credit expansion in the years prior to the crisis, and whether these variables should now be explicitly incorporated into policy objectives.¹ This paper uses insights from the former literature to sound a cautionary note in the latter debate. We show that if interest rates are constrained by the zero lower bound and monetary policy feeds back on the growth rates of target variables – such as credit or asset-price growth – then the economy may become exposed to purely self-fulfilling crises of confidence, resulting in large output losses without any ‘fundamental’ cause.

The basic intuition behind the mechanism that we highlight runs as follows. A collapse in the value of a target variable today implies it will grow in the future, as part of a subsequent recovery. But if policy is scheduled to ‘lean against’ this growth then any current collapse will imply expectations that policy in the future will be relatively tight. This in turn places downwards pressure on current inflation expectations, raising the real interest rate. If nominal rates are constrained at the zero bound, these higher real rates cannot be resisted by the central bank, and may themselves be sufficient to support the initial collapse. Interestingly, we show that this outcome can obtain in a simple dynamic New Keynesian model without capital, in which the central bank’s policy rule feeds back on the growth rate of output. Though this model is clearly not a good approximation to real-world economies, it provides a useful setting for clarifying the interactions that we wish to highlight. We then make use of the model due to Iacoviello (2005), which has been applied in a similar context recently by Mertens and Ravn (2011). This model features a real-estate sector influenced by collateralised borrowing, so contains variables corresponding to asset prices and the quantity of credit; we show that similar conclusions to the simple dynamic New Keynesian case can be reached when policy feeds back on asset-price growth or the growth rate of credit.

The main lesson we take from our exercise is that care should be taken in targeting the growth rate of any economic variable (aside from the price level) via a simple policy rule. In the context of the zero bound even rules that appear benign may have detrimental consequences. These results provide a counterweight to previous work that has suggested that interest rate rules that feed back from rates of changes may have desirable properties. In this regard, we recall first that it has been shown

¹Bloxham, Kent, and Robson (2011) provide a useful survey of this debate.



that interest rate rules that feed back from rates of change of certain variables can deliver outcomes equivalent to those under the optimal commitment policy when the zero bound is not considered. McCallum and Nelson (2004) and Stracca (2007) show this for rules involving terms in the change in the output gap. Giannoni and Woodford (2003) establish the same result, though for the case in which the objective function for policymakers is taken to include a term in the change in interest rates. Leduc and Natal (2011) show that a rule that feeds back from the rate of change in asset prices approximates the outcomes under optimal policy in a model with time-varying spreads. Finally, Blake (2012) demonstrates how to design speed limit rules that implement optimal commitment policy in the canonical New Keynesian model.

Second, Orphanides and Williams (2002) showed that interest rate rules involving terms in the change in real quantities provided immunity against mismeasuring natural rate concepts that were required to operationalise conventional Taylor rules that are informed by gaps between the level of output/unemployment relative to their natural rates. They further emphasise the benefits of such rules by noting that in the absence of knowledge about just how much uncertainty there is, it is better to err on the side of assuming that there is more.

Third, there is evidence that speed-limit rules can characterise central bank policy at times. Walsh (2003) quotes the FOMC minutes in 2000, citing evidence in support of decisions to raise rates: ‘*The [Federal Open Market] Committee remains concerned that over time, increases in demand will continue to exceed the growth in potential supply*’; and in May that year: ‘*Increases in demand have remained in excess of even the rapid pace of productivity-driven gains in potential supply*’. Some econometric evidence is provided by Mehra (2002) and Paez-Farrell (2009), who show that speed-limit rules do a reasonable job of explaining past behaviour in central bank interest rates.

Turning to the wider theoretical literature on the zero lower bound, a number of recent papers, including DelNegro, Eggertson, Ferrero, and Kiyotaki (2010) and Corsetti, Kuester, Meier, and Muller (2010), consider the implications of this bound in economies characterised by financial frictions. As in the important analysis by Woodford (2011) and Christiano, Eichenbaum, and Rebelo (2011) (both of which focus on the effects of fiscal policy when interest rates are constrained), the approach of these papers is to treat the zero bound as a restriction on fluctuations that occur *in the neighbourhood of a steady state that involves a non-zero policy rate*. The theoretical exercise is to assume shocks have occurred of a sufficient magnitude that in a fully linear system, absent any inequality constraints, the policy response would be to set a negative nominal interest rate. The focus of the analysis is to establish outcomes in the event that this is not possible. The methodological approach



commonly used owes much to the work of Eggertsson and Woodford (2003), who first considered the problem of setting optimal policy when the zero lower bound occasionally constrained outcomes.

An alternative approach to analysing the zero lower bound derives from the contribution of Benhabib, Schmitt-Grohe, and Uribe (2002). This branch of the literature emphasises the distinction between local and global inflation determinacy when the policymaker follows a Taylor rule or similar deterministic policy strategy.² The key point is that the Taylor feedback strategy inducing determinacy in the region of the ‘desirable’ steady state can no longer be pursued once it implies a negative policy rate. This fact induces a kink in the policymaker’s reaction function, which in turn permits a second, ‘undesirable’ steady state to persist, characterised by a fixed zero nominal rate, deflation and (usually) sub-optimally low output levels. Because the interest rate is fixed at zero in the neighbourhood of this steady state, the dynamics there will in general be indeterminate – in accordance with the well-known logic of Sargent and Wallace (1975). The analysis suggests that economies run the risk of becoming stuck in a deflationary steady state simply through a self-fulfilling emergence of deflationary expectations. Mertens and Ravn (2011) have extended this ‘global’ approach to studying the zero lower bound to a model with financial frictions, due originally to Iacoviello (2005). They suppose that outcomes in this economy are affected by random self-fulfilling bouts of pessimism, which cause the zero bound to bind, and the economy to move to the neighbourhood of the undesirable steady state – where it remains so long as the relevant non-fundamental (‘sunspot’) variable continues to give a pessimistic reading. They show that debt deflation and falls in collateral value, which occur in their model when pessimism arrives, magnify the falls in output associated with this ‘liquidity trap’ roughly fourfold relative to a model with no financial frictions.

Our analysis can be seen as bridging the gap between these two approaches. Like Benhabib et al., and Mertens and Ravn subsequently, one of our main findings is that large falls in output *can* be driven by self-fulfilling dynamics alone. But like Woodford, Christiano et al., Del Negro et al. and the majority of other recent studies of the zero bound, we focus only on fluctuations along a perfect foresight path in the region of the ‘desirable’ steady state, i.e. one that satisfies the usual Blanchard-Kahn conditions for local determinacy. This allows us to illustrate an important theoretical possibility that has not been given much (if any) attention in the literature: namely that poorly-chosen policy rules may, in the presence of a zero bound, permit a finite number of initial values for the economy’s choice variables to be consistent with deterministic convergence to the *same* steady state in a rational

²We use ‘determinacy’ in the weak sense here: ‘unique non-explosiveness’.



expectations equilibrium.

This seems an important possibility to highlight, since it can occur without the coordinated expectational shift upon which the Benhabib et al. and Mertens and Ravn results depend. That is, we do not require that public expectations over economic outcomes in subsequent time periods should be modelled as stochastic variables;³ even with perfect foresight our self-fulfilling dynamics can be supported, and are consistent with convergence back to the ‘desirable’ steady state. The magnitudes and co-movements of the variables in the ‘crisis’ episodes that we observe are in fact very similar to those described in the work of Mertens and Ravn (2011). But in our paper it is the *known* future policy response to contemporary outcomes that keeps real interest rates high, not exogenous ‘pessimism’.

2 A simple model

Our analysis starts with a simple New Keynesian model with Calvo pricing and without capital. We do not view this model as an empirically well-fitting description of reality, but use it as the simplest framework in which to articulate our main points. The microfoundation and derivation of the model has been extensively documented in Woodford (2003). We refer the reader to the appendix for an overview of the non-linear system, and proceed with the model linearized around a zero-inflation steady state.

2.1 Linear Model

The model is given by the following equations where \hat{y}_t , $\hat{\pi}_t$, and \hat{i}_t denote period- t log-deviation from the zero inflation steady state for output, inflation and the (gross) nominal interest rate respectively, with $t \geq 0$:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} \left(\hat{i}_t - E_t \hat{\pi}_{t+1} \right) \quad (1)$$

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} \quad (2)$$

$$\hat{i}_t = \max \left\{ \hat{i}_t^s, \beta - 1 \right\} \quad (3)$$

$$\hat{i}_t^s = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t + \alpha_{\Delta y} (\hat{y}_t - \hat{y}_{t-1}) \quad (4)$$

³Mertens and Ravn (2011) do just this, with a Markov process to determine whether expectations are ‘pessimistic’ or ‘optimistic’.



Here, $\kappa > 0$ is the composite parameter giving the slope of the Phillips curve, $\sigma > 0$ is the inverse intertemporal elasticity of substitution and $0 < \beta < 1$ is the discount factor. The monetary authority sets the nominal rate equal to the shadow rate \widehat{i}_t^s given in equation (4) as long as this does not violate the lower bound. Otherwise it sets the rate to this bound. Here, $\beta - 1$ is the log deviation in $(1 + i_t)$ required to reduce the gross nominal rate from its steady state value of β^{-1} to the lower bound of 1. We consider values for the Taylor rule coefficients $\alpha_\pi, \alpha_y, \alpha_{\Delta y}$ that imply equilibrium determinacy in the linear model without the zero lower bound restriction.

In the absence of the zero bound we would have a straightforward linear rational expectations system, with one state variable: the lagged value of the output gap. The unique non-explosive equilibrium thus implies $\widehat{y}_t = \theta_y \widehat{y}_{t-1}$, $\widehat{\pi}_t = \theta_\pi \widehat{y}_{t-1}$ and $\widehat{i}_t = \theta_i \widehat{y}_{t-1}$, where the θ coefficients are functions of the model's parameters. There are no fundamental shocks in the model, so we are free to focus on perfect foresight solution paths.

Provided \widehat{y}_{-1} is sufficiently close to zero there will always exist a perfect foresight equilibrium of this model whereby inflation, the output gap and the nominal interest rate all converge uniformly to their steady-state values, and the zero bound never binds. If we additionally assume that $\widehat{y}_{-1} = 0$ this equilibrium simply involves the system remaining in perpetual rest at steady state. Our main point is that there exists another perfect-foresight path when $\widehat{y}_{-1} = 0$,⁴ whose dynamics are far more troubling. This can be demonstrated as follows. Suppose that the zero lower bound may be binding in period 0, but is known not to bind thereafter. Then the linear responses specified by the θ parameters will apply in period 1, and the system in period 0 can be expressed as:

$$\widehat{y}_0 = \frac{1}{\kappa + \beta\theta_\pi} \widehat{\pi}_0 \tag{5}$$

$$\widehat{y}_0 = \begin{cases} \frac{(1-\beta)}{\sigma(1-\theta_y)-\theta_\pi} & \text{if } \widehat{i}_0^s < \beta - 1 \\ \frac{1}{\sigma(1-\theta_y)+\alpha_y+\alpha_{\Delta y}-\theta_\pi} (-\alpha_\pi\pi_0 + \alpha_{\Delta y}y_{-1}) & \text{otherwise} \end{cases} \tag{6}$$

Equation (5) here gives an ‘aggregate supply’ relationship, derived directly from the New Keynesian Phillips curve. Here higher output implies higher marginal costs, and thus higher current inflation (θ_π is typically positive). Equation (6) gives ‘aggregate demand’, derived from the consumption Euler equation. Importantly, the zero lower bound imparts a non-linearity to this aggregate demand curve. In general lower

⁴Alternative starting values for \widehat{y}_{-1} would also be consistent with the ‘crisis’ dynamic.

levels of output are consistent in equilibrium with higher levels of inflation, since the direct policy feedback response to higher inflation is to raise the real interest rate. But when the zero bound binds current inflation may take an arbitrarily low value without the policymaker being able to induce *any* output response.

Depending on the policy feedback parameters this ‘kink’ can have very different consequences. Figure (1) plots the aggregate demand and aggregate supply curves for two different values of the output growth feedback parameter $\alpha_{\Delta y}$, given a baseline calibration of $\sigma = 1$, $\beta = 0.99$, $\theta = 0.85$, $\phi = 2$, $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}(\sigma + \phi)$, $\alpha_\pi = 1.5$, $\alpha_y = 0$, and $\hat{y}_{-1} = 0$. In these plots, we assume that the lower bound is not binding from period two onwards, which is verified ex-post. When growth feedback is low, low values of inflation are of greater importance in the policy rule than low values of output. This means the zero bound is reached at a point on the demand curve corresponding to low inflation and high output, as in panel (a). There is a unique equilibrium, with inflation and output staying at rest in steady state, and the zero bound is not binding either in period 0 or subsequently.

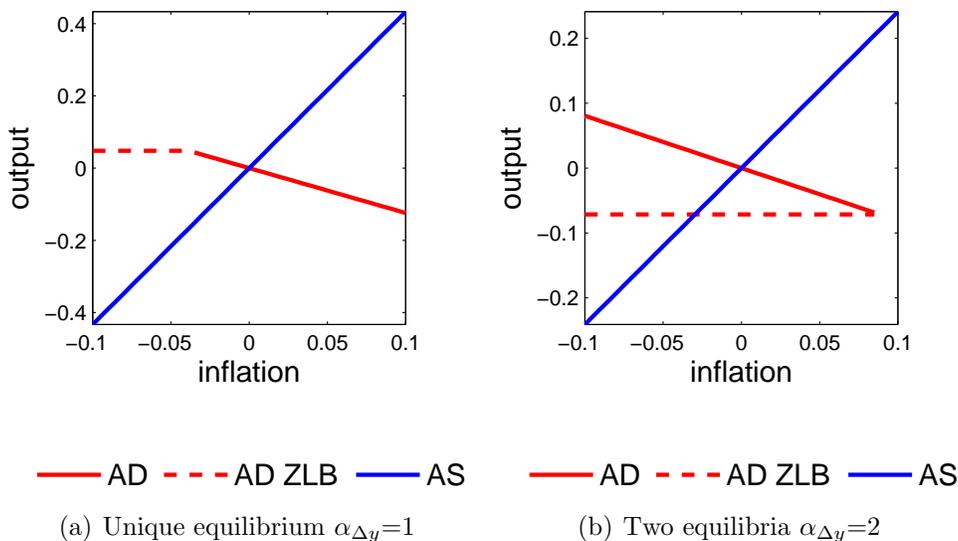


Figure 1: AD and AS curves for different values of $\alpha_{\Delta y}$

By contrast, when $\alpha_{\Delta y}$ is sufficiently large the policymaker’s relatively high concern for output means low values of the nominal interest rate arise when output is low (and inflation relatively high). The demand curve still slopes downwards, but now the cut in output that is induced as inflation increases itself results in expectations



that future output *growth* will be relatively high, as the economy returns to steady state. This in turn pulls down on current inflation expectations (via θ_π), since future output growth will induce relatively tight policy; it thus increases the real interest rate. This allows scope for nominal interest rates to be cut in the face of the inflation rise, whilst still seeing an increase in *real* interest rates. The zero bound thus binds for low values of output and high inflation, in contrast with the high-output, low-inflation case of panel (a). The consequences of this are illustrated by the changed position of the kink in panel (b). There are now two possible equilibria in period 0: rest at steady state, or a simultaneous collapse in both output and inflation. The latter is the ‘self-fulfilling crisis’ that we wish to highlight.

Figure 2 charts the dynamics of the crisis equilibrium.

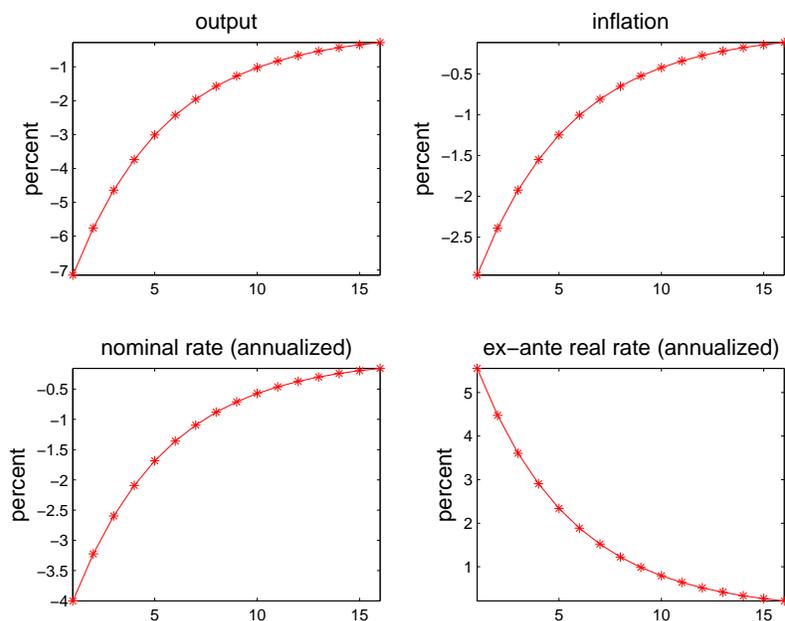


Figure 2: Self-fulfilling crisis

In period 0 output falls by just over seven per cent of its steady-state value and inflation contracts by about three percentage points. Simultaneously, and consistent with the prescribed feedback rule, the nominal interest rate is cut to its lower bound. But future output growth induces expectations of tight future policy, and thus a high current real interest rate. This ‘justifies’ the initial output collapse – that is, it ensures it is consistent with the linearised Euler equation.

To reiterate, two features of the model are essential for the existence of these crisis episodes: the zero bound on nominal interest rates and a high feedback parameter on output growth in the policy rule. Without a zero bound the policymaker would not permit real interest rates to rise in period zero when both output and inflation are below steady state. Without a large coefficient on output growth, future policy would not be expected to be particularly restrictive as the economy recovers back to steady state, and so the contemporary real interest rate would not rise by enough for the conjectured output collapse to be consistent with the Euler equation.

For this simple model, the extent of output growth feedback that is required for crises to be possible is summarized in the following proposition.

Proposition 1. *Suppose that the policy feedback parameters satisfy an augmented version of the Taylor principle: $\alpha_\pi + \frac{1-\beta}{\kappa}\alpha_y > 1$. Then a necessary condition for the system in (5) - (6) to have self fulfilling equilibria where the ZLB is binding in the initial period is:*

$$\frac{1}{\sigma}\alpha_{\Delta y} > \alpha_\pi + \frac{(1-\beta)}{\kappa}\alpha_y$$

and a sufficient condition is:

$$\frac{1}{\sigma}\alpha_{\Delta y} > \alpha_\pi + \frac{1}{\kappa}\alpha_y$$

Proof: See appendix A.

If we focus on the case of log utility and set $\alpha_y = 0$, then the condition in proposition 1 requires $\alpha_{\Delta y} > \alpha_\pi$. This is re-assuring. The empirical evidence does not suggest that central banks respond to output growth more strongly than to inflation. But we will show in later sections that one can generate very similar self-fulfilling equilibria at the zero lower bound in more complex models with much smaller coefficients on growth terms in the interest rate rule. Again, the simple model chosen here merely provides a convenient illustration of the basic principles.⁵

⁵An important qualification to this comes from a recent paper by Blake (2012), who shows that an optimal commitment strategy in the absence of the zero bound can be implemented through a ‘speed limit’ rule of the form contained in (4), assuming that the policymaker is minimising a discounted weighted sum of squared output and inflation deviations. For instance, if the preference parameter placed on the within-period output loss term relative to inflation loss is 0.2 then feedback values $\alpha_\pi = 1.43$, $\alpha_y = 0.05$ and $\alpha_{\Delta y} = 2$ are consistent with optimality, permitting self-fulfilling crises with very similar dynamic properties to those in Figure 2.

2.2 Non-linear analysis

A valid concern with the previous results is that economic variables are deviating from steady state by too much to be confident the ‘piecewise linear’ approximations to the non-linear dynamic New Keynesian model remain sound. To check this we implement a non-linear version of our solution approach, building on the extended path method for solving non-linear models under perfect foresight. Specifically, we compute the perfect-foresight solution for a finite-horizon by stacking the set of nonlinear equilibrium conditions for $t = 1, 2, \dots, K$. As a terminal condition we impose that the model is in the zero-inflation deterministic steady state in period $K + 1$, and that same steady state is imposed as an initial condition for state variables.⁶ This system is then solved via a nonlinear equation solver.⁷ Note that under the assumption of perfect foresight, the solution is exact up to machine precision without the need for any approximation.

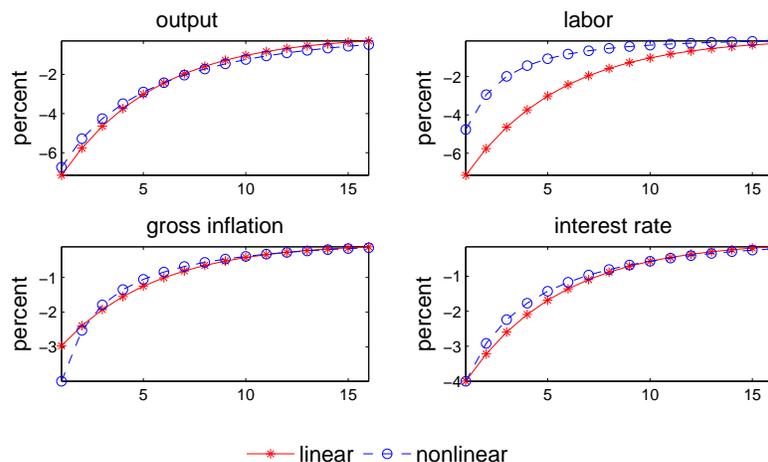


Figure 3: Nonlinear vs. linear approximation

Figure 3 compares the dynamics of this ‘crisis episode’ when applying the exact (non-linear) solution technique and the linear approximation. The values of the main model and policy parameter values are as above, but we additionally need to assign a value to the elasticity of substitution between consumption goods, ε , which is of

⁶We set $K = 100$, which is sufficiently large for the terminal conditions to have a negligible impact on initial dynamics.

⁷We set the tolerance level in MATLAB’s `fsolve` routine to 10^{-8} .

second-order importance when approximating. We set $\varepsilon = 7$, giving a steady-state mark-up of about 16 per cent on marginal costs.

Again we are able to obtain self-fulfilling crises for appropriate policy feedback parameters. The linear approximation clearly introduces some inaccuracies, but the qualitative story is not far wrong. The non-linear solution involves inflation falling by slightly more in the initial period, and labour supply by somewhat less throughout – the latter reflecting the well-known role of price dispersion in increasing the inefficiency of final goods production (a factor neglected in the linearised model), and the former then influenced by the impact of reduced wage demands when consumption is relatively low (for a given labour supply). It is notable that linearisation gives a very accurate description of output movements.

This non-linear solution technique becomes much more difficult to apply in larger models, since this increases the number of equations being associated with each time period. But we can take some comfort from having confirmed that the *qualitative* properties of the crisis dynamics appear broadly accurate when linearising, at least in this simple case. Multiple equilibria are clearly not an artifice of using a piecewise linear setup.

2.3 Interest-rate inertia

As a final piece of sensitivity analysis for this simple model we also examine how interest-rate inertia affects the existence of local self-fulfilling equilibria in the linear version of the model. We include this because inertia is a well-known feature of empirically estimated policy rules, so if its presence eliminated the possibility of a self-fulfilling crisis our results would presumably be of less practical relevance. We find that this is not the case. We first consider a feedback rule for the shadow rate of the form:

$$\widehat{i}_t^s = [1 - \rho] [\alpha_\pi \widehat{\pi}_t + \alpha_y \widehat{y}_t + \alpha_{\Delta y} (\widehat{y}_t - \widehat{y}_{t-1})] + \widehat{\rho} i_{t-1}^s$$

This links the shadow nominal interest rate in period t to the value of actual rate set at $t - 1$. We find that we are able to generate crises for exactly the same values of the α parameters when $\rho > 0$ as when $\rho = 0$. The only impact of a non-zero ρ is to increase the magnitude of the crisis, as measured by the initial falls in output and inflation.⁸

An alternative way to allow for inertia is to amend the rule as follows:

$$\widehat{i}_t^s = [1 - \rho] [\alpha_\pi \widehat{\pi}_t + \alpha_y \widehat{y}_t + \alpha_{\Delta y} (\widehat{y}_t - \widehat{y}_{t-1})] + \widehat{\rho} i_{t-1}^s$$

⁸The figures are available on request.

This links the shadow nominal rate for period t to the value of the *shadow* rate in $t - 1$. Unlike inertia in the actual rate, this has a significant qualitative impact on the crisis dynamics. What is also now the case if ρ is large enough is that the zero bound can remain binding during a crisis for more than one period. This is because the initial crisis involves a very substantial fall in the shadow nominal interest rate, unmatched by the actual rate because of the zero bound. With inertia this translates directly into a lower desired rate in the subsequent time period, potentially to the point where the zero bound continues to bind.

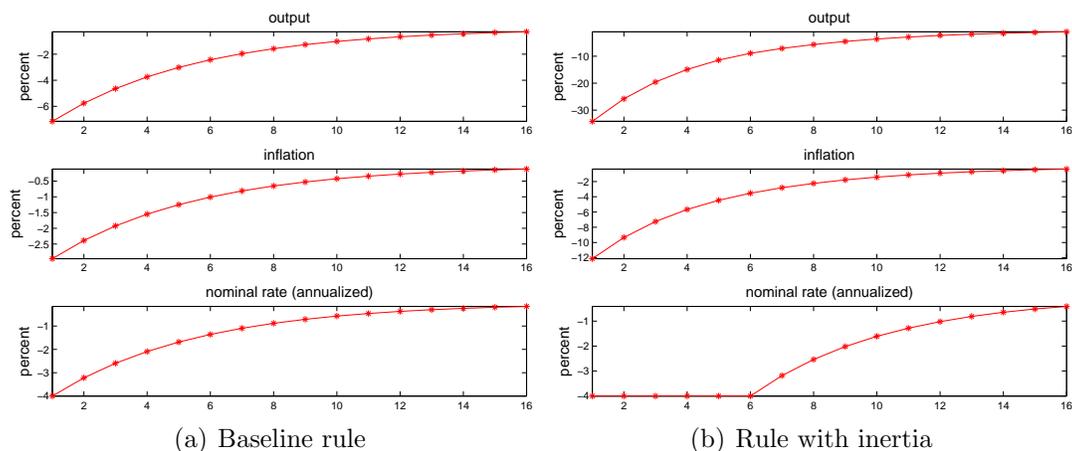


Figure 4: The role of policy inertia

Figure 4(b) plots the dynamics of this crisis episode, assuming the same values for the model’s parameters as in the baseline calibration (including $\alpha_y = 0$, $\alpha_\pi = 1.5$ and $\alpha_{\Delta y} = 2$). The inertia parameter is set to $\rho = 0.5$. Technically we solve for the equilibrium responses by conjecturing that the interest rate is at the lower bound for the first k time periods and positive thereafter. Expected values of future output and inflation in period $k + 1$ are again substituted out according to the unique rational expectations solution of the model absent the zero lower bound. We then check our conjecture ex-post by verifying that the shadow rate is below the zero bound in the periods prior to k and above the bound thereafter.⁹ For the chosen calibration we find a self-fulfilling ‘crisis’ equilibrium exists for $k = 6$, as shown in the Figure.

The main lesson we take from this analysis is that interest-rate inertia does nothing to prevent the possibility of self-fulfilling crises in the simple New Keynesian

⁹In this regard our solution approach is similar to the piecewise linear approach of Eggertsson and Woodford (2003).

model, and appears to exacerbate the negative properties of these equilibria.

3 A richer model

Whilst it is perhaps surprising that self-fulfilling equilibria of the form that we outline can be observed in a simple dynamic New Keynesian setup, that model is a restrictive and fairly unrealistic one. It is particularly restrictive for our purposes in the sense that very few variables are ultimately of relevance to outcomes, and thus the set of policy rules that could be assessed for their capacity to generate crises is limited. Given the relevance to contemporary policy discussions of models with imperfect financial markets, and the fact that these models can be used to assess the benefits of feeding back on asset-price or credit growth in particular, we now make use of one such model – due to Iacoviello (2005). As mentioned in the introduction, this model has been used recently by Mertens and Ravn (2011) to analyse the effects of exogenous bouts of ‘pessimism’ regarding the steady-state to which economic variables are converging. These authors show that financial frictions can substantially exacerbate the effects of such confidence crises.

3.1 Environment

The model contains four distinct classes of agent: households, entrepreneurs, final goods firms and a monetary policymaker. We briefly outline the setup below.

3.1.1 Households

The model economy is populated by a measure $\omega \in [0, 1]$ of households and $(1 - \omega)$ of entrepreneurs. At time t households maximise the objective function:

$$U_t^h = E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{(l_{t+s}^s)^{1+\varphi}}{1+\varphi} + \vartheta \ln(h_{t+s}) \right\} \quad (7)$$

where l_t^s is the household’s labour supply, h_t the quantity of housing it owns and c_t is the usual Dixit-Stiglitz sub-utility function across the unit-measure continuum of differential goods produced:

$$c_t = \left[\int_0^1 c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (8)$$



where ε is the elasticity of substitution across goods. If the money price of good j is $p_t(j)$, the minimum expenditure required to obtain a unit of c_t , P_t , is given by:

$$P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (9)$$

The gross rate of consumer price inflation, $\frac{P_t}{P_{t-1}}$, is denoted π_t in what follows.

Households optimise subject to the period-by-period budget constraint (expressed in real terms):

$$w_t l_t^s + D_t + \frac{(1 + i_{t-1})}{\pi_t} s_{t-1} = c_t + p_t^h (h_t - h_{t-1}) + s_t \quad (10)$$

with w_t the real wage, p_t^h the real price of housing, s_t the real quantity of saving (in nominal bonds, paying net interest i_t) and D_t dividend payments from shares that the consumer owns in final goods firms. This constraint is coupled with a usual transversality/‘no-Ponzi’ restriction.

3.1.2 Entrepreneurs

Entrepreneurs employ workers and make use of commercial real estate to produce intermediate goods, y_t , which are sold in a perfectly competitive market at price p_t^I to final goods firms. These entrepreneurs maximise a utility function expressed over consumption goods alone:

$$U_t^e = E_t \sum_{s=0}^{\infty} (\beta^e)^s \frac{(c_{t+s}^e)^{1-\sigma} - 1}{1-\sigma} \quad (11)$$

where the superscript e distinguishes their consumption of final goods from the household’s, and $\beta^e < \beta$ holds.¹⁰ This is subject to the period-by-period budget constraint:

$$b_t + p_t^I y_t - w_t l_t^d = c_t^e + p_t^h (h_t^e - h_{t-1}^e) + \frac{(1 + i_{t-1})}{\pi_t} b_{t-1} \quad (12)$$

where l_t^d denotes the entrepreneur’s labour demand, h_t^e commercial real estate (which is used in production, and whose real price is also p_t^h) and b_t real borrowing, for which entrepreneurs are charged nominal rate i_t . This is combined with an associated transversality/‘no-Ponzi’ condition, along with the collateral constraint:

¹⁰This condition is necessary to ensure a steady state in which financial leverage is observed, since the returns to leveraged investment must in general be higher than the real interest rate available to households.

$$b_t \leq m E_t \frac{\pi_{t+1}}{R_t} p_{t+1}^h h_t^e \quad (13)$$

with m interpreted as the fraction of the monetary value of next period's commercial real estate that the entrepreneur is permitted to commit to the repayment of loans, and the production function:

$$y_t = a (l_t^d)^{1-v} (h_{t-1}^e)^v \quad (14)$$

where a is a total factor productivity parameter and $(1 - v)$ is the share of the firm's revenue paid to labour.

So long as the expected returns available to entrepreneurs from holding an extra unit of commercial real estate exceed the borrowing rate, the collateral constraint must hold with equality.¹¹ In this event entrepreneurs make their intertemporal choices as if faced with a single 'composite' asset, obtained by purchasing a unit of commercial real estate that they then leverage to the maximum possible extent. One can thus show that they face an effective ex-post real rate of return on their savings, say RR_{t+1}^e , given by:

$$RR_{t+1}^e = \frac{v p_{t+1}^I \frac{y_{t+1}}{h_t^e} + p_{t+1}^h - \frac{m_t E_t \pi_{t+1} p_{t+1}^h}{\pi_{t+1}}}{\left[p_t^h - m_t E_t \left(\frac{\pi_{t+1}}{R_t} p_{t+1}^h \right) \right]} \quad (15)$$

In steady state RR^e must equal the inverse of the entrepreneurial discount factor, β^e , so that entrepreneurs have no further incentive to accumulate. Since $\beta^e < \beta$ it is only by barring households from investing in commercial real estate that we can provide the distinct rates of return that are necessary to guarantee stationary equilibrium consumption profiles for both entrepreneurs and households – despite the relative impatience of the former.¹²

3.1.3 Final goods firms

Final goods producers are monopolistically-competitive price setters owned by households, free to reset their prices only at stochastically-determined intervals – as in

¹¹If it did not then the entrepreneur could always take on an extra δ units of commercial real estate, at real price p_t^h , and borrow an extra δp_t^h (for ε sufficiently small). Given the rate of return differential this would deliver an expected welfare gain at time $t + 1$.

¹²With complete markets any general equilibrium would be Pareto efficient, and thus involve improvements upon the stationary outcome in the direction of allowing households to accumulate more and more wealth as time progresses.



Calvo (1983). Each firm has access to a linear technology, converting intermediate goods one-for-one into (differentiated) final goods. The period- t profit level of firm j , $\Pi_t(j)$, thus satisfies:

$$\Pi_t(j) = (p_t(j) - P_t p_t^I) y_t(j) \quad (16)$$

Prices are then chosen to maximise the net present value (to households) of the firm's future stream of profits, assuming a fixed probability of resetting prices equal to θ each period. The analytics of this problem are well known.

We define the Dixit-Stiglitz aggregate production of final goods y_t^f by:

$$y_t^f = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

3.1.4 Policy

The policymaker's only role is to set the nominal interest rate i_t . As in the simple New Keynesian model, we suppose i_t is set according to some 'simple rule', linking its value to contemporary realisations of the endogenous variables. Specifically, we consider rules of the following form:

$$(1 + i_t) = \max \left\{ \beta^{-1} \pi_t^{\alpha_\pi} \left(\frac{y_t^f}{y_{ss}^f} \right)^{\alpha_y} \left(\frac{y_t^f}{y_{t-1}^f} \right)^{\alpha_{\Delta y}} \left(\frac{p_t^h}{p_{t-1}^h} \right)^{\alpha_{\Delta p}} \left(\frac{b_t}{b_{t-1}} \right)^{\alpha_{\Delta b}}, 1 \right\} \quad (17)$$

The inverse of the household discount factor features here as the steady-state gross nominal interest rate because the steady-state value of inflation is set to zero. y_{ss}^f denotes the steady-state aggregate value of final output. Again we allow for the possibility of feedback on the growth rate of real output, as well as on the growth rates of house prices and aggregate credit – capturing a possible desire to 'prick' asset price bubbles or credit booms by allowing them to feed into higher rates. As before, the zero lower bound is translated into a bound on the gross nominal rate at 1.

3.2 Self-fulfilling crises and growth-rate feedback

We solve for the steady state of the model by conventional means, and study the dynamics of a corresponding approximated linear system in the neighbourhood of the steady state. The details are relegated to the appendix. We are interested to check whether feedback rules that incorporate terms in the growth rate of credit



and/or asset prices might be capable of allowing the same kinds of self-fulfilling crises that we observed in the simple New Keynesian model. The main reason for our interest in these kinds of policy rules is the ongoing debate about whether the global financial crisis might have been mitigated had policymakers taken a keener interest in asset prices and/or credit aggregates. In general those advocating that policy *should* respond to these variables argue that feedback on their growth rates should be the focus – a point noted, for instance, by Bloxham, Kent, and Robson (2011) when surveying the recent literature.¹³ But our study of the simple dynamic New Keynesian model suggests growth-rate feedback of this kind may be susceptible to self-fulfilling crises at the zero bound. It seems important to us that these potential vulnerabilities should be checked and, if indeed present, should play a role in any debate about the merits of alternative policy rules.

3.2.1 Feedback on credit growth

Our first set of simulations deals with the case in which policy feeds back on the growth rate of total ‘credit’ in the economy, where we associate credit growth with the object $\frac{b_t}{b_{t-1}}$.¹⁴ We choose the values for the structural parameters given in the table: these are carried over directly from Iacoviello (2005).¹⁵ For the policy rule we set the relevant feedback parameter $\alpha_{\Delta b}$ to 0.25, so that an increase in aggregate credit of one per cent would correspond roughly to a 25 basis-point increase in nominal rates. We additionally assume $\alpha_{\pi} = 1.5$, and $\alpha_y = \alpha_{\Delta y} = \alpha_{\Delta p} = 0$, neglecting feedback on all other variables bar inflation for simplicity.

We can then proceed as in the simple New Keynesian model: we fix the value of the nominal interest rate at the zero bound in the first time period alone, solve for the values of the endogenous variables in that and subsequent periods (under the

¹³According to these authors: “Policy concerns should mostly be about growth rates (rather than levels) of key variables, such as asset prices or credit, for a number of reasons. First, it is hard to know what constitutes a sustainable or fundamental level of such variables. Second, while high levels of indebtedness, for example, may imply greater vulnerability to adverse shocks, rapid growth may also suggest that individual, as well as system-wide, risks have not been fully appreciated, and that a larger share of exposures have yet to be tested by a period of economic weakness. Third, monetary policy cannot hope to be concerned with the level of a particular variable, such as property prices, but by altering the price of credit it can influence the willingness to service existing debts and to take on new ones.”

¹⁴Recall that b_t is the total quantity of entrepreneurial borrowing at time t , expressed in real terms. This is the closest analogue to the ‘volume of credit’ in this model.

¹⁵Our model corresponds to the ‘simple’ version presented in Section II of Iacoviello’s paper. The one departure is to set $\omega = 0.979$, a value we take from Andres, Arce, and Thomas (2010). This prevents an unrealistically low level of entrepreneurial consumption obtaining in steady state.



parameter	interpretation	value
β	household discount factor	0.99
β^e	entrepreneur discount factor	0.98
σ	inverse elasticity of intertemporal substitution	1
ϑ	weight on housing utility	0.1
φ	inverse Frisch elasticity of labour supply	0.01
ε	elasticity of substitution across final goods	21
ω	measure of household sector	0.979
v	elasticity of output with respect to CRE	0.03
θ	Calvo hazard rate	0.75
m	steady state permitted collateral ratio	0.89

Table 1: Calibration

assumption that the zero bound does not bind beyond the first period), and then check whether these realisations would indeed support zero rates at the start under the feedback rule (17).

We find that feedback on credit growth does indeed open up the possibility of a self-fulfilling crisis. That is, *given* the initial values for inflation, output and house prices that are implied by the model solution, a policymaker following rule (17) with the specified α values would wish to set nominal interest rates to zero. The dynamics of the crisis episode are illustrated in Figure 5 for output, household consumption, inflation and house prices. As can be seen, all four fall by a substantial amount in the first period: output contracts by around 3 per cent, accompanied by a severe decline of credit.



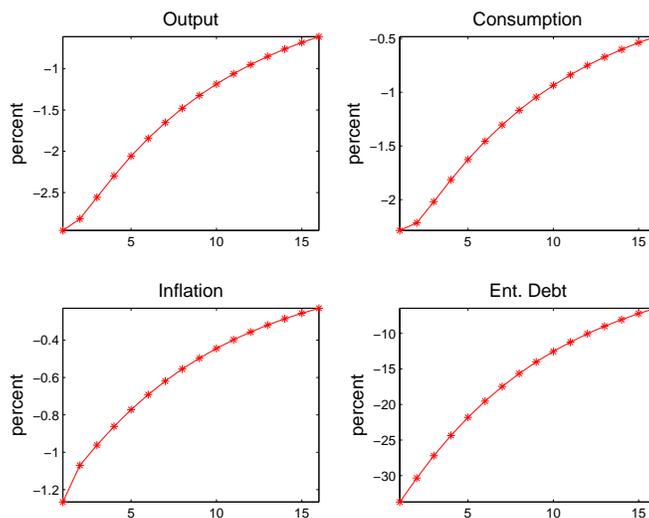


Figure 5: Self-fulfilling recession: feedback on credit growth

The logic behind the crisis occurring is really just an extension of that which applied in the New Keynesian case. In the first period a high real interest rate obtains because of deflationary expectations, with the zero lower bound preventing more expansionary monetary policy. This high real rate makes entrepreneurs reluctant to take leveraged positions in commercial real estate because of the high cost of servicing the associated debt.¹⁶ If the policymaker were not concerned for credit growth this contraction in commercial real estate would undermine the putative equilibrium, since it tends to put upwards pressure on future marginal costs in the intermediate goods sector, and hence on future inflation (reducing real interest rates).¹⁷ But when $\alpha_{\Delta b}$ is large enough the anticipated policy drive to prevent credit from returning too swiftly to its original level counteracts this inflationary effect, causing expected future marginal costs to be below their steady-state value and expected inflation to be substantially negative. That in turn generates the high real interest rates that we asserted from the start.

The main lesson of this experiment is that policymakers should exercise some caution when promising not to allow ‘credit bubbles’ to be established. It may be

¹⁶This factor is compounded by the negative wealth effect that entrepreneurs suffer through a combination of real estate price falls and low profits in the intermediate goods sector (the later due to low aggregate consumption).

¹⁷Recall that under Calvo pricing with zero steady-state inflation the current inflation level is approximately equal to a discounted sum of future marginal costs.

very hard in practice to determine what is the re-emergence of a bubble, and what is the restoration of borrowing levels back to a steady state. Advance fears that a collapse *from* the steady state would be instead be mis-interpreted as a reversion back *to* it following a ‘bubble’ could make just such a collapse more likely.

It is also interesting to note that only quite minor feedback on credit growth is needed for self-fulfilling dynamics to appear. Numerical experimentation suggests that the key threshold is satisfied for the chosen parameter set whenever $\alpha_{\Delta b} > 0.175$ – roughly equivalent to an increase in policy rates by 175 basis points for a 10-per-cent expansion in credit.

3.2.2 Feedback on house price growth

We next consider policy rules that feed back on the growth rate of house prices. We set $\alpha_{\Delta p}$ to 1.5, implying symmetric policy feedback for goods price and asset price growth.¹⁸ $\alpha_{\Delta b}$ we set to zero, and other values are carried over from the previous sub-section.

Once again self-fulfilling crises are possible. Figure 6 charts the dynamics, which are qualitatively similar to the case of credit-growth feedback, but of substantially greater magnitude. Indeed, the size of the output and house price falls are sufficiently large to challenge the validity of the linear approximation, though less dramatic (as well as much larger) falls are possible when different combinations of the policy feedback parameters are chosen.¹⁹ We additionally take some comfort from our non-linear experiments in the New Keynesian model, which showed that the possibility of self-fulfilling crises is not an artifact of linearisation.

¹⁸For the particular choices that we have made for other parameters any value for $\alpha_{\Delta p}$ from roughly 1.3 upwards will support self-fulfilling crisis dynamics.

¹⁹In general the higher the value of $\alpha_{\Delta p}$ the lower the magnitude of the crisis, provided $\alpha_{\Delta p}$ exceeds a ‘threshold’ value necessary for a crisis to occur in the first place.



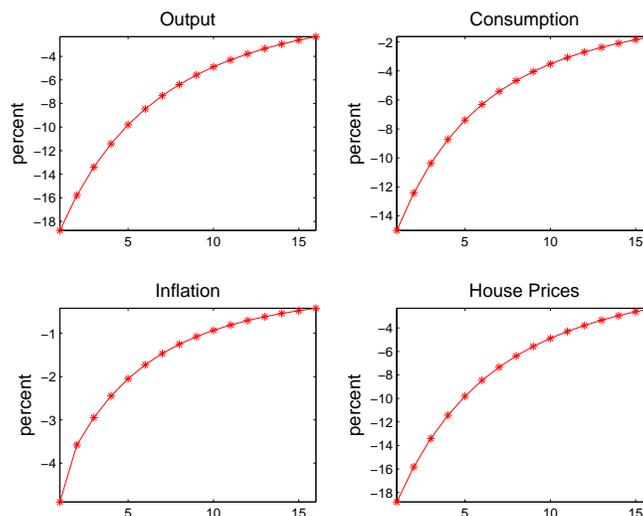


Figure 6: Self-fulfilling recession: feedback on house price growth

Once again the crisis is driven by the deflationary expectations that growth-rate feedback rules are capable of inducing. High real interest rates in the first period make entrepreneurs reluctant to invest (despite the nominal rate reaching the zero bound), and this in turn drives down the price of real estate. Knowing that the monetary policymaker will attempt to resist the return of house prices back to their steady-state level, agents in the economy rationally anticipate future economic stagnation, accompanied by deflation. These deflationary expectations justify the high real rates originally conjectured.

3.3 Sensitivity

The elasticity of intertemporal substitution is potentially a very important parameter for our results. Recall, for instance, that in the simple New Keynesian model we showed in Proposition 1 that self-fulfilling dynamics are only possible if $\alpha_{\Delta y} > \sigma\alpha_{\pi}$ (assuming $\alpha_y = 0$). Thus the greater was σ the smaller the region in the parameter space for which crises could be observed. This is because higher values of σ imply less elastic consumption, and this translates into smaller movements in output – making it harder for the growth-rate feedback to induce deflationary expectations.

It is not clear whether this sensitivity to σ translates over to the richer model of this section, when feedback is no longer simply on output growth. But our assumed value of $\sigma = 1$ (taken from Iacoviello (2005)) is certainly well below what is suggested

by microeconomic evidence, which generally points to a value in the region of $\sigma = 5$ or greater. For this reason it is important to check how sensitive our results are to the intertemporal elasticity.

We find that the existence of self-fulfilling equilibria do not depend on the value of σ very much. Figure 7 compares the different combinations of values for $\alpha_{\Delta b}$ and $\alpha_{\Delta p}$ for which crises are possible when $\sigma = 5$ and when $\sigma = 1$ in our version of the Iacoviello model – again with $\alpha_y = 0$, $\alpha_\pi = 1.5$, and all other parameters set as above.

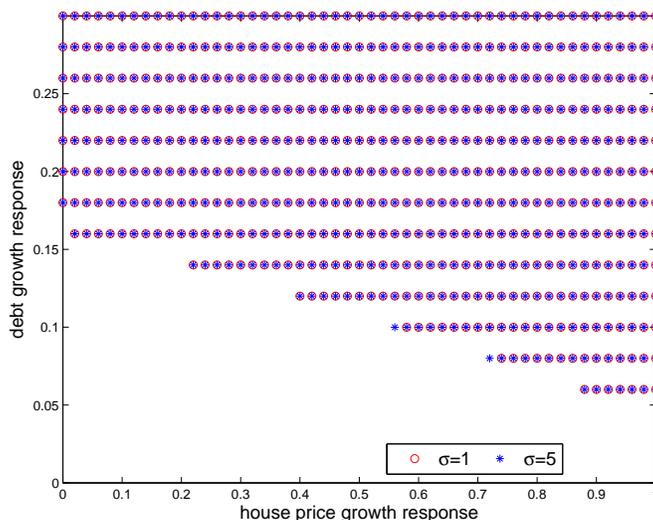


Figure 7: Parameter combinations that give rise to self-fulfilling equilibria

In other results (not reported here), we have found that self-fulfilling crises can also be induced by feeding back on the growth rate of output itself in the Iacoviello model, as in the simple New Keynesian case. Moreover, in our benchmark calibration for this model crises are possible even under pure nominal GDP targeting (i.e., when $\alpha_\pi = \alpha_{\Delta y}$, with other feedback parameters set to zero).²⁰ We have also confirmed the possibility of crises when the policymaker targets the growth rate of commercial real estate – likewise suggesting that any special focus on trends in the real estate sector should be worked carefully into policy priorities.

²⁰Recall that we needed $\alpha_{\Delta y} > \sigma\alpha_\pi$ for crises in the simple New Keynesian setup when $\alpha_y = 0$. The benchmark calibration there assumed $\sigma = 1$, so nominal GDP target was just inconsistent with crisis dynamics.

Computationally, our results rely on a guess and verify approach that is shared with many other papers in the literature as in Eggertsson and Woodford (2003). We therefore cannot rule out that there are even more equilibria of the kind we consider that we were unable to find despite an extensive search.²¹ We do not see this as shortcoming of this paper, as our main contribution is to establish that there *can* be multiple equilibria under a Taylor rule that feeds back on growth rates of endogenous variables if the lower bound is taken into account.

4 Discussion: multiple equilibria about a ‘determinate’ steady state

The dynamics here share a lot of the features (and magnitudes) of the self-fulfilling equilibria discussed by Mertens and Ravn (2011), which are likewise exhibited in a version of the Iacoviello model. But there is a crucial difference that it is worth reiterating. Those authors focus on the risk of a ‘liquidity trap’, whereby beliefs about the steady state to which the economy is converging switch at random from the ‘desirable’ state, whose local determinacy is guaranteed by satisfaction of the Taylor principle, to the alternative and locally indeterminate state (whose presence in the event of a zero lower bound was first highlighted by Benhabib, Schmitt-Grohe, and Uribe (2002)). In the Mertens and Ravn model deflationary expectations set in for exogenous reasons, and are expected to persist to the next time period with a sufficiently large probability that the policymaker is forced to accept stagnation – unable to cut real interest rates because of the zero bound. This renders the equilibrium fluctuations indeterminate and there will generally exist a continuum of paths consistent with the model.

The crisis that we highlight can instead occur even when expectations are invariant functions of the fundamental variables in the economy, and thus operates even under a ‘perfect foresight’ assumption that the economy will converge on the desirable steady state with certainty. As discussed, it does require that deflation is anticipated the period after any initial house price fall, but instead of resulting from a bout of collective pessimism these deflationary expectations are now *derived from* the relatively contractionary policy that is known to follow in periods after a house price collapse – whilst the long-term focus of agents in the economy remains

²¹We have searched by assuming that the zero lower bound is binding for periods $t = 0, \dots, K$ and nonbinding ever thereafter. The upper bound K is varied by 1 and 20. It is of course possible that other equilibria exist which have periods of a binding lower bound followed by non-binding periods and then by binding lower bounds again etc. We could not find such equilibria.



the locally ‘determinate’ steady state.

Put simply, the paper by Mertens and Ravn (2011) considers parameterizations for which their model is indeterminate and therefore possesses an infinity of equilibria. Our paper considers only perfect foresight paths. Along these paths, the exit date from any binding lower bound episode is known ex-ante and hence the model is always determinate. But the lower bound introduces a nonlinearity into the model. We show that for some parameterizations of the interest rate rule, this nonlinear model possesses a finite number of equilibria: A self-fulfilling collapse or remaining in the zero inflation steady state.

A related technical implication of our results is that for this particular policy rule the ‘determinate’ steady state is in fact associated with a form of *local* indeterminacy (so long as zero nominal rates may be considered ‘local’ to it), since more than one set of initial values for the control variables in the system can obtain, and be consistent with convergence to that steady state, absent any exogenous shocks. Unlike a linear system with an insufficient number of explosive roots, this indeterminacy does not admit a continuum of perfect-foresight solutions – in fact, it admits just two: either a crisis can start, with nominal rates dropping to zero and the dynamics above obtaining, or the economy can remain in rest at the desired steady state.

5 Conclusions

This paper highlights an important channel by which certain monetary policy feedback rules could permit self-fulfilling ‘crisis’ dynamics in the presence of a zero lower bound on nominal interest rates. Given the prominence of contemporary discussions about the appropriate treatment by monetary policymakers of asset prices and credit we place particular emphasis on our finding that if policy feeds back on the growth rates of either or both of these variables self-fulfilling crisis dynamics may inadvertently be rendered possible. Intuitively, when policy is set to be contractionary in response to the growth of a target variable it will just as readily resist its growth *back* to steady state as its growth from it. Hence a reduction in asset prices or credit today will be associated with expectations of contractionary policy tomorrow. This in turn can imply deflationary expectations, driving up the current real interest rate. If nominal interest rates are constrained at the zero bound this increase in real rates cannot be resisted by the policymaker, and permits the original asset-price or credit fall to be equilibrium-consistent.

Demonstrating the potential generality of our results, we have found that crisis episodes of this kind can be generated even in a conventional New Keynesian model without capital – which, when linearised, is one of the simplest dynamic macroe-



conomic models possible. Specifically, this occurs when policy feeds back on the growth rate of output. This model has proved useful in clarifying the intuition behind our results, and in allowing us to state formal conditions on the magnitude of growth-rate feedback required for crises to be generated. Unlike similar phenomena already discussed in the literature, the crises that we highlight are both self-fulfilling and consistent with convergence back to a zero-inflation steady state under perfect foresight.

Our results from the New Keynesian model provide a counterpoint to other work suggesting ‘speed limit’ rules of the sort we have studied can have some desirable properties: namely, that they can be devices to implement or at least mimic the optimal policy under commitment absent the zero lower bound (see Blake (2012), Giannoni and Woodford (2003), Stracca (2007), McCallum and Nelson (2004), Leduc and Natal (2011)), or that they are means to insulate the economy from the consequences of mismeasuring the levels of natural rate concepts needed as inputs to conventional monetary policy rules (see Orphanides and Williams (2002)). Since there is some evidence that these rules capture actual central bank behaviour (see, for example, Mehra (2002) and Paez-Farrell (2009)), even in that relatively simple setting our cautionary tale is of more than academic interest.

We have abstracted from the possibility that the authorities might have at their disposal other instruments to substitute for monetary policy at the zero bound (for example, fiscal tools, or unconventional monetary policy tools), and it might be expected that appropriate use of such instruments could rule out self-fulfilling attacks of the sort we have illustrated. Equally, if those instruments were also governed by concerns about growth rates, perhaps for the same reasons that the interest-rate tool were set in this way, the possibility for self-fulfilling recessions could remain. For now we leave these questions for future work.

References

- ANDRES, J., O. ARCE, AND C. THOMAS (2010): “Banking competition, collateral constraints and optimal monetary policy,” Bank of Spain Working Paper No 1001.
- BENHABIB, J., S. SCHMITT-GROHE, AND M. URIBE (2002): “Chaotic Interest-Rate Rules,” *American Economic Review*, 92(2), 72–78.
- BLAKE, A. (2012): “Determining optimal monetary speed limits,” *Economics Letters*, 116, 269–271.



- BLOXHAM, P., C. KENT, AND M. ROBSON (2011): “Asset prices, credit growth, monetary and other policies: an Australian case study,” NBER Working Paper 16845.
- CALVO, G. A. (1983): “Staggered prices in a utility-maximizing framework,” *Journal of Monetary Economics*, 12(3), 383–398.
- CHRISTIANO, L. J., M. EICHENBAUM, AND S. REBELO (2011): “When is the government spending multiplier large?,” *Journal of Political Economy*, 119, 78–121.
- CORSETTI, G., K. KUESTER, A. MEIER, AND G. MULLER (2010): “Debt Consolidation and Fiscal Stabilization of Deep Recessions,” *American Economic Review*, 100(2), 41–45.
- DELNEGRO, M., G. EGGERTSON, A. FERRERO, AND N. KIYOTAKI (2010): “The Great Escape: A quantitative evaluation of the Fed’s Non-Standard Policies,” Discussion paper, Federal Reserve Bank of New York.
- EGGERTSSON, G., AND M. WOODFORD (2003): “The Zero Interest Rate Bound and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, pp. 139–211.
- GIANNONI, M. P., AND M. WOODFORD (2003): “How Forward-Looking Is Optimal Monetary Policy?,” *Journal of Money, Credit and Banking*, 35(6), pp. 1425–1469.
- IACOVIELLO, M. (2005): “House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle,” *American Economic Review*, 95(3), 739–764.
- LEDUC, S., AND J.-M. NATAL (2011): “Should central banks lean against changes in asset prices?,” Working Paper Series 2011-15, Federal Reserve Bank of San Francisco.
- MCCALLUM, B. T., AND E. NELSON (2004): “Timeless perspective vs. discretionary monetary policy in forward-looking models,” *Review*, (Mar), 43–56.
- MEHRA, Y. P. (2002): “Level and growth policy rules and actual Fed policy since 1979,” *Journal of Economics & Business*, 54, 575–594.
- MERTENS, K., AND M. O. RAVN (2011): “Credit Channels in a Liquidity Trap,” CEPR Discussion Papers 8322, C.E.P.R. Discussion Papers.



- ORPHANIDES, A., AND J. C. WILLIAMS (2002): “Robust Monetary Policy Rules with Unknown Natural Rates,” *Brookings Papers on Economic Activity*, 33(2), 63–146.
- PAEZ-FARRELL, J. (2009): “Monetary policy rules in theory and in practice: evidence from the UK and the US,” *Applied Economics*, 41(16), 2037–2046.
- SARGENT, T. J., AND N. WALLACE (1975): “‘Rational’ Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule,” *Journal of Political Economy*, 83(2), 241–54.
- STRACCA, L. (2007): “A Speed Limit Monetary Policy Rule for the Euro Area,” *International Finance*, 10(1), 21–41.
- WALSH, C. E. (2003): “Speed Limit Policies: The Output Gap and Optimal Monetary Policy,” *The American Economic Review*, 93(1), pp. 265–278.
- WOODFORD, M. (2003): *Interest and prices: foundations of a theory of monetary policy*. Princeton University Press.
- (2011): “Simple Analytics of the Government Expenditure Multiplier,” *American Economic Journal: Macroeconomics*, 3(1), 1–35.

Appendix

A Proof of Proposition 1

Applying the familiar ‘minimum state variables’ solution technique to the piecewise-linearised New Keynesian system given by equations (1) to (4), in some period t such that the ZLB does not bind (and is not expected to do so in the future) gives:

$$\hat{y}_t = A\hat{y}_{t-1} \quad (18)$$

$$\hat{\pi}_t = B\hat{y}_{t-1} \quad (19)$$

for coefficients A and B to be determined. Taking the ‘IS’ and ‘PC’ relationships in turn, these coefficients must satisfy the restrictions:

$$A = A^2 - \frac{1}{\sigma} (\alpha_\pi B + \alpha_{\Delta y} (A - 1) + \alpha_y A - AB) \quad (20)$$



$$B = \kappa A + \beta AB \quad (21)$$

A is thus a root of the equation:

$$0 = -1 + A - \frac{\kappa}{\sigma} \alpha_\pi \frac{1}{1 - \beta A} - \frac{1}{\sigma} \alpha_{\Delta y} \left(1 - \frac{1}{A}\right) - \frac{1}{\sigma} \alpha_y + \frac{\kappa}{\sigma} \frac{A}{1 - \beta A} \quad (22)$$

whilst B solves:

$$B = \kappa \frac{A}{1 - \beta A} \quad (23)$$

Equation (22) is continuous in A for $A \in (0, \beta^{-1})$. As $A \rightarrow 0$ its right-hand side approaches $+\infty$. When $A = 1$ its right-hand side is $\frac{\kappa}{\sigma} \frac{1}{1 - \beta} (1 - \alpha_\pi) - \frac{1}{\sigma} \alpha_y$. Hence the equation has a solution with $A \in (0, 1)$ provided the augmented Taylor principle is satisfied. $A \in (0, 1)$ implies $B > 0$, by equation (23). This root implies a non-explosive dynamic for \hat{y}_t , and thus is associated with the unique non-explosive solution provided the Blanchard-Kahn conditions are satisfied.

Now move back to a period s in which the ZLB binds, but will cease to do so at $s + 1$. For the given A and B coefficients, the ‘IS’ relationship solves to give:

$$\hat{y}_s = \frac{1 - \beta}{\sigma(1 - A) - B} \quad (24)$$

And the value of $\hat{\pi}_s$ is:

$$\hat{\pi}_s = \frac{(\kappa + \beta B)(1 - \beta)}{\sigma(1 - A) - B} \quad (25)$$

We are interested in situations in which \hat{y}_s and $\hat{\pi}_s$ are negative – so that the policymaker will have an incentive to lower the nominal rate. This will occur when $\sigma(1 - A) - B < 0$. From the earlier relationships between A and B , we have:

$$\sigma(1 - A) = \frac{A - \alpha_\pi - \alpha_y \frac{A}{B}}{A - \frac{1}{\sigma} \alpha_{\Delta y}} B \quad (26)$$

Unique non-explosiveness gives $A < 1 < \alpha_\pi + \frac{1 - \beta}{\kappa} \alpha_y$. Hence the inequality we need will be satisfied so long as:

$$\frac{1}{\sigma} \alpha_{\Delta y} > \alpha_\pi + \alpha_y \frac{A}{B} \quad (27)$$

$$= \alpha_\pi + \frac{(1 - \beta A)}{\kappa} \alpha_y \quad (28)$$

Since $A \in (0, 1)$, the necessary and sufficient conditions that we state follow directly.



B Non-linear New Keynesian model

A representative consumer at time t maximises the objective:

$$U_t = E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{(l_{t+s}^s)^{1+\varphi}}{1+\varphi} \right\} \quad (29)$$

where l_t^s denotes labour supply at t and c_t a Dixit-Stiglitz aggregate across a continuum of consumption goods:

$$c_t = \left[\int_0^1 c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (30)$$

If the money price of good j is $p_t(j)$, the minimum expenditure required to obtain a unit of c_t , P_t , is given by:

$$P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (31)$$

We denote by π_t the gross rate of inflation at t , $\frac{P_t}{P_{t-1}}$. The consumer's period-by-period budget constraint is then:

$$w_t l_t^s + T_t + \frac{1+i_{t-1}}{\pi_t} s_{t-1} = c_t + s_t \quad (32)$$

with w_t the real wage, s_t the real quantity of saving (in nominal bonds, paying gross interest $(1+i_t)$), and T_t a collection of lump-sum transfers to and from profit-making firms and the government. This constraint is coupled with a usual transversality/'no-Ponzi' restriction.

Labour demand comes from a large number of perfectly competitive firms producing homogenous intermediate goods according to a linear production function:

$$y_t = a l_t \quad (33)$$

for some fixed technology parameter a . These intermediate goods are purchased by final goods firms at a price p_t^I (expressed in units of consumption aggregate), who convert them one-for-one into their respective products. These firms are in turn owned by consumers, so the generic firm j looks to maximise the objective:

$$\Pi_t(j) = E_t \sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \left[((1+\tau) p_{t+s}(j) - P_{t+s} p_{t+s}^I) y_{t+s}(j) \right] \quad (34)$$



where $p_{t+s}(j)$ is the price at which firm j sells its goods in period $t+s$, y_{t+s} is its output level that period, and τ is a proportional revenue subsidy that is paid by the government to eliminate the steady-state losses due to market power. As in Calvo (1983), these firms are permitted to reset their prices only infrequently, with a probability of resetting equal to $(1-\theta)$ each period. We denote by y_t^f the aggregate final goods output level of relevance to consumers:

$$y_t^f \equiv \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (35)$$

Policy is set according to a feedback rule of the following form:

$$(1+i_t) = \max \left\{ \beta^{-1} \pi_t^{\alpha_\pi} \left(\frac{y_t^f}{y_{ss}^f} \right)^{\alpha_y} \left(\frac{y_t^f}{y_{t-1}^f} \right)^{\alpha_{\Delta y}}, 1 \right\} \quad (36)$$

where y_{ss}^f is the steady-state aggregate level of final goods output. Note that β^{-1} is the steady-state gross nominal interest rate consistent with zero inflation.

The model is solved in the usual way, taking first-order conditions to the firm and consumer choice problems and imposing market clearing. Linearising the resulting system about a zero-inflation steady state and eliminating terms delivers the three-equation system of Section 2.1.

C Solution to Model of Section 3

We proceed to derive the basic model's key equations, considering optimal choices for households, entrepreneurs and firms in turn, followed by market clearing conditions and any further definitions.

C.1 Household optimality

Optimal dynamic choice for households implies a conventional consumption Euler equation:

$$c_t^{-\sigma} = \beta E_t c_{t+1}^{-\sigma} \frac{(1+i_t)}{\pi_{t+1}} \quad (37)$$

The optimal choice of housing similarly implies:

$$c_t^{-\sigma} p_t^h = \beta E_t c_{t+1}^{-\sigma} p_{t+1}^h + \vartheta h_t^{-1}. \quad (38)$$



Note that housing services provide a utility flow whose value to the consumer in equilibrium must exactly compensate for the fact that no financial return is paid (aside from any capital gain or loss).

Optimal intratemporal consumption-labour choice implies:

$$w_t c_t^{-\sigma} = (l_t^s)^\varphi \quad (39)$$

C.2 Entrepreneur optimality

The relevant Euler condition for entrepreneurs is:

$$(c_t^e)^{-\sigma} = \beta E_t (c_{t+1}^e)^{-\sigma} R R_{t+1}^e \quad (40)$$

We assume that the borrowing constraint binds with equality (this conjecture can then be verified *ex post*):

$$b_t = m_t E_t \frac{\pi_{t+1}}{(1+i_t)} p_{t+1}^h h_t^e \quad (41)$$

In their role as directors of intermediate goods firms, entrepreneurs must also ensure an optimal level of hiring:

$$w_t = (1-v) \frac{y_t}{l_t^I} p_t^I \quad (42)$$

Note that the right-hand side of this equation is the marginal product of labour, converted into units of the numeraire (the final good).

C.3 Firm optimality

Optimal price-setting for Calvo-constrained firms (where \tilde{P}_t is the price chosen by firms resetting at t) gives:

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^t \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \left\{ \frac{\tilde{P}_t}{P_{t+s}} - \frac{\varepsilon_t}{\varepsilon_t - 1} p_{t+s}^I \right\} P_{t+s}^{\varepsilon_t} y_{t+s}^f = 0 \quad (43)$$

The composite discount factor applied to future marginal returns here reflects the firms' ownership by households. The firm is implicitly obliged to sell as many units as are demanded at the fixed price it posts, and with a single input it has no further choice variables.



C.4 Market clearing and further definitions

Labour market clearing gives:

$$(1 - \omega) l_t^d = \omega l_t^s \quad (44)$$

Goods market clearing gives:

$$\omega c_t + (1 - \omega) c_t^e = y_t^f \quad (45)$$

Real estate market clearing gives:

$$\omega h_t + (1 - \omega) h_t^e = \bar{h} \quad (46)$$

where \bar{h} is the aggregate stock of real estate, which is held fixed for simplicity.

It is easy to show that the aggregate level of final goods output will be related to intermediate goods output according to the equation:

$$\Delta_t y_t^f = (1 - \omega) y_t \quad (47)$$

where the price dispersion index Δ_t is defined by:

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj \quad (48)$$

The consumer price index then evolves in accordance with the Calvo pricing structure:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1 - \theta) \tilde{P}_t^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (49)$$

These equations are sufficient to complete the characterisation of the non-linear model.

We solve for the steady state of the model through standard techniques, and take linear approximations to the structural equations to analyse fluctuations in the neighbourhood of this. The associated linear equations are listed in the next subsection.

C.5 Loglinear equilibrium conditions

In the region of its steady state, the model used in Section 3 onwards is described by the following linear (or piecewise linear) equations:



1. $\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta} \widehat{p}_t^I$
2. $\widehat{c}_t = \widehat{c}_{t+1} - \frac{1}{\sigma} \left(\widehat{i}_t - E_t \widehat{\pi}_{t+1} \right)$
3. $\widehat{c}_t^e = \widehat{c}_{t+1}^e - \frac{1}{\sigma} \widehat{RR}_t$
4. $\widehat{RR}_t = \frac{1}{p_{ss}^h (1-m\beta)} \left[v \frac{y_{ss}}{h_{ss}^e} p_{ss}^I E_t \left(\widehat{p}_{t+1}^I + \widehat{y}_{t+1} - \widehat{h}_t^e \right) + p_{ss}^h E_t \widehat{p}_{t+1}^h - \beta^{-1} \frac{b_{ss}}{h_{ss}^e} E_t \left(\widehat{i}_t - \widehat{\pi}_{t+1} + \widehat{b}_t - \widehat{h}_t^e \right) \right] - \frac{p_{ss}^h}{\beta^e p_{ss}^h (1-m\beta)} \left[\widehat{p}_t^h - \beta m E_t \left(\widehat{\pi}_{t+1} - \widehat{i}_t + \widehat{p}_{t+1}^h \right) \right]$
5. $p_{ss}^h c_{ss}^{-\sigma} \left(\widehat{p}_t^h - \sigma \widehat{c}_t \right) = \beta p_{ss}^h c_{ss}^{-\sigma} E_t \left(\widehat{p}_{t+1}^h - \sigma \widehat{c}_{t+1} \right) - \frac{\vartheta}{h_{ss}} \widehat{h}_t$
6. $\varphi \widehat{l}_t^s + \sigma \widehat{c}_t = \widehat{p}_t^I + \widehat{y}_t - \widehat{l}_t^s$
7. $\widehat{y}_t = (1-v) \widehat{l}_t^s + v \widehat{h}_{t-1}^e$
8. $\omega \widehat{c}_t + (1-\omega) \widehat{c}_t^e = \widehat{y}_t$
9. $\widehat{h}_t = \frac{\omega-1}{\omega} \frac{h_{ss}^e}{h_{ss}} \widehat{h}_t^e$
10. $\widehat{b}_t = E_t \left(\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^h \right) - \widehat{i}_t + \widehat{h}_t^e$
11. $m\beta p_{ss}^h h_{ss}^e E_t \left(\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^h - \widehat{i}_t + \widehat{h}_t^e \right) + v y_{ss} p_{ss}^I \left(\widehat{y}_t + \widehat{p}_t^I \right) = c_{ss} \widehat{c}_t^e + p_{ss}^h h_{ss}^e \left(\widehat{h}_t^e - \widehat{h}_{t-1}^e \right) + \beta^{-1} b_{ss} \left(\widehat{i}_{t-1} - \widehat{\pi}_t + \widehat{b}_{t-1} \right)$
12. $\widehat{i}_t = \max \left\{ \alpha_\pi \widehat{\pi}_t + \alpha_y \widehat{y}_t + \alpha_{\Delta y} \left(\widehat{y}_t - \widehat{y}_{t-1} \right) + \alpha_{\Delta p} \left(\widehat{p}_t^h - \widehat{p}_{t-1}^h \right) + \alpha_{\Delta b} \left(\widehat{b}_t - \widehat{b}_{t-1} \right), \beta - 1 \right\}$

The subscript *ss* is used to denote steady-state values of the relevant variables, and ‘hats’ to denote log deviations from steady state.

