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Abstract

We investigate the dynamics of the relationship between returns and extreme downside risk in different states of the market by combining the framework of Bali, Demirtas, and Levy (2009) with a Markov switching mechanism. We show that the risk-return relationship identified by Bali, Demirtas, and Levy (2009) is highly significant in the low volatility state but disappears during periods of market turbulence. This is puzzling since it is during such periods that downside risk should be most prominent. We show that the absence of the risk-return relationship in the high-volatility state is due to leverage and volatility feedback effects arising from increased persistence in volatility. To better filter out these effects, we propose a simple modification that yields a positive tail risk-return relationship under all states of market volatility.

Key words: Downside risk, Markov switching, financial crisis, value at risk, leverage effect, volatility feedback effect.

JEL classification: C13, C14, C58, G10, G11, G12.

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1. Introduction

The notion of tail risk, or extreme downside risk, has become increasingly prominent in the asset pricing literature. In particular, in contrast with the assumptions of the standard CAPM of Sharpe (1964) and Lintner (1965), in which portfolio risk is fully captured by the variance of the portfolio return distribution, asset returns display significant negative skewness and excess kurtosis, both of which increase the likelihood of extreme negative returns. A number of studies have examined the importance of these higher moments for asset pricing. Kraus and Litzenberger (1976) develop a three-moment CAPM, in which expected returns are determined, in part, by co-skewness with the market portfolio. This finding is supported by Harvey and Siddique (2000), who consider the role of co-skewness in a conditional asset pricing framework. Dittmar (2002) develops a non-linear pricing kernel with an endogenously determined risk factor and shows that co-kurtosis is also priced. Using moments of the return distribution implied by option prices, Conrad, Dittmar and Ghysels (2013) show that the risk-neutral skewness and kurtosis of individual securities are strongly related to their future returns. Ang, Chen and Yuhang (2006) find that co-moment risks are still significant even after general downside risk is taken into account through a downside beta measure. Other studies focus directly on the likelihood of extreme returns, rather than indirectly on the moments of the return distribution. For example, Ruenzi and Weigert (2013) use a copula-based approach to construct a systematic tail risk measure and show that stocks with high crash sensitivity, measured by lower tail dependence with the market, are associated with higher returns that cannot be explained by traditional risk factors, downside beta, co-skewness or co-kurtosis. Relatedly, Huang, Liu, Ghon Rhee and Wu (2012) propose a measure of idiosyncratic extreme downside risk based on the tail index of the generalised extreme value distribution, and show that it is associated with a premium in cross-section stock returns, even after controlling for market, size, value, momentum, and liquidity effects. Bali, Cakici and Whitelaw (2014) note the difficulties in constructing robust measures for both systematic and idiosyncratic tail risks. They introduce a hybrid tail risk measure that incorporates both market-wide and firm-specific components and show that this yields a robust and significantly positive tail risk premium.

The studies described above examine the variation in expected returns across individual stocks. An alternative strand of the literature is concerned with the variation in tail risk over time, and its impact on aggregate equity returns. This is a more challenging objective owing to potential endogeneity in the measure of tail risk that serves to obscure the risk-return relation that would be predicted by asset pricing theory. For example, since investors prefer positive skewness, an investment with higher skewness should correspond to lower expected returns. However, skewness is, by construction, associated with large positive returns, and so there will be a tendency for skewness to be positively related to returns. Additionally, owing to leverage and volatility feedback effects, high volatility tends to be associated with lower contemporaneous returns (see, for example, Black, 1976; Campbell and Hentschel, 1992). As a result, market tail risk measures such as Value-at-Risk (VaR) and Expected Tail Loss, which are positive functions of return volatility, will tend to have a negative relation with returns. Thus, while there are a number of studies that consider the cross-sectional relation between tail risk and returns for individual stocks, there is little evidence concerning tail risk at the aggregate level. Recognising this difficulty, Kelly and Jiang (2013) develop a measure of aggregate market tail risk that is based on the common component of the tail risk of individual stocks. They show that this tail risk measure is highly correlated with the tail risk implied by equity options, and that it has significant predictive power for aggregate market returns. Similarly, Allen, Bali and Tang (2012) construct an aggregate systemic tail risk measure for the financial and banking system from the returns of financial firms and show that it can robustly predict economic downturns in the US, European and Asian markets. A more direct approach to examining the intertemporal relation between stock market returns and tail risk is introduced in Bali, Demirtas and Levy (2009) (hereafter BDL). In order to circumvent the inherent endogeneity of empirical measures of tail risk described above, they measure tail risk by the previous month's one-month ahead expectation of the VaR of the market return. Using monthly data over the period July 1962 to December 2005, they show that there is a statistically and economically significant positive relation between market returns and tail risk. Moreover, the relationship between returns and tail risk is stronger than between returns and conditional volatility, and is robust to different VaR measurement methods, different VaR confidence levels, alternative measures of tail risk, different measures of the market return and the inclusion of macroeconomic control variables to control for business cycle effects.

In this paper, we investigate the nature of the relation between returns and tail risk under different market conditions. This is motivated by empirical evidence that other, closely related risks, such as co-skewness risk, affect returns differently in alternative states of the world (see, for example, Friend and Westerfield, 1980; Guidolin and Timmermann, 2008). In order to model the state-dependent relation between tail risk and return, we incorporate the BDL model into a two-state Markov switching framework. We estimate the Markov switching model using an extended sample that covers the period July 1962 to June 2013, and which includes the recent financial crisis. The two states in the estimated Markov switching model are characterised by a relatively infrequent high volatility state and a relatively frequent low volatility state. Surprisingly, we find that the positive risk-return relation documented by BDL holds in the low volatility state, but disappears in the high volatility state. To shed further light on this finding, we estimate the BDL model using two sub-samples (without Markov switching) and show that, while the risk-return relation is significantly positive during the 1962-2005 period considered by BDL, it is actually negative during the 2006-2013 period that includes the recent financial crisis. The failure of the BDL model to capture the risk-return relationship during financial crises is counter-intuitive since tail risk could be expected to be more relevant during such periods. In order to rule out omitted variable bias, we expand the set of state variables that are included in the original BDL model to control for business cycle effects. This yields a stronger and more significant positive risk-return relation in the original BDL sample, but also a stronger *negative* risk-return relation in the 2006-2013 sample. We also consider the possibility that the results are driven by the non-*iid* nature of the return generating process, and compute tail risk measures using returns that are standardised by time-varying conditional volatility. This yields a significantly positive risk-return relation in the original BDL sample, but in the 2006-2013 period, the relationship is not statistically significant.

The BDL model critically depends on the assumption that leverage and volatility feedback effects dissipate within one month, so that the one-month ahead expectation of VaR, lagged by one month, can be considered pre-determined. We show, however, that leverage and volatility feedback effects take longer to dissipate during periods of high volatility, and so the one-month ahead expectation of VaR is endogenous, even when lagged by one month. In order to circumvent the endogeneity of the tail risk measure the BDL model in the high volatility state, we consider longer horizon expectations of market VaR, at correspondingly longer lags. We show that using the two-month ahead expectation of VaR, lagged by two months, there is a statistically significant and positive relation between market returns and tail risk in both states. Using the expectations of VaR at horizons longer than two months yields similar results, which suggests that leverage and volatility feedback effects are fully dissipated within two months, even during periods of high volatility. In this way, we are able to recover a positive relationship between returns and tail risk in both low and high volatility states.

The remainder of the paper is organised as follows. Section 2 describes the methodology and the data used in the empirical analysis. Section 3 and 4 report the empirical results of our state-dependent tail risk-return relationship investigation and of our modified measures to account for leverage and volatility feedback effects. Section 5 examines the robustness of our findings. Section 6 provides a summary and offers some concluding remarks.

2. Methodology and Data

2.1. Methodology

The BDL Framework

In order to examine the dynamics of the relationship between tail risk and return, we utilise the framework of BDL, which we briefly summarise in this section. BDL measure tail risk by VaR, which, for a given cumulative distribution function of returns F_r and confidence level α, is defined as

$$\mathrm{VaR} = -F_{\mathrm{r}}^{-1}(1 - \alpha) \quad (1)$$

The impact of tail risk on returns is captured by regressing the value-weighted excess market return in month *t*+1, R_{t+1}, on the month *t* conditional expectation of VaR in month *t*+1, $E_t(VaR_{t+1})$, and a set of control variables X_t:

$$R_{t+1} = \alpha + \beta E_t(VaR_{t+1}) + \gamma X_t + \varepsilon_{t+1} \quad (2)$$

The control variables, X_t, include a range of macroeconomic variables to proxy for business cycle fluctuations, the lagged excess market return, and a dummy variable for the October 1987 crash. The risk-return relationship is reflected in the sign and the significance of the coefficient β. BDL measure VaR both parametrically and non-parametrically, using the most recent one to six months of daily market returns. Parametric VaR is obtained by fitting the Skewed Student-t distribution of Hansen (1994) to market returns over the last one month, the last two months, and so on, and calculating the corresponding quantile in each case. Non-parametric VaR is measured as the quantile of the empirical distribution of the daily market return over the past one to six months. In particular, BDL use the lowest return over the last one month (which corresponds to a VaR confidence level of 95.24%, assuming that there are 21 trading days each month), over the last two months (which corresponds to a VaR confidence level of 97.62%), and so on up to six months.

BDL estimate the conditional expectation of VaR using two approaches. First, they assume that $E_t(VaR_{t+1}) = VaR_t$, which would be equal to the true conditional expectation only if VaR follows a random walk. Second, they assume that VaR is mean-reverting and estimate an AR(4) model:

$$VaR_t = \theta_0 + \sum_{i=1}^{4} \theta_i VaR_{t-i} + v_t \tag{3}$$

The conditional expectation of VaR is then given by $E(VaR_{t+1}) = \hat{\theta}_0 + \sum_{i=1}^{4} \hat{\theta}_i VaR_{t+1-i}$. We refer to these two measures as raw VaR and AR4 VaR, respectively. BDL estimate the regression given by (2) using monthly data over the period July 1962 to December 2005, and show that there is a statistically and economically significant positive relation between market returns and tail risk. Moreover, the relationship between returns and tail risk is stronger than between returns and conditional volatility, and is robust to the different VaR measurement frameworks, different VaR confidence levels, alternative measures of tail risk and different measures of the market return.

An important aspect of the BDL approach is that they use the conditional expectation of the risk measure, rather than its realisation, in order to offset the leverage and volatility feedback effects in returns. The use of the one-month ahead expectation, lagged by one month, implicitly assumes that these leverage and volatility feedback effects are short lived, lasting no longer than a month. This subtle but important observation is the basis of our modification of the BDL framework, as detailed in Section 4.

Tail Risk in Different Market states: the Markov Switching Model

In order to examine the state-dependent dynamics of the tail risk-return relationship, we incorporate the BDL model in a Markov switching mechanism. The Markov switching mechanism has been applied in a number of different contexts to model changes in the behaviour of a time series with respect to different states of some underlying variable (see, among others, Hamilton, 1989; Hamilton, 1990; Gray, 1996; Nikolsko-Rzhevskyy, Prodan, 2012). Indeed, many studies have employed the Markov switching framework to examine the time-varying impact of volatility risk. For example, Turner, Startz and Nelson (1989) employ a Markov switching model to examine how the expectation of market volatility affects excess returns in different market conditions. Similarly, Chang-Jin, Morley and Nelson (2004) use Markov switching to directly model volatility feedback effects on returns. Given the large number of control variables in the BDL model, we choose the simplest setting with a first-order Markov process and two regimes. This is perhaps the most widely used variant of the Markov-switching model in empirical studies (see, for example, Bansal and Hao, 2002; Guidolin and Timmermann, 2006). The Markov switching BDL (hereafter MS-BDL) regression model is given by:

$$R_{t+1} = \alpha_{S_{t+1}} + \beta_{S_{t+1}} E_t(VaR_{t+1}) + \gamma_{S_{t+1}} X_t + \varepsilon_{S_{t+1}} \qquad (4)$$

where $\varepsilon_{S_t} \sim N\left(0, \sigma_{S_t}^2\right)$ and $S_t = \begin{cases} 1, if\ state\ 1\ occurs\ at\ time\ t \\ 2, if\ state\ 2\ occurs\ at\ time\ t \end{cases}$.

The coefficient β captures the risk-return relationship during periods of low volatility ($\sigma_{S_t} = \sigma_1$) and high volatility ($\sigma_{S_t} = \sigma_2$). Since the Markov switching mechanism takes into account the different volatility states of the market, we omit the October 1987 dummy variable from X_t.[1]

[1] The results and conclusions are similar if the October 1987 dummy is included.

2.2. Data

Following BDL, we use the value weighted index from the Center for Research in Security Prices (CRSP), which includes all stocks in the major US stock exchanges, to represent the return of the market. The excess market return is computed as the difference between the market return and the one-month T-bill rate obtained from Kenneth French's website. Our sample period is July 1962 to June 2013, covering the original period of July 1962 to December 2005 studied by BDL, as well as the more recent period that includes the financial crisis of 2007-08. In Table 1 we provide summary statistics (Panel A) and correlations (Panel B) for monthly excess returns and a range of realised risk measures, computed using daily returns within each month, over the full sample. The risk measures are standard deviation, mean absolute deviation, skewness, kurtosis, and maximum loss (which is the non-parametric estimate of VaR used by BDL). In Panel C, we report the estimated coefficients and corresponding t-statistics for the $AR(4)$ models of these risk measures.

[Table 1]

From Panel B of Table 1, it is clear that none of the commonly used realised tail risk measures can explain returns in a way that could be considered consistent with asset pricing theory. In particular, skewness is positively related to returns while the other measures are negatively related to returns. In unreported results, we show that these relationships hold even after controlling for state variables in a regression framework. The signs of the coefficients are not surprising: skewness is, by construction, associated with large positive returns, while the other risk measures are closely related to volatility, which is significantly negatively correlated with concurrent returns due to leverage and volatility feedback effects. It is these observations that motivate the use of expected risk measures, rather than realised risk measures, in the BDL framework.

In the regression analysis, we control for a range of state variables. The variables used by BDL are the detrended risk free rate (RFD), the change in the term structure risk premium (DTRP), the change in the credit risk premium (DCRP), and the dividend yield (DY). We construct these variables using exactly the same method and data sources as in BDL. To examine the robustness of our results, we also consider some additional macroeconomic variables that have been shown in the literature to be important determinants of aggregate equity returns, namely growth in industrial production (IPG), growth in the monetary base (MBG), the change in the inflation rate (DIF) and the change in the oil price (DO) (see, for example, Chen, Roll and Ross, 1986; Kaul, 1990; Anoruo, 2011; Aburachis and Taylor, 2012). These variables are constructed as follows. We use the monthly series of annual growth in industrial production constructed using the same method as Chen et al. (1986), the monthly growth rate of M2 measured by the logarithmic change in M2, the monthly change in inflation, and the monthly change in oil price. The industrial production and monetary supply data are obtained from the Board of Governors of the Federal Reserve System database, while the inflation rate and oil price (the WPU0561 series) are obtained from the Bureau of Labor Statistics database.

3. The relationship between tail risk and returns in different states of the market

We first examine the tail risk-return relationship in different states of the market using the MS-BDL model given by (4). Table 2 presents the estimated coefficients and t-statistics for each of the states, the variance in each state and the duration of each state, using the estimates of VaR employed by BDL: raw non-parametric VaR, raw Skewed Student-t VaR, AR4 non-parametric VaR and AR4 Skewed Student-t VaR. All measures are estimated using daily returns over the previous one month. We also estimate the model using a longer estimation sample for VaR ranging from two to six months as in BDL. This yields very similar results to those reported here. It is clear that we can identify two distinct states of the market: a relatively frequent calm state of low volatility and a relatively infrequent turbulent state of high volatility. The variance in the turbulent state is between about two and three times the level in the calm state, depending on the model estimated. The expected duration of the calm state is double that of the turbulent state. Panel A of Figure 1 plots the monthly realised volatility over the sample period. Panels B and C plot the smoothed probability of the turbulent state and the corresponding estimated state transitions, respectively, for the MS-BDL model using the AR4 Skewed Student-t tail risk measure. The state probabilities and transitions for the other models are very similar. The turbulent state covers a number of periods of market distress, including the 1973-1974 oil crisis, the October 1987 crash, the burst of the dot-com bubble in the early 2000s, and the recent financial crisis.

For all models, the coefficient on tail risk is positive and highly significant in the low volatility state. Thus it would appear that in relatively calm states of the market, there is a strong relationship between returns and tail risk, as implied by asset pricing theory. This is consistent with the results reported by BDL. However, in contrast, in the high volatility state, the coefficient on VaR is significantly *negative* for all VaR measures. In other words, in turbulent states of the market, it would appear that an increase in tail risk leads to *lower* returns in expectation.

[Table 2]

[Figure 1]

In order to shed further light on these results, we estimate the original BDL model (without Markov switching) using three samples: the original sample used by BDL (July 1962 to December 2005), the new sample (January 2006 to June 2013) and the full sample (July 1962 – June 2013). We report the results for one-month raw VaR and AR4 VaR using the non-parametric and Skewed Student-t measures in Table 3 (we obtain similar results for longer sample measures). With the original BDL sample (Panel A), we obtain results that are very close to those reported by BDL. In particular, in all cases, the estimated coefficient on the tail risk measure is significantly positive, suggesting that high tail risk is associated with high returns. However, for the new sample (Panel B), the coefficient on tail risk is, in all four cases, insignificantly positive, or even negative, suggesting a breakdown in the tail risk-return relation. As a result, using the full sample (Panel C), the coefficient on tail risk is not significant using any of the four measures. These sub-sample results suggest that the absence of a significant tail risk-return relation in the high volatility state of the Markov switching model may be attributable to a failure of the BDL model during the recent financial crisis. This is a surprising finding, since it is during episodes such as this that tail risk could reasonably be expected to be more relevant.

[Table 3]

One possible explanation for the failure of the tail risk-return relation to hold across all market states is that it reflects a bias arising from the omission of state variables that are correlated with the tail risk measure. BDL include four control variables (the detrended risk free rate, the change in the term structure risk premium, the change in the credit risk premium and the dividend yield), but it could be argued that these may be insufficient to capture the full dynamics of the economic cycle during crisis periods. Indeed, this is suggested perhaps by the fact that the BDL control variables, while significant in the original sample, are insignificant in the new sample. We therefore expand the set of state variables used by BDL to include four additional macro-variables that are commonly used in the asset pricing literature: growth in industrial production, growth in the monetary base, inflation and the change in the oil price. The estimation results including the expanded set of variables are reported in Table 4 for the three samples. The additional state variables clearly improve the overall fit of the BDL model, both in the original sample and the new sample. In particular, the R-squared coefficient increases very substantially, and in the new sample, the model explains as much as 30 percent of the variation in returns. The inclusion of the additional state variables serves to increase the magnitude and significance of the coefficient on VaR in the original sample and, consequently, it is now positive and significant in the full sample. However, in the new sample, it remains insignificant or negative.

[Table 4]

In Table 5, we report the results of estimating the Markov switching BDL model with the expanded set of state variables. The negative relationship between returns and tail risk in the high volatility state persists in most of the models. Additionally, we note that the inclusion of the additional state variables leads to a reduction in the estimated variances, especially in the second state, suggesting that they improve the overall goodness of fit of the Markov switching model. In the remaining empirical analysis, we therefore use the extended set of state variables.

[Table 5]

A second possible explanation for the failure of the risk-return relation to hold in all states is that the estimators of tail risk employed by BDL are based on the unconditional distribution of returns, and therefore implicitly assume that returns are *iid*. Ignoring the characteristics of the true dependence structure in returns, such as autocorrelation and volatility clustering, is likely to reduce the power of the regression-based tests used to identify the risk-return relation. We therefore relax the *iid* assumption and estimate tail risk using a location-scale VaR model, in which VaR is estimated using the standardised residuals of an $AR(1)$-$GARCH(1,1)$ model for daily market returns (see, for example, Berkowitz and O'Brien, 2002; Kuester, Mittnik and Paulella, 2006). Specifically, to estimate market VaR for day d, we first estimate the location-scale model using information up to day $d-1$ as:

$$r_d = \mu_d + \varepsilon_d = \mu_d + \sigma_d z_d \quad (5)$$

$$\mu_d = a_0 + a_1 r_{d-1} \quad (6)$$

$$\sigma_d^2 = c_0 + c_1\sigma_{d-1}^2 + c_2\varepsilon_{d-1}^2 \quad (7)$$

The quantile of the standardised residuals $z_d = \varepsilon_d/\sigma_d$ is transformed into an estimate of VaR using the one-step ahead forecast of the mean and volatility of returns for day d.[2] After obtaining VaR estimates for each day, we take the average of these within a period (one month to six months) to be the raw non-*iid* risk measures. This corresponds to the one-month to six-month raw VaRs in the original BDL model. We apply an AR(4) process to these raw non-*iid* measures to estimate the corresponding AR4 non-*iid* measures. We estimate the $AR(1)$-$GARCH(1,1)$ model using a five-year rolling window (1260 daily observations), and employ the Skewed Student-t distributions for the residuals. Since we must specify a distribution for the error term in the location-scale estimation, we are not able to compute a non-*iid* version of the non-parametric VaR measure. The results of estimating the Markov switching BDL model using the non-*iid* VaR measures are reported in Table 6. Allowing for the dependence structure of returns in the estimation of VaR generally leads to a strengthening of the tail risk-return relationship in MS-BDL framework. The coefficient on tail risk is now positive in the high volatility state, although it is statistically insignificant.[3]

[Table 6]

4. A Modified Measure of Expected Tail Risk

The preceding results show that the inclusion of additional state variables in the BDL model, and the use of VaR measures that explicitly allow for the dependence structure in returns, serve to improve the fit of the model and generally lead to a stronger relationship between returns and tail

[2] As a robustness check, we also employ the asymmetric GJR-GARCH model of Glosten, Jagannathan and Runkle (1993) and the EGARCH model of Nelson (1991), both of which yield similar results.

[3] In unreported results, we find that using the non-*iid* VaR measures improves the fit of the BDL model in both sub-samples, and that in the new sample, the coefficient is positive, although not significant at conventional levels.

risk in the low volatility state. However, there is still no statistically significant relationship between returns and tail risk in the high volatility state. In this section, we investigate the role of leverage and volatility feedback effects, which lead to endogeneity in realised measures of tail risk. In particular, while asset pricing theory predicts a positive relationship between returns and tail risk, realised tail risk is, by construction, associated with negative returns because high volatility (and hence high tail risk) is associated with negative returns through the leverage effect.

It is this endogeneity that motivates the use of lagged measures of expected tail risk, in place of concurrent measures of realised tail risk, in the BDL framework. However, BDL construct expected tail risk in month t, conditioning on the information set in month t-1, and so implicitly assume that volatility and leverage effects dissipate within one month. While this may be a reasonable assumption in low volatility periods, it is less likely to hold in high volatility periods. This is because high volatility is associated with higher persistence in volatility, and so leverage and volatility effects take longer to dissipate. In this case, the expected risk measure used in the BDL framework will be endogenous, thus obscuring the true relation between returns and tail risk in the high volatility state.

To investigate this idea, in Table 7 we regress the product of the conditional standard deviations of the market return in month t+1 and month t+2 (which measures volatility persistence) on the conditional variance of the market return in month t, with and without the full set of control variables. The conditional variances are obtained from the GARCH model given by (7), above, although the results using realised variances computed from daily returns are very similar. The coefficient on the conditional market variance in month t is positive and highly significant in all specifications, implying that high volatility is indeed associated with high persistence in volatility. This idea is further supported by the fact that, from Tables 5 and 6, we typically observe that the high volatility state in the MS-BDL model lasts for at least two months. As leverage and volatility feedback effects are associated with high volatility, this implies that these effects will also persist for at least two months. When these effects are prolonged, we will observe successive periods of high tail risk and low returns. As a result, the expected tail risk measures used by BDL (the one-month ahead expectations of raw VaR and AR4 VaR) will still be endogenous and negatively correlated to returns.

[Table 7]

These results suggest a simple modification of the BDL framework to account for the persistence of leverage and volatility feedback effects. In particular, we construct the following modified expected tail risk measure:

$$E_t(VaR_{t+1}) = \theta_0 + \theta_1 E_{t-1}(VaR_t) + \sum_{i=2}^{4} \theta_i VaR_{t-i} \qquad (8)$$

where θ_i $(i = 0, \ldots, 4)$ are the estimated coefficients of an $AR(4)$ model of the VaR series and $E_{t-1}(VaR_t) = \theta_0 + \sum_{i=1}^{4} \theta_i VaR_{t-1-i}$. This is similar to the $AR(4)$ measure of expected tail risk used by BDL, and differs only in that the first term on the right hand side, VaR_t, is replaced by its time $t-1$ expected value. In Table 8, we report the results of estimating the MS-BDL model using this modified measure of expected tail risk. The estimated relationship between returns and tail risk is positive and, in contrast with the results in Table 6, highly significant in both states of the world. It is also notable that the use of the modified tail risk measure leads to a change in the estimated state separation. Specifically, the high volatility state now occurs more frequently and, typically, with longer duration. Although not reported, we also observe an improvement in the log likelihood and AIC statistics using the modified expected tail risk measure relative to those obtained using the raw and AR4 measures.[4]

[Table 8]

5. Robustness Checks

Asymmetric GARCH Models for non-iid Tail Risk Measures

In the analysis above, when considering tail risk measures for non-*iid* returns, we used a simple GARCH(1,1) model for conditional volatility. Here we investigate the use of an asymmetric GARCH model that explicitly captures the leverage and feedback effects discussed in the previous section. In particular, we employ the GJR-GARCH model of Glosten at al. (1993).[5] Table 9 reports the results of estimating the MS-BDL model, using the GJR-GARCH model with a Skewed Student-t conditional distribution. As with our earlier analysis, the raw and AR4 measures of tail risk are significantly positive in the low volatility state, but insignificant in the high volatility state. In contrast, the modified measure of tail risk is significantly positive in both states.

[Table 9]

[4]In sub-sample regressions, the use of the modified expected tail risk measure yields a positive and statistically significant relation between returns and risk in both sub-samples. As a further check, we also investigated the performance of the modified expected tail risk measure by restricting the state separation in the Markov Switching estimation to be the same as that obtained using the AR4 measure. The modified expected tail risk is again significantly positive in both market states.

[5] Similar results are obtained using the EGARCH model of Nelson (1991).

Expected Tail Loss

As noted by BDL, a shortcoming of VaR is that it is not a coherent measure of risk (see Artzner, Delbaen, Eber and Heath, 1999) and so they investigate an alternative measure of risk, namely Expected Tail Loss (ETL). Under the assumption of a Normal distribution for daily market returns, $r_t \sim \mathrm{N}(\mu, \sigma^2)$, the ETL at the 100α percent confidence level is given by:

$$ETL_\alpha = \frac{1}{1-\alpha}\varphi\big(\Phi^{-1}(1-\alpha)\big)\sigma - \mu \qquad (9)$$

where φ is the standard normal probability density function and $\Phi^{-1}(1-\alpha)$ is the $(1-\alpha)$ quantile. Analogous to the *iid* and non-*iid* VaR-based measures of tail risk, we construct ETL-based raw, AR4 and modified *iid* measures of tail risk, as well as raw, AR4 and modified non-*iid* measures. Table 10 presents the results of estimating the MS-BDL model with these six ETL-based measures, and the conclusions are similar to those obtained using the corresponding VaR-based measures. In particular, the raw and AR4 measures are positive but statistically significant only in the low volatility state, while the modified measures are positive and statistically significant in the both the low volatility and high volatility states. These results are consistent with BDL, who show that the VaR-based and ETL-based measures of tail risk produce similar performance.

[Table 10]

Alternative VaR Significance Levels

In addition to the significance level of 99 percent for all parametric VaR calculations, we conduct robustness checks using significance levels of 99.9 percent, 97.5 percent, and 95 percent and obtain similar results in all cases. Thus, our inferences are robust with respect to the level of tail risk. Table 11 provides detailed results for Skewed Student-t VaR at the 95 percent significance level. The results for the 99.9 percent and 97.5 percent significance levels are available on request.

[Table 11]

Accounting for Volatility Risk

Following BDL, we investigate the incremental information content of our modified measures of expected tail risk after controlling for volatility risk. Our volatility risk measure is constructed analogously to the measure of tail risk. In particular, we calculate the average conditional variance of daily market returns from equation (7) over the corresponding period. The results from estimating the MS-BDL model including both the tail risk measure and the volatility risk measure are reported in Table 12. Consistent with the results reported by BDL, there is no statistically significant positive relationship between returns and conditional variance. Indeed, in most cases, the coefficient is negative, and in the case of the non-*iid* tail risk measure in the low volatility state, marginally significantly so.

[Table 12]

6. Conclusion

In this paper, we implement a Markov switching model to estimate the relationship between returns and tail risk documented by Bali, Demirtas, and Levy (2009), in different states of the market. We show that the relationship breaks down in the high volatility state that covers a number of financial crises. This is surprising since it is under such conditions that tail risk could be reasonably expected to be most important. We show that this result is robust to a range of features of the model, including expansion of the set of control variables, and the use of tail risk measures that account for the non-*iid* nature of market returns.

We show that the underlying reason for this finding is the heightened leverage and volatility feedback effects during crisis periods that arise as a result of increased persistence in volatility during such times. We propose a modified tail risk measure that better filters out these effects, and show that it yields a positive relation between returns and tail risk in both the low volatility and high volatility states. Moreover, this relation is robust to the use of different VaR confidence levels, alternative measures of tail risk, and after controlling for volatility risk.

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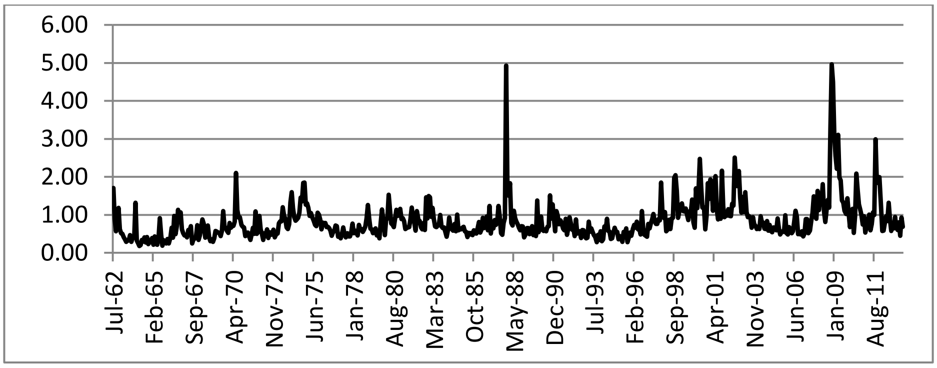
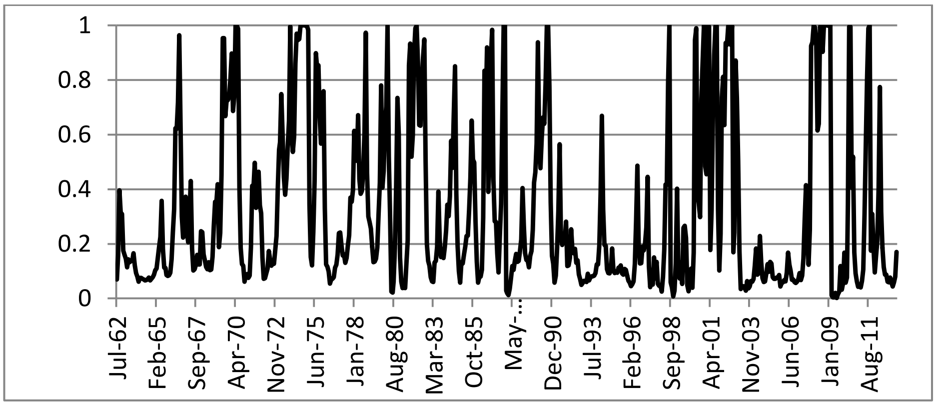
Panel A: Realised volatility of daily market returns



Panel B: Smoothed probability of turbulent state



Panel C: Markov switching state timing

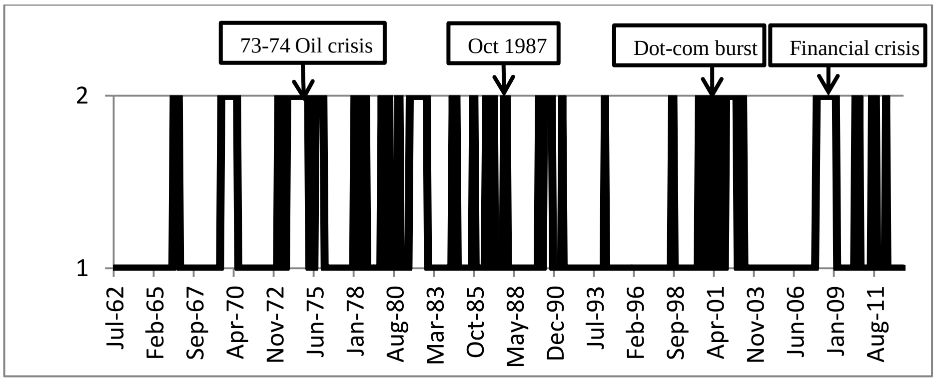


Figure 1: Market volatility and estimated states over time. Panel A shows the monthly realised volatility of market returns for the sample period (July 1962 – Jun 2013). Panels B and C show the smoothed probability of the turbulent state and the corresponding estimated state transitions using a threshold probability of 0.5, for the estimated MS-BDL model using the AR4 Skewed Student-t tail risk measure.

Table 1

Summary statistics for market returns and realised risk measures

The table reports summary statistics for the CRSP value weighted monthly excess return, and the realised standard deviation, mean absolute deviation (MAD), skewness, kurtosis, and non-parametric VaR. The realised risk measures are calculated using daily returns over one month. t-statistics are reported in parentheses. The sample period is July 1962 to June 2013.

	Monthly Excess return	Standard deviation	MAD	Skewness	Kurtosis	Non-parametric VaR
Panel A: Basic statistics						
Mean	0.50	0.84	0.64	-0.05	3.06	1.63
Median	0.86	0.70	0.54	-0.06	2.82	1.36
Standard deviation	4.49	0.52	0.39	0.58	1.13	1.27
Minimum	-23.14	0.18	0.14	-2.88	1.63	0.18
Maximum	16.05	4.96	3.79	2.51	11.71	17.13
Panel B: Cross correlation						
Monthly Excess return	1.00	-0.31	-0.30	0.08	-0.02	-0.44
Standard deviation	-0.31	1.00	0.99	0.02	0.08	0.90
MAD	-0.30	0.99	1.00	0.05	-0.03	0.85
Skewness	0.08	0.02	0.05	1.00	-0.17	-0.27
Kurtosis	-0.02	0.08	-0.03	-0.17	1.00	0.27
Nonparametric VaR	-0.44	0.90	0.85	-0.27	0.27	1.00
Panel C: Lags' coefficients in AR(4)						
Lag 1	0.09	0.56	0.61	0.07	-0.01	0.31
(t-statistic)	(2.397)	(33.964)	(30.304)	(1.889)	(-0.240)	(13.773)
Lag 2	-0.05	0.12	0.14	0.06	0.02	0.17
(t-statistic)	(-1.274)	(3.239)	(4.127)	(1.683)	(0.448)	(5.519)
Lag 3	0.03	0.11	0.03	0.14	0.11	0.22
(t-statistic)	(0.818)	(2.216)	(0.778)	(4.105)	(3.018)	(4.666)
Lag 4	0.01	-0.03	0.01	0.05	-0.01	-0.06
(t-statistic)	(0.343)	(-0.982)	(0.197)	(1.059)	(-0.197)	(-1.525)

Table 2

Extreme downside risk-return relationship in MS-BDL

The table reports the results of the MS-BDL framework for main extreme downside risk measures in BDL. The measures are calculated using daily returns over one month. Market's monthly excess return at time *t+1* is regressed on $E_t(VaR_{t+1})$ and other BDL framework's control variables at time *t*, including market's lagged excess monthly return, October 1987 dummy variable, detrended risk free rate (RFD), change in term structure risk premium (DTRP), changes in credit risk premium (DCRP), and dividend yield (DY). Within each regression, the first line shows the estimated regression coefficients, while the second line shows their corresponding t-statistics (in parentheses). All parametric VaRs are at 99% confidence level. The sample period is July 1962 to June 2013.

State	Const	$E_t(VaR_{t+1})$	Lagged return	RFD	DTRP	DCRP	DY	State variance	Expected Duration
				Raw Nonparametric VaR					
1	0.058	1.028	-0.012	-0.529	0.038	2.705	-0.008	9.399	11.531
	(0.199)	(5.934)	(-0.273)	(-2.740)	(0.170)	(1.442)	(-0.134)		
2	-2.331	-0.706	-0.019	-0.330	-1.953	5.126	0.648	29.624	5.347
	(-1.467)	(-2.318)	(-0.217)	(-0.754)	(-3.258)	(1.634)	(1.595)		
				Raw Skewed Student-t VaR					
1	-0.118	1.019	-0.018	-0.524	0.074	2.772	-0.011	8.999	7.954
	(-0.239)	(5.777)	(-0.355)	(-2.506)	(0.273)	(1.508)	(-0.122)		
2	-2.423	-0.803	-0.011	-0.382	-1.998	5.714	0.752	27.304	3.817
	(-1.514)	(-2.822)	(-0.150)	(-0.877)	(-3.358)	(1.825)	(1.840)		
				AR4 Nonparametric VaR					
1	-2.117	2.151	-0.156	-0.600	-0.763	4.665	0.305	9.401	5.642
	(-2.464)	(6.068)	(-3.622)	(-3.113)	(-2.822)	(2.758)	(1.841)		
2	-1.725	-1.202	0.144	-0.468	-0.793	2.617	0.252	20.111	2.369
	(-0.853)	(-1.623)	(1.327)	(-1.179)	(-1.354)	(0.914)	(0.614)		
				AR4 Skewed Student-t VaR					
1	-1.452	1.732	-0.076	-0.551	-0.008	2.877	0.011	8.723	7.938
	(-2.013)	(6.109)	(-1.609)	(-2.777)	(-0.066)	(1.591)	(0.090)		
2	-1.582	-1.161	0.054	-0.467	-1.965	5.171	0.683	28.068	3.788
	(-0.780)	(-1.732)	(0.569)	(-1.030)	(-3.227)	(1.605)	(1.658)		

Table 3: Main results of BDL framework in different periods

The table reports the main results of BDL framework in 3 periods: the Original period from July 1962 to December 2005, the New period from January 2006 to June 2013), and the Extended period from July 1962 to June 2013). In each regression, the market's excess monthly return at time $t+1$ is regressed on a tail risk measure $E_t(VaR_{t+1})$ and other BDL framework's control variables at time *t*, including market's lagged excess monthly return, October 1987 dummy variable, detrended risk free rate (RFD), change in term structure risk premium (DTRP), changes in credit risk premium (DCRP), and dividend yield (DY). Within each regression, the first line shows the estimated regression coefficients, the second line shows their corresponding HAC t-statistics (in parentheses). All parametric VaRs are at 99% confidence level.

	Const	$E_t(VaR_{t+1})$	Lagged return	Dummy	RFD	DTRP	DCRP	DY	Adjusted R^2
Panel A: Original period July 1962 - December 2005									
Raw NonPara VaR	-1.067	0.472	0.049	-14.684	-0.460	-0.722	3.694	0.267	3.44%
	(-1.529)	(2.098)	(1.030)	(-4.625)	(-2.500)	(-2.349)	(2.249)	(1.592)	
AR4 NonPara VaR	-2.103	1.078	0.032	-12.888	-0.454	-0.736	3.492	0.292	3.82%
	(-2.372)	(2.790)	(0.742)	(-6.398)	(-2.466)	(-2.425)	(2.118)	(1.691)	
Raw Skewed Student-t VaR	-1.068	0.413	0.047	-12.754	-0.466	-0.727	3.746	0.261	3.35%
	(-1.475)	(1.896)	(0.996)	(-4.871)	(-2.525)	(-2.360)	(2.268)	(1.546)	
AR4 Skewed Student-t VaR	-1.905	0.830	0.034	-11.621	-0.464	-0.740	3.524	0.282	3.64%
	(-2.094)	(2.386)	(0.784)	(-6.246)	(-2.500)	(-2.425)	(2.136)	(1.629)	
Panel B: New period January 2006 - June 2013									
Raw NonPara VaR	-5.724	-0.539	0.055		0.443	-0.076	-0.912	3.371	7.16%
	(-1.417)	(-0.949)	(0.442)		(0.852)	(-0.052)	(-0.329)	(1.803)	
AR4 NonPara VaR	-6.430	0.218	0.133		0.995	0.286	-2.197	2.967	5.59%
	(-1.878)	(0.277)	(1.306)		(1.332)	(0.184)	(-0.995)	(1.983)	
Raw Skewed Student-t VaR	-5.960	-0.180	0.096		0.690	0.100	-1.535	3.159	5.79%
	(-1.602)	(-0.438)	(0.762)		(1.042)	(0.065)	(-0.664)	(1.949)	
AR4 Skewed Student-t VaR	-6.767	0.423	0.149		1.206	0.413	-2.637	2.862	6.02%
	(-2.012)	(0.840)	(1.398)		(1.419)	(0.254)	(-1.303)	(2.069)	
Panel C: Extended period July 1962 - June 2013									
Raw NonPara VaR	-0.328	0.056	0.065	-8.081	-0.326	-0.689	0.715	0.228	1.44%
	(-0.451)	(0.166)	(1.449)	(-1.590)	(-1.812)	(-2.308)	(0.414)	(1.529)	
AR4 NonPara VaR	-1.183	0.496	0.073	-9.435	-0.285	-0.679	0.399	0.275	1.72%
	(-1.165)	(0.895)	(1.724)	(-3.106)	(-1.566)	(-2.279)	(0.228)	(1.813)	
Raw Skewed Student-t VaR	-0.316	0.044	0.065	-7.783	-0.327	-0.690	0.721	0.227	1.44%
	(-0.442)	(0.155)	(1.444)	(-2.197)	(-1.808)	(-2.308)	(0.410)	(1.518)	
AR4 Skewed Student-t VaR	-0.923	0.305	0.072	-8.523	-0.296	-0.682	0.450	0.262	1.61%
	(-0.950)	(0.690)	(1.718)	(-3.460)	(-1.622)	(-2.284)	(0.257)	(1.723)	

Table 4: Modification of BDL framework – Expanded set of state variables

The table reports the BDL regression using the expanded set of state variables in the 3 sub-sample periods: the Original period (July 1962 – December 2005), the New period (January 2006 – June 2013), and the Extended period (July 1962 – June 2013). In each regression, the market's excess monthly return at time $t+1$ is regressed on a tail risk measure $E_t(VaR_{t+1})$ and other extended control variables at time t, including the market's lagged excess monthly return, October 1987 dummy variable, detrended risk free rate (RFD), change in term structure risk premium (DTRP), changes in credit risk premium (DCRP), dividend yield (DY), growth in Industrial Production (IPG), growth in monetary base M2 (MBG), change in inflation rate (DIF), and change in oil price (DO). Within each regression, the first line shows the estimated regression coefficients, the second line shows their corresponding HAC t-statistics (in parentheses). Parametric VaRs are at 99% confidence level.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel A: Original Period July 1962 - December 2005								
Raw	-2.418	0.848	0.022	-21.416	-0.161	-0.455	5.093	0.378	27.867	-71.021	-0.891	-0.003	8.67%
NonPara VaR	(-3.142)	(3.409)	(0.460)	(-3.827)	(-0.863)	(-1.778)	(2.796)	(2.228)	(5.657)	(-1.214)	(-1.206)	(-0.087)	
AR4	-4.396	1.962	-0.013	-18.408	-0.122	-0.459	4.898	0.424	30.194	-69.371	-0.837	-0.007	9.82%
NonPara VaR	(-4.526)	(5.031)	(-0.339)	(-8.921)	(-0.688)	(-1.575)	(3.224)	(2.611)	(6.199)	(-1.364)	(-1.128)	(-0.202)	
Raw Skewed	-2.553	0.799	0.022	-18.583	-0.162	-0.457	5.189	0.376	28.369	-72.647	-0.789	-0.006	8.67%
Student-t VaR	(-3.605)	(3.855)	(0.495)	(-7.593)	(-0.934)	(-1.513)	(3.321)	(2.444)	(6.084)	(-1.388)	(-1.038)	(-0.185)	
AR4 Skewed	-4.358	1.664	-0.006	-16.750	-0.128	-0.458	4.903	0.424	30.829	-74.918	-0.740	-0.010	9.73%
Student-t VaR	(-4.763)	(5.239)	(-0.148)	(-9.583)	(-0.708)	(-1.549)	(3.187)	(2.628)	(6.209)	(-1.465)	(-0.993)	(-0.312)	

(Continued)

Table 4: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel B: New Period January 2006 - June 2013							
Raw	1.112	-0.759	-0.106		-2.034	-1.738	6.393	0.618	44.513	-192.284	-3.242	0.111	29.20%
NonPara VaR	(0.256)	(-1.294)	(-1.004)		(-1.560)	(-2.062)	(2.064)	(0.367)	(2.363)	(-1.245)	(-2.334)	(4.262)	
AR4	-1.894	0.018	-0.002		-0.872	-1.324	4.897	1.259	32.885	-184.180	-3.330	0.118	26.62%
NonPara VaR	(-0.411)	(0.019)	(-0.019)		(-0.662)	(-1.488)	(1.717)	(0.771)	(1.886)	(-1.214)	(-2.669)	(4.626)	
Raw Skewed	0.029	-0.360	-0.066		-1.588	-1.577	5.745	0.819	39.989	-195.006	-3.346	0.114	27.41%
Student-t VaR	(0.007)	(-0.809)	(-0.618)		(-1.269)	(-1.811)	(1.956)	(0.507)	(2.284)	(-1.256)	(-2.417)	(4.403)	
AR4 Skewed	-3.286	0.348	0.018		-0.389	-1.146	4.294	1.552	28.294	-178.398	-3.372	0.121	26.84%
Student-t VaR	(-0.721)	(0.504)	(0.187)		(-0.305)	(-1.227)	(1.587)	(0.944)	(1.724)	(-1.192)	(-2.647)	(4.481)	
						Panel C: Extended Period July 1962 - June 2013							
Raw	-1.448	0.465	0.023	-15.397	-0.237	-0.527	3.893	0.319	26.687	-97.023	-1.601	0.092	10.23%
NonPara VaR	(-2.114)	(1.839)	(0.567)	(-4.112)	(-1.584)	(-1.805)	(3.021)	(2.219)	(7.884)	(-1.890)	(-2.128)	(4.073)	
AR4	-3.152	1.374	0.008	-15.106	-0.173	-0.507	3.660	0.384	28.847	-99.912	-1.624	0.094	11.50%
NonPara VaR	(-3.509)	(3.747)	(0.197)	(-3.249)	(-0.994)	(-2.045)	(2.345)	(2.400)	(7.009)	(-1.942)	(-2.486)	(4.740)	
Raw Skewed	-1.551	0.435	0.025	-13.753	-0.230	-0.526	3.895	0.323	27.067	-97.488	-1.546	0.093	10.33%
Student-t VaR	(-2.322)	(2.087)	(0.625)	(-5.291)	(-1.522)	(-1.793)	(3.021)	(2.243)	(7.888)	(-1.883)	(-2.044)	(3.973)	
AR4 Skewed	-2.904	1.047	0.013	-13.372	-0.179	-0.509	3.622	0.380	29.010	-102.933	-1.567	0.094	11.34%
Student-t VaR	(-3.351)	(3.604)	(0.321)	(-2.956)	(-1.027)	(-2.048)	(2.317)	(2.367)	(6.998)	(-1.995)	(-2.396)	(4.753)	

Table 5: Modification of BDL framework – Expanded set of state variables: MS-BDL investigation

The table reports the results of the MS-BDL framework for one month sample risk measures using the expanded set of state variables. In each Markov switching regression, market's monthly excess return at time *t+1* is regressed on $E_t(VaR_{t+1})$ and other control variables at time *t*. The first line of each regression shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). All parametric VaRs are at 99% confidence level. The sample period is July 1962 to June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Nonparam	1	-0.657	1.198	0.018	-0.223	0.253	2.187	-0.028	17.152	-47.616	-1.693	0.030	9.276	5.345
		(-0.820)	(5.130)	(0.349)	(-0.942)	(0.937)	(1.059)	(-0.157)	(2.927)	(-0.852)	(-1.722)	(0.990)		
	2	-3.116	-0.605	-0.042	-0.359	-2.246	11.306	0.941	26.852	-129.475	-0.902	0.102	24.557	2.454
		(-1.886)	(-1.883)	(-0.377)	(-0.660)	(-3.331)	(2.929)	(2.150)	(2.662)	(-0.882)	(-0.531)	(2.168)		
AR4 Nonparam	1	-2.547	2.413	-0.054	-0.165	0.211	2.821	-0.053	18.665	-32.656	-0.656	0.003	8.982	5.531
		(-2.333)	(5.233)	(-1.108)	(-0.629)	(0.787)	(1.370)	(-0.225)	(3.037)	(-0.586)	(-0.882)	(0.092)		
	2	-4.589	0.110	0.060	-0.749	-2.310	9.277	1.055	28.866	-165.632	-2.792	0.123	25.667	2.498
		(-2.076)	(0.132)	(0.601)	(-1.336)	(-3.089)	(2.471)	(2.239)	(2.813)	(-1.187)	(-1.426)	(2.693)		
Raw Skewed Student-t	1	-0.768	1.271	0.040	-0.174	0.337	2.278	-0.118	17.512	-48.281	-1.560	0.017	8.188	4.357
		(-0.883)	(5.363)	(0.722)	(-0.544)	(1.190)	(1.041)	(-0.547)	(2.270)	(-0.869)	(-1.698)	(0.614)		
	2	-2.882	-0.484	-0.031	-0.484	-1.903	10.281	0.933	25.447	-122.962	-0.929	0.092	24.193	2.639
		(-1.959)	(-1.549)	(-0.324)	(-1.075)	(-3.176)	(3.014)	(2.368)	(2.797)	(-0.981)	(-0.638)	(2.205)		
AR4 Skewed Student-t	1	-2.513	2.338	-0.020	0.032	0.296	1.442	-0.229	22.097	-40.308	-0.502	-0.005	7.412	3.451
		(-2.013)	(4.900)	(-0.378)	(0.067)	(0.959)	(0.600)	(-0.963)	(2.347)	(-0.713)	(-0.521)	(-0.136)		
	2	-4.271	-0.064	0.053	-0.705	-1.802	8.574	1.113	26.788	-131.447	-2.317	0.115	24.055	2.295
		(-1.865)	(-0.069)	(0.576)	(-1.502)	(-2.848)	(2.591)	(2.789)	(2.559)	(-0.962)	(-1.218)	(2.705)		

Table 6: Modification of BDL framework – non-*iid* measures: MS-BDL investigation

The table reports the results of the MS-BDL framework for one month sample non-*iid* risk measures using the expanded set of state variables. In each Markov switching regression, market's monthly excess return at time *t+1* is regressed on $E_t(VaR_{t+1})$ and other control variables at time *t*. The first line of each regression shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). All parametric VaRs are at 99% confidence level. The sample period is July 1962 to June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Skewed Student-t	1	-2.184	1.452	-0.094	-0.154	0.196	3.448	0.059	22.813	-32.724	-0.472	0.002	8.941	6.446
		(-2.631)	(7.023)	(-2.055)	(-0.769)	(0.783)	(1.835)	(0.289)	(4.442)	(-0.641)	(-0.647)	(0.074)		
	2	-6.011	0.511	0.082	-0.639	-2.410	8.070	1.239	31.891	-230.519	-3.201	0.139	25.952	2.585
		(-2.924)	(1.121)	(0.860)	(-1.086)	(-3.078)	(2.003)	(2.599)	(2.667)	(-1.533)	(-1.713)	(2.951)		
AR4 Skewed Student-t	1	-3.428	1.805	-0.176	-0.218	-0.717	4.459	0.454	19.948	-36.494	-0.669	0.008	9.261	3.530
		(-3.622)	(7.211)	(-3.859)	(-0.886)	(-2.335)	(2.056)	(2.239)	(3.873)	(-0.634)	(-0.938)	(0.314)		
	2	-4.052	0.224	0.187	-0.312	-0.251	5.305	0.404	37.040	-124.934	-2.384	0.081	17.390	1.913
		(-2.398)	(0.538)	(1.953)	(-0.838)	(-0.442)	(1.788)	(1.060)	(4.117)	(-1.072)	(-1.700)	(2.017)		

Table 7: Volatility clustering in turbulent periods

The table reports how volatility induces volatility clustering. Volatility clustering is represented by the product of market's daily standard deviations in month *t+1* and *t*+2 and is regressed on daily variance of month *t*, with or without the state variables at different timings. Within each regression, the first line shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). The sample period is July 1962 to June 2013.

	Const	Conditional Variance	Market return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Regression with no state variable	0.308	0.636											49.68%
	(6.051)	(10.227)											
Regression with state variables at *t*	0.884	0.506	-0.054	-0.932	-0.099	-0.029	-0.026	-0.119	-3.886	10.032	0.015	-0.020	56.48%
	(3.680)	(7.735)	(-3.169)	(-2.535)	(-2.026)	(-0.670)	(-0.074)	(-2.589)	(-2.089)	(0.794)	(0.107)	(-1.197)	
Regression with state variables at *t*+1	0.709	0.540	-0.057	6.112	-0.068	-0.016	1.992	-0.137	-2.446	37.141	-0.341	-0.023	69.50%
	(4.570)	(10.186)	(-2.704)	(15.097)	(-1.806)	(-0.265)	(1.712)	(-3.804)	(-3.507)	(3.320)	(-1.361)	(-1.665)	
Regression with state variables at *t*+2	0.691	0.601	-0.016	1.551	-0.070	0.050	2.197	-0.115	-2.385	15.067	-0.125	-0.022	60.35%
	(5.666)	(13.303)	(-1.089)	(5.073)	(-1.893)	(0.875)	(2.324)	(-3.521)	(-2.752)	(1.496)	(-1.044)	(-1.362)	

Table 8: Modified measures: MS-BDL investigation

The table reports the results of the MS-BDL framework for the modified measures. *iid* and non-*iid* are the types of one-month raw VaRs which are used to estimate the corresponding modified measures according to formula (8). In each Markov switching regression, market's monthly excess return at time *t+1* is regressed on the modified $E_t(VaR_{t+1})$ measures and other control variables at time *t*. The first line of each regression shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). All parametric VaRs are at 99% confidence level. The sample period is July 1962 to June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
iid	1	-4.415	4.144	-0.222	-0.527	0.048	4.853	0.231	5.669	-57.487	0.200	-0.015	5.433	4.127
Nonparam		(-3.206)	(6.144)	(-4.184)	(-1.928)	(0.126)	(2.543)	(1.113)	(0.777)	(-0.999)	(0.308)	(-0.652)		
	2	-6.279	1.905	-0.044	-0.316	-0.959	5.125	0.701	32.335	-130.926	-2.757	0.135	20.893	4.917
		(-4.251)	(3.152)	(-0.704)	(-1.072)	(-2.454)	(2.111)	(2.460)	(5.309)	(-1.481)	(-2.361)	(4.307)		
non-*iid*	1	-3.084	2.728	-0.203	-0.501	0.096	4.940	0.188	5.691	-34.138	0.150	-0.013	5.812	4.579
Parametric		(-2.411)	(5.365)	(-3.849)	(-1.792)	(0.281)	(2.504)	(0.890)	(0.725)	(-0.594)	(0.210)	(-0.551)		
Skewed-t	2	-5.875	1.454	-0.037	-0.370	-1.023	4.717	0.711	31.800	-147.208	-2.872	0.136	21.618	5.167
		(-3.912)	(2.824)	(-0.585)	(-1.252)	(-2.605)	(1.920)	(2.458)	(5.123)	(-1.633)	(-2.407)	(4.332)		
non-*iid*	1	-1.965	1.683	-0.148	-0.392	0.236	5.002	-0.064	16.194	-12.016	-0.186	-0.006	7.897	5.032
Parametric		(-2.062)	(5.695)	(-3.037)	(-1.827)	(0.868)	(2.496)	(-0.306)	(2.225)	(-0.244)	(-0.293)	(-0.263)		
Skewed-t	2	-7.063	1.194	0.008	-0.196	-1.793	7.237	1.137	34.347	-200.391	-3.477	0.139	23.240	3.115
		(-3.774)	(2.445)	(0.119)	(-0.456)	(-3.100)	(2.348)	(2.933)	(3.668)	(-1.677)	(-2.255)	(3.554)		

Table 9: Modification of BDL framework – non-*iid* GJR-GARCH measures: MS-BDL investigation

The table reports the results of the MS-BDL framework for GJR-GARCH non-*iid* tail risk measures using Skewed Student-t distribution assumption for the location-scale VaR model's residuals. Market's monthly excess return at time *t+1* is regressed on a tail risk measure $E_t(VaR_{t+1})$ and other control variables at time *t*. Within each regression, the first line shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). All parametric VaRs are at 99% confidence level. The sample period is July 1962 to June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw	1	-2.297	1.518	-0.043	-0.152	0.288	3.442	0.072	24.013	-44.800	-0.424	0.002	8.817	5.984
Skewed		(-2.720)	(6.928)	(-0.938)	(-0.735)	(1.136)	(1.827)	(0.366)	(4.494)	(-0.877)	(-0.646)	(0.123)		
Student-t	2	-5.414	0.335	0.085	-0.654	-2.487	8.274	1.156	29.143	-196.573	-3.150	0.133	25.612	2.552
		(-2.710)	(0.737)	(0.887)	(-1.127)	(-3.242)	(2.103)	(2.516)	(2.544)	(-1.347)	(-1.716)	(2.879)		
AR4	1	-2.936	1.858	-0.041	-0.158	0.337	3.845	0.069	24.056	-50.548	-0.421	0.003	8.618	5.721
Skewed		(-3.181)	(6.930)	(-0.874)	(-0.729)	(1.312)	(1.985)	(0.341)	(4.313)	(-0.961)	(-0.618)	(0.109)		
Student-t	2	-5.097	0.291	0.069	-0.623	-2.402	8.236	1.087	28.767	-180.199	-3.161	0.131	25.219	2.645
		(-2.483)	(0.548)	(0.738)	(-1.144)	(-3.292)	(2.179)	(2.451)	(2.618)	(-1.264)	(-1.777)	(2.934)		
Modified	1	-2.355	1.898	-0.147	-0.408	0.252	4.804	-0.049	17.169	-16.194	-0.108	-0.006	7.655	5.240
Skewed		(-2.210)	(5.546)	(-2.987)	(-1.804)	(0.899)	(2.376)	(-0.194)	(1.922)	(-0.312)	(-0.164)	(-0.262)		
Student-t	2	-7.129	1.267	-0.004	-0.228	-1.723	6.670	1.114	32.903	-185.564	-3.499	0.141	23.183	3.466
		(-3.807)	(2.623)	(-0.074)	(-0.553)	(-3.077)	(2.226)	(2.925)	(3.887)	(-1.600)	(-2.346)	(3.794)		

Table 10: ETL measures: MS-BDL investigation

The table reports the results of the MS-BDL framework for Gaussian-ETL tail risk measures. Market's monthly excess return at time *t+1* is regressed on a tail risk measure $E_t(ETL_{t+1})$ and other control variables at time *t*. Within each regression, the first line shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). All parametric ETL are at 99% confidence level. The sample period is July 1962 to June 2013.

Measure	State	Const	$E_t(ETL_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
						Panel A: *iid* measures								
	1	-1.501	1.354	0.020	-0.160	0.369	2.827	-0.020	23.441	-72.255	-0.608	0.004	8.612	5.193
Raw *iid*		(-1.916)	(6.829)	(0.396)	(-0.704)	(1.424)	(1.409)	(-0.112)	(4.253)	(-1.381)	(-0.668)	(0.163)		
	2	-3.165	-0.395	-0.013	-0.535	-2.340	10.466	0.935	23.827	-124.926	-2.335	0.102	24.736	2.536
		(-1.930)	(-1.267)	(-0.122)	(-0.957)	(-3.478)	(2.633)	(2.329)	(2.313)	(-0.911)	(-1.007)	(2.245)		
	1	-2.700	1.970	-0.018	-0.145	0.365	3.039	-0.046	24.208	-66.155	-0.520	0.003	8.261	4.944
AR4		(-2.963)	(6.962)	(-0.396)	(-0.605)	(1.417)	(1.461)	(-0.231)	(4.120)	(-1.249)	(-0.709)	(0.118)		
iid	2	-3.624	-0.245	0.030	-0.590	-2.260	9.249	0.988	25.185	-141.181	-2.516	0.111	24.624	2.565
		(-1.943)	(-0.504)	(0.307)	(-1.132)	(-3.385)	(2.533)	(2.466)	(2.509)	(-1.071)	(-1.326)	(2.587)		
	1	-2.503	2.129	-0.172	-0.508	0.201	4.322	0.018	10.370	-36.329	0.039	-0.005	7.220	5.931
Modified		(-2.233)	(5.233)	(-3.289)	(-2.308)	(0.702)	(2.199)	(0.098)	(1.303)	(-0.682)	(0.106)	(-0.253)		
iid	2	-7.450	1.669	-0.027	-0.215	-1.495	5.401	0.953	33.060	-176.433	-3.428	0.154	22.673	4.515
		(-4.275)	(3.587)	(-0.385)	(-0.602)	(-2.911)	(1.955)	(2.841)	(4.755)	(-1.672)	(-2.572)	(4.449)		

(*Continued*)

Table 10: Continued

Measure	State	Const	$E_t(ETL_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: non-*iid* measures														
	1	-3.008	1.590	-0.173	-0.222	-0.672	4.170	0.392	20.874	-32.781	-0.695	0.007	9.449	3.924
Raw		(-3.468)	(7.411)	(-3.865)	(-0.969)	(-2.231)	(2.017)	(2.013)	(4.167)	(-0.592)	(-1.012)	(0.301)		
non-*iid*	2	-4.169	0.223	0.188	-0.349	-0.311	5.420	0.412	38.241	-135.354	-2.459	0.081	17.872	1.884
		(-2.528)	(0.605)	(1.892)	(-0.871)	(-0.507)	(1.745)	(1.045)	(3.989)	(-1.112)	(-1.661)	(1.939)		
	1	-3.674	1.915	-0.175	-0.226	-0.647	4.607	0.411	20.902	-42.473	-0.722	0.009	9.318	3.733
AR(4)		(-3.877)	(7.400)	(-3.872)	(-0.977)	(-2.140)	(2.219)	(2.105)	(4.059)	(-0.756)	(-1.026)	(0.363)		
non-*iid*	2	-3.937	0.179	0.182	-0.333	-0.324	5.338	0.370	36.985	-121.012	-2.390	0.078	17.638	1.907
		(-2.302)	(0.414)	(1.853)	(-0.857)	(-0.540)	(1.764)	(0.973)	(4.038)	(-1.020)	(-1.660)	(1.948)		
	1	-2.308	1.766	-0.149	-0.387	0.177	5.100	-0.076	17.374	-11.172	-0.220	-0.005	8.295	5.485
Modified		(-2.255)	(5.504)	(-3.019)	(-1.822)	(0.655)	(2.592)	(-0.327)	(2.504)	(-0.198)	(-0.314)	(-0.179)		
non-*iid*	2	-7.530	1.359	0.007	-0.210	-1.827	7.508	1.146	35.483	-216.296	-3.675	0.141	23.668	3.077
		(-3.797)	(2.544)	(0.091)	(-0.441)	(-3.036)	(2.328)	(2.850)	(3.583)	(-1.733)	(-2.265)	(3.468)		

Table 11: 95 percent Skewed Student-t VaR measures: MS-BDL investigation

The table reports the results of the MS-BDL framework for 95 percent Skewed Student-t VaR measures. Market's monthly excess return at time *t+1* is regressed on a tail risk measure $E_t(VaR_{t+1})$ and other control variables of time *t*. Within each regression, the first line shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). The sample period is July 1962 to June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel A: *iid* measures							
	1	-1.162	2.160	0.077	-0.082	0.417	2.319	-0.080	21.394	-70.681	-0.598	0.006	8.136	4.126
Raw *iid*		(-1.334)	(6.374)	(1.428)	(-0.294)	(1.532)	(1.034)	(-0.363)	(3.142)	(-1.277)	(-0.561)	(0.214)		
	2	-3.248	-0.634	-0.007	-0.632	-2.122	9.535	0.960	24.150	-101.546	-2.237	0.102	24.025	2.418
		(-2.098)	(-1.281)	(-0.110)	(-1.254)	(-3.256)	(2.548)	(2.443)	(2.587)	(-0.790)	(-0.908)	(2.257)		
	1	-0.778	3.328	0.080	-0.169	0.343	-0.410	-0.749	21.898	-26.518	-1.453	0.031	4.423	2.355
AR4		(-0.846)	(8.281)	(1.672)	(-0.726)	(1.308)	(-0.227)	(-3.358)	(3.996)	(-0.530)	(-1.904)	(1.329)		
iid	2	-3.369	-0.212	0.003	-0.374	-1.158	7.498	1.036	26.294	-124.415	-0.796	0.090	22.176	3.010
		(-2.549)	(-0.344)	(0.115)	(-1.324)	(-2.646)	(2.916)	(3.874)	(3.892)	(-1.376)	(-0.764)	(2.714)		
	1	-2.371	3.512	-0.195	-0.534	0.164	4.766	0.176	5.218	-38.131	0.176	-0.008	6.143	5.135
Modified		(-1.852)	(4.845)	(-3.503)	(-2.199)	(0.514)	(2.445)	(0.785)	(0.685)	(-0.640)	(0.273)	(-0.348)		
iid	2	-6.297	2.413	-0.042	-0.326	-1.107	4.399	0.757	32.640	-158.650	-2.994	0.143	21.906	5.387
		(-4.099)	(3.275)	(-0.662)	(-1.070)	(-2.696)	(1.779)	(2.563)	(5.148)	(-1.722)	(-2.487)	(4.552)		

(*Continued)*

Table 11: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel B: non-*iid* measures							
	1	-2.042	2.302	-0.091	-0.208	0.174	3.600	0.056	23.163	-47.983	-0.423	-0.001	8.944	6.464
Raw		(-2.485)	(6.979)	(-1.972)	(-1.044)	(0.688)	(1.923)	(0.262)	(4.480)	(-0.939)	(-0.668)	(-0.064)		
non-*iid*	2	-5.913	0.857	0.085	-0.618	-2.379	8.000	1.191	32.095	-232.726	-3.234	0.142	25.947	2.609
		(-2.956)	(1.177)	(0.879)	(-1.062)	(-3.024)	(1.996)	(2.553)	(2.684)	(-1.567)	(-1.752)	(3.037)		
	1	-2.499	2.647	-0.081	-0.198	0.187	3.765	0.058	23.080	-54.480	-0.441	0.001	9.057	6.657
AR(4)		(-2.992)	(7.065)	(-1.752)	(-1.009)	(0.723)	(2.058)	(0.291)	(4.527)	(-1.053)	(-0.700)	(0.068)		
non-*iid*	2	-5.621	0.722	0.081	-0.657	-2.450	8.532	1.144	31.025	-217.650	-3.195	0.137	26.214	2.625
		(-2.590)	(0.802)	(0.823)	(-1.108)	(-3.065)	(2.093)	(2.428)	(2.588)	(-1.428)	(-1.720)	(2.905)		
	1	-1.868	2.674	-0.153	-0.444	0.198	5.116	-0.056	16.228	-24.178	-0.143	-0.010	7.948	5.101
Modified		(-1.928)	(5.765)	(-3.081)	(-2.061)	(0.710)	(2.581)	(-0.223)	(2.098)	(-0.429)	(-0.185)	(-0.376)		
non-*iid*	2	-6.943	1.927	0.010	-0.187	-1.783	7.308	1.087	34.586	-201.353	-3.485	0.142	23.246	3.115
		(-3.645)	(2.395)	(0.097)	(-0.411)	(-2.980)	(2.353)	(2.737)	(3.661)	(-1.667)	(-2.215)	(3.464)		

Table 12: Modified measures with conditional variance: MS-BDL investigation

The table reports the results of the MS-BDL framework where both the modified measures and the conditional variance are included. Market's monthly excess return at time *t+1* is regressed on a modified measure $E_t(VaR_{t+1})$, the conditional variance, and other control variables at time *t*. Within each regression, the first line shows the estimated regression coefficients, the second line shows their t-statistics (in parentheses). All parametric VaRs are at 99% confidence level. The sample period is July 1962 to June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Conditional variance	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
iid	1	-4.594	4.295	-0.067	-0.217	-0.526	0.036	4.791	0.230	5.682	-57.771	0.210	-0.015	5.411	4.180
Nonparam		(-2.704)	(3.605)	(-0.147)	(-4.106)	(-1.952)	(0.089)	(2.539)	(1.097)	(0.797)	(-0.977)	(0.296)	(-0.637)		
	2	-6.586	2.146	-0.086	-0.045	-0.321	-0.964	5.063	0.694	32.170	-126.675	-2.649	0.132	21.014	4.943
		(-3.663)	(2.120)	(-0.326)	(-0.692)	(-1.088)	(-2.461)	(2.066)	(2.402)	(5.260)	(-1.409)	(-2.167)	(4.011)		
iid	1	-2.715	2.350	0.295	-0.197	-0.463	0.073	4.834	0.193	5.621	-28.392	0.098	-0.013	5.915	4.719
Skewed		(-1.803)	(2.936)	(0.719)	(-3.723)	(-1.746)	(0.210)	(2.437)	(0.930)	(0.760)	(-0.497)	(0.200)	(-0.572)		
Student-t	2	-5.918	1.484	-0.010	-0.037	-0.374	-1.025	4.791	0.711	31.960	-150.363	-2.829	0.135	21.870	5.140
		(-3.684)	(2.576)	(-0.075)	(-0.591)	(-1.242)	(-2.569)	(1.966)	(2.411)	(5.145)	(-1.649)	(-2.334)	(4.236)		
non-*iid*	1	-3.460	2.638	-0.653	-0.159	-0.374	0.297	5.453	-0.051	19.330	-19.981	-0.105	-0.007	7.678	4.885
Skewed		(-2.396)	(3.410)	(-1.306)	(-3.300)	(-1.781)	(1.069)	(2.846)	(-0.349)	(2.571)	(-0.401)	(-0.362)	(-0.315)		
Student-t	2	-6.913	1.148	-0.019	0.013	-0.177	-1.807	7.120	1.118	32.621	-186.364	-3.396	0.137	22.996	3.154
		(-3.948)	(2.524)	(-0.080)	(0.212)	(-0.417)	(-3.183)	(2.325)	(3.168)	(3.520)	(-1.596)	(-2.353)	(3.685)		