BANK OF ENGLAND

Appendix to Working Paper No. 489
Expectations, risk premia and information spanning in dynamic term structure model estimation

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Appendix

A Model Estimates

- Tables A1 and A2 show the model fit statistics for the nominal models and for the UK joint model, respectvely.
- Table A3 shows the estimates of the $\mathbb{P}$-dynamics unconditional mean of nominal interest rates for both US and UK.
- Table A4 shows the estimates of the $\mathbb{P}$-dynamics half-life (of the largest eigenvalue) of the pricing factors for both US and UK.
- Tables A5 and A6 show the sensitivity of excluding surveys from the nominal and joint models, respectively. Table A5 is an expanded version (more percentiles) of the table shown in the paper.
- The time series of the normalized range of term premia estimates as a percentage of yields are shown in Figure A1, for the US, and in Figure A2, for the UK. Because estimates with shorter samples are particularly problematic, we show the ranges for all subsamples and excluding 2002 and 2007 subsamples.For each model size N and estimation strategy we take the parameters estimated with starting sample year YY, $\hat{\Theta}^N_{YY}$, and obtain the Kalman Filter estimates of the latent factors for the entire sample (since 1972) and calculate the n-year spot nominal term premium $t\hat{p}^{N,YY}_{t,n}$. We then compute the time series of the range of term premia estimates by calculating for each date t:

$$rtp^N_{t,n} = \max_{i\in\{72:5:07\}} t\hat{p}^{N,i}_{t,n} - \min_{i\in\{72:5:07\}} t\hat{p}^{N,i}_{t,n} \tag{1}$$

- Figure A3 shows the estimates of 10 year real term premia for the UK.

Table A1: Mean absolute errors of nominal yield curve models - US and UK

	A. US Nominal Yields							
	3m	1y	2y	3y	4y	5y	7y	10y
Unconstrained								
3 factors	19.4	8.5	4.5	4.6	5.9	5.0	3.0	5.9
	20	12	6	6	8	7	3	8
4 factors	22.2	3.0	4.2	3.5	3.9	4.0	2.4	2.2
	22	4	6	5	6	5	3	3
5 factors	23.0	1.8	2.9	3.2	3.6	3.7	2.2	1.8
	23	4	6	5	6	5	3	3
With Surveys								
3 factors	19.5	8.5	4.6	4.6	5.9	5.0	3.1	5.8
	20	12	6	6	8	7	3	8
4 factors	21.6	3.0	4.2	3.5	3.9	4.0	2.4	2.2
	22	4	6	5	6	5	3	3
5 factors	23.5	3.1	4.0	3.4	3.8	3.8	2.3	1.9
	23	4	6	5	6	5	3	3
	B. UK Nominal Yields							
	6m	1y	2y	3y	4y	5y	7y	10y
Unconstrained								
3 factors	6.4	7.3	5.8	2.7	3.4	4.5	4.4	4.6
	9	10	8	4	5	6	6	7
4 factors	1.1	2.5	1.7	2.1	1.1	1.4	3.2	1.9
	2	4	3	3	2	2	5	3
5 factors	0.3	0.8	1.1	0.3	0.7	0.5	0.9	0.3
	0	1	2	0	1	1	1	1
With Surveys								
3 factors	6.4	7.3	5.8	2.7	3.4	4.5	4.4	4.6
	9	10	8	4	5	6	6	7
4 factors	1.3	2.7	1.8	2.1	1.2	1.5	3.2	2.0
	2	4	3	3	2	2	5	3
5 factors	0.3	0.8	1.1	0.4	0.6	0.5	0.9	0.4
	0.4	1.3	1.7	0.5	1.1	0.8	1.5	0.6

Notes: The table shows mean absolute errors across for each maturity for the US (top panel) and UK (bottom panel) nominal yields estimated with the entire sample (Jan 1972 - November 2010). All figures are expressed in annualized basis points. For each maturity (column) and each number of factors (rows) the absolute error for the model estimated for the full sample is shown for that maturity. The standard deviation of errors for each maturity and model is shown in the line below.

Table A2: Joint nominal-real model fit 1972

	A. UK Nominal Yields							
	6m	1y	2y	3y	4y	5y	7y	10y
Unconstrained								
3 factors	13.9	7.6	10.2	8.4	7.0	6.5	5.8	11.8
	17	10	13	11	9	9	8	15
4 factors	3.7	5.4	3.1	2.8	2.8	3.4	4.3	4.6
	5	7	4	3	4	5	6	6
5 factors	0.8	1.8	1.6	1.2	0.8	1.0	1.8	1.1
	1	3	2	2	1	1	3	2
With Surveys								
3 factors	13.9	7.5	10.2	8.3	7.0	6.5	5.7	11.6
	17	10	13	11	9	9	8	15
4 factors	3.7	5.3	3.2	2.8	2.8	3.5	4.4	4.7
	5	7	4	3	4	5	6	6
5 factors	2.9	4.2	2.8	2.1	2.4	2.8	3.1	4.1
	4	6	4	3	3	4	4	5
	B. UK Real Yields							
	1y	2y	3y	4y	5y	7y	10y	
Unconstrained								
3 factors	24.6	19.1	7.9	7.5	10.4	15.8	21.2	
	32	24	11	10	14	20	26	
4 factors	24.6	20.2	8.4	7.0	9.9	15.4	21.1	
	31	26	11	10	14	20	26	
5 factors	24.9	21.2	8.9	7.5	9.7	14.6	20.1	
	30	27	12	10	13	19	26	
With Surveys								
3 factors	24.9	19.3	8.4	7.8	10.6	16.0	21.4	
	32	25	12	11	15	21	27	
4 factors	24.0	20.4	8.7	7.3	10.0	15.5	21.4	
	31	26	12	10	14	20	26	
5 factors	6.9	4.9	3.6	3.3	3.3	2.1	6.0	
	7.9	6.2	4.7	4.2	4.3	2.9	7.8	

Notes: The table shows mean absolute errors across models for each maturity for the nominal (top panel) and real (bottom panel) yields. All figures are expressed in annualized basis points. For each maturity (column) and each number of factors (rows) the mean of the mean absolute error for the model estimated for the full sample is shown for that maturity. The standard deviation of errors for each maturity and model is shown in the line below.

Table A3: Asymptotic mean under P-dynamics

Factors	Estimation sample (starting year))							
	1972	1977	1982	1987	1992	1997	2002	2007
				A. US				
	Unrestricted							
3	4.5	3.7	2.3	1.6	2.1	1.5	1.6	-5.8
4	4.0	3.0	2.0	0.8	1.4	0.6	1.3	-5.2
5	4.3	5.7	3.2	2.2	3.1	0.0	4.4	35.8
	With Surveys							
3	5.0	4.9	4.0	3.4	4.2	4.3	4.1	6.3
4	4.9	4.9	4.3	4.2	4.5	4.5	4.4	5.6
5	4.4	4.4	4.4	4.0	4.4	4.3	4.2	9.5
	Sharpe ratio 0.5							
3	4.6	4.7	6.5	4.1	3.6	2.9	1.4	2.7
4	4.6	4.5	9.3	6.0	4.2	4.0	2.6	2.6
5	5.6	5.8	10.5	5.6	5.2	3.8	3.1	3.9
	Sharpe ratio 0.3							
3	4.7	5.3	8.3	5.3	4.0	3.3	2.5	3.6
4	7.6	7.4	10.1	6.6	5.5	4.9	3.7	3.5
5	8.6	8.4	15.5	5.6	5.6	5.5	4.9	0.4
				B. UK				
	Unrestricted							
3	6.3	7.4	8.2	5.9	5.3	2.6	2.6	2.4
4	6.3	7.0	8.1	6.5	5.6	3.2	3.2	3.4
5	6.5	7.5	8.0	6.5	5.7	2.8	3.1	3.2
	With Surveys							
3	7.3	8.1	8.7	7.2	6.9	5.4	5.0	5.1
4	6.5	6.9	8.0	7.2	7.2	5.3	5.1	5.1
5	5.8	6.0	6.5	6.5	6.3	5.4	5.0	5.0
	Sharpe ratio 0.5							
3	6.3	7.4	8.2	5.9	5.3	2.6	8.9	2.5
4	6.3	7.0	8.1	6.0	5.6	3.2	3.5	3.3
5	6.6	7.4	7.8	6.5	5.9	3.7	4.1	3.5
	Sharpe ratio 0.3							
3	6.3	7.4	8.2	5.9	5.3	2.9	6.7	3.5
4	6.8	7.5	8.9	5.4	5.8	3.8	5.1	4.2
5	6.9	7.9	8.2	5.0	6.0	4.8	4.7	4.0

Notes: The table shows the estimated unconditional mean under the $\mathbb{P}$-dynamics of nominal interest rates for US (Panel A) and UK (Panel B). For each number of factors and estimation method (blocks of rows), and each sample estimation period (columns). All figures are expressed in annualized percentage points.

Table A4: Persistence under P-dynamics

Factors	Estimation sample (starting year))							
	1972	1977	1982	1987	1992	1997	2002	2007
				A. US				
	Unrestricted							
3	6.9	8.5	4.0	5.7	2.9	1.7	3.1	2.4
4	7.9	10.1	4.4	7.5	4.4	3.4	2.4	2.2
5	7.0	7.5	2.6	4.4	0.9	2.9	0.9	0.9
	With Surveys							
3	7.0	6.5	4.3	4.9	1.8	1.3	0.9	4.8
4	6.7	6.1	4.2	3.7	2.0	2.0	1.8	3.8
5	2.6	3.2	1.8	1.9	2.2	2.1	1.8	8.8
	Sharpe ratio 0.5							
3	5.5	5.9	23.3	8.2	6.0	2.6	1.6	5.6
4	6.3	6.5	19.4	10.8	2.7	2.8	2.3	2.8
5	7.5	8.5	10.8	7.3	6.0	2.7	1.1	2.0
	Sharpe ratio 0.3							
3	6.3	7.0	11.0	7.3	2.7	1.9	3.3	5.0
4	6.5	6.0	6.9	5.5	3.2	2.3	1.5	3.2
5	6.9	5.9	13.5	3.8	2.7	2.1	2.8	0.1
				B. UK				
	Unrestricted							
3	6.9	11.1	19.5	10.6	5.5	2.9	2.9	4.0
4	6.3	9.5	19.0	10.5	4.2	2.6	1.7	1.2
5	6.8	9.8	16.6	9.7	3.5	4.6	2.0	1.6
	With Surveys							
3	7.2	10.6	16.1	7.7	8.1	1.9	1.2	1.3
4	6.1	10.9	9.8	6.1	5.6	1.7	1.4	1.6
5	5.1	5.5	4.2	5.6	6.0	1.3	1.4	1.7
	Sharpe ratio 0.5							
3	6.9	11.1	19.5	10.6	5.5	2.9	expl.	4.5
4	6.3	9.5	19.0	10.8	4.2	2.6	1.9	1.7
5	5.9	10.4	16.2	11.3	4.0	3.0	3.0	2.0
	Sharpe ratio 0.3							
3	6.9	11.1	19.5	10.6	5.5	2.4	expl.	2.1
4	7.6	7.3	16.9	28.6	5.5	1.6	9.5	1.8
5	8.5	10.9	18.6	2.5	5.8	2.3	3.7	2.0

Notes: The table shows the estimated half-life of the largest eigenvalue under the $\mathbb{P}$-dynamics for US (Panel A) and UK (Panel B). For each number of factors and estimation method (blocks of rows), and each sample estimation period (columns). All figures are expressed in years.

Table A5: Sensitivity of term premia estimates to the inclusion of surveys in filtering

Percentile	Estimation sample (starting year))							
	1972	1977	1982	1987	1992	1997	2002	2007
	A. US Nominal Yields							
3 factors								
0.5	0.0	0.0	0.0	0.1	0.1	0.1	0.1	1.1
0.9	0.8	0.9	0.8	0.7	0.5	0.5	1.4	6.0
0.95	1.2	1.4	1.2	1.1	0.7	0.8	2.3	10.0
0.99	2.3	2.6	2.2	2.0	1.3	1.2	3.3	21.1
0.999	5.0	5.4	4.7	3.3	2.2	1.7	3.8	38.3
4 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5
0.9	0.2	0.3	0.2	0.3	0.1	0.4	0.4	5.4
0.95	0.4	0.5	0.4	0.4	0.1	0.5	0.6	7.5
0.99	0.9	1.1	0.8	0.8	0.3	0.9	0.9	12.6
0.999	1.9	2.3	1.6	1.2	0.5	1.0	0.9	16.0
5 factors								
0.5	13.7	7.1	9.8	7.7	6.3	7.0	5.6	0.1
0.9	49.5	20.7	26.0	28.6	22.2	25.6	25.5	1.7
0.95	63.2	27.2	33.1	39.1	32.0	38.0	36.3	2.2
0.99	95.5	44.5	53.2	63.9	53.3	60.8	58.6	2.7
0.999	129.9	64.3	86.0	92.3	72.6	81.4	81.1	3.2
	B. UK Nominal Yields							
3 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.9	0.0	0.1	0.0	0.1	0.1	0.1	0.1	0.2
0.95	0.2	0.2	0.1	0.1	0.1	0.2	0.1	0.3
0.99	0.6	0.4	0.2	0.2	0.1	0.3	0.2	0.4
0.999	0.9	0.6	0.3	0.3	0.2	0.4	0.3	0.4
4 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.9	2.5	3.1	3.8	0.6	1.1	0.4	0.0	0.1
0.95	11.9	12.3	12.4	1.3	1.7	0.5	0.1	0.1
0.99	29.8	28.2	25.3	2.1	2.6	0.8	0.1	0.2
0.999	58.5	55.3	42.0	2.9	3.6	0.9	0.1	0.2
5 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.9	1.7	2.3	2.2	0.7	0.4	0.3	0.3	0.7
0.95	8.5	9.5	5.1	1.4	0.7	0.4	0.5	1.1
0.99	22.1	20.2	10.3	2.5	1.3	0.8	0.9	2.2
0.999	40.5	34.4	15.3	3.4	1.7	1.1	1.0	2.2

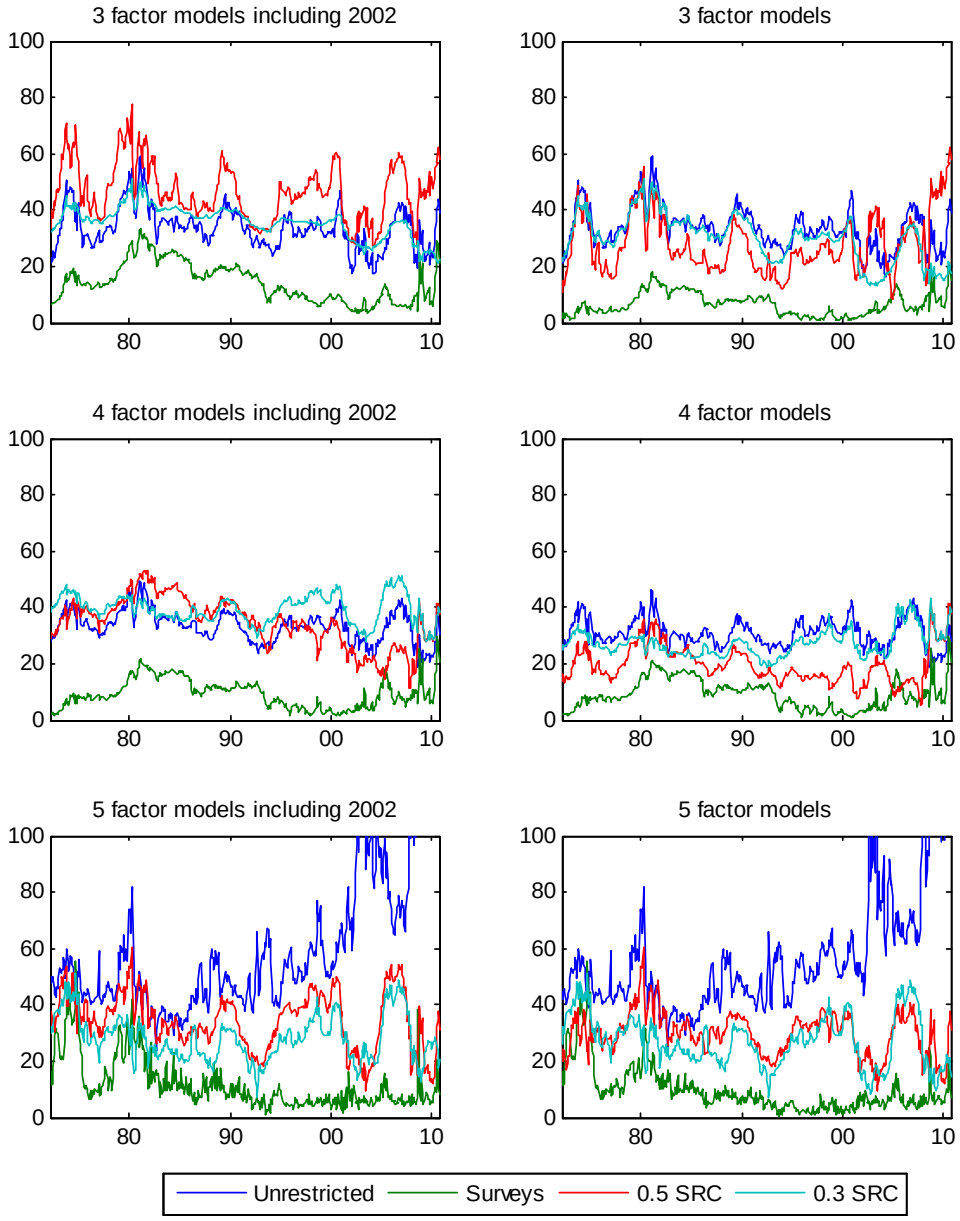
Notes: The table shows the percentiles of the absolute differences between the estimates of spot term premia for the models estimated using survey forecasts when the surveys are not used in filtering the states. For each number of factors (blocks of rows), and each sample estimation period (columns), the percentiles of the absolute difference between the term premia estimates for maturities from 10 years to 20 years, with and without surveys used in filtering, are shown along the rows for each block. All figures are expressed in annualized basis points.

Table A6: Sensitivity of joint model term premia estimates to the inclusion of surveys in filtering

Percentile	Estimation sample (starting year))							
	1972	1977	1982	1987	1992	1997	2002	2007
	A. UK Nominal Yields							
3 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.9	0.5	0.5	0.8	0.1	0.1	0.3	0.6	0.1
0.95	1.1	1.2	2.0	0.1	0.2	0.5	1.1	0.1
0.99	2.9	2.6	4.3	0.3	0.5	1.6	2.2	0.2
0.999	10.1	8.9	5.5	0.6	0.8	2.5	3.4	0.2
4 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3
0.9	0.3	0.4	0.4	0.6	0.6	0.3	0.5	1.5
0.95	0.6	0.7	0.6	1.6	0.9	0.7	1.0	1.7
0.99	1.5	1.2	1.4	2.9	1.5	1.6	1.7	3.0
0.999	4.9	5.0	5.8	4.9	2.3	3.0	2.0	3.1
5 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
0.9	0.2	0.3	0.3	0.6	0.6	0.5	0.6	0.7
0.95	0.6	0.7	1.0	1.2	0.9	0.9	0.9	1.4
0.99	1.8	1.9	3.8	4.9	1.5	2.1	1.4	2.2
0.999	4.1	3.9	7.1	10.1	2.0	3.8	1.7	2.5
	B. UK Real Yields							
3 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.9	0.2	0.2	0.4	0.1	0.1	0.1	0.1	0.3
0.95	0.5	0.5	0.9	0.4	0.2	0.2	0.2	0.3
0.99	1.4	1.4	2.0	0.7	0.3	0.6	0.4	0.5
0.999	2.3	2.3	2.5	1.3	0.4	1.1	0.7	0.5
4 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
0.9	0.1	0.1	0.1	0.2	0.3	0.1	0.6	1.6
0.95	0.2	0.2	0.2	0.4	0.4	0.2	0.9	1.8
0.99	0.7	0.8	0.7	0.9	0.7	0.4	1.6	3.1
0.999	2.7	2.9	2.8	1.7	1.1	0.9	2.0	3.1
5 factors								
0.5	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
0.9	0.3	0.4	0.7	1.6	0.3	0.7	0.4	0.3
0.95	0.6	0.8	1.1	4.5	0.4	1.1	0.7	0.5
0.99	1.7	2.0	2.5	15.4	0.7	2.2	1.0	0.9
0.999	3.2	3.3	5.0	27.0	0.9	3.4	1.3	1.1

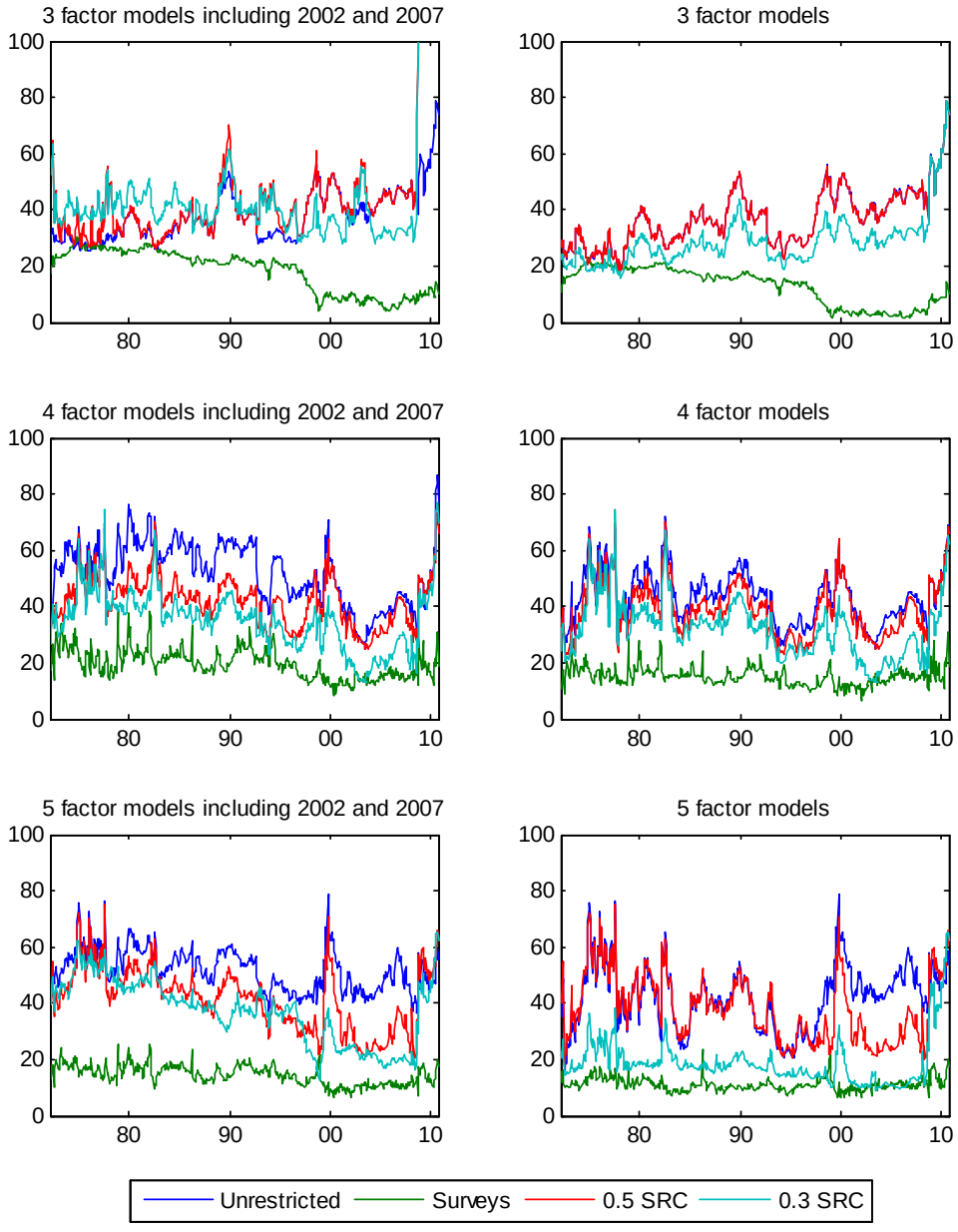
Notes: The table shows the percentiles of the absolute differences between the estimates of spot nominal (top panel) and real (bottom panel) term premia for the models estimated using survey forecasts when the surveys are not used in filtering the states. For each number of factors (blocks of rows), and each sample estimation period (columns), the percentiles of the absolute difference between the term premia estimates for maturities from 10 years to 20 years, with and without surveys used in filtering, are shown along the rows for each block. All figures are expressed in annualized basis points.

Figure A1: Range of 10 year nominal spot term premia as a proportion of fitted yields for US



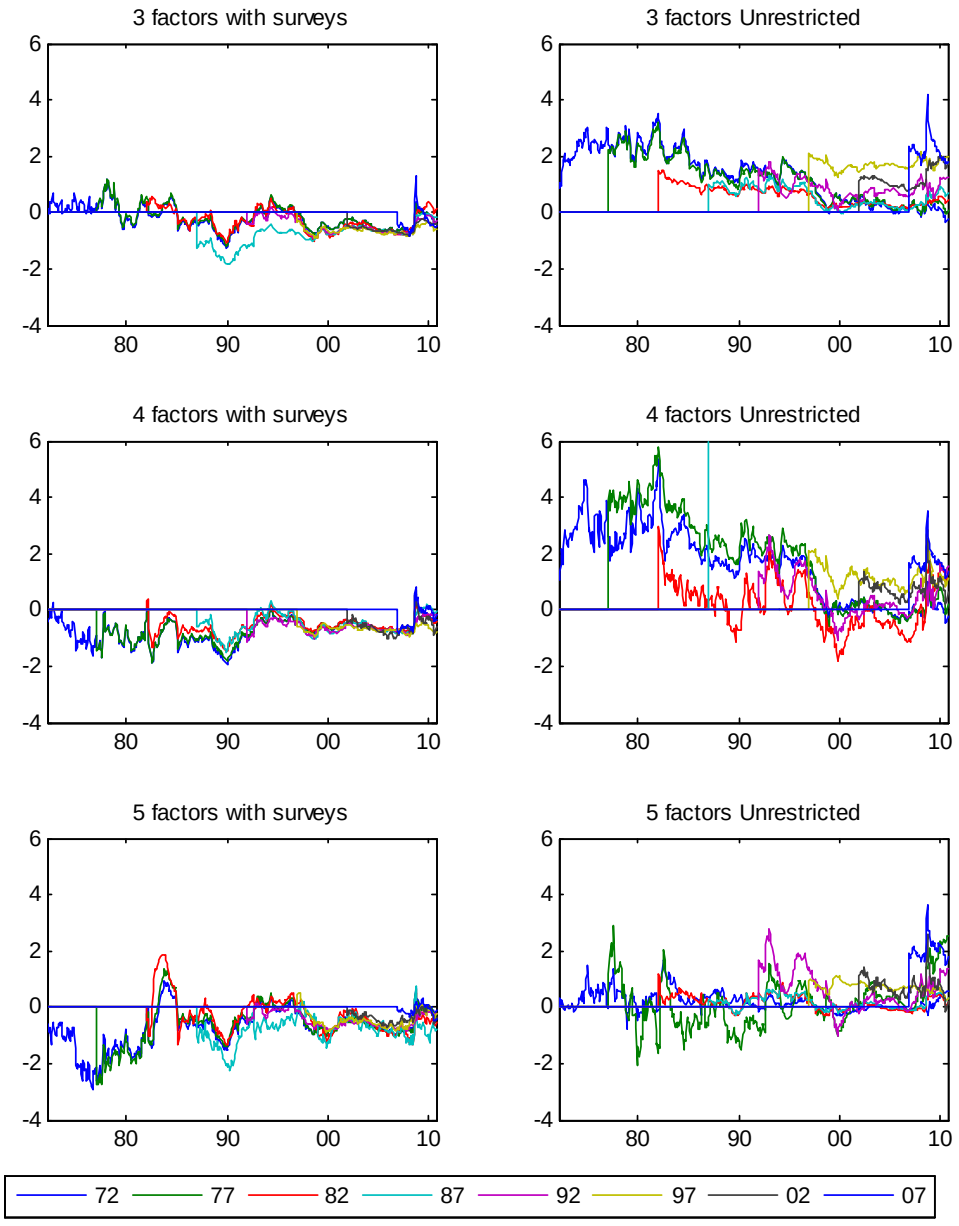
Notes: The figure shows the range of the 10 year spot term premium estimates for the US nominal government bond yields for the entire sample as a proportion of the fitted yield (see Equation (1) and the description in page 1). Each chart shows the range of estimates for the four different estimation strategies (unrestricted, using surveys and with a 0.5 average constraint on the maximum Sharpe ratio) for a given number of factors. In each chart, the range for each estimation strategy is calculated across the 7 different estimation samples (with starting dates 1972:5:2002) with that strategy.

Figure A2: Range of 10 year nominal spot term premia as a proportion of fitted yields for UK



Notes: The figure shows the range of the 10 year spot term premium estimates for the UK nominal government bond yields for the entire sample as a proportion of the fitted yield (see Equation (1) and the description in page 1). Each chart shows the range of estimates for the three of the different estimation strategies (unrestricted, using surveys and with a 0.5 average constraint on the maximum Sharpe ratio) for a given number of factors. In each chart of the right column, the range for each estimation strategy is calculated across the 6 different starting dates (1972:5:1997) with that strategy, while the charts on the left column also include the estimates with samples starting in 2002 and 2007.

Figure A3: UK 10 year real term premium with and without surveys



Notes: The figure shows the 10 year spot real term premium estimates for the UK government bond yields for a total of 48 estimated models. All figures are in percentage points per annum. The models vary by sample, with 8 different samples shown in each chart. The samples vary by start date, starting every 5 years from 1972 to 2007, with all samples ending in Dec 2010. The 3 models varying by number of factors (3 to 5) are displayed along the rows. The models using surveys are displayed in the left column and the unrestricted models in the right column. The forecasts for 1, 2 and 3 years ahead Bank Rate from the Bank of England's Survey of External Forecasters and the forecasts for inflation from Consensus Forecasts for 1, 2, 3, 4, 5 years ahead and the average between 6 and 10 years ahead were used for estimation of the models with surveys.

B Monte Carlo

- The description of the design of these Monte Carlo experiments is in the Appendix of the paper.
- Cramér-von Mises test P-values: Tables B1 through B3 show the same median p-values for the Cramér-von Mises test for the Monte Carlo experiments MC2 - MC4, respectively, as the table for MC1 in the paper.
- Bias: Tables B4 through B6 show the same statistics for the bias in unconditional mean and half-lifes of the largest eigenvalue of Φ for the Monte Carlo experiments MC2 - MC4, respectively, as the table for MC1 in the paper.

Table B1: Monte Carlo P-values for Alternative Time Series and Cross-Section Estimates (MC2)

A. Estimates from Time Series

	40	50	70	100	200	300	500	700	1000
0.5	0%	0%	1%	24%	55%	76%	89%	98%	99%
1	0%	0%	0%	2%	11%	26%	42%	70%	88%
2	0%	0%	0%	0%	0%	1%	5%	19%	36%
3	0%	0%	0%	0%	0%	0%	1%	3%	10%
5	0%	0%	0%	0%	0%	0%	0%	0%	0%
7	0%	0%	0%	0%	0%	0%	0%	0%	0%
10	0%	0%	0%	0%	0%	0%	0%	0%	0%
15	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	0%	0%	0%	0%	0%	0%	0%	0%	0%
50	0%	0%	0%	0%	0%	0%	0%	0%	0%

B. Estimates with Cross-Section Forecasts

	Like Q			Data US			Data UK		
	Noise	Survey	Yields	Noise	Survey	Yields	Noise	Survey	Yields
0.5	1%	100%	100%	0%	53%	100%	0%	1%	16%
1	0%	99%	100%	0%	42%	100%	0%	0%	10%
2	0%	99%	100%	0%	7%	85%	0%	0%	1%
3	0%	99%	100%	0%	0%	32%	0%	0%	0%
5	0%	97%	100%	0%	0%	2%	0%	0%	0%
7	0%	42%	100%	0%	0%	2%	0%	0%	0%
10	0%	10%	100%	0%	0%	100%	0%	0%	0%
15	0%	0%	100%	0%	0%	0%	0%	0%	0%
20	0%	0%	99%	0%	0%	0%	0%	0%	0%
30	0%	0%	71%	0%	0%	0%	0%	0%	0%
50	0%	0%	13%	0%	0%	0%	0%	0%	0%

Notes: The table shows the median of the p-values from the pairwise Cramér-von Mises test for common estimated dynamics from the Monte Carlo experiment model 'MC2' (described in Appendix of the paper). This is a 3 factor VAR, with largest eigenvalue of 0.9997. The test is applied to all pairwise combinations of forecasts from the 1000 estimates. The forecasts are generated using actual estimated factors from UK data (the same from which the true parameters were taken) for different forecast horizons (rows).

Table B2: Monte Carlo P-values for Alternative Time Series and Cross-Section Estimates (MC3)

A. Estimates from Time Series

	40	50	70	100	200	300	500	700	1000
0.5	3%	7%	40%	74%	86%	95%	97%	99%	99%
1	0%	0%	7%	32%	49%	67%	78%	89%	95%
2	0%	0%	0%	4%	11%	22%	33%	49%	64%
3	0%	0%	0%	0%	1%	5%	10%	20%	32%
5	0%	0%	0%	0%	0%	0%	1%	2%	5%
7	0%	0%	0%	0%	0%	0%	0%	0%	1%
10	0%	0%	0%	0%	0%	0%	0%	0%	0%
15	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	0%	0%	0%	0%	0%	0%	0%	0%	0%
50	0%	0%	0%	0%	0%	0%	0%	0%	0%

B. Estimates with Cross-Section Forecasts

	Like Q			Data US			Data UK		
	Noise	Survey	Yields	Noise	Survey	Yields	Noise	Survey	Yields
0.5	69%	99%	100%	29%	87%	99%	23%	74%	98%
1	33%	99%	100%	3%	61%	99%	2%	40%	97%
2	6%	98%	100%	0%	26%	88%	0%	7%	96%
3	1%	98%	100%	0%	14%	75%	0%	1%	67%
5	0%	98%	100%	0%	4%	66%	0%	0%	3%
7	0%	85%	100%	0%	1%	70%	0%	0%	0%
10	0%	13%	99%	0%	0%	54%	0%	0%	0%
15	0%	0%	47%	0%	0%	1%	0%	0%	0%
20	0%	0%	4%	0%	0%	0%	0%	0%	0%
30	0%	0%	0%	0%	0%	0%	0%	0%	0%
50	0%	0%	0%	0%	0%	0%	0%	0%	0%

Notes: The table shows the median of the p-values from the pairwise Cramér-von Mises test for common estimated dynamics from the Monte Carlo experiment model 'MC3' (described in Appendix of the paper). This is a 4 factor VAR, with largest eigenvalue of 0.9914. The test is applied to all pairwise combinations of forecasts from the 1000 estimates. The forecasts are generated using actual estimated factors from US data (the same from which the true parameters were taken) for different forecast horizons (rows).

Table B3: Monte Carlo P-values for Alternative Time Series and Cross-Section Estimates (MC2)

A. Estimates from Time Series

	40	50	70	100	200	300	500	700	1000
0.5	3%	9%	53%	92%	99%	99%	99%	100%	100%
1	0%	0%	13%	55%	79%	90%	96%	99%	99%
2	0%	0%	0%	11%	27%	41%	54%	73%	87%
3	0%	0%	0%	1%	5%	11%	19%	35%	54%
5	0%	0%	0%	0%	0%	0%	0%	1%	5%
7	0%	0%	0%	0%	0%	0%	0%	0%	0%
10	0%	0%	0%	0%	0%	0%	0%	0%	0%
15	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	0%	0%	0%	0%	0%	0%	0%	0%	0%
50	0%	0%	0%	0%	0%	0%	0%	0%	0%

B. Estimates with Cross-Section Forecasts

	Like Q			Data US			Data UK		
	Noise	Survey	Yields	Noise	Survey	Yields	Noise	Survey	Yields
0.5	38%	99%	100%	17%	81%	100%	15%	31%	60%
1	16%	99%	100%	2%	53%	100%	1%	11%	51%
2	2%	99%	100%	0%	9%	78%	0%	1%	29%
3	0%	99%	100%	0%	1%	31%	0%	0%	10%
5	0%	85%	100%	0%	0%	1%	0%	0%	0%
7	0%	67%	100%	0%	0%	1%	0%	0%	0%
10	0%	48%	100%	0%	0%	99%	0%	0%	0%
15	0%	0%	96%	0%	0%	0%	0%	0%	0%
20	0%	0%	51%	0%	0%	0%	0%	0%	0%
30	0%	0%	3%	0%	0%	0%	0%	0%	0%
50	0%	0%	0%	0%	0%	0%	0%	0%	0%

Notes: The table shows the median of the p-values from the pairwise Cramér-von Mises test for common estimated dynamics from the Monte Carlo experiment model 'MC4' (described in Appendix of the paper). This is a 4 factor VAR, with largest eigenvalue of 0.9998. The test is applied to all pairwise combinations of forecasts from the 1000 estimates. The forecasts are generated using actual estimated factors from UK data (the same from which the true parameters were taken) for different forecast horizons (rows).

Table B4: Monte Carlo (MC2) Percentiles of the Bias for Estimates of Unconditional Mean and Half-life

Model	40	50	100	200	300	400	500	700	1000
	A. Estimates from Time Series								
	Asymptotic Mean in percentage points (DGP = -24.4)								
0.01	-109.2	-127.6	-144.8	-300.5	-111.0	-72.1	-56.1	-48.9	-51.4
0.05	-23.5	-22.2	-32.7	-37.0	-37.9	-33.2	-30.8	-30.5	-27.8
0.10	-14.7	-15.8	-20.3	-27.6	-25.0	-24.3	-22.3	-21.5	-20.7
0.50	0.6	1.2	1.2	0.7	1.0	1.1	1.3	2.3	1.5
0.90	16.5	18.3	21.7	25.3	26.4	28.3	27.2	27.2	23.8
0.95	26.3	25.7	29.0	35.4	37.3	37.6	36.5	37.0	30.3
0.99	152.3	216.8	113.9	100.6	107.1	88.0	83.9	102.9	48.6
	Half-life in years (DGP=208)								
0.01	-207	-206	-205	-202	-201	-200	-198	-191	-185
0.05	-206	-205	-203	-201	-198	-196	-192	-184	-175
0.10	-205	-205	-203	-199	-195	-191	-186	-179	-167
0.50	-202	-201	-196	-183	-174	-166	-156	-140	-121
0.90	-164	-167	-152	-97	-85	-66	-65	-22	7
0.95	expl.	411	-2	198	46	35	21	86	92
0.99	expl.	expl.	expl.	expl.	expl.	expl.	4216	2413	597

Model	Like Q			Data US			Data UK		
	Noise	Survey	Yields	Noise	Survey	Yields	Noise	Survey	Yields
	B. Estimates with Cross-Section Forecasts								
	Asymptotic Mean in percentage points (DGP = -24.4)								
0.01	-57.2	-51.5	-7.0	-87.1	-130.2	-155.2	-96.2	-151.3	-81.6
0.05	-19.7	-28.2	-3.9	-22.8	-39.2	-95.4	-26.9	-25.2	-23.3
0.10	-14.3	-22.8	-3.0	-14.8	-20.9	-68.6	-15.5	-15.9	-14.9
0.50	0.4	-1.5	0.3	0.5	0.8	4.2	0.6	0.3	0.7
0.90	17.0	15.1	4.3	16.2	25.0	88.6	16.6	16.3	15.9
0.95	27.8	21.3	5.6	24.6	58.5	111.0	25.0	24.8	24.6
0.99	140.8	57.7	8.3	96.6	146.2	161.8	163.7	52.8	51.1
	Half-life in years (DGP=208)								
0.01	-205	-187	-54	-206	-201	-181	-206	-206	-204
0.05	-202	-174	-30	-205	-199	-149	-205	-204	-202
0.10	-201	-162	-17	-203	-199	-115	-204	-203	-201
0.50	-197	-2	0	-197	-189	-2	-198	-198	-198
0.90	expl.	2790	0	expl.	expl.	3	expl.	expl.	expl.
0.95	expl.	expl.	8	expl.	expl.	expl.	expl.	expl.	expl.
0.99	expl.	expl.	146	expl.	expl.	expl.	expl.	expl.	expl.

Notes: The table shows the percentiles of the bias in the estimated unconditional mean of interest rates $(\delta_0 + \delta_1 (I_N - \Phi)^{-1} \mu)$ and half-life of the largest eigenvalue of Φ $(\frac{1}{12} \frac{\ln(0.5)}{\ln(\max(eig(\Phi)))})$ from the Monte Carlo experiment (model 'MC2').

Table B5: Monte Carlo (MC3) Percentiles of the Bias for Estimates of Unconditional Mean and Half-life

Model	40	50	100	200	300	400	500	700	1000
	A. Estimates from Time Series								
	Asymptotic Mean in percentage points (DGP = 4.94)								
0.01	-4.7	-4.1	-2.7	-2.0	-1.6	-1.4	-1.2	-1.0	-0.8
0.05	-3.2	-3.0	-2.0	-1.4	-1.1	-0.9	-0.8	-0.7	-0.6
0.10	-2.5	-2.2	-1.6	-1.1	-0.9	-0.7	-0.6	-0.5	-0.5
0.50	0.0	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0
0.90	2.2	2.1	1.4	1.0	0.8	0.7	0.7	0.5	0.5
0.95	2.7	2.7	1.9	1.3	1.2	1.0	0.8	0.7	0.6
0.99	4.8	4.0	2.5	1.9	1.7	1.3	1.2	1.0	0.9
	Half-life in years (DGP=6.7)								
0.01	-5.7	-5.6	-4.9	-3.9	-3.5	-3.1	-2.7	-2.3	-2.0
0.05	-5.5	-5.3	-4.3	-3.4	-2.8	-2.5	-2.2	-1.8	-1.5
0.10	-5.2	-5.0	-3.9	-2.9	-2.4	-2.1	-1.8	-1.5	-1.3
0.50	-3.6	-3.2	-1.8	-1.0	-0.8	-0.5	-0.4	-0.3	-0.2
0.90	1.1	1.4	1.9	1.8	1.7	1.6	1.3	1.1	1.0
0.95	3.5	3.9	3.7	2.8	2.4	2.2	1.9	1.7	1.5
0.99	13.9	10.9	7.2	5.0	4.2	3.9	3.3	2.5	2.2

Model	Like Q			Data US			Data UK		
	Noise	Survey	Yields	Noise	Survey	Yields	Noise	Survey	Yields
	B. Estimates with Cross-Section Forecasts								
	Asymptotic Mean in percentage points (DGP = 4.94)								
0.01	-2.0	-0.5	-0.2	-3.6	-0.9	-0.4	-4.1	-2.5	-1.7
0.05	-1.2	-0.3	-0.1	-2.6	-0.5	-0.3	-3.1	-1.6	-1.1
0.10	-1.0	-0.2	-0.1	-2.0	-0.4	-0.2	-2.3	-1.2	-0.9
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.90	0.8	0.2	0.1	1.8	0.4	0.2	2.2	1.1	0.9
0.95	1.1	0.3	0.1	2.4	0.5	0.3	2.9	1.5	1.1
0.99	1.7	0.4	0.1	3.7	0.8	0.4	4.8	2.3	1.5
	Half-life in years (DGP=6.7)								
0.01	-5.2	-2.8	-0.9	-5.3	-4.8	-2.5	-5.3	-5.0	-4.4
0.05	-4.2	-2.1	-0.5	-4.6	-3.6	-1.7	-4.6	-4.2	-3.6
0.10	-3.7	-1.6	-0.3	-4.1	-3.0	-1.3	-4.1	-3.7	-3.1
0.50	-1.5	0.0	0.0	-1.8	-1.1	0.0	-1.8	-1.5	-1.2
0.90	1.2	0.9	0.2	1.4	0.9	0.7	1.7	1.3	2.2
0.95	2.4	1.6	0.5	4.0	1.7	1.3	4.3	3.4	3.9
0.99	6.8	2.9	1.1	14.5	3.4	3.2	21.9	7.9	8.4

Notes: The table shows the percentiles of the the bias in estimated unconditional mean of interest rates $(\delta_0 + \delta_1 (I_N - \Phi)^{-1} \mu)$ and half-life of the largest eigenvalue of Φ $(\frac{1}{12} \frac{\ln(0.5)}{\ln(\max(eig(\Phi)))})$ from the Monte Carlo experiment (model 'MC3').

Table B6: Monte Carlo (MC4) Percentiles of the Bias for Estimates of Unconditional Mean and Half-life

Model	40	50	100	200	300	400	500	700	1000
	A. Estimates from Time Series								
	Asymptotic Mean in percentage points (DGP = 14.35)								
0.01	-139.1	-97.6	-105.6	-50.8	-32.6	-31.9	-24.8	-19.3	-16.6
0.05	-27.9	-30.6	-29.2	-25.4	-20.3	-19.2	-15.7	-13.3	-11.7
0.10	-18.3	-21.0	-18.7	-17.4	-15.5	-15.1	-12.3	-10.8	-9.0
0.50	0.7	0.8	1.3	0.4	0.0	0.0	0.2	-0.3	-0.4
0.90	19.5	21.1	22.4	19.2	17.0	14.3	12.5	10.3	9.1
0.95	29.8	33.7	30.9	26.8	21.3	18.8	16.6	13.3	11.4
0.99	131.8	165.3	65.2	69.2	43.1	30.9	24.9	20.3	17.5
	Half-life in years (DGP=51.4)								
0.01	-49.5	-49.1	-47.6	-45.7	-44.5	-43.6	-43.1	-42.2	-39.4
0.05	-48.7	-48.2	-46.5	-44.2	-43.0	-41.9	-41.1	-39.1	-33.0
0.10	-48.2	-47.5	-45.7	-43.1	-41.9	-40.3	-39.0	-34.7	-29.7
0.50	-44.3	-43.0	-40.6	-35.1	-29.3	-24.9	-20.0	-15.2	-11.6
0.90	-3.8	3.4	-13.3	3.5	9.6	14.9	19.5	21.5	17.7
0.95	expl.	expl.	42.3	39.2	34.7	42.1	39.0	40.6	32.6
0.99	expl.	expl.	expl.	4111.3	273.6	193.0	103.8	89.9	63.1

Model	Like Q			Data US			Data UK		
	Noise	Survey	Yields	Noise	Survey	Yields	Noise	Survey	Yields
	B. Estimates with Cross-Section Forecasts								
	Asymptotic Mean in percentage points (DGP = 14.35)								
0.01	-87.0	-44.9	-5.5	-105.7	-98.6	-183.3	-122.0	-103.5	-87.2
0.05	-28.2	-15.1	-2.9	-27.0	-31.2	-38.0	-27.8	-28.4	-27.5
0.10	-19.3	-11.2	-2.1	-18.9	-19.5	-24.7	-18.4	-19.3	-18.3
0.50	0.7	0.7	0.1	0.7	0.5	0.7	0.5	0.4	0.9
0.90	20.4	11.7	2.2	19.9	20.2	31.6	19.8	20.2	20.1
0.95	34.1	15.6	2.8	31.5	29.5	41.4	29.3	30.8	32.9
0.99	132.3	87.6	4.8	153.2	119.7	68.5	155.4	129.8	256.5
	Half-life in years (DGP=51.4)								
0.01	-48.1	-41.8	-22.4	-49.0	-46.5	-41.9	-49.0	-48.8	-48.3
0.05	-46.7	-41.0	-15.0	-47.9	-44.4	-40.8	-48.1	-47.9	-47.2
0.10	-46.0	-40.6	-10.4	-47.3	-43.0	-39.9	-47.4	-47.2	-46.2
0.50	-38.7	-15.3	-0.1	-41.4	-37.3	-0.4	-42.2	-41.4	-40.0
0.90	expl.	281.2	0.3	expl.	expl.	0.7	302.9	expl.	expl.
0.95	expl.	expl.	10.3	expl.	expl.	expl.	expl.	expl.	expl.
0.99	expl.	expl.	52.2	expl.	expl.	expl.	expl.	expl.	expl.

Notes: The table shows the percentiles of the the bias in estimated unconditional mean of interest rates $(\delta_0 + \delta_1 (I_N - \Phi)^{-1} \mu)$ and half-life of the largest eigenvalue of Φ $(\frac{1}{12} \frac{\ln(0.5)}{\ln(\max(eig(\Phi)))})$ from the Monte Carlo experiment (model 'MC4').