BANK OF ENGLAND

Staff Working Paper No. 553

Some unpleasant properties of loglinearized solutions when the nominal rate is zero

R Anton Braun, Lena Boneva and Yuichiro Waki

September 2015

BANK OF ENGLAND

Staff Working Paper No. 553

Some unpleasant properties of loglinearized solutions when the nominal rate is zero

R Anton Braun,[1] Lena Boneva[2] and Yuichiro Waki[3]

Abstract

Does fiscal policy have large and qualitatively different effects on the economy when the nominal interest rate is zero? An emerging consensus in the New Keynesian (NK) literature is that the answer to this question is yes. Evidence presented here suggests that the NK model's implications for fiscal policy at the zero bound may not be all that different from its implications for policy away from it. For a range of empirically relevant parameterizations, employment increases when the labour tax rate is cut and the government purchase multiplier is less than 1.05.

Key words: Zero lower bound, fiscal policy, New Keynesian model.

JEL classification: E52, E62.

---

(1) Federal Reserve Bank of Atlanta. Email: r.anton.braun@gmail.com
(2) Bank of England and London School of Economics. Email: lena.koerber@bankofengland.co.uk
(3) University of Queensland. Email: y.waki@uq.edu.au

The views expressed in this paper are those of the authors, and not necessarily those of the Federal Reserve System or of the Bank of England. This paper was previously circulated under the title of 'Small and orthodox fiscal multipliers at the zero bound'. We thank participants from seminars at Australian National University, Bank of Japan, Bank of Portugal, European Central Bank, Federal Reserve Bank of Atlanta, University of Tokyo, University of Groningen, London School of Economics and Vanderbilt University and conference participants at the 7th Dynare Conference, 29th Annual Congress of the European Economic Association, 19th Spring Meeting of Young Economists and the 20th Conference in Computing in Economics and Finance for their comments. We would also like to thank Larry Christiano, Isabel Correira, Luca Fornaro, Pedro Teles and Tao Zha for their helpful comments.

Information on the Bank's working paper series can be found at
www.bankofengland.co.uk/research/Pages/workingpapers/default.aspx

Publications Team, Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 7601 4030 Fax +44 (0)20 7601 3298 email publications@bankofengland.co.uk

ISSN 1749-9135 (on-line)

1 Introduction

The recent experiences of Japan, the United States and Europe with zero/near-zero nominal interest rates have raised new questions about the conduct of monetary and fiscal policy in a liquidity trap. A large and growing body of new research has emerged that provides answers using New Keynesian (NK) frameworks that explicitly model the zero bound on the nominal interest rate. Modeling the zero bound on the nominal interest rate is particularly important in the NK model because the interest rate policy of the monetary authority plays a central role in stabilizing the economy. Very low nominal interest rates constrain the ability of monetary policy to respond to shocks and this may result in macroeconomic instability.

Recent research has found that fiscal policy has very different effects on the economy when the nominal interest rate is zero. Eggertsson (2011) finds that employment falls in response to a cut in the labor tax rate, a property that he refers to as the "paradox of toil." Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011) conclude that the size of the government purchase multiplier is close to two or even larger. These results have sharp implications for the conduct of fiscal policy in low interest rate environments. If supply-side stimulus is contractionary and demand-side fiscal policies are particularly potent then governments should rely exclusively on demand side fiscal stimulus when the central bank's actions are constrained by the zero lower bound.

This paper proposes and solves a tractable stochastic nonlinear NK model that honors the zero bound on the nominal interest rate and that also reproduces the large output and small inflation declines observed during the U.S. Great Recession. We encounter some parameterizations of the model that are consistent with previous results. However, the novel contribution of our paper is that we find other empirically relevant parameterizations of the model where the government purchase multiplier is about one or less and the response of employment to a cut in the labor tax rate is positive.

These new findings are important because they raise the possibility that there might also be a role for using supply side policies to stabilize the economy in low interest rate environments. On the one hand, the case for demand side measures is weaker since their efficacy is small. On the other hand, the case for supply-side measures is stronger because they are expansionary.

Why are the results presented here different from previous findings? One reason is the solution method. Previous results are based on a solution method that models the nonlinearity induced by the zero bound on the nominal interest rate but loglinearizes the other equilibrium conditions about a zero inflation steady-state. This solution method zeroes out the resource costs of price adjustment which affects the local dynamics of the model at the zero bound. A comparison of loglinear (LL) and nonlinear (NL) solutions reveals that the LL solution sometimes incorrectly predicts that supply side stimulus is contractionary when in fact it is expansionary.

A second and distinct reason for our findings is the parameterization of the model. The GR was associated with a 7% decline in GDP but only a 1% decline in the annualized inflation rate (see Christiano, Eichenbaum, and Rebelo (2011)). We calibrate the model to these targets and this has implications for the size of the government purchase multiplier. Intuitively, government purchases are primarily a demand shifter and reproducing the GR targets results in a relatively flat aggregate supply schedule. At the zero bound, the government purchase multiplier can still be large in this situation. Indeed, Woodford (2011) has found that the government purchase multiplier can be arbitrarily large in the neighborhood of a point that can be indexed by the expected duration of zero interest rates.[1] This region of the parameter space is small under our calibration scheme. If the expected duration of zero interest rates exceeds 7 quarters or is less than 5 quarters, the government purchase multiplier is small using either the LL or the NL equilibrium conditions.

These points are made in a nonlinear stochastic NK model with Rotemberg (1982) quadratic price adjustment costs. Rotemberg adjustment costs are widely used when studying the zero lower bound (Benhabib, Schmitt-Grohe, and Uribe (2001), Evans, Guse, and Honkapohja (2008), Aruoba and Schorfheide (2013), Eggertsson, Ferrero, and Raffo (2013)) because the dimension of the state space is small. In our setup, output and inflation in the zero lower bound state solve a system of two nonlinear equations, which are the nonlinear analogues of what Eggertsson and Krugman (2012) refer to as "aggregate demand" and "aggregate supply" schedules. Some merits of this approach are that we can provide a graphical representation of the NL equilibrium conditions, an analytical characterization of the model's key properties and an easy and accurate strategy for computing all equilibria. This final merit is important because we encounter multiple zero bound equilibria. LL solution methods, in contrast, have the property that aggregate supply and aggregate demand have a single crossing point at the zero bound.

Many NK models use Calvo price adjustment instead. The standard LL solution method also zeroes out the resource costs of price dispersion in Calvo models of price adjustment. Omitting this term can also create similar biases under Calvo price adjustment. Section 5.3 and the Online Appendix provide evidence that this is the case using a tractable (but stylized) model of Calvo price adjustment. In this sense, our findings are likely to apply to a large class of NK models.

Our research is closest to research by Christiano and Eichenbaum (2012) who consider related questions in a similar model. They show that imposing a particular form of E-learnability rules out one of the two equilibria that occur in their model, and find that the qualitative properties of the remaining equilibrium are close to the LL solution. Our main

[1] Intuitively, the government purchase multiplier is large because the aggregate demand and aggregate supply schedules are nearly parallel.

conclusions about the size and sign of fiscal multipliers do not rely on multiplicity of zero bound equilibrium. In fact, some of our most interesting results occur in regions of the parameter space where equilibrium is unique and the aggregate demand and aggregate supply schedules have their conventional slopes. We also describe some problems with applying their E-learning equilibrium selection strategy. It does not omit all forms of multiplicity and sometimes selects equilibria that are not empirically relevant while ruling out other equilibria that reproduce observations from the GR.

Our research is also related to recent work by Mertens and Ravn (2010) who consider zero bound sunspot equilibria. A major advantage of our setup is that it is straightforward to find all equilibria by finding the zeros of an equation. We encounter new cases of multiplicity, most significantly the possibility of multiple zero bound equilibria. Ascertaining the presence of multiple zero bound equilibria is a daunting task in richer NK models such as those considered by Gust, Lopez-Salido, and Smith (2012), Aruoba and Schorfheide (2013) or Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez (2012). Results presented here offer guidance about the regions of the parameter/shock space where multiplicity is most likely to arise in medium-scale NK models.

The remainder of our analysis proceeds in the following way. Section 2 describes the model and equilibrium concept. Section 3 explains how the model is parameterized. Section 4 characterizes equilibrium using the NL and the LL equilibrium restrictions. Section 5 documents that fiscal multipliers may be small and orthodox at the zero lower bound. Finally, Section 6 concludes.

2 Model and equilibrium

We consider a stochastic NK model with Rotemberg (1982) quadratic costs of price adjustment faced by intermediate goods producers. Monetary policy follows a Taylor rule when the nominal interest rate is positive but is restricted from falling below zero. The equilibrium analyzed here is the Markov equilibrium proposed by Eggertsson and Woodford (2003).

2.1 The model

Households The representative household chooses consumption c_t, labor supply h_t, and bond holdings b_t to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} d_j \right) \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\nu}}{1+\nu} \right\} \tag{1}$$

subject to the budget constraint

$$b_t + c_t = \frac{b_{t-1}(1 + R_{t-1})}{1 + \pi_t} + (1 - \tau_{w,t})w_t h_t + T_t. \tag{2}$$

where ν governs the elasticity of labor supply and σ is the curvature parameter for consumption.[2] R_t and π_t are the net nominal interest rate and the net inflation rate, respectively, and the after-tax real wage is $(1 - \tau_{w,t})w_t$. The preference discount factor from period t to $t + 1$ is βd_{t+1}, and d_t is a preference shock. We assume that the value of d_{t+1} is revealed at the beginning of period t. The variable T_t includes transfers from the government and profit distributions from the intermediate producers. The optimality conditions for consumption and labor supply choices are

$$c_t^\sigma h_t^\nu = w_t(1 - \tau_{w,t}), \tag{3}$$

and

$$1 = \beta d_{t+1} E_t \left\{ \frac{1 + R_t}{1 + \pi_{t+1}} \left(\frac{c_t}{c_{t+1}} \right)^\sigma \right\}. \tag{4}$$

Final good producers Perfectly competitive final good firms use a continuum of intermediate goods $i \in [0, 1]$ to produce a single final good with the technology: $y_t = [\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}$. The profit maximizing input demands for final goods firms are

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} y_t, \tag{5}$$

where $P_t(i)$ denotes the price of the good produced by firm i and P_t the price of the final good. Thus $\pi_t = P_t/P_{t-1} - 1$. The price of the final good satisfies $P_t = [\int_0^1 P_t(i)^{1-\theta} di]^{1/(1-\theta)}$.

Intermediate goods producers Intermediate good i is produced according to $y_t(i) = z_t h_t(i)$, where z_t, the state of technology, is common to all producers. Labor is homogeneous and thus real marginal cost for all firms is w_t/z_t. Producer i sets prices to maximize

$$E_0 \sum_{t=0}^{\infty} \lambda_{c,t} \left[(1 + \tau_s) \frac{P_t(i)}{P_t} y_t(i) - \frac{w_t}{z_t} y_t(i) - \frac{\gamma}{2} \big(\frac{P_t(i)}{P_{t-1}(i)} - 1 \big)^2 y_t \right] \tag{6}$$

subject to the demand function (5). Producers take the stochastic discount factor, $\lambda_{c,t} \equiv \beta^t (\prod_{j=0}^t d_j) c_t^{-\sigma}$, as given. The sales subsidy τ_s satisfies $(1 + \tau_s)(\theta - 1) = \theta$, or that profits are zero in a steady-state with zero inflation. The final term in brackets is the cost of price adjustment. We assume it is proportional to aggregate production y_t, so that the

[2]Our preferences over leisure make no distinction between the number of days worked in a period and the number of hours worked per day. Formally, we are treating the two margins as perfect substitutes from the perspective of the representative household.

share of price adjustment costs in the aggregate production depends only on inflation. The optimality condition for intermediate producers in a symmetric equilibrium with $(P_t(i), y_t(i), h_t(i)) = (P_t, y_t, h_t)$ for all i is

$$\pi_t(1+\pi_t) = \frac{\theta}{\gamma}\left(\frac{w_t}{z_t} - 1\right) + \beta d_{t+1} E_t \left\{ \left(\frac{c_t}{c_{t+1}}\right)^{\sigma} \frac{y_{t+1}}{y_t} \pi_{t+1}(1+\pi_{t+1}) \right\}. \tag{7}$$

Monetary policy Monetary policy follows a Taylor rule that respects the zero lower bound on the nominal interest rate:

$$R_t = \max(0, r_t^e + \phi_\pi \pi_t + \phi_y \widehat{gdp}_t), \tag{8}$$

where $r_t^e \equiv 1/(\beta d_{t+1}) - 1$ and $\widehat{gdp}_t$ is the log deviation of GDP from its steady-state value.[3]

The aggregate resource constraint is given by

$$c_t = (1 - \kappa_t - \eta_t) y_t, \tag{9}$$

where $\kappa_t \equiv (\gamma/2)\pi_t^2$ is the resource cost of price adjustment and where government purchases are $g_t = \eta_t y_t$. GDP in our economy, gdp_t, is

$$gdp_t \equiv (1 - \kappa_t) y_t = c_t + g_t. \tag{10}$$

This definition of GDP assumes that the resource costs of price adjustment are intermediate inputs and are consequently subtracted from gross output when calculating GDP.

The term κ_t plays a central role in the analysis that follows. Section 4 shows that loglinearizing equation (10) around a zero inflation steady-state can result in incorrect inferences about the local dynamics of this economy at the zero lower bound and relates this result to κ_t. Whenever the inflation rate changes, κ_t also changes and (10) implies that GDP and labor input h_t move differently, possibly even in opposite directions. However, if equation (10) is loglinearized about a zero inflation steady-state κ_t disappears and GDP and labor input are identical. A term like κ_t occurs in many NK models. For instance, the resource cost of price dispersion is an analogous term that appears in the resource constraint under Calvo pricing (see Yun (2005)). Thus, loglinearizing the resource constraint about a zero inflation rate under Calvo pricing creates the same potential biases. We present results for a model with Calvo price setting in Section 5.3 that illustrate this point.

[3]The assumption that monetary policy responds directly to variations in d_t is made to facilitate comparison with other papers in the literature.

2.2 Markov equilibrium with zero interest rates

Following Eggertsson and Woodford (2003), we analyze the zero bound using a two state Markov equilibrium concept. Let s_t denote the state of the economy which is either low or high, $s_t \in \{L, H\}$. The initial state, s_0, is L and s_t evolves according to a time-homogeneous Markov rule. The transition probability from state L to L is $p < 1$ and H is an absorbing state. All exogenous variables including the preference shock d_{t+1}, technology shock z_t, and fiscal policy $\{\tau_{w,t}, \eta_t\}$ change if and only if s_t changes: $\{d_{t+1}, z_t, \tau_{w,t}, \eta_t\}$ equals $\{d^L, z^L, \tau_w^L, \eta^L\}$ when $s_t = L$, and $\{1, z, \tau_w, \eta\}$ when $s_t = H$.

Under these assumptions, the equilibrium is characterized by two distinct values for prices and quantities. Prices and quantities in state L are denoted with the superscript L and prices and quantities in state H have no superscript. In state H the economy rests in a steady-state with a zero inflation rate and a positive nominal interest rate. More formally, $h = \{(1-\tau_w)/(z^{\sigma-1}(1-\eta)^\sigma)\}^{1/(\sigma+\nu)}$ and $\pi = 0$ if $s_t = H$.

Equilibria with a zero nominal interest rate in state L are subsequently referred to as *zero bound equilibria*.[4]

2.3 Employment and inflation in a zero bound equilibrium

An attractive feature of the model is that the equilibrium conditions for employment and inflation state L can be summarized by two equations in these two variables.[5] These equations are nonlinear versions of what Eggertsson and Krugman (2012) refer to as "aggregate supply" (AS) and "aggregate demand" (AD) schedules. In what follows, we adopt the same shorthand when referring to these equations.

The AS schedule summarizes intermediate goods firms' price setting decisions, the household's intratemporal first order condition, and the aggregate resource constraint. To obtain the AS schedule, start with (7) and substitute out the real wage using (3). Then use (9) to replace consumption with labor input. In a zero bound Markov equilibrium, the AS schedule in state L is

$$\pi^L(1+\pi^L) = \frac{\theta}{\gamma(1-p\beta d^L)}\left[\frac{\left(1-\kappa^L-\eta^L\right)^\sigma (h^L)^{\sigma+\nu}}{(1-\tau_w^L)(z^L)^{1-\sigma}} - 1\right] \tag{11}$$

where $\kappa^L = (\gamma/2)(\pi^L)^2$.

The AD schedule summarizes the household's Euler equation and the resource constraint. It is obtained by substituting consumption out of the household's intertemporal

[4] Other restrictions are that $p\beta d^L < 1$, to guarantee that utility is finite, and that prices and quantities are non-negative.

[5] In this model there is no meaningful distinction between hours and employment. Henceforth the term employment is used because employment is the focal point of policy discussions. Also we express the schedules in terms of employment rather than output because it is easier to ascertain the response of employment to a cut in the labor tax rate.

Euler equation (4) using the resource constraint (9). The resulting AD schedule in a zero bound Markov equilibrium in state L is

$$\frac{h^L}{h} = \frac{z}{z^L} \frac{1-\eta}{1-\kappa^L-\eta^L} \left(\frac{1-p\beta d^L/(1+\pi^L)}{(1-p)\beta d^L} \right)^{\frac{1}{\sigma}}. \tag{12}$$

3 Parameterization of the model

A principal claim of this paper is that LL solutions of the NK model can break down in empirically relevant situations. This section describes our strategy for producing empirically relevant parameterizations of the model.

3.1 Parameters that are not specific to the zero bound

An object of central interest is the slope of the conventional loglinear New Keynesian Phillips Curve which is given by $slope(NKPC) \equiv \theta(\sigma + \nu)/\gamma$. Its value influences the slope of the AS schedule using both the LL and the NL equilibrium conditions. There are many combinations of these parameters that can make $slope(NKPC)$ big or small. The baseline parameterization of the model fixes some of these parameters and estimates others using Bayesian methods.

Preferences over consumption are assumed to be logarithmic ($\sigma = 1$) because this is a common reference point in the DSGE literature. It is also well known that β is not well identified in DSGE models. Consequently, β is fixed at 0.997 which implies an annual rate of time preference of 1.2% (see also Denes, Eggertsson, and Gilbukh (2013)). The parameter θ is set to 7.67, which implies a markup of 15%.[6] This choice of θ lies midway between previous estimates from disaggregate and aggregate data. Broda and Weinstein (2004) find that the median value of this parameter ranges from 3 to 4.3 using 4-digit industry level data for alternative country pairs. Denes, Eggertsson, and Gilbukh (2013) estimate θ to be about 13 in a NK model that is similar to ours.

The government purchase share parameter η is fixed at 0.2 and the labor tax rate τ_w at 0.2. This leaves ν, the curvature parameter for leisure and γ, the adjustment cost parameter and the coefficients of the Taylor rule. These parameters are estimated using Bayesian methods on quarterly U.S. data on inflation, the output gap and the Federal Funds rate over a sample period that extends from 1985:I through 2007:IV. The estimated posterior mode of γ is 458.4 and 90% of its posterior mass lies between 315 and 714. This value is larger than Ireland (2003) who estimates a value of 162, and Gust, Lopez-Salido, and Smith (2012) who estimate $\gamma = 94$. Experiments with a tighter prior on γ produces estimates of γ that lie in the range of 100 to 150 but most posterior mass lies outside

[6] We found that θ and γ are not individually identified by our estimation procedure. Given the central role played by γ in the NK model, we decided to fix θ and estimate γ.

of the prior. Instead of ruling out these other lower estimates of γ, we report results for values of γ that range from 100 to 600. The posterior mode for ν, which governs the curvature of the disutility of work, is 0.28 with 90% of its posterior mass lying in the interval 0.08 and 0.63. These estimates in conjunction $\theta = 7.67$ and $\sigma = 1$ imply that $slope(NKPC) = 0.0214$ which is close to the value of 0.024 estimated by Rotemberg and Woodford (1997).

Finally, the posterior modes of the Taylor rule parameters are $\phi_\pi = 3.46$ and $\phi_y = 1.63$. A complete set of estimation results can be found in Section F of the Online Appendix.

3.2 Parameters and shocks that are specific to the zero bound

The remaining parameters are the shocks in the zero bound state, $\{d^L, z^L, \tau_w^L, \eta^L\}$, and the persistence parameter p which governs the expected duration of the zero bound. This final parameter is important for the local dynamics of the model at the zero bound and results will be reported for a large range of values of $p \in [0.05, 0.95]$. For each choice of p, d^L and z^L are chosen to hit output and inflation targets from the U.S. GR.[7] The specific targets are taken from Christiano, Eichenbaum, and Rebelo (2011). They provide empirical evidence that the U.S. financial crisis that ensued after the collapse of Lehman Brothers in the third quarter of 2008 produced a decline in output of 7% and a decline in the inflation rate of 1%.

Using these targets the resource costs of price adjustment constitute 0.14% of gross output at the baseline value of γ. It is difficult to directly measure the overall magnitude of the resource cost of price adjustment but a rough idea of the potential magnitude of this cost is provided by Levy, Bergen, Dutta, and Venable (1997) who find that menu costs constitute 0.7% of revenues of supermarket chains.

4 Characterization of equilibrium

This section compares and contrasts the dynamics of the model in the zero bound state using the LL and the NL equilibrium conditions. The NK model has very rich dynamics at the zero bound and much of this richness is lost using the standard LL solution method. In particular, the AD and AS schedules can have conventional local slopes at the zero bound and multiple zero bound equilibria can occur.

[7] Fiscal policy is fixed at its steady-state value, i.e. $\{\tau_w^L, \eta^L\} = \{\tau_w, \eta\}$ and Section G of the Online Appendix shows that there is a unique mapping from these two targets to $\{d^L, z^L\}$.

4.1 Characterization of zero bound equilibria using the loglinearized equilibrium conditions

An important strand of the previous literature has worked with equilibrium conditions that are loglinearized around a perfect foresight steady-state with zero inflation and a positive nominal interest rate. We start by briefly highlighting this and several other key properties of the LL equilibrium in the zero bound state.[8]

The most salient properties of the LL solution at the zero bound are that the slopes of both the AD schedule, $slope(AD^{LL}) \equiv (1-p)\sigma/p$, and the AS schedule, $slope(AS^{LL}) \equiv slope(NKPC)/(1-p\beta)$, are positive.[9] Thus, only two types of zero bound equilibria can occur. In a *Case I* zero bound equilibrium *the AD schedule is steeper* whereas, in a *Case II* zero bound equilibrium *the AS schedule is steeper*.

An example of a Case I equilibrium is shown in Figure 1a. It reports the configuration of the AD^{LL} and AS^{LL} schedules in the low state with no shocks, L_{ns}, and in the low state with shocks to preferences and technology, L.[10] Most of the recent literature on the zero bound has focused exclusively on Case I equilibria. Equilibrium is globally unique in Case I equilibria (see Proposition 4 in Braun, Körber, and Waki (2012)). When the shocks are set to their steady-state levels, the AD and AS schedules cross at the steady-state ($R > 0$). However, the AD schedule has a kink that arises from imposing the zero bound restriction on the Taylor rule. If the shocks are instead set to hit the GR targets, the schedules cross in the region where $R = 0$ and $slope(AD^{LL})$ is larger than $slope(AS^{LL})$.

Case I equilibria are associated with relatively low values of p. As the expected duration of zero interest rates increases, AD^{LL} rotates to the right. A longer episode of zero interest rates means a longer expected duration of deflation and this in turn has a stronger contractionary effect on current demand. A larger value of p rotates AS^{LL} to the left. Firms recognizing that the expected duration of low demand is longer are willing to take on bigger price cuts (see also Eggertsson (2011)). When p is sufficiently large $slope(AS^{LL})$ becomes larger than $slope(AD^{LL})$ and the zero bound equilibrium switches to Case II.

Figure 1b provides an example of a Case II zero bound equilibrium. The AD schedule now crosses the AS schedule twice when shocks are set to their steady-state values. The upper crossing point occurs at the steady-state ($R > 0$) and a second lower crossing point occurs when $R = 0$. Bullard (2010), Mertens and Ravn (2010) and Aruoba and Schorfheide (2013) introduce sunspot variables into Case II equilibria that allow for switches from the steady-state equilibrium to the zero bond equilibrium with no shocks to fundamentals.

[8] A more complete discussion of the properties of LL solutions at the zero bound can be found in Woodford (2011) and Braun, Körber, and Waki (2012).

[9] These slope definitions are specific to the situation where $R = 0$.

[10] We thank a referee for suggesting that we use this type of figure. Similar figures are also used in Mertens and Ravn (2010).

We are interested in Case II equilibria that can reproduce the GR targets and for this reason we allow for shocks to preferences and technology. This induces shifts in the AD schedule as shown in Figure 1b. Under weak conditions described in Section C of the Online Appendix there is a single $R = 0$ crossing point of AD^{LL} and AS^{LL} and it follows that the zero bound equilibrium is always unique using the LL equilibrium conditions.

Case I and Case II equilibria have different local dynamics and thus very different implications for fiscal policy (Mertens and Ravn, 2010) in low interest rate environments. In a Case I equilibrium supply shocks have have unorthodox effects on output. For instance, a cut in the labor tax rate shifts the equilibrium down and inflation and employment (and output) fall. However, in a Case II equilibrium employment and inflation increase in response to a labor tax rate cut. Demand shocks such as an increase in government purchases can have potent effects on output in Case I equilibria. The government purchase multiplier always exceeds one in Case I equilibria and is sometimes much larger than one. In Case II equilibria, the government purchase multiplier is always smaller than one and may even be negative. To understand these results, consider, for instance, how the size of the government purchase multiplier varies with p. When p is small $slope(AS^{LL})$ is also small and the government purchase multiplier, which is primarily a demand shifter, is small. The size of the government purchase multiplier increases with p up to the bifurcation point which occurs when $slope(AD^{LL}) = slope(AS^{LL})$. Beyond this point the equilibrium switches to a Case II equilibrium and it follows that the government purchase multiplier is less than one.

From this brief summary one can understand why Case I equilibria have received so much attention in the literature. In this type of equilibrium conventional supply side stimulus is contractionary and should be avoided at the zero bound. Policies that stimulate aggregate demand though are particularly effective.

4.2 Characterization of zero bound equilibria in the NK model

We now characterize equilibrium of the model using the NL equilibrium conditions. Two new results emerge. The AD schedule can be downward sloping at the zero bound and for some parameterizations of the model there are multiple zero bound equilibria.

4.2.1 Slopes of Aggregate Demand and Aggregate Supply

The slopes of the AD and AS schedules vary with the size of the shocks when one uses the NL equilibrium conditions. However, it is straightforward to derive analytical expressions for their *local* slopes in the neighborhood of the zero bound equilibrium by loglinearizing around (h^L, π^L) instead of the zero inflation steady-state.[11]

[11] The restrictions $c^L > 0$ and $h^L > 0$ imply that attention can be restricted to $\{\pi^L : 1 - \kappa^L - \eta^L > 0$, $1 - p\beta d^L/(1 + \pi^L) > 0$, and $(1 - p\beta d^L)\gamma\pi^L(1 + \pi^L) + \theta > 0\}$.

At the zero bound, the slope of the AD schedule in the neighborhood of (h^L, π^L) is

$$slope(AD) \equiv \left[\frac{1}{\sigma} \frac{p\beta d^L/(1+\pi^L)^2}{1 - p\beta d^L/(1+\pi^L)} + \frac{(\kappa^L)'}{1-\kappa^L-\eta^L} \right]^{-1}. \tag{13}$$

Slope(AD) consists of the inverse of the two terms in brackets. Note that the first term captures the same tradeoffs between employment and inflation as $slope(AD^{LL})$. It is positive and simplifies to $slope(AD^{LL})^{-1}$ when $(d^L, \pi^L) = (1/\beta, 0)$. What is new is the second term in equation (13) which reflects the fact that price adjustment on the margin is costly and absorbs resources.[12] It is negative in a deflationary zero bound equilibrium and acts like a leaky bucket, driving a wedge between what is produced and what is available for consumption.

The effect of the resource costs of price adjustment on AD is most pronounced when the expected duration of zero interest rates is short. Inspection of equation (13) reveals that at $p = 0$, $slope(AD)$ is unambiguously negative. Increasing p from zero rotates the AD schedule to the right for the reasons described in Section 4.1 and it becomes steeper until its slope eventually turns positive. Thus, $slope(AD^{LL})$ and $slope(AD)$ will have different signs when the expected duration of zero interest rates is sufficiently short and will both be positive when the expected duration is sufficiently long.

The parameters σ and γ are also important for the sign of $slope(AD)$. A larger value of σ reduces the size of the first term in equation (13), and a larger value of γ increases the resource cost of price adjustment and thus the magnitude of the second term. It follows that $slope(AD) < 0$ will be negative for a larger range of values of p when σ and/or γ are large.

The slope of the AS schedule can also be negative at the zero bound. Loglinearizing the nonlinear AS schedule at (h^L, π^L) yields

$$slope(AS) \equiv \left[\frac{1 - p\beta d^L}{slope(NKPC)} \frac{1+2\pi^L}{mc^L} + \frac{\sigma}{\sigma+\nu} \frac{(\kappa^L)'}{1-\kappa^L-\eta^L} \right]^{-1} \tag{14}$$

where marginal cost in state L is given by $mc^L = \pi^L(1+\pi^L)(1-p\beta d^L)\gamma/\theta + 1$.

Slope(AS) is given by the inverse of the two terms in brackets. The first term is unambiguously positive and reflects the same factors that determine $slope(AS^{LL})$. Note that this term simplifies to $slope(AS^{LL})^{-1}$ when evaluated at $(d^L, \pi^L) = (1, 0)$. The second term is negative in a deflationary zero bound equilibrium but disappears at $(d^L, \pi^L) = (1, 0)$. This term reflects how the resource cost of price adjustment affects the supply of goods. Less deflation reduces the amount of resources that are absorbed by costly price adjustment. This creates a positive wealth effect that puts downward pressure on labor supply. If this effect is strong enough employment can fall even though the wage has risen.

[12] Note that the second term of (13) disappears when $slope(AD)$ is evaluated at $(d^L, \pi^L) = (1/\beta, 0)$.

The value of p also plays an important role in determining $slope(AS)$. Suppose that $(\kappa^L)' = \gamma\pi^L$ is sufficiently small so that $slope(AS) > 0$ when $p = 0$.[13] Then as p is increased the AS schedule rotates to the left becoming steeper until its slope turns negative. Consequently, the signs of $slope(AS^{LL})$ and $slope(AS)$ are most likely to agree at smaller values of p.

The analysis so far is useful for understanding the slopes of the AD and AS schedules in the neighborhood of a crossing point but it is silent about the number of crossing points at the zero bound. The solution to the NL equilibrium conditions at the zero bound can be reduced to finding the zeros of a nonlinear function in the inflation rate. We show next that the NK model may have multiple zero bound equilibria with distinct local configurations of AD and AS.

4.2.2 New types of zero bound equilibria in the NK model

A conventional configuration of AD and AS The results from the previous section suggest that the NK model has rich dynamics at the zero bound and that some of this richness is lost when one loglinearizes the equilibrium conditions at a steady-state with zero inflation. Perhaps the most important new case is that of a *downward sloping AD* and an *upward sloping AS schedule* at the zero bound. Figure 2c shows that this case occurs using the baseline parameterization of the model. The shocks are chosen to reproduce the Great Recession GDP and inflation targets and $p = 0.4$. This case, which is subsequently referred to as *Case III*, has the property that equilibrium is globally unique.

Multiple zero bound equilibria The AD^{LL} and AS^{LL} schedules generically have a unique intersection at the zero bound. But in the true model, there are regions of the parameter space where multiple zero bound equilibria occur. Figure 2d provides an example of this situation using the baseline parameterization with $p = 0.88$. Under this configuration of parameters, there are *two zero bound equilibria* and one equilibrium with a positive nominal interest rate. We refer to this situation as a *Case MZB* zero bound equilibrium.[14] The two zero bound equilibria have different local properties. In this example, the zero bound equilibrium that hits the GR targets exhibits $slope(AS) > slope(AD) > 0$. At the second zero bound equilibrium $0 > slope(AS) > slope(AD)$. The inflation rate in this second equilibrium is implausibly small at -16.8% per annum. However, the two zero bound equilibria are much closer for other choices of p. If, for instance, p is set to 0.86, the inflation rate in the nontargeted zero bound equilibrium is -0.8% and GDP declines by -6.7%. These magnitudes are similar to those in the

[13] This condition is satisfied for all of the results we report below.

[14] Case MZB can be divided into a number of sub cases that vary according to the slope of the targeted and the non-targeted zero bound equilibria. The number of distinct types of equilibrium is large and we thus choose to bundle them together in a single case.

targeted equilibrium which reproduces an inflation rate of -1% and a 7% decline in GDP. In this example, the targeted equilibrium has $slope(AD) > slope(AS) > 0$ and the second equilibrium has $slope(AS) > slope(AD) > 0$.

These two examples illustrate that the local properties of Case MZB zero bound equilibria are very sensitive to the particular choice of p in this region of the parameter space. Increasing p by 0.02 results in equilibria with very different local properties. The reason why the local dynamics are so complicated in this region is because it includes the bifurcation point where $slope(AD) = slope(AS)$.

Case I and II equilibria also occur using the NL equilibrium conditions (Figures 2a and 2b). In other words, there are regions of the parameter space in the NK model where $slope(AD) > slope(AS) > 0$ and equilibrium is globally unique (Case I) and there are other regions of the parameter space where there is one zero bound equilibrium with $slope(AS) > slope(AD) > 0$ and a second equilibrium with a positive interest rate equilibrium (Case II).[15]

4.3 When and how do LL solutions fail?

Figure 3 reports the regions where each of the four cases occur using the NL equilibrium conditions (first row) and the regions where the two cases occur using the LL solution (second row). The figures in the first column report results for alternative values of $p \in [0, 0.945]$ ranging and for $\sigma \in [0.5, 2]$ and the results in the second column consider alternative values of p and $\gamma \in [100, 600]$. For each parameterization the shocks $\{d^L, z^L\}$ are adjusted to reproduce the GR inflation and GDP targets and in situations with multiple zero bound equilibria only the targeted equilibrium is reported.

Consider the baseline parameterization which is denoted with a solid black line. Sections 4.1 – 4.2 establish that $slope(AD) < 0$ for smaller values of p but that $slope(AD^{LL}) > 0$ for all p. A comparison of rows 1 and 2 of Figure 3 reveals that this breakdown in the LL solution occurs for all values of $p \leq 0.56$.

A second breakdown of the LL solution occurs in the interval $p \in [0.857, 0.890]$ using the baseline parameterization. For both solutions this interval contains a bifurcation point.[16] However, the LL solution fails to register the fact that there are multiple zero bound equilibria. The qualitative properties of the targeted NL equilibrium and the LL equilibrium are generally the same for $p \in [0.857, 0.890]$. But the magnitudes of the slopes of the AD and AS schedules can be quite different.

A disturbing aspect of these results is that the bifurcation occurs at values of p that are close to values maintained in previous research. Denes, Eggertsson, and Gilbukh (2013)

[15]Cases I, II, III and MZB do not exhaust all of the possible configurations of $slope(AD)$ and $slope(AS)$. Most notably, none of the examples have $slope(AS) < 0$. This situation only occurs at very high values of $p > 0.98$ using the baseline parameterization of the model.

[16]Using the NL solution the bifurcation occurs at $p \approx 0.861$ and using the LL solution it occurs at $p \approx 0.866$.

report an estimate of $p = 0.86$ and Christiano and Eichenbaum (2012) posit a value of $p = 0.775$. Our results indicate that small variations in p in this region can have a large effect on the nature of the zero bound equilibrium.

Section 4.2 shows that the size of region when $slope(AD) < 0$ is increasing in σ and γ. Recall that a higher value of σ reduces both $slope(AD)$ and $slope(AS)$. Figure 3 shows that increasing σ has a particularly large effect on the size of the Case III region. It increases from $[0, 0.56]$ when $\sigma = 1$ to $[0, 0.71]$ when $\sigma = 2$. Varying γ, which only reduces $slope(AS)$, has a smaller effect on the size of the Case III region but a larger effect on the location of the bifurcation point and thus the size of the Case II and Case MZB regions. It was pointed out above that our estimated baseline value of γ is higher than some other estimates. If a value of $\gamma = 100$ is used instead, the Case III region only includes $p \leq 0.22$. However, equilibrium is globally indeterminate (either Case MZB or Case II) for all $p \geq 0.77$.

5 Small and orthodox fiscal multipliers at the zero bound

This section establishes that the NK model can be used to build a case for the efficacy of supply side fiscal stimulus in a low interest rate environment. The argument is developed in two steps. We start by showing that a labor tax cut may increase employment using empirically relevant parameterizations of the NK model. Then we demonstrate that for these same parameterizations of the model, the government purchase multiplier is sometimes close to one. Taken together these two points strengthen the case for supply side policies and weaken the case for demand side policies in low interest rate environments.

5.1 Labor Tax Multiplier

Eggertsson (2011) has found that a reduction in the labor tax rate lowers employment in the NK model at the zero bound and refers to his result as a "paradox of toil." Eggertsson and Krugman (2012) using a similar line of reasoning argue that labor tax rate cuts should be avoided when the economy is in a liquidity trap. These results are derived using LL solutions and attention is limited to equilibria that are globally unique. It follows from the results in Section 4.1 that *all* zero bound equilibria satisfy $slope(AD^{LL}) > slope(AS^{LL}) > 0$. In other words, all zero bound equilibria are Case I equilibria and the labor tax multiplier is positive. Cutting the labor tax rate shifts the AS schedule outward along a stable AD schedule and employment falls. However, the results in Section 4.2 show that if one uses the NL equilibrium conditions instead, limiting attention to equilibria that are globally unique is not sufficient to rule out a negative labor tax multiplier. Equilibrium is also unique in Case III equilibria and they have the property that $slope(AS) > 0 > slope(AD)$ and thus that supply side stimulus is expansionary.

Figure 4 provides information that allows the reader to easily discern when the paradox of toil occurs in the NK model and when it does not by partitioning the parameter space according to the sign and magnitude of the labor tax multiplier. The upper panels report labor tax multipliers using the NL equilibrium conditions and configurations of the shocks that reproduce the GR inflation and GDP targets. Baseline values of σ and γ are denoted with a line and the targeted equilibrium is reported in situations where there are multiple zero bound equilibria. For purposes of comparison the lower panels report results based on the LL solution using the same shocks.

The labor tax multiplier is negative in the red region. In the lower panels (LL solution) this only occurs when $p > 0.86$. This region of the parameter space is where Case II zero bound equilibrium occur (see Figure 3). They have the property that $slope(AS^{LL}) > slope(AD^{LL}) > 0$ but as discussed above, the equilibrium is not globally unique.

The two upper panels of the figure, in contrast, have two disjoint red regions where the labor tax multiplier is negative. The leftmost red region in the first row of Figure 4 is of particular interest because equilibrium is globally unique. It corresponds to Case III (see Figure 3). The AD and AS schedules have their conventional slopes and it follows that the labor tax multiplier is negative. For the baseline parameterization this region obtains for $p \in [0, 0.57]$. When $\sigma = 2$, the labor tax multiplier is negative for p as high as 0.71. The leftmost red region is smallest when $\gamma = 100$. However, a reduction in the size of the left red region is offset by an increase in the size of the red region on the right, which consists of Case MZB equilibria and Case II equilibria.[17] Lower values of γ can have a big impact on inference. For instance, Eggertsson (2011) produces a paradox of toil with $p = 0.77$. As can be seen in Figure 4 there is no paradox of toil at this choice of p when $\gamma \leq 150$.

One of the reasons previous research on the zero bound has attracted so much attention is because the magnitude of the paradox of toil is large. Denes, Eggertsson, and Gilbukh (2013), for instance, report a median posterior labor tax multiplier of 0.1, i.e. employment *increases* by 0.1% when the labor tax is *raised* by one percentage point, in a loglinearized NK model that is calibrated to the GR. Figure 4 also reports information on the size of the paradox of toil in regions of the parameter space where it occurs. The paradox of toil is generally small using the NL equilibrium conditions. It only exceeds 0.1 in the blue region which is close but just to the left of the bifurcation point where the $slope(AD) = slope(AS)$. In this region both the size and the sign of the labor tax multiplier is very sensitive to small changes in the value of p.[18]

[17] For the Case MZB equilibria, the targeted equilibrium has a slope configuration with $slope(AS) > slope(AD) > 0$.

[18] Section D of the Online Appendix shows that the labor tax multiplier is inversely related to $slope(AD) - slope(AS)$.

5.2 Government purchase multipliers

A number of papers including most notably Christiano, Eichenbaum, and Rebelo (2011) have found that the fiscal multiplier is large in the NK model at the zero bound. The government purchase multiplier is also large for some parameterizations of our model. However, it is close to one and sometimes is even less than one for a range of empirically relevant parameterizations of the model. Moreover, there is considerable overlap in the regions of the parameter space with small government purchase multipliers and negative labor tax multipliers.

Government purchase multipliers using the NL equilibrium conditions are reported in the upper panels of Figure 5. Government purchase multipliers that are less than one are colored red. They only occur to the right of the bifurcation ($p \geq 0.861$) and correspond to either Case II or targeted Case MZB equilibria. In either case the zero bound equilibrium satisfies $slope(AS) > slope(AD) > 0$ which immediately implies that the sign of the labor tax multiplier is negative. In principal, the government purchase multiplier could also be negative in this region. In practice though this only occurs in a very small neighborhood just to the right of the bifurcation point. This region of the parameter space has two other notable properties. An increase in government purchases is deflationary and equilibrium is globally indeterminate.

The remaining parameterizations of the model have government purchase multipliers that are larger than 1 but in some cases they are very close to 1. For instance, using the baseline values of σ and γ, the government purchase multiplier is less than 1.05 when $p \in [0, 0.73]$ (green) and it only exceeds 1.5 in the very small blue region that is just to the left of the bifurcation point. Combining these results with the previous findings on the labor tax multiplier implies that for all $p \in [0, 0.56]$. there is a unique zero bound equilibrium and it has the properties that the government purchase multiplier is small and the labor tax multiplier has an orthodox sign.

A comparison of the upper and lower panels of Figure 5 shows that the LL solution works reasonably well in predicting the size of the government purchase GDP multiplier. There are large green regions with small government purchase multipliers less than 1.05 using either solution and the region where the government purchase multiplier exceeds 1.5 is small in both the upper and lower panels.

Since the government purchase multiplier is primarily a demand shifter, one might be concerned that our finding stems from the fact that the slope of the AS schedule is very flat and that this is due to our setting of $slope(NKPC)$. Section A of the Online Appendix illustrates how the results change when $slope(NKPC)$ is increased from its baseline value of 0.0214 to 0.06. The size of the green region is smaller but the size of the blue region with multipliers in excess of 1.5 continues to be very small.

Finally, note that smaller values of γ reduce $slope(NKPC)$ and this increases the size of the region where the government purchase multiplier is less than one. Christiano and Eichenbaum (2012) set $\gamma = 100$ and choose $p = 0.775$. Panel B) of Figure 5 indicates that with these choices our model produces a government purchase multiplier of less than 1 for the GR.

5.3 Discussion

Expected duration of zero interest rates We have found that the expected duration of zero interest rates plays a central role in determining the properties equilibrium at the zero bound. Given the importance of the magnitude of p, it is worthwhile to discuss grounds for entertaining large and small values of p. The NK model produces large positive fiscal multipliers when p is slightly less than 0.861 using the baseline parameterization. However, the local dynamics of the NK model change if p exceeds this value and it is hard to rule out larger values of p on empirical grounds. The actual duration of zero interest rates has been much longer than 7 quarters in the U.S. and Japan. Interest rates have been close to zero since the fourth quarter of 2008 in both countries.[19] Case II is the most common type of equilibrium for $p > 0.861$ in Figure 3. And in this type of equilibrium the government purchase multiplier is less than 1 and employment increases in response to a cut in the labor tax rate.

The other region where supply side policies are likely to be most effective is when the expected duration of zero interest rates is short. Should one take seriously small values of p? First, p does not have to be all that small to obtain a small government purchase multiplier or a negative labor tax multiplier. For instance, if p is set to 0.775 as in Christiano and Eichenbaum (2012), the government purchase GDP multiplier is 1.09 using the baseline parameterization of our model and it drops to 1.05 if σ is set to 2 instead. Similarly, when $\sigma = 2$ the labor tax multiplier is negative for p as large as 0.705.

Second, it is difficult to rule out even very short expected durations of zero interest rates on a priori grounds. In Aruoba and Schorfheide (2013), for instance, the expected duration of zero interest rates is often only one quarter. Their model requires a long sequence of negative monetary policy shocks to account for the fact that the U.S. policy rate has been about zero since 2008.[20]

A distinct reason for ruling out low values of p is that values around 0.4 or less require positive technology shocks and large values of d^L to reproduce the GR targets. A positive technology shock does not play a central role in our findings. Section A of the Online

[19] Japan has had two other episodes of very low interest rates: March 1999 – July 2000 and March 2001 – June 2006 (see Hayashi and Koeda (2013)).

[20] A number of other recent papers consider medium-scale NK models with zero bound contraints and find it difficult to solve/estimate specifications that can reproduce the long periods of zero interest rates experienced by Japan and the U.S. (see e.g. Gust, Lopez-Salido, and Smith (2012) or Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez (2012)). Thus, it appears that the experiences of the U.S., Japan and other countries with long episodes of low interest rates are a puzzle for most of the recent NK literature.

Appendix contains results that repeat our analysis, holding technology fixed and varying θ instead, and the size of the regions with small and orthodox fiscal multipliers increase.

However, a large value of the preference shock is essential if this simple model is to reproduce a 7% decline in GDP when the expected duration of state L is very short. For instance, at $p = 0.4$ a value of $d^L = 1.0445$ is required to reproduce the GR with the baseline parameterization. It is beyond the scope of this paper to determine what caused the GR. But we do feel that it is important to use observations from the GR to discipline the model.

Calvo price adjustment The analysis has used Rotemberg price setting. We believe that our finding that the LL solution fails at the zero bound is not specific to the form of costly price adjustment. As described above price dispersion using Calvo price setting also reduces the resources that are available for private and public consumption. In particular, if Calvo price setting is used instead the term κ in the resource constraint becomes $\kappa_t \equiv (x_t - 1)/x_t$ where x_t summarizes the relative price dispersion described in Yun (2005).[21]

Unfortunately, x_t is an endogenous state variable and the zero bound equilibrium becomes much more complicated to compute. To give the reader an indication about what might happen under Calvo price adjustment Section B of the Online Appendix derives results for a stylized but tractable model with Calvo price adjustment. In this model x_t is only allowed to take on two distinct values: $x_t = x^L$ in state L and $x_t = 1$ in state H. This assumption is valid if the LL solution is accurate because x_t is constant at 1 when loglinearized around the zero inflation steady-state.

Figure 6 compares the AD and the AS schedules under this version of Calvo pricing with the baseline model using a value of $p = 0.4$. The figure shows that the two models of price adjustment are almost indistinguishable in the neighborhood of the equilibrium. In particular, the equilibrium using Calvo pricing also has conventionally sloped AS and AD schedules. Section B of the online Appendix illustrates that this version of the Calvo pricing also has very similar properties to Rotemberg at higher values of p. For instance, the Calvo model also has a Case I equilibrium when $p = 0.8$ and a Case II equilibrium when $p = 0.9$.[22]

Other shocks Our message that supply side stimulus can be expansionary at the zero bound has implications for other supply shocks. For instance, Christiano, Eichenbaum,

[21] A complete description of the equilibrium condition in the Calvo model is provided in the Online Appendix.

[22] To draw AS and AD under Calvo pricing, the probability that a firm is unable to change its price (α) is calibrated such that the loglinearized New Keynesian Phillips curve has the same slope as in the Rotemberg model with our baseline parameterization. The implied value is $\alpha = 0.88$. In the Rotemberg model, the slope of the New Keynesian Phillips curve is given by $\theta(\sigma + \nu)/\gamma$ while in the Calvo model it is $(1 - \alpha)(1 - \beta\alpha)(\sigma + \nu)/\alpha$.

and Rebelo (2011) find that the response of output to an improvement in technology is contractionary at the zero bound. This finding runs counter to empirical evidence in Wieland (2013) that suggests improvements in technology are also expansionary at the zero bound. In our model positive technology shocks are expansionary in Case III and Case II equilibria when $\sigma = 1$ (see Section D.2 of the Online Appendix).

Our finding that the LL solution works well when computing the government purchase multiplier depends on the size of the shocks. The LL solution exhibits much larger biases if the shocks are calibrated to observations from the Great Depression instead. The interested reader is referred to Braun and Körber (2011) for more details.

Other parameterizations We have described how our results vary with p, σ and γ but it is possible that other values of the parameters that we have held fixed matter. In Section A of the Online Appendix we report additional results that are designed to address this concern. We continue to find regions of the parameter space that have small government purchase multipliers and negative labor tax multipliers. Perhaps the most important new finding is that multiple zero bound equilibria occur in much larger regions of the parameter space and, in particular, in regions where p is very small (see Figures 8 and 11 of the Online Appendix).

Equilibrium selection In situations with multiplie equilibria, we have adopted the convention of reporting the targeted zero bound equilibrium that reproduces the GR calibration target. Is this a reasonable way to proceed? Christiano and Eichenbaum (2012) propose using an E-learning criterion to rule out multiple equilibria. Applying that criterion here does not always resolve the issue of multiple zero bound equilibria in our model. This is because in some cases both the targeted and the non-targeted zero bound equilibrium are E-learnable.[23] What is perhaps more troubling is that when the E-learning criterion is applied to Case II equilibria, it rules out the zero bound equilibrium and selects the positive interest rate equilibrium which fails to hit the GR targets. This selection works in the same way using either the NL or the LL solution. We are not convinced that it is reasonable to use an equilibrium selection criterion that rejects the zero bound equilibrium in Figure 2c and instead selects the positive interest rate equilibrium. We think it makes more sense to select the zero bound equilibrium because it reproduces the GR facts and to rule out the positive interest rate equilibrium instead.

[23] For instance, when $p = 0.865$ the non-targeted zero bound equilibrium that produces a 13% decline in GDP and an annualized rate of deflation of 5% is also E-learnable. This example also has a positive interest rate equilibrium that is also E-learnable. It has the property that GDP falls by 0.6% and that employment is 5% above its steady-state level.

6 Conclusion

In this paper we have documented the properties of a tractable nonlinear New Keynesian model that honors the zero lower bound on the nominal interest rate and also reproduces the large output and small inflation declines that occurred during the U.S. Great Recession. Some parameterizations of the model support the contention that supply side fiscal policies should be avoided in low interest rate environments and that demand side policies should be relied on instead. However, two of the principal arguments underlying this contention (labor tax cuts are contractionary and the government purchase multiplier is large) are not robust. Other empirically relevant parameterizations of the same NK model have much smaller government purchase multipliers and also provide a rationale for supply side measures such as a labor tax cut. In particular, one cannot dismiss the possibility that supply side policies are expansionary at the zero bound and solving the NK model using nonlinear methods plays an important role in reaching this conclusion.

References

Aruoba, S. B., and F. Schorfheide (2013): "Macroeconomic Dynamics Near the ZLB: A Tale of Two Equilibria," NBER Working Papers 19248, National Bureau of Economic Research, Inc. 1, 4.1, 5.3

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001): "Monetary Policy and Multiple Equilibria," *American Economic Review*, 91(1), 167–186. 1

Braun, R. A., L. Körber, and Y. Waki (2012): "Some unpleasant properties of log-linearized solutions when the nominal interest rate is zero," *Federal Reserve Bank of Atlanta Working Paper 2012-5a.* 4.1, 8

Braun, R. A., and L. M. Körber (2011): "New Keynesian Dynamics in a Low Interest Rate Environment," *Journal of Economic Dynamics and Control*, 35(12), 2213–2227, Federal Reserve Bank of Atlanta Working Paper. 5.3

Broda, C., and D. Weinstein (2004): "Globalization and the gains from variety," Staff Reports 180, Federal Reserve Bank of New York. 3.1

Bullard, J. (2010): "Seven Faces of "The Peril"," *Federal Reserve Bank of St. Louis Review*, 92(5), 339–352. 4.1

Christiano, L., and M. Eichenbaum (2012): "Notes on Linear Approximations, Equilibrium Multiplicity and E-learnability in the Analysis of the Zero Bound," mimeo. 1, 4.3, 5.2, 5.3, 5.3

Christiano, L., M. Eichenbaum, and S. Rebelo (2011): "When is the Government Spending Multiplier Large?," *Journal of Political Economy*, 119(1), 78–121. 1, 3.2, 5.2, 5.3

Denes, M., G. B. Eggertsson, and S. Gilbukh (2013): "Deficits, Public Debt Dynamics and Tax and Spending Multipliers," *Economic Journal*, 0, 133–163. 3.1, 4.3, 5.1

Eggertsson, G. (2011): "What Fiscal Policy is Effective at Zero Interest Rates?," in *NBER Macroeconomics Annual 2010, Volume 25*, NBER Chapters, pp. 59–112. National Bureau of Economic Research, Inc. 1, 4.1, 5.1

Eggertsson, G., A. Ferrero, and A. Raffo (2013): "Can Structural Reforms Help Europe?," Unpublished manuscript, Brown University. 1

Eggertsson, G., and P. Krugman (2012): "Debt, Deleveraging and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *Quarterly Journal of Economics*, 127(3). 1, 2.3, 5.1

Eggertsson, G., and M. Woodford (2003): "The Zero Interest Rate Bound and Optimal Monetary Policy," *Brookings Panel on Economic Activity.* 2, 2.2

Evans, G. W., E. Guse, and S. Honkapohja (2008): "Liquidity traps, learning and stagnation," *European Economic Review*, 52(8), 1438–1463. 1

Fernandez-Villaverde, J., G. Gordon, P. A. Guerron-Quintana, and J. Rubio-Ramirez (2012): "Nonlinear Adventures At The Zero Lower Bound," *NBER Working Paper 18058*. 1, 20

Gust, C., D. Lopez-Salido, and M. E. Smith (2012): "The empirical implications of the interest-rate lower bound," Finance and Economics Discussion Series 2012-83, Board of Governors of the Federal Reserve System (U.S.). 1, 3.1, 20

Hayashi, F., and J. Koeda (2013): "A Regime-Switching SVAR Analysis of Quantitative Easing," CARF F-Series CARF-F-322, Center for Advanced Research in Finance, Faculty of Economics, The University of Tokyo. 19

Ireland, P. N. (2003): "Endogenous money or sticky prices?," *Journal of Monetary Economics*, 50(8), 1623–1648. 3.1

Levy, D., M. Bergen, S. Dutta, and R. Venable (1997): "The Magnitude of Menu Costs: Direct Evidence from Large U. S. Supermarket Chains," *The Quarterly Journal of Economics*, 112(3), 791–824. 3.2

Mertens, K., and M. O. Ravn (2010): "Fiscal Policy in an Expectations Driven Liquidity Trap," CEPR Discussion Papers 7931, C.E.P.R. Discussion Papers. 1, 4.1, 10

Rotemberg, J., and M. Woodford (1997): "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," in *NBER Macroeconomics Annual 1997, Volume 12*, NBER Chapters, pp. 297–361. National Bureau of Economic Research, Inc. 3.1

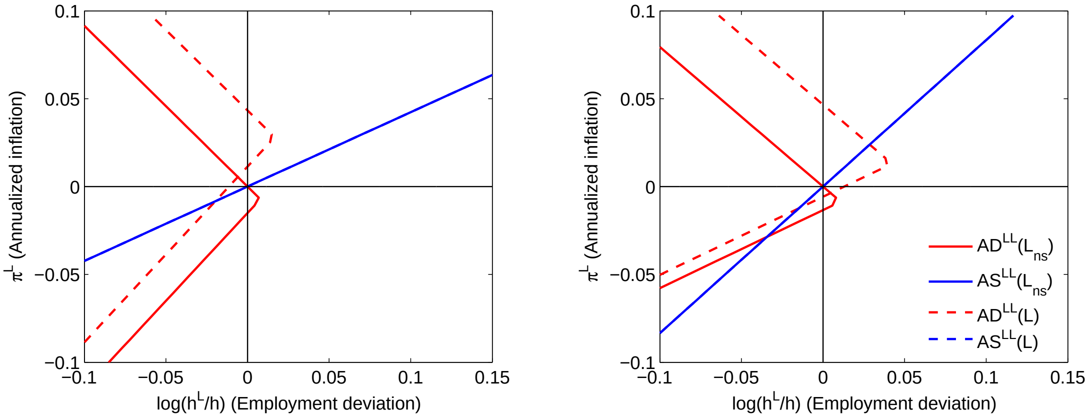
Rotemberg, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90(6), 1187–1211. 1, 2

Wieland, J. (2013): "Are Negative Supply Shocks Expansionary at the Zero Lower Bound?," Unpublished manuscript, UC Berkeley. 5.3

Woodford, M. (2011): "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, 3(1), 1–35. 1, 8

Yun, T. (2005): "Optimal Monetary Policy with Relative Price Distortions," *American Economic Review*, 95(1), 89–109. 2.1, 5.3

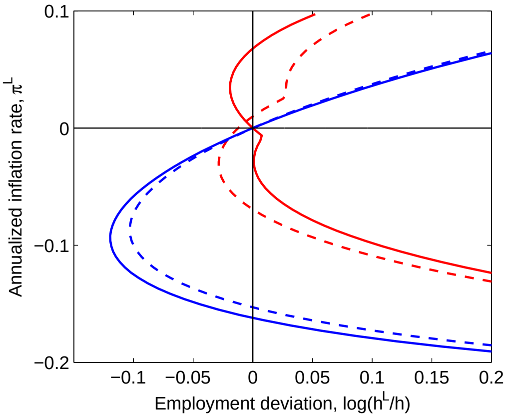
Figure 1: Zero-Bound Equilibria Using loglinearized Equilibrium Conditions



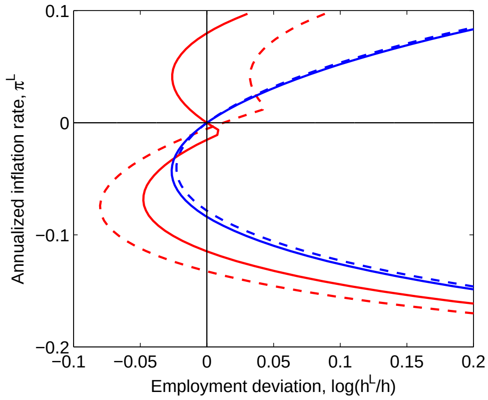
(A) Case I, $(p = 0.8)$ (B) Case II, $(p = 0.9)$

Notes: The plots use the baseline parameterization of the model. The schedules labeled L use values of $\widehat{d}^L$ and $\widehat{z}^L$ that reproduce the Great Recession targets using the nonlinear equilibrium conditions and the schedules labeled L_{ns} set the shocks to their steady-state values. The loglinearized AS schedule is the same in states L and L_{ns} because $\sigma = 1$.

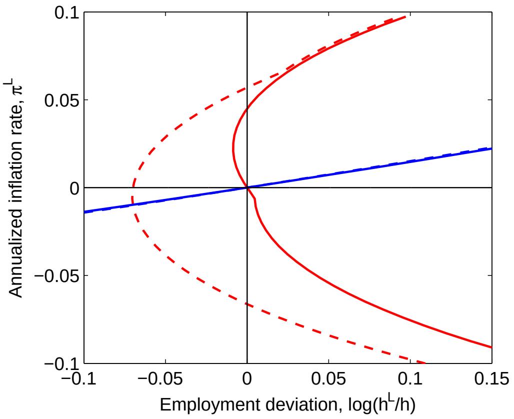
Figure 2: Zero-Bound Equilibrium That Can be Found Using Nonlinear Solution Methods.



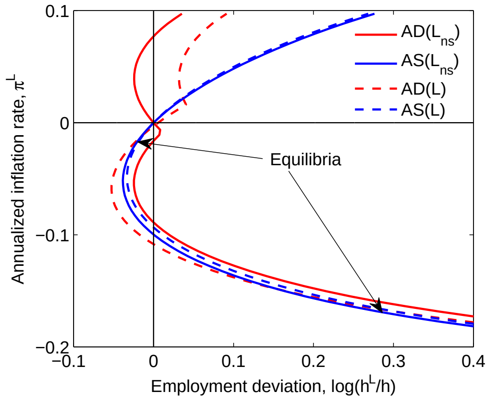
(a) Case I, $(p = 0.8)$



(b) Case II, $(p = 0.9)$



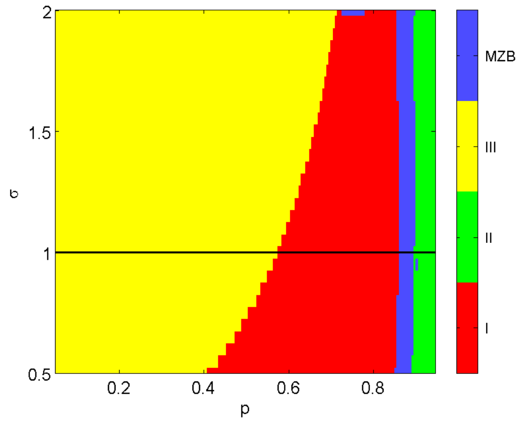
(c) Case III, $(p = 0.4)$



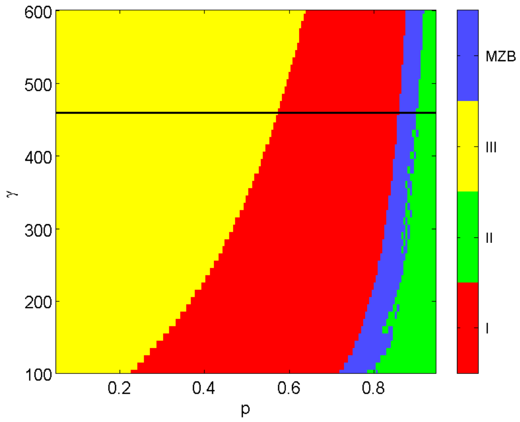
(d) Case MZB, $(p = 0.88)$

Notes: The plots are based on our baseline parameterization. The schedules labeled L_{ns} set all shocks to their steady-state variables. The schedules labeled L use shocks $\widehat{d}^L$ and $\widehat{z}^L$ that reproduce our Great Recession targets for GDP and inflation using the nonlinear equilibrium conditions.

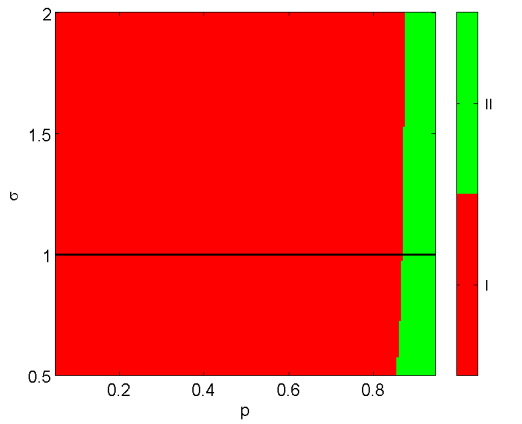
Figure 3: Types of Zero Bound Equilibria For Alternative Values of Price Adjustment Costs and Risk Aversion.



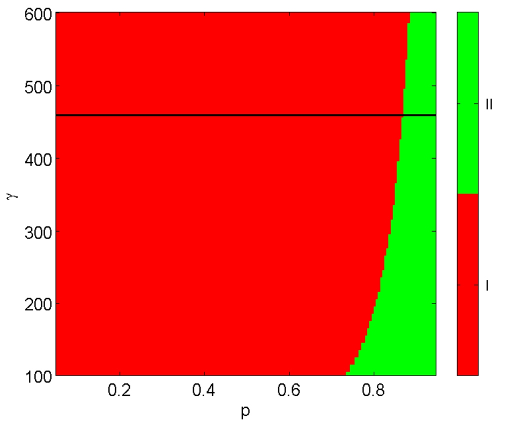
(a) NL Equilibrium: Alternative Combinations of p and σ.



(b) NL Equilibrium: Alternative Combinations of p and γ.



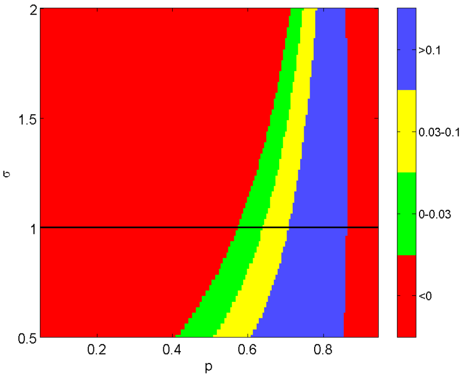
(c) LL Equilibrium: Alternative Combinations of p and σ.



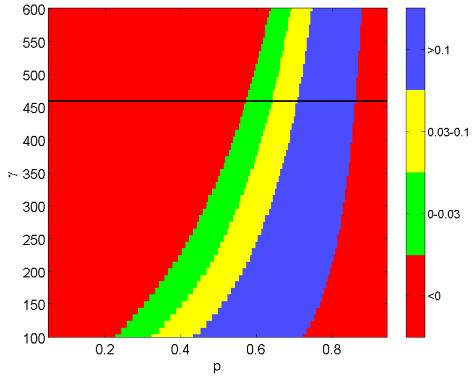
(d) LL Equilibrium: Alternative Combinations of p and γ.

Notes: Red: Case I (slope(AD)>0>slope(AS)); Light Green: Case II (slope(AS)>slope(AD)>0); Yellow: Case III (slope(AS)>0>slope(AD)); Blue: Case MZB (multiple zero bound equilibria); The baseline parameterization of the model is denoted with a black line.

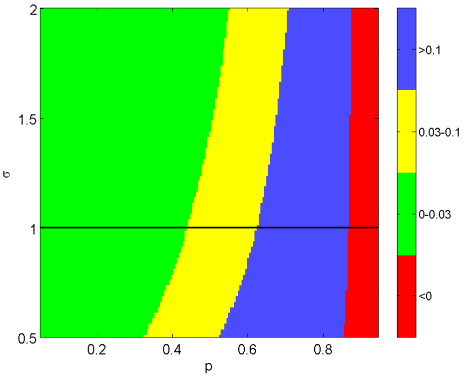
Figure 4: The labor tax fiscal multiplier at alternative values of p, σ and γ.



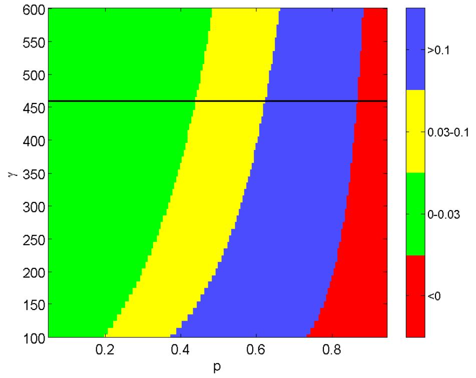
(a) Alternative values of p and σ NL solution.



(b) Alternative values of p and γ NL solution.



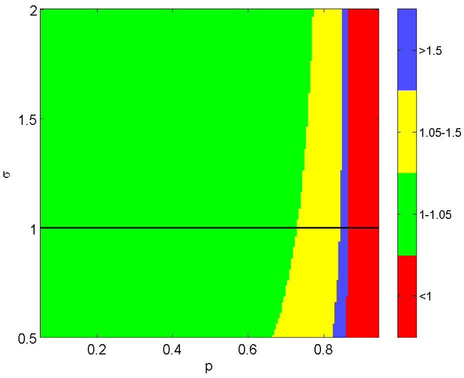
(c) Alternative values of p and σ LL solution.



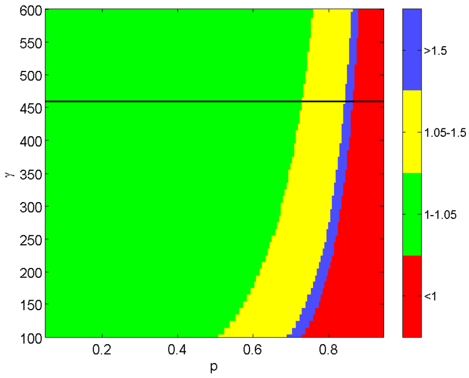
(d) Alternative values of p and γ LL solution.

Notes: Red: Labor tax multiplier is negative (employment increases when the labor tax is cut); Green: Labor tax multiplier is in $[0, 0.03]$; Yellow: labor tax multiplier is in $(0.03, 0.1]$; Blue: labor tax multiplier exceeds 0.1. The black line denotes the baseline value of each parameter.

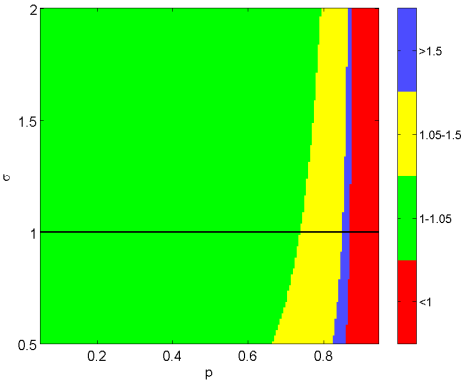
Figure 5: Response of output to an increase in government purchases at alternative values of p, σ and γ.



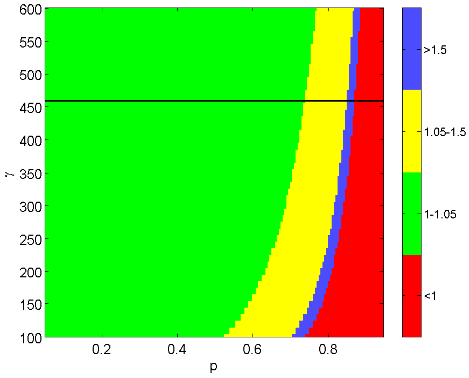
(a) Alternative values of p and σ NL Solution.



(b) Alternative values of p and γ NL solution.



(c) Alternative values of p and σ LL solution.



(d) Alternative values of p and γ LL solution.

Notes: Red: the government-purchase-GDP-multiplier is less than 1; Green: the multiplier is in $[1, 1.05]$; Yellow: the multiplier is in $[1.05, 1.5]$, blue: the multiplier exceeds 1.5. The baseline parameterization is denoted with a line.

Figure 6: Zero-Bound Equilibria in the Calvo vs. Rotemberg model ($p = 0.4$)