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Long-term unemployment and convexity in the Phillips curve

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Abstract

The notion that the long-term unemployed are relatively detached from the labour market and therefore exert only little downward pressure on wage inflation has regained significant traction recently. This paper investigates whether the conclusion that long-term unemployment is only weakly related to inflation depends on the assumption of linearity in the Phillips curve. Specifically, once convexity is allowed for during the estimation process, long-term unemployment appears to have a significant negative influence on wage inflation, whereas in a linear Phillips curve model it is only the short-term unemployment rate that matters for wage dynamics. The intuition is simple; by the time the long-term unemployment rate rises during a recession, the economy may have already transitioned into a relatively flat region of the Phillips curve, generating the misperception that the marginal effect of long-term unemployment on wage inflation is smaller than that of short-term unemployment. Linear models which do not capture state dependence in the slope of the Phillips curve would therefore bias downwards the estimated importance of long-term unemployment in explaining wage dynamics if the true Phillips curve is convex.

Key words: Phillips curve, convexity, natural rate of unemployment, Kalman filter, long-term unemployment, hysteresis.

JEL classification: C22, E24, E32, J64.

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Summary

The relationship between wage growth and unemployment is a key trade-off concerning monetary policy makers, as labour costs form a critical part of the inflationary transmission mechanism. One important question is how the composition of the unemployment pool, and specifically the share of long-term unemployment, affects that trade-off. Detachment from the labour force is likely to increase with unemployment duration, so that the long-term unemployed search less actively for jobs and therefore exert less downward pressure on wages. If so, short-term unemployment may pull down on wage inflation more than long-term unemployment does. In this situation, policymakers might anticipate a period of high wage growth if short-term unemployment starts to fall to low levels even if the long-term unemployment rate remains elevated.

But there may be complications arising from the integral dynamics of unemployment. In this paper it emerges that the estimated disinflationary effects of long-term unemployment hinge on whether or not wage growth becomes less sensitive to unemployment as the latter rises – a form of non-linearity. One reason why the negative relationship between wages and unemployment might become flatter at high levels of unemployment is that workers may tend to resist cuts in their nominal wages. When unemployment is low, wage growth tends to be high as firms compete for a scarce pool of resources. But due to worker resistance to wage cuts the reverse might not hold to the same extent, with a relatively large increase in unemployment needed to reduce wage growth during a recession.

Why does this non-linearity matter for the measured effect of long-term unemployment on wage growth? It is because long-term unemployment inevitably lags behind movements in short-term unemployment as it takes time for the new unemployed to move into the long-term category. So high levels of long-term unemployed are only associated with lengthy periods of high unemployment. A flattening off of the relationship between wages and unemployment at high levels of unemployment would then imply that long-term unemployment does little to reduce wage inflation further. The apparently different effects of short and long-term unemployment on wage inflation could therefore be merely as a result of timing rather than labour market detachment among the long-term unemployed.

By modifying statistical models of labour market dynamics to incorporate this insight, this paper finds that there appears to be much less difference between the short and long-term unemployed in terms of their marginal influence on wage behaviour than is suggested by the recent literature. When the non-linearity described above is not taken into account, estimation results corroborate the finding already established in the literature that it is predominantly the short-term unemployed that matter for wage inflation. Long-term unemployment in this specification tends to have no statistically significant effect on wage inflation. When the non-linearity is taken into account, long-term unemployment has a much larger effect on wage inflation. For some of the specifications considered, the data fail to reject the hypothesis that short and long-term unemployment rates have equal effects on inflation. In some instances, the models even suggest that long-term unemployment creates more of a drag on wage growth than short-term unemployment does, all else equal. Statistical uncertainty makes it difficult to draw a very precise conclusion, but the results in this paper caution against excluding long-term unemployment from estimates of aggregate labour market slack as is suggested by much of the recent literature. Both the short-term unemployment rate and the long-term unemployment rate are likely to contain useful information for judging the degree of wage pressure in the economy.

Contents

1 Introduction

The latest Phillips curve puzzle which has captured the attention of central banks and researchers is the so-called "missing disinflation" of the post-2008 recession period in the United States. The term comes from the tendency of Phillips curves estimated on pre-2008 data to predict, conditional on subsequently observed labour market conditions, a period of significant disinflation following the 2008 recession, with actual inflation outturns remaining notably higher than predicted on the basis of such estimates (Ball and Mazumder, 2011).

This apparent breakdown in the Phillips trade-off precipitated a substantial effort to reconcile the model with the latest data. Ball and Mazumder (2011) conjectured that the effect of long-term unemployment on the Phillips curve is likely to be an important area of future research, citing previous work by Llaudes (2005) which demonstrated that long-term unemployment appeared to exert much less influence on inflation than short-term unemployment for a variety of countries. A tough econometric issue arises in the U.S. data due to the high degree of collinearity between short- and long-term unemployment rates. Kiley (2014) takes a closer look at this issue by exploiting regional data and finds no evidence that long-term unemployment is irrelevant for inflation. This finding stands in contrast to a number of other studies, in addition to that of Llaudes (2005), which argue that the long-term unemployment rate does not determine inflation and the post-2008 Phillips curve turns out to be stable if long-term unemployment is omitted from the slack variable (Krueger et al. 2014, Gordon 2013, Watson 2014 and Linder et al. 2014). As Paul Krugman recently phrased it, "[t]he suggestion that only the short-term unemployed matter for wage determination, that the long-term unemployed have been written off, is rapidly congealing into orthodoxy."[1] Perhaps so much so that theoretical treatments of the optimal monetary policy implications of the subject are starting to emerge (Rudebusch and Williams, 2014).

This paper contributes to this rapidly expanding literature by investigating whether the conclusion that long-term unemployment is only weakly related to inflation depends on the assumption of linearity in the Phillips curve. It is argued that non-linearity in the Phillips curve distorts the relationship between long-term unemployment and wage inflation in a way which makes it seem like long-term unemployment is irrelevant for wage dynamics in linear statistical models. When a convex specification of the Phillips curve is estimated, it turns out that the long-term unemployment rate tends to be significantly negatively related to wage inflation.

Non-linearities are always difficult to disentangle, but an attempt is made to offer a very simple potential explanation. Conditional on there being a convex inflation-unemployment trade-off, the fact that the rise in the long-term unemployment rate lags the rise in the short-term unemployment rate during a recession implies that the economy may already be in a relatively flat region of the Phillips curve by the time long-term unemployment has risen. Fluctuations in short- and long-term unemployment rates are out of phase with each other. This phase difference implies that differential inflationary effects associated with short- and long-term unemployment can arise purely because of state dependence in the slope of the Phillips curve rather than any intrinsic difference between the short- and long-term unemployed, such as their respective degrees of labour market attachment. Controlling for this non-linearity tends to overturn the result in the existing literature that the long-term unemployed are far less relevant, or even completely irrelevant, for wage determination.

Previous research mainly characterises inflation-unemployment dynamics as a linear process, with the notable exception of Linder et al. (2014) who estimate a log-linear version of the Phillips curve but do not consider the issues raised here and indeed even argue that the omission of long-term unemployment improves the predictive capability of the model. The approach taken to introduce non-linearity into the inflation-unemployment trade-off in this study is based on the work of Laxton et al. (1999), who impose a hyperbolic shape on the Phillips equation. It is, of course, possible to introduce non-linearity in a wide variety of ways, including more general methods which do not impose convexity prior to estimation. The assumptions

[1] See his blog post entitled "Low Inflation and Structural Illusions", April 9th 2014.

made in this study regarding functional forms are designed to keep the non-linearities in the model as simple as possible. The research question addressed here is therefore whether long-term unemployment still remains unimportant for inflation when the inflation-unemployment trade-off is convex instead of linear.

This study does not directly address the (perhaps more important) issue of whether convexity in the Phillips curve is a realistic property to begin with. Testing for the presence of non-linearity has been the focus of much previous empirical work and warrants separate investigation.[2] There is nonetheless a clear theoretical justification for assuming state dependence of the inflation-unemployment trade-off. Daly and Hobijn (2014) have recently shown how the presence of downward nominal wage rigidity "bends" the Phillips curve. In fact, the idea that the Phillips curve flattens off at lower rates of inflation can be traced back to the original Phillips (1958) article, in which it is stated in the opening paragraph that "[t]he relation between unemployment and the rate of change of wage rates is [...] likely to be highly non-linear."

In addition to Kiley (2014), Hooper et al. (2014) is another study which uncovers significant downward wage pressure from long-term unemployment using state level data for the US. However, the interesting finding in Hooper et al. (2014) in relation to the present study is their observation that long-term unemployment tends to be associated with downward wage pressure only when the short-term unemployment gap has closed, an empirical fact which is consistent with the non-linear dynamics outlined above. However, the explanation they provide is still based on the idea that the long-term unemployed are passive participants in the labour market relative to the short-term unemployed; when the short-term unemployment gap closes, the long-term unemployed become "more relevant for filling job openings and begin to exert downward pressure on wages". The alternative explanation offered in this paper is that non-linearity in the Phillips curve can be responsible for generating these dynamics rather than inherent differences in the labour force attachment of the short- and long-term unemployed.

There is a clear connection between the renewed discussion on the absence of a disinflationary effect of long-term unemployment in Phillips curves and the corresponding decades old debate on the effects of unemployment duration in the wage curve literature. The failure of high unemployment to moderate UK wages in the 1980s prompted discussion on whether long-term unemployment reduces the disinflationary pressure associated with excess labour supply at high levels of aggregate unemployment.[3] Blanchflower and Oswald (1990) stress the presence of non-linearities in the microeconomic data and how unemployment does not depress pay beyond a certain limit. They found that the share of long-term unemployment does not have an additional effect on wage levels once non-linearities in the unemployment rate are controlled for. The current study fills a gap in the recent applied Phillips curve literature by addressing concerns about unemployment duration and non-linearities which were previously considered to be important in the wage curve literature given the prevailing macroeconomic climate at the time. The results in this study could be interpreted as being complementary to, and consistent with, the earlier cross-sectional findings of Blanchflower and Oswald (1990) in the sense that non-linearity in the Phillips curve is also found to imply that unemployment duration does not matter for wage inflation. The potential significance of this finding owes to it obtaining despite the conceptual differences (which Blanchflower and Oswald (1995) make clear) between the wage and Phillips curves and the different data and modelling techniques used in the different frameworks.[4]

The remainder of the paper proceeds as follows. The model and estimation methods are outlined in the following section. Estimation results are reported and discussed in section three. The fourth section briefly considers a simple extension allowing hysteresis in the labour market to distort the time path of the natural

[2] See Yates (1998) for an overview and some empirical results for the UK.

[3] See Nickell (1987) and Blackaby et al. (1991).

[4] The wage curve "is an equilibrium locus that is a description neither of inherently temporary phenomena nor of transitory dynamics" (Blanchflower and Oswald, 1995). In contrast, the Phillips curve, for which the microfounded derivation by Gali (2011) makes clear, describes a disequilibrium process. Fluctuations in wage inflation are driven by divergences between actual and desired wage mark-ups caused by nominal rigidity.

rate. Section five concludes.

2 The Model

2.1 Outline

The general form of the Phillips curve that is used in this study is a simple expression relating wage inflation to a lagged indexation variable and a measure of the unemployment gap. It is based on the reduced form specification outlined in Gali (2011), who also provides a more rigorous theoretical treatment of the wage Phillips curve in a New Keynesian setting. This study's empirical analysis can be viewed as an extension of the empirical component of Gali (2011), which also focuses on a reduced form (linear) wage equation but with a constant natural rate of unemployment (considered here as a special case).

The starting point is the following simple relationship between wage inflation and unemployment,

$$\pi_t^w = \alpha + \gamma \pi_{t-1}^p - \lambda \left(u_t - u_t^n \right) \tag{1}$$

where π_t^w and π_t^p are wage and price inflation, respectively, $u_t - u_t^n$ is the difference between unemployment and its (time-varying) natural rate and $\{\alpha, \gamma, \lambda\}$ is a parameter vector. This is the augmented Phillips curve in Gali's (2011) terminology - that is, augmented to include wage indexation to past price inflation. As noted by Gali (2011), π_t^p need not correspond to period-on-period price inflation, but rather can assume the form of a smoother index such as a moving average of some specified length.

Following Llaudes (2005), this study considers a generalised version of (1) which allows short- and long-term unemployment rates to enter into the Phillips curve with potentially different weights. *Effective* unemployment is defined as

$$\widetilde{u}_t = \theta u_t^s + (1 - \theta) u_t^l$$

with u_t^s and u_t^l denoting the short- and long-term unemployment rates. The parameter θ, to be determined by estimation, is the object of primary concern in this study since it quantifies the extent to which the downward pressure on wage inflation associated with labour market slack is a function of unemployment duration. Accordingly, the effective unemployment gap is defined as the sum of two separate gaps, one each for the short- and long-term unemployment rates. The effective unemployment gap is expressed as

$$\widetilde{u}_t - \widetilde{u}_t^n = \theta \left(u_t^s - u_t^{s,n} \right) + (1 - \theta) \left(u_t^l - u_t^{l,n} \right) \tag{2}$$

so that separate natural rates - or trends - are defined for u_t^s and u_t^l. This results in the following duration dependent Phillips curve,

$$\pi_t^w = \alpha + \gamma \pi_{t-1}^p - \lambda \left(\widetilde{u}_t - \widetilde{u}_t^n \right) + \epsilon_t \tag{3}$$

where $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$ is i.i.d., so that the standard model is nested when $\theta = 0.5$.

Convexity is introduced into the Phillips equation by following the simple approach of Laxton et al. (1999). The convex version of equation (3) can then be expressed as

$$\pi_t^w = \alpha + \gamma \pi_{t-1}^p - \lambda \frac{\widetilde{u}_t - \widetilde{u}_t^n}{\widetilde{u}_t - a} + \epsilon_t \tag{4}$$

where the asymptote a can be thought of as representing capacity constraints in the labour market, or the level of effective unemployment below which inflationary pressure becomes unbounded. Equation (4) *postulates* a convex relationship between π_t^w and $\widetilde{u}_t$. However, the degree of convexity is controllable, in the sense that an asymptote above zero can be imposed and can be time-varying, as in Laxton et al. (1999). In this paper, a is treated as a parameter to be estimated. The Phillips equations are augmented with an additional explanatory variable representing productivity shocks in order to capture business cycle fluctuations in wages that are not entirely driven by aggregate demand.[5]

2.2 Different approaches to the filtering problem

Two standard filtering methods are considered in order to estimate the time paths of $u_t^{s,n}$ and $u_t^{l,n}$. The first is a simple univariate Hodrick-Prescott (H-P) filter. Two different calibrations for the smoothing parameter are considered. For the first, the smoothing parameter is set to the standard value of 1600, applied ubiquitously to smooth quarterly macroeconomic time series data. In this case, the unemployment gap is pre-determined in the sense that it is not estimated jointly with wage dynamics. For the second calibration exercise, an effort is made to relax the degree of arbitrariness associated with controlling model smoothness by specifying a data-based criterion for penalising variability in $u_t^{s,n}$ and $u_t^{l,n}$. Specifically, the smoothing parameter is chosen so as to maximise the fit of the Phillips equation as measured by its R^2. To do this, a search for the optimal (in this simple sense) H-P smoothing parameter value is carried out over a wide range from 100 to 100000. This calibration is referred to as the "optimised H-P filter".

The econometric strategy most frequently adopted in the modern literature on Phillips curve estimation is an application of the Kalman smoother, which is the approach also taken by Llaudes (2005) in his estimation of duration dependent Phillips curves. The convention in the literature is to assume that the unobserved component of unemployment follows a random walk. In the effective unemployment model outlined above this implies that $u_t^{s,n}$ and $u_t^{l,n}$ both follow random walks,

$$\begin{aligned} u_t^{s,n} &= u_{t-1}^{s,n} + \nu_t^s \\ u_t^{l,n} &= u_{t-1}^{l,n} + \nu_t^l \end{aligned} \tag{5}$$

where $\nu_t^i \sim N\left(0, \sigma_{\nu^i}^2\right)$ for $i = \{s, l\}$ and it is further assumed that the error terms ν_t^s and ν_t^l have zero correlation structure with ϵ_t and with each other.[6] Together with the Phillips equations (3) and (4), the resulting systems can be cast straightforwardly in state space form and the parameters estimated by maximum likelihood through a standard application of the Kalman filter.

The major practical issue with the Kalman filter is that the maximum likelihood estimate of the variance of the non-stationary component of unemployment is biased downwards if its true value is close to zero.[7] In order to deal with this issue, it has become common to fix the signal-to-noise ratio prior to estimation. This might be viewed as a serious limitation of this particular filtering process since ideally the optimal degree of variation in the permanent component of unemployment would be determined as part of the estimation process itself rather than imposed in a somewhat arbitrary manner. Nevertheless, Gordon (1997) argues that it is reasonable to form an economic prior regarding the time series behaviour of $\widetilde{u}_t^n$ - or more precisely in the current model, of both $u_t^{s,n}$ and $u_t^{l,n}$. In order for the concept of a natural rate of unemployment to be useful from a policy perspective, the signal-to-noise ratio should be low enough such that large quarter-on-quarter fluctuations in $\widetilde{u}_t^n$ are avoided. This prior is rationalised on the basis that $\widetilde{u}_t^n$ is meant to vary only quite slowly over time as institutions evolve or in response to significant structural change. Here results are reported for a baseline signal-to-noise ratio of 0.05 for both $u_t^{s,n}$ and $u_t^{l,n}$. Prior to the application of the Kalman filter, the parameter vector, apart from θ, is initialised based on the results of a simple non-linear least squares regression of (3) or (4) using the (standard) H-P filtered data for the unemployment gaps. The initial value for θ is always 0.5. The unobserved state variables are initialised at the starting trend values

[5] This is a deviation-from-trend variable for productivity which is calculated as the change in the ratio of productivity per hour to a quadratic time trend, similarly to Gordon (1997).

[6] A more general error structure with non-zero covariance between ν_t^s and ν_t^l is also possible, and would seem logical. However, attempts to estimate the covariance returned statistically insignificant results for both the linear and convex Phillips curves. Therefore, the model is kept simple by imposing zero covariance between ν_t^s and ν_t^l.

[7] This is often referred to as the pile-up problem. See Basistha and Startz (2008).

obtained from H-P filtered unemployment data (with smoothing parameter 1600).

2.3 Data

Wage inflation is measured by the log quarterly growth rate of Average Weekly Earnings for the whole economy, for which a historical series is constructed by the Office for National Statistics, divided by average weekly hours. Price inflation is represented by the log quarterly change in the consumer price index. A claimant count measure of unemployment is used which is based on the measure of individuals who claim job search benefits, sorted by duration and expressed as a percentage of the aggregate labour force using employment data from the Labour Force Survey.[8] Short-term unemployment is defined as those claiming benefits for up to six months while long-term unemployment is any duration above that.

3 Results

3.1 Linear model

Table 1 presents results from the estimation of the linear Phillips curve using the different filtering techniques described above and also including a specification in which the natural rate of unemployment is constant. Qualitatively, the overall impression from Table 1 reflects results typically obtained in the recent literature on linear Phillips curves. The long-term unemployment rate does not appear to moderate wage pressure. Point estimates of the weight on short-term unemployment, $\widehat{\theta}$, in both the H-P and Kalman filtered versions are essentially unity. This is slightly higher than the value of 0.88 obtained by Llaudes (2005) for the UK using a Kalman filtering approach, although the difference in our findings is not statistically meaningful. The parameter estimates for the standard and optimised H-P filter specifications are very similar. It turns out that the optimal H-P smoothing parameter is 2736, which does not yield unemployment gap dynamics that are substantially different to the more standard calibration. Figure 1 plots the R^2 from the Phillips regressions against the smoothing parameter. This shows that the degree of variation is not large; raising the smoothing parameter from a low of 100 to its optimal value improves the fit of the model by about 7%, but further increases in the smoothing parameter do little to the fit of the model.

The estimation results for the constant natural rate model suggest that $\widehat{\theta}$ is significantly higher than one, implying that long-term unemployment appears to have a significant *positive* effect on inflation. This effect is also present in the Kalman filtered version of the linear Phillips curve, although only to a statistically insignificant extent. What could rationalise $\theta > 1$? If long-term unemployment acts as a negative supply shock in the labour market, perhaps because people who experience long spells of unemployment find it difficult to re-enter employment, then the labour market would effectively be tighter, for a given level of aggregate unemployment, when the long-term unemployment share is relatively high. This idea is not new and is based on the hypothesis of hysteresis effects associated with rising unemployment duration and is considered more carefully in section 4.

In sum, the linear specifications with time variation in the natural rate of unemployment replicate the standard finding in the literature that there appears to be little or no role for long-term unemployment in affecting inflation dynamics. When the natural rate is held constant, long-term unemployment appears to be inflationary. The notable difference between the constant and time-variable natural rate specifications indicates that the manner in which unemployment gap dynamics are modelled has quantitatively important implications for the estimation of the relative importance of long-term unemployment in influencing inflation dynamics.

[8] I am especially grateful to Philip King for sharing his historical data on the claimant count by duration, constructed using Department of Employment archives to extend backwards the more recent claimant count data as published by the ONS.

This last point alludes to a potential sensitivity in the calibration of the signal-to-noise ratio in the Kalman filter specification. Figure 2 plots $\widehat{\theta}$ for signal-to-noise ratios ranging from 0 to 1. It turns out that $\widehat{\theta}$ is actually quite sensitive to assumptions made about the volatility of the non-stationary component of unemployment. The baseline value of 0.05 for the signal-to-noise ratio assumed previously is very close to the value of 0.04 assumed by Llaudes (2005) and somewhat lower than the value of 0.16 used by Greenslade et al. (2003). Figure 2 shows that variation within these bounds, which are representative of the existing literature, does not influence the conclusion regarding the absence of downward wage pressure from long-term unemployment. Even at a signal-to-noise ratio of 0.2, the point estimate of θ is still above 0.9. However, once the signal-to-noise ratio is calibrated at significantly higher values which are not typically found in the literature, then $\widehat{\theta}$ begins to fall and uncertainty around the point estimate increases. Conversely, with a signal-to-noise ratio equal to zero, $\widehat{\theta}$ is 1.76, consistent with the constant natural rate model shown in Table 1.

3.2 Convex model

Estimation results for the convex Phillips equation are reported in Table 2. The major difference to the standard linear specification is that the point estimates of the weight on short-term unemployment are significantly lower across all specifications apart from when the natural rate is held constant. The difference between the linear and convex specifications is smaller when the H-P filter as opposed to the Kalman filter is used to generate the unemployment gap. But even then the difference is notable. Using the standard H-P filter, $\widehat{\theta}$ falls from 0.96 in the linear model to 0.64 in the convex model. The optimal H-P filter smoothing parameter is now substantially higher at 66231 and $\widehat{\theta}$ falls to 0.76 from about 1 in the linear version. The asymptote is not found to be statistically different to zero when using H-P filtered unemployment data but is estimated to be around 0.5% with the Kalman filter.

Perhaps somewhat surprisingly, $\widehat{\theta}$ is below 0.5 when the Kalman filter is used. As usual, statistical uncertainty cautions against drawing too bold a conclusion; a Wald test for the null that $\theta = 0.5$ in this specification returns a χ^2 statistic equal to 2.71 (p value of 0.0999). The Wald test for the standard H-P filter model in Table 2 rejects the null that $\theta = 1$ with better than 10% confidence (p value of 0.0665). On the whole, whilst not definitive, these results are suggestive that the quantitative implications of imposing a convex inflation-unemployment trade-off for the relative importance of long-term unemployment are meaningful. Even though the point estimates vary across different filtering methods, there is a clear tendency for the weight on long-term unemployment to increase materially when switching from a linear to a convex wage inflation equation. The exception is the model in which the natural rate of unemployment is held constant. However, since the modern literature on Phillips curve estimation emphasises the importance of time-variation in the natural rate of unemployment, it is those specifications which warrant greater attention.

It is possible to think of mechanisms which might rationalise the result that $\widehat{\theta} < 0.5$. The argument commonly put forward for why $\theta > 0.5$ is that as the long-term unemployed become marginalised from the labour force, they are less suitable competitors for those in employment (Krueger et al., 2014). However, this embeds the assumption that what matters primarily for wage pressure is search intensity or effectiveness. In particular, it neglects another channel through which long-term unemployment can potentially affect wage dynamics, namely that the bargaining position of the typical long-term unemployed worker deteriorates relative to the short-term unemployed. It seems reasonable for long-term unemployed workers to have worse outside options than the more recently unemployed. The long-term unemployed might be willing to do the same amount of work for less and might also experience earnings losses because of moving to a job that is worse than the one they were previously occupied in. This channel puts downward pressure on wages in addition to the marginal effect on labour market congestion. Deterioration of the typical worker's outside option could have a moderating effect on aggregate wage inflation which is distinct from the congestion effect.

Long-term unemployment could therefore possibly have a stronger restraining effect on wage inflation than does short-term unemployment because of such a channel.

Another robustness check which is worth mentioning is the sensitivity of the estimation results to the choice of the indexation variable, π^p_{t-1}. The results reported in Tables 1 and 2 are based on a four quarter moving average of lagged price inflation. Using the lagged inflation rate instead of a moving average does not alter the conclusions drawn about the consequences of convexity. Estimation with one-period lagged inflation tends to lower $\widehat{\theta}$ across all specifications, such that switching from a linear to a convex model still tends to be associated with a significant reduction in $\widehat{\theta}$. The difference is not large for the Kalman specification; the linear and convex models predict a $\widehat{\theta}$ of 0.93 and 0.16, respectively, which are close to the baseline estimates reported previously. For the standard H-P model, $\widehat{\theta}$ falls from 0.68 in the linear model to 0.16 in the convex model and in the optimised H-P model the fall is from 0.87 to 0.17.[9] Therefore, the conclusion that the long-term unemployment gap gains importance under a non-linear specification remains valid despite the point estimates of θ falling across specifications.

3.3 Discussion

Why do the long-term unemployed have an effect on wage inflation in the convex model but not in the linear model? Figure 3 shows that the share of the long-term unemployed tends to lag movements in aggregate unemployment; by the time long-term unemployment starts to rise, aggregate unemployment is already high. The long-term unemployment share actually falls on impact of a negative shock. But when aggregate unemployment is high, the labour market has already transitioned into a relatively flat region of the convex Phillips curve. So it *appears* as though the long-term unemployed are not doing much to bring wages down any further. Conversely, when aggregate unemployment starts to fall, long-term unemployment again lags short-term unemployment so that falls in long-term unemployment tend to occur in the relatively steep region of a convex Phillips curve, generating upward pressure on wages.

Can we be sure that a convex relationship between wage inflation and aggregate unemployment does not arise precisely *because* the long-term unemployed are irrelevant for wage determination and the long-term unemployment share tends to be higher when aggregate unemployment is high? If this were the correct interpretation, the weight on the long-term unemployment gap would be expected to be statistically insignificant even in the convex Phillips equation. The fact that rises in long-term unemployment lag rises in short-term unemployment implies that the partial derivative of inflation with respect to long-term unemployment will fall even before the long-term unemployment rate begins to rise, so that by the time that it does start to rise, it exerts relatively little further restraint on wages. This will not be the case for the short-term unemployed since short-term unemployment leads long-term unemployment. The linear model, which does not estimate the parameters conditional on allowing for the partial derivative of the short- or long-term unemployment rate to be a function of the aggregate unemployment rate, cannot capture these particular dynamics.

It is possible to gain a more quantitative understanding of this mechanism by calculating how $\partial\pi^w_t/\partial u^s_t = \theta\partial\pi^w_t/\partial\widetilde{u}_t$ and $\partial\pi^w_t/\partial u^l_t = (1-\theta)\,\partial\pi^w_t/\partial\widetilde{u}_t$ evolved over the 2008 recession using the available parameter estimates. From (4), it follows that

$$\frac{\partial\pi^w_t}{\partial\widetilde{u}_t} = -\lambda\left(\frac{1}{\widetilde{u}_t - a} - \frac{\widetilde{u}_t}{(\widetilde{u}_t - a)^2}\right) - \lambda\frac{\widetilde{u}^n_t}{(\widetilde{u}_t - a)^2}$$

For illustration, consider the Kalman filter specification. Figure 4 shows the partial derivatives of inflation with respect to short- and long-term unemployment for $2008.1 - 2009.2$, representing the period of time from

[9] It is worth noting that in the H-P models the asymptotes are significantly different to zero at 0.6% when one period lagged inflation is used as the indexation variable.

which unemployment began to rise to the quarter after which short-term unemployment displayed a marked correction. Panel (a) of the figure shows the extent to which the influence of long-term unemployment is distorted by the short-term unemployment rate by keeping the latter fixed at its 2008.1 value. Clearly, the partial derivative of inflation with respect to long-term unemployment is dramatically reduced by the spike in short-term unemployment at the onset of the recession, even before long-term unemployment actually sets in. The long-term unemployment rate reached an initial peak only in 2010.1, but the marginal effect of long-term unemployment on inflation had already fallen by sixty percent even before 2009.1, mainly due to the rise in short-term unemployment. By the third quarter of 2008, the long-term unemployment rate had not even begun to rise, but the short-term unemployment rate had already increased by enough to reduce the downward pull of long-term unemployment on inflation by about a third. By the end of 2008, controlling for the distortion arising from short-term unemployment, long-term unemployment had only risen by enough to reduce its marginal impact by just twenty percent. Although increases in long-term unemployment also reduce the marginal impact of short-term unemployment, the distortion is not present to nearly the same extent, as shown in panel (b).

To summarise, if the true θ were such that the long-term unemployed exerted very little influence on wages, then this would also plausibly result in a convex relationship between inflation and aggregate unemployment. As the share of long-term unemployment rises, the aggregate unemployment rate would generate less effective downward pressure on wages at the margin, giving rise to a convex shape. However, estimation results suggest that convexity in the aggregate is not the result of a zero derivative of π_t^w with respect to u_t^l, as the linear model would suggest. Therefore, the convex model overturns the results of the linear framework in the sense that it predicts a statistically significant downward sloping relationship between long-term unemployment and wage inflation.

3.4 Time path(s) of the natural rate(s) of unemployment

The paths of the natural rates of short- and long-term unemployment are shown in Figures 5 and 6 for the linear and convex wage inflation equations, respectively. As anticipated from the estimation results in Table 1, given the statistically insignificant weight on long-term unemployment in the linear Phillips curve, the Kalman filter is unable to extract a meaningful signal about the path of $u_t^{l,n}$. This is reflected in panel (b) of Figure 5 by the fact that the estimated path of $u_t^{l,n}$ from the Kalman filter is basically flat. In the linear model, recall that only the short-term unemployment gap is relevant. Consider then the natural rate of short-term unemployment, displayed in panel (a). The path is very similar for both the standard and optimised H-P filters and is somewhat different to the path obtained using a Kalman smoother. The different filtering methods imply broadly qualitatively similar dynamics up until the 2008 recession, although the Kalman filter picks up more of the inflation volatility of the 1970s and 1980s. The natural rate of short-term unemployment obtained using a Kalman smoother does not reach as high a peak in the 1980s as implied by the H-P filter, but all specifications imply a protracted decline in the natural rate of short-term unemployment over the 1990s and early 2000s.

The Kalman filter predicts that the short-term unemployment rate could have been about half a percentage point lower just before the 2008 recession without generating upward inflationary pressure in the labour market. The short-term unemployment rate still seemed to be trending downwards over the period from 2001 to 2007, and it is unclear whether u_t^s would have converged to $u_t^{s,n}$ had the 2008 shock not happened. It is important to note, however, that the path of $u_t^{s,n}$ is sensitive to the inclusion of additional regressors in the Phillips curve. For example, there was a clear step change in wage inflation around 1992 (it is possible to see this from Figure 8, which is discussed in the next section). Adding an intercept dummy to (3) in an attempt to capture this potential structural change reduces the extent to which the model tends to account for the slowdown in wage inflation with a widening of the short-term unemployment gap. The resultant path for $u_t^{s,n}$ (not reported) implies that the unemployment gap is very close to zero before the 2008 recession as $u_t^{s,n}$ is estimated to have fallen at a slower pace as short-term unemployment recovered over the 1990s. The inclusion of the intercept dummy does not change the estimated value of θ, which at 1.16 remains practically the same as in the baseline. The coefficient on the dummy variable turns out to be statistically insignificant, however, and so is not retained in the rest of the analysis. Another sensitivity of the path of $u_t^{s,n}$ concerns the starting value used to initialise the Kalman filter. Raising this starting value relative to the baseline calibration shifts the path of $u_t^{s,n}$ without affecting its shape or the parameter estimates.

The Kalman filter and H-P models differ markedly in their implications for $u_t^{s,n}$ during the 2008 recession. The Kalman filter estimates the unobserved component of unemployment through a signal equation based on the dynamics of wage inflation. Consequently, the period of exceptionally weak wage growth following 2008 is partially interpreted by the Kalman filter as consistent with a fall in the natural rate of short-term unemployment, whereas this mechanism is absent from a simple univariate time series filter. On the face of it, this would suggest that substantial reductions in unemployment would be feasible prior to the emergence of significant inflationary pressure from the UK labour market. However, the reality is that the statistical methods applied here, despite constituting standard practice, are liable to capture at least some cyclicality in the implied fluctuations of the natural rate of unemployment.

The corresponding paths of the natural rates of unemployment derived from the convex specification are shown in Figure 6. Consider panel (a) first, which shows the path of the natural rate of short-term unemployment. Convexity has a notable effect on the paths obtained using the optimised H-P and Kalman filters, which are substantially smoother compared to the linear models. The Kalman filter now also predicts a meaningful role for the long-term unemployment gap, and the dynamics of $u_t^{l,n}$ are displayed in panel (b). The main qualitative difference to the linear model in the Kalman specification is that negative unemployment gaps are very rare, occurring only at very low levels of unemployment. Laxton et al. (1999) stress that in the presence of convexity, the average unemployment rate in a stochastic economy is expected to be larger than the natural rate of unemployment due to the curvature of the Phillips curve. Figure 6 suggests that the UK labour market has spent very little time below the natural rate of unemployment over the past four decades, which is perhaps reasonable given the protracted periods of high unemployment during the 1980s and 1990s.[10]

3.5 Which model of wage inflation?

The foregoing analysis raises an obvious question, which has long been asked in the applied literature on the nature of the inflation-unemployment trade-off; is non-linearity actually present? And, if so, to what extent? Although the implications of non-linearity for stabilisation policy and welfare are clear, as Laxton et al. (1999) stress, econometric methods designed to identify convexity typically have low power, especially if the degree of convexity is conservative and the range of variation in inflation and unemployment over the sample period is not large. The aim of the current paper is not to prove the case for convexity but rather to investigate what the inflationary consequences of long-term unemployment are conditional on non-linearity in the Phillips curve. To this end, a simple empirical exercise is taken up in this section, which is to examine the fit of the estimated wage inflation equations during the recent recession.

The results from this exercise are displayed in Figure 7. The actual hourly wage inflation data is smoothed by taking a four quarter moving average. Panel (a) shows the fit of the linear specifications. Quarterly UK nominal wage inflation has remained at 0.5% or below for the last two years of the sample. All of the linear wage equations predict significant nominal wage deflation over 2009, overstating the observed weakness in

[10] This conclusion is robust to raising the starting values for the initial states, although when $u_0^{s,n}$ and $u_0^{l,n}$ are set to values materially higher than actual unemployment in 1971, implausible parameter estimates sometimes obtain (e.g. $\widehat{\theta} < 0$) and the maximised likelihood tends to fall.

wages. This is consistent with the finding in Gali (2011) for US data, who concludes that fitted (linear) wage Phillips curves generate excessively weak wage growth over the 2008 recession. The linear models in Figure 7 then predict some unwinding of the weakness in wages after 2009, with the constant natural rate and H-P specifications predicting a recovery in wage inflation by the end of the sample, which is grossly at odds with the data. The Kalman filter does a better job at generating persistent weakness in wage inflation. It was demonstrated previously in Figure 5 that the linear Kalman model achieves this weakness through a fall in the time-varying natural rate of short-term unemployment. The H-P models also counterfactually predict excessively strong wage inflation immediately prior to the recession, since this filtering method generates a negative unemployment gap just before the 2008 shock.

The fit of the convex models is shown in panel (b) of Figure 7. In contrast to the linear specifications, the convex models do not tend to predict excessive nominal wage deflation at the onset of the recession. The H-P filters still predict an excessively rapid rebound in wage growth from 2010 onwards as the unemployment gaps close in these models, although the difference between the standard and optimised calibrations is now more notable. The Kalman filter specification tracks the persistent weakness in wages reasonably well, without the initial overshooting observed for the linear specification. The persistent slowdown in wage growth is now accounted for by the rise in unemployment, and in particular long-term unemployment, over the recession, in contrast to the linear specification which also relies on a fall in the permanent component of unemployment to generate the protracted weakness in wage growth.

On balance, it would therefore seem as though the convex specification has perhaps performed somewhat better than the linear specification during the recent recession since it avoids excessive nominal wage deflation during the onset of the recession. Curvature in the Phillips curve prevents nominal wage inflation from turning significantly negative. For some wider context, the fitted wage equations for the entire sample are shown in Figure 8. The models' fit is reasonable over the the longer-term, including the secular slowdown following 1992. Furthermore, it can also be seen that the convex specification tends to better match the large inflationary spike during the 1970s.

4 Hysteresis

There exists the possibility that the natural rate of unemployment may tend to gravitate towards actual unemployment over time. If this were the case, unemployment would display hysteresis, so that long-run equilibrium unemployment is at least partially influenced by temporary fluctuations. Concerns over hysteresis seem all the more pressing following the most recent recession which caused a spell of high unemployment that has lasted for five years since 2009. The channel through which hysteresis was initially thought to operate was based on the insider-outsider theory of wage formation (Blanchard and Summers, 1986), but more recent thinking on the subject stresses the importance of human capital loss and labour market detachment that is associated with long-term unemployment (Ball, 2009). Whether or not hysteresis is present in the labour market has important implications for monetary policy. If, as Ball (2009) states, there do exist mechanisms which pull the natural rate toward the actual unemployment rate, then long-run equilibrium unemployment can be affected by monetary policy. More recently, Farmer (2013) has also argued that the idea of a NAIRU that is independent of aggregate demand is "past its sell by date". Farmer's work has at its core the idea that multiple equilibria exist and investors' beliefs select among the alternative paths. Farmer argues that a model of this type, which contains no inherent mechanism that forces the unemployment rate back to its natural rate, tends to outperform a standard New Keynesian Phillips curve. The analysis in this section takes up the suggestion by Ball (2009) that accounting explicitly for unemployment duration in the analysis of hysteresis is likely to be an important avenue for research.

The baseline results suggested that the natural rate of unemployment has not risen by much during the most recent recession, and indeed might have even fallen below its historical minimum towards the very end of the sample. However, the point estimates of θ were above 1 for some specifications, which could be indicative of hysteresis in the labour market induced by long-term unemployment. Modifying the dynamics of the baseline model to explicitly allow for hysteresis in unemployment, the equations in (5) governing movement in $u_t^{s,n}$ and $u_t^{l,n}$ are now expressed as

$$u_t^{i,n} = \rho^i u_{t-1}^i + \left(1 - \rho^i\right) u_{t-1}^{i,n} + \nu_t^i \tag{6}$$

for $i = \{s, l\}$. The parameter ρ^i determines the extent to which hysteresis is present, allowing for different degrees of hysteresis in the short- and long-term unemployment rates. Equation (6) clarifies the potential connection between hysteresis and the estimated value of θ; if long-term unemployment is an important source of hysteresis, then rather than placing a negative or small positive weight on u_t^l to generate relatively little downward pressure on inflation from long-term unemployment, the model with equation (6) could instead achieve a similar effect by shrinking the long-term unemployment *gap* for a given θ during recessions if $\rho^l > 0$.

Estimation results for the hysteresis model, under both the linear and convex specifications, are presented in Table 3. The point estimates of ρ^i are statistically insignificant for both specifications and the estimation results do not differ meaningfully from the baseline results reported in Tables 1 and 2. On the basis of this simple test, it is concluded that there is no immediately supportive evidence of hysteresis in UK unemployment although the issue certainly warrants further consideration.

5 Conclusion

The tendency of recent statistical models to imply that there is no significant disinflationary effect of long-term unemployment appears to be related to the assumption of a linear inflation-unemployment trade-off that is typically made. The business cycle fact that long-term unemployment lags short-term unemployment implies that linear models could bias downwards the effect of long-term unemployment on inflation if the true Phillips curve is convex. The intuition is simple; by the time the long-term unemployment rate starts to rise, the economy has already transitioned into a significantly flatter region of the Phillips curve due to a spike in short-term unemployment at the onset of a downturn. This mechanism could provide an alternative explanation for why the long-term unemployed appear to exert relatively little downward pressure on wages, as opposed to common appeals to search theory which rest on the idea of labour market detachment.

The findings in this paper have the potential to reconcile the result in the macroeconometric literature that the long-term unemployed are irrelevant for inflation dynamics with the observation that “the long-term unemployed look basically the same as other unemployed people in term of their occupations, educational attainment and other characteristics” (Yellen, 2014). As Rudebusch and Williams (2014) note, the issue of whether policy makers should simply focus on the short-term unemployment gap hinges on whether or not there are compositional effects associated with unemployment duration that are relevant for understanding inflation. If, as has been argued elsewhere in the literature, the long-term unemployed are indeed marginalised, then significant inflationary pressure would be expected to arise as the short-term unemployment gap normalises following a recession. Conversely, according to the results of this study, if the true model were convex there could remain significant downward wage pressure from long-term unemployment even after the short-term unemployment gap has closed. This disinflationary pressure, however, would be at least partially offset by a worsening inflation-unemployment trade-off at the margin as unemployment declines in a convex economy. Furthermore, the convex model would predict a significant reduction in disinflationary pressure as the long-term unemployment gap closes, which would be absent altogether from a linear model in which the long-term unemployment rate does not matter for inflation in the first place.

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Table 1: Linear estimation results

	Standard H-P filter		Optimised H-P filter		Kalman filter		Cons. natural rate	
Intercept (α)	.006	(.001)	.006	(.001)	.013	(.007)	.023	(.003)
Lagged inflation (γ)	.994	(.070)	.995	(.070)	.681	(.106)	1.04	(.071)
Slope (λ)	-1.43	(.232)	-1.29	(.208)	-.939	(.188)	-.605	(.112)
Short-term weight (θ)	.957	(.159)	1.02	(.157)	1.12	(.233)	1.76	(.136)
Productivity deviation	.372	(.088)	.371	(.088)	.311	(.062)	.307	(.088)
Log likelihood	541.23		541.36		547.48		537.69	

Note: Standard errors in parentheses.

Table 2: Convex estimation results

	Standard H-P filter		Optimised H-P filter		Kalman filter		Cons. natural rate	
Intercept (α)	.005	(.001)	.005	(.001)	.034	(.009)	-.012	(.003)
Lagged inflation (γ)	.943	(.070)	.914	(.071)	.274	(.150)	1.00	(.070)
Slope (λ)	-.035	(.013)	-.021	(.007)	-.031	(.009)	$5e^{-4}$	($9e^{-5}$)
Short-term weight (θ)	.643	(.195)	.758	(.180)	.264	(.143)	1.80	(.117)
Productivity deviation	.370	(.086)	.349	(.085)	.276	(.064)	.317	(.088)
Asymptote (a)	.000	(.006)	.003	(.004)	.005	(.002)	-	-
Log likelihood	544.32		545.22		566.86		538.25	

Note: Standard errors in parentheses. A zero asymptote is imposed in the constant natural rate model for identification.

Table 3: Estimation results of the hysteresis model

	Phillips curve specification			
	Linear		Convex	
Intercept (α)	.014	(.006)	.032	(.009)
Lagged inflation (γ)	.632	(.158)	.238	(.151)
Slope (λ)	-.945	(.198)	-.029	(.008)
Short-term weight (θ)	1.12	(.248)	.250	(.135)
Productivity deviation	.310	(.061)	.275	(.064)
Asymptote (a)	-	-	.005	(.001)
Degree of hysteresis in short-term unemp. (ρ^s)	-.011	(.025)	-.008	(.013)
Degree of hysteresis in long-term unemp. (ρ^l)	-.001	(.102)	-.004	(.017)
Log likelihood	547.74		567.23	

Note: Standard errors in parentheses.

Figure 1: Optimal H-P smoothing parameters

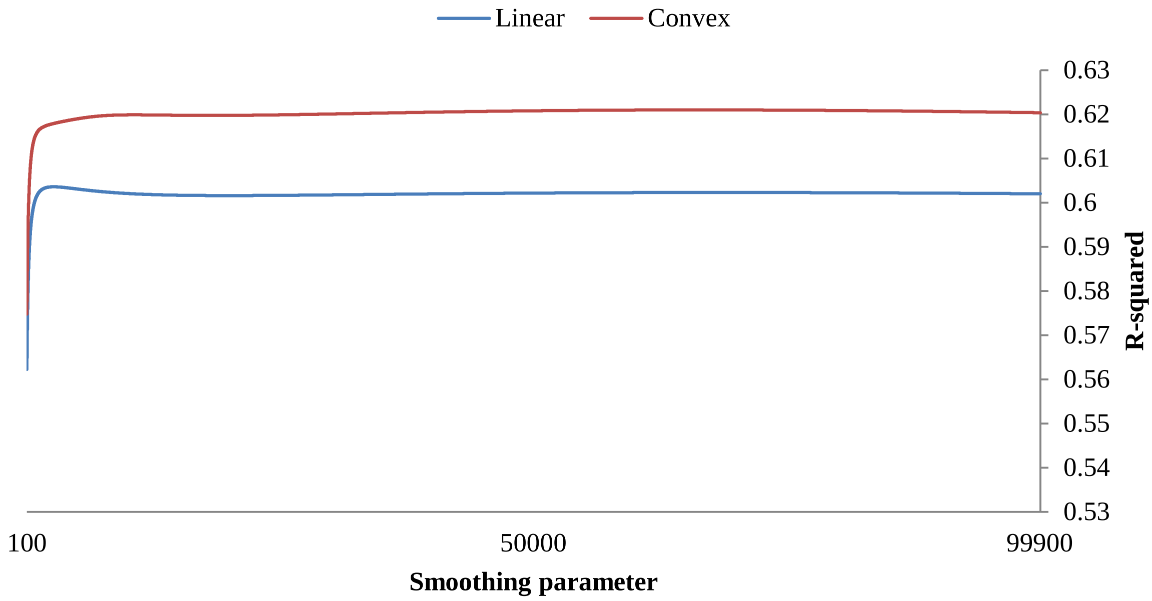
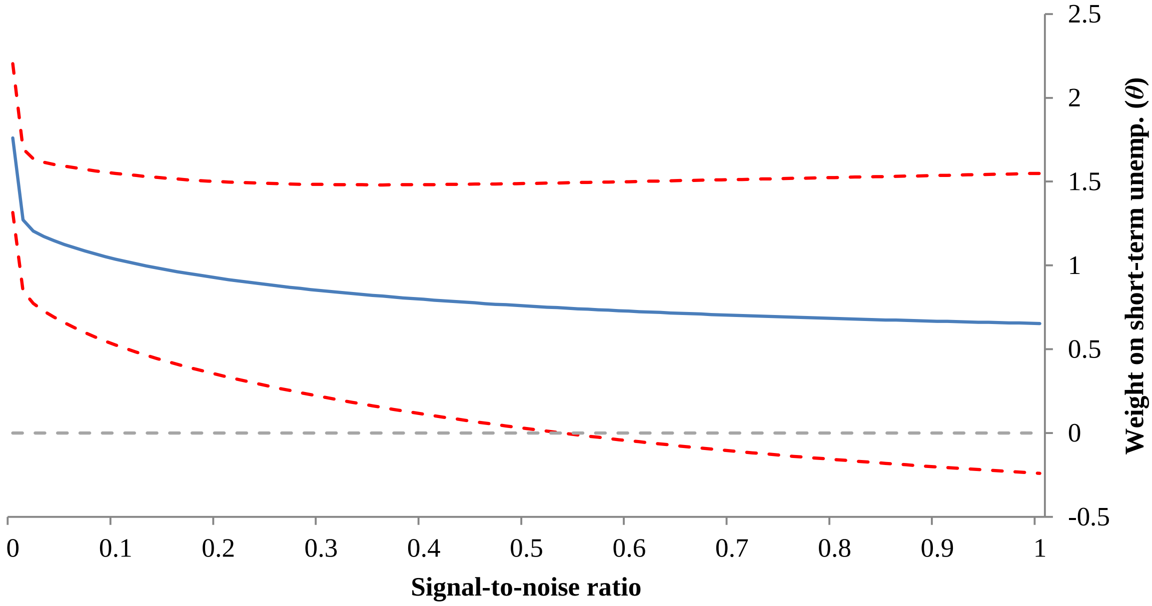


Figure 2: Sensitivity of the relative unemployment weights to the signal-to-noise ratio



Note: dotted lines indicate two standard error bands.

Figure 3: Fluctuations in aggregate unemployment and the long-term share

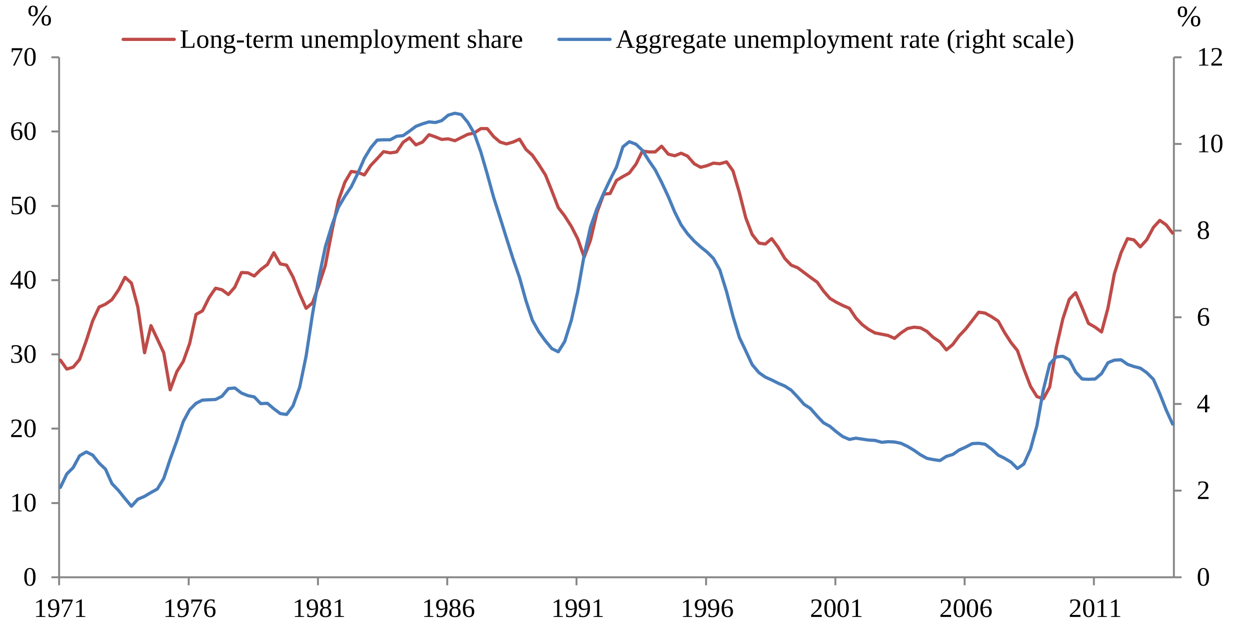
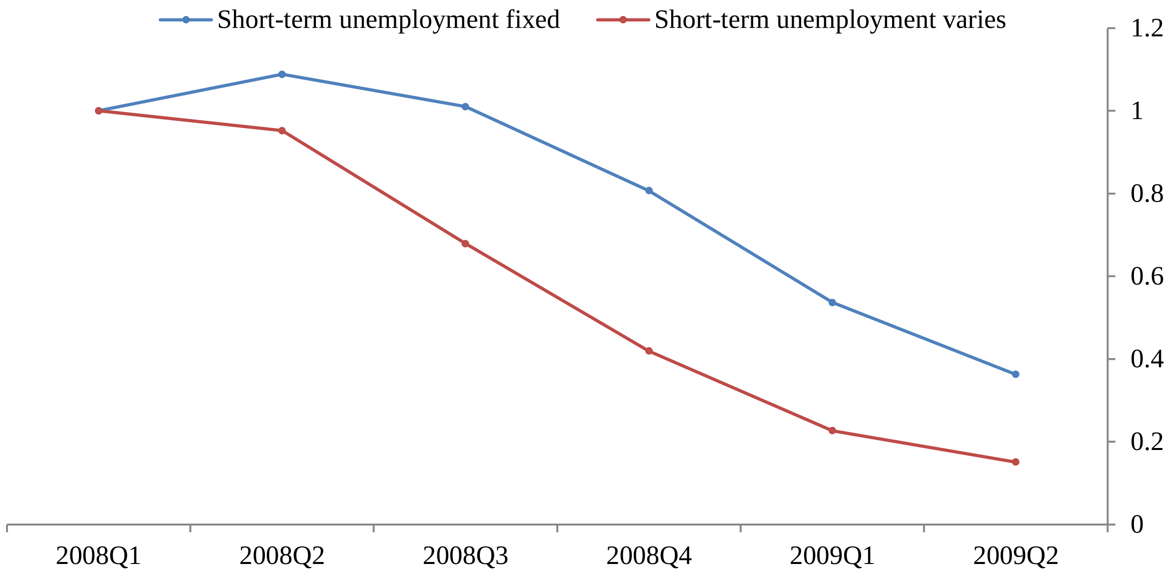
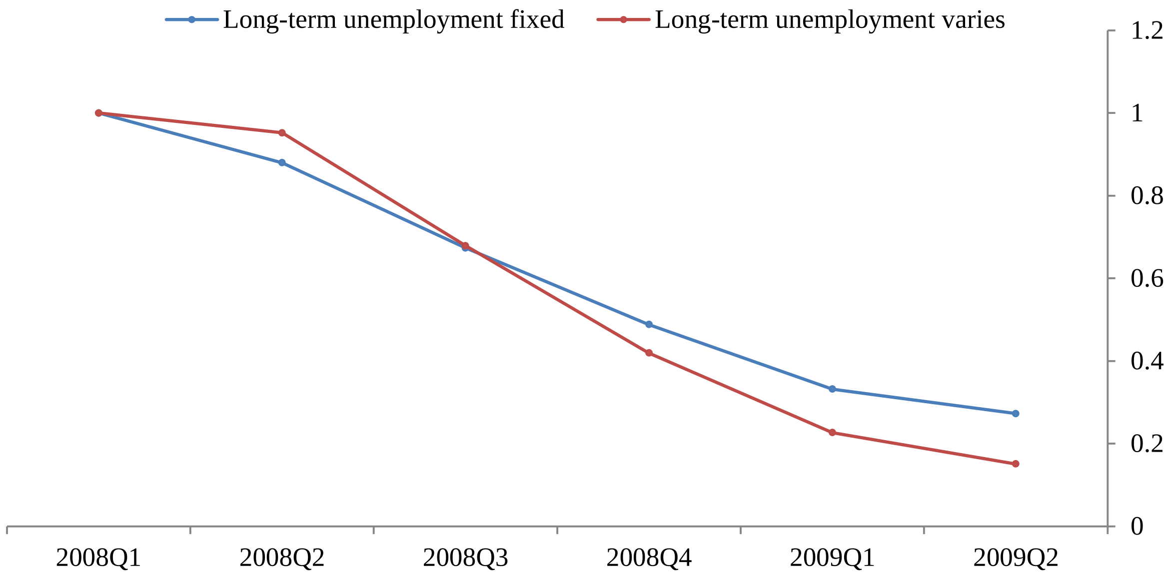


Figure 4: Derivative of the Phillips curve with respect to short- and long-term unemployment

(a) Derivative with respect to long-term unemployment



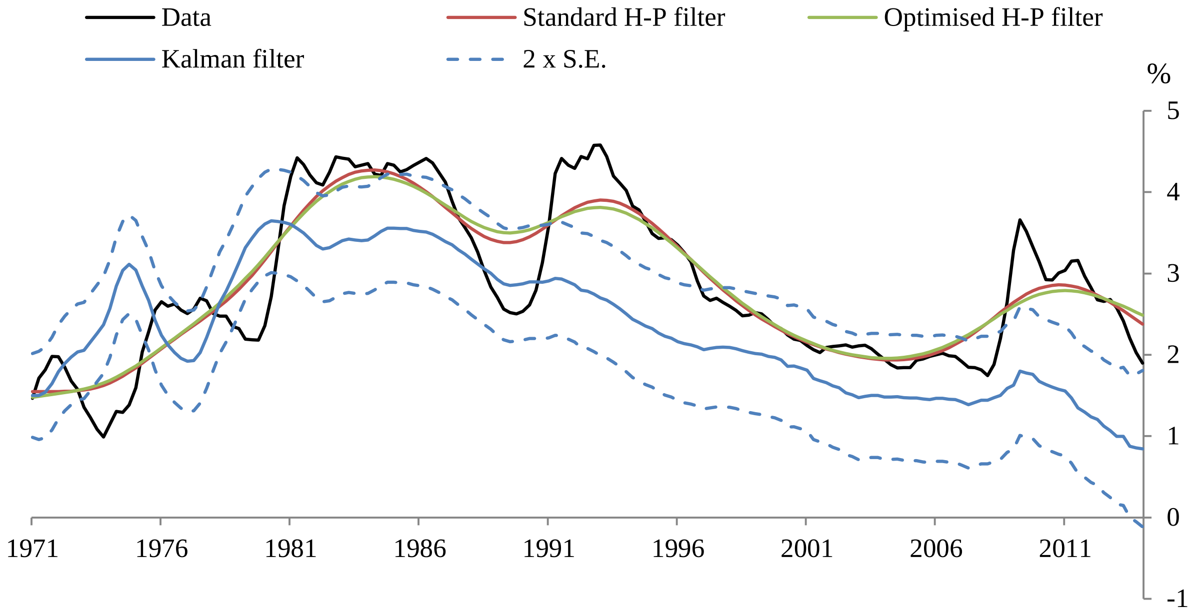
(b) Derivative with respect to short-term unemployment



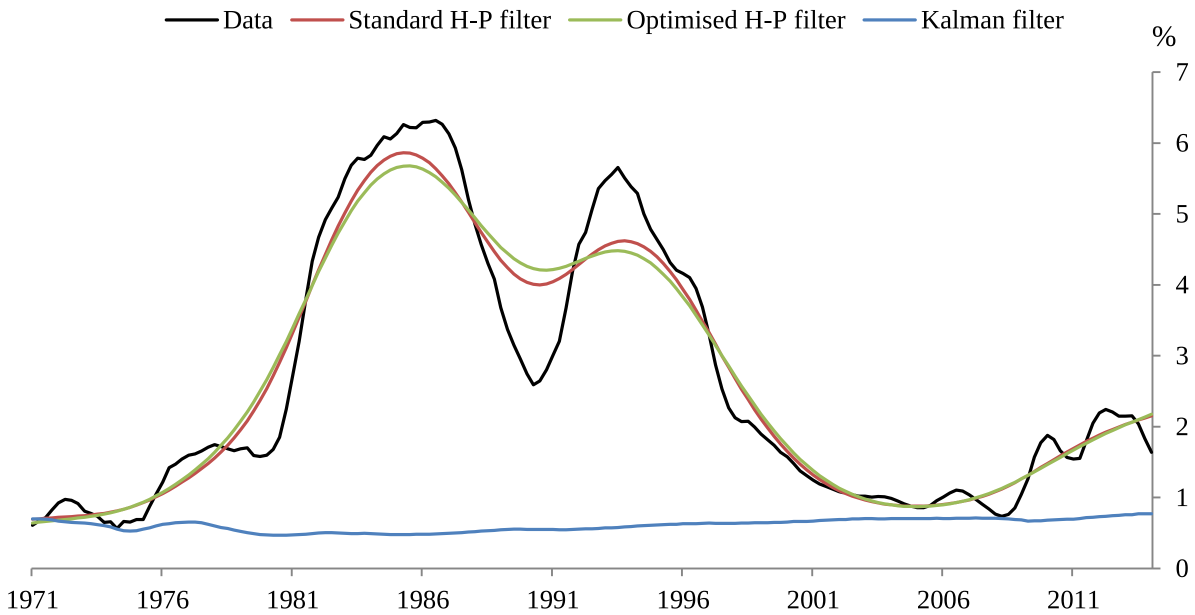
Note: derivatives normalised to 1 in 2008Q1.

Figure 5: Natural rates of unemployment in the linear model

(a) Short-term unemployment



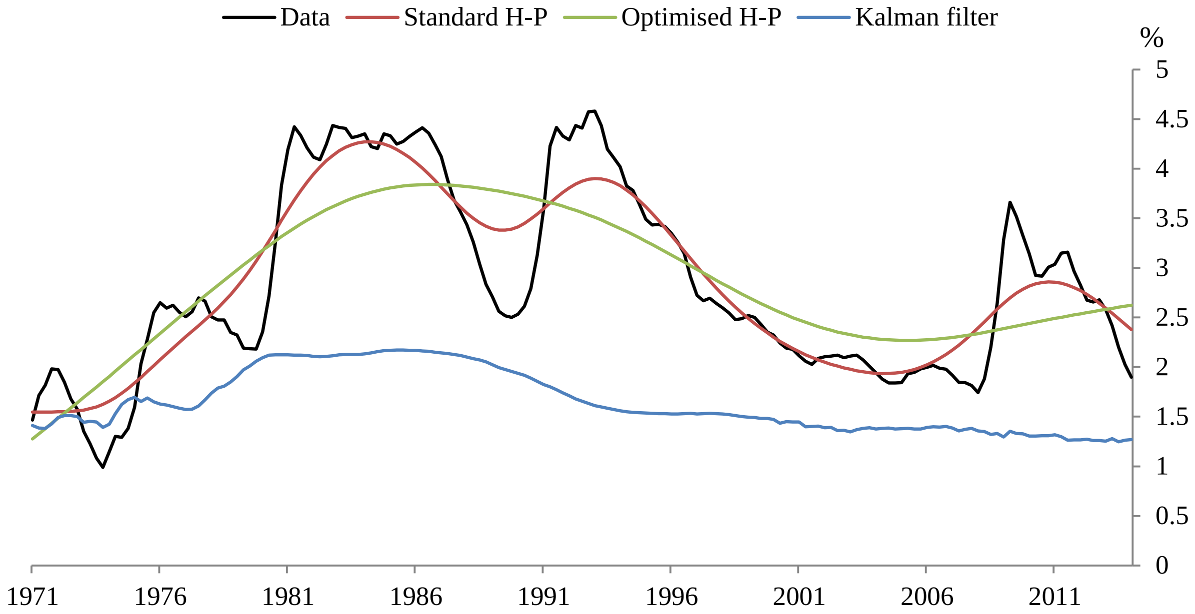
(b) Long-term unemployment



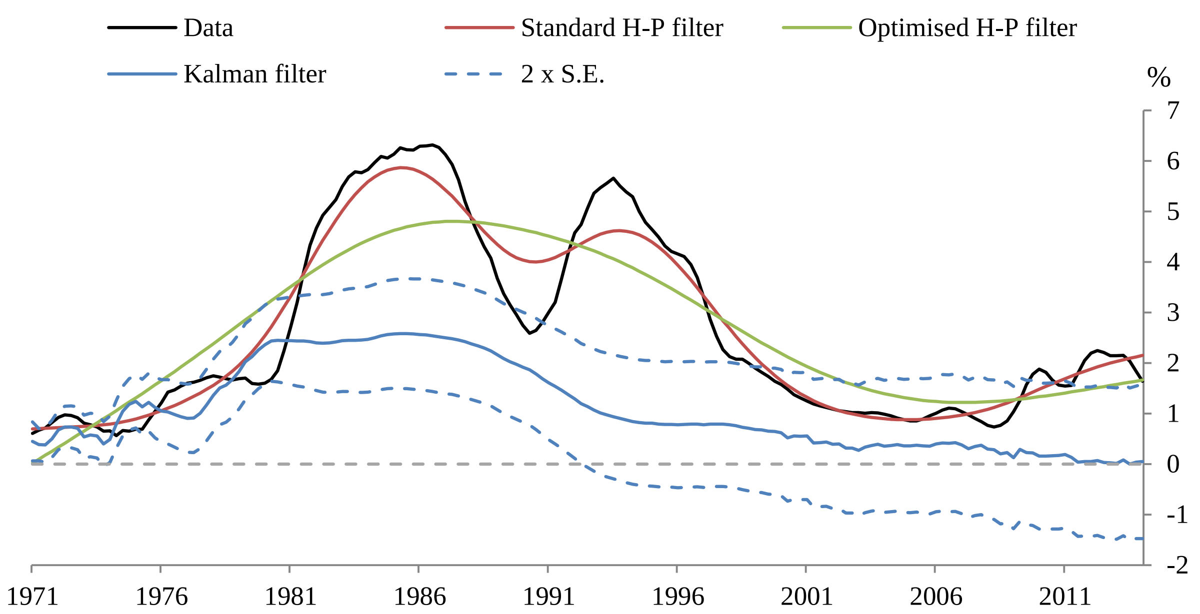
Note: error bands in panel (b) omitted.

Figure 6: Natural rates of unemployment in the convex model

(a) Short-term unemployment



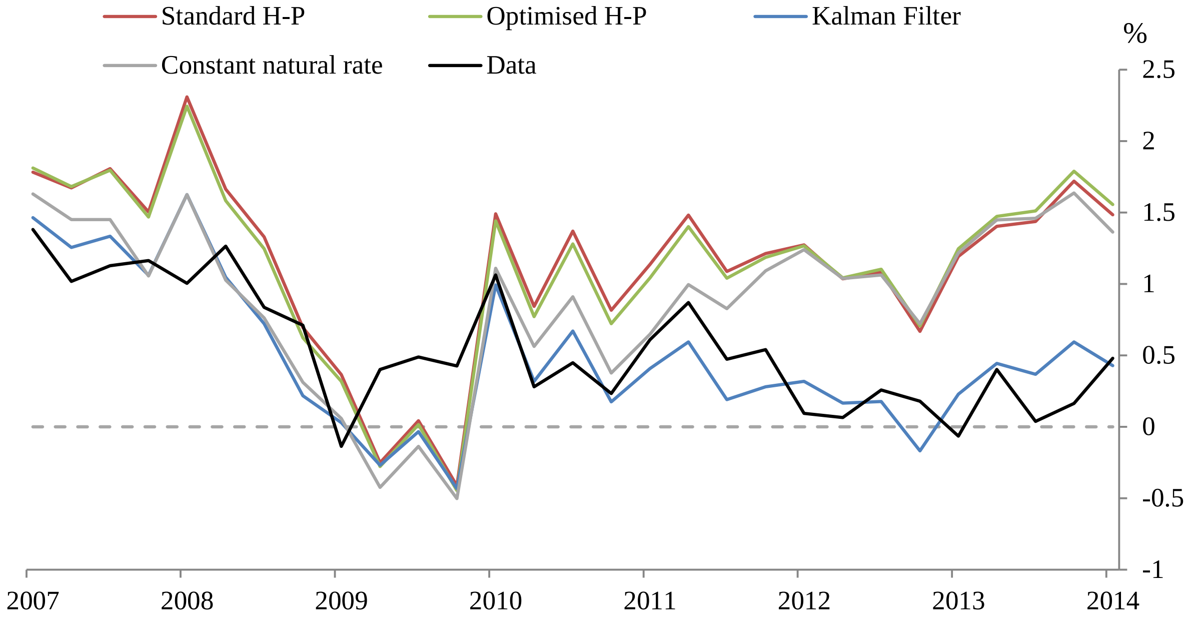
(b) Long-term unemployment



Note: error bands in panel (a) omitted.

Figure 7: Fitted wage inflation dynamics over the 2008 recession

(a) Linear model



(b) Convex model

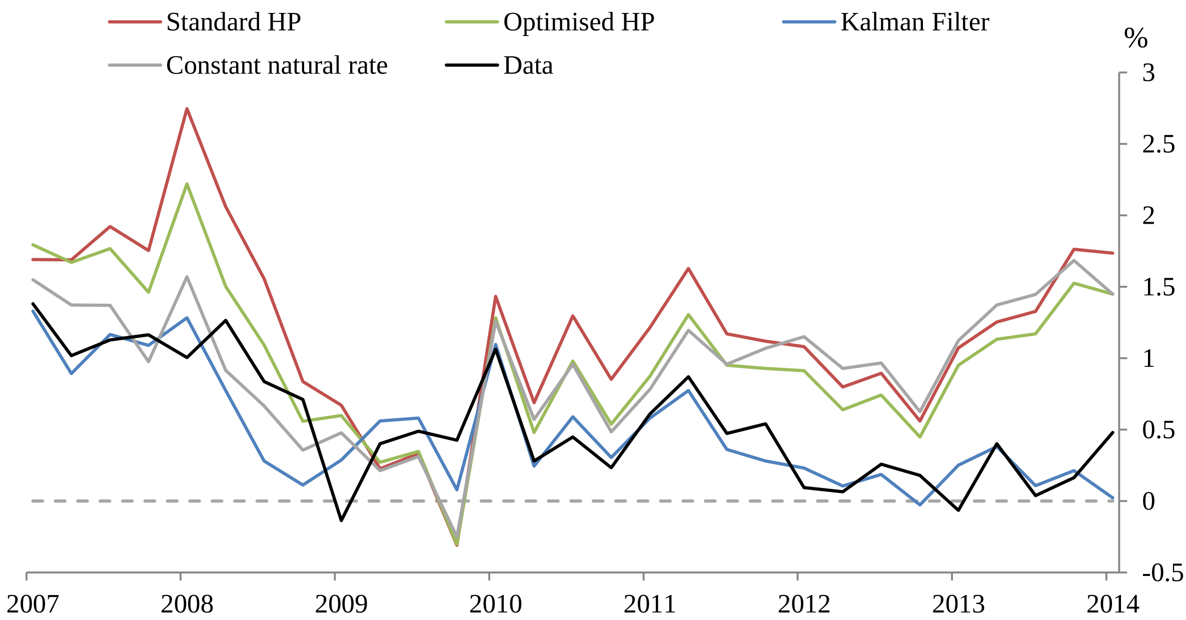
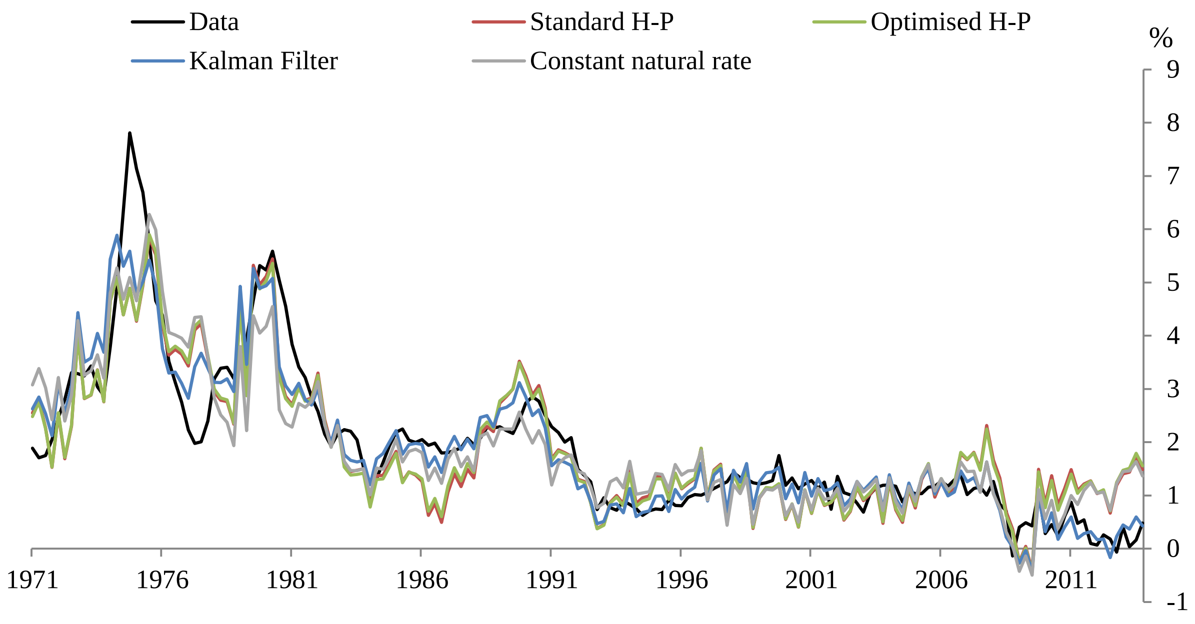


Figure 8: Fitted wage inflation dynamics over 1971-2014

(a) Linear model



(b) Convex model