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Bank leverage, credit traps and credit policies

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Abstract

We construct an overlapping generations macroeconomic model with which to study the causes, consequences and remedies to 'credit traps' — prolonged periods of stagnant real activity accompanied by low productivity, financial sector undercapitalisation, and the misallocation of credit. In our model, credit traps arise when shocks to bank equity capital tighten banks' borrowing constraints, causing them to allocate credit to easily collateralisable but low productivity projects. Low productivity weakens bank capital generation, reinforcing tight borrowing constraints, sustaining the credit trap steady state. We use the model to study policy options, both *ex ante* (avoiding credit traps) and *ex post* (escaping them). *Ex ante*, restrictions on bank leverage can help to enhance the economy's resilience to the shocks that can cause credit traps. Further, a policymaker focused on maximising the economy's resilience to credit traps would set leverage countercyclically, allowing an expansion of leverage in minor downturns and reducing leverage in upswings. However, *ex post*, relaxing a leverage cap will not help escape the trap. Instead, a range of unconventional policies are needed. We study publicly intermediated lending, discount window lending, and recapitalisation, and compare the efficacy of these policies under different conditions.

Key words: Unconventional credit policy, leverage regulation, financial intermediation, financial crisis.

JEL classification: E58, G01, G21.

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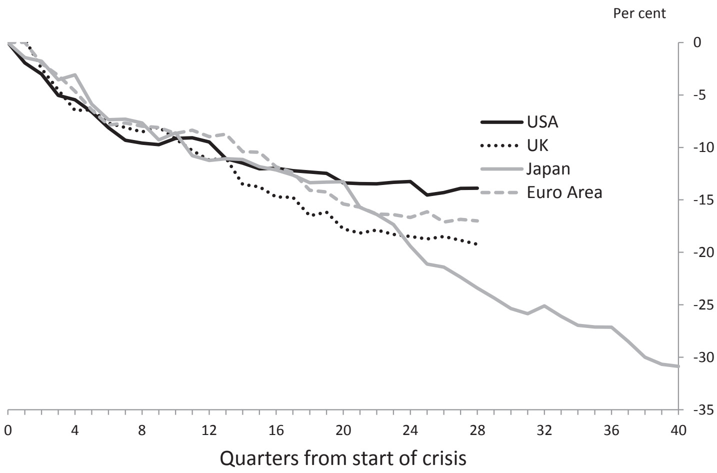
1 Introduction

Financial crises tend to have severe negative effects on real activity, and recoveries following crises tend to be weak and slow (e.g. Claessens & Kose (2013)). In Japan, for example, real GDP remained some 30 per cent below its pre-crisis trend 10 years after the onset of its financial sector distress in 1991 (Figure 1). In the UK, the gap between realised real GDP and the level implied by the pre-crisis trend was around 20 per cent five years after the onset of the crisis in 2007. And in the USA, Japan, the UK and the euro-area, the rate of credit growth collapsed around the onset of the crises. In Japan, anaemic credit growth continued for at least a decade (Figure 2).

Figure 1: Level of real GDP relative to trend: USA, UK, Japan, Euro Area



Trend growth is calculated as mean quarterly increase in real GDP over 1950:1–2006:4 (USA); 1955:2–2006:4 (UK); 1980:2–1991:4 (Japan); 1996:2–2006:4 (Euro area). Source: Datastream.

These consequences have triggered various policy responses. On the one hand, reform of financial regulation continues apace. Across jurisdictions, macroprudential policy authorities have been established and tasked with conducting system-wide prudential policy, including the use of countercyclical bank capital requirements.[1] At the same time, central banks and governments have introduced a range of 'unconventional' monetary and credit policies, including asset purchases, policies to support bank funding, and recapitalisation of financial institutions. In light of these sweeping changes to the policy landscape, there is a real need to understand the mechanisms, costs and benefits of these interventions, and the conditions under which they can be effective. This paper enhances the understanding of such 'credit policies' – both ex-ante

[1] Leading examples include the Financial Policy Committee at the Bank of England, and the European Systemic Risk Board in the euro area.

Figure 2: Credit growth around crises: USA, UK, Japan, Euro Area

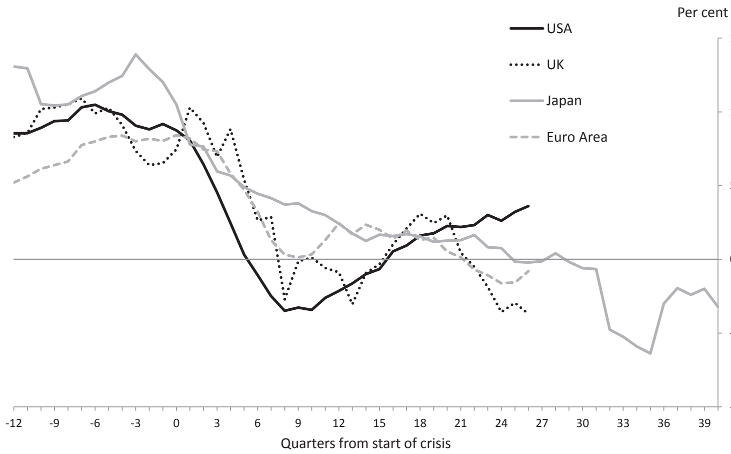


Chart shows year-on-year growth in nominal lending to the private sector. Source: BIS

(to avoid credit crises), and ex-post (to escape their consequences) – by presenting a novel, tractable macroeconomic model to understand their effects.

We do this by constructing a simple overlapping generations model featuring financial intermediation and credit frictions, and use it to study the credit policies mentioned above. The key feature of our model that makes it particularly useful for studying these policies is its ability to generate a 'credit trap' steady state – that is, a steady state of the economy that features low real activity, low productivity, low bank capital, and weak bank profitability. In our model, the borrowing constraints facing banks depend on the health of the banking system: when the net worth of the banking system is low, banks' ability to finance productive investment through borrowing is severely constrained. The economy enters a 'credit trap' when a large unanticipated shock to bank assets reduces bank capital below a critical threshold, causing banks' funding conditions to tighten, inducing them to invest in less productive assets that, nonetheless, have higher pledgeability to creditors. Thus, even a temporary shock can have extremely persistent effects if it causes a large reduction in bank capital. And it is the possibility of entering a credit trap that has profound implications for policy that have not been examined by existing work in this area.

The credit trap steady state is Pareto-dominated by the 'good' steady state. Our first contribution is therefore to ask how a bank leverage cap (which limits the ratio of a bank's assets to its equity capital) might be used to prevent the economy falling into a credit trap ex-ante. We show that a leverage cap has two effects. Reducing leverage means larger shocks are required to send the economy from the good steady state into the credit trap – improving the economy's 'resilience'. But lower leverage also reduces output in the good steady state of the economy as it restricts credit supply. Thus, in setting the leverage cap, the policymaker trades off the resilience benefit against the output cost. Interestingly, however, at low levels of leverage no trade-off exists. When leverage is very low, low lending volumes reduce bank net worth, bringing bank capital closer to the critical threshold below which the economy falls into a trap. Allowing leverage to rise from this level would boost lending and output, causing bank net worth to grow, taking the economy away from the critical threshold and enhancing the resilience of the economy to the credit trap.

We show that there exists a 'resilience-maximising' level of leverage which trades off the expansionary effect that higher leverage has on credit supply, bank profits and capital, with the contractionary effect higher leverage has through raising the economy's vulnerability to shocks. We show that even a policymaker concerned only with maximising resilience would adopt countercyclical policies, allowing an expansion of leverage in minor downturns and reducing leverage in upswings. For example, following a minor downturn, the economy comes closer to the credit trap threshold. By relaxing binding leverage restrictions, the policymaker boosts activity and therefore bank capital, *raising* resilience. But once a large shock pushes the economy into a credit trap, the relaxation of leverage restrictions will not be effective in getting the economy out of it. In a credit trap, the low net worth of banks severely restricts their ability to fund productive investments, causing them instead to seek out pledgeable but less productive assets, misallocating credit. In this context, relaxing a leverage cap either has no effect (if the leverage restriction does not bind relative to what the market would allow[2]), or only encourages more funds to be channelled to the unproductive sector (if the restriction did bind), restoring net worth to at most its depressed laissez-faire level. Thus, once an economy is in a credit trap, relaxing a leverage requirement cannot boost bank capital to the extent necessary to escape the trap, and the banks will continue to invest in the less productive assets.

Our second contribution is to show that, instead, to escape the trap, an 'unconventional' credit policy of some form is required. We compare the effects of direct publicly intermediated lending to the private sector ('direct lending'), public lending to the financial intermediary sector ('discount window lending'), and bank recapitalisation[3] (e.g. Gertler & Kiyotaki (2010)). For

[2]This result can arise in the model because the market-based leverage constraint becomes tighter following a negative shock to the banking system, and hence a regulatory constraint that binds pre-crisis may no longer bind following a sufficiently large shock. This tightening of the market-based constraint is consistent with the deleveraging observed by UK banks following the crisis, see Bank of England (2015) Chart 5.

[3]For simplicity we focus on government capital injections, but we acknowledge that distressed banks may be

simplicity, we only focus on the ex post effect of these policies in our paper, but acknowledge that some of these policies-for instance public capital injections-could cause ex ante moral hazard, which is not considered here. For comparability across these policies, and to keep the model tractable, we study their effects assuming each is financed with the issuance of risk-free government debt, which represents a perfect substitute for bank deposits. Hence these policies entail some crowding out of bank funding. But we show that, despite this, and despite potential efficiency costs associated with these policies, they can be successful in raising the aggregate capital stock and output from a credit trap equilibrium, and may do so to the extent necessary to escape the trap. In particular, we find a 'pecking order' for the efficacy of these policies which depends on their relative efficiency costs and the state of the world. The tractability of our OLG framework is particularly useful at establishing these conditions.

If all three policies entail equal per unit efficiency costs (Case (i)), bank recapitalisation is strictly preferred to direct lending, which in turn is preferred to discount window lending, on the basis of the ability of these policies to raise the physical capital stock (and therefore output) on impact (Table 1). The intuition is that in a credit trap, when each policy has the same unit efficiency cost, direct lending to the private sector is preferred to discount window lending to banks because the latter is subject to an additional financial friction between the government/central bank and financial intermediaries – namely, like private bank creditors, the government is imperfectly able to enforce repayment from banks, even if it is better at doing so than private creditors. Thus, each unit of public debt finances a greater increase in credit when lending is undertaken directly to the private sector than through the banks. In light of this, recapitalisation is more expansionary still. The intuition is that as well as adding to the stock of credit, recapitalisation helps to ameliorate the financial friction that exists between bank creditors and banks – effectively *crowding in* bank funding (deposits). This means that if the unit efficiency costs are equal, recapitalisation is preferred to direct lending and is the dominant policy.

We also consider an alternative ordering of efficiency costs (Case (ii)). It is plausible, for example, that the government/central bank is more efficient at undertaking discount window lending than it is at performing recapitalisation or direct lending, as it is closer in nature to the

able to raise equity from other sources, particularly given the creation of the resolution regime for failing banks and the development of structures for loss absorbing capacity that must be available to resolve banks. We also note that the possibility of causing ex ante moral hazard - which is not considered here - creates an argument against public capital injections, even if it is ex post efficient in a situation where a large segment of the banking system is unable to raise new equity.

Table 1: Pecking order for ex-post policies depending on efficiency cost and state of the economy

	Efficiency cost	*Pecking order*
Case (i)	Window $\geq$ Direct $\geq$ Recap	Recap $\succ$ Direct $\succ$ Window
Case (ii)	{Direct, Recap} $>$ Window	
– Mild credit crunch		Window $\succ$ Direct, Recap
– Severe credit crunch		Direct, Recap $\succ$ Window

'Window' refers to discount window lending; 'Recap' to recapitalisation; and 'Direct' to direct public lending. The symbol $\succ$ denotes strict preference on the basis of the policy's ability to raise the physical capital stock on impact.

central bank's standard open market operations requiring collateral management. Recapitalisation using public funds could also be undesirable as it could encourage moral hazard, although we do not model this explicitly. When the inefficiencies have this relative size, the pecking order of policies depends on the state of the economy. In a mild downturn, where banks can issue debt relatively easily, the pecking order follows the relative ordering of efficiency costs: discount window lending is preferred to the other two policies. But in a more severe downturn, when bank net worth is particularly low and they face tight borrowing constraints, discount window lending is dominated by the other policies. The intuition for this result is that discount window lending works through the banking system, so its efficacy is contingent on the severity of the credit friction between banks and their creditors. In a severe crunch, this friction is also severe, attenuating the expansionary effects of discount window lending relative to direct lending, which circumvents the friction entirely, or recapitalisation, which helps to ameliorate it, in addition to expanding lending directly. This finding that the appropriate policy response can vary with the state of the world is consistent with the range of policies implemented as the crisis played out.

The rest of this paper is organised as follows. In the next subsection we relate our work to the literature. Section 2 sets out the model. Section 3 shows how an unexpected shock to productivity can send the economy into a credit trap. Section 4 considers the role of leverage restrictions in avoiding credit traps (ex-ante). Section 5 considers the effectiveness of leverage and unconventional credit policies in escaping a credit trap (ex-post). Section 6 concludes.

1.1 Related literature

Two key features of the model contribute to its ability to generate credit traps. First, similar to Benmelech & Bergman (2012), the collateral value of bank assets depends on the aggregate health of the financial sector (see also e.g. Shleifer & Vishny (1992)). This makes bank equity capital the key state variable in the economy as it determines the severity of the credit friction that exists between banks and their creditors. As bank capital deteriorates, an intermediary's ability to issue debt – to achieve leverage – is diminished, shrinking the supply of credit to finance productive activity. The focus of our paper is on the balance sheet health of the financial intermediary sector, whereas Benmelech & Bergman (2012) focus on collateral constraints in the real sector. The mechanism in our paper therefore bears resemblance to Benmelech & Bergman (2012) (and the Shleifer & Vishny (1992) channel they use), but also shares the focus of recent work by Gertler & Kiyotaki (2010) and Gertler & Karadi (2011) on credit constrained financial intermediaries. This is natural given our focus on how credit policies – including leverage regulation – work through this sector in particular.[4]

Second, similar to Matsuyama (2007), we allow for heterogeneity in the composition of credit, distinguishing investment opportunities – 'projects' – according to their inherent productivity and according to the pledgeability of the cash flows that they generate. The idea is that some projects, like lending to small firms, have high productivity but, because of their relative opacity, have a collateral value that is particularly sensitive to the net worth of the intermediary sector. When the banking sector is healthy, bank creditors are willing to finance productive but opaque projects because they know that, if the worst came to the worst, they could seize assets and sell them to another lender to manage. When the banking sector has low net worth, by contrast, the resale value of these projects may be low because the ability of other buyers to absorb such assets is limited. In that case, creditors seek out pledgeable returns, which encourages banks to invest in other, lower productivity assets, like loans to firms with established but less innovative technologies, or liquid assets like government bonds or central bank reserves. This, in turn, depresses output.

The mechanism which generates persistent credit misallocation in our model is different from 'evergreening' of loans examined by Peek and Rosengren (2005) and Caballero et al (2008) in the context of Japan's 'lost decade'. According to their analysis, undercapitalised Japanese banks had the incentive to continue rolling over loans to weak and unproductive firms in order to prevent realisation of credit losses through foreclosure. In our model, by contrast, banks favour less productive but more collateralisable investments following a negative shock to their equity capital because their creditors demand more collateral.[5] This phenomenon was widely

[4]Other papers examining the impact of bank leverage regulation in a macroeconomic context include Angeloni & Faia (2013), Christensen et al. (2011), Gertler et al. (2012), Christiano & Ikeda (2013). The welfare implications of interventions in credit markets are examined in Bianchi (2011) and Lorenzoni (2008).

[5]The modeling approach in this paper is also very different to Caballero et al. (2008) who consider a micro

observed after the recent global financial crisis which was triggered by system-wide dry-up of wholesale funding.[6]

2 Model

2.1 Introduction

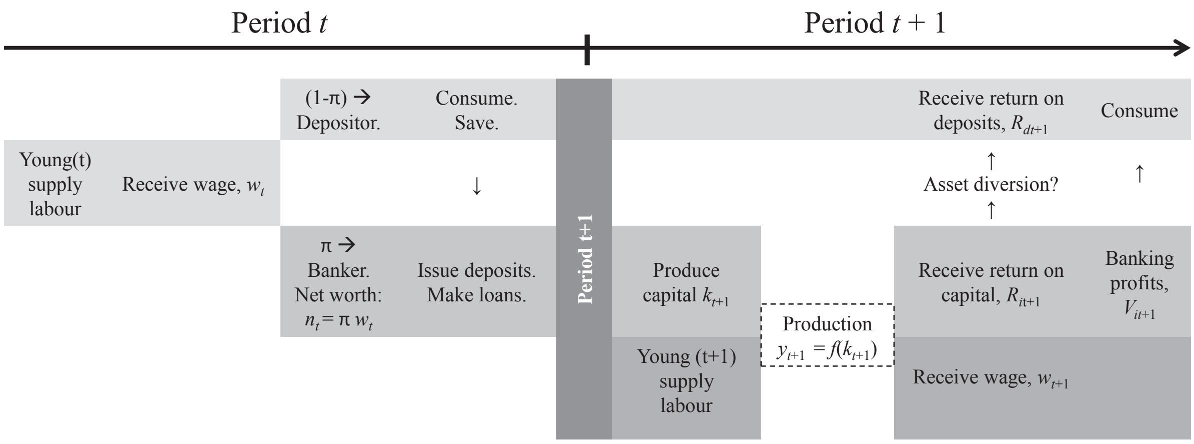
We begin with a brief overview of the model, with a timeline of the economy shown in Figure 3. Mass 1 of identical households are born each period. The life of a household is divided into two subperiods: 1, when the household is young (in period t), and 2, when the household is old (in period $t+1$). In the first period, each 'young' household receives a labour endowment of unity, which they sell in return for wage income w_t denominated in final consumption goods. At the end of period 1, a fraction $1-\pi$ of households become depositors, while a fraction π become bankers. Thus, households divide $(1-\pi)\, w_t$ between period 1 consumption and saving via deposits, whereas $n_t \equiv \pi w_t$ is used as bank equity to start a household bank. Banks combine their net worth with deposits to invest in one of two physical capital producing technologies. In the following period, the physical capital held by the banks is combined with the labour endowment of the *next* generation, producing output goods. Banks receive the return on capital with which they pay back depositors, returning any profits lump-sum to the now old households, and the banks then exit. The new young workers, having received their wage, then form their own set of banks (which have no direct link to the previous banks) and the process summarised in Figure 3 repeats itself.

The OLG structure is employed purely for tractability, helping us to obtain analytic expressions throughout. It should not be inferred that the intended model period is thus a generation, or around 30 years as is often the case with OLG models. Whilst we do not match this model to the data, the intended model length throughout is of the order of one year. In the following sub-sections we describe the model in more detail.

model focusing on the dynamic path of firm entry and exit with and without subsidised lending to unprofitable 'Zombie' firms. In particular the bank lending decision is not modeled. By contrast, we develop a dynamic macro model in which an explicitly modeled banking sector chooses to invest in different sectors. This allows bank lending decisions to feed back into bank profitability and in turn the bank lending decisions, giving rise to the possibility of multiple steady states in the model and a permanent impact of temporary negative shocks (a 'credit trap'). By contrast, in the model of Caballero et al. (2008) there is a unique steady state which the economy eventually returns to once a negative shock is unwound.

[6] Arguably, our model is less applicable to the case of Japan in the 1990s. Because all forms of deposits and uninsured debt were fully guaranteed in practice in all bank failures after 1996, there is little evidence that the credit misallocation there was driven by creditors' demand for more collateral; rather, the existing research suggests that the credit misallocation was primarily driven by banks trying to prevent realisation of losses through evergreening. See Nelson and Tanaka (2014) for a summary.

Figure 3: Timeline of events in the model



2.2 Households

Lifetime utility for households is given by

$$U_t = \log c_{1t} + \beta \log c_{2t}, \tag{1}$$

where $\beta \leq 1$ is the household's discount factor, and c_{jt} denotes consumption in period $j = 1, 2$ of the household born in period t. The budget constraints facing the household in each period are

$$c_{1t} + d_{i,t} \leq (1 - \pi)\, w_t, \quad c_{2t} \leq R^d_{i,t+1} d_{i,t} + V_{i,t+1}, \tag{2}$$

where $d_{i,t}$ denotes the household's saving via bank deposits, $R^d_{i,t+1}$denotes gross return on deposits[7], and $V_{i,t+1}$ denotes the profits obtained from banking activities, where $i = \{A, B\}$ denotes the sector in which the bank invests.

2.3 Banking and output production

Part of the household's initial wealth is used to capitalise a bank with net worth $n_t \equiv \pi w_t$. The bank takes deposits from (other) households and combines these with its own net worth to invest in capital-producing projects. There are two sectors that banks can invest in, $i = \{A, B\}$. The differences between these two sectors are described in detail in section 2.4 below. Here we describe a bank's problem for a generic sector i.

[7] The rate paid on deposits, $R^d_{i,t+1}$, is agreed at time t, and is not state contingent. This rate is dated $t+1$ to reflect when deposits are repaid.

If the bank invests in sector i, then its balance sheet reads:

$$s_{i,t} = n_t + d_{i,t},$$

where $s_{i,t}$ denotes the stock of loans in sector i. If $n_t + d_{i,t}$ final goods are invested in period t, physical capital produced in period $t+1$ is

$$k_{t+1} = x_i \left(n_t + d_{i,t} \right), \quad i = \{A, B\}, \tag{3}$$

where x_i denotes the productivity of investment in sector i.[8]

In this section, deposit contracts are signed with both banks and depositors assuming that x_A and x_B are non-stochastic. We will later consider in Section 3 what happens when the economy is hit by an *unanticipated* productivity shock.

In each period, final goods are produced using physical capital (financed by bank capital and deposits of the 'old') and labour provided by the 'young', using Cobb-Douglas production technology:

$$y_{t+1} = f(l_{t+1}, k_{t+1}) = l_{t+1}^{1-\alpha} k_{t+1}^{\alpha} = k_{t+1}^{\alpha}, \quad 0 \le \alpha < 1. \tag{4}$$

Labour and capital receive their respective marginal products, such that the wage of the 'young' is given by $w_{t+1} = (1-\alpha)k_{t+1}^{\alpha}$ while the marginal product of capital is given by $f'(k_{t+1}) = \alpha k_{t+1}^{\alpha-1}$. This implies that the bank's net worth in $t+1$ is given by:

$$n_{t+1} = \pi(1-\alpha)k_{t+1}^{\alpha}. \tag{5}$$

For simplicity, we assume that capital stock depreciates fully after each period. Thus, banks' investments in sector i at t generate gross return $R_{i,t+1}$ in terms of final output, given by:

$$R_{i,t+1} = x_i f'(k_{t+1}) = x_i \alpha k_{t+1}^{\alpha-1}.$$

Bank profits from investing in sector i, after repaying depositors gross interest rate $R_{i,t+1}^{d}$, are:

$$V_{i,t+1} = R_{i,t+1} \left(n_t + d_{i,t} \right) - R_{i,t+1}^{d} d_{i,t}. \tag{6}$$

[8] There is only one *type* of capital, but there are two *technologies*, A and B, for producing it from output goods.

These are returned lump-sum to the generation that financed the bank's activities, who are old in period $t+1$.

2.4 Credit market frictions

Banks are subject to a borrowing constraint which depends on the project in which they invest. The borrowing constraint arises because bankers can abscond with a fraction $1-\lambda_i$ of gross project returns (e.g. by paying an unwarranted bonus to themselves). As a result, only a fraction λ_i of the gross return from investment in sector i is pledgeable to creditors. Thus, λ_i can be interpreted as a borrowing constraint imposed by the market, with lower λ_i implying a tighter borrowing constraint or, equivalently, lower *asset liquidity* (see Holmström & Tirole (2011)). In order to guarantee repayment of $R^d_{i,t+1}d_{i,t}$, creditors demand that:

$$\lambda_i R_{i,t+1}\left(n_t+d_{i,t}\right) \geq R^d_{i,t+1}d_{i,t}, \tag{7}$$

such that total pledgeable returns do not fall short of period $t+1$ final output owed to creditors.

We can also interpret λ_i in terms of leverage. Leverage $L_{i,t+1}$ (mark-to-market) is given by

$$L_{i,t+1}=\frac{R_{i,t+1}\left(n_t+d_{i,t}\right)}{R_{i,t+1}\left(n_t+d_{i,t}\right)-R^d_{i,t+1}d_{i,t}}.$$

From (7),

$$L_{i,t+1} \leq \frac{R_{i,t+1}\left(n_t+d_{i,t}\right)}{R_{i,t+1}\left(n_t+d_{i,t}\right)\left(1-\lambda_i\right)}=\frac{1}{1-\lambda_i}. \tag{8}$$

Thus, $(1-\lambda_i)^{-1}$ is the maximum leverage the market allows when the bank invests in sector i.

The two sectors that banks can invest in differ in both their productivity and pledgeability to creditors. We maintain the following assumptions:

Assumption 1 (Productivity): $x_A > x_B$.

Assumption 1 says that for a given input of final goods in period t, more capital is produced in period $t+1$ from investing in sector A than in sector B. Sector A is thus more productive than sector B. One could interpret sector A as loans to small firms and B as an alternative use of bank funds such as holding cash, central bank reserves, or buying government bonds, which do not contribute as much to the growth of the economy.

Assumption 2 (Pledgeability): $\lambda_i=\lambda_i(n_t)$, with $\lambda'_A(n_t) > \lambda'_B(n_t) \geq 0$.

The first part of Assumption 2 makes bank asset pledgeability endogenous to *aggregate* bank net worth ($\lambda_i = \lambda_i(n_t)$), with higher net worth increasing asset pledgeability ($\lambda_i'(n_t) > 0$). This captures the idea that with larger equity capital, bank creditors are more confident that the funds they lend to banks will be repaid. This in turn captures the notion that when the aggregate banking sector is in better health, the 'second best' buyers of a bank's assets – namely other banks – are better able to ensure that a bank's assets remain liquid (e.g. Benmelech & Bergman (2012); Shleifer & Vishny (1992)).

The second part of Assumption 2 concerns its treatment of sectoral heterogeneity. In particular, $\lambda_A'(n_t) > \lambda_B'(n_t) \geq 0$ says that the liquidity of sector A assets is more sensitive to balance sheet health than is the liquidity of sector B assets. This captures the fact that the resale value of some assets depends more on specialist 'second-best' buyers than others. For example, the liquidity of mortgages (collateralised with real estate) may depend more on the health of the banking system than the liquidity of government bonds, on account of the larger number of 'second best' buyers of the latter than the former. In particular, as less monitoring and expertise are required to hold government bonds than to service mortgages, there will be a greater number of potential buyers of government bonds, including many beyond the banking system, making their liquidity relatively less dependant on banking system health. As we saw during the financial crisis, the liquidity of government debt markets held up far better than the liquidity of the RMBS market.

The relative attractiveness of sectors A and B depends on the health of the economy. Sector A offers higher returns than sector B but becomes relatively less attractive when the banking system is more fragile, with the liquidity of sector B holding up relatively better. The latter effect can dominate in a banking crisis leading sector B (e.g. government bonds) to be more attractive for an individual bank than sector A (e.g. real economy lending) giving rise to a credit trap, as we detail later.[9] Because of the centrality of Assumption 2, we next consider its theoretical and empirical underpinnings.

[9] An alternative channel that could result in banks investing in low risk projects when the banking system is more fragile would operate through risk aversion. In particular, if banks became more risk averse as their net worth decreased, a similar result to the one modeled here could obtain. However, it may not be straightforward to obtain such a result if the banks have limited liability as well as risk aversion. As Gollier et al. (1997) show, the combination of risk aversion and limited liability results in a 'bet for resurrection', with maximal risk chosen, when net worth is sufficiently low. The model presented here abstracts from this channel with banks instead being risk neutral and investing in projects on the assumption that any shocks to their returns occur with probability 0.

2.4.1 Assumption 2: Theory

In the Appendix we show that Assumption 2 naturally arises in a variant of the model in which banks face liquidity shocks, meaning only a subset can invest in a given period. The remaining banks remain on-hand to buy up assets on the secondary market in the event of a breakdown in bargaining between creditors and investing banks. Anticipation that such bargaining can later take place pins down asset pledgeability ex ante. The greater the balance sheet capacity of the stand-by banks, the larger is creditors' bargaining power, raising asset pledgeability. In the Appendix we formalise this, showing $\lambda_i'(n_t) > 0$.

In this setting, the remaining banks – the 'second best buyers' – must verify the investment projects they take on, using a fraction ρ of their net worth to do so. It is natural to assume that $\rho_A > \rho_B$: the real economy loans of sector A are relatively more opaque and require more resources to verify than the more liquid assets of sector B. This makes the pledgeability of sector A loans relatively more sensitive to the net worth of the 'second best buyers'. Formally we show that this set-up implies that $\lambda_A'(n_t) > \lambda_B'(n_t)$, given $\rho_A > \rho_B$.

2.4.2 Assumption 2: Evidence

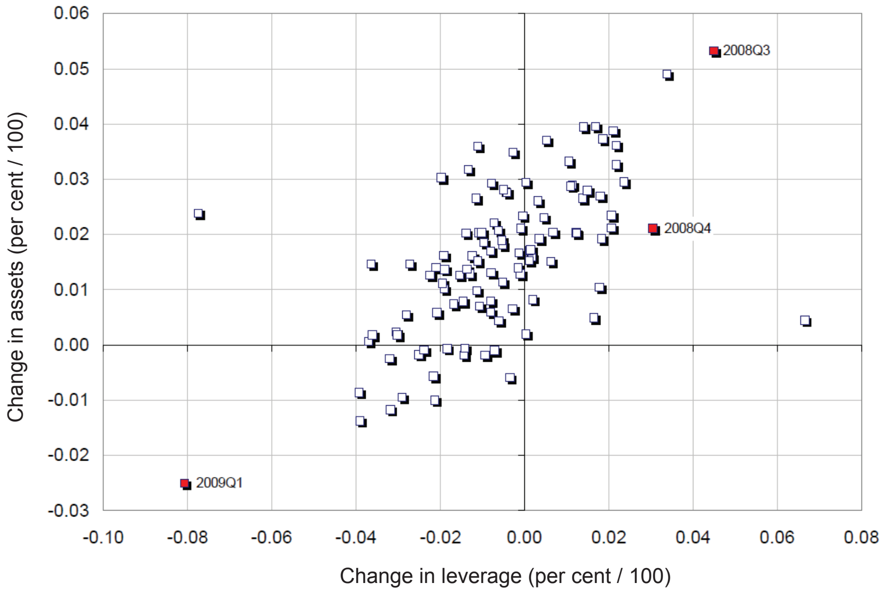
It can be shown that in our model, when there is a positive spread ($R_{i,t+1} > R^d_{i,t+1}$), the total value of assets held by banks is given by

$$\alpha \left[\frac{n_t}{\pi} x_i \left(\frac{\pi + \beta\lambda_i(n_t)}{1 + \beta\lambda_i(n_t)} \right) \right]^{\alpha}.$$

From the assumption that $\lambda_i'(n_t) > 0$ it follows that banks' asset holdings increase in n_t. Further, with a positive spread, the bank leverage limit binds and so banking system leverage is given by $(1 - \lambda_i)^{-1}$. Thus, an implication of the assumption is that as n_t rises, leverage and bank asset holdings both increase. In other words, our model predicts *procyclical leverage*, as has been documented empirically by Adrian et al. (2012) for financial institutions in the US (see Figure 4).

Turning to the second part of Assumption 2, from $\lambda_A'(n_t) > \lambda_B'(n_t)$ we would expect that as the equity of banks increases, they shift towards riskier real economy lending. Further, looking at a cross-section of banks, we would expect that the greater the increase in equity, the greater the shift towards riskier lending. We test this implication of the model using data from Capital IQ. We first define the average risk weight as the ratio of risk weighted assets to total assets. We

Figure 4: Quarterly asset growth and quarterly leverage growth of US commercial banks, 1984:1–2010:2. Leverage defined as ratio of sector assets to sector equity. Source: Adrian et al. (2012)



then plot changes in average risk weights from 2006-2011 against equity growth over the same period for over 300 banks in Figure 5. The upwards sloping fitted regression line is consistent with the prediction of the assumption: banks with a greater increase in equity tended to exhibit a greater shift towards riskier assets.

One interpretation of sector B is that it represents cash or other very liquid assets. As a second test of $\lambda'_A(n_t) > \lambda'_B(n_t)$ we examine whether banks hold relatively less cash as their equity increases. In Figure 6 we plot the change in the cash to assets ratio for banks against their equity growth over the period 2006-2011. The fitted regression line is downwards sloping, showing that banks with a bigger drop in equity ended up holding relatively more cash, providing further corroboration of the assumption.

2.4.3 Assumption 2: Summary

Taken together, the theory and evidence provide plausible support for Assumption 2. In what follows, we maintain a slightly stronger variant of Assumption 2 in which the pledgeability of loans to sector B is independent of aggregate bank worth, namely:

Assumption $2'$ (Pledgeability): $\lambda_i = \lambda_i(n_t)$, $\lambda_B(n_t) = \lambda_B$, with $\lambda'_A(n_t) > \lambda'_B(n_t) = 0$.

This assumption is made for expositional simplicity, though it is stronger than is necessary to prove our results.

Figure 5: 5-year change in average risk-weight against 5-year change in equity, 2006–11, for a sample of 300 banks. Source: authors' calculations and Capital IQ-see Appendix for Disclaimer of Liability.

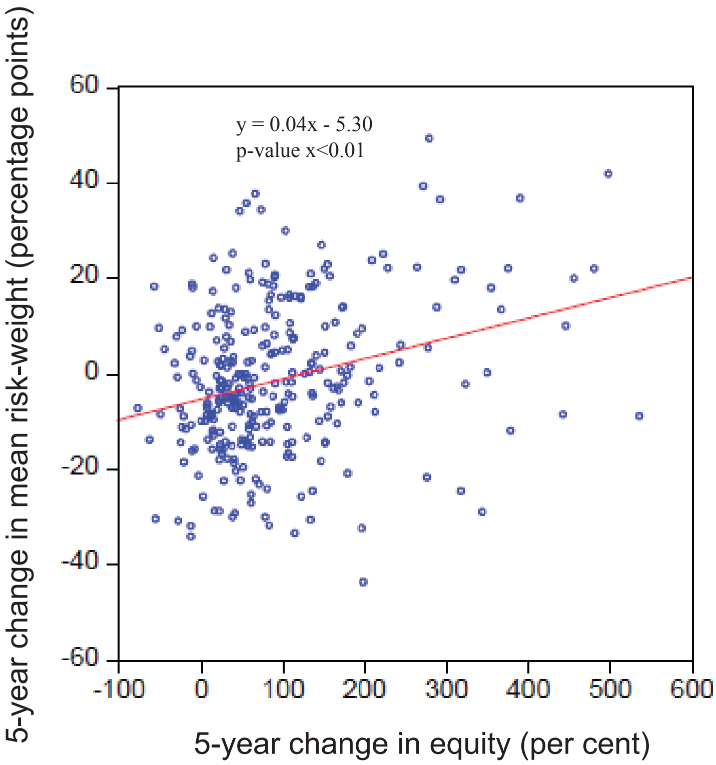
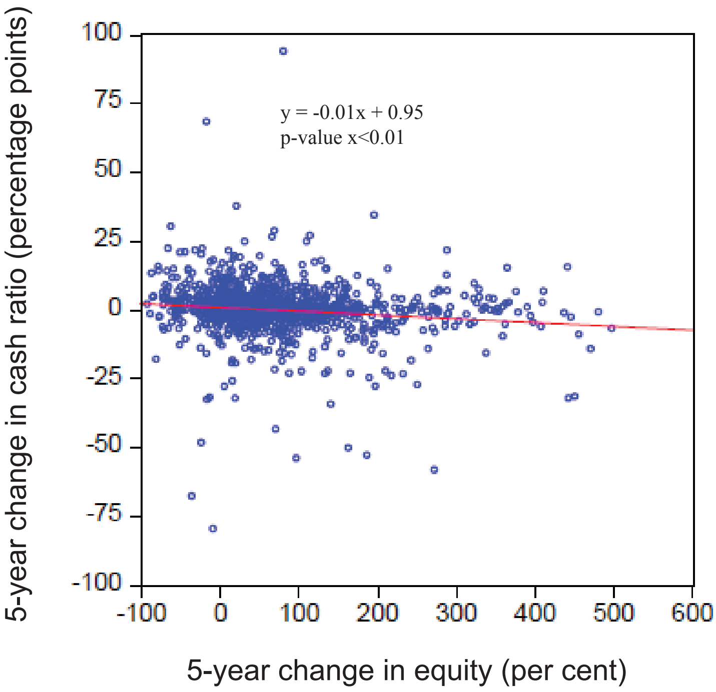


Figure 6: 5-year change in cash ratio against 5-year change in equity, 2006–11, for a sample of 1333 banks. Source: authors' calculations and Capital IQ-see Appendix for Disclaimer of Liability.



2.5 Credit market equilibrium

In order to derive equilibrium in the credit market, we first derive the households' supply of deposits. The household's optimal consumption-saving decision is governed by the first-order condition using (1) and (2):

$$\beta\frac{R^d_{i,t+1}}{c_{2t}} = \frac{1}{c_{1t}},$$

which gives optimal saving of:

$$d_t = \frac{\beta}{1+\beta}\left(1-\pi\right)w_t - \frac{1}{1+\beta}\frac{V_{i,t+1}}{R^d_{i,t+1}}. \tag{9}$$

The following series of events then determines deposit market equilibrium. First, depositors determine their deposit supply schedules, taking into account the different levels of pledgeable returns delivered by banks' portfolios. Second, conditional on these deposit supply schedules, banks maximise their profits by choosing their debt issuance and the composition of their asset portfolios.

We begin with the bank's optimisation problem. For banks that invest in sector i, raising deposits to invest will be profitable as long as

$$R_{i,t+1} > R^d_{i,t+1}. \tag{10}$$

When this is the case, the bank will borrow up until the point at which its borrowing constraint (7) binds. Thus, banks' demand for funds for investing in sector i is given by (suppressing the argument of $\lambda_i(n_t)$):

$$d_{i,t} = \frac{\lambda_i R_{i,t+1}}{R_{d,t+1} - \lambda_i R_{i,t+1}} n_t. \tag{11}$$

Using (6) and (11), banks' profits from investing in sector i are given by:

$$\begin{aligned} V_{i,t+1} &= \left(R_{i,t+1} - R^d_{i,t+1}\right) d_{i,t} + R_{i,t+1} n_t \\ &= \frac{1-\lambda_i}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} R^d_{i,t+1} R_{i,t+1} n_t. \end{aligned} \tag{12}$$

These profits are returned lump-sum to households in period 2. Thus the deposit supply of households conditional on a bank investing in sector i is given by:

$$d_{i,t} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{1}{1+\beta}\frac{1-\lambda_i}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} R_{i,t+1} n_t. \tag{13}$$

In equilibrium, deposit supply (13) must equal deposit demand (11). Using $n_t = \pi w_t$, the equilibrium deposit quantity when the bank invests in sector i is given by:

$$d^*_{i,t} = \frac{\lambda_i \beta}{1+\lambda_i \beta}(1-\pi)w_t. \tag{14}$$

Note that equilibrium deposits are increasing in λ_i, the pledgeability of the bank's returns. That is, alleviating the financial friction would raise the amount of saving, and hence investment, in the economy. The intuition for this is that a greater degree of asset pledgeability reassures bank creditors that their deposits will be safe, so they are willing to expand the equilibrium quantity of saving.

Given (3) and (14), capital produced at $t+1$ when the bank invests in sector i is given by:

$$\begin{aligned} k^*_{i,t+1} &= x_i \left[\pi w_t + \frac{\lambda_i \beta}{1+\lambda_i \beta}(1-\pi)w_t\right] \\ &= x_i \frac{(1-\alpha)k_t^\alpha}{1+\lambda_i \beta}(\pi + \lambda_i \beta). \end{aligned} \tag{15}$$

It can be shown that, in equilibrium, the return on the bank's investment in sector i is given by (see Appendix):

$$R^*_{i,t+1} = \frac{\alpha x_i^\alpha}{\left[\frac{\pi+\lambda_i\beta}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}. \tag{16}$$

Using the equilibrium condition that deposit supply (13) must equal deposit demand (11), the equilibrium deposit rate, given that the bank invests in sector i, is given by:

$$\begin{aligned} R^{d*}_{i,t+1} &= \lambda_i R_{i,t+1}\left(1 + \frac{n_t}{d^*_{i,t}}\right) \\ &= \frac{\alpha x_i^\alpha (1+\lambda_i\beta)^{1-\alpha}(\pi+\lambda_i\beta)^\alpha}{\beta(1-\pi)\left[(1-\alpha)k_t^\alpha\right]^{1-\alpha}}. \end{aligned} \tag{17}$$

Note that higher bank capital relative to debt (i.e. higher π) increases the equilibrium incentive-compatible interest rate that depositors can charge. Further, the deposit rate paid within a given sector is increasing in the productivity of that sector and its pledgeability. The pledgeability and productivity of the two sectors are thus crucial in determining the sector that yields better returns for depositors. It can be shown that condition (10) holds in equilibrium as long as:

$$\beta(1-\pi) > \pi + \lambda_i \beta. \tag{18}$$

In what follows, we assume that (18) holds for both sectors. It can also been shown that $R^{d*}_{i,t+1} > \lambda_i R^{*}_{i,t+1}$ so the financial constraint binds in equilibrium.

2.6 Credit trap

Given that the borrowing constraint (7) binds for banks in equilibrium and banks compete with each other for deposits, households choose to deposit in banks that can offer the highest deposit rate. In the Appendix, we show that, given (18) holds, it will be profit- maximising for banks to invest in the sector that pays depositors the highest return, rather than taking no deposits and investing always in sector A as long as:

$$x_B(1-\lambda_B)\frac{\pi+\lambda_B\beta}{1+\lambda_B\beta} \geq x_A\pi. \tag{19}$$

Then:

Definition 1 *A credit trap is a situation in which banks invest perpetually in the unproductive sector (sector B).*

From (17), $R^{d*}_{A,t+1} < R^{d*}_{B,t+1}$ and banks invest in sector B when, for some n:

$$x_A^{\alpha}(1+\lambda_A(n)\beta)^{1-\alpha}(\pi+\lambda_A(n)\beta)^{\alpha} \leq x_B^{\alpha}(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^{\alpha}.$$

We can therefore show that banks invest in sector B when the net worth of the banking system falls below a critical threshold:

Lemma 1 *Under conditions (18) and (19), the bank invests in sector B at time t when $n_t < \tilde{n}$, where $\tilde{n}$ solves:*

$$x_A^{\alpha}(1+\lambda_A(\tilde{n})\beta)^{1-\alpha}(\pi+\lambda_A(\tilde{n})\beta)^{\alpha} = x_B^{\alpha}(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^{\alpha}. \tag{20}$$

Thus, the bank invests in sector A and the credit market equilibrium is given by $(d^*_{A,t}, R^{d*}_{A.t+1})$ when $n_t > \tilde{n}$; it invests in sector B and the credit market equilibrium is given by $(d^*_{B.t}, R^{d*}_{B.t+1})$ when $n_t \leq \tilde{n}$.

Proof. See Appendix. ■

This establishes that when banks' net worth falls below a critical threshold $\tilde{n}$ creditors become unwilling and banks become unable to invest in sector A. Because sector A is more productive than sector B, a higher return on sector B can only arise if there is more investment in it, i.e. a greater amount of leverage. When the banking system is healthy, high leverage when investing in sector A will be possible, making it more attractive. Only when the banking system is sufficiently impaired and banks cannot borrow enough to finance loans to sector A will investment flow to B. We next establish the aggregate consequences of these investment decisions.

In the general equilibrium of the economy, the capital stock evolves according to equation (3). Using equilibrium deposits and bank capital, the law of motion for physical capital can be expressed as:

$$k_{t+1} = x_i \frac{\pi + \lambda_i \beta}{1 + \lambda_i \beta} (1 - \alpha) k_t^{\alpha}, \quad i = \{A, B\}. \tag{21}$$

Note that in general, tomorrow's capital stock will be larger the less severe the financial friction (higher λ_i), and the higher is bank capital relative to debt (higher π). We can then establish:

Lemma 2 *Conditional on bank portfolios being allocated to sector B, the steady state level of physical capital converges to*

$$k_B^* = \left(x_B \frac{\pi + \lambda_B \beta}{1 + \lambda_B \beta} (1 - \alpha) \right)^{\frac{1}{1-\alpha}}, \tag{22}$$

which is the unique, stable steady state under investment in sector B. Conditional on bank portfolios being allocated to sector A, the steady states of A (possibly multiple) satisfy

$$k_A^* = \left(x_A \frac{\pi + \lambda_A \left(\pi(1-\alpha) k_A^{*\alpha} \right) \beta}{1 + \lambda_A \left(\pi(1-\alpha) k_A^{*\alpha} \right) \beta} (1 - \alpha) \right)^{\frac{1}{1-\alpha}}. \tag{23}$$

Proof. It is straightforward to demonstrate this using (5) and (21). ■

In the following analysis we assume that sector A has a unique stable steady state when $n_t > \tilde{n}$.[10] This ensures that if sector A is invested in, the economy will converge to k_A^* absent any shocks.

We now establish a proposition under which the economy features a credit trap.

[10]The shape of $\lambda_A(n_t)$ is relevant for this. Conditions on this functional form can be given that ensure there is a unique stable state in sector A for $n_t > \tilde{n}$.

Proposition 1 *Suppose (18), (19) hold. Let n_B^* be the steady state level of banker net worth when sector B is invested in:*

$$n_B^* = \pi(1-\alpha)\left(x_B \frac{\pi + \lambda_B \beta}{1 + \lambda_B \beta}(1-\alpha)\right)^{\frac{\alpha}{1-\alpha}}.$$

Then the economy features a credit trap if

$$x_A^\alpha (1 + \lambda_A(n_B^*)\beta)^{1-\alpha}(\pi + \lambda_A(n_B^*)\beta)^\alpha < x_B^\alpha (1 + \lambda_B \beta)^{1-\alpha}(\pi + \lambda_B \beta)^\alpha. \tag{24}$$

Proof. Given (18) and (19), banks invest in sector B rather than sector A iff

$$x_A^\alpha (1 + \lambda_A(n_t)\beta)^{1-\alpha}(\pi + \lambda_A(n_t)\beta)^\alpha < x_B^\alpha (1 + \lambda_B \beta)^{1-\alpha}(\pi + \lambda_B \beta)^\alpha.$$

Hence if

$$x_A^\alpha (1 + \lambda_A(n_B^*)\beta)^{1-\alpha}(\pi + \lambda_A(n_B^*)\beta)^\alpha < x_B^\alpha (1 + \lambda_B \beta)^{1-\alpha}(\pi + \lambda_B \beta)^\alpha,$$

the banks will invest in sector B when $n_t = n_B^*$ i.e. they invest in sector B in the steady state of B. This is thus a steady state equilibrium and without shocks the economy will invest in sector B for the rest of time, so is stuck in a credit trap. ■

An economy in which this holds is shown in Figure 7.[11] The critical value of banking system net worth at which sector A is invested in is given by $\widetilde{n}$. Above this level of banking system health, the economy invests exclusively in sector A, and the economy converges to the 'good' steady state (n_A^*), featuring high levels of capital, output and income. If the banking system is sufficiently impaired with $n_t < \widetilde{n}$, sector B is invested in, and the economy converges to the 'bad' steady state (n_B^*), featuring low levels of capital, output and bank lending. This is indeed

[11] Note that there will always be a jump in the law of motion at $\widetilde{n}$. To see this, at the trap threshold, the return paid on deposits in A and B is the same, so after rearranging

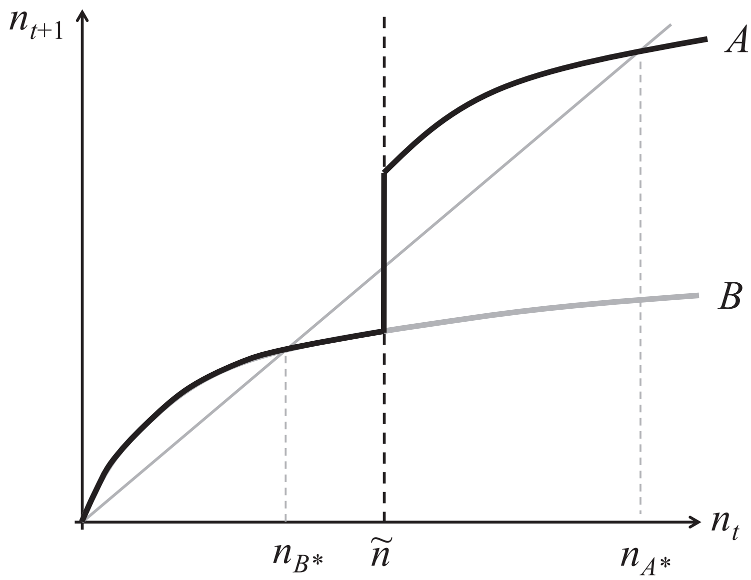
$$\begin{aligned} x_A^\alpha \left(\frac{\pi + \lambda_A(\widetilde{n})}{1 + \lambda_A(\widetilde{n})}\right)^\alpha &= x_B^\alpha \left(\frac{\pi + \lambda_B}{1 + \lambda_B}\right)^\alpha \left(\frac{1 + \lambda_B \beta}{1 + \beta \lambda_A(\widetilde{n})}\right) \\ &> x_B^\alpha \left(\frac{\pi + \lambda_B}{1 + \lambda_B}\right)^\alpha \end{aligned}$$

The last part follows as we must have $\lambda_B > \lambda_A(\widetilde{n})$, given $x_A > x_B$. Applying (21) and (5) its clear that at $\widetilde{n}$, n_{t+1} is greater when A is invested in.

For the economy to have a steady state with investment in sector A we require the jump in the law of motion for n_{t+1} at $\widetilde{n}$ to be sufficiently large that it surpasses the 45 degree line. For this we require that

$$\pi(1-\alpha)\left(x_A \left(\frac{\pi + \lambda_A(\widetilde{n})\beta}{1 + \lambda_A(\widetilde{n})\beta}\right)(1-\alpha) k_t^\alpha\right)^\alpha > \widetilde{n}$$

Figure 7: Aggregate law of motion in an economy with a credit trap



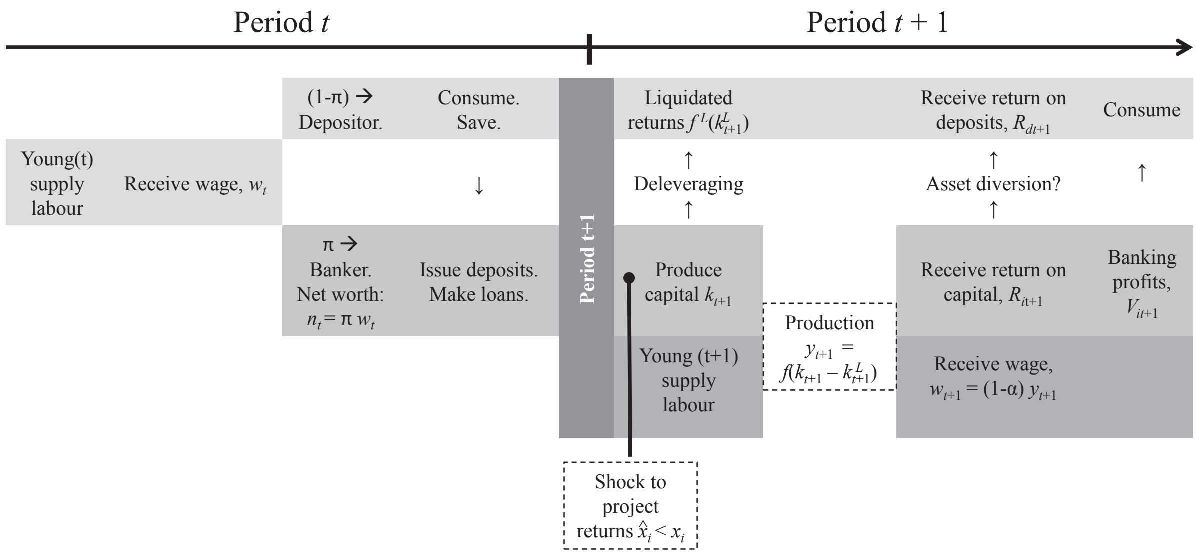
a steady state when banks invest in sector B when $n_t = n_B^*$, for which we require $n_B^* < \widetilde{n}$, which is ensured by (24).

The intuition for the credit trap is as follows. When the health of the banking system is high, the collateral value of financial assets is also high, allowing banks to have large leverage when investing in sector A, making it more attractive than sector B (by allowing them to pay higher returns to depositors). A is productive and so delivers high returns, resulting in high net worth in the banking system in the next period, which keeps them investing in A. Conversely, when the financial system is severely impaired, sector B is more attractive than sector A due to the low leverage permitted on financial assets. Crucially, because the banks invest in the unproductive sector B, their net worth remains low in future periods, keeping them investing in B.

The economy's entry to the credit trap has implications for interest rate spreads. Upon entry there is a discrete decrease in the gross rate of return banks receive on their investments, $R_{i,t+1}$, as they switch from investing in the productive sector A to the unproductive sector B. Recall the return from investing in sector i is given by $R_{i,t+1} = x_i a k_{t+1}^{a-1}$. Whilst the decrease in output[12] the economy experiences as investment is switched to sector B decreases k_{t+1}, pushing

[12]See footnote 10 above.

Figure 8: Timeline of events: case of a financial shock



up the return, this effect is dominated by the reduction in productivity, x_i.[13] By contrast, there is no change in the interest rate paid on deposits as the credit trap is entered. This is because at the trap threshold, $\widetilde{n}$, the deposit rate is the same regardless of the sector the banks invest in (as given by (20)). Thus, on entering the credit trap, the spread between $R_{i,t+1}$ and $R^d_{i,t+1}$ narrows.

3 A financial crisis

We now illustrate how an unexpected productivity shock, which sharply reduces banks' net worth, can send the economy from the good equilibrium to the bad one. A revised timeline for the economy is shown is Figure 8.

[13]Formally, we can write

$$R^*_{i,t+1} = \frac{\alpha x_i^\alpha}{\left[\frac{\pi+\lambda_i\beta}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}} = \frac{\alpha x_i^\alpha (1+\lambda_i\beta)^{1-\alpha}(\pi+\lambda_i\beta)^\alpha}{(\pi+\lambda_i\beta)((1-\alpha)k_t^\alpha)^{1-\alpha}}$$

Thus, at the trap threshold $\widetilde{n}$, $R^*_{B,t+1}(\widetilde{n}) < R^*_{A,t+1}(\widetilde{n})$ iff

$$\frac{\alpha x_B^\alpha (1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha}{(\pi+\lambda_B\beta)((1-\alpha)k_t^\alpha)^{1-\alpha}} < \frac{\alpha x_A^\alpha (1+\lambda_A(\widetilde{n})\beta)^{1-\alpha}(\pi+\lambda_A(\widetilde{n})\beta)^\alpha}{(\pi+\lambda_A(\widetilde{n})\beta)((1-\alpha)k_t^\alpha)^{1-\alpha}}$$

Applying (20) this holds iff

$$\frac{1}{(\pi+\lambda_B\beta)} < \frac{1}{(\pi+\lambda_A(\widetilde{n})\beta)}$$

This follows as given (20) and $x_A > x_B$ we must have $\lambda_A(\widetilde{n}) < \lambda_B$.

3.1 Set up

Suppose that in period t, the economy is in the good steady state in which banks invest in sector A. When deposit contracts were signed, households did not think bank asset returns $R_{A,t+1}$ were stochastic, expecting productivity to be x_A with certainty. Suppose that instead, after deposits have been collected and investment in sector A is made, an *unexpected* negative productivity shock hits at the start of period $t+1$, such that realised productivity, $\hat{x}_A$, is less than the level expected with certainty: $\hat{x}_A < x_A$. Given the realised shock, the actual capital produced is less than the amount assumed when contracts were signed (3), and is given by:

$$\hat{k}_{t+1} = \hat{x}_A \left(n_t + d^*_{A,t} \right).$$

This implies that bankers would default on deposits at the end of period $t+1$ if left to themselves, since (7) no longer holds under the realised return

$$\hat{R}_A = \frac{\alpha \hat{x}_A^{\alpha}}{\left[\frac{\pi+\lambda_A\beta}{1+\lambda_A\beta}(1-\alpha)k_t^{\alpha} \right]^{1-\alpha}} < R^*_A.$$

Banker default would occur because when asset returns are at the level households anticipated, banks have exactly the required level of pledgeable assets to repay depositors fully.[14] Thus, with any reduction in the value of their assets, banks have an insufficient amount of pledgeable assets and (7) is violated. Realising this, depositors will withdraw their funds until (7) holds again, as we discuss in the next sub-section.

Intuitively, following the shock, the value of the banks' assets has dropped, but their liabilities (what they promised to depositors) are unchanged. Without an adjustment to their balance sheet, their leverage will then increase. However, at the expected level of asset returns, (8) holds with equality and bank leverage is just low enough that they can pledge the required amount to depositors. Thus, following the negative shock, bank leverage is too high to fully repay depositors.

3.2 Depositor run and asset liquidation

Realising that they will not be repaid fully if they wait until the end of period $t+1$, depositors withdraw their funds, forcing partial liquidation of the project, by seizing capital $k^L_{t+1} \leq \hat{k}_{t+1}$

[14] Given our assumption (18), the pledgeability constraint (7) binds.

from banks at the start of $t+1$. Here we are capturing the idea that following a negative shock to asset values, deleveraging is required to bring leverage back to its original level. Unlike the standard output-producing technology (4), the interim *liquidation technology* uses *only capital* to produce output: 'old' households (depositors) seize physical capital from banks before banks can use it to produce final output, but since 'old' households do not have labour endowment, they use their own unproductive 'cottage' technology to turn the capital seized from banks into final output goods. The liquidation technology has the following form:

$$\hat{y}^L_{t+1} = f^L(\hat{k}_{t+1}, k^L_{t+1}), \tag{25}$$

where k^L_{t+1} is the amount of capital being liquidated by the depositors and $f^L(\hat{k}_{t+1}, 0) = 0$. We allow that the technology may depend on the aggregate amount of capital in the economy, $\hat{k}_{t+1}$. The aggregate output produced after the negative productivity shock and liquidation, $\hat{y}_{t+1}$, is given by the sum of the output produced by 'old' households using liquidation technology (25), $\hat{y}^L_{t+1}$, and the output produced by bankers with the remaining capital using the standard technology (4), $\hat{y}^P_{t+1}$:

$$\hat{y}_{t+1} = \hat{y}^P_{t+1} + \hat{y}^L_{t+1} = (\hat{k}_{t+1} - k^L_{t+1})^\alpha + f^L(\hat{k}_{t+1}, k^L_{t+1}).$$

Once the unexpected productivity shock is realised, depositors withdraw their money from the bank and invest the proceeds into the liquidation technology until bank leverage falls to the point where banks can credibly promise to repay the remaining deposit liabilities. We consider a benchmark case in which the total final output available for consumption of the 'old' is invariant to the size of liquidation, k^L_{t+1}. Even in this benchmark case, liquidation by the 'old' depositors imposes costs on the 'young', who faces lower wages: they have less physical capital to work with and hence see their marginal product of labour reduced:

$$\hat{w}_{t+1} = (1-\alpha)\hat{y}^P_{t+1} = (1-\alpha)(\hat{k}_{t+1} - k^L_{t+1})^\alpha.$$

This in turn implies that liquidation by the 'old' depositors also reduces bank capital in the next period:

$$\hat{n}_{t+1} = \pi(1-\alpha)\hat{y}^P_{t+1} = \pi(1-\alpha)(\hat{k}_{t+1} - k^L_{t+1})^\alpha. \tag{26}$$

In this benchmark case, as the consumption of the 'old' is invariant to the extent of bank liquidation, the burden of liquidation by the 'old' is imposed entirely on the 'young' and the subsequent generations, who need to work with less capital and thus face lower wages and consumption. Liquidation gives rise to negative intergenerational externalities. In the Appendix we show that, in the benchmark case, the output produced using young labour following a negative shock is given by

$$\hat{y}_{t+1}^{P*} = \frac{\hat{x}_A^{\alpha} - \lambda_A(n_t)x_A^{\alpha}}{1-\lambda_A(n_t)}\left[\frac{\pi + \lambda_A(n_t)\beta}{1+\lambda_A(n_t)\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha}. \tag{27}$$

Clearly, $\hat{y}_{t+1}^{P*} < y_{t+1}$, where y_{t+1}(given by (4)) is the level of output that would have occurred in the absence of the negative technology shock.

A key question of interest is whether an economy ends up in a credit trap following a negative productivity shock. We know from Lemma 1 that this crucially depends on the size of the reduction in bank capital following the shock. Specifically, if bank capital only experiences a relatively small shock, such that $\hat{n}_{t+1}$ remains above $\tilde{n}$ (given by (20)), then the economy converges back to the 'good' steady state k_A^*. But if the shock to bank capital is sufficiently large such that $\hat{n}_{t+1} \leq \tilde{n}$, then the economy will converge to the credit trap equilibrium and remain stuck at k_B^*. In the next section we consider what leverage policy can do to help the economy to avoid falling into credit traps.

4 Avoiding a credit trap: Leverage restrictions

In this section we consider how a leverage ratio cap – which limits the amount that banks can borrow to finance their investments – could be set to increase the resilience of the economy against the risk of falling into a credit trap. Consider a leverage ratio cap, λ_r, where 'r' denotes 'regulatory', which limits the amount of bank borrowing as follows:[15]

$$\lambda_r R_{i,t+1}(n_t + d_{i,t}) \geq R_{i,t+1}^d d_{i,t}.$$

[15]In equilibrium bank leverage is $(1-\lambda)^{-1}$, so by choosing λ, the regulator also chooses the banking leverage ratio.

4.1 Leverage and resilience

Assume that the economy at t starts with physical capital $k_t > \tilde{k}$, such that banks invest in sector A.[16] Suppose now that the regulator imposes a leverage ratio cap, $\lambda_r < \lambda_A(n_A^*)$, where n_A^* is the level of bank capital in a 'good' steady state. To keep things simple, assume that the leverage ratio does not bind on sector B: $\lambda_r > \lambda_B$. This implies that the leverage requirement does not alter the threshold $\tilde{n}$ for bank capital below which the economy falls into a credit trap.

We define a threshold level of the productivity realisation $\tilde{x}_t(\lambda_r)$ which reduces bank capital sufficiently in the next period such that banks start investing in sector B and the economy gets stuck in a credit trap. This threshold is a function of the regulatory leverage ratio cap (see Appendix for derivation):

$$\tilde{x}_t(\lambda_r) \equiv \left(\lambda_r x_A^\alpha + \frac{\widetilde{n}(1-\lambda_r)}{(1-\alpha)\pi\left[\frac{\pi+\lambda_r\beta}{1+\lambda_r\beta}(1-\alpha)k_t^\alpha\right]^\alpha} \right)^{\frac{1}{\alpha}}. \tag{28}$$

The economy falls into a credit trap whenever $\hat{x}_A \leq \tilde{x}_t(\lambda_r)$. Thus, $\tilde{x}_t(\lambda_r)$ is a measure of the *resilience* of the financial system: the lower $\tilde{x}_t(\lambda_r)$, the more resilient the financial system, in the sense that the economy avoids getting into a credit trap for a larger range of negative productivity shocks.

It can be shown that, under mild conditions, $\tilde{x}_t(\lambda_r)$ is U-shaped, reaching its minimum at $\lambda_r = \lambda^{\min} \in (0,1)$. This is demonstrated in Figure 9. Formally, we have the following proposition.

Proposition 2 *Suppose*

$$(1-\alpha)\pi x_A^\alpha \left[\frac{\pi+\beta}{1+\beta}(1-\alpha)k_t^\alpha\right]^\alpha > \widetilde{n},$$

and

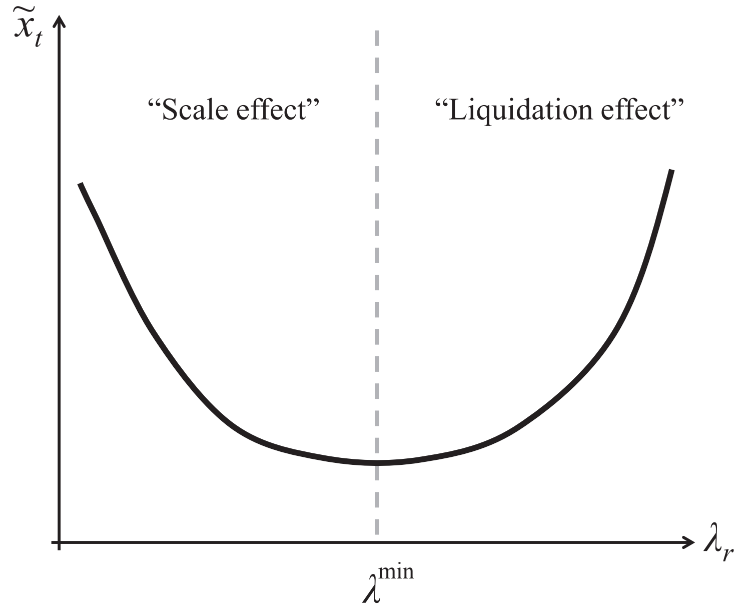
$$x_A^\alpha < \frac{\widetilde{n}}{(1-\alpha)\pi((1-\alpha)k_t^\alpha)^\alpha}\left(\frac{\alpha\beta(1-\pi)+\pi}{\pi^{1+\alpha}}\right).$$

Then

$$\exists \lambda^{min} \in (0,1) : \frac{d\tilde{x}_t(\lambda_r)}{d\lambda_r} \left\{ \begin{array}{c} < 0 \; for \; \lambda_r \in [0, \lambda^{min}) \\ = 0 \; for \; \lambda_r = \lambda^{min} \\ > 0 \; for \; \lambda_r \in (\lambda^{min}, 1] \end{array} \right\}.$$

[16] $\widetilde{k}$ corresponds to $\widetilde{n}$, the threshold above which banks invest in sector A. Specifically, $\widetilde{n} = \pi(1-\alpha)\widetilde{k}^\alpha$.

Figure 9: Resilience and leverage: the scale effect and the liquidation effect



Further, λ^{min} is unique and $\tilde{x}_t(\lambda_r)$ reaches a unique minimum at $\lambda_r = \lambda^{min}$.

Proof. See Appendix. ■

Remark 1 *The first condition states that when there are no shocks ($\hat{x}_A = x_A$) and $\lambda_r = 1$, the economy avoids the credit trap, i.e. it is possible for the economy to avoid the trap when there are no shocks.*[17] *The second condition ensures that $d\tilde{x}_t(0)/d\lambda_r < 0$, that is, when $\lambda_r = 0$, and banks take no deposits, increasing leverage increases the resilience of the economy.*

The U-shape reflects the two opposing effects of leverage on resilience. On the one hand, for any given initial level of capital k_t and productivity realisation $\hat{x}_A$, banks would have produced more capital in period $t+1$ when leverage was high at t (λ_r is high): other things equal, this puts the economy farther away from the credit trap threshold $\tilde{n}$ and hence increases resilience. We call this the *scale effect.* On the other hand, depositors liquidate a greater proportion of the capital produced following the shock at $t+1$ the greater leverage at t:[18] other things equal, this makes it more likely that the economy falls into a credit trap and hence reduces resilience. We call this the *liquidation effect.* We can demonstrate that when leverage is low ($\lambda_r < \lambda^{\min}$), the scale effect dominates, such that allowing banks to increase leverage will increase resilience.

[17]The condition is implied by the necessary condition in footnote 9 for the economy to have multiple steady states.

[18]This is because for a given negative shock, the reduction in net worth is greater the higher the initial leverage ratio.

Over this range, *there is no trade-off between output and resilience*: increasing leverage increases both. But when leverage is high ($\lambda_r > \lambda^{\min}$), the liquidation effect dominates, such that allowing banks to increase leverage will reduce resilience.

Under the conditions given, $\lambda^{\min} > 0$. The leverage ratio associated with this is given by $(1 - \lambda^{\min})^{-1} > 1$. Thus, under the conditions given in the proposition, the leverage ratio that maximises resilience is greater than 1. This implies that even a "resilience nutter" – who focused only on the resilience of the financial system – would implement a leverage ratio greater than 1.[19] This is due to the scale effect.

It is interesting to examine how the desirability of leverage policy varies with the state of the economy. First, it is clear that following a small negative shock to the financial system, the economy will recover faster to its steady state if a binding leverage policy is relaxed. This is because doing so allows more deposits to flow into the banking system, raising the amount of investment and future output. It may be thought that this is at the cost of lowering resilience, by letting weaker banks take on higher leverage. But the proposition below shows that, on the contrary, the leverage ratio that maximises resilience is *countercyclical*:

Proposition 3 *Suppose the conditions of Proposition (2) hold. Then*

$$\frac{d\lambda^{min}}{dk_t} < 0.$$

The proposition shows that when the state of the economy deteriorates – a decrease in k_t – the λ_r that maximises resilience *increases*. Thus, even a "resilience nutter" would allow greater leverage in a downturn. This is because the scale effect becomes relatively more important when k_t is lower. With n_t closer to the trap threshold $\tilde{n}$, it is desirable to allow more investment to help banks improve their balance sheets. Even if a policymaker cared only about resilience, they would conduct counter-cyclical leverage policy in this setting.

4.2 Leverage restriction and the steady state

The leverage ratio cap not only affects the resilience of the system but also the steady state level of capital and hence output if the leverage ratio is kept constant across periods. From (21) and (4), we know that when the leverage ratio cap is kept at λ_r, the level of output in the

[19] A "resilience nutter" is the macroprudential equivalent of the "inflation nutter" in monetary policy.

'good' steady state is given by:

$$y_A^*(\lambda_r) = k^*(\lambda_r)^\alpha = \left[x_A \frac{\pi + \lambda_r\beta}{1+\lambda_r\beta}(1-\alpha)\right]^{\frac{\alpha}{1-\alpha}}. \tag{29}$$

It is straightforward to demonstrate that $\partial y_A^*(\lambda_r)/\partial\lambda_r > 0$: output in the 'good' steady state will be higher the higher the leverage ratio cap is set. This is intuitive, as banks can invest more in productive projects the greater is their permitted leverage.

Substituting (29) into (28), the threshold level of productivity realisation which tips the economy into a credit trap when the economy is initially at the regulated steady state given by (29) can be expressed as follows:

$$\tilde{x}_A^*(\lambda_r) = \left(\lambda_r x_A^\alpha + \frac{\widetilde{n}(1-\lambda_r)}{\pi(1-\alpha)^{\frac{1}{1-\alpha}} x_A^{\frac{\alpha^2}{1-\alpha}} \left(\frac{\pi+\lambda_r\beta}{1+\lambda_r\beta}\right)^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}.$$

As before, it can be shown that, under mild conditions, $\tilde{x}_A^*(\lambda_r)$ is U-shaped, and reaches its minimum when $\lambda_r = \lambda^{\min*} \in (0,1)$.

Proposition 4 *Suppose*

$$x_A^{\frac{\alpha}{1-\alpha}}(1-\alpha)^{\frac{1}{1-\alpha}}\pi\left(\frac{\pi+\beta}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}} > \widetilde{n}. \tag{30}$$

Suppose

$$x_A^\alpha < \frac{\widetilde{n}\left[\alpha\beta(1-\pi)+(1-\alpha)\pi\right]}{(1-\alpha)^{\frac{2-\alpha}{1-\alpha}}\pi^{\frac{2-\alpha}{1-\alpha}} x_A^{\frac{\alpha^2}{1-\alpha}}}. \tag{31}$$

Then

$$\exists\lambda^{min*} \in (0,1) : \frac{d\tilde{x}_A^*(\lambda_r)}{d\lambda_r}\left\{\begin{array}{c} < 0 \text{ for } \lambda_r \in \left[0,\lambda^{min*}\right) \\ = 0 \text{ for } \lambda_r = \lambda^{min*} \\ > 0 \text{ for } \lambda_r \in \left(\lambda^{min*},1\right] \end{array}\right\}.$$

Further, λ^{min*} is unique and $\tilde{x}_A^*(\lambda_r)$ reaches a unique minimum at λ^{min*}.

Proof. *The proof follows the method similar to the proof of Proposition 2* ■

Remark 2 *The first condition states that when there are no shocks ($\hat{x}_A = x_A$) and $\lambda_r = 1$, the economy avoids the credit trap in the steady state. The second condition ensures that $d\tilde{x}_A^*(0)/d\lambda_r < 0$ that is, when $\lambda_r = 0$ and so banks take no deposits, increasing leverage increases the resilience of the economy in the steady state.*

4.3 Leverage restriction: summary

In summary, leverage policy can be effective in reducing the chance of the economy falling into a credit trap. If the privately-determined leverage ratio is greater than that associated with $\lambda^{\min}$, resilience could be improved by implementing this as a leverage cap (and in this case it would bind too). After a small negative shock that does not result in the economy falling into the trap, and at which the original leverage ratio still binds, relaxing this leverage limit would be desirable. Doing so helps the economy recover faster and will increase the economy's resilience against falling into the trap following a further shock.

5 Escaping a credit trap: leverage caps and other credit policies

In this section we consider what policy can do to get the economy out of a trap following a sufficiently large negative shock. We begin by considering the role of countercyclical leverage requirements, showing that they cannot help the economy escape.

5.1 Relaxing the leverage ratio cap

Proposition 5 *Suppose (18), (19) hold. Suppose with regulatory leverage ratio λ_r in place the economy is stuck investing in sector B. Then relaxing λ_r will not help the economy escape from the credit trap.*

Proof. Given (18) and (19), banks invest in sector B rather than sector A iff

$$x_A^{\alpha}(1+\lambda_A(n_t)\beta)^{1-\alpha}(\pi+\lambda_A(n_t)\beta)^{\alpha} < x_B^{\alpha}(1+\lambda\beta)^{1-\alpha}(\pi+\lambda\beta)^{\alpha}, \tag{32}$$

where $\lambda = \min\{\lambda_r, \lambda_B\}$. In the trap with banks investing in sector B, (32) must hold. As $x_A > x_B$, it must be that $\lambda > \lambda_A(n_t)$. In other words, permitted leverage when investing in B must exceed permitted leverage when investing in A. If $\lambda_r \geq \lambda_B$, the regulator permits higher leverage than the market so relaxing the regulatory constraint will not alter equilibrium. If $\lambda_B > \lambda_r$, the regulatory constraint binds, and relaxing it permits higher leverage in B. But this only enhances the attractiveness of investing in B relative to A. Thus, in both cases, relaxing λ_r will not direct investment towards A. ■

The intuition for the proof is simple. As sector A is inherently more productive, a higher rate on deposits can only be paid when investing in B (making it more attractive) if the volume of lending in B is greater. Thus, with policy in place, more leverage is possible in sector B than in A, and relaxing the policy constraint either has no effect (if the constraint is not binding) or allows an even greater volume of investment in B (if it is), making investment in B assets only more attractive. Neither helps with the reallocation of funds towards the more productive sector, which is needed to escape the trap. Thus while countercyclical leverage policy can be beneficial in facilitating recovery after a small shock, it is not helpful if the shock is sufficiently large to result in a credit trap.

5.2 Unconventional credit policies

In this section we ask whether alternative government/central bank policies could be used to help the economy escape a credit trap (e.g. Gertler & Kiyotaki (2010)). We consider three *unconventional credit policies*: direct lending to the private sector by the government; discount window lending to banks by the central bank; and recapitalisation of the banking system. Variants of all three policies were employed during the financial crisis in the US, the UK and the euro-area.[20] We conduct the following analysis under the weaker form of Assumption 2 in which λ_B is increasing in banking system net worth n rather than invariant to it (as with Assumption 2$'$) stated above.

The government's source of funding for all three policies is assumed to be risk-free bonds, which are perfect substitutes for bank deposits, and hence pay the same return. In addition to the transfer of resources entailed under government intervention, each policy entails some efficiency cost, designed to capture something of the inefficiency associated with the government's activities in credit markets. Given implementation of policy j, the consolidated public sector's budget constraint is given by:

$$(1 + \tau_j)s_{j,t} = d_{g,t} - R^d_t d_{g,t-1} + R_j s_{g,t-1},$$

where $\tau_j > 0$ represents the efficiency cost of implementing the policy, $s_{j,t}$ denotes the volume of policy j conducted, $R^d_t d_{g,t-1}$ is the total paid out on government bonds issued in the previous period and $R_j s_{g,t-1}$ is the return made on implementing the policy in the previous period.

We conduct the analysis under some simplifying conditions. First, suppose that the gov-

[20] It may appear unusual to explore short-term unconventional credit policies in an OLG model. However, as discussed in Section 2, the OLG structure used in the model is purely for tractability, with the intended length of a period being of the order of a year.

ernment has no outstanding debt ($d_{g,t-1} = 0$), and did not conduct any policies previously ($s_{g,t-1} = 0$), reducing the budget constraint to:

$$s_{j,t} = \frac{d_{g,t}}{1 + \tau_j}. \tag{33}$$

Equation (33) demonstrates clearly the impact of the inefficiency cost of policy: the greater τ_j, the less policy can be implemented for a given amount of bonds issued. Second, any revenues earned by the government on its unconventional policies are returned to next generation's young. Under these assumptions, with $d_{g,t}$ government bonds issued, the household sector's supply of funds for deposits is given by:

$$d_{i,t} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{(1-x_g)}{1+\beta}\frac{V_{i,t+1}}{R^d_{i,t+1}} - d_{g,t}, \tag{34}$$

where $x_g \in [0, 1]$ represents the equity stake in banks following any equity injection ($x_g = 0$ if there is no equity injection). We note how this contrasts to (9), the case of no policy intervention. Third, because the economy is initially stuck in a credit trap it won't escape from by itself and welfare will be higher out of the trap, we take escape from the trap as the goal of policy intervention. Escaping a trap ultimately requires that physical capital and hence bank equity capital increase, so we take increases in the capital stock on impact of the policy to be our criterion for 'success'.

5.3 Direct lending

Under direct lending, government funds are loaned directly to the private sector. In principle, the government could lend directly to the most productive projects available (sector A). In order to enhance the comparability of the exercise across policies, and to take a 'conservative' benchmark, suppose that the government is constrained to invest in the equilibrium sector – which would be sector B in a trap.

If $s_{g,t}$ government loans are made, the capital stock the following period given investment in i is:

$$k_{t+1} = x_i(n_t + d_{i,t}) + x_i(s_{g,t}). \tag{35}$$

The amount of output goods the government invests in capital production, $s_{g,t}$, augments the amount invested by the banking sector, $n_t + d_{i,t}$. But the supply of deposits, $d_{i,t}$ is affected by the amount of government bonds issued, from (34). Following similar analysis to the basic model, it can be shown that the equilibrium amount of deposits supplied is given by

$$d_{i,t}^{*} = \frac{\beta\lambda_i(n_t)}{1+\beta\lambda_i(n_t)}(1-\pi)w_t - d_{g,t}\frac{(1+\beta)\lambda_i(n_t)}{1+\beta\lambda_i(n_t)}. \tag{36}$$

Comparing (36) with (14) we see that government policy partially crowds-out private sector deposits (i.e. deposits are smaller with policy), however the net effect on capital can be positive due to direct impact of the government's investment. To derive the law of motion for k_{t+1} we combine (35) and (36), giving:

$$k_{t+1} = x_i\left[\left(\frac{\pi+\lambda_i(n_t)\beta}{1+\lambda_i(n_t)\beta}\right)(1-\alpha)k_t^{\alpha}\right] + x_i d_{g,t}\left[\frac{1}{1+\tau_g} - \frac{\lambda_i(n_t)(1+\beta)}{1+\beta\lambda_i(n_t)}\right], \tag{37}$$

where τ_g is the efficiency cost of direct lending. This reduces to (21), the case of no policy, when $d_{g,t}=0$. The second term represents the net impact of policy.

Given a credit trap in sector B, direct lending turns out to be effective in raising k_{t+1} iff

$$\tau_g < \frac{1-\lambda_B(n_t)}{\lambda_B(n_t)(1+\beta)}. \tag{38}$$

Note that the right-hand side of (38) is decreasing in λ_B: direct lending is less effective when the economy is healthier. Further, it can be that direct lending raises k_{t+1} following a financial crash, but *lowers* it when the economy is healthy. These points are formalised in the following lemma.

Lemma 3 *The effectiveness of the direct lending policy is decreasing in* λ_i *:*

$$\frac{\partial^2 k_{t+1}}{\partial\lambda_i\partial d_{g,t}} < 0.$$

Further, suppose that in the credit trap, $n_t = \underline{n}$ *whilst, in the high output steady state of sector* A, $n_t = \overline{n} > \underline{n}$. *Suppose further that (with* $\lambda_A(\overline{n}) > \lambda_B(\underline{n})$*)*

$$\frac{1-\lambda_A(\overline{n})}{(1+\beta)\lambda_A(\overline{n})} < \tau_g < \frac{1-\lambda_B(\underline{n})}{(1+\beta)\lambda_B(\underline{n})}.$$

Then policy is effective in raising k_{t+1} *following a crash (*$n_t = \underline{n}$*), but lowers* k_{t+1} *in the good state of the economy (*$n_t = \overline{n} > \underline{n}$*).*

Proof. The proof of the second part is immediate from (38). For the first part note that

$$\frac{\partial k_{t+1}}{\partial d_{g,t}} = x_i \left[\frac{1}{1+\tau_g} - \frac{\lambda_i(1+\beta)}{1+\beta\lambda_i} \right].$$

So

$$\begin{aligned} \frac{\partial^2 k_{t+1}}{\partial \lambda_i \partial d_{g,t}} &= -x_i(1+\beta)\frac{(1+\beta\lambda_i) - \lambda_i\beta}{(1+\beta\lambda_i)^2} \\ &= -x_i \frac{1}{(1+\beta\lambda_i)^2} < 0. \end{aligned}$$

■

The intuition for these results is straightforward. Direct government intervention always has a positive impact on the economy, directly boosting k_{t+1}. However, this is paid for by displacing deposits by government bonds (the 'crowding out' effect), reducing the funding of the banking system. This is exacerbated by the inefficiency of government intervention ($\tau_g > 0$), requiring extra deposits to be displaced to fund a given level of direct lending. When the financial friction is very tight (λ_i low), deposit levels are low,[21] thus there is little displacement effect, and the direct benefit to the economy outweighs the negative crowding out effect. However, with a looser financial friction in a stronger economy (λ_i high), deposit levels are higher, so there is a larger cost from crowding out, which can then dominate the positive effect (whose size does not change with λ_i). Thus, whilst this policy may be very effective during a credit crunch, it does not follow that it would be desirable for the government to entirely displace the financial sector when the economy is healthy.

5.4 Discount window lending

Under discount window lending, the central bank lends directly to the banks. Let m_t be the amount lent to the banking sector, so with inefficiency cost τ_m, $m_t = d_{g,t}/(1+\tau_m)$. Then the total amount invested by the banking system (ie its total liabilities) is given by:

$$n_t + d_{i,t} + m_t.$$

Through discount window lending, the central bank can enforce repayment of its loans better than individual bank creditors can. In particular, the fraction of discount window-funded asset

[21]Without policy, $d^*_{i,t} = \frac{\lambda_i\beta(1-\pi)w_t}{1+\beta\lambda_i}$, which is increasing in λ_i.

returns that banks can divert is:

$$(1-\lambda_i)(1-\omega_g),$$

with the constant $\omega_g \in (0,1)$. When $\omega_g = 0$, the central bank faces no advantage over the private sector in the pledgeability of its loans. We map this into the bank's borrowing constraint by writing:

$$\lambda_i R_{i,t+1}(n_t + d_{i,t} + \omega m_t) \geq R^d_{i,t+1} d_{i,t} + R^m_{t+1} m_t,$$

where R^m_{t+1} is the (endogenous) rate paid on loans from the government and $\omega \equiv 1+\omega_g (1-\lambda_i) \lambda_i^{-1} > 1$ represents the greater *pledgeability* of assets financed at the discount window. The total pledgeability on loans from the government is given by $\omega\lambda_i < 1$.

Faced with two sources of funding (deposits and the discount window), the banks have a portfolio choice problem. Profits are:

$$V_{i,t+1} = R_{i,t+1}(n_t + d_{i,t} + \omega m_t) - R^d_{i,t+1} d_{i,t} - R^m_{t+1} m_t,$$

to be maximised subject to the leverage constraint.[22] As greater leverage is allowed when borrowing from the government, an endogenous 'penalty wedge' arises on discount window lending: $R^m_{t+1} > R^d_{t+1}$. The result of this portfolio choice problem is the following wedge between funding sources:

$$R^m_{t+1} = R^d_{i,t+1} + \frac{\lambda_i}{1-\lambda_i}(\omega - 1)(R_{i,t+1} - R^d_{i,t+1}).$$

Following the usual steps in the derivation, equilibrium deposit supply is given by

$$d^*_{i,t} = \frac{\lambda_i \beta}{1+\lambda_i\beta}(1-\pi)w_t - \frac{(1-\lambda_i)}{1+\lambda_i\beta}\left(\frac{R_{i,t+1} - R^m_{t+1}}{R_{i,t+1} - R^d_{i,t+1}}\right) m_t - \frac{\lambda_i(1+\beta)d_{g,t}}{1+\lambda_i\beta}.$$

We show in the Appendix that in equilibrium $\frac{R_{i,t+1}-R^m_{t+1}}{R_{i,t+1}-R^d_{i,t+1}} = \frac{1-\omega\lambda_i}{1-\lambda_i}$, which combined with the law of motion for capital $k_{t+1} = x_i(n_t + d_{i,t} + m_t)$ gives, for sector i:

$$k_{t+1} = x_i \left[\left(\frac{\pi + \lambda_i(n_t)\beta}{1+\lambda_i(n_t)\beta}\right)(1-\alpha)k_t^\alpha\right] + x_i d_{g,t}\left[\frac{1-(1-\omega_g)\left(\frac{1-\lambda_i(n_t)}{1+\lambda_i(n_t)\beta}\right)}{1+\tau_m} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta}\right]. \quad (39)$$

It can be shown that discount window lending is effective in raising k_{t+1} given a credit trap in

[22] A full derivation of the results in this section is given in the Appendix.

sector B if and only if:

$$\tau_m < w_g \frac{1-\lambda_B(n_t)}{\lambda_B(n_t)(1+\beta)}. \tag{40}$$

This expression is identical to (38) save for the inefficiency τ_m and the $w_g \in (0,1)$ term, representing the financial friction the central bank faces on its loans to the private sector banks. Thus, as with direct lending, policy can be effective when the economy is in a credit crunch, but ineffective, reducing k_{t+1} when the economy is healthy. We introduce a lemma analogous to Lemma 3:

Lemma 4 *The effectiveness of discount window lending is decreasing in λ_i:*

$$\frac{\partial^2 k_{t+1}}{\partial \lambda_i \partial d_{g,t}} < 0.$$

Further, suppose that in the credit trap, $n_t = \underline{n}$ whilst, in the high output steady state of sector A, $n_t = \overline{n} > \underline{n}$. Suppose further that (with $\lambda_A(\overline{n}) > \lambda_B(\underline{n})$)

$$\omega_g \frac{(1-\lambda_A(\overline{n}))}{(1+\beta)\lambda_A(\overline{n})} < \tau_m < \omega_g \frac{(1-\lambda_B(\underline{n}))}{(1+\beta)\lambda_B(\underline{n})}.$$

Then policy is effective in raising k_{t+1} following the crash, but lowers k_{t+1} in the good state of the economy.

Proof. The proof of the second part is immediate from (40). For the first part note that

$$\frac{\partial k_{t+1}}{\partial d_{g,t}} = x_i \left[\frac{1-(1-\omega_g)\left(\frac{(1-\lambda_i)}{(1+\lambda_i\beta)}\right)}{1+\tau_m} - \frac{\lambda_i(1+\beta)}{1+\lambda_i\beta} \right].$$

So

$$\frac{\partial^2 k_{t+1}}{\partial \lambda_i \partial d_{g,t}} = x_i \frac{1+\beta}{(1+\lambda_i\beta)^2} \left(\frac{1-\omega_g}{1+\tau_m} - 1 \right) < 0.$$

■

As with the direct lending case, when λ_i is higher, the negative effect on k_{t+1} from the crowding out of deposits becomes larger, as there are more deposits made when the banking system is healthier. While the overall effectiveness of policy decreases as the economy recovers, this decrease can occur at a different rate compared to direct lending, owing to the presence of $\omega_g \in (0,1)$. As we discuss below, this can result in direct lending being more effective following a very severe credit crunch, with discount window lending more effective for less severe crunches.

5.5 Bank recapitalisation

For simplicity we focus on government capital injections in this section, but we acknowledge that distressed banks may be able to raise equity from other sources, particularly given the creation of the resolution regime for failing banks and the development of structures for loss absorbing capacity that must be available to resolve banks.[23] We also note that the possibility of causing ex ante moral hazard - which is not considered here - creates an argument against public capital injections, even if it is ex post efficient in a situation where a large segment of the banking system is unable to raise new equity.

As with the other two policy interventions, the government is inefficient in investing in equity, with inefficiency cost τ_{gn}. Thus the amount of equity invested by the government, $n_{g,t}$, satisfies:

$$n_{g,t} = \frac{d_{g,t}}{1+\tau_{gn}}.$$

In return for its injection of resources to the banking system, the government obtains x_g fraction of bank equity, resulting in optimal household saving given by (34), with the equity share in the bank 'diluted' to $1 - x_g$. (Recall, the government's proceeds from the intervention are returned to the next generation.)

A very important direct effect of the recapitalisation is that λ_i increases, as it is now based on $n_t + n_{g,t} : \lambda_i(n_t + n_{g,t}) > \lambda_i(n_t)$. This direct effect of the recapitalisation, all else equal, *crowds in* depositors: with the financial friction reduced, they are willing to supply more deposits, raising investment. This goes beyond the usual effect of higher net worth allowing more deposits to be taken at a fixed leverage ratio. Here the leverage ratio rises too. To derive the equilibrium law of motion for k_{t+1} we follow the usual steps, first determining equilibrium in the banking sector.

With the banks' leverage constraints binding they demand deposits,[24]

$$d_{i,t} = \frac{\lambda_i\left(n_t + n_{g,t}\right) R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i\left(n_t + n_{g,t}\right) R_{i,t+1}}.$$

[23] More information on the resolution regime in the UK can be found here: http://www.bankofengland.co.uk/financialstability/Documents/resolution/apr231014.pdf

[24] Note the addition of $n_{g,t}$ which is absent with no equity injection.

Bank profits are given by[25]

$$V_{i,t+1} = \left(R_{i,t+1} - R^d_{i,t+1}\right) d_{i,t} + R_{i,t+1}(n_t + n_{g,t}).$$

Following the usual steps, equilibrium deposits are given by

$$d^*_{i,t} = \frac{\lambda_i(n_t + n_{g,t})\beta(1-\pi)w_t}{(1+\beta)\lambda_i(n_t+n_{g,t}) + (1-x_g)(1-\lambda_i(n_t+n_{g,t}))} - \frac{(1+\beta)d_{g,t}\lambda_i(n_t+n_{g,t})}{(1+\beta)\lambda_i(n_t+n_{g,t}) + (1-x_g)(1-\lambda_i(n_t+n_{g,t}))}.$$

The impact of policy is notably different to the other two cases, as the rise in λ_i, and dilution through the $x_g > 0$ term, increase the fraction of first period resources saved, $\frac{\beta(1-\pi)w_t}{1+\beta}$.[26] To determine the overall effect of a recapitalisation on $d^*_{i,t}$ we need to specify the relationship between x_g and $n_{g,t}$, i.e. how much equity the government gets in return for its investment. We consider the general form weighting the banks' current equity with factor $\gamma > 0$:

$$x_g = \frac{n_{g,t}}{n_{g,t} + \gamma n_t}. \tag{41}$$

We give two examples of γ:

1. The fraction the government obtains reflects the banks' current equity ($\gamma = 1$)

$$x_g = \frac{n_{gt}}{n_{gt} + n_t}.$$

For example, if the net worth of the banking system at time t is 100 units of output goods and the government invests 100 units, it ends up owning half the equity of the banking system.

2. The fraction the government obtains reflects the pdv of the banking system

$$x_g = \frac{n_{gt}}{n_{gt} + \frac{V_{i,t+1}}{R^d_{i,t+1}}}.$$

In the Appendix we show that without government intervention, $\frac{V_{i,t+1}}{R^d_{i,t+1}} = \frac{n_t(1-\lambda_i)\beta(1-\pi)}{(1+\beta\lambda_i)\pi}$.

[25]The formula (save for the $n_{g,t}$ term) for bank profits has not changed here. What changes is *who* gets the profits once realised, i.e. the split between households and the government.

[26]The 'crowding in' effect occurs through households anticipating lower dividends from the banking system when old due to dilution of their equity stakes, inducing them to save more to better spread consumption.

So

$$x_g = \frac{n_{gt}}{n_{gt} + n_t\left[\frac{(1-\lambda_i)\beta(1-\pi)}{(1+\beta\lambda_i)\pi}\right]},$$

and $\gamma = \frac{(1-\lambda_i(n_t))\beta(1-\pi)}{(1+\beta\lambda_i(n_t))\pi}$. In this case, the share is not based on the net worth the bank currently has, but the discounted value of what their lifetime profits. This is the value households place on the bank. Under this scheme, if the bank has current net worth of 100, but discounted profits of 400, and the government invests 100, they end up owning 20% of the banking system.

With this general form (41), we can re-write equilibrium deposits (with details in the Appendix) in a way to make the effect of policy comparable to direct and discount window lending. Combined with the law of motion for capital $k_{t+1} = x_i(n_t + n_{g,t} + d_{i,t})$, this gives

$$k_{t+1} = x_i\left(\frac{\pi + \lambda_i(n_t)\beta}{1+\lambda_i(n_t)\beta}w_t\right) + x_i d_{g,t}\left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta}\right] \tag{42}$$

$$+ x_i\frac{\lambda_i(\hat{n}_t) - \lambda_i(n_t)}{[1+\lambda_i(\hat{n}_t)\beta](1+\lambda_i(n_t)\beta)}[w_t\beta(1-\pi) - d_{g,t}(1+\beta)]$$

$$+ \frac{x_i d_{g,t}\lambda_i(\hat{n}_t)[1-\lambda_i(\hat{n}_t)][\beta(1-\pi)w_t - (1+\beta)d_{g,t}]}{(1+\tau_{gn})[1+\beta\lambda_i(\hat{n}_t)]\left[\frac{d_{g,t}}{1+\tau_{gn}}(1+\beta)\lambda_i(\hat{n}_t) + (1+\beta\lambda_i(\hat{n}_t))\gamma n_t\right]}, \tag{43}$$

where:

$$\hat{n}_t \equiv n_t + \frac{d_{g,t}}{1+\tau_{gn}}.$$

Written in this form, we can see the separate effects of the recapitalisation. As usual, the first term captures what k_{t+1} would have been absent policy, with the second term capturing the trade off between the crowding out effect and direct investment in the economy (the extra equity is automatically invested). The third term is new, capturing the 'crowding in of depositors', representing the fact that the recapitalisation increases λ_i, which induces more deposits to flow into the banking system. As $w_t\beta(1-\pi) - d_{g,t}(1+\beta) > 0$ this term is positive. Finally, the fourth term captures the impact of watering down households' equity stakes, which also draws deposits into the banking system.

With direct and discount window lending, the effect of policy is linear in the amount of government borrowing $d_{g,t}$. This is not the case here, making it more difficult to establish when policy is effective. Rather, we focus on the marginal impact when $d_{g,t} = 0$, i.e. $\{dk_{t+1}/d(d_{g,t})\}_{d_{g,t}=0}$. The following lemma provides the required condition for a bank recapitalisation to be effective when the economy is in a credit trap, investing in sector B (with proof in the Appendix).

Lemma 5 *With bank recapitalisation, the marginal effect of policy in a credit trap at* $d_{g,t} = 0$ *is positive (i.e.* $\{dk_{t+1}/d(d_{g,t})\}_{d_{g,t}=0} > 0$*) iff*

$$\tau_{gn} < \frac{1-\lambda_B(n_t)}{\lambda_B(n_t)(1+\beta)}\left[1+\frac{\left[n_t\lambda_B'(n_t)+\frac{\lambda_B(n_t)(1-\lambda_B(n_t))}{\gamma}\right][w_t\beta(1-\pi)]}{(1-\lambda_B(n_t))(1+\lambda_B(n_t)\beta)n_t}\right].$$

Note this is of a similar form as (38) and (40) with the addition of two positive terms, the first due to $\lambda_B'(n_t) > 0$, representing the crowding in of depositors, the second the watering down of shareholders (this second effect disappears when $x_g = 0$ (which can be seen as $\gamma \to \infty$), in which case households are not watered down).

Unlike the two prior policies, with a bank recapitalisation it need not be the case that the effectiveness of policy decreases uniformly as the economy recovers. In particular, if $\lambda_i(.)$ has a convex region, a recapitalisation will be particularly effective in this region, resulting in a large increase in bank leverage. However under certain conditions, when the economy is sufficiently healthy (λ_i is sufficiently large) the impact of policy decreases as the economy recovers further. Hence, if the marginal impact is negative when $d_{g,t} = 0$, policy will reduce k_{t+1} for all positive $d_{g,t}$. With this we can provide conditions under which bank recapitalisation reduces k_{t+1} in the good steady state when investment flows to sector A. This is summarised in the following lemma.

Lemma 6 *Let net worth in the good steady state of the economy be* $\overline{n}$ *where the economy invests in sector* A*. Suppose*

$$\lambda_A''(\overline{n}) < \frac{2\beta\left[\lambda_A'(\overline{n})\right]^2}{(1+\lambda_A(\overline{n})\beta)}, \tag{44}$$

and

$$\lambda_A(\overline{n}) > \frac{-1+\sqrt{1+\beta(2+\beta)}}{\beta(2+\beta)}. \tag{45}$$

Then $dk_{t+1}/dd_{g,t}$ *is maximised at* $d_{g,t} = 0$*. Further, if*

$$\tau_{gn} > \frac{1-\lambda_A(\overline{n})}{\lambda_A(\overline{n})(1+\beta)}\left[1+\frac{\left[n_t\lambda_A'(\overline{n})+\frac{\lambda_A(\overline{n})(1-\lambda_A(\overline{n}))}{\gamma}\right][w_t\beta(1-\pi)]}{(1-\lambda_A(\overline{n}))(1+\lambda_A(\overline{n})\beta)\overline{n}}\right],$$

then, in the good steady state, bank recapitalisation lowers k_{t+1} *for all* $d_{g,t} > 0$.

Remark 3 $\frac{-1+\sqrt{1+(\beta(2+\beta))}}{\beta(2+\beta)} < \frac{1}{2}$.

Remark 4 *A sufficient condition for (44) holding is* $\lambda_A''(\overline{n}) < 0$*, that is, in the good steady state of the economy, the increase of* λ_A *in banking system net worth happens at a decreasing rate.*

In summary, for bank recapitalisation, as with the other two policies, it can be effective in raising k_{t+1} when the economy is in bad health, but ineffective (lowering k_{t+1}) when the economy recovers.

5.6 Comparison of policies

We have shown that all three policies can be effective in raising k_{t+1} on impact during a credit trap. Here we compare the effectiveness of these, asking which delivers the largest increase in k_{t+1} for a given amount of spending $d_{g,t}$.

5.6.1 Case (i) $\tau_m \geq \tau_g \geq \tau_{gn}$

We first suppose that the inefficiencies in direct lending are at least as great as those with bank recapitalisation, and those with discount window lending are at least as great as those with direct lending. Here we have a clear prediction about the relative effectiveness of the policies.

Proposition 6 *Suppose* $\tau_m \geq \tau_g \geq \tau_{gn}$*, then for common* $d_{g,t}$[27]

$$k_{t+1}^{equity} > k_{t+1}^{direct} > k_{t+1}^{discount}.$$

Further, if discount window lending raises k_{t+1} *then so does direct lending, though the reverse is not true. If direct lending raises* k_{t+1}*, then so too does recapitalisation, though the reverse is not true.*[28]

[27]This is for feasible $d_{g,t}$ i.e. those less than the total amount households want save via deposits and government bonds. Note that the result does not depend on the specific γ used in the equity pricing rule.

[28]An alternative characterisation of the result is that under these conditions the required amount of government bonds needed to be raised to escape the trap will be lowest for bank recapitalisation and highest for discount window lending.

Proof. For the first part of the proof, from the above formulas it's clear we need to establish that

$$\frac{1}{(1+\tau_{gn})} + \frac{\lambda_i(1-\lambda_i)\left[\beta(1-\pi)w_t - (1+\beta)d_{g,t}\right]}{(1+\tau_{gn})(1+\beta\lambda_i)\left[\frac{d_{g,t}}{(1+\tau_{gn})}(1+\beta)\lambda_i + (1+\beta\lambda_i)\gamma n_t\right]}$$
$$+ \frac{\left[\lambda_i\left(n_t + \frac{d_{g,t}}{1+\tau_{gn}}\right) - \lambda_i(n_t)\right]}{\left(1+\lambda_i\left(n_t + \frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)(1+\lambda_i(n_t)\beta)}\left[w_t\beta(1-\pi) - d_{g,t}(1+\beta)\right]$$
$$> \frac{1}{1+\tau_g} > \frac{1-(1-\omega_g)\left(\frac{(1-\lambda_i)}{(1+\lambda_i\beta)}\right)}{(1+\tau_m)}.$$

The first inequality clearly follows from $\beta(1-\pi)w_t > (1+\beta)d_{g,t}$ and $\tau_g \geq \tau_{gn}$. The second inequality follows from $\tau_m \geq \tau_g$ and $\omega_g < 1$.

For the second part of the proof, we first need to establish that

$$\tau_m < w_g\frac{(1-\lambda_i)}{\lambda_i(1+\beta)} \Rightarrow \tau_g < \frac{1-\lambda_i}{\lambda_i(1+\beta)}.$$

This is clear as then $\tau_g \leq \tau_m < w_g\frac{(1-\lambda_i)}{\lambda_i(1+\beta)} < \frac{(1-\lambda_i)}{\lambda_i(1+\beta)}$. It is clear that the reverse implication does not hold as $w_g < 1$.

For the second, suppose that direct lending is effective:

$$\tau_g < \frac{1-\lambda_i}{\lambda_i(1+\beta)}.$$

Then $\tau_{gn} \leq \tau_g < \frac{1-\lambda_i}{\lambda_i(1+\beta)}$ so $\left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta}\right] > 0$. From (42) its clear that-as the other two terms are positive-k_{t+1} is raised with bank recapitalisation. It is clear that the reverse implication does not hold. This completes the proof. ■

We've shown that if the inefficiencies are the same for the three policies, bank recapitalisation will raise k_{t+1} the most, with discount window lending raising it the least. Further, the recapitalisation will be effective in raising k_{t+1} for the largest range of states of the economy (i.e. the largest range of λ_i) and discount window lending the smallest range of states of the economy. Thus, in a mild banking crisis, it may be that discount window and direct lending are ineffective, but the recapitalisation is still effective.

The reason for these differences is intuitive. All three policies crowd out deposits in a similar way through the issuance of government bonds. With direct lending, the money raised is invested directly into the economy without any frictions. This is more effective than discount window lending when $\omega_g < 1$ because then the central bank still faces a friction when lending to banks, resulting in a smaller increase in output than the amount invested. Thus, if discount window lending is at least as inefficient as direct lending ($\tau_m \geq \tau_g$) direct lending will be more effective. The recapitalisation resembles direct lending in that the amount invested directly adds to the capital stock. This is because it shows up as bank equity, so no financial friction is faced by the government, unlike with discount window lending. In addition, by raising λ_i directly, depositors are crowded in. A further positive impact from the recapitalisation arises from the watering down of households' bank equity stakes. These last two effects both result in more deposits, and a higher k_{t+1}. Thus, when direct lending is at least as inefficient as bank recapitalisation ($\tau_g \geq \tau_{gn}$) k_{t+1} will be higher with bank recapitalisation .

We next show that when the inefficiencies do not follow the order $\tau_m \geq \tau_g \geq \tau_{gn}$, which policy is most effective can depend on the state of the economy.

5.6.2 Case (ii): discount window lending most efficient $\tau_g, \tau_{gn} > \tau_m$

Here we consider the case in which discount window lending is inherently less inefficient to implement than the other two policies. It is plausible that this could be the case because discount window lending is closer in line with the specialities of a central bank/government, compared to equity investing or originating loans directly to the private sector. We first consider the case in which discount window lending is more efficient than direct lending, i.e. $\tau_m < \tau_g$. We have:

Proposition 7 *Suppose*

$$\tau_m < \tau_g - (1+\tau_g)(1-\omega_g)\frac{(1-\lambda_i)}{(1+\lambda_i\beta)}.$$

Then discount window lending is more effective in raising k_{t+1} than direct lending.

Proof. Discount window lending is more effective in raising k_{t+1} than direct lending when

$$\frac{1-(1-\omega_g)\left(\frac{(1-\lambda_i)}{(1+\lambda_i\beta)}\right)}{(1+\tau_m)} > \frac{1}{1+\tau_g} \text{ iff}$$
$$\tau_g - (1+\tau_g)(1-\omega_g)\left(\frac{(1-\lambda_i)}{(1+\lambda_i\beta)}\right) > \tau_m.$$

■

We note that the left-hand side of this is increasing in λ_i and so could hold for a large λ_i and fail for a small λ_i. We thus have a corollary:

Corollary 1 *Consider two credit crunches with associated banking system net worth* n_1, n_2 *with* $n_1 > n_2$, *so* n_2 *is the more severe credit crunch. Suppose*

$$\tau_g - (1+\tau_g)(1-\omega_g)\left(\frac{(1-\lambda_i(n_1))}{(1+\lambda_i(n_1)\beta)}\right) > \tau_m,$$
$$\tau_g - (1+\tau_g)(1-\omega_g)\left(\frac{(1-\lambda_i(n_2))}{(1+\lambda_i(n_2)\beta)}\right) < \tau_m.$$

Then direct lending is more effective in raising k_{t+1} *in the more severe credit crunch* (n_2), *whilst discount window lending is more effective in the milder credit crunch* (n_1).

Proof. Immediate. ■

The corollary highlights an interesting trade-off that can arise. With a mild shock to the banking system, discount window lending can be more effective due to the lower inherent inefficiency it involves (resulting in fewer crowded-out deposits). But with a sufficiently severe shock to the banking system, λ_i will be low and this policy will be less effective. This is because this policy must work *through* the banking system, and when the health of intermediary balance sheets is impaired, the central bank also faces a large credit friction when lending to these firms. Here, circumventing the banking system, and lending directly to the economy can be more effective.

Consider a similar case in which discount window lending is inherently more efficient than bank recapitalisation, i.e. $\tau_m < \tau_{gn}$. Here we also note that discount window lending can be more effective in a mild downturn, with bank recapitalisation more effective in a more severe banking crisis.

Proposition 8 *Suppose* $\omega_g < \frac{1+\beta}{2+\beta}$ *and we have the second equity pricing rule in which the government's equity share is based on the PDV of the banks.*[29] *Consider two credit crunches with associated banking system net worth* n_1, n_2 *with* $n_1 > n_2$, *so* n_2 *is the more severe crunch.*

[29] That is, $\gamma = \frac{(1-\lambda_i)\beta(1-\pi)}{(1+\beta\lambda_i)\pi}$. The exact form of γ does not matter for the result, only simplifies the exposition.

Suppose (44) and (45) hold for n_1, and further that

$$\frac{(1+\tau_{gn})\left[(1+\beta\lambda_i(n_1))-(1-\omega_g)(1-\lambda_i(n_1))\right]}{\left[1+(1+\beta)\lambda_i(n_1)+\frac{\lambda_i'(n_1)w_t\beta(1-\pi)}{(1+\lambda_i(n_1)\beta)}\right]}-1>\tau_m,$$

$$\frac{(1+\tau_{gn})\left[(1+\beta\lambda_i(n_2))-(1-\omega_g)(1-\lambda_i(n_2))\right]}{\left[1+(1+\beta)\lambda_i(n_2)+\frac{\lambda_i'(n_2)w_t\beta(1-\pi)}{(1+\lambda_i(n_2)\beta)}\right]}-1<\tau_m.$$

Then bank recapitalisation is more effective in raising k_{t+1} in the more severe credit crunch (n_2), for a range of $d_{g,t}>0$, whilst discount window lending is more effective in the milder credit event (n_1) for all $d_{g,t}>0$.

Remark 5 *The $\omega_g<\frac{1+\beta}{2+\beta}$ condition is required so that the impact of discount window lending closely follows the health of the economy.*

Proof. See Appendix. ■

In the more severe crunch, discount window lending is less effective as it has to work through the banking system, and with low λ_i, the fraction of government lending that makes it through to the real economy is limited. By contrast, bank recapitalisation directly boosts output as the equity is directly invested in sector i. Further, the increase in λ_i can have a large positive impact on k_{t+1}, crowding in depositors. These large positive benefits outweigh the greater inherent inefficiency associated with bank recapitalisation. In a less severe crunch, the benefit from increasing λ_i will not be as large, and with higher λ_i, discount window lending will become relatively more effective. Consequently, the lower inefficiency of this type of policy can result in it being more effective overall.

5.7 Summary

When the inefficiencies of the three policies are equal, we have a clear ranking in terms of the effectiveness of raising output in the economy: bank recapitalisation is the most effective policy whilst discount window lending is the least effective. When discount window lending is more efficient than the other two policies, it can be more effective in a milder banking crisis, but less effective than direct lending and bank recapitalisation in a severe banking crisis.[30] Table 1 in the Introduction summarises these results.

[30] As mentioned above, bank recapitalisation may be undesirable as it could create ex ante moral hazard, though this channel is not captured in our model.

The effectiveness of the unconventional credit policies studied here depends on the state of the economy. While they can be highly effective in a credit crunch, these policies could damage a healthy economy. The model naturally rejects the conclusion that it is desirable for the government to fully replace the banking sector in all states of the world. As is intuitive, interventions are most valuable when the economy is depressed and credit frictions are at their worst.

6 Concluding remarks

The recent financial crisis has raised the question of whether there is something fundamentally different about economic recovery following a severe financial crisis and, if so, how macroprudential policy tools should be used both before and after a crisis. Most modern macroeconomic models are unsuitable for addressing this question, with their economies quickly returning to health once a negative shock is unwound. In this context macroprudential policy tools play the role of reducing volatility, rather than avoiding a catastrophe or supporting the recovery from a crisis. By contrast, in this paper we explicitly consider a model in which the economy can become trapped in a steady state featuring permanently lower output, bank credit and productivity following a sufficiently severe financial shock, a confluence of characteristics we call a *credit trap*.

In this paper we have developed a simple, tractable OLG model for analysing credit traps. We have examined the effectiveness of policy both at preventing a credit trap occurring, and helping the economy to escape (which becomes necessary as it will not recover without intervention). Our analysis shows that a leverage ratio cap is effective in increasing the resilience of the economy against shocks and reducing the probability of a credit trap. However, this comes at the cost of lowering the level of output in the 'good' steady state, and hence the policymaker needs to set the cap to trade off these costs and benefits. Relaxing the leverage ratio cap is effective in encouraging faster recovery after a negative productivity shock, provided that the shock is sufficiently small. But if the shock is large enough to tip the economy into a credit trap, then relaxing the leverage ratio cap will not help the economy get out of it.

To escape a credit trap other policies are needed, and we consider the efficacy of a set of 'unconventional' credit policies: direct lending; bank recapitalisation; and discount window lending. These policies present rich, realistic trade-offs which vary with their relative efficiency costs. Their effectiveness depends on the state of the economy, with all more effective when the economy is weaker.

In future work, it would be interesting to analyse more thoroughly the optimal leverage cap that would be set by a policymaker in advance of a trap. We have shown that the level of the leverage cap that maximises resilience is countercyclical: it would be interesting to analyse numerically if the optimal level of the leverage cap is too, and whether this would vary with the state of the economy in a non-linear way. This would be particularly interesting when the economy is just above the trap threshold, and the policymaker has to trade-off rebuilding the health of the banking system with the possibility of further negative shocks.

TECHNICAL APPENDIX

A Bargaining over distressed assets: microfoundation for $\lambda_i(n_t)$

In this section we discuss a microfoundation for Assumption 2, that $\lambda'_A(n_t) > \lambda'_B(n_t) \geq 0$ based on a small change to the baseline model environment and the introduction of a bargaining problem between depositors and bankers.

A.1 Set up

At the beginning of period t, banks receive liquidity shocks which determine whether or not they are able to invest. With probability $\gamma \in (0, 1)$ a bank is able to invest in period t. With probability $1 - \gamma$, no investment opportunity arises until period $t + 1$. We refer to the banks that can invest in t as 'early types' and the banks that cannot invest until $t + 1$ as 'late types'.

If early types raise deposits and invest in period t, a return of $R_{i,t+1}$ per unit invested is realised in period $t + 1$. As a result, the total surplus to be divided between depositors and early types is $R_{i,t+1}(\gamma n_t + d_{i,t})$. Early types and depositors bargain over this surplus.

In period $t+1$, late types become able to invest. If they choose to make loans on their own, a fraction $\rho \in (0, 1)$ of their net worth is lost in establishing lending operations. The fraction ρ captures the idea that a bank needs to spend some resources in resuming its lending operations to a particular sector after a period of inactivity (e.g. it needs to reestablish mechanisms to assess borrowers' creditworthiness and to assess borrowers' collateral values). The fraction ρ can also be interpreted as the cost required to verify the project they are investing in. Given this cost, the surplus they earn when investing is $R_{i,t+1}(1 - \rho)(1 - \gamma)n_t$.

Finally, at the beginning of period $t + 1$, depositors face a choice. They can roll over their claims on early types' asset returns, or they can withdraw their funds and transfer them to late types. If they withdraw their funding they face a penalty, and only receive fraction $\delta \in (0, 1)$ of what they are owed when they remove their deposits. Withdrawing their deposits brings funding to late types together with knowledge of loan market collateral values, relieving late types of the need to spend ρn_t on discovering this information for themselves. Under this outcome, the total surplus is therefore $R_{i,t+1}[(1 - \gamma)n_t + \delta d_{i,t+1}]$. For late types to be willing to do this (ie to satisfy late types' participation constraints) it must be that the fraction of the surplus $\sigma \in (0, 1)$ that they receive satisfies:

$$\sigma R_{i,t+1}[(1 - \gamma)n_t + \delta d_{i,t}] \geq R_{i,t+1}(1 - \rho)(1 - \gamma)n_t$$

This leaves depositors with $(1 - \sigma)R_{i,t+1}[(1 - \gamma)n_t + \delta d_{i,t}]$. Solving for the σ that just satisfies late types' participation constraint gives depositors outside options of $R_{i,t+1}[\rho(1 - \gamma)n_t + \delta d_{i,t}]$.

A.2 Bargaining outcome

Now consider the bargaining problem between depositors and early types. Let depositors receive a fraction $\lambda \in (0, 1)$ of the surplus and have bargaining power $\theta \in (0, 1)$, while early types receive the remainder and have the complementary bargaining power. The bargaining solution solves:

$$\max_{\lambda} \pi_d^{\theta} \times \pi_b^{1-\theta}$$
$$\pi_d \equiv \lambda R_{i,t+1}(\gamma n_t + d_{i,t+1}) - R_{i,t+1}[\rho(1 - \gamma)n_t + \delta d_{i,t}]$$
$$\pi_b \equiv (1 - \lambda)R_{i,t+1}(\gamma n_t + d_{i,t+1}) - R_{i,t+1}\gamma n_t$$

The first-order condition is:

$$\frac{\theta}{1 - \theta} = \frac{\pi_d}{\pi_b}$$

Solving for the optimal λ, which we write as $\lambda^*(n_t)$ gives:

$$\lambda^*(n_t) = \frac{d_{i,t}\left[\theta + (1-\theta)\delta\right] + (1-\theta)\rho(1-\gamma)n_t}{\gamma n_t + d_{i,t}}$$

For $\lambda^*(n_t) < 1$, we require that $\gamma > \frac{\left[(1-\theta)\rho - \frac{d_{i,t}}{n_t}(1-\theta)(1-\delta)\right]}{(1+(1-\theta)\rho)}$. Then:

Lemma 7 *$\lambda^*(n_t)$ is increasing in n_t for*

$$\gamma < \frac{(1-\theta)\rho}{\left[\theta + (1-\theta)\delta\right] + (1-\theta)\rho}$$

Proof. The derivative of $\lambda^*(n_t)$ in n_t is

$$\lambda'(n_t) = \frac{(1-\theta)\rho(1-\gamma)d_{i,t} - d_{i,t}\left[\theta + (1-\theta)\delta\right]\gamma}{(\gamma n_t + d_{i,t})^2}$$

Thus $\lambda'(n_t) > 0$ for $(1-\theta)\rho(1-\gamma) > \left[\theta + (1-\theta)\delta\right]\gamma$ which can be rearranged to give the condition. ■

Further:

Lemma 8 *The sensitivity of $\lambda^*(n_t)$ to n_t is increasing in ρ.*

Proof. The second derivative is:

$$\frac{\partial^2 \lambda^*(n_t)}{\partial\rho\partial n_t} = \frac{(1-\theta)(1-\gamma)d_{i,t}}{(\gamma n_t + d_{i,t})^2} > 0$$

so a higher ρ makes $\lambda^*(n_t)$ more sensitive to bank net worth. ■

It follows that for $\rho_A > \rho_B$ we'll have $\lambda'_A(n_t) > \lambda'_B(n_t)$.

B Proof from Section 2: Model

B.1 Households

Lemma 9 *Households optimal saving is given by*

$$d_t = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{1}{1+\beta}\frac{V_{t+1}}{R_{d,t+1}} \tag{46}$$

Proof. The household problem is

$$\max_{c_{1t},c_{2t}} \log c_{1t} + \beta \log c_{2t} : c_{1t} + d_t \leq (1-\pi)w_t$$
$$c_{2t} \leq R_{d,t+1}d_t + V_{t+1}$$

Optimally both constraints will bind so the problem can be rewritten as: $\max_{d_t} \log((1-\pi)w_t - d_t) + \beta\log(R_{d,t+1}d_t + V_{t+1})$. With a strictly concave objective function, the FOC is sufficient for a global maximum. $FOC : \frac{-1}{(1-\pi)w_t - d_t} + \frac{\beta R_{d,t+1}}{R_{d,t+1}d_t + V_{t+1}} = 0$. Rearranging the FOC gives the result. ■

B.2 Deposit market equilibrium

We consider different cases here, beginning with a positive spread in equilibrium followed by zero spread. We then summarise the results.

B.2.1 Positive Spread: $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$

Lemma 10 *Suppose sector i is invested in. If $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$ then the equilibrium supply of deposits from households is given by*

$$d^*_{i,t} = \frac{\lambda_i \beta}{1+\lambda_i \beta}(1-\pi)w_t$$

Proof. When $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$ the pledgeability constraint holds with equality.[31] Thus

$$\lambda_i R_{i,t+1}(n_t + d_{i,t}) = R_{d,t+1} d_{i,t} \tag{47}$$

Rearranging this gives the deposit demand of banks:

$$d_{i,t} = \frac{\lambda_i R_{i,t+1} n_t}{R_{d,t+1} - \lambda_i R_{i,t+1}} \tag{48}$$

To calculate the deposit supply of households we must look at the lump sum transfer households receive from banks: $V_{i,t+1} := (R_{i,t+1} - R_{d\,t+1})d_{i,t} + R_{i,t+1} n_t = \frac{R_{i,t+1} n_t}{R_{d,t+1} - \lambda_i R_{i,t+1}} R_{d,t+1}(1-\lambda_i)$. Thus, from (46) deposit supply is given by

$$d_{i,t} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{1}{1+\beta}\frac{V_{i,t+1}}{R_{d,t+1}} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{1}{1+\beta}\frac{(1-\lambda_i)R_{i,t+1} n_t}{R_{d,t+1} - \lambda_i R_{i,t+1}} \tag{49}$$

In equilibrium of the deposit market, deposit supply (48) equals deposit demand (49), so

$$\frac{\lambda_i R_{i,t+1} n_t}{R_{d,t+1} - \lambda_i R_{i,t+1}} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{1}{1+\beta}\frac{(1-\lambda_i)R_{i,t+1} n_t}{R_{d,t+1} - \lambda_i R_{i,t+1}}$$

Given $n_t = \pi w_t$ this can be rearranged to give

$$\frac{R_{i,t+1}}{R_{d,t+1} - \lambda_i R_{i,t+1}} = \frac{\beta(1-\pi)}{\pi\left(\beta\lambda_i + 1\right)} \tag{50}$$

Hence, in equilibrium, when sector i is invested in

$$V_{i,t+1} = R_{d,t+1}(1-\lambda_i)w_t \frac{\beta(1-\pi)}{(\beta\lambda_i + 1)} \tag{51}$$

The equilibrium level of deposits can be found by substituting (50) into (49), giving the result. ■

Lemma 11 *In equilibrium with sector i invested in and $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$*

$$R^*_{d,t+1} = R_{i,t+1}\frac{\pi + \lambda_i \beta}{\beta(1-\pi)}$$

Proof. Given $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$,(47) holds so: $R_{d,t+1} = \lambda_i R_{i,t+1}\left(\frac{n_t}{d_{i,t}} + 1\right)$. From the prior lemma, using $n_t = \pi w_t$: $d^*_{i,t} = \frac{\lambda_i \beta}{1+\lambda_i \beta}(1-\pi)\frac{n_t}{\pi}$ so $\frac{n_t}{d^*_{i,t}} = \frac{(1+\lambda_i\beta)\pi}{\lambda_i\beta(1-\pi)}$. Thus $R^*_{d,t+1} = \lambda_i R_{i,t+1}\left(\frac{n_t}{d^*_{i,t}} + 1\right) = \frac{R_{i,t+1}}{\beta(1-\pi)}\left(\pi + \lambda_i\beta\right)$. ■

Corollary 2 *The above equilibrium indeed satisfies $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$ (so is consistent) if $\beta(1-\pi) > \pi + \lambda_i\beta$*

[31] $R_{i,t+1} > R_{d,t+1}$ ensures the bank takes as many deposits as they can. $R_{d\,,t+1} > \lambda_i R_{i,t+1}$ ensures that they are constrained by the pledgability constraint.

Proof. The condition ensures that $\frac{\pi+\lambda_i\beta}{\beta(1-\pi)} < 1$ and so $R^*_{d,t+1} < R_{i,t+1}$.

For the second inequality $R_{d,t+1} > \lambda_i R_{i,t+1}$ iff $\frac{R_{i,t+1}}{\beta(1-\pi)}(\pi + \lambda_i\beta) > \lambda_i R_{i,t+1}$ iff $\pi(1+\beta\lambda_i) > \beta\lambda_i - \beta\lambda_i = 0$. This always holds. ■

Lemma 12 *If $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$, and sector i is invested in, then*

$$k_{t+1} = x_i \frac{\pi + \lambda_i\beta}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha$$

Proof. The amount of capital produced next period is given by $k_{t+1} = x_i(n_t + d^*_{i,t})$. When $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$, $d^*_{i,t} = \frac{\lambda_i\beta}{1+\lambda_i\beta}(1-\pi)w_t$. Thus $k_{t+1} = x_i(\pi w_t + \frac{\lambda_i\beta}{1+\lambda_i\beta}(1-\pi)w_t) = x_i\frac{w_t}{1+\lambda_i\beta}\left(\pi\left(1+\lambda_i\beta\right) + \lambda_i\beta(1-\pi)\right) = x_i\frac{(1-\alpha)k_t^\alpha}{1+\lambda_i\beta}\left(\pi + \lambda_i\beta\right)$, where we have used $w_t = (1-\alpha)k_t^\alpha$ given Cobb-Douglas technology. ■

Lemma 13 *If $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$ and sector i is invested in then*

$$R_{i,t+1} = \frac{\alpha x_i^\alpha}{\left[\frac{\pi+\lambda_i\beta}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}$$

$$R_{d,t+1} = \frac{\alpha x_i^\alpha(1+\lambda_i\beta)^{1-\alpha}(\pi+\lambda_i\beta)^\alpha}{\beta(1-\pi)\left[(1-\alpha)k_t^\alpha\right]^{1-\alpha}}$$

Proof. We assume full depreciation of capital during output production for tractability so $R_{i,t+1} = x_i f'(k_{t+1}) = \frac{\alpha x_i}{k_{t+1}^{1-\alpha}}$.This expression gives the gross return on output goods invested in sector i. Each unit of output goods invested produces x_i units of capital goods next period, each of which earns the return to capital from output, which is the marginal product of capital. Using the prior lemma: $R_{i,t+1} = \frac{\alpha x_i}{\left[x_i\frac{(1-\alpha)k_t^\alpha}{1+\lambda_i\beta}(\pi+\lambda_i\beta)\right]^{1-\alpha}} = \frac{\alpha x_i{}^\alpha}{\left[\frac{(\pi+\lambda_i\beta)}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}$. For the deposit rate expression, note that from a prior lemma, given $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$ we have $R^*_{d,t+1} = R_{i,t+1}\frac{\pi+\lambda_i\beta}{\beta(1-\pi)} = \frac{\alpha x_i^\alpha}{\left[\frac{(\pi+\lambda_i\beta)}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}\left(\frac{\pi+\lambda_i\beta}{\beta(1-\pi)}\right) = \frac{\alpha x_i^\alpha(\pi+\lambda_i\beta)^\alpha(1+\lambda_i\beta)^{1-\alpha}}{\beta(1-\pi)[(1-\alpha)k_t^\alpha]^{1-\alpha}}$ ■

B.2.2 Zero Spread: $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}$

We establish an analogous series of results to the positive spread case.

Lemma 14 *Suppose sector i is invested in. If $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}$ then the equilibrium supply of deposits from households is given by*

$$d^*_{i,t} = \frac{w_t}{1+\beta}\left(\beta(1-\pi) - \pi\right)$$

Proof. *When $R_{i,t+1} = R_{d,t+1}$, $V_{i,t+1} = R_{i,t+1}n_t = R_{d,t+1}n_t$. Thus, from (46) $d_{i,t} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{1}{1+\beta}\frac{R_{d,t+1}n_t}{R_{d,t+1}} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{\pi w_t}{1+\beta} = \frac{w_t}{1+\beta}\left(\beta(1-\pi)-\pi\right)$.* ■

Lemma 15 *The above equilibrium indeed satisfies $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}$ (so is consistent) if $\beta(1-\pi) \leq \pi + \beta\lambda_i$.*

Proof. The pledgeability constraint requires that $\lambda_i R_{i,t+1}(n_t + d_{i,t}) \geq R_{d,t+1}d_{i,t}$.With zero spread, this becomes $\lambda_i(n_t + d_{i,t}) \geq d_{i,t}$ or $\lambda_i n_t \geq d_{i,t}(1-\lambda_i)$. From the prior lemma, we must have $\lambda_i n_t \geq \frac{w_t}{1+\beta}\left(\beta(1-\pi)-\pi\right)(1-\lambda_i)$ this holds iff $\lambda_i\pi \geq \frac{1}{1+\beta}\left(\beta(1-\pi)-\pi\right)(1-\lambda_i)$ iff $\pi + \beta\lambda_i \geq \beta\left(1-\pi\right)$. ■

Lemma 16 *If $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}$, and sector i is invested in, then*

$$k_{t+1} = \frac{x_i \beta}{1+\beta}(1-\alpha)k_t^\alpha$$

Proof. The amount of capital produced next period is given by the product of the amount of output goods invested and the realised level of technology $x_i : k_{t+1} = x_i(n_t + d^*_{i,t})$. When $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}, d^*_{i,t} = \frac{w_t}{1+\beta}(\beta(1-\pi) - \pi)$. Thus $k_{t+1} = x_i(\pi w_t + \frac{w_t}{1+\beta}(\beta(1-\pi) - \pi)) = \frac{x_i w_t}{1+\beta}(\pi(1+\beta) + (\beta(1-\pi) - \pi)) = \frac{x_i \beta}{1+\beta}(1-\alpha)k_t^\alpha$. ■

Lemma 17 *If $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}$ and sector i is invested in then*

$$R_{i,t+1} = \frac{\alpha x_i^\alpha}{\left[\frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}$$

$$R_{d,t+1} = R_{i,t+1} = \frac{\alpha x_i^\alpha}{\left[\frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}$$

Proof. We assume full depreciation of capital during output production for tractability so $R_{i,t+1} = \frac{\alpha x_i}{k_{t+1}^{1-\alpha}}$. Using the prior lemma: $R_{i,t+1} = \frac{\alpha x_i{}^\alpha}{\left[\frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}$. As there is zero spread, the gross return from investment in sector i is equal to the deposit rate and so $R_{d,t+1} = R_{i,t+1} = \frac{\alpha x_i^\alpha}{\left[\frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}}$. ■

B.2.3 Other Potential Cases

So far we have considered two mutually exclusive cases: $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$ and $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}$. We now consider other possible cases.

Lemma 18 *In any equilibrium we must have $R_{d,t+1} > \lambda_i R_{i,t+1}$*

Proof. Suppose this doesn't hold, then we have $R_{i,t+1} > \lambda_i R_{i,t+1} \geq R_{d,t+1}$. The pledgeability constraint requires that $\lambda_i R_{i,t+1}(n_t + d_{i,t}) \geq R_{d,t+1} d_{i,t}$. This always holds in this case as $\lambda_i R_{i,t+1}(n_t + d_{i,t}) \geq R_{d,t+1}(n_t + d_{i,t}) \geq R_{d,t+1} d_{i,t}$. Hence, in this case the constraint is satisfied for all $d_{i,t}$. Further, as $0 < \lambda_i < 1$ there is a positive spread and so the bank wants to take as many deposits as possible. Thus, optimally it sets $d_{i,t} = \infty$, which cannot be an equilibrium as there is a finite amount of potential deposits from households. ■

Lemma 19 *Suppose $\beta(1-\pi) \geq \pi$. Then in any equilibrium we must have*

$$R_{i,t+1} \geq R_{d,t+1}$$

The condition is the same condition that ensures that in the case of zero spreads, the households want to make non-negative deposits. This is not trivial in the model as the households can consume in the second period even if they don't make deposits, due to their equity stake in the bank which is paid out in the second period of their life.

Proof. Suppose this condition does not hold. Then $R_{i,t+1} < R_{d,t+1}$ and the banks lose money on every unit of deposits taken. Optimally they thus set $d_{i,t} = 0$. This fails to be an equilibrium if the households want to make deposits at these prices. Given the banks set $d_{i,t} = 0$, it follows that $V_{i,t} = n_t R_{i,t+1}$ with bank returns just coming from them trading on their own account. From (46) we then have $d_{i,t} = \frac{\beta}{1+\beta}(1-\pi)w_t - \frac{1}{1+\beta}\frac{\pi w_t R_{i,t+1}}{R_{d,t+1}} = \frac{w_t}{1+\beta}\left(\beta(1-\pi) - \frac{\pi R_{i,t+1}}{R_{d,t+1}}\right) > \frac{w_t}{1+\beta}(\beta(1-\pi) - \pi) \geq 0$ so $d_{i,t} > 0$.Under these conditions we do not have an equilibrium as deposit supply is greater than deposit demand. ■

B.2.4 Summary For sector i

We now establish a summary proposition for the deposit market equilibrium.

Proposition 9 *Suppose $\beta(1-\pi) \geq \pi$. Then in equilibrium in the deposit market we have*

$$R_{i,t+1} \geq R_{d,t+1} > \lambda_i R_{i,t+1}$$

There are two cases:

(i) If $\beta(1-\pi) > \pi + \lambda_i \beta$ then $R_{i,t+1} > R_{d,t+1} > \lambda_i R_{i,t+1}$ and the unique equilibrium is given by

$$\begin{aligned} d^*_{i,t} &= \frac{\lambda_i \beta}{1+\lambda_i \beta}(1-\pi)w_t \\ R_{i,t+1} &= \frac{\alpha x_i^\alpha}{\left[\frac{\pi+\lambda_i\beta}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}} \\ R_{d,t+1} &= \frac{\alpha x_i^\alpha (1+\lambda_i\beta)^{1-\alpha}(\pi+\lambda_i\beta)^\alpha}{\beta(1-\pi)\left[(1-\alpha)k_t^\alpha\right]^{1-\alpha}} \\ k_{t+1} &= x_i \frac{\pi+\lambda_i\beta}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha \end{aligned}$$

(ii) If $\beta(1-\pi) \leq \pi + \lambda_i\beta$ then $R_{i,t+1} = R_{d,t+1} > \lambda_i R_{i,t+1}$ and the unique equilibrium is given by

$$\begin{aligned} d^*_{i,t} &= \frac{w_t}{1+\beta}\left(\beta(1-\pi)-\pi\right) \\ R_{d,t+1} = R_{i,t+1} &= \frac{\alpha x_i^\alpha}{\left[\frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha\right]^{1-\alpha}} \\ k_{t+1} &= \frac{x_i\beta}{1+\beta}(1-\alpha)k_t^\alpha \end{aligned}$$

Proof. From the prior lemmas with $\beta(1-\pi) \geq \pi$ we have $R_{i,t+1} \geq R_{d,t+1} > \lambda_i R_{i,t+1}$. As shown above, when $\beta(1-\pi) > \pi + \lambda_i\beta$ we have an equilibrium with a positive spread and no equilibrium with a zero spread. Further, when $\beta(1-\pi) \leq \pi + \lambda_i\beta$ we have an equilibrium with zero spread and no equilibrium with a positive spread. ■

Corollary 3 *We have a positive spread in sector i if*

$$\lambda_i < \frac{\beta(1-\pi)-\pi}{\beta}$$

In particular, we are guaranteed a positive spread in both sectors in all states of the economy if

$$\begin{aligned} \lambda_B &< \frac{\beta(1-\pi)-\pi}{\beta} \\ \overline{\lambda}_A &< \frac{\beta(1-\pi)-\pi}{\beta} \end{aligned}$$

where $\overline{\lambda}_A$ is the maximum value $\lambda_A(n_t)$ takes.

Proof. This is immediate from the previous proposition. ■

B.3 Sector Invested In

In our specification, depositors dictate the sector that is invested in, based on which will pay a higher return to them. For this to be an equilibrium we require that bankers prefer to do this than take no deposits and invest in the other sector. Here we examine conditions that ensure the banks have no incentive to deviate from the derived equilibrium.[32]

Lemma 20 *Suppose $\beta(1-\pi) > \pi + \lambda_i\beta$ $(i = A, B)$ so that there would be positive spreads in both sectors were they invested in. Further, suppose that*

$$x_B(1-\lambda_B)\left(\frac{\pi+\lambda_B}{1+\beta\lambda_B}\right) \geq \pi x_A$$

Then the banks invest in sector A iff $R^A_{d,t+1} > R^B_{d,t+1}$. Here the banks always take deposits and invest in the sector the depositors want rather than taking no deposits and investing by themselves.

Proof. Under the given conditions, there is a positive spread when both sectors are invested in. Thus $V_{i,t+1} = R_{d,t+1}(1-\lambda_i)w_t\frac{\beta(1-\pi)}{(\beta\lambda_i+1)}$ and $R_{d,t+1} = \frac{\alpha x_i^\alpha(1+\lambda_i\beta)^{1-\alpha}(\pi+\lambda_i\beta)^\alpha}{\beta(1-\pi)[(1-\alpha)k_t^\alpha]^{1-\alpha}}$. Combining these gives equilibrium bank profits when sector i is invested in *and deposits are taken:* $V^*_{i,t+1} = \alpha x_i^\alpha(1-\lambda_i)\left(\frac{(\pi+\lambda_i\beta)}{(1+\beta\lambda_i)}\right)^\alpha((1-\alpha)k_t^\alpha)^\alpha$. Consider the bank profits that one deviating bank would make if they switched to investment in sector $j \neq i$, taking no deposits: $V^{*nd}_{j,t+1} = R_{j,t+1}n_t = \frac{\alpha x_j n_t}{k_{t+1}^{1-\alpha}}$. *Crucially* as the deviating bank is infinitesimal, the total capital next period is unaltered: it is the level of investment in capital from sector i that determines returns next period. Expected capital next period is given by $k_{t+1} = x_i\frac{\pi+\lambda_i\beta}{1+\lambda_i\beta}(1-\alpha)k_t^\alpha$. Thus $V^{*nd}_{j,t+1} = \frac{\alpha x_j\pi((1-\alpha)k_t^\alpha)^\alpha}{\left(x_i\frac{\pi+\lambda_i\beta}{1+\lambda_i\beta}\right)^{1-\alpha}}$. We consider two cases: (i) $R^A_{d,t+1} > R^B_{d,t+1}$ Then a potential deviating bank chooses not to deviate iff $V^*_{A,t+1} \geq V^{*nd}_{B,t+1}$ iff $\alpha x_A^\alpha(1-\lambda_A)\left(\frac{(\pi+\lambda_A\beta}{1+\lambda_A\beta}\right)^\alpha((1-\alpha)k_t^\alpha)^\alpha \geq \frac{\alpha x_B\pi((1-\alpha)k_t^\alpha)^\alpha}{\left(x_A\frac{\pi+\lambda_A\beta}{1+\lambda_A\beta}\right)^{1-\alpha}}$ iff $x_A(1-\lambda_A)\frac{\pi+\lambda_A\beta}{1+\lambda_A\beta} \geq x_B\pi$. As $\beta(1-\pi) > \pi+\lambda_A\beta$ we have $(1-\lambda_A)\frac{\pi+\lambda_A\beta}{1+\lambda_A\beta} > \pi$. Thus we have that $x_A(1-\lambda_A)\frac{\pi+\lambda_A\beta}{1+\lambda_A\beta} > x_A\pi > x_B\pi$. Hence $V^*_{A,t+1} > V^{*nd}_{B,t+1}$. Note we do not need the condition for this to hold. The intuition in this case is simple: when investing in sector A there are higher gross returns on each unit (given that $x_A > x_B$ and *there is the same amount of capital next period in both cases*) and more units are invested as deposits are taken. Further, there is a positive spread, so profit is made on each extra deposit taken and invested.

We now consider the other case: (ii) $R^A_{d,t+1} < R^B_{d,t+1}$ Here depositors want the bank to invest in sector B. It is optimal for a bank to not deviate from this iff $V^*_{B,t+1} \geq V^{*nd}_{A,t+1}$ iff $x_B(1-\lambda_B)\frac{\pi+\lambda_B\beta}{1+\lambda_B\beta} \geq x_A\pi$.which holds given the condition in the lemma. In this case we need a condition as there is a trade off for the banks: they get a higher gross return on each unit when investing in A, but they invest a greater volume when investing in B. If this volume is great enough and the profit margin is too, it is optimal for the bank to take deposits and invest in B. ■

Lemma 21 *Suppose $\beta(1-\pi) > \pi + \lambda_i\beta$ $(i = A, B)$, then $R^A_{d,t+1} > R^B_{d,t+1}$ iff*

$$x_A^\alpha(1+\lambda_A\beta)^{1-\alpha}(\pi+\lambda_A\beta)^\alpha > x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$$

Proof. Given the above conditions both sectors will have positive spreads were they invested in. Thus $R^i_{d,t+1} = \frac{\alpha x_i^\alpha(1+\lambda_i\beta)^{1-\alpha}(\pi+\lambda_i\beta)^\alpha}{\beta(1-\pi)[(1-\alpha)k_t^\alpha]^{1-\alpha}}$. Hence $R^A_{d,t+1} > R^B_{d,t+1}$ iff $x_A^\alpha(1+\lambda_A\beta)^{1-\alpha}(\pi+\lambda_A\beta)^\alpha > x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$. ■

[32]Note: given $(1-\pi)\beta > \pi$ so that households always wish to make deposits, the cases of banks not taking deposits are not equilibria. The work here verifies that our proposed equilibria are indeed equilibria.

Lemma 22 *Suppose* $x_B(1-\lambda_B)\left(\frac{\pi+\lambda_B}{1+\beta\lambda_B}\right) \geq \pi x_A$ *and* $\beta(1-\pi) > \pi + \lambda_i\beta$ $(i = A, B)$*. Further, suppose that*

$$\lambda'_A(n_t) > 0 \ \forall n_t \geq 0;$$
$$x_A > x_B;$$
$$\lambda_A(0) = \underline{\lambda}_A \in [0, \lambda_B);$$
$$\lim_{n_t\to\infty} \lambda_A(n_t) = \overline{\lambda}_A \in (\underline{\lambda}_A, 1);$$
$$x_A^\alpha(1+\underline{\lambda}_A\beta)^{1-\alpha}(\pi+\underline{\lambda}_A\beta)^\alpha < x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha;$$
$$x_A^\alpha(1+\overline{\lambda}_A\beta)^{1-\alpha}(\pi+\overline{\lambda}_A\beta)^\alpha > x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$$

Then there exists a unique level of banker net worth $\widetilde{n}$ *:bankers invest in A iff* $n_t > \widetilde{n}$[33]*. This is defined implicitly by*

$$x_A^\alpha(1+\lambda_A(\widetilde{n})\beta)^{1-\alpha}(\pi+\lambda_A(\widetilde{n})\beta)^\alpha = x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$$

Proof. With the given conditions $R^i_{d,t+1} = \frac{\alpha x_i^\alpha(1+\lambda_i\beta)^{1-\alpha}(\pi+\lambda_i\beta)^\alpha}{\beta(1-\pi)[(1-\alpha)k_t^\alpha]^{1-\alpha}}$, and banks invest in sector A rather than sector B iff $R^A_{d,t+1} > R^B_{d,t+1}$.Let $g(n_t) := x_A^\alpha(1+\lambda_A(n_t)\beta)^{1-\alpha}(\pi+\lambda_A(n_t)\beta)^\alpha - x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$. Then banks invest in sector A iff $g(n_t) > 0$. By the above conditions, $g(0) < 0$. Further, $\lim_{n_t\to\infty} g(n_t) > 0$. Thus, for sufficiently large n_t, $g(n_t) > 0$. As $\lambda_A(.)$ is differentiable on $[0,\infty)$, it is continuous on the same interval and hence so too is $g(.)$. Thus, by the Intermediate Value Theorem, $\exists \widetilde{n} : g(\widetilde{n}) = 0$. Further, as $\lambda'_A(n_t) > 0 \ \forall n_t \geq 0$, $g'(n_t) > 0 \ \forall n_t \geq 0$. Hence, $\widetilde{n}$ is unique, and $g(n_t) > 0$ iff $n_t > \widetilde{n}$. ■

B.4 Credit Trap Condition

Corollary 4 *Suppose* $x_B(1-\lambda_B)\left(\frac{\pi+\lambda_B}{1+\beta\lambda_B}\right) \geq \pi x_A$ *and* $\beta(1-\pi) > \pi + \lambda_i\beta$ $(i = A, B)$*. Let* n_B^* *be the steady state value of banker net worth when sector B is invested in. The economy features a credit trap if*

$$x_A^\alpha(1+\lambda_A(n_B^*)\beta)^{1-\alpha}(\pi+\lambda_A(n_B^*)\beta)^\alpha < x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$$

Proof. From the above lemmas, banks invest in sector B rather than sector A iff $x_A^\alpha(1+\lambda_A(n_t)\beta)^{1-\alpha}(\pi+\lambda_A(n_t)\beta)^\alpha < x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$. Hence if $x_A^\alpha(1+\lambda_A(n_B^*)\beta)^{1-\alpha}(\pi+\lambda_A(n_B^*)\beta)^\alpha < x_B^\alpha(1+\lambda_B\beta)^{1-\alpha}(\pi+\lambda_B\beta)^\alpha$ the banks will invest in sector B when $n_t = n_B^*$. i.e. they invest in the B in the steady state of B. This is thus a steady state equilibrium and without shocks the economy will invest in sector B for the rest of time, so is stuck in a credit trap. ■

C Proofs from Section 3: Financial Crisis

C.1 Deleveraging

In terms of the impact on the macroeconomy of deleveraging, we are interested in the output produced by the standard productive technology as this links to the wages of the next generation. This is given by

$$y^P_{t+1} = (k_{t+1} - k^L_{t+1})^\alpha$$

[33]This $\widetilde{n}$ is time-invariant so long as the expected level of technology in sector A is constant:x_A is constant over time.

This is the quantity we focus on when examining how the leverage of the banking sector affects the resilience of the economy: the resilience is higher the higher this quantity.

The leverage limit will hold for the expected level of capital next period, k^e_{t+1}. Capital k_{t+1} has value $V(k_{t+1})$ in terms of output where $V(k_{t+1}) = \alpha k^{\alpha}_{t+1}$. Given that they owe depositors $R_{d,t+1}\overline{d}$ units of output goods, their net worth, in terms of output goods, is $\alpha k^{\alpha}_{t+1} - R_{d,t+1}\overline{d}$. We thus have the following relationship holding for the expected amount of capital next period[34]:

$$\frac{\alpha(k^e_{t+1})^{\alpha}}{\alpha(k^e_{t+1})^{\alpha} - R_{d,t+1}\overline{d}} = \frac{1}{1-\lambda_A} \tag{52}$$

If $k_{t+1} < k^e_{t+1}$ then the leverage limit will be exceeded and depositors will withdraw deposits until it holds. To consider how much the bank may have to deleverage, it is useful to consider their net worth as a function of initial capital holdings k_{t+1} and the amount of capital liquidation they do k^L_{t+1} :

$$\begin{aligned} &NW(k_{t+1}, k^L_{t+1}) = \alpha(k_{t+1} - k^L_{t+1})^{\alpha} + L(k_{t+1}, k^L_{t+1}) - R_{d,t+1}\overline{d} \\ &\textit{with } L(k_{t+1}, 0) = 0 \end{aligned}$$

To emphasise, if the bankers initially hold k_{t+1} units of capital and liquidate k^L_{t+1} units, then the value of their remaining capital holdings in terms of output is $\alpha(k_{t+1} - k^L_{t+1})^{\alpha}$.

In general, if less capital is produced than expected, we require that

$$\frac{\alpha(k_{t+1} - k^L_{t+1})^{\alpha}}{\alpha(k_{t+1} - k^L_{t+1})^{\alpha} + L(k_{t+1}, k^L_{t+1}) - R_{d,t+1}\overline{d}} = \frac{1}{1-\lambda_A}$$

This implies that

$$\begin{aligned} \alpha(k_{t+1} - k^L_{t+1})^{\alpha}(1-\lambda_A) &= \alpha(k_{t+1} - k^L_{t+1})^{\alpha} + L(k_{t+1}, k^L_{t+1}) - R_{d,t+1}\overline{d} \text{ so} \\ R_{d,t+1}\overline{d} - L(k_{t+1}, k^L_{t+1}) &= \lambda_A \alpha(k_{t+1} - k^L_{t+1})^{\alpha} \end{aligned}$$

This condition states that the amount owed to depositors after deleveraging is equal to the pledgeable return bankers can promise with their remaining capital.

To further analyse this expression, we note that from (52) we have that

$$\begin{aligned} \alpha(k^e_{t+1})^{\alpha}(1-\lambda_A) &= \alpha(k^e_{t+1})^{\alpha} - R_{d,t+1}\overline{d} \text{ so} \\ R_{d,t+1}\overline{d} &= \lambda_A \alpha(k^e_{t+1})^{\alpha} \end{aligned}$$

Substituting this into the above expression gives

$$\lambda_A \alpha(k^e_{t+1})^{\alpha} - L(k_{t+1}, k^L_{t+1}) = \lambda_A \alpha(k_{t+1} - k^L_{t+1})^{\alpha} \tag{53}$$

We note that, of course, if $k_{t+1} = k^e_{t+1}$ then this has solution $k^L_{t+1} = 0$, i.e. no deleveraging. For general $L(.,.)$ there will be no analytic solution to this. Below we consider a special case in which net worth is constant as the bank deleverages.

C.2 Benchmark case: net worth constant with deleveraging

This is a natural benchmark as it isolates the impact of deleveraging per se, without 'fire sale' costs. Using the above expressions $NW(k_{t+1}, k^L_{t+1}) = \alpha(k_{t+1} - k^L_{t+1})^{\alpha} + L(k_{t+1}, k^L_{t+1}) - \lambda_A \alpha(k^e_{t+1})^{\alpha}$. Net worth is constant with deleveraging iff $\frac{\partial L(k_{t+1}, k^L_{t+1})}{\partial k^L_{t+1}} = \frac{\alpha^2}{(k_{t+1} - k^L_{t+1})^{1-\alpha}}$ which requires that $L(k_{t+1}, k^L_{t+1}) = -\alpha(k_{t+1} - k^L_{t+1})^{\alpha} + C$ where C is a constant. Given $L(k_{t+1}, 0) =$

[34] That is the amount produced when the capital producing technology has its expected value: $\widehat{x}_A = x_A$

0, $C = \alpha k_{t+1}^{\alpha}$. Thus our liquidation technology that gives constant net worth is given by $\widetilde{L}(k_{t+1}, k_{t+1}^{L}) = \alpha k_{t+1}^{\alpha} - \alpha(k_{t+1} - k_{t+1}^{L})^{\alpha}$. With this, $NW(k_{t+1}, k_{t+1}^{L}) = \alpha k_{t+1}^{\alpha} - \lambda_A \alpha (k_{t+1}^{e})^{\alpha}$. Further, (53) becomes

$$\lambda_A \alpha (k_{t+1}^{e})^{\alpha} - \left(\alpha k_{t+1}^{\alpha} - \alpha(k_{t+1} - k_{t+1}^{L})^{\alpha}\right) = \lambda_A \alpha (k_{t+1} - k_{t+1}^{L})^{\alpha}$$

so $(k_{t+1} - k_{t+1}^{L})^{\alpha} = \frac{k_{t+1}^{\alpha} - \lambda_A (k_{t+1}^{e})^{\alpha}}{(1-\lambda_A)}$. Now $k_{t+1} = \widehat{x}_A \frac{\pi + \lambda_A \beta}{1+\lambda_A \beta}(1-\alpha)k_t^{\alpha}$, hence the output of productive technology, y_{t+1}^{P} is given by:

$$y_{t+1}^{P} = \frac{(\widehat{x}_A^{\alpha} - \lambda_A x_A^{\alpha})}{(1-\lambda_A)} \left[\frac{\pi + \lambda_A \beta}{1 + \lambda_A \beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha}$$

Proposition 10 *With the benchmark liquidation technology and sector A invested in*

$$n_{t+1} = (1-\alpha)\pi \frac{(\widehat{x}_A^{\alpha} - \lambda_A(n_t) x_A^{\alpha})}{(1-\lambda_A(n_t))} \left[\frac{\pi + \lambda_A(n_t)\beta}{1+\lambda_A(n_t)\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha} \quad \textit{if } \widehat{x}_A < x_A$$
$$n_{t+1} = (1-\alpha)\pi \widehat{x}_A^{\alpha} \left[\frac{\pi + \lambda_A(n_t)\beta}{1+\lambda_A(n_t)\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha} \quad \textit{if } \widehat{x}_A \geq x_A$$

Proof. The next generation wages are based on the amount of productive output: $n_{t+1} = (1-\alpha)\pi y_{t+1}^{P}$

If $\widehat{x}_A < x_A$ then liquidation takes place and $y_{t+1}^{P} = \frac{(\widehat{x}_A^{\alpha} - \lambda_A x_A^{\alpha})}{(1-\lambda_A)}\left[\frac{\pi+\lambda_A\beta}{1+\lambda_A\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha}$

If $\widehat{x}_A \geq x_A$ then no liquidation takes place (as the leverage limit is not violated) and $y_{t+1}^{P} = \widehat{x}_A^{\alpha}\left[\frac{\pi+\lambda_A\beta}{1+\lambda_A\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha}$ ■

D Proofs from Section 4: Policy Options to Avoid Credit Traps

Derivation of $x_A^T(\lambda)$

When the regulatory requirement λ is imposed, we know that the economy will fall into a credit trap whenever bank equity falls below $\tilde{n}$. This condition is given by: $\hat{n}_{t+1} = \pi(1-\alpha)\frac{(\widehat{x}_A^{\alpha} - \lambda x_A^{\alpha})}{(1-\lambda)}\left[\frac{\pi+\lambda\beta}{1+\lambda\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha} \leq \tilde{n}$. Solving the above for $\hat{x}_A$ $\widehat{x}_A^{\alpha} \leq \lambda x_A^{\alpha} + \frac{\tilde{n}(1-\lambda)}{(1-\alpha)\pi\left[\frac{\pi+\lambda\beta}{1+\lambda\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha}}$. Hence the threshold productivity shock below which the economy falls into a credit trap in the next period is given by:

$$x_A^T(\lambda) := \left[\lambda x_A^{\alpha} + \frac{\widetilde{n}(1-\lambda)}{(1-\alpha)\pi\left[\frac{\pi+\lambda\beta}{1+\lambda\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha}}\right]^{\frac{1}{\alpha}}$$

We now demonstrate the "u-shaped" resilience proposition from the text.

Proof of Propostion 2. We first introduce some notation to simplify the exposition of the proof. Let $z(\lambda) := \lambda x_A^{\alpha} + \frac{\widetilde{n}(1-\lambda)}{(1-\alpha)\pi\left[\frac{\pi+\lambda\beta}{1+\lambda\beta}(1-\alpha)k_t^{\alpha}\right]^{\alpha}}$.Then $x_A^T(\lambda) \equiv (z(\lambda))^{\frac{1}{\alpha}}$. Now $\frac{dx_A^T(\lambda)}{d\lambda} = \frac{1}{\alpha}(z(\lambda))^{\frac{1}{\alpha}-1} z'(\lambda) > 0$ iff $z'(\lambda) > 0$. Further, $\frac{d^2 x_A^T(\lambda)}{d\lambda^2} = \frac{1}{\alpha}(\frac{1}{\alpha}-1)(z(\lambda))^{\frac{1}{\alpha}-2}(z'(\lambda))^2 + \frac{1}{\alpha}(z(\lambda))^{\frac{1}{\alpha}-1} z''(\lambda)$. Hence, if $z''(\lambda) > 0$ then $\frac{d^2 x_A^T(\lambda)}{d\lambda^2} > 0$. Given these results, in the following steps of the proof we can work with $z(\lambda)$. We introduce further notation: let $h(\lambda) := \frac{(1-\lambda)}{\left[\frac{\pi+\lambda\beta}{1+\lambda\beta}\right]^{\alpha}}$. Then $z(\lambda) = \lambda x_A^{\alpha} + \frac{\widetilde{n}h(\lambda)}{(1-\alpha)\pi[(1-\alpha)k_t^{\alpha}]^{\alpha}}$.The proof now proceeds via a series of steps.

(i) $\frac{dx_A^T(\lambda)}{d\lambda} > 0$ for λ close to 1. We show $z'(\lambda) > 0$ for λ close to $1. z'(\lambda) = x_A^{\alpha} + \frac{\widetilde{n}h'(\lambda)}{(1-\alpha)\pi[(1-\alpha)k_t^{\alpha}]^{\alpha}}$. Now $h'(\lambda) = -(1+\lambda\beta)^{\alpha}(\pi+\lambda\beta)^{-\alpha} - \alpha\beta(1-\pi)(1-\lambda)(1+\lambda\beta)^{\alpha-1}(\pi+$

$\lambda\beta)^{-\alpha-1}$. Thus $\lim_{\lambda\to1} z'(\lambda) = x_A^\alpha - \frac{\widetilde{n}}{(1-\alpha)\pi[(1-\alpha)k_t^\alpha]^\alpha}\left(\frac{1+\beta}{\pi+\beta}\right)^\alpha$. This is positive so long as $x_A^\alpha(1-\alpha)\pi\left[(1-\alpha)k_t^\alpha\right]^\alpha\left(\frac{\pi+\beta}{1+\beta}\right)^\alpha > \widetilde{n}$. Thus, given our assumed condition $\lim_{\lambda\to1} z'(\lambda) > 0$. However, $z'(\lambda)$ is continuous so $\exists\lambda^* < 1 : z'(\lambda) > 0\ \forall\lambda\in[\lambda^*, 1)$. Thus $\frac{dx_A^T(\lambda)}{d\lambda} > 0\ \forall\lambda\in[\lambda^*, 1)$.

(ii) $\frac{d^2x_A^T(\lambda)}{d\lambda^2} > 0\ \forall\lambda\in[0,1]$. It is sufficient to show that $z''(\lambda) > 0\ \forall\lambda\in[0,1]$. Now $z''(\lambda) = \frac{\widetilde{n}h''(\lambda)}{(1-\alpha)\pi[(1-\alpha)k_t^\alpha]^\alpha}$. Using the expression for $h'(\lambda)$ from step (i) it can be shown that

$$\frac{h''(\lambda)}{\alpha\beta} = \left(\frac{1+\lambda\beta}{\pi+\lambda\beta}\right)^\alpha\left[\frac{-1}{1+\lambda\beta}+\frac{1}{\pi+\lambda\beta}\right]$$
$$+(1-\pi)\left(\frac{1+\lambda\beta}{\pi+\lambda\beta}\right)\left[\frac{1}{(1+\lambda\beta)(\pi+\lambda\beta)}+\frac{(1-\lambda)\beta(1-\alpha)}{(1+\lambda\beta)^2(\pi+\lambda\beta)}+\frac{(1-\lambda)\beta(1+\alpha)}{(1+\lambda\beta)(\pi+\lambda\beta)^2}\right]$$

This expression is positive-for the first term note that $1 > \pi$. Hence $z''(\lambda) > 0\ \forall\lambda\in[0,1]$.

(iii) We now use steps (i), (ii) to prove the proposition. The second condition in the proposition gives $\frac{dx_A^T(0)}{d\lambda} < 0$. From step (i) $\exists\lambda^* < 1 : \frac{dx_A^T(\lambda^*)}{d\lambda} > 0$. Now we must have $\lambda^* > 0$, for otherwise, given $\frac{d^2x_A^T(\lambda)}{d\lambda^2} > 0$,we'd have $\frac{dx_A^T(0)}{d\lambda} > 0$, a contradiction. As $\frac{dx_A^T(\lambda)}{d\lambda}$ is continuous, by the Intermediate Value Theorem, $\exists\widehat{\lambda} : \frac{dx_A^T(\widehat{\lambda})}{d\lambda} = 0$. Further, as $\frac{d^2x_A^T(\lambda)}{d\lambda^2} > 0$ $\widehat{\lambda}$ is unique. The following then holds

$$\frac{dx_A^T(\lambda)}{d\lambda}\begin{Bmatrix} <0 \text{ for } \lambda\in[0,\widehat{\lambda}) \\ =0 \text{ for } \lambda=\widehat{\lambda} \\ >0 \text{ for } \lambda\in(\widehat{\lambda},1] \end{Bmatrix}$$

And so $\frac{dx_A^T(\lambda)}{d\lambda}$ reaches a unique minimum at $\lambda = \widehat{\lambda}$. ■

D.1 Countercyclical Maximum Resilience Policy

Proof of Proposition 3. Using the above notation: $\frac{dx_A^T(\lambda)}{d\lambda} = 0$ iff $z'(\lambda) = 0$ iff $x_A^\alpha = -\frac{\widetilde{n}h'(\lambda)}{(1-\alpha)\pi[(1-\alpha)k_t^\alpha]^\alpha}$ iff $\frac{x_A^\alpha(1-\alpha)\pi[(1-\alpha)k_t^\alpha]^\alpha}{\widetilde{n}} = \left(\frac{1+\lambda\beta}{\pi+\lambda\beta}\right)^\alpha\left[1+\frac{\alpha\beta(1-\lambda)(1-\pi)}{(1+\lambda\beta)(\pi+\lambda\beta)}\right]$. This last equation implicitly defines $\widehat{\lambda}$. The RHS is decreasing in λ. Increasing $\widetilde{n}$ decreases the LHS, so decreases the RHS, so increases $\widetilde{\lambda}$ (which maintains equality between the two sides of the equation). Thus $\frac{d\widehat{\lambda}}{d\widetilde{n}} > 0$. By a similar argument $\frac{d\widehat{\lambda}}{dk_t} < 0$. ■

E Proofs from Section 5: Unconventional Credit Policy

We derive the laws of motion for k_{t+1} for each of the three policies separately.

E.1 Direct Lending

With $d_{g,t}$ government bonds issued, households' saving is given by

$$d_{i,t} = \frac{\beta}{1+\beta}\left[(1-\pi)(1-\alpha)k_t^\alpha\right] - \frac{1}{1+\beta}\frac{V_{i,t+1}}{R_{i,t+1}^d} - d_{g,t} \tag{54}$$

With a positive spread, the banks' borrowing constraint will bind giving $d_{i,t} = \frac{\lambda_i R_{i,t+1}n_t}{R_{i,t+1}^d-\lambda_i R_{i,t+1}}$ and following the prior proofs in the Appendix, we have

$$\frac{V_{i,t+1}}{R_{d,t+1}} = \frac{R_{i,t+1}n_t}{R_{d,t+1}-\lambda_i R_{i,t+1}}(1-\lambda_i)$$

In banking system equilibrium, deposit demand is equal to deposit supply giving

$$\frac{\lambda_i R_{i,t+1} n_t}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} = \frac{\beta}{1+\beta}\left[(1-\pi)(1-\alpha)k_t^{\alpha}\right] - \frac{1}{1+\beta}\frac{R_{i,t+1} n_t (1-\lambda_i)}{R_{d,t+1} - \lambda_i R_{i,t+1}} - d_{g,t}$$

After rearranging, this gives (36) in the text. Following the steps given there results in (37).

E.2 Discount Window Lending

The bank has two sources of funding: deposits and government loans, and maximises its profits with respect to these subject to its combined leverage constraint. We have the following Lagrangian:

$$\begin{aligned} L = {} & R_{i,t+1}(n_t + d_{i,t} + m_t) - R^d_{i,t+1} d_{i,t} - R^m_{t+1} m_t \\ & + \mu \left[\lambda_i R_{i,t+1}(n_t + d_{i,t} + \omega m_t) - R^d_{i,t+1} d_{i,t} - R^m_{t+1} m_t\right] \end{aligned}$$

FOCs:

$$\begin{aligned} d_{i,t} : \mu &= \frac{R_{i,t+1} - R^d_{i,t+1}}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} \\ m_t : \mu &= \frac{R_{i,t+1} - R^m_{i,t+1}}{R^m_{i,t+1} - \omega \lambda_i R_{i,t+1}} \end{aligned}$$

Combining the two gives

$$\frac{R_{i,t+1} - R^d_{i,t+1}}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} = \frac{R_{i,t+1} - R^m_{i,t+1}}{R^m_{i,t+1} - \omega \lambda_i R_{i,t+1}} \tag{55}$$

We proceed to derive equilibrium through the usual series of steps.

Banks' Demand for Deposits
With a binding borrowing constraint, we have, after rearranging

$$d_{i,t} = \frac{\lambda_i R_{i,t+1}}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} n_t - \frac{(R^m_{t+1} - \omega \lambda_i R_{i,t+1})}{(R^d_{i,t+1} - \lambda_i R_{i,t+1})} m_t$$

Applying (55) we have

$$d_{i,t} = \frac{\lambda_i R_{i,t+1}}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} n_t - \frac{(R_{i,t+1} - R^m_{i,t+1})}{(R_{i,t+1} - R^d_{i,t+1})} m_t$$

Bank Profits
The profits for the bank are given by

$$V_{i,t+1} = (R_{i,t+1} - R^d_{i,t+1}) d_{i,t} + R_{i,t+1} n_t + (R_{i,t+1} - R^m_{i,t+1}) m_t$$

Substituting in the expression for deposits and rearranging gives

$$V_{i,t+1} = \frac{R_{i,t+1} R^d_{i,t+1} (1-\lambda_i) n_t}{R^d_{i,t+1} - \lambda_i R_{i,t+1}}$$

Household Deposit Demand

The equation for this is also given by (54), thus substituting in bank profits, we have household deposit demand given by

$$d_{i,t} = \frac{\beta}{1+\beta}\left[(1-\pi)w_t\right] - \frac{1}{1+\beta}\frac{R_{i,t+1}(1-\lambda_i)n_t}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} - d_{g,t}$$

Deposit Market Equilibrium
To determine we equate the supply and demand for deposits:

$$\begin{aligned}&\frac{\lambda_i R_{i,t+1}}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} n_t - \frac{(R_{i,t+1} - R^m_{i,t+1})}{(R_{i,t+1} - R^d_{i,t+1})} m_t \\ &= \frac{\beta}{1+\beta}\left[(1-\pi)w_t\right] - \frac{1}{1+\beta}\frac{R_{i,t+1}(1-\lambda_i)n_t}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} - d_{g,t}\end{aligned}$$

Solving, and rearranging gives

$$d^*_{i,t} = \frac{\lambda_i\beta}{1+\lambda_i\beta}(1-\pi)w_t - \frac{\lambda_i(1+\beta)}{1+\lambda_i\beta}d_{g,t} - \frac{(1-\lambda_i)}{1+\lambda_i\beta}\left(\frac{R_{i,t+1} - R^m_{t+1}}{R_{i,t+1} - R^d_{i,t+1}}\right) m_t \tag{56}$$

Lemma 23 *In equilibrium*

$$\frac{R_{i,t+1} - R^m_{t+1}}{R_{i,t+1} - R^d_{i,t+1}} = \frac{1-\omega\lambda_i}{1-\lambda_i}$$

Proof. We first show that, in equilibrium,

$$\begin{aligned}R^d_{i,t+1} &= \psi^d_t \lambda_i R_{i,t+1} \\ R^m_{t+1} &= \left(\frac{(1-\omega\lambda_i)\psi^d_t + \omega - 1}{1-\lambda_i}\right)\lambda_i R_{i,t+1}\end{aligned}$$

Where

$$\psi^d_t := \frac{\left(\frac{n_t}{d_{i,t}} + 1 + \frac{1-\omega\lambda_i}{1-\lambda_i}\frac{m_t}{d_{i,t}}\right)}{\left(1 + \frac{1-\omega\lambda_i}{1-\lambda_i}\frac{m_t}{d_{i,t}}\right)}$$

To show this, first note that from the binding borrowing constraint

$$R^d_{i,t+1} = \lambda_i R_{i,t+1}\left(\frac{n_t}{d_{i,t}} + 1\right) - \left(R^m_{t+1} - \omega\lambda_i R_{i,t+1}\right)\frac{m_t}{d_{i,t}}$$

Rearranging (55) gives

$$R^m_{t+1} = \frac{(1-\omega\lambda_i)}{1-\lambda_i}R^d_{i,t+1} + \frac{(\omega-1)\lambda_i R_{i,t+1}}{1-\lambda_i}$$

Thus, the deposit rate satisfies

$$R^d_{i,t+1} = \lambda_i R_{i,t+1}\left(\frac{n_t}{d_{i,t}} + 1 + \omega\frac{m_t}{d_{i,t}}\right) - \left(\frac{(1-\omega\lambda_i)}{1-\lambda_i}R^d_{i,t+1} + \frac{(\omega-1)\lambda_i R_{i,t+1}}{1-\lambda_i}\right)\frac{m_t}{d_{i,t}}$$

Solving for $R^d_{i,t+1}$:

$$\begin{aligned}R^d_{i,t+1} &= \lambda_i R_{i,t+1}\left(\frac{n_t}{d_{i,t}} + 1 + \frac{(1-\omega\lambda_i)m_t}{(1-\lambda_i)d_{i,t}}\right)\left(1 - \frac{1-\omega\lambda_i}{1-\lambda_i}\frac{m_t}{d_{i,t}}\right)^{-1} \\ &= \psi^d_t \lambda_i R_{i,t+1}\end{aligned}$$

Further,

$$R^m_{t+1} = \frac{(1-\omega\lambda_i)}{1-\lambda_i}\psi^d_t\lambda_i R_{i,t+1} + \frac{(\omega-1)\lambda_i R_{i,t+1}}{1-\lambda_i}$$
$$= \lambda_i R_{i,t+1}\left(\frac{(1-\omega\lambda_i)\psi^d_t + \omega - 1}{1-\lambda_i}\right)$$

We now use these two results to establish the lemma:

$$\frac{R_{i,t+1} - R^m_{t+1}}{R_{i,t+1} - R^d_{i,t+1}} = \frac{1-\omega\lambda_i}{1-\lambda_i} \text{ iff}$$
$$\frac{\left[1 - \frac{\lambda_i}{1-\lambda_i}\left((1-\omega\lambda_i)\psi^d_t + \omega - 1\right)\right]}{\left[1-\psi^d_t\lambda_i\right]} = \frac{1-\omega\lambda_i}{1-\lambda_i}\text{iff}$$
$$1 - \lambda_i\left[(1-\omega\lambda_i)\psi^d_t + \omega\right] = \left[1-\psi^d_t\lambda_i\right](1-\omega\lambda_i)$$

But the LHS can be written

$$1 - \lambda_i\left[(1-\omega\lambda_i)\psi^d_t + \omega\right] = -(1-\omega\lambda_i)\lambda_i\psi^d_t + (1-\omega\lambda_i) = (1-\omega\lambda_i)(1-\lambda_i\psi^d_t) = RHS$$

This completes the proof of the lemma. ■

Given this (56) becomes

$$d^*_{i,t} = \frac{\lambda_i\beta}{1+\lambda_i\beta}(1-\pi)w_t - \frac{\lambda_i(1+\beta)}{1+\lambda_i\beta}d_{g,t} - \frac{(1-\omega\lambda_i)\,m_t}{1+\lambda_i\beta}$$

Now,

$$k_{t+1} = x_i(n_t + d_{i,t}) + x_i m_t$$

Thus we can write the law of motion for k_{t+1} as (noting $m_t = \frac{d_{g,t}}{1+\tau_m}$)

$$k_{t+1} = x_i\left[n_t + \frac{\lambda_i\beta}{1+\lambda_i\beta}(1-\pi)w_t\right] + x_i d_{g,t}\left(\frac{\left[1-\frac{(1-\omega\lambda_i)}{1+\lambda_i\beta}\right]}{1+\tau_m} - \frac{\lambda_i(1+\beta)}{1+\lambda_i\beta}\right)$$

The first term simplifies to k_{t+1} absent policy, in the usual way.
Further, given that $\omega = 1 + \frac{\omega_g(1-\lambda_i)}{\lambda_i}$we can write

$$1-\omega\lambda_i = 1 - \lambda_i\left(1 + \frac{\omega_g(1-\lambda_i)}{\lambda_i}\right) = 1 - \lambda_i - \omega_g(1-\lambda_i) = (1-\lambda_i)(1-\omega_g)$$

Substituting this in results in the expression for k_{t+1} in the text.

E.3 Bank recapitalisation

E.3.1 Derivation of Law of Motion

When the government obtains x_g fraction of bank equity, optimal household saving is then given by

$$d_{i,t} = \frac{\beta}{1+\beta}\left[(1-\pi)w_t\right] - \frac{(1-x_g)}{(1+\beta)}\frac{V_{i,t+1}}{R^d_{i,t+1}} - d_{g,t} \quad (57)$$

To derive the equilibrium law of motion for k_{t+1} we follow the usual steps, first determining equilibrium in the banking sector.

With the banks' leverage constraints binding they demand deposits,[35]

$$d_{i,t} = \frac{\lambda_i R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}}$$

Bank profits are given by[36]

$$V_{i,t+1} = \left(R_{i,t+1} - R^d_{i,t+1}\right) d_{i,t} + R_{i,t+1}(n_t + n_{g,t})$$

Following the usual steps, with the binding constraint

$$V_{i,t+1} = \frac{R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} R^d_{i,t+1}(1 - \lambda_i)$$

Then, from (57) deposit supply is given by

$$d_{i,t} = \frac{\beta}{1+\beta}\left[(1-\pi)w_t\right] - \frac{(1-x_g)(1-\lambda_i)}{(1+\beta)} \frac{R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} - d_{g,t}$$

In deposit market equilibrium the supply and demand for deposits are equal

$$\frac{\lambda_i R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} = \frac{\beta}{1+\beta}\left[(1-\pi)w_t\right] - \frac{(1-x_g)(1-\lambda_i)}{(1+\beta)} \frac{R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} - d_{g,t}$$

Rearranging

$$\frac{R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} \left[\lambda_i + \frac{(1-x_g)(1-\lambda_i)}{(1+\beta)}\right] = \frac{\beta}{1+\beta}\left[(1-\pi)w_t\right] - d_{g,t}$$

$$\frac{R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} \left[(1+\beta)\lambda_i + (1-x_g)(1-\lambda_i)\right] = \beta\left[(1-\pi)w_t\right] - (1+\beta)d_{g,t}$$

$$\frac{R_{i,t+1}(n_t + n_{g,t})}{R^d_{i,t+1} - \lambda_i R_{i,t+1}} = \frac{\beta\left[(1-\pi)w_t\right] - (1+\beta)d_{g,t}}{(1+\beta)\lambda_i + (1-x_g)(1-\lambda_i)}$$

Then from the banks' deposit demand equation, equilibrium deposits are given by

$$d^*_{i,t} = \frac{\lambda_i \beta(1-\pi)w_t - (1+\beta)d_{g,t}\lambda_i}{(1+\beta)\lambda_i + (1-x_g)(1-\lambda_i)}$$

This reduces to the no-policy equilibrium level of deposits when $d_{g,t} = 0$ and $x_g = 0$.

Finally, $k_{t+1} = x_i\,(n_t + n_{g,t} + d_{i,t})$, so using $\frac{d_{g,t}}{(1+\tau_{gn})} = n_{g,t}$ we have

$$k_{t+1} = x_i \left(n_t + \frac{\lambda_i \beta(1-\pi)w_t}{(1+\beta)\lambda_i + (1-\lambda_i)(1-x_g)}\right) + d_{g,t}x_i\left(\frac{1}{1+\tau_{gn}} - \frac{(1+\beta)\lambda_i}{(1+\beta)\lambda_i + (1-\lambda_i)(1-x_g)}\right) \tag{58}$$

The presence of the policy term x_g on the denominator makes this expression harder to compare to the other two policy cases, so we re-write it to put it into a comparable form.

Note that

$$\frac{1}{(1+\beta)\lambda_i + (1-\lambda_i)(1-x_g)} = \frac{1}{1+\beta\lambda_i} + \left[\frac{(1-\lambda_i)x_g}{\left((1+\beta)\lambda_i + (1-\lambda_i)(1-x_g)\right)(1+\beta\lambda_i)}\right]$$

[35] Note the addition of $n_{g,t}$ which is absent with no equity injection.

[36] The formula (save for the $n_{g,t}$ term) for bank profits has not changed here. What changes is *who* gets them once they're realised, i,e, the split between households and the government.

Thus, we can write

$$k_{t+1} = x_i \left(n_t + \frac{\lambda_i \beta (1-\pi) w_t}{(1+\beta\lambda_i)} \right) + \frac{x_i(1-\lambda_i) x_g \lambda_i \beta (1-\pi) w_t}{((1+\beta)\lambda_i + (1-\lambda_i)(1-x_g))(1+\beta\lambda_i)}$$
$$+ d_{g,t} x_i \left(\frac{1}{1+\tau_{gn}} - \frac{(1+\beta)\lambda_i}{(1+\beta\lambda_i)} \right) - \frac{d_{g,t} x_i (1-\lambda_i) x_g (1+\beta)\lambda_i}{((1+\beta)\lambda_i + (1-\lambda_i)(1-x_g))(1+\beta\lambda_i)}$$

After simplifications, this can be written as

$$k_{t+1} = x_i \left(\left(\frac{\pi + \lambda_i \beta}{1+\lambda_i \beta} \right) (1-\alpha) k_t^{\alpha} \right) + x_i d_{g,t} \left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i (1+\beta)}{1+\lambda_i \beta} \right] \tag{59}$$
$$+ x_g \frac{x_i \lambda_i (1-\lambda_i) \left[\beta(1-\pi) w_t - (1+\beta) d_{g,t} \right]}{\left[(1+\beta)\lambda_i + (1-\lambda_i)(1-x_g) \right] (1+\beta\lambda_i)}$$

An additional effect of equity is directly raising λ_i, it being a function of $n_t + n_{g,t}$:

$$\lambda_i(n_t + n_{g,t})$$

Then, the impact of an equity injection can be written as

$$k_{t+1} = x_i \left(\left(\frac{\pi + \lambda_i(n_t+n_{g,t})\beta}{1+\lambda_i(n_t+n_{g,t})\beta} \right) (1-\alpha) k_t^{\alpha} \right)$$
$$+ x_i d_{g,t} \left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t+n_{g,t})(1+\beta)}{1+\lambda_i(n_t+n_{g,t})\beta} \right]$$
$$+ \frac{x_i d_{g,t} \lambda_i(n_t+n_{g,t})(1-\lambda_i(n_t+n_{g,t})) \left[\beta(1-\pi) w_t - (1+\beta) d_{g,t} \right]}{(1+\tau_{gn})(1+\beta\lambda_i(n_t+n_{g,t})) \left[\frac{d_{g,t}}{(1+\tau_{gn})}(1+\beta)\lambda_i(n_t+n_{g,t}) + (1+\beta\lambda_i(n_t+n_{g,t}))\gamma n_t \right]}$$

We note that policy directly affects the first term, 'crowding in' depositors. We re-write the expression to make it comparable to the baseline case.

After some algebra, we can show that:

$$\frac{\pi + \lambda_i(n_t+n_{g,t})\beta}{1+\lambda_i(n_t+n_{g,t})\beta} = \frac{\pi + \lambda_i(n_t)\beta}{1+\lambda_i(n_t)\beta} + \frac{\beta(1-\pi)\left[\lambda_i(n_t+n_{g,t}) - \lambda_i(n_t) \right]}{(1+\lambda_i(n_t+n_{g,t})\beta)(1+\lambda_i(n_t)\beta)}$$

Further

$$\frac{\lambda_i(n_t+n_{g,t})}{1+\lambda_i(n_t+n_{g,t})\beta} = \frac{\lambda_i(n_t)}{1+\lambda_i(n_t)\beta} + \frac{\lambda_i(n_t+n_{g,t}) - \lambda_i(n_t)}{\left[1+\lambda_i(n_t+n_{g,t})\beta \right] \left[1+\lambda_i(n_t)\beta \right]}$$

Thus, we can write

$$x_i \left(\frac{\pi + \lambda_i(n_t+n_{g,t})\beta}{1+\lambda_i(n_t+n_{g,t})\beta} \right) w_t - x_i d_{g,t} \frac{\lambda_i(n_t+n_{g,t})(1+\beta)}{1+\lambda_i(n_t+n_{g,t})\beta}$$
$$= x_i \frac{\pi + \lambda_i(n_t)\beta}{1+\lambda_i(n_t)\beta} w_t - x_i d_{g,t} \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta}$$
$$+ x_i \frac{\left[\lambda_i(n_t+n_{g,t}) - \lambda_i(n_t) \right]}{(1+\lambda_i(n_t+n_{g,t})\beta)(1+\lambda_i(n_t)\beta)} \left(w_t \beta (1-\pi) - d_{g,t}(1+\beta) \right)$$

Thus, in full we can write

$$k_{t+1} = x_i \left(\frac{\pi + \lambda_i(n_t)\beta}{1+\lambda_i(n_t)\beta} w_t \right) + x_i d_{g,t} \left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta} \right]$$
$$+ x_i \frac{\left[\lambda_i\left(n_t + \frac{d_{g,t}}{1+\tau_{gn}}\right) - \lambda_i(n_t)\right]}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)(1+\lambda_i(n_t)\beta)} [w_t\beta(1-\pi) - d_{g,t}(1+\beta)]$$
$$+ x_i d_{g,t} \frac{\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\left(1-\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right)}{(1+\tau_{gn})\left(1+\beta\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right)}$$
$$\cdot \frac{[\beta(1-\pi)w_t - (1+\beta)d_{g,t}]}{\left[\frac{d_{g,t}}{(1+\tau_{gn})}(1+\beta)\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right) + \left(1+\beta\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right)\gamma n_t\right]}$$

This gives expression (43) in the text.

E.3.2 Other Results

We first establish an expression for the impact of policy:

$$\frac{dk_{t+1}}{d(d_{g,t})}$$

We go through the various components of (43) step by step.

The first term is straightforward with derivative

$$x_i \left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta} \right]$$

The derivative for the second term is given by

$$x_i \frac{\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)[w_t\beta(1-\pi) - d_{g,t}(1+\beta)]}{(1+\tau_{gn})\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)^2} - x_i \frac{\left[\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right) - \lambda_i(n_t)\right](1+\beta)}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)(1+\lambda_i(n_t)\beta)}$$

To ease notation, let

$$f(d_{g,t}) := \frac{\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\left(1-\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right)}{(1+\tau_{gn})\left(1+\beta\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right)}$$
$$\cdot \frac{[\beta(1-\pi)w_t - (1+\beta)d_{g,t}]}{\left[\frac{d_{g,t}}{(1+\tau_{gn})}(1+\beta)\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right) + \left(1+\beta\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right)\gamma n_t\right]}$$

Then the third term can be written as $x_i d_{g,t} f(d_{g,t})$.

It has derivative

$$x_i f(d_{g,t}) + x_i d_{g,t} f'(d_{g,t})$$

Thus, we have

$$\frac{dk_{t+1}}{d\left(d_{g,t}\right)} = x_i\left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta}\right] + x_i\frac{\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\left[w_t\beta(1-\pi)-d_{g,t}(1+\beta)\right]}{(1+\tau_{gn})\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)^2} \tag{60}$$

$$- x_i\frac{\left[\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)-\lambda_i(n_t)\right](1+\beta)}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)(1+\lambda_i(n_t)\beta)} + x_i f(d_{g,t}) + x_i d_{g,t} f'(d_{g,t})$$

Corollary 5

$$\left\{\frac{dk_{t+1}}{d\left(d_{g,t}\right)}\right\}_{d_{g,t=0}} = x_i\left[\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta}\right] + x_i\frac{\lambda_i'\left(n_t\right)w_t\beta(1-\pi)}{(1+\tau_{gn})\left(1+\lambda_i\left(n_t\right)\beta\right)^2}$$
$$+ x_i\frac{\lambda_i\left(n_t\right)\left(1-\lambda_i\left(n_t\right)\right)\beta(1-\pi)w_t}{(1+\tau_{gn})\left(1+\beta\lambda_i\left(n_t\right)\right)\left[\left(1+\beta\lambda_i\left(n_t\right)\right)\gamma n_t\right]}$$

We now prove Lemma 5.
Proof. From the preceding line,

$$\left\{\frac{dk_{t+1}}{d\left(d_{g,t}\right)}\right\}_{d_{g,t=0}} > 0$$
$$\text{iff}\frac{1}{(1+\tau_{gn})} - \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta} + \frac{\lambda_i'\left(n_t\right)\left[w_t\beta(1-\pi)\right]}{(1+\tau_{gn})\left(1+\lambda_i\left(n_t\right)\beta\right)^2}$$
$$+\frac{\lambda_i\left(n_t\right)\left(1-\lambda_i(n_t)\right)\beta(1-\pi)w_t}{(1+\tau_{gn})\left(1+\beta\lambda_i\left(n_t\right)\right)^2\gamma n_t} > 0$$

This holds iff

$$\frac{1}{(1+\tau_{gn})}\left[1+\frac{\lambda_i'\left(n_t\right)\left[w_t\beta(1-\pi)\right]}{\left(1+\lambda_i\left(n_t\right)\beta\right)^2} + \frac{\lambda_i\left(n_t\right)\left(1-\lambda_i(n_t)\right)\beta(1-\pi)w_t}{\left(1+\beta\lambda_i\left(n_t\right)\right)^2\gamma n_t}\right] > \frac{\lambda_i(n_t)(1+\beta)}{1+\lambda_i(n_t)\beta}\text{iff}$$
$$\frac{1+\lambda_i(n_t)\beta}{\lambda_i(n_t)(1+\beta)}\left[1+\frac{\left[\gamma n_t\lambda_i'\left(n_t\right)+\lambda_i\left(n_t\right)\left(1-\lambda_i(n_t)\right)\right]\left[w_t\beta(1-\pi)\right]}{\left(1+\lambda_i\left(n_t\right)\beta\right)^2\gamma n_t}\right] - 1 > \tau_{gn} \text{ iff}$$
$$\frac{1-\lambda_i(n_t)}{\lambda_i(n_t)(1+\beta)} + \frac{\left[\gamma n_t\lambda_i'\left(n_t\right)+\lambda_i\left(n_t\right)\left(1-\lambda_i(n_t)\right)\right]\left[\beta(1-\pi)\right]}{\lambda_i(n_t)(1+\beta)\left(1+\lambda_i\left(n_t\right)\beta\right)\gamma\pi} > \tau_{gn}$$

This condition can be written:

$$\tau_{gn} < \frac{1-\lambda_i(n_t)}{\lambda_i(n_t)(1+\beta)}\left[1+\frac{\left[\gamma n_t\lambda_i'\left(n_t\right)+\lambda_i\left(n_t\right)\left(1-\lambda_i(n_t)\right)\right]\left[\beta(1-\pi)\right]}{(1-\lambda_i(n_t))(1+\lambda_i(n_t)\beta)\gamma\pi}\right]$$

■

We now establish the sufficient conditions for the maximum marginal impact of an equity injection to be at $d_{g,t}=0$, first establishing a useful lemma.

Lemma 24 *Suppose* $\lambda_i(n_t) > \frac{-1+\sqrt{1+(\beta(2+\beta))}}{\beta(2+\beta)}$
Then

$$f'(d_{g,t}) < 0$$

Proof. It is clear that, treating λ_i as a constant, increasing $d_{g,t}$ decreases $f(d_{g,t})$. Now $d_{g,t}$ increases λ_i, so it's enough to show that $f(d_{g,t})$ is decreasing in λ_i. We write the relevant part as

$$\frac{\lambda(1-\lambda)}{(1+\beta\lambda)\left[(\alpha\lambda+(1+\beta\lambda)\gamma n_t\right]}=\frac{\lambda-\lambda^2}{(1+\beta\lambda)\left[\lambda\left[\alpha+\beta\gamma n_t\right]+\gamma n_t\right]}$$

where $\alpha := \frac{d_{g,t}}{(1+\tau_{gn})}(1+\beta)$

Then, taking the derivative wrt λ :

$$\frac{(1-2\lambda)(1+\beta\lambda)\left[\lambda\left[\alpha+\beta\gamma n_t\right]+\gamma n_t\right]-\lambda(1-\lambda)\left[\beta\left[\lambda\left[\alpha+\beta\gamma n_t\right]+\gamma n_t\right]+\left[\alpha+\beta\gamma n_t\right](1+\beta\lambda)\right]}{(1+\beta\lambda)^2\left[\lambda\left[\alpha+\beta\gamma n_t\right]+\gamma n_t\right]^2}$$

$$=-\left[\frac{\lambda^2\left(\alpha(1+\beta)+\beta n_t\gamma(2+\beta)\right)+2\lambda\gamma n_t-\gamma n_t}{(1+\beta\lambda)^2\left[\lambda\left[\alpha+\beta\gamma n_t\right]+\gamma n_t\right]^2}\right]$$

This expression is then negative iff $\lambda > \frac{-2\gamma n_t+\sqrt{4\gamma^2 n_t^2+4\gamma n_t(\alpha(1+\beta)+\beta n_t\gamma(2+\beta))}}{2(\alpha(1+\beta)+\beta\gamma n_t(2+\beta))}$
iff

$$\lambda > \frac{-2\gamma n_t+\sqrt{4\gamma^2 n_t^2+4\gamma n_t\left(\frac{d_{g,t}}{(1+\tau_{gn})}(1+\beta)^2+\beta\gamma n_t(2+\beta)\right)}}{2(\frac{d_{g,t}}{(1+\tau_{gn})}(1+\beta)^2+\beta\gamma n_t(2+\beta))}$$

Note that the RHS is decreasing in $d_{g,t}$ hence it's sufficient that λ is greater than the expression when $d_{g,t}=0$

Evaluated at $d_{g,t}=0$, we require

$$\lambda > \frac{-2\gamma n_t+\sqrt{4\gamma^2 n_t^2+4\gamma n_t\left(\beta\gamma n_t(2+\beta)\right)}}{2\beta\gamma n_t(2+\beta)}$$
$$=\frac{-1+\sqrt{1+(\beta(2+\beta))}}{\beta(2+\beta)}$$

Note the required $\lambda < \frac{1}{2}$ ■

Proposition 11 *Suppose.*

$$\lambda_i''\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right) < \frac{2\beta\left[\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right]^2}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)}$$

and

$$\lambda_i(n_t) > \frac{-1+\sqrt{1+\beta(2+\beta)}}{\beta(2+\beta)}$$

then

$$\frac{dk_{t+1}}{d\left(d_{g,t}\right)} \text{ is maximised at } d_{g,t}=0$$

Further, if

$$\tau_{gn} > \frac{1-\lambda_i(n_t)}{\lambda_i(n_t)(1+\beta)}\left[1+\frac{\left[\gamma n_t\lambda_i'(n_t)+\lambda_i(n_t)(1-\lambda_i(n_t))\right]\left[w_t\beta(1-\pi)\right]}{(1-\lambda_i(n_t))(1+\lambda_i(n_t)\beta)\gamma n_t}\right]$$

Then an equity injection lowers k_{t+1} for all $d_{g,t}>0$.

Proof. First consider the following term:

$$\frac{\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)^2}$$

Its derivative is negative iff

$$\lambda_i''\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\frac{1}{1+\tau_{gn}}\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)^2$$
$$<2\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\frac{\beta}{1+\tau_{gn}}$$

iff

$$\lambda_i''\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)<\frac{2\beta\left[\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\right]^2}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)}$$

We now show the following term is increasing in $d_{g,t}$:

$$\frac{\left[\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)-\lambda_i(n_t)\right]}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)}$$

It's derivative is positive iff

$$\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\frac{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)}{1+\tau_{gn}}$$
$$>\left[\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)-\lambda_i(n_t)\right]\lambda_i'\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\frac{\beta}{1+\tau_{gn}}$$

iff

$$1+\lambda_i(n_t)\beta>0$$

Thus, it follows that the following term is decreasing in $d_{g,t}$.

$$-x_i\frac{\left[\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)-\lambda_i(n_t)\right](1+\beta)}{\left(1+\lambda_i\left(n_t+\frac{d_{g,t}}{1+\tau_{gn}}\right)\beta\right)(1+\lambda_i(n_t)\beta)}$$

Consider (60). Under the given conditions the first three terms are all decreasing in $d_{g,t}$. This leaves $x_i f(d_{g,t})+x_i d_{g,t} f'(d_{g,t})$. As $f'(d_{g,t})<0$ under the given conditions, the first term is also decreasing in $d_{g,t}$. Finally, as $f'(d_{g,t})<0$, $x_i d_{g,t} f'(d_{g,t})$ takes it's maximum value for non-negative $d_{g,t}$ at $d_{g,t}=0$.

From Lemma 5 given

$$\tau_{gn}>\frac{1-\lambda_i(n_t)}{\lambda_i(n_t)(1+\beta)}\left[1+\frac{[\gamma n_t\lambda_i'(n_t)+\lambda_i(n_t)(1-\lambda_i(n_t))][w_t\beta(1-\pi)]}{(1-\lambda_i(n_t))(1+\lambda_i(n_t)\beta)\gamma n_t}\right] \tag{61}$$

$\left\{\frac{dk_{t+1}}{d(d_{g,t})}\right\}_{d_{g,t}=0}<0$ which implies that $\left\{\frac{dk_{t+1}}{d(d_{g,t})}\right\}_{d_{g,t}=0}<0$ for all $d_{g,t}>0$ under the conditions given here. ■

E.4 Comparison of Policies

Here we prove the proposition comparing the efficacy of an equity injection and discount window lending.

Proof of Proposition 8. Under the given conditions, in the milder credit crunch, n_1, the marginal impact of an equity injection on k_{t+1} is greatest at $d_{g,t}=0$. As the impact of discount window lending is linear in $d_{g,t}$ it is more effective in raising k_{t+1} for all $d_{g,t}$ than an equity injection if the marginal impact is greater at $d_{g,t} = 0$:

$$\frac{1-(1-\omega_g)\left(\frac{(1-\lambda_i(n_1))}{(1+\lambda_i(n_1)\beta)}\right)}{(1+\tau_m)} > \frac{1}{(1+\tau_{gn})} + \frac{\lambda_i'(n_1)\,w_t\beta(1-\pi)}{(1+\tau_{gn})\,(1+\lambda_i(n_1)\,\beta)^2} + \frac{\lambda_i(n_1)\,(1-\lambda_i(n_1))\,\beta(1-\pi)w_t}{(1+\tau_{gn})\,(1+\beta\lambda_i(n_1))^2\,\gamma n_1}$$

With the second equity pricing rule this reduces to

$$\frac{1-(1-\omega_g)\left(\frac{(1-\lambda_i(n_1))}{(1+\lambda_i(n_1)\beta)}\right)}{(1+\tau_m)} > \frac{1}{(1+\tau_{gn})}\left[\frac{1+(1+\beta)\lambda_i(n_1)}{(1+\beta\lambda_i(n_1))} + \frac{\lambda_i'(n_1)\,w_t\beta(1-\pi)}{(1+\lambda_i(n_1)\,\beta)^2}\right]$$

Rearranging this condition gives

$$\begin{aligned}&(1+\tau_{gn})\left[1-(1-\omega_g)\left(\frac{(1-\lambda_i(n_1))}{(1+\lambda_i(n_1)\beta)}\right)\right]\\ &> (1+\tau_m)\left[\frac{1+(1+\beta)\lambda_i(n_1)}{(1+\beta\lambda_i(n_1))} + \frac{\lambda_i'(n_1)\,w_t\beta(1-\pi)}{(1+\lambda_i(n_1)\,\beta)^2}\right] \text{ iff}\\ &\frac{(1+\tau_{gn})\left[(1+\beta\lambda_i(n_1))-(1-\omega_g)(1-\lambda_i(n_1))\right]}{\left[1+(1+\beta)\lambda_i(n_1)+\frac{\lambda_i'(n_1)w_t\beta(1-\pi)}{(1+\lambda_i(n_1)\beta)}\right]}\\ &> (1+\tau_m)\end{aligned}$$

An equity injection will be more effective for a range of $d_{g,t} > 0$ in the more severe credit crunch, n_2, if it is more effective at $d_{g,t} = 0$. This condition reduces to

$$\begin{aligned}&\frac{(1+\tau_{gn})\left[(1+\beta\lambda_i(n_2))-(1-\omega_g)(1-\lambda_i(n_2))\right]}{\left[1+(1+\beta)\lambda_i(n_2)+\frac{\lambda_i'(n_2)w_t\beta(1-\pi)}{(1+\lambda_i(n_2)\beta)}\right]}\\ &< (1+\tau_m)\end{aligned}$$

We look for conditions under which the LHS is increasing in λ_i, so this inequality can hold for n_2 and fail for n_1. It will be increasing iff

$$\begin{aligned}&[\beta+1-\omega_g]\left[1+(1+\beta)\lambda_i(n_t)+\frac{\lambda_i'(n_t)\,w_t\beta(1-\pi)}{(1+\lambda_i(n_t)\,\beta)}\right]\\ &> \left[(1+\beta\lambda_i(n_t))-(1-\omega_g)(1-\lambda_i(n_t))\right]\left[(1+\beta)+\frac{d}{d\lambda_i}\left(\frac{\lambda_i'(n_t)\,w_t\beta(1-\pi)}{(1+\lambda_i(n_t)\,\beta)}\right)\right]\\ &= \left[1+\lambda_i(n_t)\,(\beta+(1-\omega_g))-(1-\omega_g)\right]\left[(1+\beta)+\frac{d}{d\lambda_i}\left(\frac{\lambda_i'(n_t)\,w_t\beta(1-\pi)}{(1+\lambda_i(n_t)\,\beta)}\right)\right]\end{aligned}$$

Given $\omega_g < \frac{1+\beta}{2+\beta}$, $\omega_g < 1+\beta$. Suppose n_t is sufficiently large that $\lambda_i''(n_t) < 0$, then a sufficient condition for the RHS increasing in λ_i is

$$[\beta+1-\omega_g]\,(1+(1+\beta)\lambda_i(n_t)) > \left[1+\lambda_i(n_t)\,(\beta+(1-\omega_g))-(1-\omega_g)\right](1+\beta)$$

This reduces to the condition we assume:

$$\omega_g < \frac{1+\beta}{2+\beta}$$

■

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