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Timothy S Fuerst, Charles T Carlstrom and
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Timothy S Fuerst,[1] Charles T Carlstrom[2] and Matthias Paustian[3]

Abstract

This paper derives the optimal lending contract in the financial accelerator model of Bernanke, Gertler and Gilchrist (BGG). The optimal contract includes indexation to the aggregate return on capital, household consumption, and the return to internal funds. This triple indexation results in a dampening of fluctuations in leverage and the risk premium. Hence, compared to the contract originally imposed by BGG, the privately optimal contract implies essentially no financial accelerator.

Key words: Financial accelerator, optimal contracts, aggregate risk.

JEL classification: E32, C32.

(1) University of Notre Dame and Federal Reserve Bank of Cleveland. Email: tfuerst@nd.edu
(2) Federal Reserve Bank of Cleveland. Email: charles.t.carlstrom@clev.frb.org
(3) Federal Reserve Board. Email: matthias.o.paustian@frb.gov

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Publications Team, Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 7601 4030 Fax +44 (0)20 7601 3298 email publications@bankofengland.co.uk

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Summary

Frictions in credit markets are widely known to amplify business cycles. The mechanism typically works via leverage (the ratio of debt to net worth) of the borrower. Typically, an adverse macroeconomic shock reduces the value of the assets of credit constrained borrowers. The resulting fall in borrowers' net worth increases leverage. In turn, higher leverage makes an underlying credit friction more severe and raises credit spreads. As a consequence, demand for investment falls by more than would happen in a world without credit market frictions, depressing asset values further. This sets in motion a feedback loop between rising spreads and falling asset prices that is at the heart of the financial accelerator.

The most prominent paper incorporating the financial accelerator mechanism in a quantitative macroeconomic model was published in 1999 by Ben Bernanke, Mark Gertler and Simon Gilchrist (BGG). We revisit the debt contract they employed and highlight how the financial accelerator depends on the treatment of aggregate risk in the debt contract. BGG study the optimal financial contract in a world where borrower and lender have asymmetric information about firm-specific productivity. Lenders can only observe the return of the firm's project by paying a monitoring cost. In addition, there is aggregate macroeconomic risk that is costlessly observable by everyone. The key assumption is that the return to the lender does not depend on the realization of aggregate risk.

In this paper, we derive the optimal financial contract and show that the return to the lender varies with the realization of aggregate risk. Consequently, the interest rate in the optimal debt contract is contingent on aggregate macro variables, much as the coupon payment in an inflation-indexed bond is linked to the particular realization of aggregate inflation. This 'state contingency' in the optimal contract is, however, rather complex. We show that the lender return varies with shocks to household consumption, the aggregate return on capital and the marginal value of internal funds of the borrower.

A key feature of the state contingent debt contract is that it limits fluctuations in leverage and greatly reduces the financial accelerator. When an unexpected adverse macroeconomic shock reduces the return on borrowers' investments, the loan contract calls for a reduction in the borrowers' interest rate. As a result, fluctuations in net worth and leverage are limited and much of the adverse feedback loop described above is avoided. Ultimately, aggregate risk is shared between households (lenders) and entrepreneurs (borrowers), rather than falling predominantly on the borrowing constrained firm as in BGG. In a model calibrated to match US data, we show that this contract implies a welfare improvement for both parties. Furthermore, amplification from credit frictions is negligible.

It is an open question to what extent actual contracts are state contingent in the way our analysis suggests. At face value, it seems that such contingency is very rare. Our primary contribution is to derive the optimal debt contract in the BGG model, not to state that financial frictions in the data cannot amplify macro shocks. But the analysis also enables us to quantify the welfare cost of financial frictions. We find that the costs of frictions are small, increasing in adjustment costs.

1 Introduction

The financial accelerator model of Bernanke, Gertler, and Gilchrist (2000), hereafter BGG, is widely used as a convenient mechanism for integrating financial factors into an otherwise standard DSGE model. The BGG model embeds the costly state verification (CSV) model of Townsend (1979) in an environment with risk neutral entrepreneurs, risk averse households, and aggregate risk. Appealing to insurance concerns, BGG assume that the lending contract between the entrepreneur and lender is characterized by a lender return that is invariant to innovations in aggregate variables. These aggregate innovations then feed directly into entrepreneurial net worth which is crucial in the BGG model because agency costs are diminished by increases in net worth. For example, since the lender's return is fixed, a positive productivity shock shifts wealth to entrepreneurs and thus lowers agency costs. This sets in motion a contemporaneous financial accelerator, a virtuous circle in which higher net worth drives up the price of capital, which in turn increases net worth, etc. This process thus amplifies the effect of the shock. But the vital first step in this amplification is the assumption that the lender's return is pre-determined. The importance of this insurance assumption is well known. For example, consider the following comment of Chari (2003) on BGG:

> *These authors have an economy with risk neutral agents called entrepreneurs and risk averse agents called households. They claim that an optimal contract in the presence of aggregate risk has the return paid by entrepreneurs to be a constant, independent of the current aggregate shock. I have trouble understanding this result. Surely, entrepreneurs should and would provide insurance to households against aggregate shocks. One way of providing such insurance is to provide a high return to households when their income from other sources is low and a low return when their income from other sources is high. My own guess is that if they allowed the return to households to be state contingent, then aggregate shocks would have no effects on the decisions of households and would be absorbed entirely by entrepreneurs. Before we push this intriguing financial accelerator mechanism much further, I think it would be wise to make sure that we get the microeconomics right.*

In this paper, we take up the task suggested by Chari (2003) and attempt to get “the microeconomics right”. In particular, we solve for the privately optimal contract (POC) in the original BGG framework. We then contrast this POC with the contract assumed in BGG, and analyze both the normative and the positive implications for macroeconomics fluctuations.

Our principle results include the following. First, as anticipated by Chari (2003), the financial contract imposed in the BGG model is not privately optimal. That is, lenders and borrowers would both prefer a different loan contract. Second, the POC has the loan repayment vary in response to innovations in the observed aggregate shocks, but not quite in the manner anticipated by Chari (2003). In particular, the POC has debt repayment linked to innovations in three observables: the return on capital, the level of household consumption, and the entrepreneur's valuation of net worth.[1] The consumption insurance portion of the POC was anticipated by Chari (2003). But the POC also provides a hedge to entrepreneurs who prefer greater net worth when the return to internal funds is high. Taken together, we show that the POC is a state-contingent debt contract that dampens fluctuations in leverage and thus the risk premium.

The mechanism is as follows. Consider a positive aggregate shock to the return on capital, for example due to higher productivity. In BGG, this shock leads to a large increase in the entrepreneur's net worth because the lender's return is predetermined. Ceteris paribus, higher net worth lowers leverage and sets in motion the financial accelerator. Under the POC, the promised repayment and thus the lender's return on the loan are positively indexed to these innovations, so that the innovation to productivity is shared by the lender and the entrepreneur. This more modest movement in net worth largely eliminates the financial accelerator.

Quantitatively, our results are important. In our benchmark calibration the conditional welfare cost of BGG compared to the POC is a 0.023% perpetual increase in the annual flow of household consumption. Turning to amplification, in an economy with sticky prices the response of output on impact to monetary policy shocks is more than 100% larger with BGG con-

[1] To develop intuition, we focus on how the debt contract is indexed to these three distinct observables. But this is a partial equilibrium argument. In general equilibrium, all of these observables are functions of the underlying states so that one cannot so easily separate the indexation into three distinct components. (The entrepreneur's valuation of internal funds is observable as we show below that it is the discounted sum of future leverage and the risk-free rate.)

tracts than with the POC. For TFP shocks, the difference is about 60% larger on impact. In both cases, leverage and the risk-premium are nearly constant under the POC such that the economy mimics its frictionless counterpart (apart from a steady state distortion).

Our results on the POC are related to Krishnamurthy (2003). Krishnamurthy (2003) introduces insurance markets into a three-period model where borrowing is secured by collateral as in Kiyotaki and Moore (1997). These insurance markets allow for state contingent debt that is indexed to aggregate shocks. Krishnamurthy (2003) proves that such insurance eliminates any feedback from collateral values to investment, and thus reduces collateral amplification to zero. Similarly, Di Tella (2013) shows that allowing for contracts that condition on the aggregate state completely eliminates the financial amplification resulting from productivity shocks in the infinite horizon model of Brunnermeier and Sannikov (2014). We have a similar result here: the POC has the level of debt repayment indexed one-for-one to the aggregate return on capital, so that bankruptcy rates do not respond to innovations in the return to capital. By itself, this indexation dramatically reduces the financial accelerator.

An interesting question is how risk-aversion on the part of entrepreneurs would change our results. We cannot satisfactorily answer this question since the optimality of the debt contract in the underlying CSV framework relies on the assumption that the entrepreneur's payoff is linear. Mookherjee and Png (1989) and Winton (1995) have shown that the optimal contract with a risk-averse principal and risk-averse agent is generally not simple debt. But if we simply assume a debt contract with risk-averse entrepreneurs, we conjecture that the POC will still have the level of debt repayment indexed one-for-one to the aggregate return on capital, but depending on the level of risk-aversion the consumption insurance that entrepreneurs provide is mitigated (or even eliminated). In any event, we leave these questions for future work.

The paper proceeds as follows. The next section outlines the competitive equilibrium of the model. Section 3 contrasts the contract indexation to BGG. The quantitative analysis including welfare implications are carried out in Section 4. Section 5 provides some sensitivity analysis on the financial accelerator by examining the privately optimal contract and the BGG contract in a model with sticky prices and more exogenous shocks. Concluding comments are provided in Section 6.

2 The Model

2.1 Households

The typical household consumes the final good (C_t) and sells labour input (L_t) to the firm at real wage w_t. Preferences are given by

$$U(C_t, L_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - B\frac{L_t^{1+\eta}}{1+\eta}$$

The household budget constraint is given by

$$C_t + D_t \leq w_t L_t + R_t^D D_{t-1} + \Pi_t$$

The household chooses the level of deposits (D_t) which are then used by the lender to fund the entrepreneurs (more details below). Note that the gross real return on time $t-1$ deposits (D_{t-1}) is realized at time t (R_t^d). The household owns shares in the final goods firms, capital-producing firms, and the lender. Only the capital-producing firms will generate profits (Π_t) in equilibrium. The household's optimization conditions include:

$$-U_L(t)/U_c(t) = w_t \tag{1}$$

$$E_t M_{t+1} R_{t+1}^d = 1 \tag{2}$$

where $M_{t+1} \equiv \beta\frac{U'(C_{t+1})}{U'(C_t)}$, is the pricing kernel.

2.2 Final goods firms

Final goods are produced by competitive firms who hire labour and rent capital in competitive factor markets at real wage w_t and rental rate r_t. The production function is Cobb-Douglas where A_t is the random level of total factor productivity:

$$Y_t = A_t\left(K_t^f\right)^\alpha (L_t)^{1-\alpha} \tag{3}$$

The realization of total factor productivity is publicly observed at the beginning of time t. The variable K_t^f denotes the amount of capital available for time t production. This is different from the amount of capital at the end of the previous period as some is lost because of monitoring costs. The optimization conditions include:

$$mpl_t = w_t \tag{4}$$

$$mpk_t = r_t \tag{5}$$

where mpl_t and mpk_t denote the marginal products of labour and capital, respectively.

2.3 New Capital Producers

The production of new capital is subject to adjustment costs. In particular, investment firms take $I_t\vartheta(I_t)$ consumption goods and transform them into I_t investment goods that are sold at price Q_t. Their profits are thus given by $Q_tI_t - I_t\vartheta(I_t)$, where the function ϑ is convex. We find it convenient to normalize $\vartheta(I_{ss}) = 1,\ \ \vartheta'(I_{ss}) = 0$ and $\vartheta''(I_{ss}) = \psi$, where I_{ss} is the steady-state level of investment. Variations in investment lead to variations in the price of capital, which is the key to the financial accelerator mechanism.

2.4 Lenders

The representative lender accepts deposits from households and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time $t+1$. The gross real return on these loans is denoted by R^L_{t+1}. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans only aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, dividends are given by $Div_{t+1} = R^L_{t+1}D_t - R^d_{t+1}D_t$. The lending market is competitive so that the lender takes as given the rates of return. This then implies that in equilibrium $R^L_{t+1} = R^d_{t+1}$, and the lender earns zero profits.

2.5 Entrepreneurs and the Loan Contract

There is a continuum of entrepreneurs with preferences that are linear in consumption. The entrepreneurs discount the future at rate β, and are the sole accumulators of physical capital. The time $t+1$ rental rate and capital price are denoted by r_{t+1} and Q_{t+1}, respectively, implying that the gross return to holding capital from time t to time $t+1$ is given by:

$$R^k_{t+1} \equiv \frac{r_{t+1} + (1-\delta)\,Q_{t+1}}{Q_t}. \tag{6}$$

At the end of period t, the entrepreneurs sell all of their accumulated capital, and then re-purchase it along with any net additions to the capital stock. This purchase is financed with entrepreneurial net worth and external financing from a lender.

External financing is subject to a one-period CSV problem. In particular, one unit of capital purchased at the end of time-t is transformed into ω_{t+1} units of capital in time $t+1$, where ω_{t+1} is an idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$ and a mean of one. The realization of ω_{t+1} is directly observed by the entrepreneur at the beginning of time $t+1$, but the lender can observe the realization only if a monitoring cost is paid, a cost that destroys part of the capital produced by the project. We assume that this monitoring cost is linear in the project size, $\mu\omega_{t+1}R^k_{t+1}K_{t+1}$. The assumption of linearity allows for aggregation, but is non-standard in the optimal contracting literature.[2]

Under the assumption that payoffs are linear in the project outcome, Townsend (1979) demonstrates that the optimal contract between the entrepreneur and lender is risky debt in which monitoring only occurs if the promised payoff is not forthcoming.[3] In the present context, we proceed as follows. The appendix demonstrates that if the entrepreneur's value function is linear, the debt contract is optimal. To complete the logical circle, we will show that if risky debt is the optimal contract, then the entrepreneur's value function is linear. In particular, we conjecture that the entrepreneur's value function is linear and given by $V_t NW^i_t$, where V_t is constant across entrepreneurs, and NW^i_t denotes the net worth of the representative entrepreneur at the beginning of time t, after the loans from the previous period have been settled. We will verify this conjecture below.

Under the assumption that risky debt is optimal, the loan contract is characterized by a reservation value of the idiosyncratic shock that separates repayment from default. Debt repayment does not occur for sufficiently low values of the idiosyncratic shock, $\omega^i_{t+1} \leq \bar{\omega}^i_{t+1}$. Let Z^i_{t+1} denote the promised gross rate of return so that Z^i_{t+1} is defined by

$$Z^i_{t+1}(Q_t K^i_{t+1} - NW^i_t) \equiv \bar{\omega}^i_{t+1} R^k_{t+1} Q_t K^i_{t+1}. \tag{7}$$

[2] We follow BGG and Carlstrom and Fuerst (1997) by assuming that there is enough inter-period anonymity so that today's contract cannot be based on previous realizations of the idiosyncratic shock. As noted by these authors, this assumption vastly simplifies the analysis, for otherwise the optimal contract would depend upon the entire history of each entrepreneur (see, for example Gertler (1992)).

[3] The risky debt result also assumes that the equilibrium contract must be in pure strategies, i.e., no random audits.

We find it convenient to express this in terms of the leverage ratio $\overline{\kappa}_t^i \equiv \left(\frac{Q_t K_{t+1}^i}{NW_t^i}\right)$ so that (7) becomes

$$Z_{t+1}^i \equiv \bar{\omega}_{t+1}^i R_{t+1}^k \frac{\overline{\kappa}_t^i}{\overline{\kappa}_t^i - 1} \tag{8}$$

Let $f(\bar{\omega}_{t+1}^i)$ and $g\left(\bar{\omega}_{t+1}^i\right)$ denote the entrepreneur's share and lender's share of the project outcome:

$$f\left(\bar{\omega}\right) \equiv \int_{\bar{\omega}}^{\infty} \omega\phi\left(\omega\right) d\omega - [1 - \Phi\left(\bar{\omega}\right)]\bar{\omega} \tag{9}$$

$$g\left(\bar{\omega}\right) \equiv [1 - \Phi\left(\bar{\omega}\right)]\,\bar{\omega} + (1-\mu)\int_{0}^{\bar{\omega}} \omega\phi\left(\bar{\omega}\right) d\omega. \tag{10}$$

The entrepreneur's net worth NW_t^i is leveraged into a project size of $Q_t K_{t+1}^i$, so that the entrepreneur's payoffs, R_{t+1}^E, and lender returns, R_{t+1}^L, are given by:

$$R_{t+1}^E \equiv R_{t+1}^k Q_t K_{t+1}^i f\left(\bar{\omega}_{t+1}^i\right) = R_{t+1}^k f\left(\bar{\omega}_{t+1}^i\right)\overline{\kappa}_t^i NW_t^i \tag{11}$$

$$R_{t+1}^L \equiv \frac{R_{t+1}^k g\left(\bar{\omega}_{t+1}^i\right) Q_t K_{t+1}^i}{\left(Q_t K_{t+1}^i - NW_t^i\right)} = R_{t+1}^k g\left(\bar{\omega}_{t+1}^i\right)\frac{\overline{\kappa}_t^i}{\overline{\kappa}_t^i - 1}. \tag{12}$$

We assume that the distribution of aggregate shocks has a bounded support. We further assume that these bounds are tight enough so that the entrepreneur's expected return to internal funds, $E_t R_{t+1}^k f\left(\bar{\omega}_{t+1}^i\right)\overline{\kappa}_t^i$, will always exceed $1/\beta$. Consequently, the entrepreneur will postpone consumption indefinitely. To avoid self-financing in the long run by accumulating sufficient internal funds, we assume that fraction (1-γ) of the entrepreneurs die each period, where dying means eating their accumulated net worth and exiting the economy. They are then replaced by an equal number of new entrepreneurs. These new entrepreneurs each need a trivial amount of initial net worth to begin activity. We assume that this comes from a lump sum transfer from the existing entrepreneurs. But since this transfer can be arbitrarily small, and since only aggregate net worth matters in this setting, we neglect these transfers in what follows.

In summary, the representative entrepreneur sets $c_t^{e,i} = NW_t^i$ with probability (1-γ), and $NW_{t+1}^i = R_{t+1}^k f\left(\bar{\omega}_{t+1}^i\right)\overline{\kappa}_t^i NW_t^i$ with probability γ. The Bellman equation for the representative entrepreneur is thus given by:

$$V_t NW_t^i = (1-\gamma)c_t^{e,i} + \gamma\beta max_{\overline{\kappa}_t^i, \bar{\omega}_{t+1}^i} E_t V_{t+1} NW_{t+1}^i \tag{13}$$

where the maximization is subject to the lender's participation constraint (equation (16) below). Substituting in the trivial consumption decision, we have

$$V_t = (1-\gamma) + \gamma\beta max_{\overline{\kappa}_t^i, \bar{\omega}_{t+1}^i} \overline{\kappa}_t^i E_t V_{t+1} R_{t+1}^k f\left(\bar{\omega}_{t+1}^i\right) \tag{14}$$

On the other side of the contract, we have the lender whose opportunity cost is linked to the return on deposits. We can thus write the end of time-t contracting problem as:

$$max_{\overline{\kappa}_t^i, \bar{\omega}_{t+1}^i} \overline{\kappa}_t^i \gamma\beta E_t V_{t+1} R_{t+1}^k f\left(\bar{\omega}_{t+1}^i\right) \tag{15}$$

subject to

$$E_t M_{t+1} R_{t+1}^k g\left(\bar{\omega}_{t+1}^i\right) \overline{\kappa}_t^i \geq \left(\overline{\kappa}_t^i - 1\right) \tag{16}$$

After some re-arrangement, the optimization conditions include:

$$\gamma\beta V_{t+1} f'\left(\bar{\omega}_{t+1}^i\right) + \Lambda_t^i M_{t+1} g'\left(\bar{\omega}_{t+1}^i\right) = 0 \tag{17}$$

$$\gamma\beta E_t V_{t+1} R_{t+1}^k f\left(\bar{\omega}_{t+1}^i\right) + \Lambda_t^i \left[E_t M_{t+1} R_{t+1}^k g\left(\bar{\omega}_{t+1}^i\right) - 1\right] = 0 \tag{18}$$

$$E_t R_{t+1}^k M_{t+1} g\left(\bar{\omega}_{t+1}^i\right) \frac{\overline{\kappa}_t^i}{\overline{\kappa}_t^i - 1} = 1 \tag{19}$$

where Λ_t^i denotes the multiplier on the constraint (16). Combining (17)-(18), we have that $\bar{\omega}_{t+1}^i = \bar{\omega}_{t+1}$, is constant across entrepreneurs. Next, expression (19) then implies that $\bar{\omega}_t^i = \bar{\omega}_t$, is also constant across entrepreneurs. Returning to the Bellman equation (14), if the optimal $\bar{\omega}_{t+1}^i$ and $\overline{\kappa}_t^i$ are constant across entrepreneurs, then so is V_t. Hence, we have confirmed our conjecture that the value function is linear with a time-varying slope coefficient V_t that is constant across entrepreneurs.[4] Further, note that: (i) V_t is the discounted sum of future leverage and returns to capital, and (ii) the entrepreneur would prefer a contract in which NW_{t+1} is positively correlated with V_{t+1}. Using (18) we can write the Bellman equation as

$$V_t = (1-\gamma) + \Lambda_t \tag{20}$$

[4] In a related environment, Krishnamurthy (2003) demonstrates that although borrowers are risk-neutral in consumption, they may be risk-averse in net worth. This risk-aversion arises in Krishnamurthy (2003) because the borrower's production technology is concave, and the collateral constraint is not always binding. In the BGG model studied here, the production technology is linear and the need for external finance is a permanent feature of the CSV framework, so the entrepreneurs' payoff is linear in net worth. However, the entrepreneurs care about the covariance of net worth with aggregate shocks.

so that the entrepreneur's valuation of net worth is tightly linked to the shadow value of the contract constraint. Hence, the entrepreneur's valuation of net worth is the weighted sum of the unit payoff of eating upon death, and the gain to holding on to net worth if the agent survives. The linearity of the value function implies that we need only track aggregate net worth. Summing over all entrepreneurs we have $NW_{t+1} = \gamma R^k_{t+1} f\left(\bar{\omega}_{t+1}\right) \overline{\kappa}_t NW_t$. We henceforth drop the entrepreneur index i.

Returning to (17), the privately optimal contract (POC) is thus described by the $\bar{\omega}_{t+1}$ that satisfies:

$$\frac{\Lambda_t M_{t+1}}{\beta\gamma\left[1-\gamma+\Lambda_{t+1}\right]} = \frac{-f'\left(\bar{\omega}_{t+1}\right)}{g'\left(\bar{\omega}_{t+1}\right)} \equiv F\left(\bar{\omega}_{t+1}\right), \tag{21}$$

where the second order condition requires $F'\left(\bar{\omega}_{t+1}\right) > 0$. Expression (21) implies that the default cut-off is indexed to aggregate variables in a natural way. When household consumption is low (M_{t+1} is high), the optimal $\bar{\omega}_{t+1}$ (and thus the lender's return) increases as a form of consumption insurance to the household. Similarly, when the cost of external finance is high (Λ_{t+1} is high), the contract calls for a lower $\bar{\omega}_{t+1}$ so that the entrepreneur will hold on to more net worth (NW_{t+1} positively covaries with V_{t+1}).

2.6 Market Clearing and Equilibrium

In equilibrium, household deposits fund the entrepreneurs' projects, $D_t = Q_t K_{t+1} - NW_t$. Net of monitoring costs, the amount of capital available for production is given by $K^f_t = \Upsilon\left(\bar{\omega}_t\right) K_t$, where $\Upsilon_t \equiv f\left(\bar{\omega}_t\right) + g\left(\bar{\omega}_t\right) = 1 - \mu \int_0^{\bar{\omega}_t} x\phi\left(x\right) dx$. The competitive equilibrium is defined by the variables $\{C_t, L_t, I_t, K_{t+1}, \bar{\omega}_t, \Lambda_t, \overline{\kappa}_t, C^e_t, Q_t, R^d_{t+1}\}$ that satisfy

$$\begin{aligned}
E_t R^d_{t+1} M_{t+1} &= 1 && (22)\\
-U_L(t)/U_c(t) &= mpl_t && (23)\\
E_t R^k_{t+1} M_{t+1} g\left(\bar{\omega}_{t+1}\right) \frac{\overline{\kappa}_t}{\left(\overline{\kappa}_t - 1\right)} &= 1 && (24)\\
\frac{\Lambda_{t-1} M_t}{\beta\gamma\left[1-\gamma+\Lambda_t\right]} &= \frac{-f'\left(\bar{\omega}_t\right)}{g'\left(\bar{\omega}_t\right)} && (25)\\
\Lambda_t &= \beta\gamma E_t[(1-\gamma)+\Lambda_{t+1}] R^k_{t+1} f\left(\bar{\omega}_{t+1}\right)\overline{\kappa}_t && (26)\\
Q_t K_{t+1} &= \gamma\left[Q_t\left(1-\delta\right)+mpk_t\right] f\left(\bar{\omega}_t\right) K_t \overline{\kappa}_t && (27)\\
K_{t+1} &= \left(1-\delta\right)\Upsilon\left(\bar{\omega}_t\right) K_t + I_t && (28)\\
C_t + I_t \nu\left(\frac{I_t}{I_{ss}}\right) + C^e_t &\leq A_t((\bar{\omega}_t) K_t)^{\alpha}(L_t)^{1-\alpha} && (29)\\
C^e_t &= (1-\gamma)\left[Q_t\left(1-\delta\right)+mpk_t\right] f\left(\bar{\omega}_t\right) K_t && (30)\\
Q_t &= \nu\left(I_t\right) + \left(I_t\right)\nu'(I_t) && (31)
\end{aligned}$$

where we have used $M_{t+1} \equiv \beta \frac{U'(c_{t+1})}{U'(c_t)}$, $\overline{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right)$, and $R^k_{t+1} \equiv \frac{mpk_{t+1}+(1-\delta)Q_{t+1}}{Q_t}$. The marginal products are defined as: $mpl_t = \left(1-\alpha\right) Y_t/L_t$, and $mpk_t = \alpha Y_t/(\Upsilon\left(\bar{\omega}_t\right) K_t)$, with $Y_t \equiv A_t(\Upsilon\left(\bar{\omega}_t\right) K_t)^{\alpha}(L_t)^{1-\alpha}$.

The underlying friction in this agency cost model is explicit in (24). Compared to a frictionless real business cycle model, the investment margin (24) is distorted by the term: $g\left(\bar{\omega}_{t+1}\right)\frac{\overline{\kappa}_t}{(\overline{\kappa}_t-1)}$. This suggests that there may be welfare gains to minimizing fluctuations in this distortion. As we will see below, the POC will vary repayment rates in such a way as to dampen fluctuations in this distortion.

3 Comparing the POC to BGG

Because of BGG's prominence in the literature, it is particularly useful to compare the POC contract and the contract imposed by BGG. The differences are particularly transparent in log-linear form, so we look at the linearized versions of POC and BGG. In log-linear form (lower case letters), equations (17)-(19) for the POC are given by:

$$\Psi\bar{\omega}_t = m_t + \lambda_{t-1} - \beta\lambda_t \tag{32}$$

$$\kappa_t + E_t\left(r^k_{t+1} + \theta_f\bar{\omega}_{t+1}\right) = \lambda_t - \beta E_t\lambda_{t+1} \tag{33}$$

$$E_t\left(r^k_{t+1} + m_{t+1} + \theta_g E_t\bar{\omega}_{t+1}\right) = \left(\frac{1}{\kappa - 1}\right)\kappa_t \tag{34}$$

where $\Psi = \frac{\bar{\omega}_{ss}F'(\bar{\omega}_{ss})}{F(\bar{\omega}_{ss})} > 0$, $\theta_g \equiv \frac{\bar{\omega}_{ss}g'(\bar{\omega}_{ss})}{g(\bar{\omega}_{ss})}$, $0 < \theta_g < 1$, and $\theta_f \equiv \frac{\bar{\omega}_{ss}f'(\bar{\omega}_{ss})}{f(\bar{\omega}_{ss})} < 0$. In the steady state, we have that $\frac{\theta_f}{\theta_g} = (1 - \kappa)$. Scrolling (32) forward, and combining with (33)-(34), we have a convenient expression for the spread between the return on capital and the lender's return:

$$E_t\left(r^k_{t+1} - r^L_{t+1}\right) = \left[\frac{\Psi}{(\kappa - 1)\Psi - \kappa\theta_f}\right]\kappa_t \equiv \nu\kappa_t \tag{35}$$

Under the POC, this spread (35) is precisely the log-linearized version of the expected investment distortion, $E_t g\left(\bar{\omega}_{t+1}\right)\frac{\overline{\kappa}_t}{(\overline{\kappa}_t - 1)}$. Note that increases in leverage are associated with increases in this distortion. From (8) and (12), the promised payment and lender's return are given by:

$$z_t = \bar{\omega}_t + r^k_t - \frac{1}{\kappa - 1}\kappa_{t-1} \tag{36}$$

$$r^l_t = -\frac{1}{(\kappa - 1)}\kappa_{t-1} + \theta_g\bar{\omega}_t + r^k_t \tag{37}$$

Combining the previous, we can express the POC in log-linear form:

$$\bar{\omega}^{POC}_t = \left(\frac{1 - \nu(\kappa - 1)}{\theta_g(\kappa - 1)}\right)\kappa_{t-1} + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{38}$$

$$z^{POC}_t = E_{t-1}r^d_t + \frac{(1 - \theta_g)\left[1 - \nu(\kappa - 1)\right]}{\theta_g(\kappa - 1)}\kappa_{t-1} + \left(r^k_t - E_{t-1}r^k_t\right) + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{39}$$

$$r^{l,POC}_t = E_{t-1}r^d_t + \left(r^k_t - E_{t-1}r^k_t\right) + \frac{\theta_g}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta\theta_g}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{40}$$

$$\lambda_t = E_t\sum_{j=0}^{\infty}\beta^j\left(\kappa\nu\kappa_{t+j} + r^d_{t+j+1}\right) \tag{41}$$

The POC has three key characteristics.[5] First, the promised repayment and lender's return are scaled one-for-one by innovations in r_t^k so that the default cut-off is sterilized from these innovations. This is quite natural. There are two sources of uncertainty within the underlying CSV problem: unobserved idiosyncratic shocks and the observed aggregate return on capital. Bankruptcy and costly monitoring are part of the optimal debt contract as the mechanism to ensure truthful revelation of the idiosyncratic shock. But there is no need for such a deterrent for observed aggregate shocks. A second key feature of the POC is that it provides consumption insurance to the household in that the lender's return is increasing when the marginal utility of consumption is unexpectedly high (m_t is high). The higher lender return is then passed on to the household via increases in the deposit rate. Third, the POC provides a hedge to the entrepreneur in that when the return to internal funds is high (λ_t is high), the repayment to the lender declines so that the entrepreneur can build up net worth.

In sharp contrast to the state-contingent POC, the original BGG model imposed a contract in which the lender's return and deposit rate are predetermined. This is not an implication of the modelling framework, but is instead an assumption imposed on the model. As BGG write, "Since entrepreneurs are risk neutral, we *assume* that they bear all the aggregate risk associated with the contract" (BGG, page 1385, emphasis added). There are two problems with this assumption. First, the household's risk is linked to the marginal utility of consumption, not to the return on capital. Second, the entrepreneur cares about the covariance between net worth and the return on capital. The POC includes both of these motivations in (38)-(41), while the contract assumed by BGG does not. The behaviour of bankruptcy rates in BGG is given implicitly by [6] and

$$g\left(\bar{\omega}_t\right) = \frac{R_{t-1}^d(\overline{\kappa}_{t-1} - 1)}{R_t^k \overline{\kappa}_{t-1}}. \tag{42}$$

[5] As noted earlier, this is a partial equilibrium argument because in general equilibrium, all of these observables are functions of the underlying states.

[6] The full set of BGG contract conditions are given by (42) and $\beta\gamma f'\left(\bar{\omega}_{t+1}\right) + \frac{\Lambda_{t+1}}{R_t^d} g'\left(\bar{\omega}_{t+1}\right) = 0$ and $\beta\gamma E_t R_{t+1}^k f\left(\bar{\omega}_{t+1}\right)\overline{\kappa}_t = E_t \Lambda_{t+1}$

We can express the BGG contract in log-linear form:

$$\bar{\omega}_t^{BGG} = \frac{[1-\nu(\kappa-1)]}{\theta_g(\kappa-1)}\kappa_{t-1} - \frac{1}{\theta_g}(r_t^k - E_{t-1}r_t^k) \tag{43}$$

$$z_t^{BGG} = r_{t-1}^d + \frac{(1-\theta_g)\,[1-\nu\,(\kappa-1)]}{\theta_g(\kappa-1)}\kappa_{t-1} + \left(\frac{\theta_g-1}{\theta_g}\right)\left(r_t^k - E_{t-1}r_t^k\right) \tag{44}$$

$$r_t^{l,BGG} = r_{t-1}^d \tag{45}$$

Unlike the POC, default will depend upon innovations in r_t^k. For the calibration used below, $\theta_g \approx 1$, so that the BGG default rates fall sharply with innovations in r_t^k. This then implies that the promised repayment declines modestly with innovations in r_t^k. But the key expression is (45): the lender's return is pre-determined. This implies that innovations in r_t^k are entirely absorbed by entrepreneurial net worth, and that the contract is missing the household and entrepreneurial hedging motives of the POC.

Although BGG and POC differ only by innovations, the inertial dynamics of net worth imply that these differences will have persistent consequences. Linearizing the behaviour of aggregate net worth and using the two contracts we have

$$\begin{aligned} nw_t^{POC} =& nw_{t-1}^{POC} + E_{t-1}r_t^d + \kappa\nu\kappa_{t-1} + \left(r_t^k - E_{t-1}r_t^k\right) \\ &+ \frac{\theta_f}{\Psi}\left(m_t - E_{t-1}m_t\right) - \frac{\beta\theta_f}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \end{aligned} \tag{46}$$

$$nw_t^{BGG} = nw_{t-1}^{BGG} + r_{t-1}^d + \kappa\nu\kappa_{t-1} + \kappa\left(r_t^k - E_{t-1}r_t^k\right). \tag{47}$$

As with the lender's return, the differences in the two contracts differ by the response of net worth to innovations in three aggregate variables: the return on capital, the household's marginal utility of consumption, and the entrepreneur's valuation of net worth. Here we will focus on the magnitude of these effects. Under our baseline calibration, $\kappa = 2$, so that the BGG contract responds to r_t^k by twice as much as POC. As for the other innovations, the POC provides insurance both to the household (consumption insurance) and the entrepreneur (financing insurance) in a symmetric fashion. These insurance effects are non-trivial: our baseline calibration implies $\frac{\theta_f}{\Psi} = -2.17$.

The difference between the POC and BGG are most clearly evident for the case of a net worth shock. That is, suppose we append (46)-(47) with an innovation in entrepreneurial net worth that is realized at the beginning of the period. There are three fundamental effects coming from such a shock.

First, an innovation in net worth leads to an increase in the price of capital (because entrepreneurs are the purchasers of capital) and thus an innovation in r_t^k. Second, the increase in net worth ameliorates the agency distortion so that investment spending increases and consumption declines, i.e., a positive innovation in m_t. Third, the higher level of net worth lowers the marginal valuation of net worth, i.e., a negative innovation in λ_t. Under the POC, the latter two effects work against the r_t^k innovation so that there is essentially no movement in net worth (we will see this quantitatively below). But under BGG, the latter two effects are entirely absent, and the r_t^k effect is magnified because of leverage. In summary, under POC an innovation in net worth is largely sterilized by movements in the required repayment rate, while under BGG, the net worth innovation sets in motion a contemporaneous financial accelerator that magnifies this initial shock.

The POC thus contains two insurance or hedging motives. Which of these two POC insurance effects dominates? The two insurance terms in (39)-(40) can be combined, by noting that in general equilibrium, innovations in the household's pricing kernel are linked to innovations in the deposit rate. The household's pricing kernel implies

$$(m_t - E_{t-1}m_t) = \sigma(c_t - E_{t-1}c_t) = -\left(E_t\sum_{j=0}^{\infty} r_{t+j+1}^d - E_{t-1}\sum_{j=0}^{\infty} r_{t+j+1}^d\right) \tag{48}$$

Assuming $\beta = 1$, we can use (41) to substitute the above expression into (39)-(40) so that the repayment expressions becomes:

$$\begin{aligned} z_t^{POC} =& E_{t-1}r_t^d + \frac{(1-\theta_g)\left[1-\nu(\kappa-1)\right]}{\theta_g(\kappa-1)}\kappa_{t-1} + \left(r_t^k - E_{t-1}r_t^k\right) \\ & - \frac{\kappa\nu}{\Psi}\left(E_t\sum_{j=0}^{\infty}\kappa_{t+j} - E_{t-1}\sum_{j=0}^{\infty}\kappa_{t+j}\right) \end{aligned} \tag{49}$$

$$r_t^{l,POC} = E_{t-1}r_t^d + \left(r_t^k - E_{t-1}r_t^k\right) - \frac{\kappa\nu\theta_g}{\Psi}\left(E_t\sum_{j=0}^{\infty}\kappa_{t+j} - E_{t-1}\sum_{j=0}^{\infty}\kappa_{t+j}\right) \tag{50}$$

Thus in general equilibrium, the combined effect of these two insurance motives result in one strategy: dampen fluctuations in leverage. That is, when leverage is unexpectedly high, the repayment levels decline so that net worth rises and leverage is pulled back down. In terms of asset prices, the time t-1 risk premium in the model is given by $E_{t-1}(z_t^{POC} - r_t^d)$. From (49), this is proportional to leverage. Hence, by dampening movements in leverage, the POC also dampens movements in the risk premium. In sharp contrast, under BGG, the lender repayment is pre-determined so that movements in leverage and the risk premium are magnified.

4 Quantitative Analysis

Our benchmark calibration largely follows BGG. The discount factor β is set to 0.99. Utility is assumed to be logarithmic in consumption ($\sigma = 1$), and the elasticity of labour is assumed to be 3 ($\eta = 1/3$). The production function parameters include $\alpha = 0.35$, investment adjustment costs $\psi = 0.50$, and quarterly depreciation is $\delta = .025$. We parameterize the adjustment cost so that in the steady state the price of capital is equal to unity. As for the credit-related parameters, we calibrate the model to be consistent with: (i) a steady state spread between Z and R^d of 200 bp (annualized), (ii) a quarterly bankruptcy rate of .75%, and (iii) a leverage ratio of $\kappa = 2$. These values imply a survival rate of $\gamma = 0.94$, a standard deviation of the idiosyncratic productivity shock of 0.28, and a monitoring cost of $\mu = 0.63$. In the linearized model (see appendix), this then implies an agency cost elasticity of $\nu = 0.188$.[7]

Our baseline analysis assumes that total factor productivity follows an AR (1) process with $\rho^A = 0.95$. But first to develop intuition, Figure 1 presents impulse response functions for the case of a 1% iid TFP shock, $\rho^A = 0$. Note that the POC responds to this iid shock in something of an iid fashion. That is, there is very little persistence. In particular, leverage (and thus the risk premium) is essentially unchanged. This arises because of the POC's indexed repayment (see (40) and (50)). The TFP shock leads to a positive innovation in r_t^k, a negative innovation in λ_t (as consumption rises), and a negative innovation in m_t (because the movement in net worth is larger than the movement in desired capital). Quantitatively, these latter two insurance motives largely cancel as leverage hardly moves under the POC, and the lender's return responds one-to-one to innovations in r_t^k. Matters

[7] BGG calibrated $(R^k - R^d)$ to 200 bp. But the model's risk premium is $(Z - R^d)$, so we calibrate this spread to 200 bp. Our calibration leads to a larger BGG financial accelerator in that the BGG calibration implies $\nu = 0.04$, while our calibration has $\nu = 0.188$.

are different with BGG. Because the lender's rate is pre-determined, the TFP shock leads to a sharp increase in net worth and a symmetric decline in leverage. This net worth expansion leads to an amplification (relative to the POC) of output and investment. These effects diminish only slowly as entrepreneurial net worth returns to normal levels.

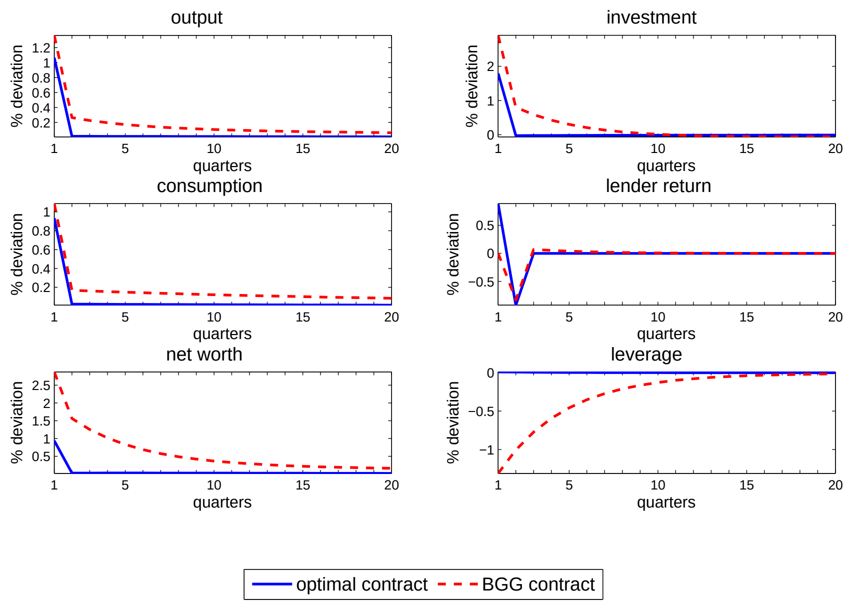

Figure 1: Impulse response to a 1% TFP shock with flexible prices (no serial correlation).

Figure 2 looks at the case of an auto-correlated TFP shock. Compared to the POC, BGG shifts net worth and thus consumption towards the entrepreneur. The persistent movement in net worth leads to a persistent decline in leverage and the risk premium and hence an amplification of investment and output. But under the POC, the lender repayment moves on impact to largely eliminate all movements in leverage and the risk premium. Hence, the POC dynamics will be roughly identical to a frictionless model

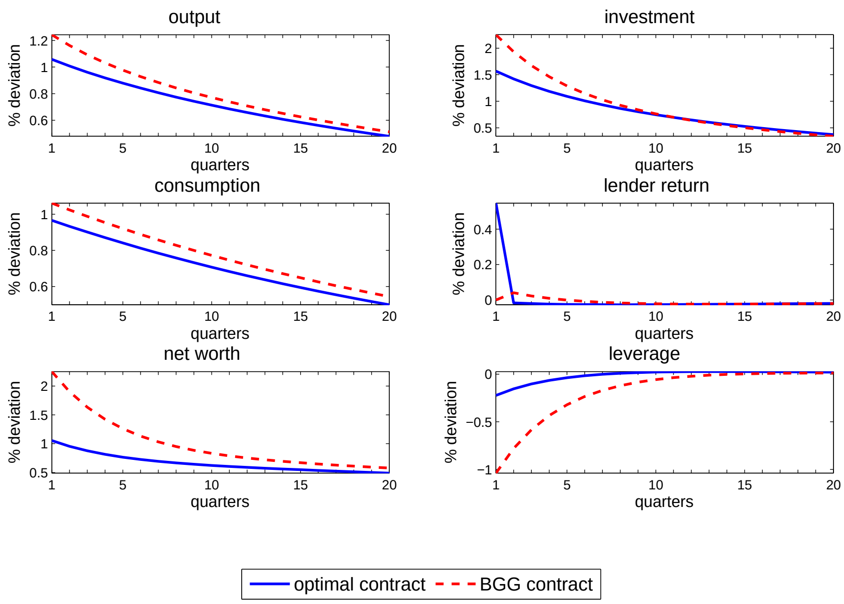

Figure 2: Impulse response to a 1% TFP shock with flexible prices (serial correlation=0.95).

(in which leverage and the risk premium are unchanged by the shock).

Table 1 provides a welfare analysis of the POC and BGG under the baseline calibration and sensitivity analysis over the persistence in the shock process and the degree of investment adjustment costs. The standard deviation of TFP is set to 0.01. Under the POC, this generates a standard deviation of household consumption equal to 0.037 (and a SD of 0.037 for aggregate consumption). This standard deviation of household consumption increases to 0.038 under BGG (and a SD of 0.041 for aggregate consumption).

The welfare measures we report are computed based on a second-order approximation to the nonlinear equilibrium conditions of the model and the two contracts. Our preferred welfare measure is the household and entrepreneur's value function where the state variables are evaluated at the deterministic steady state (which is the same across contracts). Thus, we employ a conditional welfare measure. We follow closely the computational strategy outlined in Schmitt-Grohé and Uribe (2005). Since the entrepreneur's value function is linear, we consider a representative entrepreneur that holds the entire stock of net worth. To turn these utility flows into consumption equivalents, we divide this utility difference by the steady state marginal utility of consumption. The total welfare gain is the sum of the household and entrepreneur's welfare gain, expressed in consumption units and then taken as a fraction of aggregate consumption.

	$\rho = 0.95$ $\psi = 0.5$	$\rho = 0$ $\psi = 0.5$	$\rho = 0.99$ $\psi = 0.5$	$\rho = 0.95$ $\psi = 0.25$	$\rho = 0.95$ $\psi = 2$
Household	0.013	0.023	0.008	0.007	0.024
Entrepreneur	-0.001	0.003	0.000	0.001	-0.005
Total	0.023	0.048	0.016	0.014	0.036

Table 1: Welfare gain of POC relative to BGG. The entries represent the difference in the household and entrepreneurial value function under the two contracts (POC-BGG) in consumption equivalents.

For the baseline calibration, the welfare gain is fairly small, a perpetual annual flow of 0.023% of household consumption. But as point of comparison, Lucas (1987) estimates that the welfare cost of consumption variability is a consumption flow of 0.068% (assuming consumption variability of 0.037). Hence, the welfare cost of the BGG contract (compared to POC) is of the same order of magnitude as the welfare cost of business cycle variability. The primary driver of the welfare cost comes from changes in household behaviour. Under the BGG contract, employment and investment over-respond to TFP shocks because of the financial accelerator. In contrast, the entrepreneur is essentially indifferent to the two contracts, although general equilibrium effects lead to a modest decline in lifetime utility under the POC for the baseline calibration.

The welfare losses of the BGG contract double for the case of iid shocks. The efficient response to an iid shock is a temporary increase in employment and investment. But under BGG, the initial boom in net worth leads to a protracted increase in output. An increase in investment adjustment costs implies an increase in the financial accelerator that is operative under the

BGG contract. Hence, the welfare cost of BGG is also larger for the higher level of adjustment costs.

5 Sensitivity to other shocks

As a form of sensitivity analysis on the positive aspects of the model, we investigate adding sticky prices and other exogenous shocks to the analysis. The focus is on the financial accelerator and how it is affected by the two alternative loan contracts, BGG and POC. We integrate sticky prices via the familiar Dynamic New Keynesian (DNK) methodology. Note that there is no nominal stickiness between the lender and entrepreneur so that the POC is unchanged. The DNK model is standard so we dispense with the formal derivation, see, for example, Woodford (2003) for details. Imperfect competition distorts factor prices so that the marginal productivity of capital and labour in (4) and (5) are pre-multiplied by marginal cost. Marginal cost in turn is affected by the path of inflation so that in log deviations we have a relationship between inflation (π_t) and marginal cost (ζ_t):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \zeta_t \tag{51}$$

The model is then closed with the familiar Fisher equation linking real and nominal interest rates:

$$i_t = E_t \pi_{t+1} + \sigma(E_t c_{t+1} - c_t) \tag{52}$$

and an interest rate policy for the central bank. In log deviations the policy rule is given by:

$$i_t = \phi_\pi \pi_t + \epsilon_t^m. \tag{53}$$

where ϵ_t^m is an exogenous policy movement with autocorrelation $\rho^m = 0.50$. We also consider a shock to the financial market. In particular, we augment the evolution of net worth with an exogenous and persistent shock that takes the form of a lump-sum transfer from households to entrepreneurs that occurs at the beginning of time-t. By altering the path of net worth, this shock will alter leverage and risk premia. We assume that this net worth shock is persistent with $\rho^{NW} = 0.70$. All three of the aggregate shocks (TFP, monetary policy, and net worth) are observed at the beginning of the period. The DNK parameter calibration is standard with $\phi_\pi = 1.5$ and $\kappa = 0.025$.

Figures 3-5 report the impulse response functions to a 1% TFP shock ($\rho^A = 0.95$), a 25 bp (quarterly) monetary policy shock, and a 1% net worth shock. The key to understanding all three experiments is to focus on the behaviour of net worth. The indexation under the POC leads to sharp changes in the lender's return so that the change in net worth is quite modest when compared to BGG. And as noted earlier, the persistence in net worth implies that the initial change in net worth drives all subsequent dynamics. As suggested earlier, the poster boy for this mechanism is the net worth shock. In the BGG model, a 1% exogenous shock to net worth leads to a 10% movement in net worth (recall that leverage = 2) because the financial accelerator kicks in: higher net worth boosts the price of capital, the higher price of capital boosts net worth, etc. This implies a sharp decline in leverage and the risk premium. But these effects are entirely absent in POC. In fact, net worth actually declines on impact so that leverage (and the risk premium) increases. The POC thus involves over-shooting of leverage and the risk premium as it transitions back to the steady state. But in comparison to BGG, these leverage movements are trivial.

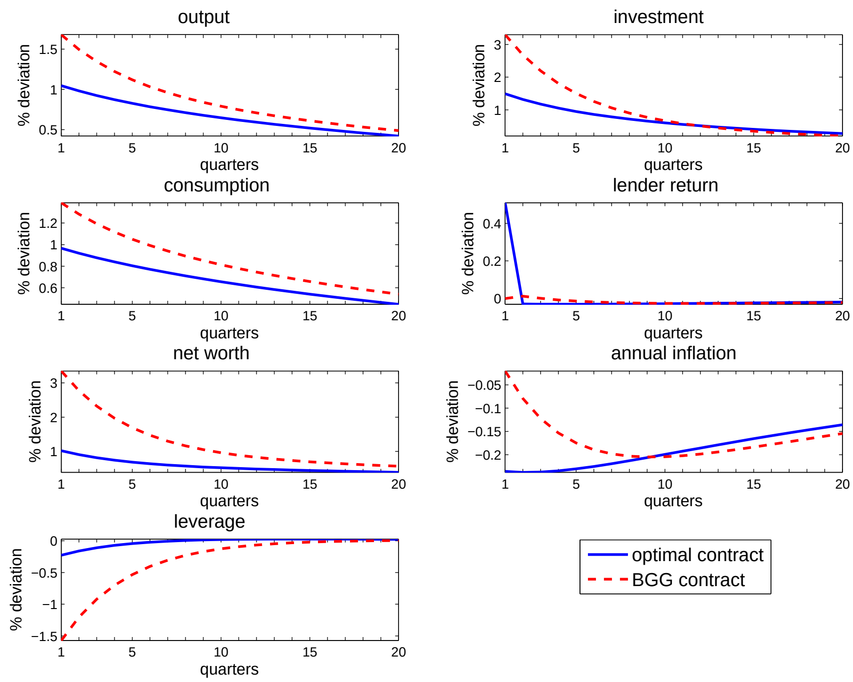

Figure 3: Impulse response to 1% TFP shock with sticky prices (serial correlation = 0.95).

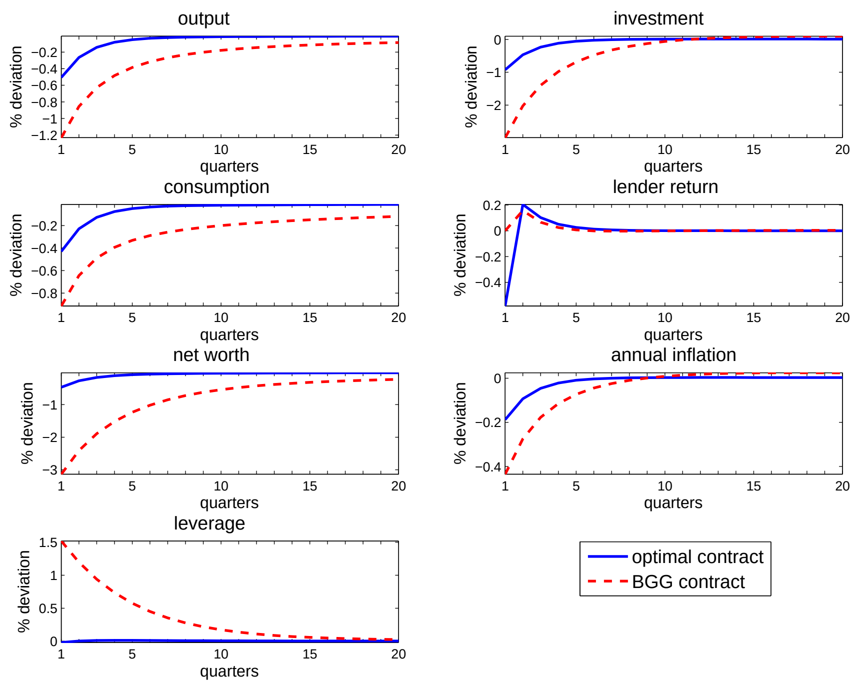

Figure 4: Impulse response to 1% monetary policy shock with sticky prices (serial correlation = 0.5).

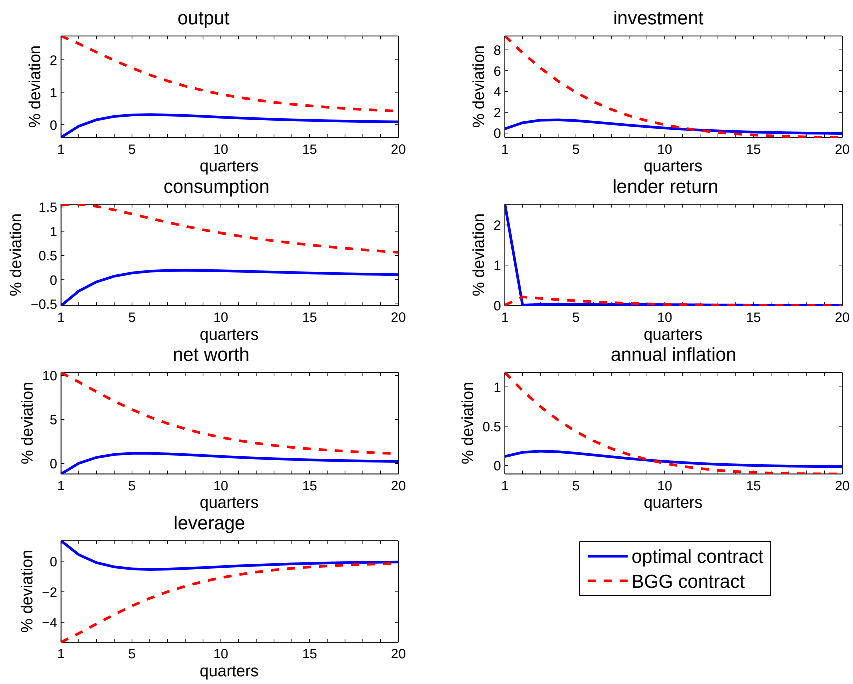

Figure 5: Impulse response to 1% net worth shock with sticky prices (serial correlation = 0.7).

In summary, the BGG model delivers sharp amplification of all three shocks, but only because the non-optimality of the BGG contract leads to sharp movements in net worth, leverage and the risk premium. This amplification is even manifested in the comparison of the TFP response in the flexible price model (Figure 2) vs. the sticky price model (Figure 3). With the BGG contract and sticky prices, the innovation in net worth leads to such a stimulus to investment that the TFP "supply" shock is augmented with "demand" characteristics so that marginal cost rises on impact.[8] This endogenous decline in mark-ups in the DNK model leads to a larger increase in the rental rate and thus the return on capital. Hence, net worth moves

[8] The behaviour of marginal cost can be inferred from the time path of inflation and the Phillips curve.

by more with sticky prices than it does with flexible prices. This net worth movement leads to a further amplification of real activity in the sticky price model. In contrast, under the POC, the net worth movement in both flexible and sticky prices is quite comparable so that the real response to a TFP shock is modestly dampened by the addition of sticky prices as marginal cost falls and mark-ups rise. This is the typical effect of sticky prices in DNK models.

6 Conclusion

Two basic functions of financial markets are to intermediate between borrowers and lenders, and to provide a mechanism to hedge risk. Both of these motivations are present here. The risky debt contract is the optimal method of intermediation as it mitigates the informational asymmetries arising from the CSV problem. An important feature of the optimal contract between lenders and borrowers is indexation to observed aggregate shocks. The optimal level of indexation provides for both consumption insurance for households and a net worth hedge for entrepreneurs. The original analysis of BGG ignored both of these effects, which implied a sub-optimal amplification of aggregate shocks that is not part of competitive behaviour. In contrast, the POC model derived here incorporates these optimal indexation effects.[9] The financial accelerator is thus sharply muted when compared to BGG.

It is an open empirical question to assess the degree of contract indexation in the data. At face value, it may appear that indexation is scarce.[10] But there are two cautions to this assertion. First, less explicit features of lending relationships may serve a similar purpose. Second, the myriad layers of hedging devices in financial markets may result in macro behaviour that is more akin to the indexation under the POC. Carlstrom, Fuerst, Ortiz, and Paustian (2013) use familiar Bayesian methods to estimate a BGG-style macro model in which the degree of repayment indexation is a parameter to estimate. Although the specific results vary depending on observables, they estimate a significant level of debt indexation in the model. Further, the estimated level of indexation implies a trivial financial accelerator.

[9] In a companion paper, Carlstrom, Fuerst, and Paustian (2013) explore the Pareto efficiency implications of the POC.

[10] Halonen-Akatwijuka and Hart (2013) try to explain why optimal contracts may not condition on all verifiable aggregate states and hence remain imperfectly indexed.

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A APPENDIX

A.1 Linearized Model with POC

$$E_t(r^k_{t+1} - r^l_{t+1}) = \nu\kappa_t \tag{A.1}$$

$$nw^{POC}_t = nw^{POC}_{t-1} + E_{t-1}r^d_t + \kappa\nu\kappa_{t-1} + \left(r^k_t - E_{t-1}r^k_t\right) + \frac{\theta_f}{\Psi}\left(m_t - E_{t-1}m_t\right) - \frac{\beta\theta_f}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A.2}$$

$$r^{l,POC}_t = E_{t-1}r^d_t + \left(r^k_t - E_{t-1}r^k_t\right) + \frac{\theta_g}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta\theta_g}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A.3}$$

$$z^{POC}_t = E_{t-1}r^d_t + \frac{(1-\theta_g)\left[1-\nu\left(\kappa-1\right)\right]}{\theta_g(\kappa-1)}\kappa_{t-1} + \left(r^k_t - E_{t-1}r^k_t\right) + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A.4}$$

$$\bar{\omega}^{POC}_t = \left(\frac{1-\nu(\kappa-1)}{\theta_g\left(\kappa-1\right)}\right)\kappa_{t-1} + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A.5}$$

$$\lambda_t = E_t\sum_{j=0}^{\infty}\beta^j\left(\kappa_{t+j} + r^d_{t+j}\right) \tag{A.6}$$

$$r^k_t = \varepsilon q_t + (1-\varepsilon)\,mpk_t - q_{t-1} \tag{A.7}$$

$$\kappa_{t-1} = (q_{t-1} + k_t - nw_{t-1}) \tag{A.8}$$

$$\sigma c_t + \eta l_t = \alpha k_t + a_t - \alpha l_t \tag{A.9}$$

$$E_t r^d_{t+1} = \sigma(E_t c_{t+1} - c_t) \tag{A.10}$$

$$q_t = \psi i_t \tag{A.11}$$

$$k_{t+1} = \delta i_t + (1-\delta)\,k_t \tag{A.12}$$

$$c_{ss}c_t = -c^e_{ss}nw_t - i_{ss}i_t + a_t + \alpha k_t + (1-\alpha)\,l_t \tag{A.13}$$

where $\varepsilon \equiv \frac{1-\delta}{mpk_{ss}+(1-\delta)}$. Also we have $\bar{\kappa} \equiv K_{SS}/NW_{ss}$, Q_{ss}= 1, $R^s_{ss} = 1/\beta$. Finally, in the linear model we set $\mu\int_0^{\bar{\omega}_{ss}}\omega\phi\left(\omega\right)d\omega \approx 0$ so that monitoring costs do not appear above.

A.2 The optimality of the debt contract

In this appendix we elabourate on the link between linearity and the implication that the optimal contract is risky debt. By "risky debt," we mean a contract in which the payoff does not vary with individual productivities over a no-monitoring range, and involves confiscation of the entire project when monitoring does occur. In this appendix we show that if the entrepreneur's value function is linear in net worth, then the optimal contract is risky debt. Using risky debt, the text then demonstrates that the entrepreneur's value function is in fact linear, thus validating the initial linearity assumption and completing the logical circle.

The typical entrepreneur begins the period with net worth NW_t^i. We assume that the entrepreneur's value function is linear in net worth, given by $V_t NW_t^i$. In equilibrium, the return on internal funds will always be high enough such that he will postpone consumption until death. With probability (1-γ) he dies and eats $c_t^e = NW_t^i$. With probability γ, he lives and uses all of his net worth to fund a project, with leverage rate $\overline{\kappa}_t$. The typical entrepreneur's project size is thus given by $\overline{\kappa}_t NW_t^i$, so that $(\overline{\kappa}_t - 1)NW_t^i$, is the amount funded by the lender. The linear return on the entrepreneur's project is $\omega_{t+1}^i R_{t+1}^k$, where ω has a unit mean and is iid. The entrepreneur reports ω_{t+1}^i to the lender, and R_{t+1}^k is observed by all. Because of this linearity and observability, we can without loss of generality normalize the repayment function as $R_{t+1}^k R(\omega_{t+1}^i; R_{t+1}^k)$, for some function $R(\cdot)$. Incentive compatibility implies that $R(\omega_{t+1}^i; R_{t+1}^k)$ is independent of ω_{t+1}^ion the no-monitoring set. The monitoring interval is given without loss of generality by $A_{t+1} = [0, \bar{\omega}_{t+1}]$, where $R(\omega_{t+1}^i; R_{t+1}^k) = R_{t+1}$ for $\omega_{t+1}^i \geq \bar{\omega}_{t+1}$. The time $t+1$ payoffs are given by:

$$E_t ent_{t+1} = E_t V_{t+1} \overline{\kappa}_t NW_t^i R_{t+1}^k \left[E_t \left(\omega_{t+1}^i \right) - R_{t+1} \left(1 - \Phi_{t+1} \right) - X_{t+1} \right] \quad \text{(B.1)}$$

$$E_t len_{t+1} = E_t M_{t+1} \overline{\kappa}_t NW_t^i R_{t+1}^k \left[R_{t+1} \left(1 - \Phi_{t+1} \right) + X_{t+1} - \mu \Phi_{t+1} \right] \quad \text{(B.2)}$$

where Φ_{t+1} is the CDF evaluated at $\bar{\omega}_{t+1}$, M_{t+1} is the household's intertemporal pricing kernel, and we define

$$X_{t+1} \equiv \int_0^{\bar{\omega}_{t+1}} R(\omega) \phi(\omega) d\omega$$

The linearity of the entrepreneur's value function is exploited in (B1). That is, we can pull the expectations operator into the expression and use $E_t\left(\omega_{t+1}^i\right) = 1$. This would not be possible if the entrepreneur's payoff were nonlinear or if V_{t+1} or R_{t+1}^k were correlated with ω_{t+1}^i. The optimal contract maximizes $E_t ent_{t+1}$, subject to the lender's participation constraint:

$$E_t len_{t+1} \geq (\overline{\kappa}_t - 1) NW_t^i$$

We have already noted that incentive compatibility implies that the repayment function must be constant over the no-monitoring set. The final step is to demonstrate that when monitoring does occur, the lender seizes the entire project, that is $R\left(\omega_{t+1}^i\right) = \omega_{t+1}^i$ for all $\omega_{t+1}^i \in A_{t+1}$. But this result is standard given the linear expressions (B1) and (B2). See, for example, section 4.2 of Freixas and Rochet (2008), for a general discussion of the CSV problem and the optimality of risky debt.