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Abstract

In a world of interconnected financial markets it is plausible that risk appetite — an important factor in asset pricing — is determined globally. By constructing an estimate of variance risk premia (VRP) for UK, US and euro-area equity markets, we are able to estimate international variance risk premia and use them to construct a proxy for global risk aversion. The impact of this time-varying risk aversion proxy on bond risk premia is then analysed within an arbitrage-free term structure model of UK interest rates, where it is introduced explicitly as a pricing factor. By linking VRP to a stochastic discount factor, we find that the risk aversion factor has significantly affected UK government bond yields. The changes in the risk aversion factor have been particularly important in the period of the 2008–09 financial crisis, with medium maturity yields being affected the most.

Key words: Affine term structure models, option implied volatility, realized volatility, risk aversion, stochastic discount factor, variance risk premium, volatility forecasting.

JEL classification: C22, C52, G12, E43.

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1 Introduction

Extracting measures of global market sentiment and relating them to asset risk premia is the main focus of this paper. In particular, we analyse the impact of a time-varying proxy of global risk aversion on bond risk premia within an arbitrage-free term structure model of UK interest rates.

The importance of global risk sentiment for explaining developments in financial markets is well documented elsewhere in the literature. Changes in global risk aversion are found to be a key determinant of emerging market sovereign bond spreads (e.g., among many others, Caceres, Guzzo, and Segoviano, 2010), and they also appear to be a significant factor influencing capital flow movements (e.g. Forbes and Warnock, 2012). But although it is well-accepted that global risk aversion is time varying (e.g. Guiso, Sapienza and Zingales, 2013), its unobservability poses a considerable obstacle for macroeconomic and financial analysis.

To circumvent the problem of unobservability, it has been a common practice to draw on proxies for risk aversion. Many such proxies are based on financial market data. For example, Baek, Bandopadhyaya and Du (2005) compute a 'risk appetite index' as the Spearman's rank correlation measure between monthly stock market index returns and historic realised stock market volatility. On a more theoretically consistent basis, Ait-Sahalia and Lo (1998) show that a measure of investors' risk aversion can be derived by comparing option-implied return distributions to return distributions estimated from the realised movements of the underlying asset prices. Probably the most popular proxy for time-varying risk aversion among practitioners has been a measure of the implied volatility derived from 1-month options on the S&P 500 US equity index, the VIX. However, apart from reflecting market risk pricing, the VIX also captures market expectations of next month's realised volatility in the equity index. More recently, the so-called variance risk premium (VRP) has been considered, which adjusts a VIX-based measure for market volatility expectations (e.g. Bollerslev, Tauchen, and Zhou, 2009). And Bollerslev, Marrone, Xu, and Zhou (2014) showed that to better capture global risk sentiment, it might be preferable to use a measure that covers a broader set of advanced economy markets.

In this paper we estimate a proxy for global risk aversion from option prices similarly to Bollerslev, Marrone, Xu, and Zhou (2014). To obtain global rather than US-focussed estimates, we use options on three major international equity indices, namely the S&P 500 (US), FTSE 100 (UK) and EuroStoxx 50 (euro area). We start by modelling the expected variance of returns on the underlying index. This allows us to obtain VRP estimates as the difference between corresponding implied variances (squared implied volatilities) and the measures of expected realised variances. By extracting such VRP measures from US, UK and euro area option prices over the 2000-2014 period, we are able to combine information from UK and euro-area VRPs with the US VRP and obtain a proxy for global risk aversion, either as the market capitalisation weighted average, or as the first principal component of the individual US, UK and euro-area VRPs.

Finally, we analyse the impact of time-varying risk aversion on bond risk premia within an arbitrage-free model of interest rates. Specifically, we estimate an affine term structure model for monthly interest rates derived from UK government bond yields. The term structure is assumed to be determined by factors based on i) the principal components of zero-coupon yields and ii) our global risk aversion factor. We find that the risk aversion factor has significantly affected UK government bond yields. The impact of changes in the risk aversion factor has been particularly important in the period of the recent financial crisis. The 2008-2009 increase in market risk aversion had an impact of up to 90 basis points on medium- to long-term maturity UK yields and accounted for up to 70% of the term premium component of yields, with 5-7 year yields being affected the most.

The remainder of the paper is organised as follows: Section 2 explains the available methods for modelling the time-varying variance of asset returns, describes the data, introduces the econometric methodology, and discusses the variance forecasting performance of the models. The VRP as a proxy for risk aversion is discussed in Section 3. Finally, Section 4 presents a term structure model with time-varying risk aversion and analyses its impact on bond risk premia. Section 5 concludes our analysis.

2 Modelling the variance risk premium and the variance of asset returns

To obtain VRP estimates we follow the methodology of Bollerslev, Tauchen and Zhou (2009). The first step is to model the expected conditional variance of returns of an underlying equity index, $E_t(V_{t+1}^{equity})$.[1] After we have modelled the conditional variance, we can get the VRP as the difference between the corresponding implied variances derived from option prices (squaring option-implied volatility) and the measures of expected realised variances:

$$VRP_t = IV_t^2 - E_t(V_{t+1}^{equity}) \tag{1}$$

where IV_t^2 is the squared one-month horizon implied volatility from options and V_t^{equity} is the realised variance.[2]

2.1 Realised variance

Different methods for estimating the dynamics of conditional variances have evolved in the literature. The main obstacle in modelling conditional variance, however, is the fact that not only the expected dynamics, but even the true volatility of returns cannot be observed directly. Therefore the actual volatility also needs to be modelled. A basic measure of the volatility of returns is the rolling standard deviation of daily returns, a method still widely employed by market practitioners. However, this measure is subject to numerous critiques, e.g. it is sensitive to the chosen length of rolling window. The financial econometric literature, starting from the ARCH model by Engle (1982) and the GARCH model by Bollerslev (1986), tried to address the critiques and to come up with superior models of volatility. Subsequently, applications and extensions of the ARCH/GARCH approach have become the norm in more sophisticated volatility modelling. Finally, the latest literature on volatility forecasting stresses the importance of model-free measures of the 'realised variance' of returns.

[1] As the vast empirical literature on the volatility of asset returns reveals, the variances of asset returns fluctuate over time: large changes in returns tend to cluster, which is a key stylised fact about asset returns, known as volatility clustering and observed across a range of assets and frequencies (e.g. Bollerslev, Chou and Kroner, 1992).

[2] As $V_{t+1}^{equity} - IV_t^2$ is the return to buying variance in a variance swap, VRP_t is technically the negative of variance risk premium. As that number is mostly negative, we follow the definition as in Bekaert and Hoerova (2014).

The concept of 'realised volatility', introduced by Andersen and Bollerslev (1998), is based on using high-frequency data and provides a more precise estimate of the daily volatility of asset returns. The idea is simple: the daily realised volatility of a single asset return is measured via the sample variance of high frequency data, such as 5-minute returns data. This method has been applied to equity returns in Andersen, Bollerslev, Diebold, and Ebens (2001) and to exchange rates in Andersen, Bollerslev, Diebold, and Labys (2001). The resulting realised variance is not latent, but observed, and a sample average of V_{t+1}^{equity} can be used for computing the *unconditional* variance of returns. It has also been shown that models of the *conditional* variance, $E_t(V_{t+1}^{equity})$, produce more accurate forecasts when based on these realised volatility measures (see Chen and Ghysels, 2012).

2.2 Methodology

To estimate the conditional variance, we follow a popular method in the literature (e.g. Bekaert and Hoerova, 2014) of empirical regression-based projections of realised variance on the prior information set comprising a wide set of possible predictive variables, including lagged realised variance components, most recent return variances (e.g. weekly or daily), negative return shocks, and lagged squared option-implied volatility.

For each of the equity indices, we base our econometric models of $E_t(V_{t+1}^{equity})$ on the latest findings of variance forecasting literature, which emphasises that:

1. variance is persistent, so current realised variances predict next period variance realisations (e.g. Chernov, 2007);
2. there can be information in the most recent return variances (e.g. Corsi, 2009);
3. implied variance contains information about future realised variance, so can be used as a predictor (e.g. Christensen and Prabhala, 1998);
4. there can be different predictive information in jump versus continuous variance components (e.g. Andersen, Bollerslev, and Diebold, 2007); and

5. variance is asymmetric, so that good news and bad news have different predictability for future variance (e.g. Engle and Ng, 1993).

More recently, there has been a focus on monetary policy rate uncertainty as a possible factor affecting volatility. In particular, market participants suggest that the lower volatility of financial markets implied by derivatives prices post-crisis reflected reduced uncertainty around the path of monetary policy (see e.g. Bank of England, 2014). There are several possible reasons behind this explanation. First, monetary policy rates in the United Kingdom, the United States and the euro area have been constrained by effective lower bounds, limiting the scope for interest rates to fall further and truncating their distribution in the near term. Second, the Bank of England, Federal Reserve and ECB have also provided considerable information on their reaction functions through forward guidance, reducing uncertainty about the path of policy in the future.[3] More generally, given that the policy rate is a key factor for pricing many capital market and derivative assets, a reduction in the uncertainty that investors ascribe to the future path of monetary policy rate may help to lower uncertainty in pricing financial asset classes more broadly. To analyse this channel, we investigate if a variable reflecting uncertainty about short-term interest rates has an explanatory power and can affect the expected variance of equity returns.

We therefore consider model specifications that include all the features discussed above. In its most general form, our forecasting model is:

$$\begin{aligned} RV_t^{(22)} &= \beta_0 + \beta_1 IV_{t-22}^2 + \beta_2 RV_{t-22}^{(22)} + \beta_3 RV_{t-22}^{(5)} + \beta_4 RV_{t-22}^{(1)} + \beta_5 C_{t-22}^{(22)} + \beta_6 J_{t-22}^{(22)} + \\ &\quad \beta_7 C_{t-22}^{(5)} + \beta_8 J_{t-22}^{(5)} + \beta_9 C_{t-22}^{(1)} + \beta_{10} J_{t-22}^{(1)} + \beta_{11} r_{t-22}^{(22)-} + \beta_{12} MP_{t-22}^{uncert} + \varepsilon_t, \end{aligned} \tag{2}$$

where the different elements are defined as follows.

The dependent variable is the monthly realised variance, $RV_t^{(22)}$, which is the sum of the daily realised variances over 22 trading days. Following Sheppard, Liu and Patton (2013), who show that realised variance based on 5-minute data is the best estimator of the realised variance

[3] For example, the Bank of England's Monetary Policy Committee (MPC) introduced forward guidance in August 2013. The MPC stated that it intended, at a minimum, to maintain the exceptionally accommodative stance of monetary policy until economic slack had been substantially reduced, provided that this did not put at risk either price stability or financial stability. See Bank of England (2013).

across different assets, we estimate daily realised variances from squared 5-minute intraday returns.

As independent variables, we include lagged values of the monthly, $RV^{(22)}_{t-22}$, weekly, $RV^{(5)}_{t-22}$, and daily, $RV^{(1)}_{t-22}$, realised variances. Lagged implied variance is included as IV^2_{t-22}, which is the implied volatility expressed as monthly percentage squared. For example, for the S&P 500 we use the VIX and $IV^2_{t-22} = \frac{VIX^2_{t-22}}{12}$. Our next set of independent variables split realised variance into continuous and 'jump' components at monthly, weekly and daily frequencies. This follows Andersen, Bollerslev, and Diebold (2007), who find that the jump component of volatility is less persistent than the continuous component, and that splitting them out improves out-of-sample volatility forecasting. As in Bekaert and Hoerova (2014), we identify the jump component using the threshold bipower variation proposed by Corsi, Pirino, and Renò (2010), which significantly reduces the small-sample bias in the standard bipower variation estimates (e.g. Barndorff-Nielsen and Shephard, 2004). Using the methodology from Corsi, Pirino, and Renò (2010), we define the jump, J, in the daily realised variance, as:

$$J_t = \max\left[RV_t - TBPV_t, 0\right] \tag{3}$$

where $TBPV_t$ is the threshold power variation. We then define the continuous component as

$$C_t = RV_t - J_t \tag{4}$$

To get weekly and monthly continuous and jump components, we average the daily components and express them in monthly units: $J^{(h)}_t = \frac{22}{h}\sum_{j=1:h} J_{t-j+1}$ and $C^{(h)}_t = \frac{22}{h}\sum_{j=1:h} C_{t-j+1}$ for $h = 5$ for weekly units and $h = 22$ for monthly units.[4] The so-called 'leverage effect' is where a negative return increases leverage, making the security more risky and so increasing its volatility (e.g. Campbell and Hentschel, 1992, and Bekaert and Wu, 2000). To incorporate this effect, we follow Corsi and Renò (2012) in adding average monthly negative returns as an independent variable. This is defined as $r^{(22)-}_t = \min\left[r^{(22)}_t, 0\right]$ where $r^{(22)}_t = \sum_{j=1:22} r_{t-j+1}$, for daily returns r_t. Finally, we proxy monetary policy rate uncertainty, MP^{uncert}_{t-22}, using lagged squared interest rate implied volatility based on options on short-term interest rate futures. We also check the sensitivity of results by using a squared spread between 1-month forward LIBOR rates and the policy rate as an alternative proxy.

[4] When using the model with jumps, one can only have two of RV, C and J as $RV = C + J$. So we omit RV in these cases.

In total, we consider 52 different model specifications based on different combinations of independent variables listed above and estimate them by OLS.

2.3 Data

We obtain S&P 500, FTSE 100, Euro Stoxx 50 equity indices and corresponding implied volatility indices (VIX for S&P 500, VFTSE for FTSE 100 and VSTOXX for Euro Stoxx 50) from Bloomberg. To get V_t^{equity}, we use daily realised variance of 5 minute returns data from the underlying equity indices, as published by the Oxford-Man Institute (Heber, Lunde, Shephard and Sheppard, 2009). For our proxy of monetary policy uncertainty, we use three-month implied volatility from options on three-month interest rate futures. Our data are from 4 January 2000 to 31 December 2014, excluding public holidays. We have a total of 3692 daily observations for the case of S&P 500, 3693 for the FTSE, and 3723 for the Euro Stoxx.

2.4 Model selection

In this subsection, we select optimal models for variance forecasting from the 52 models outlined above by analysing their relative out-of-sample forecasting performance. Different estimation schemes can be employed to perform out-of-sample forecast evaluation, known as the fixed, rolling, and recursive schemes. Under the fixed scheme the parameters are estimated once and this point estimate is used throughout the out-of-sample forecasting subsample. In the rolling and recursive schemes, the parameters are re-estimated every time a forecast is made. The recursive scheme uses all past observations for the estimation, whereas the rolling scheme only uses a fixed number of the most recent observations. We estimate the competing models and then use them to produce one-step ahead forecasts under each estimation scheme.

For the fixed evaluation scheme, we estimate the models using the first 70% of the sample, which involves the data starting from 4 January 2000 and going up to 15 June 2010 for the US; up to 30 June 2010 for the UK; and up to 7 July 2010 for the euro area data. Without re-estimation, we produce and evaluate their forecasts on the remaining 30% of the sample.

Table 1 shows the statistics and the ranking of all best performing models for each of the indices (i.e. models for which the forecasting performance is not significantly different from the model with the lowest mean squared forecasting errors (MSFE).[5]

Table 1. Best performing variance forecasting models for equity indices

	Model									MSFE
	IV^2_{t-22}	$RV^{(22)}_{t-22}$	$RV^{(5)}_{t-22}$	$RV^{(1)}_{t-22}$	$C^{(22)}_{t-22}$	$C^{(5)}_{t-22}$	$C^{(1)}_{t-22}$	$r^{(22)-}_{t-22}$	MP^{unc}_{t-22}	
UK					+	+	+		+	0.86**
					+	+			+	0.88**
	+				+	+	+		+	0.88**
					+	+	+			0.87**
					+	+				0.89**
US	+				+	+				0.82***
	+				−	+	+			0.83***
					+	+	+			0.85***
	+	+	+						+	0.88***
	+	+								0.85***
					+	+	+	−		0.86**
	+	+							+	0.89***
	+				+					0.86***
Euro	+							−	+	0.86***
area	+				−	+	+		+	0.89*
					+	+	+	−	+	0.89***
	+				+	+	+	−	+	0.87***
	+				+	+	+	−		0.89*
	+				+	+		−	+	0.88**
	+								+	0.86***
	+	−	+	+				−		0.88***
	+	−	+						+	0.87***

Note: Last column shows the mean squared forecasting errors (MSFE) relative to a Random Walk over the out-of-sample period from February 2010 to December 2014 for individual forecasting models insignificantly different from the models with the lowest mean squared forecasting errors. A value lower than one means that the model outperforms the benchmark. *** or ** mark significant difference in performance at 5% or 1% significance level by Giacomini-White test. Model specifications with $C^{(i)}_{t-22}$ also contain $J^{(i)}_{t-22}$."-" and "+" denote estimated coefficient signs.

[5] The results from rolling (window length is half of the sample) and recursive estimations provide identical model ranking, so we do not provide separate tables for them.

For the case of the FTSE 100 return variance, the best performing models explain next period 1-month variance by jumps and continuous realised variance components and by our proxy for monetary policy interest rate uncertainty, which is captured by lagged squared three-month interest rate implied volatility based on options on three-month LIBOR interest rate futures. The results show that UK short-term interest rate implied volatility has a significant positive predictive power for volatility in the UK equity index. The results hold also in the case of the alternative proxy for monetary policy rate uncertainty, which is the squared spread between one-month forward LIBOR and the policy rate. A model with a lagged squared option-implied volatility included as an additional predictor also turns out to be among top performers. As expected in this case, we find that the current implied volatility is positively related to the next period 1-month realised volatility.

In the US, the best performing models are based on various combinations of jumps and continuous realised variance components, lagged squared option-implied volatility, and again a proxy for monetary policy rate uncertainty. Monetary policy rate uncertainty is also important for euro-area equity return volatility, and, as in the case of the FTSE 100 and S&P 500 indices, is positively related to uncertainty in equity markets. The low volatility levels in equity markets during 2013-2014 should therefore not be totally surprising, as it partially reflects lower uncertainty about short-term interest rates. This may have been caused by unconventional monetary policy and forward guidance in the US, UK and euro area, leading investors to expect policy rates to remain close to the zero lower bound for longer.

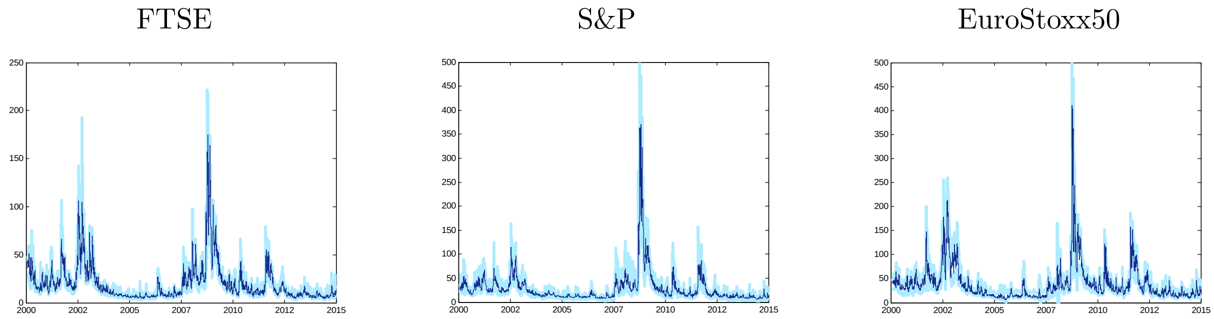
Consistent with the volatility literature, we also find that negative returns (i.e. associated with bad news) play an important role in predicting the variance of EuroStoxx 50 and S&P 500 equity indices. This confirms the well-known result that volatility is affected more by bad news than by good news. Thus an asymmetric framework should be considered when modelling equity volatility dynamics.

Given that out-of-sample criteria could be unstable and sensitive to the data sample chosen for the selection, we also evaluated the in-sample performance of the models by estimating the models using the full sample of daily data between March 8, 2000 and December 31, 2014. Based on standard information criteria based on mean squared errors, such as AIC and BIC, we find that the top performing conditional variance models are rather similar for the different equity indices, emphasising the importance of including variables to capture jumps and negative returns. We also find that for all cases the proxies for monetary policy uncertainty have a significant and positive predictive power for the 1-month ahead equity return variance. However, all the models based on the in-sample criteria selected are quite complex, containing from five to nine regressors on top of the constant, confirming a well known fact that as more complexity is added to a model the better will the model fit the data in-sample. Interestingly, most of our best out-of-sample performing models are also complex, i.e. they rely on a large number of predictors. In general, an over-parameterised model tends to do worse than a more parsimonious model out-of-sample, as in an out-of-sample comparison it will take a great deal of luck for an over-parameterised model to offset its disadvantage relative to a simpler model. Therefore, when the complex models are found to outperform a simpler model out-of-sample, it gives us a stronger evidence in favor of the larger models, than had the same result been found only in-sample.

For each equity index, our variance forecasting models produce similar forecasts and we are unable to identify a single model producing significantly superior forecasts. So we take a simple average of a number of best performing models for each index to produce forecasts of the conditional variances.[6] In particular, we choose the average combination of the three models specified for the UK in Table 1. Analogously, we average variance forecasts from US and euro-area equity index models shown in Table 1. The models produce broadly similar forecasts and the forecast combinations could be depicted by narrow swathes as shown by Figure 1.

[6] Here we follow Stock and Watson (2003), who show that simple model averaging can be superior to other model combination methods, and Smith and Wallis (2009), who suggest that the underlying problem of constructing optimally weighted forecast combination lies in the estimation error in the estimated weights, in particular when the size of the gains from optimal combination is small relatively to estimation error (Elliott, 2011).

Figure 1: **Expected 1-month ahead conditional variance.**



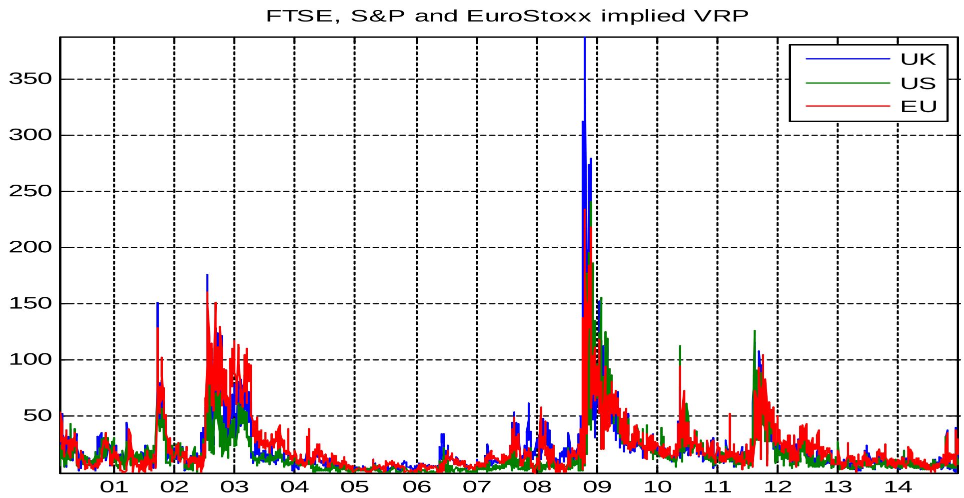
Note: Dark blue line shows the average of the best models from Table 1. The light blue swathe is constructed by the minimum and maximum of expected variances produced by Table 1 models

3 VRP as a proxy for investor risk aversion

3.1 VRP estimates and interpretations

The model selection procedure outlined in the previous section allows us to select the optimal models of conditional variances for the case of each of the three international indices. At the next step, we obtain VRPs as a difference between corresponding implied variances (e.g. squared VIX) and the measures of expected realised variances using equation (1). The resulting VRP estimates are displayed in Figure 2.

Figure 1: **Daily VRP estimates for FTSE100, S&P500, and EuroStoxx50.**



Several observations follow. First, VRPs appear to be time varying. Second, the VRP components also explain a significant share of corresponding of implied variances. In particular, the contribution of VRP components was greatest in the aftermath of the global financial crisis in 2009. This is consistent with Corradi, Distaso, and Mele (2013) who find that the volatility risk premium is strongly countercyclical, even more so than stock volatility: VRP, which is typically not very volatile, may increase in bad times to extremely high levels, and quite quickly.

These findings suggest that temporal variation in expected volatility and variance risk premia both play an important role in determining implied volatility internationally. The former is not surprising, as expected volatility is a measure of the uncertainty, or quantity of risk, expected by the markets. The interpretation of volatility risk premia may seem to be less intuitive though. Primarily, the difference between the implied and expected equity volatilities is the compensation investors are willing to pay for assets that perform well when stock market volatility is high and rises beyond their expectations. Volatility risk premia for US equity indices have therefore been interpreted as indicators of investors' risk aversion (see Rosenberg and Engle, 2002; Corradi, Distaso, and Mele, 2013; Bali and Zhou, 2015; Bekaert, Engstrom, and Xing, 2009; and Drechsler and Yaron, 2011, among others). And according to Todorov (2010), willingness to pay for jump risk insurance significantly increases after jumps are observed in the markets, which is consistent with time-varying risk aversion being primarily driven by large market moves.

Theoretically, several model features can generate time varying risk aversion, e.g. models with habit persistence (Constantinides, 1990; Campbell and Cochrane, 1999). Thus it should be possible to analyse the links between a stochastic volatility risk premium and agents' risk aversion. Indeed, Bollerslev, Gibson, and Zhou (2011) demonstrate a direct link between the stochastic volatility risk premium and the coefficient of risk aversion for the representative investor within the standard intertemporal asset pricing framework. They assume a linear volatility risk premium, with an affine stochastic volatility model and a representative agent with power utility. In this framework, they find that the constant relative risk aversion coefficient is directly proportional to the volatility risk premium and the relationship still applies in an approximate sense if investors' risk aversion is time-varying and follows a diffusion process. Within a simple representative agent setting and more general volatility model, Bakshi and Madan (2006) show that the volatility spread may be expressed as a nonlinear function of the aggregate degree of risk aversion when the physical volatility distribution is negatively skewed and leptokurtic.

Subsequently, given the availability and relative computational ease of VRP measures, they have become widely-used proxies for fluctuations in the representative agent's risk aversion in a series of reduced form empirical models (e.g. Bekaert and Hoerova, 2014; Forbes and Warnock, 2012).

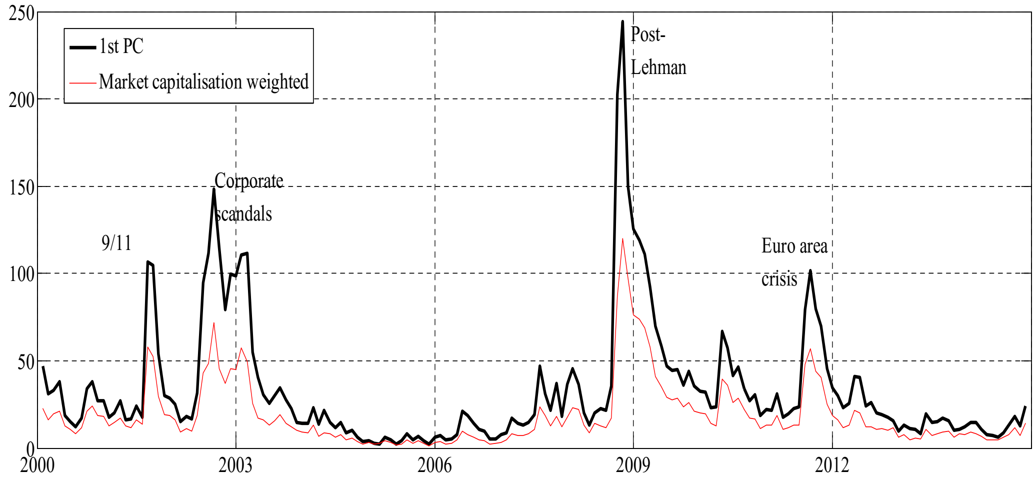
The notion of time varying risk aversion is perhaps more intuitive outside the representative agent framework, as changing regulation and the market share of a specific type of investors – e.g. institutional investors – could affect aggregate market attitude to risk. Taking the interpretation of VRP as time-varying risk aversion at face value, it is perhaps unsurprising that the three measures are highly correlated – pairwise daily correlations are 0.79, 0.79, and 0.87 for UK-US, EA-US, and UK-EA respectively. Indeed, the share of international institutional investors in each of the advanced economy indices is large and financial markets are increasingly driven by the behaviour of global asset managers (IMF, 2014; Haldane, 2014).[7] Thus, it should be unsurprising that UK and euro-area risk aversion indicators are very similar to the one extracted from the VIX.

3.2 Global risk aversion

But this raises an interesting question: which VRP measure should we use as a risk aversion indicator when pricing assets internationally? On the one hand, the US asset market is considered to be a driver of asset markets globally, "making the weather for the rest" (e.g. Miranda Agrippino and Rey, 2015), and hence the VRP extracted from the US VIX should also matter for foreign stock market returns, as shown by Londono (2011). On the other hand, by constructing VRPs for different markets, we may improve the US-implied measure as we are able to use more of available information across international financial markets and so get a better a better understanding of recent trends in global market sentiment. For

[7] For example, the proportion of UK shares owned by overseas investors stood at an estimated 53.2% of the value of the UK stock market at the end of 2012, up from 30.7% in 1998 and 43.4% in 2010. Moreover, international ownership is mostly represented by financial institutions. Unit trusts, banks, pension funds and other financial institutions own more than 82% of international holdings of UK stocks. Source: Office for National Statistics.

Figure 2: **Alternative "global" VRP measures at monthly frequency.**



example, Bollerslev, Marrone, Xu, and Zhou (2014) construct a simple “global” variance risk premium proxy, defined as the market capitalisation weighted average of the individual country variance risk premia. They show that using this proxy for the “global” variance risk premium provides a more accurate measure for all of the countries, and may be interpreted as a proxy for aggregate risk aversion. Given the dominant size of the US market (up to 70% of total market capitalisation) however, the resulting global measure is largely skewed towards the US VRP. Indeed, in our case this monthly market capitalisation weighted global proxy is highly correlated with US-based measure (correlation coefficient of 0.98) and less so with euro-area and UK based measures (0.88 and 0.89, respectively).

Alternatively, we can abstract from idiosyncratic country-specific shocks by extracting a common component of international VRP measures, which could potentially deliver a cleaner measure of global market risk aversion. For example, summarising a common component of VRPs extracted from US, UK and euro-area option prices with the help of principal component analysis (PCA), we find that the first principal component (PC) explains around 88% of the total variance of international VRPs at a daily frequency and produces correlations with country specific measures in the relatively narrow range of 0.88-0.95.

The two alternative “global” variance risk premium measures - market capitalisation based and PC based - are highly correlated (correlation coefficient of 0.96). In Figure 3 we plot the monthly series for both. The measures are relatively stable for the 2004-2007 and 2013-2014 periods. The peaks in the measures coincide with commonly accepted "market fear" events:

the largest peaks are observed in the aftermath of the Lehmans failure, around US corporate scandals/the Iraq war and after the 9/11 terrorist attack, consistent with Corradi, Distaso, and Mele (2013) and the idea that these VRP measures could be interpreted as a proxy for global risk aversion.

To sum up, we have several possible candidate risk aversion proxies for analysing asset prices internationally: the US VRP, a local currency-based VRP, and the two variants of the global VRP measure.

3.3 Excess return regressions

When market participants become more risk averse, they value any risky asset less and attach a higher price for bearing a given amount of risk. So if the VRP measures do reflect market risk aversion, they should matter for the pricing of risky assets. To give a more credible illustration of the possible role of the VRP as a proxy for risk aversion and at the same time to pick the best performing measure, we examine their predictive power for risk premia.

First, we test predictive power of the proposed measures in the case of equity excess returns, as Bekaert and Hoerova (2014) do for the VIX-implied VRP. In particular, we regress equity excess returns (rx_{t+h}) on lagged values of our alternative VRP estimates:

$$rx_{t+h} = \alpha + \beta VP_{it} + \varepsilon_{t+h}, \tag{5}$$

where VP_{it} could stand for individual US, UK and euro-area VRP estimates, or our global VRP estimates. We implement the regression analysis using the sample of monthly data from 2000 to 2014, and holding periods of 1, 6, 12 and 24 months for the excess returns on the S&P 500, FTSE 100 and EuroStoxx 50 equity indices. Excess returns are calculated as the difference in log prices of the equity index over the holding period, less the risk-free interest rate over that holding period (proxied by government bond yields).

The regression results are in Table 2. Several things stand out. First, for each equity market, the squared VIX never delivers a superior performance with respect to the VRP measures, consistent with the findings by Bollerslev, Marrone, Xu, and Zhou (2014): the degree of predictability afforded by the VRP measure easily exceeds that of the implied variance measures when included in isolation.

Second, we also find that US VRP is a significant predictor for UK excess returns, outperforming the domestic VRP measure. This result is consistent with Londono (2011), who showed that in a two-country general equilibrium model, the VRP generated in a "leader" economy plays a key role in explaining the time variation in equity returns in all countries. Therefore, the VRP of the leader country (which would be the US in many cases) outperforms the follower country VRP in predicting equity returns.

Table 2. Estimated coefficients on VRP measures from equity index excess returns regressions.

	1-month	6-month	12-month	24-month
S&P				
Global (PC)	1.15 (0.2 ; 0.00)	27.14 (1.9 ; 0.04)	51.15 (2.9 ; 0.07)	108.25 (3.6 ; 0.14)
Global (Market Capitalisation Average)	1.22 (0.2 ; 0.00)	29.20 (2.1 ; 0.05)	50.43 (2.8 ; 0.06)	111.50 (3.8 ; 0.15)
US VRP	1.12 (0.2 ; 0.00)	29.38 (2.3 ; 0.05)	47.57 (2.6 ; 0.06)	110.86 (3.9 ; 0.14)
VIX^2	−3.56 (−0.8 ; 0.01)	9.65 (0.7 ; 0.01)	28.43 (1.6 ; 0.02)	76.31 (2.5 ; 0.07)
FTSE 100				
Global (PC)	2.26 (0.8 ; 0.00)	17.29 (1.4 ; 0.02)	38.02 (2.2 ; 0.05)	79.50 (3.1 ; 0.10)
Global (Market Capitalisation Average)	2.37 (0.8 ; 0.00)	19.23 (1.5 ; 0.03)	37.98 (2.0 ; 0.05)	79.68 (3.1 ; 0.10)
US VRP	2.36 (0.7 ; 0.00)	19.79 (1.6 ; 0.03)	36.23 (1.8 ; 0.04)	77.38 (3.0 ; 0.09)
Local VRP	1.90 (0.7 ; 0.00)	11.78 (1.0 ; 0.01)	29.42 (1.8 ; 0.03)	65.04 (2.7 ; 0.07)
VIX^2	−0.30 (−0.1 ; 0.00)	6.27 (0.5 ; 0.00)	26.07 (1.5 ; 0.02)	57.02 (2.4 ; 0.05)
EuroStoxx 50				
Global (PC)	0.61 (0.1 ; 0.00)	5.87 (1.2 ; 0.01)	27.88 (1.2 ; 0.01)	67.11 (1.9 ; 0.04)
Global (Market Capitalisation Average)	0.62 (0.1 ; 0.00)	4.30 (0.9 ; 0.01)	23.83 (0.9 ; 0.01)	60.11 (1.7 ; 0.03)
US VRP	0.55 (0.1 ; 0.00)	2.84 (0.6 ; 0.00)	19.16 (0.7 ; 0.01)	52.53 (1.4 ; 0.02)
Local VRP	1.44 (0.3 ; 0.00)	5.74 (1.3 ; 0.01)	41.81 (1.8 ; 0.03)	89.18 (2.5 ; 0.07)
VIX^2	−3.27 (−0.8 ; 0.00)	5.11 (1.0 ; 0.01)	9.74 (0.4 ; 0.00)	30.49 (0.9 ; 0.01)

Note: Each row corresponds to a version of regression (5), containing a constant and an alternative lagged VRP measure. Newey-West t-statistics and R^2 are in brackets.

Third, each of the global VRP measures serves as a highly significant predictor variable for all of the different country returns, with t-statistics systematically in excess of 1.8 at the horizons from 6 months and longer in the case of the UK and US indices. Finally, for the FTSE 100, global VRP-based regressions result in stronger average predictability than do the US-VRP based regressions at longer horizons. This result holds for both global VRP measures, suggesting that by pooling information from international markets we could improve the estimation of a common "global" risk aversion proxy. Although it is important for the interpretation, in this reduced form statistical analysis we do not take a stand on whether our global VRP measure is reflecting better the US influence globally or it is capturing better the impact of global economic conditions on local country markets. The results are less strong for the EuroStoxx 50 excess returns, though these are somewhat sensitive to the choice of euro area risk-free rate.

The results show that a "global" variance risk premium is a relatively good predictor of future excess returns and in many cases outperforms the commonly used US specific VRPs.[8] The results also confirm earlier findings in the literature that both US-based and the global VRP estimates have a superior predictive power for equity excess returns internationally. This helps support using them as a proxy for global risk aversion when analysing equity markets.

Market risk attitude is crucial for asset pricing in general, not just for equities. If global market risk aversion matters for risk pricing, and asset markets are not isolated, it should be reflected in the risk premia of broader asset classes. For example, VRP measures should also be significant in explaining future bond excess returns.

Earlier work by Mueller, Vedolin, and Zhou (2011) demonstrates that the equity variance risk premium heavily loads on short-term US bond risk premia. We replicate their regression analysis on UK, US, and German one-month excess bond returns and find that they are significantly and positively loaded on all three VRP measures (Table 3). In our case, we also show that the global VRP measure estimated as a first PC of international equity based VRPs shows the superior predictive ability.

[8] However, in absolute terms, the statistical evidence shown by R^2s are not very strong, suggesting that all of our VRP measures are only weakly statistically significant in explaining future stock market returns. As discussed by Bekaert and Hoerova (2014), apart from the small sample problem, another reason for this is the well-known fact that equity risk premia are probably driven by multiple state variables so that the univariate regressions are necessarily mis-specified.

Interestingly, the 10-year bond excess returns, especially in the US and German cases, are explained less by these risk aversion measures. The statistical evidence does not show a strong significance of the VRP measures in explaining future one-month excess returns on these bonds, where thr R-squared never exceeds 3%. This could be consistent with the special role of government bonds at those maturities: because they are highly liquid assets that provide a reliable store of value, flight to safety flows into these bonds could partly offset and mitigate the direct impact of increased risk aversion on bond risk pricing.

The risk aversion measures appear to be more relevant in predicting short-run excess returns on UK medium maturity bonds, where the R-squared for the predictive regressions reaches 7-8%. As in the case of excess stock returns, the R-squared for the UK regressions is always larger for the global VRPs than for the "local" VRPs, with the PC-based global VRP delivering slightly better results in general.

Table 3. Estimated coefficients on VRP measures from bond excess returns regressions.

	3-year	5-year	10-year
US			
Global (PC)	0.09 (1.7 ; 0.01)	0.18 (1.6 ; 0.02)	0.45 (1.4 ; 0.03)
Global (Market Capitalisation Average)	0.08 (1.5 ; 0.01)	0.15 (1.3 ; 0.01)	0.34 (1.1 ; 0.01)
US VRP	0.07 (1.2 ; 0.01)	0.11 (1.0 ; 0.01)	0.23 (0.8 ; 0.01)
UK			
Global (PC)	0.18 (2.8 ; 0.08)	0.27 (2.9 ; 0.07)	0.44 (2.2 ; 0.04)
Global (Market Capitalisation Average)	0.17 (2.9 ; 0.07)	0.25 (2.8 ; 0.06)	0.37 (2.0 ; 0.03)
US VRP	0.16 (2.8 ; 0.06)	0.22 (2.7 ; 0.05)	0.30 (1.6 ; 0.02)
Germany			
Global (PC)	0.12 (3.4 ; 0.04)	0.15 (2.3 ; 0.02)	0.21 (1.4 ; 0.01)
Global (Market Capitalisation Average)	0.11 (3.1 ; 0.03)	0.13 (2.0 ; 0.02)	0.15 (1.0 ; 0.01)
US VRP	0.10 (2.8 ; 0.03)	0.11 (1.6 ; 0.01)	0.10 (0.7 ; 0.00)

Note: Each row corresponds to a regression of one-month excess returns on bonds with 3, 5, and 10 year maturities on a constant and an alternative lagged VRP measure. Newey-West t-statistics and R^2 are in brackets.

In sum, all three of the proposed global VRP based measures perform relatively well in forecasting risk premia across assets and internationally, which favours their interpretation as global risk aversion indicators. The slightly better relative performance of the PC-based VRP measure overall may hint at it being a preferable indicator.

However, we acknowledge that there have been also several alternative interpretations of VRPs. For example, Adrian and Shin (2010) document that VRPs are affected by broker dealers' funding liquidity. Consequently, financially/liquidity constrained intermediaries could behave *as if* they have become extremely risk averse, even if their intrinsic attitude to risk has not changed. And although structurally the two interpretations - changed risk attitude or imposed financial constraints - are different, we do not discriminate between them here. The main question at hand is to measure the deviations from risk neutrality in financial markets and explore the influence of fluctuations in observed market risk attitude, whether it is intrinsic or induced by constraints, on asset pricing. Relatedly, Barras and Malkhozov (2014) claim that the daily option-based VRP could be disproportionately influenced by the risk-bearing capacity of financial intermediaries in options markets, and spikes in the option-based VRP can arise when these intermediaries are in a deleveraging phase. In such cases, the changes in option-based VRP estimates would not necessarily represent changes in the risk attitude of equity investors. However, the authors also show that, although the deviations of option-based VRP estimates from their fundamental values can be large, they are only temporary. Over longer horizons and frequencies, the option based estimates deliver reliable estimates of equity investors' risk attitude.

4 Impact of changes in risk aversion on the term structure of interest rates

Most recent models of the term structure of interest rates rely on a no-arbitrage assumption, such as no-arbitrage affine term structure models (ATSMs). Since Duffie and Kan (1996), ATSMs have become something of the industry standard in the empirical finance literature. These models assume that bond yields are driven by a small number of factors and that there are no arbitrage opportunities from trading across bonds of different maturities. The factors driving yields could be either unobserved (e.g. Dai and Singleton, 2002), or observed statistical yield components (e.g. Adrian, Crump, and Moench, 2013), or observed macroeconomic variables (e.g. Ang and Piazzesi, 2003).

The prices of risk in the ATSMs are imposed to be linearly related to the yields (and/or observed macroeconomic variables, as in Ang and Piazzesi, 2003). The ATSM method does not explicitly account for the fact that investors' risk preference, or risk aversion, can change as a result of some non-fundamental or catastrophic events, like terrorist attacks or market crashes. These kind of events can generate an exaggerated perception of risk and spikes in risk aversion, which may not be captured by current macroeconomic variables, although the higher risk aversion and pessimism among investors could eventually negatively influence businesses and overall macro-economic conditions. In contrast, as we showed in the previous section, equity and option markets may provide timely evidence on the risk attitude of financial market investors. Therefore, linking the prices of risk directly to the measures of investors risk attitude could result in more plausible bond pricing models.

This idea would be consistent with the novel approach by Chen, Collin-Dufresne, and Goldstein (2008), who allow for the market price of risk to be driven by counter-cyclical risk aversion motivated by the Campbell and Cochrane (1999) external habit model. By calibrating the market prices of risk and pricing kernel to equity returns and aggregate consumption data, they are able to match the historical levels and time variation of corporate Baa–Aaa credit spreads and thus resolve the "credit spread puzzle".

Here we propose to explicitly account for time varying risk aversion in an arbitrage-free term structure model of interest rates on UK government bonds by introducing a risk aversion factor in the model of the market prices of risk. In particular, in our application of the otherwise standard ATSM framework, we assume that UK yields can be determined by two groups of factors: factors extracted from UK yields and the observed risk aversion factor measured by the VRP. In that way, we can test and evaluate the importance of the factor on bond yields and the prices of risk, as will be shown in the rest of this section.

4.1 Benchmark ATSM

In the standard no-arbitrage ATSM framework, the short rate r_t is assumed to be an affine function of an $(N \times 1)$ vector of pricing factors X_t,

$$r_t = \delta_0 + \delta_1 X_t,$$

where δ_0 is a scalar and δ_1 is an $(1 \times N)$-vector of coefficients. The state vector X_t is assumed to follow a Gaussian vector autoregressive process:

$$X_t = C + \Phi X_{t-1} + u_t, \quad u_t \sim N(0, Q)$$

The assumption of no-arbitrage implies there is a pricing kernel M_t such that $P_t^{(n)} = E_t[M_{t+1} P_{t+1}^{(n-1)}]$, where $P_t^{(n)}$is the price of an n-period bond at time t. The pricing kernel is assumed to have the following form:

$$M_{t+1} = \exp(-r_t - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' Q^{-\frac{1}{2}} u_t) \tag{6}$$

where the vector of market prices of risk λ_t is assumed to be an essentially affine function of the state vector:

$$\lambda_t = Q^{-\frac{1}{2}}(\lambda_0 + \lambda_1 X_t) \tag{7}$$

Given these assumptions, bond yields $y_t^{(n)} \equiv -\frac{1}{n} \log P_t^{(n)}$ and excess holding returns $rx_{t+1}^{(n)} \equiv (\log P_{t+1}^{(n-1)} - \log P_t^{(n)}) - r_t$ are affine functions of the state vector, e.g.

$$y_t^{(n)} = a_n + b_n X_t \tag{8}$$

where the coefficients a_n, b_ncan be estimated recursively (see, for example, Malik and Meldrum, 2014) and depend on parameters δ_0, δ_1,$C, \Phi, Q, \lambda_0, \lambda_1$.

To estimate this model, we can adapt the Adrian, Crump, and Moench (2013) regression based estimation method and use principal components of yields as observed yield curve factors. The model is estimated on the term structure of zero coupon interest rates, derived from UK government bonds, at monthly frequency over the period from January 2000 to December 2014. Since the inclusion of yields of more maturities improves the precision of the parameter estimates, we use a wide range of maturities for the estimation, comprising all intermediate monthly maturities starting from 1 to 10 years, so the number of observed maturities is $K = 109$. However, the lack of continuous data for very short maturities means that rates along this portion of the curve cannot be identified. For the short rate r_t we use the Bank of England policy rate.

First, we establish a required number of factors. Results from a principal component analysis (PCA) are presented in Table 4, suggesting that first three components explain 99.99% of the yield curve. This is consistent with other empirical studies showing that more than 99% of the movement of various government bond yields are captured by three factors (consistently with Litterman and Scheinkman, 1991, and many others since them).

Table 4. PC analysis of UK term structure of nominal yields.

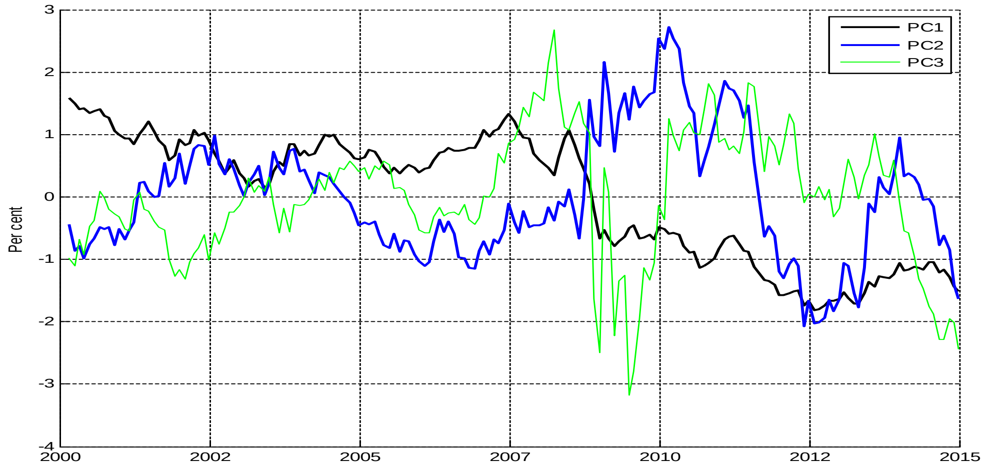
		Selected yield loadings						
	EV	1	2	3	4	5	7	10
PC1	97.95	0.09	0.09	0.10	0.10	0.10	0.10	0.09
PC2	1.96	0.19	0.13	0.09	0.05	0.01	-0.06	-0.15
PC3	0.085	-0.30	-0.05	0.07	0.11	0.10	0.02	-0.14
PC4	0.004	0.26	-0.08	-0.12	-0.05	0.04	0.10	-0.21

Note: Table presents the principal component (PC) loadings for the term structure of yields from January 2000 to December 2014 for selected yields. Column EV shows the proportion of the total variance explained by each PC.

Additionally, the estimation approach by Adrian, Crump, and Moench (2013), which uses excess returns regressions to fit the cross section of yields, allows for direct tests of the number of pricing factors: the Anderson (1951) and Wald statistics, which correspondingly follow $\chi^2(K-N+1)$ and $\chi^2(K)$ distributions. The corresponding test statistics in the case of the model specification with three factors are 652.93 with P-value = 0.00 and 605.93 with P-value = 0.00, and in the case of four factors are 567.10 with P-value = 0.00 and 99.66 with P-value = 0.78 (for test details, see Adrian, Crump, and Moench (2013)). The tests support a three-factor specification, while the evidence of the importance of the fourth factor is less clear.

Therefore, for the initial analysis, we use the three-factor model based on the first three principal components of yields as a baseline model specification. Therefore we denote the state vector $X_t = [PC1_t,\ PC2_t,\ PC3_t]'$, where $PC1$, $PC2$, and $PC3$ represent the first, second and third principal components of the yield curve. The factors are presented in Figure 4. Event though the factors are statistical and are extracted without any prior economic context, the structure of their loadings across yields with different maturities suggests some possible economic interpretations. PC1, which is positively loaded on all maturities, can be interpreted as the level of the yield curve: indeed, its correlation with the 10-year yield is

Figure 3: **PC factors of the UK yield curve.**



extremely high: 0.96. The loading structure for PC2 suggests its close relationship to the slope of the yield curve (the correlation between PC2 and the difference between the 10-year yield and 1-month yield is more than 0.6). And the U-shape yield loadings for PC3 means it can be interpreted as the curvature of the yield curve.

4.2 ATSM with risk aversion proxy

Next, we review an ATSM specification that uses the global risk aversion factor (derived from international one-month VRPs and standardised) as an additional factor. We first test the assumption that the factor is spanned by checking whether corresponding factor loadings are equal to zero using Wald tests.

The Wald test declines a specification with three PC factors and an observed risk aversion factor (Wald statistics is 0.4 with P-value equal to 1). However, we note that the risk aversion factor could be reflected in the several PC factors of the yield curve. In particular, there is a significant positive relationship between the global risk aversion factor and PC2: the correlation is 0.32. This positive correlation of the slope-related factor with the risk aversion factor may explain why the slope factor tends to account for a large fraction of bond risk premia and also can forecast stock returns, as emphasized by Fama and French (1989). The correlations between other two factors and risk aversion are small and negative (around -0.14).

We can therefore decompose the slope factor into a global risk aversion factor, G_t, and a residual, and so include our global risk aversion factor explicitly in the term structure model as an additional observable factor. The resulting model is based on four factors $X_t = [PC1_t,\ rPC2_t, PC3_t, G_t]'$, where $rPC2_t$ is the residual, $resid_t$, from the regression

$$PC2_t = \alpha + \beta G_t + resid_t$$

estimated on the whole sample from January 2000 to December 2014. As a result, the state-space representation of the four-factor yield curve model in discrete time can be written as:

$$\text{State} \quad : \qquad X_t = C_{4x1} + \Phi_{4x4} X_{t-1} + u_t, \quad u_t \sim N(0_{4x1}, Q_{4x4}) \qquad (9)$$

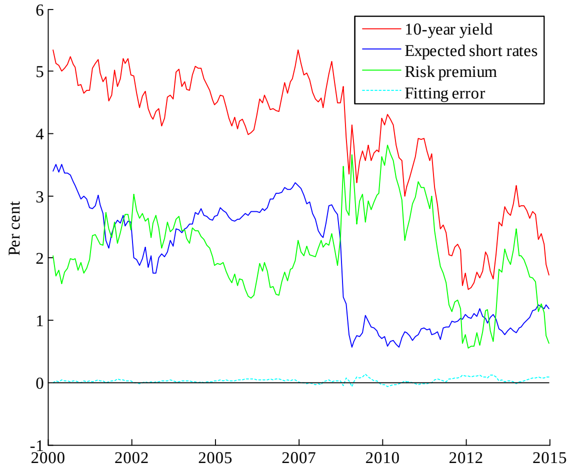
$$\text{Space} \quad : \qquad y_t^{(n)} = a_n + b_n X_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma),\ n \in [12:120]. \qquad (10)$$

The estimation of the model is identical then to Malik and Meldrum (2014), so we omit the estimation details here.

Estimation results. The estimation results indicate that the model with the observed proxy for risk aversion performs well. Figure 5 shows the model-implied decomposition of the 10-year yield into average expectations of the short rates over the next ten years, and the bond risk premium. The yield pricing errors are small, not exceeding 14 basis points in absolute value. The bond risk premium is broadly countercyclical. It rose sharply during the financial crisis, as investors reappraised their expectations for the path of short rates given the possible depth of the recession. And the bond risk premium rose again in 2013 around the time of the so-called 'taper tantrum'. Expectations of average short term interest rates over the next ten years have dropped significantly from their pre-crisis level (2.7% on average) at the end of 2008 to an average of just 0.9% in the post-2009 period.

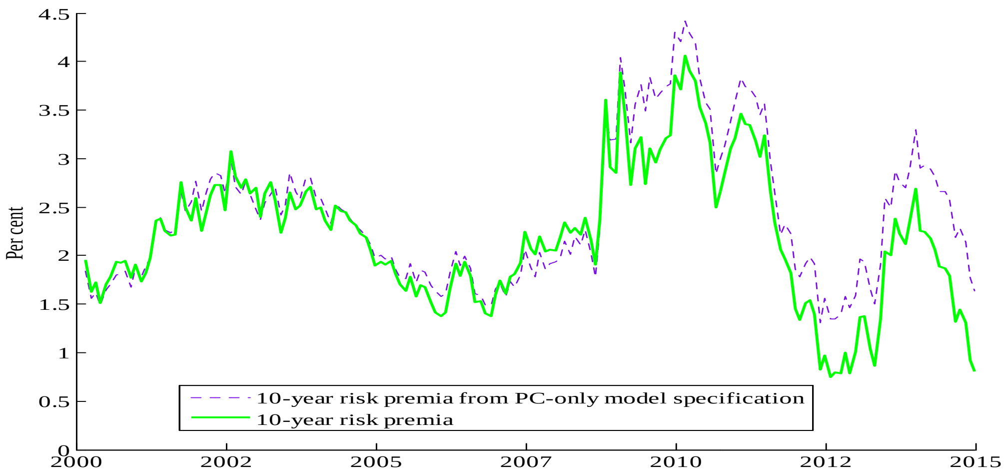
The bond risk premium from Figure 5 is shown alongside the premium estimated from a three-factor benchmark ATSM (i.e. without the global risk aversion proxy) in Figure 6. Given the construction of our model with risk aversion, it is unsurprising that these bond risk premia estimates follow a similar pattern and are almost identical before 2009. The larger divergence between the two alternative estimates of bond risk premia in the post-2009 part of the sample reflects the difference in the implied paths for risk-free rates produced by the two models: the PC-only model specification implies average policy rates expected to be around 0.2% over next ten years starting from 2009, which we believe to be less plausible than 0.9% and inconsistent

Figure 4: **Model-implied decomposition of 10-year UK yields.**



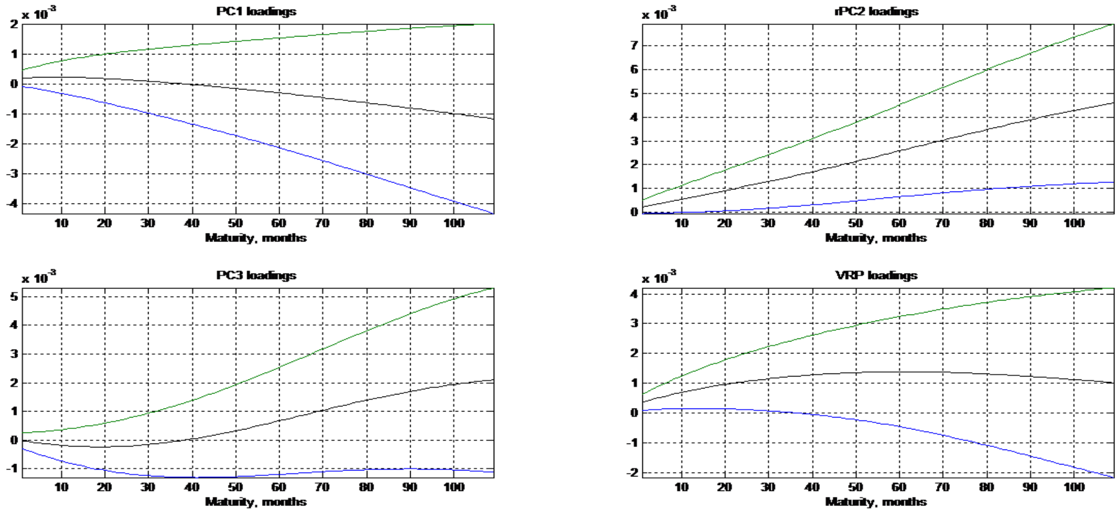
with survey expectations (e.g. Bank of England's survey of external forecasters or the Reuters surveys), which all imply a flat or increasing future path of Bank Rate from its 0.5% level, where the policy rate has stayed since 2009.

Figure 5: **Estimated 10-year bond risk premia.**



Model implied factor loadings for the predictive excess returns of various maturities are depicted in Figure 7. The $rPC2_t$ factor loadings are positive and significant for all maturities. The loadings are also positive on the risk aversion factor, G_t, for all maturities. However, they appear to be significant only for maturities up to 3 years and they are decreasing after 5 year maturities, possibly suggesting that risk aversion may exhibit opposite forces on bond returns at higher maturities, namely "risk-off" and "flight to safety" effects. "Risk-off" describes a positive relationship between excess returns and risk aversion, as agents require higher excess returns due to increased risk aversion. "Flight to safety", instead, is an observed characteristics of some advanced countries government bonds, like those of the US, UK, and Germany, and implies a negative relationship between returns on such bonds and risk aversion, as investors tend to buy more of these bonds, driving yields lower, when they become more risk averse. For example, Adrian, Crump and Vogt (2015) show that when the VIX rises above its median value, investors tend to reallocate from stocks to bonds, leading to an increase in expected returns for stocks and a compression of expected returns for bonds.

Figure 6: **Factor loadings for the predictive excess return regressions.**



Note: Black lines show the factor loading estimates, while green and blue lines denote corresponding 95% confidence intervals.

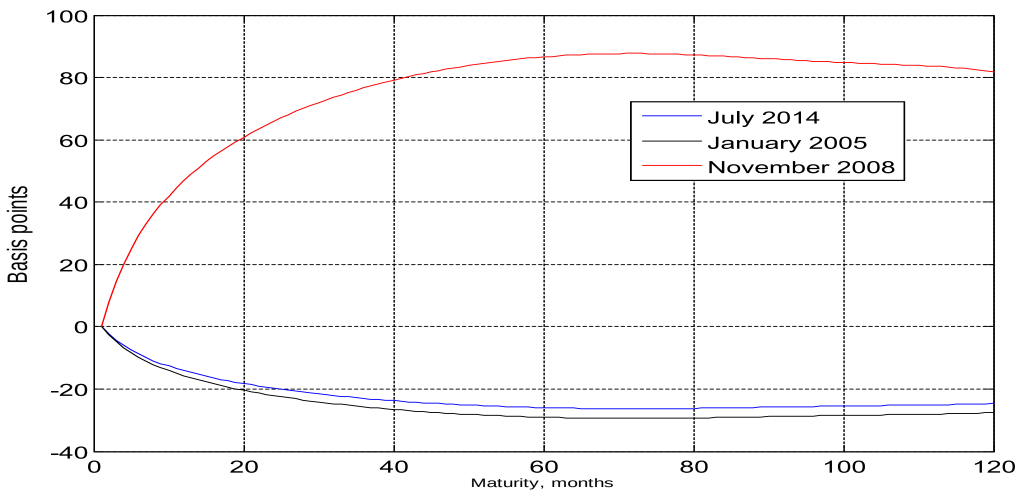
The estimated short term interest rate function,

$$r_t = \delta_0 + \delta_1 X_t = 2.92 + 2.04PC1_t - 0.57rPC2_t - 0.28PC3_t - 0.16G_t,$$

implies a negative, albeit relatively small, loading on the risk aversion factor, suggesting that in times of heightened risk aversion, monetary policy has a role to reduce market fears by taking a looser stance. A similar interpretation was given by Bekaert, Hoerova, and Lo Duca (2013), who established in VAR analysis that monetary policy tend to accommodate risk aversion and uncertainty shocks, but the relationship is not statistically significant.

Risk aversion and bond risk premia. We can also use the model to explore the impact of changes in risk aversion on bond risk premia. Figure 8 shows the contribution of time varying global risk aversion to bond premia, calculated as a difference between model-implied premia for the case when risk aversion is time varying and for the case when it is held constant (which is assumed to be equal to the average of the risk aversion proxy over the available sample). Changes in risk aversion pushed bond risk premia down during the great moderation of 2004-2006 and put significant upward pressure on them during the 2008-2009 financial crisis: during the crisis, the contribution of our global risk aversion proxy to bond risk premia was up to 90 basis points, with 5 to 7-year maturities affected the most. In contrast, more recent data show that, similarly to the so-called "Great Moderation", bond risk premia in the summer of 2014 were around 25 basis points lower at maturities longer than 3 years due to decreased risk aversion.

Figure 7: **Contribution of changes in time varying global risk aversion to UK bond risk premia across maturities.**



Finally, we have to point out that we use a measure of risk aversion derived from stock returns to capture the discount factor of bond investors and UK bond risk premia. One caveat with this approach is that the investor base in stock markets and the UK government bond market may well differ. Hence, the aggregate risk aversion of the stock investors may not be equal to the risk aversion of UK government bond investors. Indeed, although the main players in advanced economy equity and bond markets are the same institutional investors (pension funds, life insurers, banks and other financial institutions), their relative shares are different and change over time. In particular, UK pension funds and life insurers are key investors in both UK government bonds and equities, however, they seem to be more represented in the UK government bonds market than in the equity market. For example, in 2012, the share of UK equities owned by UK pension funds and life insurers was just over 10% (having decreased from around 40% in the early 2000s), while they held around 26% of the UK gilts (down from

64% in 2000) and remain the largest UK government bond investors.[9] Another important and more recent player in the UK government bond market is the Bank of England, which buys UK government bonds as part of its QE programs but does not actively participate in the international stock market. Therefore the stochastic discount factor derived from gilt yields may be influenced by the preferences of these two types of investors more than by the risk aversion of more prominent equity investors. However, given that our specification of the discount factor of bond investors is quite general, the specifics of UK government bond investors should be captured by the remaining factors, $PC1_t$, $rPC2_t$, and $PC3_t$. Recent literature also supports our approach. For instance, Bekaert, Engstrom, and Grenadier (2010) show that stochastic risk aversion plays an important role in explaining positive stock-bond return correlations: in their model, similarly to Wachter (2006), increases in risk aversion increase both equity and bond premiums.

In summary, our measure of risk aversion over a one month horizon, derived from equity markets, appears to be helpful in explaining bond risk premia on credit-risk free government bonds at maturities out to 10 years. These findings are consistent with the previous studies of the determinants of emerging market sovereign risk premia, which showed that global investors' risk aversion drives time variation in the risk premia and explains the remarkable narrowing of emerging market spreads between 2002 and 2006 (Remolona, Scatigna, and Wu, 2008) and the significant drop in 2012 (Heinz and Sun, 2014). But, to our knowledge, this is the first study to directly relate the time varying proxy of risk aversion to the stochastic discount factor and evaluate the importance of global risk aversion for the term structure of UK government bonds.

5 Concluding Remarks

In this paper we extract a global component of international variance risk premia and use it as a proxy for global risk aversion. Our contribution is three-fold. First, we improve the forecasting performance of available models of conditional variances of international equity indices' returns. In particular, we show that, along with other more traditional explanatory

[9] Sources: DMO and http://www.bankofengland.co.uk/publications/Documents/news/2014/dp310714.pdf

variables, monetary policy uncertainty could be an important factor determining equity returns' volatility. Second, based on implied volatilities extracted from option prices and on our preferred models of conditional international variances, we estimate international variance risk premia and use them to construct a global risk aversion factor. Third, we show that the global risk aversion factor plays an important role in determining bond risk premia through market prices of risk.

To our knowledge, this is the first study that attempts to introduce time-varying risk aversion explicitly as a factor into a term structure model. As a result of this assumption, domestic interest rates become dependent on global market sentiment. For the case of UK bonds, we find that the risk aversion factor significantly affected the level and dynamics of interest rates. We find that the risk aversion factor is mostly reflected in the so-called slope factor of the yield curve, which tends to explain a large fraction of bond risk premia.

In our partial equilibrium model, global risk aversion is introduced as an exogenous factor. But what factors explain global risk appetite? Understanding what determines the global risk appetite factor remains an important question on our agenda. Various explanations were proposed in the literature. Danielsson, Shin, and Zigrand (2010) incorporate endogenous risk into a standard asset-pricing model and show that risk appetite is linked to beliefs about future outcomes, so that market participants appear to become "more risk-averse" in response to deteriorating market outcomes. Similarly, Bruno and Shin (2015) claim that greater risk-taking by banks is linked to their local currency appreciation. Using the empirical study for the United States, Bekaert, Hoerova, and Lo Duca (2013) document that risk aversion decreases after about six months of loose US monetary policy. Therefore exploring the nature of our global risk appetite factor and relating it to global and local fundamentals, monetary policy and exchange rate factors could be a fruitful extension of our work. One way of dealing with this question would be to jointly model the term structure of US, euro-area and UK interest rates, including a global risk aversion factor. This would allow us to understand how channels such as policy expectations and international spillover effects could affect risk taking and what impact this may have on long-term interest rates.

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