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Zhuoshi Liu,[1] Elisabetta Vangelista,[2] Iryna Kaminska[3] and Jon Relleen[4]

Abstract

Market-based measures of inflation expectations can be derived either from the difference between yields on nominal and inflation-linked government bonds or from inflation swap rates. These measures are important indicators of the outlook for inflation and are monitored regularly by the United Kingdom's Monetary Policy Committee (MPC), alongside other measures of inflation expectations such as those based on surveys. However, the market rates we observe are not perfect measures of expected future inflation. Moreover, in the United Kingdom inflation-linked market instruments reference RPI inflation, whereas the MPC's target is CPI inflation of 2%. To better extract useful information about expectations for CPI inflation, we develop a no-arbitrage term structure model to decompose the forward inflation curve into: measures of CPI inflation expectations; the expected spread between expected RPI and CPI inflation (the RPI/CPI inflation 'wedge'); and estimates of risk premia. We then further decompose risk premia estimates into inflation risk premia and liquidity risk premia. We show that long-horizon expectations of CPI inflation, as implied by our model, fell in the 1990s after the introduction of inflation targeting and the creation of the MPC and have since remained fairly stable at around 2%. Our model also suggests that the large falls in measures of implied inflation based on index-linked gilts after the financial crisis were to a large extent the result of changes in liquidity premia in inflation-linked gilt prices.

Key words: Affine arbitrage-free dynamic term structure model, breakeven inflation, inflation expectations, risk premia, funding liquidity, survey expectations.

JEL classification: C40, E31, E43, E52, G12.

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1 Introduction

The yield spread between conventional and inflation-linked government bonds (known as "gilts" in the UK), commonly referred to as the 'break-even inflation' (BEI) rate is used as an indicator of inflation expectations. And the development of the inflation derivatives market, particularly around 10 years ago, provided an alternative set of instruments from which to extract market-based inflation expectations, to the extent that BEI rates from inflation swaps are now often used in market commentaries as a more reliable indicator of inflation expectations.

These measures are useful indicators of the outlook for inflation and are monitored regularly by the UK Monetary Policy Committee (MPC), alongside other measures of inflation expectations such as those based on surveys. BEI rates have also become increasingly used in central bank publications, regular market commentaries and research. The relevant literature for UK BEI rates has also become increasingly rich, e.g. see Joyce, Lildholdt, and Sorensen (2010), Guimaraes (2014), Abrahams, Adrian, Crump, and Moench (2013), D'Amico, Kim, and Wei (2014) and Pflueger and Viceira (2013).

However, BEI rates are imperfect indicators of expected inflation and UK BEI rates reference RPI inflation, whereas the MPC targets CPI inflation. In addition to expectations of future CPI inflation rates, which is the key measure for the UK Monetary Policy Committee, UK BEI rates may contain three additional non-trivial components: (1) an inflation risk premium to compensate for uncertainty about future inflation; (2) liquidity risk premia; (3) the spread between RPI inflation, to which market instruments are indexed, and CPI inflation, which is the measure that the MPC's inflation target refers to.

In this paper, we develop an affine term structure model of BEI rates, which allows us to better extract information from both gilt and inflation swap measures by addressing these issues. The model decomposes market-implied BEI rates into measures of inflation expectations and risk premia using a no-arbitrage framework. It is common to analyse the BEI term structure using joint affine term structure models of nominal and real interest rates. The novelty of our paper lies in the fact we model the term structure of UK BEI rates directly. This approach simplifies the model greatly and allows us to model both gilt and inflation swap BEI rates jointly with relative ease. It also avoids dealing with the zero lower bound (ZLB) for nominal yields, which is not accounted for in the affine term structure model framework. In addition, our model makes use of professional forecast survey data, which help to identify the dynamics of the pricing factors and provides a reliable way to obtain robust decompositions (as shown in Joyce, Lildholdt, and Sorensen (2010), Kim and Orphanides (2012), and Guimaraes (2014) among others).

The two main contributions of the paper are: (1) modelling liquidity premia in gilt BEI rates by making use of inflation swap BEI rates; and (2) estimating expectations of the wedge between RPI and CPI inflation rates and so reflecting expectations of CPI inflation. We will discuss each of these in more detail below.

First, we explicitly model the liquidity risk premium embedded in gilt BEI rates, which is driven by the relative illiquidity of index-linked (or inflation-linked[1]) gilts compared to conventional gilts. Previous evidence (mostly for the US market) suggests that the liquidity premium in Treasury inflation-protected securities (TIPS) yields can be substantial and vary over time. Nonetheless, there are large differences in the liquidity premium estimates available in the literature. Christensen and Gillan (2012) estimated a TIPS liquidity premium of the order of 30 to 40 basis points on average, ranging between 2 and 123 bps for a 10-year yield. Pflueger and Viceira (2013) estimated this liquidity premium being about 70 bps for TIPS and 25 basis points for 10-year UK inflation linked gilts, with estimates generally being positive[2] but declining over time. D'Amico, Kim, and Wei (2014) model liquidity as a latent factor in no-arbitrage term structure models of nominal and TIPS yields and estimate an average 50 basis points liquidity premium for TIPS; applying their models to the UK data, they find that liquidity premia in index-linked gilt yields were fairly low (and smaller than liquidity premiums in TIPS) prior the crisis, but they spiked to nearly 250 basis points at the height of the crisis.

Our approach to estimating the liquidity premium makes use of inflation swap BEI rates. Liquidity premia in inflation swap BEI rates are likely to be smaller than those in gilt BEI rates for two main reasons. First, swap contracts do not require large upfront payments, as is the case for bond investments. Hence, leveraged investors face lower capital constraints when gaining exposure to inflation-linked cashflows using inflation swaps compared to using index-linked bonds. As a result, inflation swap BEI rates may be less affected by market liquidity conditions than gilt BEI rates, since illiquid markets may be associated with higher funding costs and capital constraints. Second, studies have found fairly large bid-ask spreads and liquidity premia at certain times in government index-linked bond markets (see Bauer, 2015; Christensen and Gillan, 2012; and Fleckenstein, Longstaff, and Lustig, 2014). D'Amico, Kim, and Wei (2014) also show that their model index-linked gilt liquidity premium estimates can be linked to such observable measures of index-linked gilt liquidity as the difference between index-linked and conventional gilt asset swap spreads and the difference between the 10-year inflation swap rate and the 10-year gilt BEI rate.

In line with the literature, we assume that liquidity premia are present in gilt BEI rates but that liquidity premia are negligible for inflation swap BEI rates. Using the spread between gilt BEI and inflation swap BEI rates, we can therefore gain insights into liquidity conditions in bond markets.

Second, given that UK inflation-linked financial instruments are linked to RPI inflation, whereas the MPC's target is CPI inflation, we seek to model not only expectations for RPI inflation but also for CPI inflation. In practice, the difference between RPI and CPI inflation reflects a range of factors, such as different components included in the calculation, different weights applied to the basket of goods and formula effects (geometric versus

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[1] In this paper index-linked and inflation-linked are used interchangeably.

[2] A positive liquidity premium in index linked gilts is a negative liquidity premium in BEI rates, if conventional gilt yields are assumed 'liquid'

arithmetic averages). By jointly modelling RPI and CPI inflation we can also generate estimates for the future 'wedge' between RPI and CPI inflation that is priced into RPI BEI rates. In one specification of our model we also try including professional survey expectations for both indices, which may help to identify model parameters that would otherwise be very imprecisely estimated and help to anchor the dynamics of the pricing factors.

Our results show that, after the introduction of inflation targeting in 1992 and the creation of the MPC in 1997, the significant falls in BEI rates reflected a fall in both inflation expectations and a fall in the inflation risk premium, suggesting that investors placed confidence in the new monetary policy framework. Both CPI and RPI inflation expectations have been reasonably well anchored at medium and long horizons since the introduction of the Monetary Policy Committee in 1997.

The results also suggest that the negative sign of the risk premium in gilt BEI rates during the recent crisis was, to a large extent, the result of negative liquidity premia, which we conclude was driven by periods of illiquidity in the market for index-linked gilts. Our estimates of inflation risk premia for long-term gilt and inflation swap BEI rates, which are required by investors to compensate them for uncertainty about future inflation, have been modestly positive during most of the sample.

In addition, we show that the expected CPI and RPI inflation wedge is quite volatile at short horizons but is more stable at longer horizons, converging to around 66 basis points. At face value this suggests that the estimates of long-term RPI expectations can be transformed to estimates of long-term CPI expectations via a constant adjustment. However, we recognise that the model's estimate of the wedge is a little below some other forecasts of the long-run RPI/CPI wedge. For example, the Bank's discussions with market participants suggest that they generally expect the wedge will average around 80-100 basis points in the long-term. In part the difference may reflect methodological changes in 2010 by the ONS, which our model will not fully capture given the relatively short sample period after it.

The rest of the paper is structured as follows: Section 2 explains the model setup and specifications, Section 3 shows the data and preliminary analysis, Section 4 discusses the empirical results, Section 5 presents the sensitivity analysis of the model and finally Section 6 concludes the paper.

2 Affine term structure models of breakeven inflation rates

2.1 Modelling CPI and RPI inflation rates

As is standard in affine term structure models, we assume that the one-period nominal risk-free interest rate (r_t) is an affine function of a $K^f \times 1$ vector of unobserved factors, $\mathbf{f}_t$:[3]

[3] The intercept is not present in the nominal short rate equation as the factors have non-zero means. The same logic applies to the real short rate equation below.

$$r_t = \boldsymbol{\delta}_f \mathbf{f}_{t,}$$

where $\boldsymbol{\delta}_f$ is a $1 \times K^f$ vector of constant factor loadings. The one-period risk-free real CPI-linked short rate $r_t^{*,CPI}$ is also driven by the same factor vector $\mathbf{f}_t$:

$$r_t^{*,CPI} = \boldsymbol{\delta}_f^{*,CPI} \mathbf{f}_t,$$

where $\boldsymbol{\delta}_f^{*,CPI}$ is a $K^f \times 1$ vector of scalars.

Hence, by the Fisher equation, the short-term (i.e. one-period) CPI breakeven inflation rate is given as

$$\pi_{t,1}^{CPI} = r_t - r_t^{*,CPI} = \left(\boldsymbol{\delta}_f - \boldsymbol{\delta}_f^{*,CPI}\right)\mathbf{f}_t \equiv \mathbf{J}\mathbf{f}_t \qquad (1)$$

We normalise the factor vector in the above equation so that the coefficients are units, i.e. $\mathbf{J}$ = $= [\underbrace{1,1 \ldots 1}_{K^f}]$. We also follow Guimaraes (2014) to assume, for the sake of simplicity and parsimony, that the one-period ahead expected CPI inflation rate at time t ($\pi_t^{e,CPI}$) is deterministic and equals the short-term breakeven inflation rate:

$$\pi_t^{e,CPI} = \pi_{t,1}^{CPI} = \mathbf{J}\mathbf{f}_t$$

The one-period RPI-linked risk-free real short rate $r_t^{*,RPI}$, is assumed to be driven by both $\mathbf{f}_t$ and an inflation wedge factor q_t:

$$r_t^{*,RPI} = \boldsymbol{\delta}_f^{*,RPI} \mathbf{f}_t + q_t = \left(\boldsymbol{\delta}_f^{*,CPI} + \boldsymbol{\theta}_f^*\right)\mathbf{f}_t + q_t \qquad (2)$$

where $\boldsymbol{\theta}_f^* = [\underbrace{\theta_{f1}^*, \ldots \theta_{fn}^*}_{K^f}]$ is a 1 by K^f vector, which represents the difference between $\boldsymbol{\delta}_f^{*,RPI}$ and $\boldsymbol{\delta}_f^{*,CPI}$. The short-term RPI breakeven inflation rate ($\pi_{t,1}^{RPI}$), which equals the expected RPI inflation rate at time t ($\pi_t^{e,RPI}$), is modelled as the difference between the nominal and real short rate in a similar manner as the CPI breakeven inflation rate:

$$\begin{aligned}\pi_{t,1}^{RPI} = \pi_t^{e,RPI} = r_t - r_t^{*,RPI} &= \boldsymbol{\delta}_f \mathbf{f}_t - \left(\boldsymbol{\delta}_f^{*,CPI} + \boldsymbol{\theta}_f^*\right)\mathbf{f}_t - q_t \\ &= \mathbf{J}\mathbf{f}_t - \boldsymbol{\theta}_f^* \mathbf{f}_t - q_t\end{aligned} \qquad (3)$$

As discussed above, the liquidity risk premia components in some RPI-linked instruments (i.e. RPI-linked gilts) can be significant and need to be modelled. We do this by assuming that the short-term real rate used to price RPI-linked gilts ($r_t^{*b,RPI}$) is adjusted by a liquidity spread[4], so we have

$$r_t^{*b,RPI} = r_t^{*,RPI} + l_t = \boldsymbol{\delta}_f^{*,RPI} \mathbf{f}_t + q_t + l_t$$

where l_t denotes the one-period liquidity premium. The short-term RPI-linked gilt implied breakeven inflation rate is thus given as

$$\begin{aligned}\pi_{t,1}^{b,RPI} = r_t - r_t^{*b,RPI} = r_t - r_t^{*,RPI} - l_t &= \pi_{t,1}^{RPI} - l_t \\ = \mathbf{J}\mathbf{f}_t - \boldsymbol{\theta}_f^* \mathbf{f}_t - q_t - l_t\end{aligned} \qquad (4)$$

We stack the latent factor $\mathbf{f}_t$, the RPI-CPI wedge factor q_t and the liquidity spread l_t to get $\mathbf{x}_t = [\mathbf{f}_t{}', q_t, l_t]'$, which follows a first-order Gaussian VAR under the risk-neutral measure ($\mathbb{Q}$):

[4] As shown in D'Amico, Kim, and Wei (2014), Bauer (2015) and Fleckenstein, Longstaff, and Lustig (2014), there are non-trivial liquidity premium in the government index-linked bond market.

$$\mathbf{x}_{t+1} = \boldsymbol{\kappa}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}^{\mathbb{Q}}_{t+1}$$
$$\boldsymbol{\varepsilon}^{\mathbb{Q}}_{t+1} \sim \boldsymbol{\mathcal{N}}(\mathbf{0}, \mathbf{I}) \quad (5)$$

where $\boldsymbol{\kappa}^{\mathbb{Q}} = [\kappa^{\mathbb{Q}}_{\infty}, \underbrace{0 \ldots 0}_{K^f-1}, \kappa^{\mathbb{Q}}_q, \kappa^{\mathbb{Q}}_l]'$ is a K by 1 vector where $K = K^f + 2$; $\boldsymbol{\Phi}^{\mathbb{Q}} = diag[\xi_1, \ldots, \xi_K]$, is a K by K matrix ; $\boldsymbol{\Sigma}$ is a K by K lower triangular matrix. The long-term mean of the CPI inflation rate under $\mathbb{Q}$ is given as $\kappa^{\mathbb{Q}}_{\infty}/(1 - \xi_1)$.

Given that the short term CPI and RPI breakeven inflation rates are correspondingly equal to one-period ahead expected CPI ($\pi^{e,CPI}_t$) and RPI ($\pi^{e,RPI}_t$) inflation rates, we can write them as affine functions of $\mathbf{x}_t$:

CPI short breakeven $$\pi^{CPI}_{t,1} = \pi^{e,CPI}_t = r_t - r^{*,CPI}_t = \mathbf{J}\mathbf{f}_t = \boldsymbol{\delta}\mathbf{x}_t \quad (6)$$

RPI short breakeven $$\pi^{RPI}_{t,1} = \pi^{e,RPI}_t = r_t - r^{*,RPI}_t = \mathbf{J}\mathbf{f}_t - \boldsymbol{\theta}^*_f\mathbf{f}_t - q_t = \bar{\boldsymbol{\delta}}\mathbf{x}_t \quad (7)$$

where $\boldsymbol{\delta} = [\mathbf{J}, 0, 0]$ and $\bar{\boldsymbol{\delta}} = [\mathbf{J} - \boldsymbol{\theta}^*_f, -1, 0]$.

Similarly, we can rewrite equation (4) for the gilt implied short-term breakeven inflation rate as an affine function of $\mathbf{x}_t$:

$$\pi^{b,RPI}_{t,1} = \mathbf{J}\mathbf{f}_t - \boldsymbol{\theta}^*_f\mathbf{f}_t - q_t - l_t = \bar{\boldsymbol{\delta}}^{\boldsymbol{b}}\mathbf{x}_t$$

where: $\bar{\boldsymbol{\delta}}^{\boldsymbol{b}} = [\mathbf{J} - \boldsymbol{\theta}^*_f, -1, -1]$.

Therefore the expected one period ahead RPI-CPI wedge (w_t) is

$$w_t = \pi^{e,RPI}_t - \pi^{e,CPI}_t = -\boldsymbol{\theta}^*_f\mathbf{f}_t - q_t = (\bar{\boldsymbol{\delta}} - \boldsymbol{\delta})\mathbf{x}_t$$

Also the short-term liquidity premium (l_t) is given as

$$l_t = \pi^{RPI}_{t,1} - \pi^{b,RPI}_{t,1} = (\bar{\boldsymbol{\delta}} - \bar{\boldsymbol{\delta}}^{\boldsymbol{b}})\mathbf{x}_t$$

2.2 *Term structure of breakeven inflation rates*

Let $P_{t,n}$ denote the price of an n-period nominal zero-coupon conventional bond at time t, while let $P^*_{t,n}$ denote the real price of an n-period synthetic index-linked bond at time t. Under the assumption of no-arbitrage, we have the following bond pricing equations under the risk-neutral measure $\mathbb{Q}$:

$$P_{t,n} = E^{\mathbb{Q}}_t[\exp(-r_t)P_{t+1,n-1}]$$
$$P^*_{t,n} = E^{\mathbb{Q}}_t[\exp(-r^*_t)P^*_{t+1,n-1}]$$

The ratio of the nominal and real bond prices of the same maturity can be expressed as the ratio of their expected prices in one period adjusted for the short-term breakeven inflation rate (see the appendix A1 for more details):

$$\frac{P_{t,n}}{P^*_{t,n}} = \frac{\exp(-r_t)E^{\mathbb{Q}}_t[P_{t+1,n-1}]}{\exp(-r^*_t)E^{\mathbb{Q}}_t[P^*_{t+1,n-1}]} \approx \exp[-\pi_{t,1}]E^{\mathbb{Q}}_t\left(\frac{P_{t+1,n-1}}{P^*_{t+1,n-1}}\right) \quad (8)$$

Given these assumptions, we can show that the ratio of the n-period nominal and real bond prices is an exponentially affine function of $\mathbf{x}_t$:

$$\frac{P_{t,n}}{P_{t,n}^*} = \exp(a_n + \mathbf{b}_n \mathbf{x}_t),$$

where the scalar a_n and $1 \times K$ scalar vector $\mathbf{b}_n$ can be solved with the recursive equations:

$$a_n = a_{n-1} + \mathbf{b}_{n-1}\boldsymbol{\kappa}^{\mathbb{Q}} + 0.5\mathbf{b}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathbf{b}_{n-1}' + a_1 \quad (9)$$

$$\mathbf{b}_n = \mathbf{b}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}} + \mathbf{b}_1 \quad (10)$$

At its maturity, the price of a nominal bond is given as £1 while the real price of an index-linked bond equals 1 unit of goods. Therefore the ratio of the nominal and real bond prices is equal to one at the maturity, which gives the boundary conditions for the recursion equations above: $a_O = 0$ and $\mathbf{b}_0 = -\mathbf{0}$.

Therefore the n-period synthetic CPI BEI rate ($\pi_{t,n}^{CPI}$), inflation swap based RPI BEI rate ($\pi_{t,n}^{RPI}$) and RPI linked gilt BEI rate ($\pi_{t,n}^{b,RPI}$) can all be derived with the following general equation:

$$\pi_{t,n}^i = -\frac{1}{n}\ln\frac{P_{t,n}}{P_{t,n}^{*,i}} = -\frac{1}{n}\left(a_n^i + \mathbf{b}_n^i\mathbf{x}_t\right),$$

where i in $\pi_{t,n}^i$ represents one of the three different types of breakeven rates as mentioned above, and $P_{t,n}^{*,i}$ denotes the real price of an n-period corresponding index-linked bond at time t. Also the scalar a_n^i and the 1 by K scalar vectors $\mathbf{b}_n^i$ can be derived recursively as shown in Eq (9) and (10), where we just replace the previous initial conditions (a_1 , $\mathbf{b}_1$) with the new ones for different breakeven rates:

$$\pi_{t,n}^{CPI}: \ a_1^{CPI} = 0, \mathbf{b}_1^{CPI} = -\boldsymbol{\delta};$$
$$\pi_{t,n}^{RPI}: \ a_1^{RPI} = 0, \mathbf{b}_1^{RPI} = -\bar{\boldsymbol{\delta}};$$
$$\pi_{t,n}^{b,RPI}: a_1^{b,RPI} = 0, \mathbf{b}_1^{b,RPI} = -\bar{\boldsymbol{\delta}}^{\boldsymbol{b}}.$$

We follow the new identification scheme proposed by Joslin, Singleton and Zhu (2011) (JSZ henceforth) to normalise the short rate and the drift of the Q-dynamics, given that it allows for more efficient estimation of the parameters under both the real-world ($\mathbb{P}$) and the risk-neutral ($\mathbb{Q}$) measures (see Guimarães, 2014). We also follow JSZ by carrying out the transformation to get the new portfolio factors, which can be principal components of a panel of time series data. In the original JSZ paper, the real world dynamics of the portfolio factors can be estimated with OLS independently of the risk neutral dynamics estimation. Although this will not be the case for the Kalman Filter estimation that we perform in this paper (see section 2.5 for more discussion of the estimation), we use the observed portfolio factors to find good starting values for the latent portfolio factors in the Kalman Filter estimation.

Let $\mathbf{z}_t$ denote the latent portfolio factors, which are constructed to match the first K principal components of RPI gilt BEI rates, short term CPI and the RPI breakeven inflation rates. We assume the portfolio factor vector $\mathbf{z}_t$ follows a VAR(1) process under both $\mathbb{P}$ and

$\mathbb{Q}$. The latent factors $\mathrm{x_t}$ (as defined in section 2.1) can be recovered from $\mathbf{z_t}$ via the inverse transformation. The linear transformation from the original latent factors $\mathbf{x_t}$ to the portfolio factors $\mathbf{z_t}$ is given as below:

$$\mathbf{z}_t = \mathbf{a}_g + \mathbf{B}_g\mathbf{x}_t$$

where $\mathbf{a}_g = \mathbf{G}\cdot\mathbf{a}$, $\mathbf{B}_g = \mathbf{G}\cdot\mathbf{B}$, $\mathbf{G}$ is the loading matrix used to construct the principal component vector $\mathbf{z}_t$, and $\mathbf{a}/\mathbf{B}$ are defined in Appendix A2.

The real-world dynamics of $\mathbf{z}_t$ is given by the following transition equation:

$$\mathbf{z}_{t+1} = \boldsymbol{\kappa}^z + \boldsymbol{\Phi}^{\mathbf{z}}\mathbf{z}_t + \boldsymbol{\Sigma}^{\mathbf{z}}\boldsymbol{\varepsilon}^{\mathbf{z}}_{t+1}$$

$$\boldsymbol{\varepsilon}^z_{t+1} \sim \boldsymbol{\mathcal{N}}(\mathbf{0},\mathbf{I})$$

where $\boldsymbol{\kappa}^{\mathbf{z}}$ is a K by 1 vector, $\boldsymbol{\Phi}^{\mathbf{z}}$ is a K by K matrix ; $\boldsymbol{\Sigma}^{\mathbf{z}}$ is a K by K lower triangular matrix.

As shown in Appendix A2, we model the inflation swap breakeven inflation rate ($\pi^{RPI}_{t,n}$) and index-linked bond breakeven inflation rate ($\pi^{b,RPI}_{t,n}$) as affine functions in terms of the new state vector $\mathbf{z}_t$ plus measurement errors:

$$\pi^{RPI}_{t,n} = -\frac{1}{n}(a^{RPI}_n + \mathbf{b}^{RPI}_n\mathbf{z}_t) + e_{t,n} \qquad (11)$$

$$\pi^{b,RPI}_{t,n} = -\frac{1}{n}\left(a^{b,RPI}_n + \mathbf{b}^{b,RPI}_n\mathbf{z}_t\right) + e^b_{t,n}, \qquad (12)$$

where we assume the error terms $e_{t,n}$ and $e^b_{t,n}$ both follow independent nominal distribution $\boldsymbol{\mathcal{N}}(0,\omega)$ with the same volatility.

2.3 *Inflation projections*

Kim and Orphanides (2012) suggest that the typically short time series available for estimating dynamic term structure models lead to problems identifying the $\mathbb{P}$ dynamics of the factors and suggest the use of survey data to help with the identification. In our case, we use survey data to provide more information on expected inflation in the future. As discussed in previous sections (see equations (6), (7), (11) and (12)), the expected one-period CPI and RPI inflation rates are given by

$$\pi^{e,CPI}_t = -a^{z,CPI}_1 - \mathbf{b}^{z,CPI}_1\mathbf{z}_t;$$

$$\pi^{e,RPI}_t = -a^{z,RPI}_1 - \mathbf{b}^{z,RPI}_1\mathbf{z}_t.$$

Expected CPI and RPI inflations between n and n+1 periods ahead at time t under $\mathbb{P}$ are respectively given by

CPI: $$\pi^{e,CPI}_{t,t+n} = E_t\left(\pi^{e,CPI}_{t+n}\right) = -a^{z,CPI}_1 - \mathbf{b}^{z,CPI}_1 E_t(\mathbf{z}_{t+n}) = -a^{z,e,CPI}_n - \mathbf{b}^{z,e,CPI}_n\mathbf{z}_t$$

RPI: $$\pi^{e,RPI}_{t,t+n} = E_t\left(\pi^{e,RPI}_{t+n}\right) = -a^{z,RPI}_1 - \mathbf{b}^{z,RPI}_1 E_t(\mathbf{z}_{t+n}) = -a^{z,e,RPI}_n - \mathbf{b}^{z,e,RPI}_n\mathbf{z}_t$$

where

$$a^{z,e,CPI}_n = a^{z,CPI}_1 + \mathbf{b}^{z,CPI}_1(\mathbf{I} - \boldsymbol{\Phi}^z)^{-1}(\mathbf{I} - (\boldsymbol{\Phi}^z)^n)\boldsymbol{\kappa}^z$$

$$\mathbf{b}_n^{z,e,CPI} = \mathbf{b}_1^{z,CPI}(\boldsymbol{\Phi}^z)^n$$
$$a_n^{z,e,RPI} = a_1^{z,RPI} + \mathbf{b}_1^{z,RPI}(\mathbf{I} - \boldsymbol{\Phi}^z)^{-1}(\mathbf{I} - (\boldsymbol{\Phi}^z)^n)\boldsymbol{\kappa}^z$$
$$\mathbf{b}_n^{z,e,RPI} = \mathbf{b}_1^{z,RPI}(\boldsymbol{\Phi}^z)^n$$

Given that the surveys refer to annual inflation expectations rather than monthly inflation, we derive the model-implied annual inflation as

$$\pi_{t,t+n}^{a,e,CPI} = \sum_{i=1}^{12} \pi_{t,t+n+i}^{e,CPI} + e_{t,n}^{a,CPI} = \sum_{i=1}^{12} \left(-a_{n+i}^{z,e,CPI} - \mathbf{b}_{n+i}^{z,e,CPI}\mathbf{z}_t\right) + e_{t,n}^{a,CPI}$$

$$\pi_{t,t+n}^{a,e,RPI} = \sum_{i=1}^{12} \pi_{t,t+n+i}^{e,RPI} + e_{t,n}^{a,RPI} = \sum_{i=1}^{12} \left(-a_{n+i}^{z,e,RPI} - \mathbf{b}_{n+i}^{z,e,RPI}\mathbf{z}_t\right) + e_{t,n}^{a,RPI}$$

where the survey forecasts over n horizon are measured with a Normally and independently distributed error term $e_{t,n}^{a,CPI} \sim \boldsymbol{\mathcal{N}}(0, \omega^{a,CPI})$ and $e_{t,n}^{a,RPI} \sim \boldsymbol{\mathcal{N}}(0, \omega^{a,RPI})$.

2.4 Breakeven inflation decomposition

The fitted BEI rate can be decomposed into two components: expectations for future inflation and a risk premium. For the inflation swap BEI ($\hat{\pi}_{t,n}^{RPI}$), we assume that the risk premium consists of only an inflation risk premium, given our assumption that the liquidity premium embedded in inflation swap rates is generally very small and difficult to identify. But for the fitted gilt BEI rate ($\hat{\pi}_{t,n}^{b,RPI}$), the risk premium includes both an inflation risk premium and a liquidity premium. The expectations component and the inflation risk premium components should be the same for both inflation swap and gilt BEI rates.

The decompositions for fitted values of inflation swap (IS) and gilt BEI rates are given below

IS BEI: $\hat{\pi}_{t,n}^{RPI} = exp_{t,n} + tp_{t,n}^{is} = exp_{t,n} + rp_{t,n}$

Gilt BEI: $\hat{\pi}_{t,n}^{b,RPI} = exp_{t,n} + tp_{t,n}^{gilt} = exp_{t,n} + rp_{t,n} + lp_{t,n}^{gilt}$

where we have:

Expected inflation: $exp_{t,n} = \frac{1}{n}\sum_{i=1}^{n} \pi_{t,t+i}^{e,RPI}$

Inflation risk premium: $rp_{t,n}$

Bond liquidity premium: $lp_{t,n}^{gilt} = \hat{\pi}_{t,n}^{b,RPI} - \hat{\pi}_{t,n}^{RPI}$

risk premium for inflation swap BEI: $tp_{t,n}^{is} = rp_{t,n}$

risk premium for gilt BEI: $tp_{t,n}^{gilt} = rp_{t,n} + lp_{t,n}^{gilt}$

One of the key assumptions made in this paper is that the difference between bond and swap BEI rates represents a liquidity premium. This is mainly because swaps and bonds have different characteristics, among which the most important is that swaps do not require large upfront payments as would be required for bond investments. Hence leveraged investors would face lower capital constraints to gain exposures to inflation-linked cashflows using inflation swaps compared to using index-linked bonds. These constraints will generally affect the ability to arbitrage between conventional and inflation linked bonds and it will tend to be priced as a charge e.g. a liquidity premium that makes the bond yield higher and hence the BEI rate lower. This characteristic is often referred to as 'shadow cost of capital' (see Garleanu and Pedersen (2011)). Some other reasons that may make swaps more liquid than bonds include the more flexible nature of cash flow in swaps that means that it is less likely for a swap to become "special"[5] in the way that government bonds may. In addition, it can be difficult and costly to short physical bonds at some time. But, it is generally as easy to sell inflation protection as it is to buy protection in inflation swap markets.

2.5 *State-space system and Kalman Filter*

We can summarise the above models for the RPI bond breakeven rates, RPI and CPI breakeven rates according to the following state-space system below.

$$\mathbf{z}_{t+1} = \boldsymbol{\kappa}^z + \boldsymbol{\Phi}^z \mathbf{z}_t + \mathbf{w}_{t+1}, \mathbf{w}_{t+1} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}^z \boldsymbol{\Sigma}^{z\prime}) \quad (13)$$

$$\mathbf{y}_t = \mathbf{a} + \mathbf{B}\mathbf{z}_t + \mathbf{e}_t, \mathbf{e}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega}\boldsymbol{\Omega}') \quad (14)$$

where

$$\mathbf{y}_t = \begin{pmatrix} \boldsymbol{\pi}_t^{b,RPI} \\ \boldsymbol{\pi}_t^{RPI} \\ \boldsymbol{\pi}_t^{a,e,CPI} \\ \boldsymbol{\pi}_t^{a,e,RPI} \\ \pi_{t,1}^{CPI} \\ \pi_{t,1}^{RPI} \\ \pi_{t,1}^{b,RPI} \end{pmatrix}, \mathbf{a} = \begin{pmatrix} -\mathbf{a}^{zb,RPI} \\ -\mathbf{a}^{z,RPI} \\ -\mathbf{a}^{z,e,CPI} \\ -\mathbf{a}^{z,e,RPI} \\ -a_1^{z,CPI} \\ -a_1^{z,RPI} \\ -a_1^{zb,RPI} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -\mathbf{B}^{zb,RPI} \\ -\mathbf{B}^{z,RPI} \\ -\mathbf{B}^{z,e,CPI} \\ -\mathbf{B}^{z,e,RPI} \\ -\mathbf{b}_1^{z,CPI} \\ -\mathbf{b}_1^{z,RPI} \\ -\mathbf{b}_1^{zb,RPI} \end{pmatrix}$$

and

$$\mathbf{a}^{zb,RPI} = \left(\frac{1}{n_1} a_{n_1}^{zb,RPI}, \frac{1}{n_2} a_{n_2}^{zb,RPI}, \dots \frac{1}{n_N} a_{n_N}^{zb,RPI}\right)'$$

$$\mathbf{a}^{z,RPI} = \left(\frac{1}{n_1} a_{n_1}^{z,RPI}, \frac{1}{n_2} a_{n_2}^{z,RPI}, \dots \frac{1}{n_N} a_{n_N}^{z,RPI}\right)'$$

$$\mathbf{a}^{z,e,CPI} = \left(\sum_{i=1}^{12} a_{n_1+i}^{z,e,CPI}, \sum_{i=1}^{12} a_{n_2+i}^{z,e,CPI}, \dots \sum_{i=1}^{12} a_{n_N+i}^{z,e,CPI}\right)'$$

$$\mathbf{a}^{z,e,RPI} = \left(\sum_{i=1}^{12} a_{n_1+i}^{z,e,RPI}, \sum_{i=1}^{12} a_{n_2+i}^{z,e,RPI}, \dots \sum_{i=1}^{12} a_{n_N+i}^{z,e,RPI}\right)'$$

[5] The specialness of a specific instrument refers to the difference between the specific collateral rate for this instrument and a general collateral rate (i.e. a "normal" interest rate), which can be due to the inability or high cost of supplying that instrument. See Duffie (1996).

$$\mathbf{B}^{zb,RPI} = \begin{pmatrix} \frac{1}{n_1}\mathbf{b}_{n_1}^{zb,RPI} \\ \frac{1}{n_2}\mathbf{b}_{n_2}^{zb,RPI} \\ \dots \\ \frac{1}{n_N}\mathbf{b}_{n_N}^{zb,RPI} \end{pmatrix}, \mathbf{B}^{z,RPI} = \begin{pmatrix} \frac{1}{n_1}\mathbf{b}_{n_1}^{z,RPI} \\ \frac{1}{n_2}\mathbf{b}_{n_2}^{z,RPI} \\ \dots \\ \frac{1}{n_N}\mathbf{b}_{n_N}^{z,RPI} \end{pmatrix}, \mathbf{B}^{z,e,CPI} = \begin{pmatrix} \mathbf{b}_{n_1}^{z,e,CPI} \\ \mathbf{b}_{n_2}^{z,e,CPI} \\ \dots \\ \mathbf{b}_{n_N}^{z,e,CPI} \end{pmatrix}, \mathbf{B}^{z,e,RPI} = \begin{pmatrix} \mathbf{b}_{n_1}^{z,e,RPI} \\ \mathbf{b}_{n_2}^{z,e,RPI} \\ \dots \\ \mathbf{b}_{n_N}^{z,e,RPI} \end{pmatrix}$$

$$\boldsymbol{\Omega} = \mathrm{diag}(\underbrace{\omega, \dots, \omega}_{for\ \boldsymbol{\pi}_t^{b,RPI}, \boldsymbol{\pi}_t^{RPI}}, \underbrace{\omega^{a,CPI} .. \omega^{a,CPI}}_{for\ \boldsymbol{\pi}_t^{a,e,CPI}}, \underbrace{\omega^{a,RPI}, \dots, \omega^{a,RPI}}_{for\ \boldsymbol{\pi}_t^{a,e,RPI}}, \underbrace{\omega, \omega, \omega}_{for\ \pi_{t,1}^{CPI} / \pi_{t,1}^{RPI} / \pi_{t,1}^{b,RPI}})$$

The state equation (14) shows the real-world dynamics of the state vector, $\mathbf{z}_t$. The measurement equation (14) gives the mapping between the observed variables and the state vector, where the observed variables include: RPI bond breakeven $\pi_{t,n}^{b,RPI}$; RPI inflation swap rates $\pi_{t,n}^{RPI}$; annual CPI survey expectations $\pi_{t,t+n}^{a,e,CPI}$; RPI survey expectations $\pi_{t,t+n}^{a,e,RPI}$. We also add one-month breakeven inflation rate ($\pi_{t,1}^{CPI}$, $\pi_{t,1}^{RPI}$ and $\pi_{t,1}^{b,RPI}$ for CPI, inflation swap and gilt RPI respectively) to pin down the short end of the breakeven rate curve.

We estimate the complete model (13) and (20) using maximum log-likelihood estimation, where the Kalman Filter is used to filter the factors. The log-likelihood function to be maximised is given as below:

$$\log \mathcal{L}(\boldsymbol{\Theta}; \mathbf{y}_{t=1,\dots,T}) = \sum_{t=1}^{T} \log f(\mathbf{y}_t | \mathbf{y}_{t-1}, \boldsymbol{\Theta})$$

where $\boldsymbol{\Theta}$ is the parameter set that include all the parameters to be estimated, i.e. $\boldsymbol{\Theta} = \{\boldsymbol{\kappa}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\theta}_f^*, \boldsymbol{\kappa}^z, \boldsymbol{\Phi}^z, \boldsymbol{\Sigma}^z, \boldsymbol{\Omega}\}$. The parameters $\boldsymbol{\kappa}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}$ are defined in equation (5), $\boldsymbol{\kappa}^z, \boldsymbol{\Phi}^z, \boldsymbol{\Sigma}^z$ in (13), $\boldsymbol{\theta}_f^*$ in equation (2), and $\boldsymbol{\Omega}$ in equation (14).

3 Data and preliminary analysis

Our sample period spans October 1992 to December 2013 with data observations at a monthly frequency. The main reason for starting the sample period from October 1992 is to match a major change in the monetary policy framework in the UK, which adopted inflation targeting in October 1992, and hence to avoid a possible structural break in the data.

Gilt BEI rates (Chart 1. A) are computed as the difference between continuously compounded nominal and real spot rates (i.e. yields on zero-coupon bonds), which are estimated using the Variable Roughness Penalty (VRP) model by Anderson and Sleath

(2001) and published by the Bank of England. For bond breakeven rates, we use 3-, 4-, 5-, 7- and 10-year maturities from October 1992 to December 2013, sampled at monthly frequency on the 21st day of the month in line with the CPI and RPI data releases.

Inflation swap implied BEI rates[6] (Chart 1. B) are also obtained for the same bond maturities. Unfortunately, inflation swaps are only available from 2004. Our estimation methodology, which is based on the Kalman Filter, is able to deal with the missing data problem given that the estimation of state vectors will not be seriously affected by the missing data issue. In the Kalman Filter, the observable variables (the BEI rates and other inflation data) are used to improve the first-round estimate of the state vectors, rather than working as a direct input for calculating the state vectors.

The exclusion of maturities shorter than three years is due to the lack of good quality data at the short end of the BEI curves. According to Anderson and Sleath (2001), constraints are applied to the VRP model to guarantee stability at the short end of the curve by omitting index-linked bonds with short maturities or bonds that are unsuitable due to the small number available in the specific curve segment. This creates gaps in the time series of real spot rates, and hence of BEI rates, at shorter maturities.

In order to address the issue of a lack of short maturity data, we also include proxies for one-month CPI and RPI breakeven rates (i.e. $\pi_{t,1}$ and $\bar{\pi}_{t,1}$) in the model (see data plot in Chart 1. C). These are approximated by regressing the month-on-month CPI and RPI inflation on the lagged year-on-year CPI and RPI inflation rates (Chart 1. D). The UK CPI and RPI price index data we used are non-seasonally adjusted, published monthly by the Office for National Statistics (ONS). The realised rate of year-on-year inflation is calculated as the annual log change of the price index. We use lagged year-on-year inflation rates as explanatory variables instead of lagged month-on-month inflation rates to avoid seasonality exhibited in month-on-month inflation time series, which is highly undesirable. Therefore the one-month CPI and RPI inflation breakeven rates are approximated by a linear function of lagged year-on-year inflation rates in our paper.

Our proxy for the one-month RPI breakeven rate is useful for identifying the short end of inflation swap BEI curves. For the gilt BEI curve, we need to adjust for the liquidity premium, as discussed in the previous sections. A proxy for the one-month bond breakeven rate is derived as the one-month RPI breakeven rates adjusted for a short-term liquidity spread, estimated by regressing bond-swap breakeven spreads (i.e. liquidity premium) on the corresponding maturities at each period. This assumes the term structure of liquidity premia follows a straight line and the value of the short term liquidity spread can be inferred by extending this line to the one-month maturity. We apply this short term liquidity spread adjustment to the one-month RPI breakeven rate for the period after 2004. We cannot do the same adjustment for the period before 2004 due to the lack of inflation swap data. Therefore, we approximate the one-month gilt BEI inflation rate using the one-

[6] See Hurd and Relleen (2006) for details of estimation.

month RPI inflation rate for the period before 2004. This is a simplistic assumption, but may not be unreasonable given that inflation was fairly stable over the period in question.

We supplement the dataset with survey data for CPI and RPI inflation expectations 1 to 10 years ahead by Consensus Economics (Chart 1 E-F). They are available semi-annually from April 2004 for CPI and from April 1990 for RPI.

One issue with the RPI survey data is that the forecast is actually given for the RPIX inflation (RPI inflation excluding mortgage interest payments). Following Joyce, Lildholdt, and Sorensen (2010) who noted that at medium to long horizons the RPI/ RPIX wedge is likely to be small, we do not take into account the difference between the two indices.

As our JSZ portfolio weights are chosen to be the same as the loadings used to construct the principal components, we carried out principal component analysis following standard practice in the term structure literature, to identify the number of factors required to explain the variance in BEI rates, CPI inflation and RPI inflation (Table 1). The analysis shows that 5 principal components are required to explain 99.79% of the data variance for gilt BEI, CPI and RPI inflation for the sample period from 1992 to 2013. We did a similar analysis for both gilt and inflation swap BEI rates, but excluding CPI and RPI inflation, with a data sample from 2004 to 2013. In this case we need at least 4 principal components to explain 99.94% of the variance. A portfolio of inflation swap BEI rates, and CPI and RPI inflation data from 2004 to 2014 would only require 3 factors. So, overall, we need at least 5 factors for our model in order to fit the gilt and inflation swap BEI rates as well as RPI and CPI inflation. This also shows that modelling breakeven rates directly instead of from a joint nominal and real curve estimation made our specifications more parsimonious given that we need at least 6 or 7 factors to explain the same proportion of variance of a portfolio of BEI rates, nominal rates and CPI and RPI inflation.

4 Results

Based on the principal component analysis, our preferred model has 5 factors and it is estimated for the sample period between October 1992 and the end of December 2013. The model fits bond and swap BEI rates well at all maturities. For example, the fitting errors of 5 and 10 year BEI rates are less than 20 basis points (see Chart 2) in absolute terms. In Table 2 we report the estimated model parameters: $\boldsymbol{\Theta} = \left\{\boldsymbol{\kappa}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\theta}_f^*, \boldsymbol{\kappa}^{\mathbf{z}}, \boldsymbol{\Phi}^{\mathbf{z}}, \boldsymbol{\Sigma}^{\mathbf{z}}, \boldsymbol{\Omega}\right\}$. We found that the diagonal parameters in $\boldsymbol{\Phi}^{\mathbb{Q}}$ are all significant with the largest element very close to 1 (i.e. 0.990), showing the high persistency of the dynamics of factors under $\mathbb{Q}$. We find that the largest eigenvalue for the matrix $\boldsymbol{\Phi}^{\mathbf{z}}$ is 0.988, which is also very high. This suggests that the factors are also highly persistent under $\mathbb{P}$.

4.1 Gilt and swap BEI rates decomposition

As Chart 3 (panel A/B) shows, both long and medium term (i.e. 10 and 5-year respectively) gilt BEI rates fell significantly after the introduction of inflation targeting in 1992 and drifted downward during the 1990s. This is partially accounted for by a fall in inflation expectations. The fall in breakeven rates was also associated with a significant fall in inflation risk premia, suggesting investors had more confidence in the new monetary policy framework and/or were less uncertain about future inflation. These results are consistent with the earlier findings by Joyce, Lildholdt, and Sorensen (2010) and Abrahams, Adrian, Crump, and Moench (2013).

With the exception of Q3-2008 when both 5 and 10-year RPI inflation expectations peaked at around 3.5% and the subsequent falls in inflation expectations during the height of the financial crisis, our measures of medium and long-term RPI inflation expectations are reasonably stable and average 2.8% since 1998.

Estimates of the 10-year gilt BEI risk premium (Chart 3.A) were generally positive and decreasing across the sample period, averaging at around 1% until 1997, 20 basis points between 1997 and 2008 and minus 10 basis points thereafter, in line with the estimates by Abrahams, Adrian, Crump, and Moench (2013). Chart 3.B shows that the 5-year gilt BEI risk premium also exhibits a similar downward trend over the sample.

The decomposition of this risk premium for the 10-year gilt BEI rate after 2004 (Chart 3.C)[7] shows that the inflation risk premium was, on average, 15 basis points. The maximum level was reached in October 2009 at 75 basis points. It went down to -40 basis points in Q4-11. Chart 3.D shows the 5-year risk premium decomposition where the inflation risk premium is slightly negative (-6 basis points on average after the crisis) but the liquidity premium is much lower (-44 basis points on average after the crisis). This is rather different from the estimates found in previous studies, such as Guimaraes (2014), which found large negative inflation premia in the medium and long-term gilt BEI rates since the crisis. This may be because those estimates not only include the inflation risk premium, which is driven by uncertainty about future inflation risk, but also a liquidity premium. Therefore our model suggests that the negative sign of gilt BEI risk premia since the crisis is more the result of market liquidity factors rather than a strongly negative inflation risk premium.

Our estimates of the liquidity premium explains a large part of the total risk premium in some periods, especially at time of crisis as in 2008, when it accounted for 98% of the total risk premium and its absolute value was as high as 80 basis points for the 10-year gilt BEI rate. These estimates are in the range with those found earlier in the literature, e.g. between negligible estimates by Pflueger and Viceira (2013) and 200bp by D'Amico, Kim, and Wei (2014). We believe that the relatively high liquidity premium estimate reflects a combination of funding constraints in the market for inflation-linked gilts in that period and exceptional movements driven by flight to quality effects towards conventional gilts and the unwinding of derivatives positions by institutional investors, following the failure of Lehman Brothers. The liquidity premium otherwise averaged -30 basis points after 2009 at the 10 years maturity and stabilised at around -20 basis points after 2012. The rise in risk premia after September 2012 was instead primarily driven by inflation risk premia rather

[7]It is hard to distinguish between two risk premia components in gilt BEI– liquidity and inflation premium- prior 2004 due to the lack of the inflation swap data.

than liquidity premia, as the latter remained fairly constant. We also found that the estimated liquidity factor turns out to be very similar to both the funding illiquidity proxy in Malkhozov, Mueller, Vedolin, and Venter (2014) and the liquidity premium estimates in Pflueger and Viceira (2013). This lends further support for the robustness of the liquidity premium estimation.

On average the term structure of inflation risk premia is upward sloping (Chart 4) in line with the existing literature. Intuitively this is because inflation uncertainty is likely to be increasing with the time horizon, although the slope of our estimates varies over time. We found that in 2000-2004 the term structure was flat (where 10-year inflation risk premium is about the same as that of 5-year) and that after 2004 it became upward sloping again (with the 10-year inflation risk premium lying above that of 5-year). The term structure of liquidity premia (Chart 4) is flat and positive between 2004 and 2008, and downward sloping but negative thereafter.

Inflation swap BEI rates (Chart 5) are generally less volatile than corresponding maturity bond breakeven rates, which may be more significantly affected by liquidity conditions. This would also imply that their movements are more driven by changes in inflation expectations. This may corroborate views from the Bank's market contacts that swap BEI rates represents a more reliable indicator of inflation expectations, compared to bond BEI rates.

4.2 Estimated CPI expectations and RPI-CPI wedge

Our estimates of RPI and CPI inflation expectations for 2, 3 5 and 10 years horizons are reported in Chart 6. The key finding is that the long-term (i.e. 10-year) RPI and CPI inflation expectations are very stable and well anchored with the latter close to the MPC's 2% CPI inflation target. But the estimates of RPI and CPI expectations at shorter horizons are rather volatile with the former the most volatile (Chart 6.A).

Estimates of long-term CPI expectations average 2.3% over the whole sample. We also found that after the crisis long-term expectations for both CPI and RPI are slightly more volatile than the period before the crisis but after the independence of the Bank of England (i.e. from 1997 to 2008).

As regards estimated expectations for the RPI-CPI 'wedge' (i.e. the spread between RPI and CPI inflation), we can distinguish three periods which each exhibit significantly different features (Chart 7). The first period is between 1992 and 1997 (see Chart 7A), where more than 50% of the expected RPI-CPI wedge term structures are downward sloping. The second period is between 1998 and 2007 (see Chart 7B), where more than 75% of the expected RPI-CPI wedge term structures are downward sloping. The last period is between 2008 and 2013 (see Chart 7C), where more than 75% of the expected RPI-CPI wedge term structures are upward sloping. The contrast between 2nd and 3rd period is especially large. This suggests, in general, the market expected a higher than average RPI-CPI wedge at short horizons before the 2008 crisis but a lower wedge after the crisis. Therefore the short term RPI inflation expectation is a better proxy for CPI expectations after the crisis than it was before the crisis.

We also observe that the dispersion of expectations for the RPI and CPI wedge was largest for shorter horizons. The expected wedge appears to mean-revert beyond 4 to 5 years. Chart 8 further demonstrates this point by showing the term structure of the wedge dispersion, where the dispersion starts at a high level at the shortest maturity and then falls quickly to a very low level after 3 years.

Over longer horizons, the expected RPI/CPI wedge appears fairly stable at around 66 basis points (Chart 7). At face value this suggests that estimates of long-term RPI expectations can be transformed to estimates of long-term CPI expectations via a constant adjustment. In other words, we could approximate long-term CPI inflation expectations by subtracting a constant wedge of 66 basis points from the measure of RPI inflation expectations. We cannot apply a similar constant adjustment to short-term RPI inflation measures, however, given that our estimates of the expected RPI-CPI wedge change significantly from month to month at shorter horizons (e.g the 2-5 year horizon).

It is also worth noting that our estimates for the expected long-run RPI-CPI wedge are a little lower than some other estimates. For example, the Bank's latest discussions with market participants suggest that they generally expect the wedge will average around 80-100 basis points in the long-term (see Domit and Roberts-Sklar, 2015). However, in some cases these forecasts were adjusted up following methodological changes in 2010 by the ONS and our model is unlikely to fully capture that yet, given the short sample period since 2010. So the model may slightly underestimate current expectations for the future RPI/CPI wedge.

5 Sensitivity analysis

In this section we test the robustness of our results to the choice of sample period, to the inclusion of survey data and to the liquidity assumption. We carry out various exercises and report the results in Charts 9-12.

Sensitivity to the sample period

We estimate the model across different periods (with the same ending date but different starting date) in order to check the model sensitivity to two possible structural breaks: (1) the introduction of inflation targeting in 1992, and (2) the creation of an independent MPC at the Bank of England in 1997. The longest sample period goes back to 1989 when CPI data are available for the first time. We find that the model is fairly robust to the choice of the sample period as the estimated BEI rates, term premia and expectations are all very similar to each other for different sample periods (Chart 9).

We also estimated the model by gradually expanding the end date of the sample by 1 year from 2006 with the starting date in Oct 1992. We find that expanding the data sample stabilises both CPI and RPI inflation expectations in the post-1998 period (Chart 10), as the range of projections narrows over time. This suggests that our preferred sample period (from 1992 to 2013) is long enough to guarantee the stable estimation of the decomposition.

Impact of inclusion of survey data

We re-estimated the preferred 5-factor model without survey data. The results (Chart 11) show that the estimates of CPI expectations become very sensitive to the sample selection if the model excludes survey data. This is also true for the estimates of RPI expectations[8]. Therefore the inclusion of the survey data helps to improve the robustness of the estimation of inflation expectations to different sample choices.

Impact of liquidity assumption

To test the impact of the liquidity assumption, we re-estimated the model without the inclusion of inflation swap BEI data so that the liquidity premium cannot be identified in the model. The principal component analysis suggests that 4 factors are enough for the new dataset which includes gilt BEI rates but NOT inflation swap data.

We found that excluding inflation swap BEI data (but still including survey data) mainly affects the inflation premia estimation that become negative after 2004 (Chart 12), in line with Guimaraes (2014). This is due to the fact that the new estimation of inflation premium also includes the unidentified liquidity premium component. Therefore, the impact of liquidity assumption affects mostly the estimation of the inflation risk premium rather than the expectation.

6 Conclusion

The breakeven inflation rates implied from traded financial instruments (in particular index-linked gilts and inflation swaps), should contain rich information on inflation expectations. However, UK BEI rates cannot be interpreted as market forecasts of future CPI inflation, which is the measure of inflation targeted by the UK MPC. This is because BEI rates in the UK refer to RPI rather than CPI inflation and also because BEI rates include risk premia, which compensate for inflation risk and also liquidity risk in some cases.

To address these limitations and extract more information from BEI rates, we develop a no-arbitrage term structure model to decompose breakeven inflation rates into CPI inflation expectations, expecations for the 'wedge' between RPI and CPI inflation and risk premia. We further decompose estimates of risk premia in gilt BEI rates into inflation risk and liquidity premia components.

There are a few novel features in our model. First, we model BEI rates directly without jointly modelling nominal and real yields as many previous studies have done. Second, we model both bond and inflation swap BEI rates jointly, allowing us to identify the liquidity

[8] Results are available upon requests.

premium in gilt BEI rates. Third, we incorporate professional survey data on inflation forecasts in our model to improve the estimation of the real world dynamics. The plausibility tests carried out demonstrate the robustness of our model estimation to the sample choice, the impact of liquidity assumption, and also show the importance of including survey data.

We find that our estimates for both CPI and RPI inflation expectations have been reasonably stable at medium and long-term horizons since 1997. But long-term expectations for both CPI and RPI are slightly more volatile after the crisis compared to the decade just before the crisis (i.e. the period between 1997 and 2008).

The term structure of inflation risk premia is found to be upward sloping on average, in line with the existing literature, consistent with inflation uncertainty increasing with time horizon. Liquidity premia in gilt BEI rates are found to explain a large part of the total risk premium in gilt BEI rates during certain periods, especially in the crisis period after 2008. The results suggest that the negative sign of the risk premium in gilt BEI rates during these periods was, to a large extent, the result of negative liquidity premia, which we conclude were driven by periods of illiquidity in the market for index-linked gilts. This also suggests that inflation swap BEI rates may be a more reliable indicator of inflation expectations, compared to bond BEI rates.

Finally, our model suggests that expectations for the wedge between CPI and RPI inflation are quite volatile for short horizons but very stable (converging to 66 basis points) at longer horizons. At face value this suggests that our estimates for long-term RPI expectations can be transformed to get a view on long-term CPI inflation expectations via a simple constant adjustment. We also note, however, that our estimates for the long-run RPI-CPI wedge are a little lower than some other recent forecasts. For example, the Bank's latest discussions with market participants suggest that they generally expect the wedge will average around 80-100 basis points in the long-term. In some cases forecasts of the long-run wedge were adjusted upwards following methodological changes in 2010 by the ONS, which our model will not fully capture given the short sample period afterwards, so the model may underestimate current expectations for the future RPI/CPI wedge.

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Appendix

A1 Breakeven Rates Approximation

The appendix explains how we can derive the approximation relationship in (8).

First we will show the following relationship holds

$$\frac{E_t^{\mathbb{Q}}(P_{t+1,n-1})}{E_t^{\mathbb{Q}}(P_{t+1,n-1}^*)} = \exp(c_n)\, E_t^{\mathbb{Q}}\left(\frac{P_{t+1,n-1}}{P_{t+1,n-1}^*}\right) \tag{15}$$

Let's start by denoting $P_{t,n}/P_{t,n}^*$ as $P_{t,n}^{\pi}$, which gives the inflation breakeven rate as $\pi_{t,n} = -P_{t,n}^{\pi}/n$. We then take log on the left-hand-side of the above equation and obtain the following.

$$\begin{aligned}
&\ln\left(\frac{E_t^{\mathbb{Q}}(P_{t+1,n-1})}{E_t^{\mathbb{Q}}(P_{t+1,n-1}^*)}\right)\\
&\qquad = E_t^{\mathbb{Q}}(\ln P_{t+1,n-1}) + 0.5V_t^{\mathbb{Q}}(\ln P_{t+1,n-1}) - E_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^*)\\
&\qquad - 0.5V_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^*)\\
&\qquad = E_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^* + \ln P_{t+1,n-1}^{\pi}) + 0.5V_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^* + \ln P_{t+1,n-1}^{\pi})\\
&\qquad - E_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^*) - 0.5V_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^*)
\end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned}
&\ln\left(\frac{E_t^{\mathbb{Q}}(P_{t+1,n-1})}{E_t^{\mathbb{Q}}(P_{t+1,n-1}^*)}\right)\\
&\qquad = E_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^{\pi}) + 0.5V_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^{\pi})\\
&\qquad + COV_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^*, \ln P_{t+1,n-1}^{\pi})
\end{aligned} \tag{16}$$

Given that we have $\ln(E_t^{\mathbb{Q}}(P_{t+1,n-1}^{\pi})) = E_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^{\pi}) + 0.5V_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^{\pi})$, the constant term c_n in Eq(15) thus equals the covariance term in the above equation

$$c_n = COV_t^{\mathbb{Q}}(\ln P_{t+1,n-1}^*, \ln P_{t+1,n-1}^{\pi})$$

Second, we argue that the constant term c_n only plays an insignificant role and can be dropped in (15). Therefore the following approximation will hold:

$$\frac{P_{t,n}}{P_{t,n}^*} = \frac{\exp(-r_t)E_t^{\mathbb{Q}}[P_{t+1,n-1}]}{\exp(-r_t^*)E_t^{\mathbb{Q}}[P_{t+1,n-1}^*]} \approx \exp[-\pi_{t,1}]E_t^{\mathbb{Q}}\left(\frac{P_{t+1,n-1}}{P_{t+1,n-1}^*}\right)$$

We believe it is justifiable to assume $c_n \approx 0$ for the following reasons: (1) The covariance term c_n is negligibly small compared to the sum of the first two terms in Eq(16) for all maturities that we have used to fit the model (i.e. 3 years to 10 years). preliminary results show that one month realised covariance terms (calculated using daily breakeven and real yield data) between 3, 6, and 10 year $\ln P_{t,n}^{\pi}$ and $\ln P_{t,n}^*$ are either under or just above 0.01% of the sum of the expectation and the variance terms in Eq(16) although they are calculated under $\mathbb{P}$ rather than $\mathbb{Q}$. (2) Our model is an inflation only model which does not include any nominal or real yield data. As a result, the covariance term will be estimated using extra nominal/real yield data if we are to include this term. This adds unnecessary complexity without bringing any real benefits. (3) The assumption on covariance term (c_n) will have no impact on any dynamic analysis. This is because the covariance term is a constant and will not change with time. Therefore, it has no impact on any dynamic analysis such as how expectation/term premium components change over time.

A2 JSZ transformation

As discussed in the main text, $\mathrm{z_t}$ is constructed to match the first K principal components of RPI linked gilt implied breakeven rates, short term CPI and the RPI breakeven inflation rates. The linear transformation from the original latent factors $\mathrm{x_t}$ to the portfolio factors $\mathrm{z_t}$ is given as below:

$$\mathbf{z}_t = \mathbf{G}\cdot\begin{pmatrix}\boldsymbol{\pi}_t^{b,RPI}\\ \pi_{t,1}^{CPI}\\ \pi_{t,1}^{RPI}\end{pmatrix} = \mathbf{G}\begin{pmatrix}\mathbf{a}^{b,RPI}+\mathbf{B}^{b,RPI}\mathbf{x}_t\\ \boldsymbol{\delta}\mathbf{x}_t\\ \overline{\boldsymbol{\delta}}\mathbf{x}_t\end{pmatrix} = \mathbf{G}(\mathbf{a}+\mathbf{B}\mathbf{x}_t)$$

where

$$\boldsymbol{\pi}_t^{b,RPI} = \begin{pmatrix}\pi_{t,n1}^{b,RPI}\\ \dots\\ \pi_{t,nN}^{b,RPI}\end{pmatrix}, \mathbf{a} = \begin{pmatrix}\mathbf{a}^{b,RPI}\\ 0\\ 0\end{pmatrix}, \mathbf{B} = \begin{pmatrix}\mathbf{B}^{b,RPI}\\ \boldsymbol{\delta}\\ \overline{\boldsymbol{\delta}}\end{pmatrix}, \mathbf{a}^{b,RPI} = \begin{pmatrix}a_{n1}^{b,RPI}\\ \dots\\ a_{nN}^{b,RPI}\end{pmatrix}, \mathbf{B}^{b,RPI} = \begin{pmatrix}\mathbf{b}_{n1}^{b,RPI}\\ \dots\\ \mathbf{b}_{nN}^{b,RPI}\end{pmatrix}$$

Following JSZ, we specify the dynamics of $\boldsymbol{z}_t$ under $\mathbb{Q}$ as

$$\mathbf{z}_{t+1} = \boldsymbol{\kappa}^{z\mathbb{Q}} + \boldsymbol{\Phi}^{z\mathbb{Q}}\mathbf{z}_t + \boldsymbol{\Sigma}^z\boldsymbol{\varepsilon}_{t+1}^{z\mathbb{Q}}$$

$$\boldsymbol{\varepsilon}_{t+1}^{z\mathbb{Q}} \sim \mathcal{N}(\mathbf{0},\mathbf{I})$$

where the parameters in the above equation can be inferred from those in equation (5) as

$$\boldsymbol{\kappa}^{z\mathbb{Q}} = \mathbf{B}_g\boldsymbol{\kappa}^{\mathbb{Q}} + \mathbf{a}_g - \boldsymbol{\Phi}^{z\mathbb{Q}}\mathbf{a}_g$$
$$\boldsymbol{\Phi}^{z\mathbb{Q}} = \mathbf{B}_g\boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{B}_g^{-1}$$
$$\boldsymbol{\Sigma}^z = \mathbf{B}_g\boldsymbol{\Sigma}$$

We can derive the following general pricing model for CPI breakeven inflation rates with regard to $\boldsymbol{z}_t$:

$$\pi_{t,n}^{CPI} = -\frac{1}{n}\left(a_n^{z,CPI} + \mathbf{b}_n^{z,CPI}\mathbf{z}_t\right) \quad (17)$$

where

$$a_n^{z,CPI} = a_{n-1}^{z,CPI} + \mathbf{b}_{n-1}^{z,CPI}\boldsymbol{\kappa}^{z\mathbb{Q}} + 0.5\mathbf{b}_{n-1}^{z,CPI}\boldsymbol{\Sigma}^z\boldsymbol{\Sigma}^{z\prime}\mathbf{b}_{n-1}^{z,CPI\prime} + a_1^{z,CPI} \quad (18)$$

$$\mathbf{b}_n^{z,CPI} = \mathbf{b}_{n-1}^{z,CPI}\boldsymbol{\Phi}^{z\mathbb{Q}} + \mathbf{b}_1^{z,CPI} \quad (19)$$

where $a_1^{z,CPI} = \boldsymbol{\delta}\mathbf{B}_g^{-1}\mathbf{a}_g$ and $\mathbf{b}_1^{z,CPI} = -\boldsymbol{\delta}\mathbf{B}_g^{-1}$**,** which are derived by solving the following equation:

$$\pi_{t,1}^{CPI} = -\left(a_1^{z,CPI} + \mathbf{b}_1^{z,CPI}\mathbf{z}_t\right) = \boldsymbol{\delta}\mathbf{x}_t. \quad (20)$$

Similarly, the n-period RPI breakeven inflation rate $\pi_{t,n}^{RPI}$ and that adjusted for the liquidity premium $\pi_{t,n}^{b,RPI}$ are given as:

$$\pi_{t,n}^{RPI} = -\frac{1}{n}\left(a_n^{z,RPI} + \mathbf{b}_n^{z,RPI}\mathbf{z}_t\right)$$
$$\pi_{t,n}^{b,RPI} = -\frac{1}{n}\left(a_n^{zb,RPI} + \mathbf{b}_n^{zb,RPI}\mathbf{z}_t\right)$$

where the scalar $a_n^{z,CPI}/\,a_n^{zb,RPI}$ and vector $\mathbf{b}_n^{z,RPI}$ and $\mathbf{b}_n^{zb,RPI}$ can be derived recursively as shown in equations (18) and (19) the following initial conditions:

$$a_n^{z,RPI} = \overline{\boldsymbol{\delta}}\mathbf{B}_g^{-1}\mathbf{a}_g, \mathbf{b}_n^{z,RPI} = -\overline{\boldsymbol{\delta}}\mathbf{B}_g^{-1};$$

$$a_n^{zb,RPI} = \bar{\boldsymbol{\delta}}^b \mathbf{B}_g^{-1}\mathbf{a}_g, \mathbf{b}_n^{zb,RPI} = -\bar{\boldsymbol{\delta}}^b \mathbf{B}_g^{-1}.$$

The initial conditions are derived by solving the following equations:

$$\pi_{t,1}^{RPI} = -\left(a_1^{z,RPI} + \mathbf{b}_1^{z,RPI}\mathbf{z}_t\right) = \bar{\boldsymbol{\delta}}\mathbf{x}_t \tag{21}$$

$$\pi_{t,1}^{b,RPI} = -\left(a_1^{zb,RPI} + \mathbf{b}_1^{zb,RPI}\mathbf{z}_t\right) = \bar{\boldsymbol{\delta}}^b\mathbf{x}_t. \tag{22}$$

For the purposes of estimation, we assume that inflation swap breakeven inflation ($\pi_{t,n}^{RPI}$) and index linked bond breakeven inflations ($\pi_{t,n}^{b,RPI}$) are measured with errors:

$$\pi_{t,n}^{RPI} = -\left(a_n^{z,RPI} + \mathbf{b}_n^{z,RPI}\mathbf{z}_t\right)/n + e_{t,n} \tag{23}$$

$$\pi_{t,n}^{b,RPI} = -\left(a_n^{zb,RPI} + \mathbf{b}_n^{zb,RPI}\mathbf{z}_t\right)/n + e_{t,n}^b, \tag{24}$$

where we the error terms $e_{t,n}$ and $e_{t,n}^b$ both follow independent nominal distribution $\mathcal{N}(0, \omega)$ with the same volatility.

Tables

Table 1 Principal Components of a portfolio of BEI rates (3 to 10-year maturity), CPI and RPI inflation

	Gilt BEI, CPI and RPI inflation (from 1992 - 2013)		**IS BEI, CPI and RPI inflation (from 2004 - 2013)**		**IS and gilt BEI (from 2004 - 2013)**	
Principal component	% variance explained	Cumulative %	% variance explained	Cumulative %	% variance explained	Cumulative %
1	83%	83.00%	76.00%	76.000%	76.30%	76.30%
2	10%	93.00%	13.2%	89.200%	13.50%	89.80%
3	4.2%	97.20%	10.2%	99.400%	7.91%	97.71%
4	2.5%	99.70%	0.54%	99.940%	1.99%	99.70%
5	0.09%	99.79%	0.02%	99.960%	0.30%	100.00%

Note: Maturities of bond and swap BEI rates are 3, 4, 5 and 10 years

Table 2 Estimated parameters

Parameter	Estimation	Parameter	Estimation	Parameter	Estimation
$\boldsymbol{\kappa}^{\mathbb{Q}}$		$\boldsymbol{\Phi}^{z}$		$\boldsymbol{\Sigma}^{z}$	
κ_{∞}^{Q}	3.03E-05 (0.000245)	$\Phi_{1,1}^{z}$	0.964621** (0.010718)	$\Sigma_{1,1}^{z}$	0.000422** (1.71E-05)
κ_{q}^{Q}	-4.16E-07 (0.000312)	$\Phi_{2,1}^{z}$	-0.02783 (0.060369)	$\Sigma_{2,1}^{z}$	-8.27E-05** (3.22E-05)
κ_{l}^{Q}	2.86E-10 (2.52E-07)	$\Phi_{3,1}^{z}$	0.234792** (0.084033)	$\Sigma_{3,1}^{z}$	-1.62E-05** (6.97E-06)
$\boldsymbol{\Phi}^{\mathbb{Q}}$		$\Phi_{4,1}^{z}$	-0.21543 (0.281361)	$\Sigma_{4,1}^{z}$	4.66E-05** (5.50E-06)
ξ_1	0.990392** (0.00494)	$\Phi_{5,1}^{z}$	0.001544 (0.884889)	$\Sigma_{5,1}^{z}$	-4.66E-06 (2.86E-06)
ξ_2	0.980790** (0.008495)	$\Phi_{1,2}^{z}$	0.029709** (0.015132)	$\Sigma_{2,2}^{z}$	0.000331** (2.67E-05)
ξ_3	0.878017** (0.018632)	$\Phi_{2,2}^{z}$	0.977231** (0.068983)	$\Sigma_{3,2}^{z}$	6.00E-05** (2.21E-05)
ξ_4	0.986378** (0.063425)	$\Phi_{3,2}^{z}$	-0.14111 (0.116641)	$\Sigma_{4,2}^{z}$	3.46E-05** (9.94E-06)
ξ_5	0.988050** (.000783)	$\Phi_{4,2}^{z}$	0.296531 (0.308671)	$\Sigma_{5,2}^{z}$	-3.73E-05** (2.69E-06)
$\boldsymbol{\theta}_{f}^{*}$		$\Phi_{5,2}^{z}$	0.000354 (0.961827)	$\Sigma_{3,3}^{z}$	8.15E-05 (1.56E-05)
θ_{f1}^{*}	0.055388 (0.219931)	$\Phi_{1,3}^{z}$	-0.00461 (0.004413)	$\Sigma_{4,3}^{z}$	-1.53E-06** (5.74E-06)
θ_{f2}^{*}	0.000297 (0.286948)	$\Phi_{2,3}^{z}$	0.023694 (0.020434)	$\Sigma_{5,3}^{z}$	-4.06E-06** (1.61E-06)
θ_{f3}^{*}	-0.000465 (0.285091)	$\Phi_{3,3}^{z}$	0.920517** (0.038479)	$\Sigma_{4,4}^{z}$	5.96E-05 (7.78E-06)
$\boldsymbol{\kappa}^{z}$		$\Phi_{4,3}^{z}$	-0.08653 (0.096722)	$\Sigma_{5,4}^{z}$	-1.60E-05** (1.58E-06)
κ_{1}^{z}	0.000150 (0.000106)	$\Phi_{5,3}^{z}$	0.000432 (0.299253)	$\Sigma_{5,5}^{z}$	5.17E-06 (3.64E-06)
κ_{2}^{z}	1.95E-05 (0.000129)	$\Phi_{1,4}^{z}$	-0.0051 (0.00327)		
κ_{3}^{z}	-5.35E-05* (3.14E-05)	$\Phi_{2,4}^{z}$	-0.01177 (0.019835)		
κ_{4}^{z}	5.75E-06 (3.30E-05)	$\Phi_{3,4}^{z}$	0.035233 (0.026455)		
κ_{5}^{z}	-1.88E-06 (1.23E-05)	$\Phi_{4,4}^{z}$	0.888555** (0.086601)		
$\boldsymbol{\Omega}$		$\Phi_{5,4}^{z}$	0.000179 (0.292131)		
ω	5.54E-05** (5.60E-07)	$\Phi_{1,5}^{z}$	-6.99E-07 (0.001412)		
$\omega^{a,CPI}$	0.000275** (1.96E-05)	$\Phi_{2,5}^{z}$	0.001904 (0.005389)		
$\omega^{a,RPI}$	0.000322** (1.52E-05)	$\Phi_{3,5}^{z}$	1.95E-05 (0.011348)		
		$\Phi_{4,5}^{z}$	0.0012 (0.030619)		
		$\Phi_{5,5}^{z}$	0.959584** (0.08411)		

Note: 1. Significance level: **5%; *10%. Numbers in parenthesis are standard deviations, which are calculated by using the outer product of the scores, as explained in Greene (2011) on his discussion of the BHHH estimator[9].

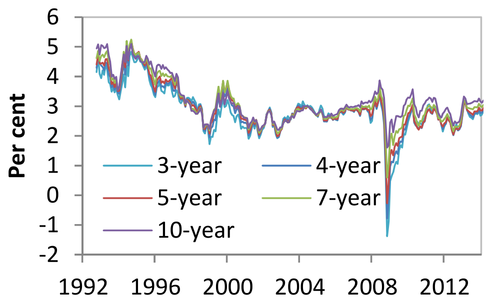
2. The largest eigenvalue for the $\boldsymbol{\Phi}^{z}$ matrix is 0.988.

[9] Please see Berndt, Hall, Hall and Hausman (1974).

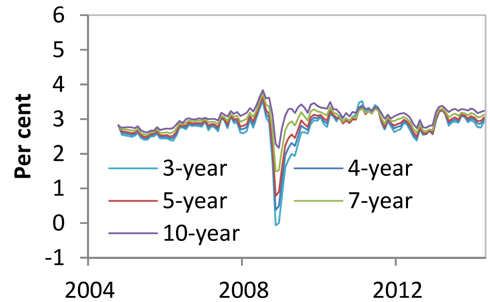
Charts

Chart 1. Inflation breakeven rates, realised inflation and surveys

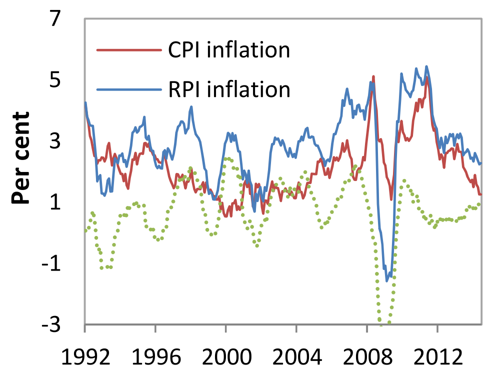
A. Bond RPI breakeven inflation rates



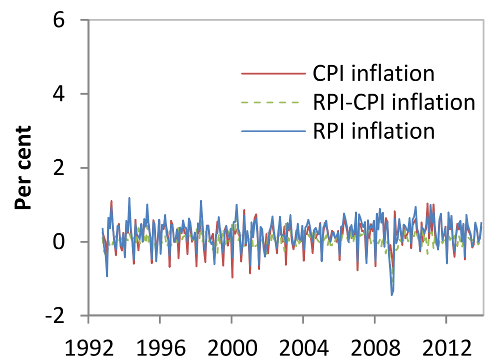
B. Swap RPI breakeven inflation rates



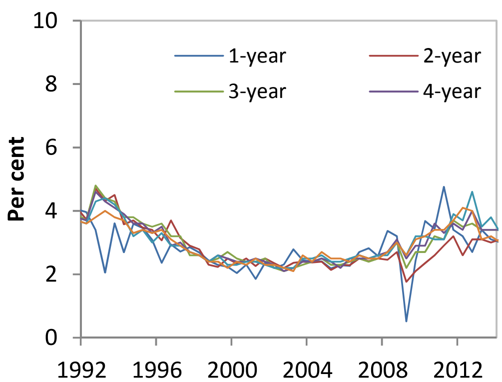
C. Year-on-year inflation rates



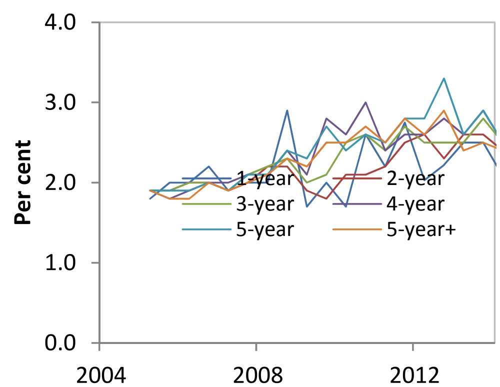
D. Month-on-month inflation rates



E. RPIX inflation survey data at different horizon



F. CPI inflation survey data at different horizon

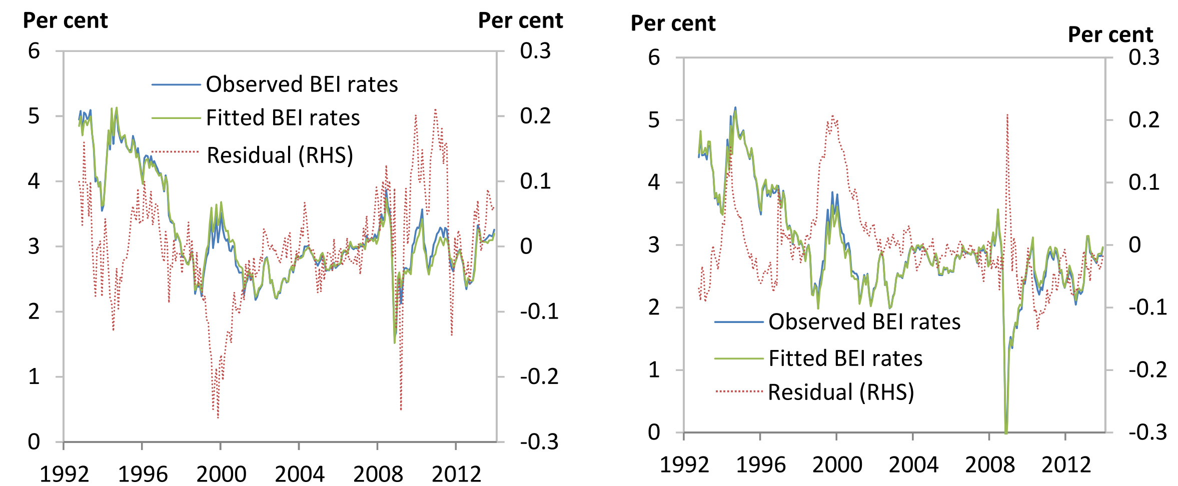


Note: Bond RPI breakeven inflation rates are computed as the difference between continuously compounded nominal and real spot yields published by the Bank of England. RPI Swaps breakeven inflation rates are by the Bank of England. CPI and RPI inflation surveys are from Consensus Economics. CPI surveys and RPI swaps are only available after October 2004. Monthly inflation rates used in estimation have been annualised and obtained by regressing month-on-month inflation data corresponding to the actual date for price on year-on-year inflation data that have a 1-month lag and correspond to release date.

Chart 2. Actual and fitted bond and swap spot breakeven rates at selected maturities

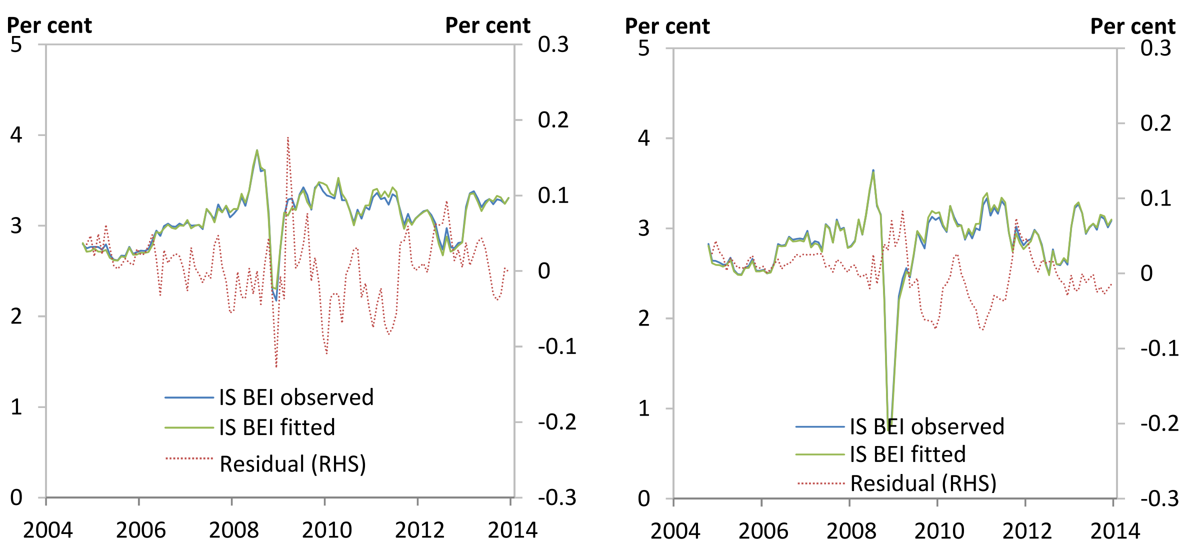
A. 10-year bond BEI rates

B. 5-year bond BEI rates



C. 10-year inflation swap BEI rates

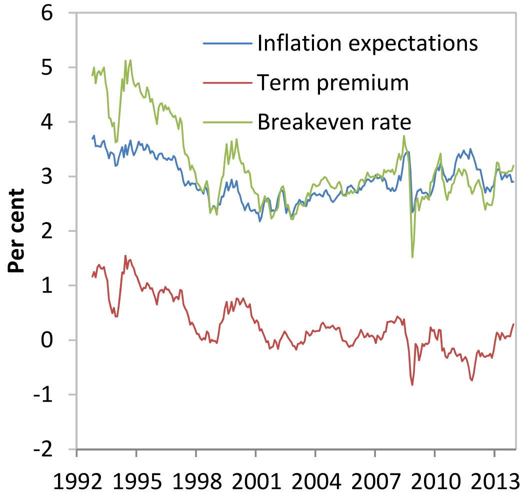
D. 5-year inflation swap BEI rates



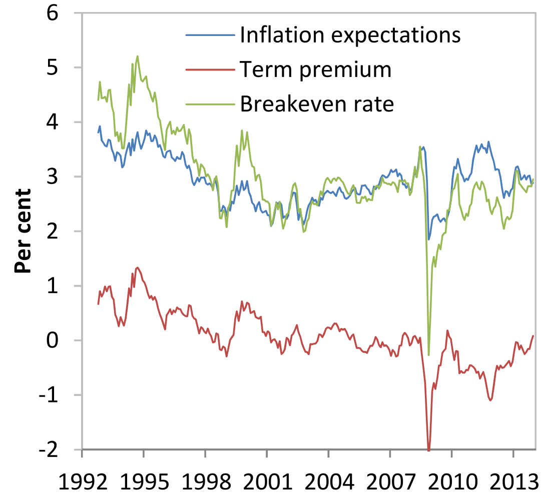
Note: IS BEI stands for Inflation Swap Breakeven rates. The sample period of the preferred model is October 1992 and December 2013. Swap data are only available after May 2004. All data are by the Bank of England. The observed bond and inflation swap breakeven rates are plotted with reference to the left hand axis. Residuals are plotted with reference to the right hand axis in percentage points.

Chart 3 Bond spot BEI rates decomposition - preferred model

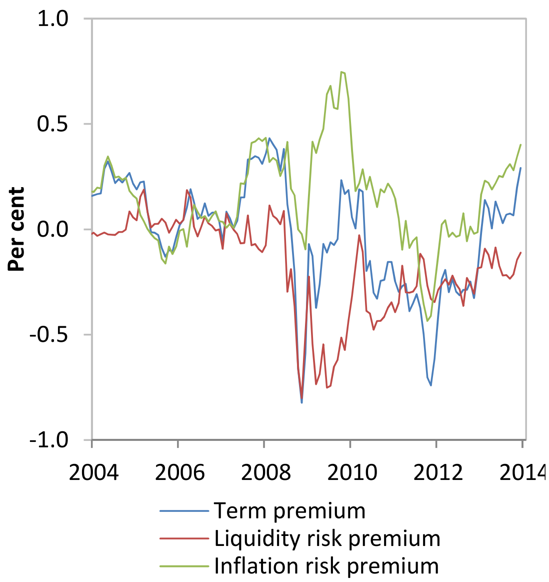
A. 10-year maturity breakeven rate



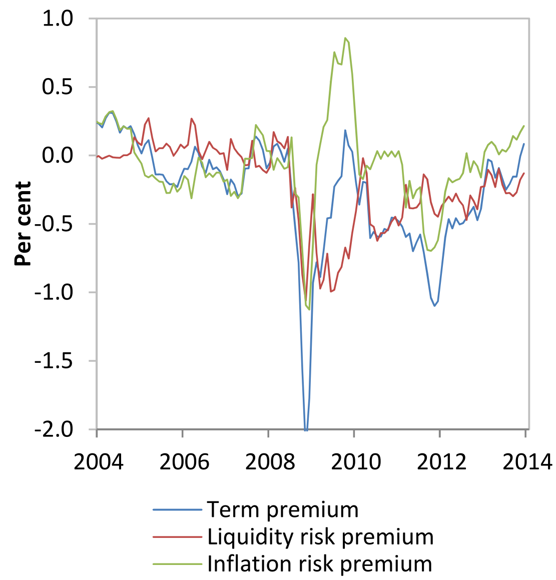
B. 5-year maturity breakeven rate



C. Term premium decomposition at 10-year maturity



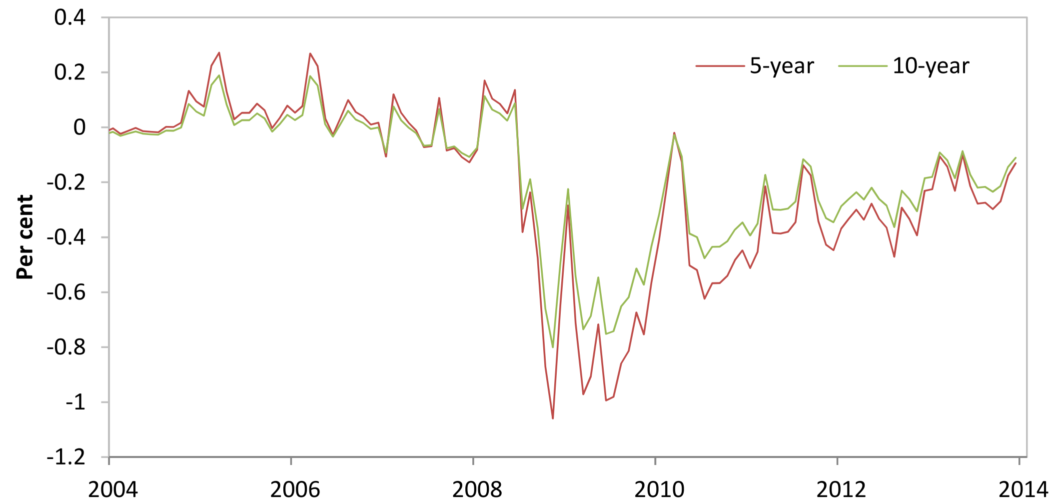
D. Term premium decomposition at 5-year maturity



Note: Bond breakeven rates are observed. Term premium is the sum of liquidity and inflation premia. There is no swap data before 2004, therefore it is not possible to distinguish between inflation and liquidity premium before then. CPI bond and swap data are not available in UK, hence decomposition only refers to RPI inflation and RPI linked bond and swap data.

Chart 4 Term structure of liquidity and inflation risk premium in bond BEI rates

A. Liquidity risk premium at 5 and 10 year maturity



B. Inflation risk premium at 5 and 10-year maturity

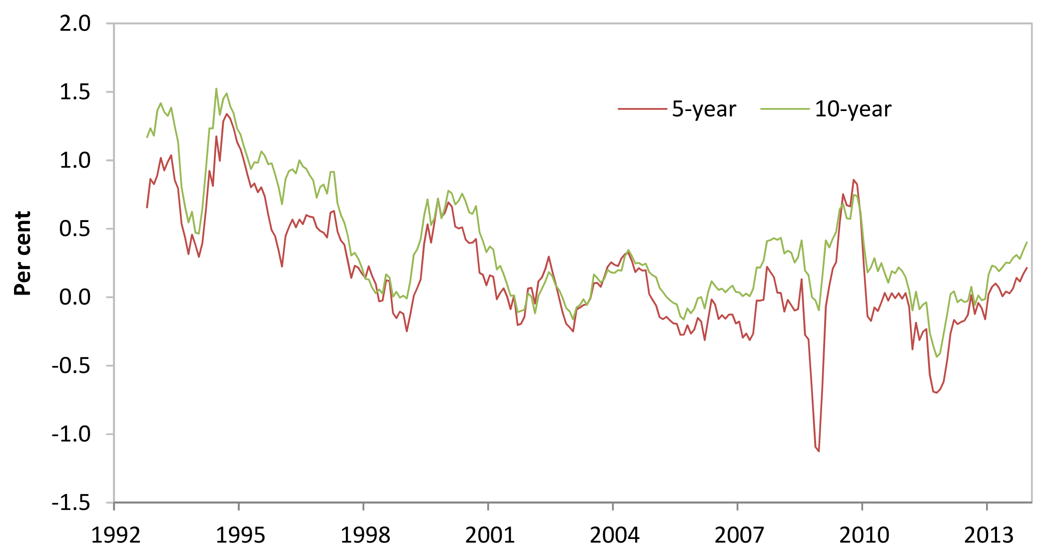
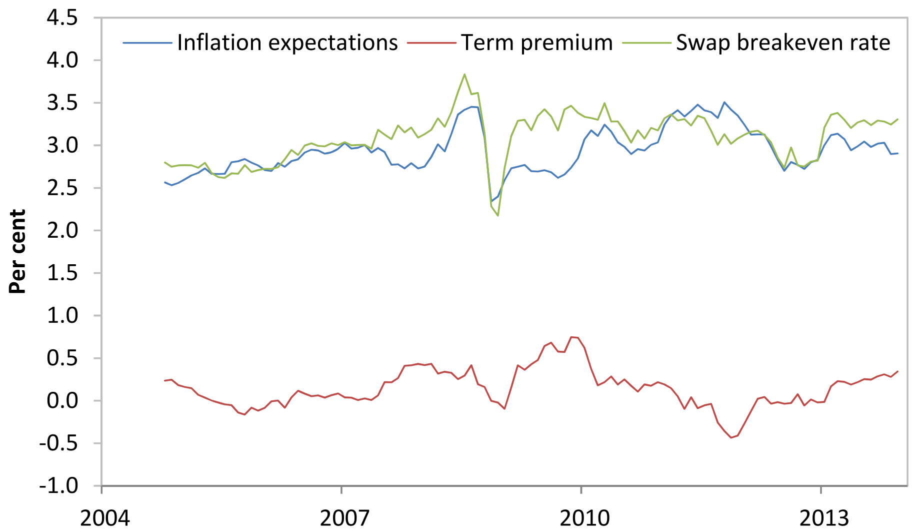
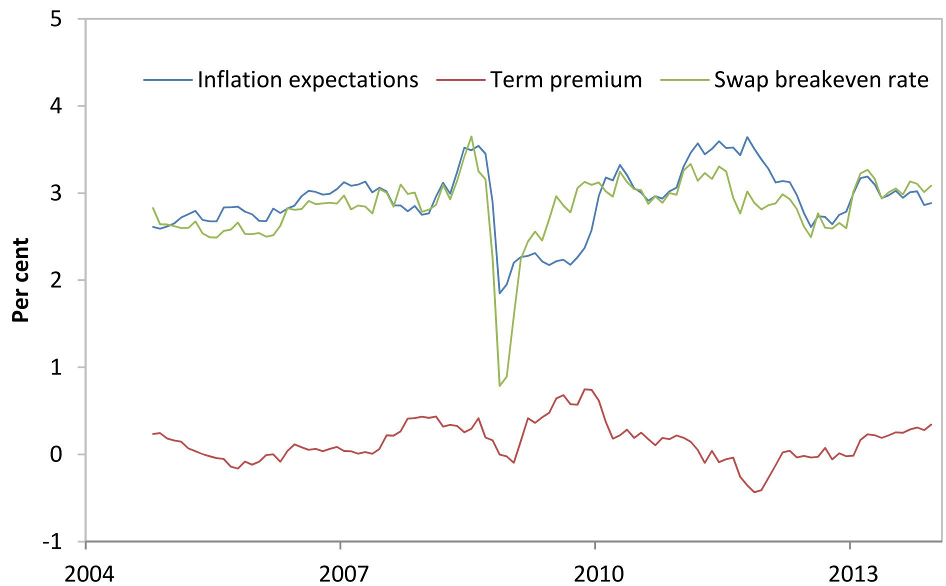


Chart 5 Inflation swap spot BEI rates decomposition - the preferred model

A. 10-year maturity



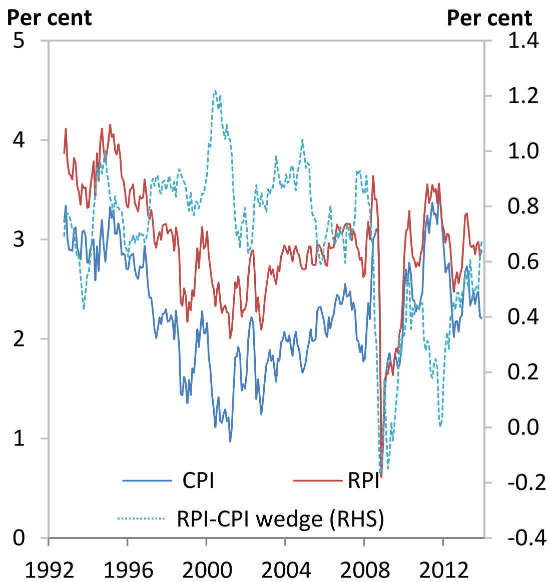
B. 5-year maturity



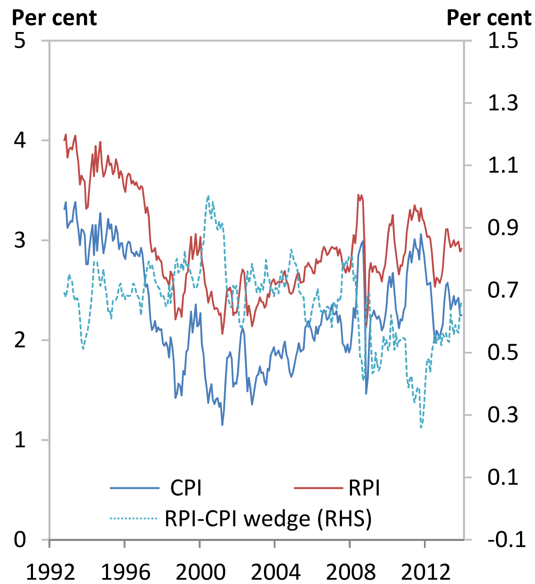
Note: Swap breakeven rates are observed. RPI inflation expectations and inflation risk premia in swap and bond breakeven rates are the same; term premium in swap breakeven rates is the same as inflation risk premium.

Chart 6 Expected 1-year CPI and RPI inflation rates over various horizons

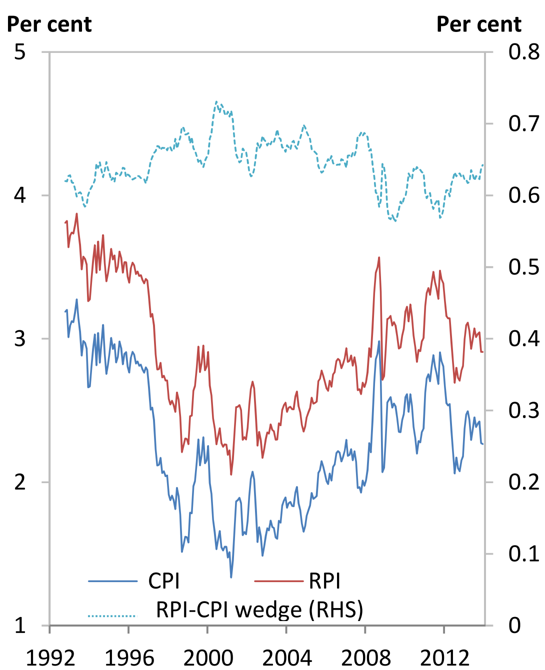
A. 2-year horizon



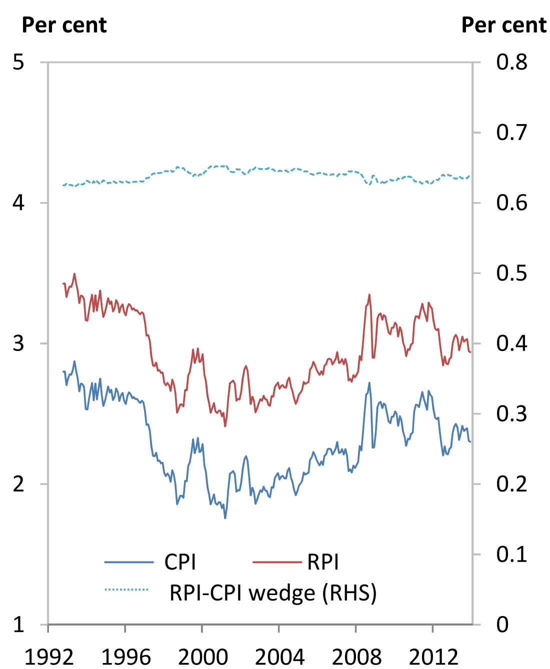
B. 3-year horizon



C. 5-year horizon



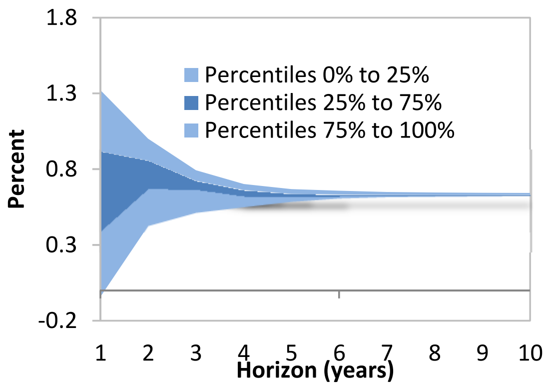
D. 10-year horizon



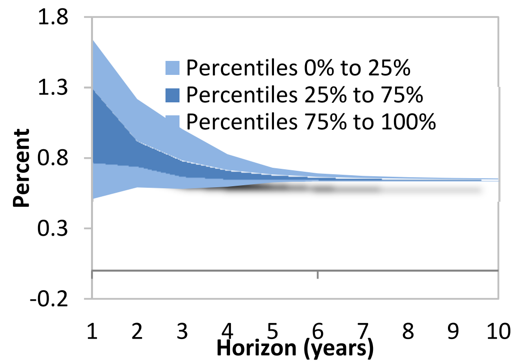
Note: CPI and RPI inflation rates are plotted with reference to the left hand axis. The RPI-CPI inflation difference is plotted with reference to the right hand axis (RHS).

Chart 7. Dispersion of expected annual RPI-CPI inflation wedge over a 10-year horizon over three different periods

A. 1992-1997



B. 1998-2007



C. 2008-2013

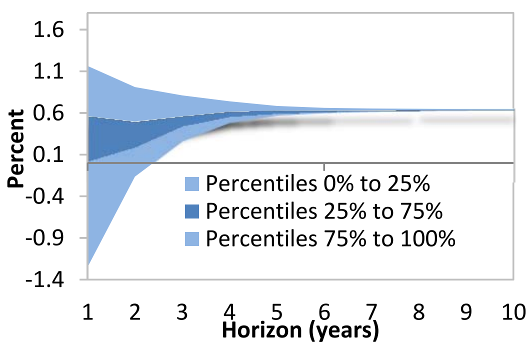


Chart 8. Annual RPI-CPI wedge dispersion over different forecast horizons (i.e. Standard deviation of RPI-CPI inflation difference)

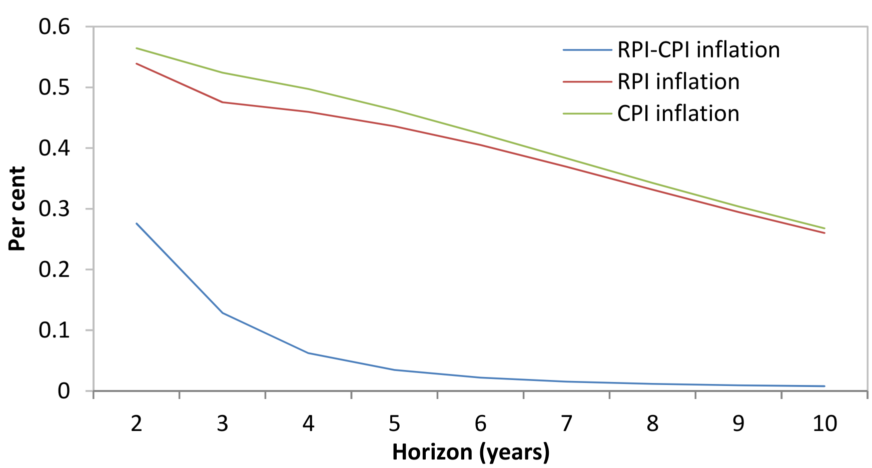
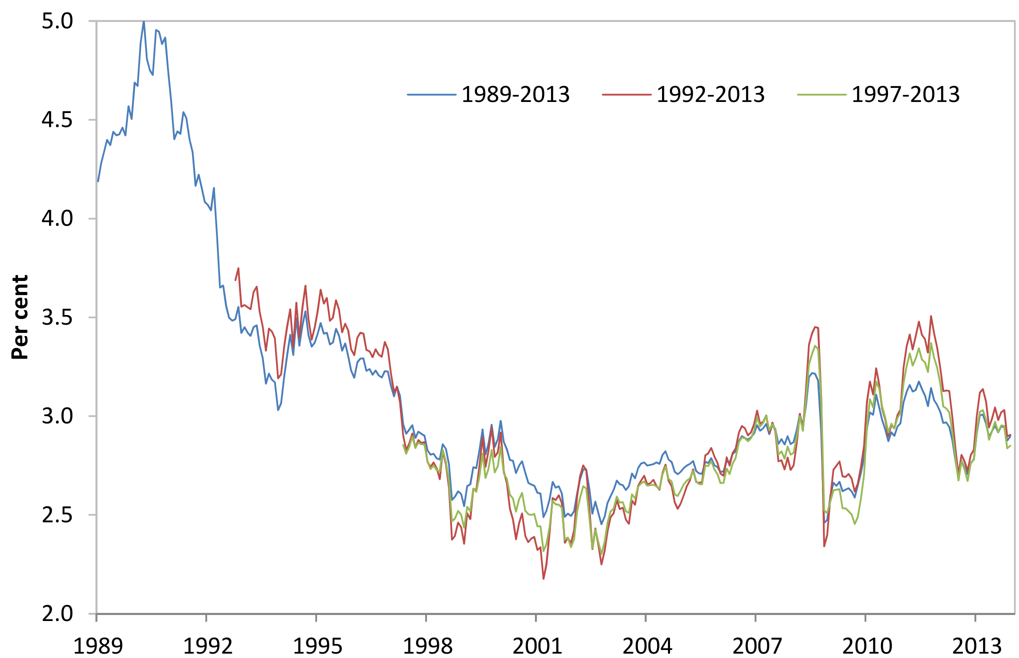


Chart 9. Sensitivity analysis for different sample periods with the same end date by different start dates.

A 10-year RPI inflation expectation estimates



B 10-year RPI-CPI inflation expectation estimates

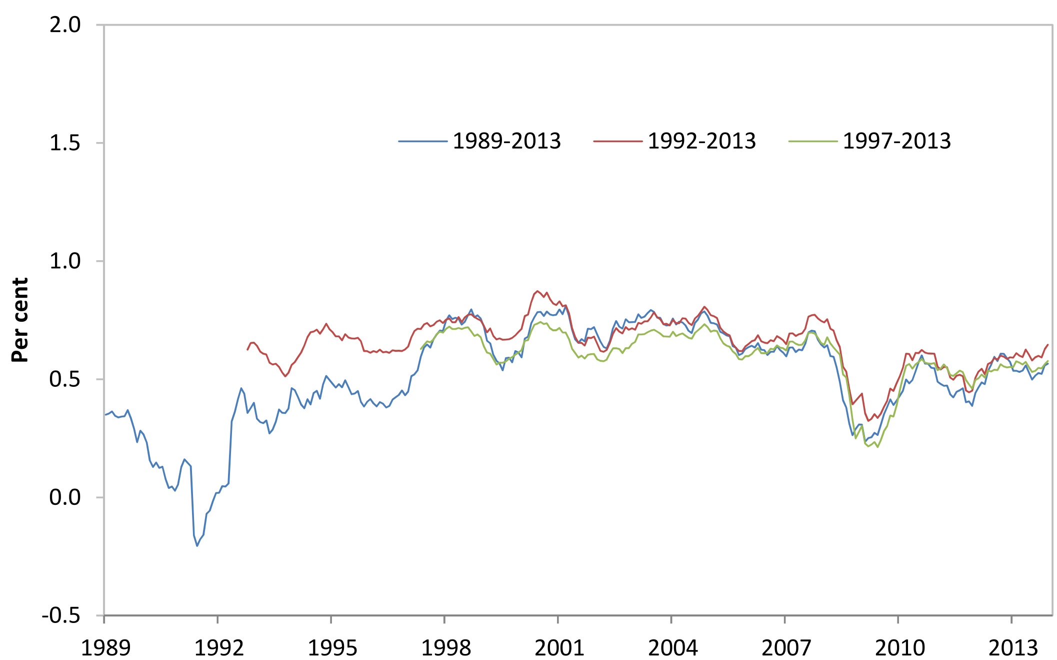
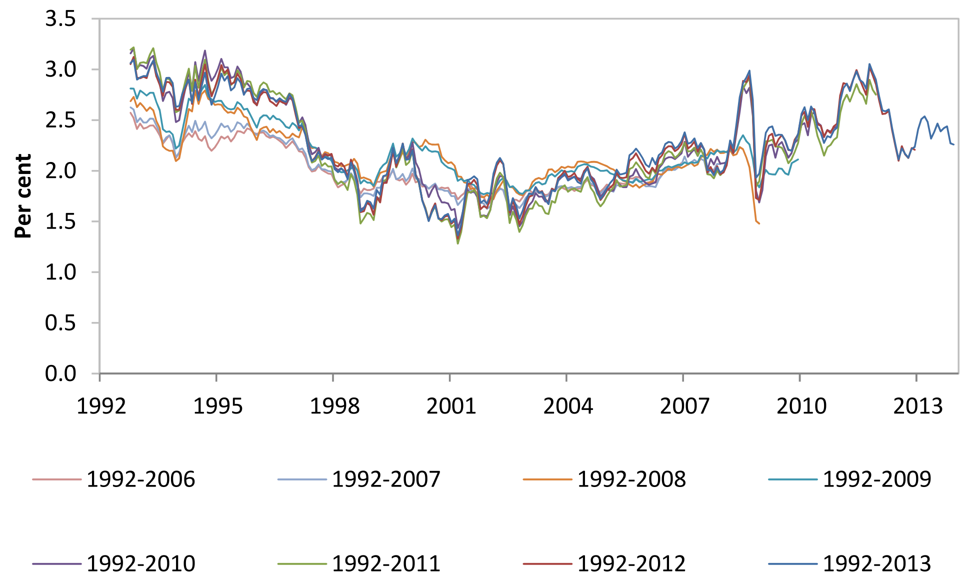


Chart 10. Sensitivity analysis for different sample periods with the same start date by different end dates.

A 10-year CPI inflation expectation estimates



B 10-year RPI-CPI inflation expectation estimates

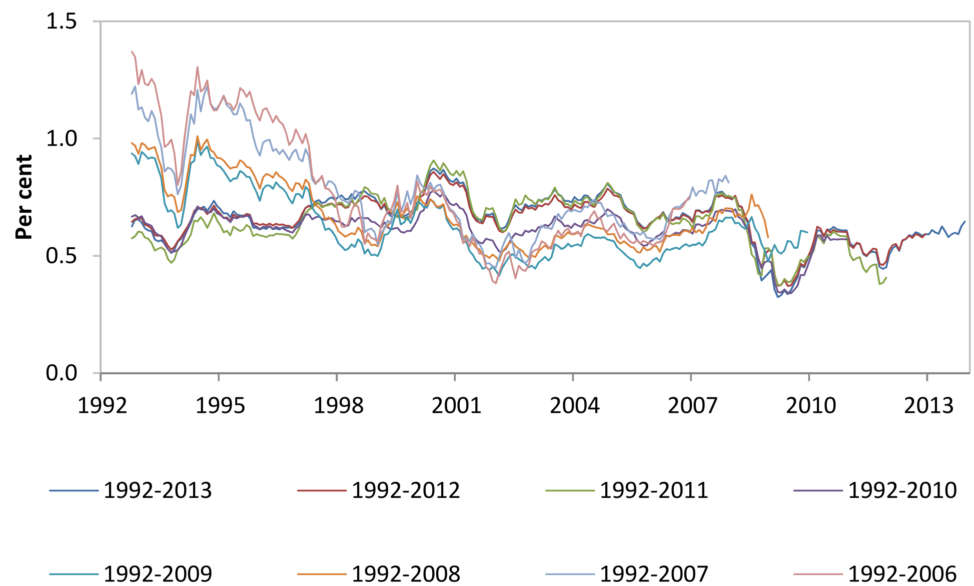
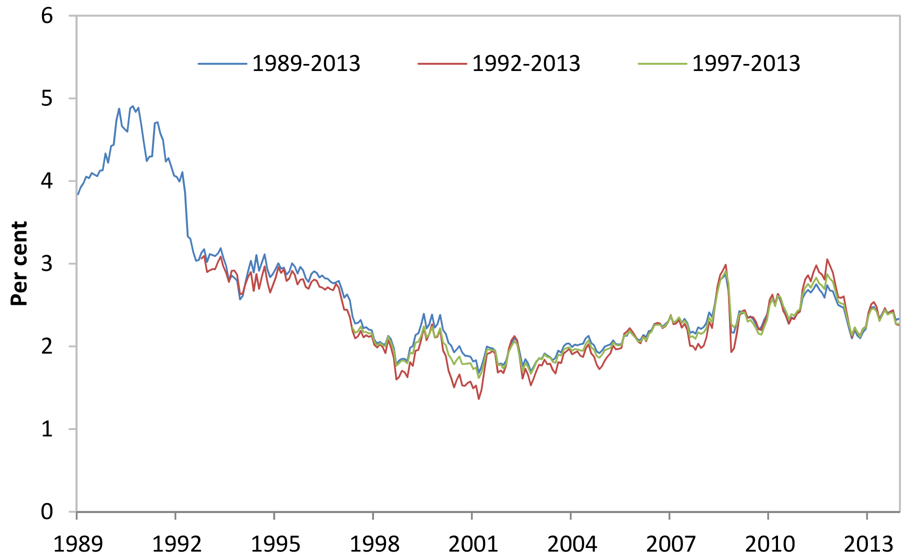
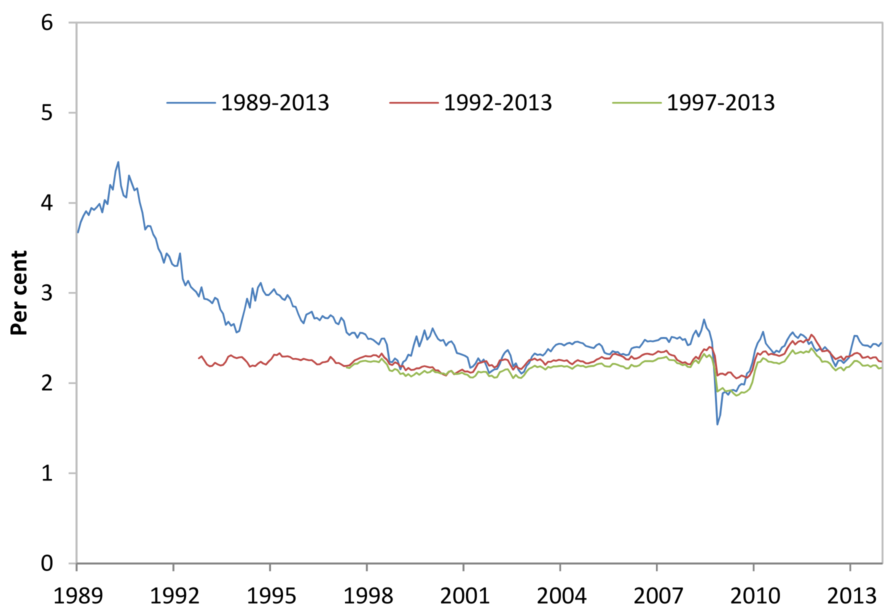


Chart 11. Sensitivity analysis for the impact of survey data.

A 10-year CPI inflation expectation estimates with survey data – preferred model



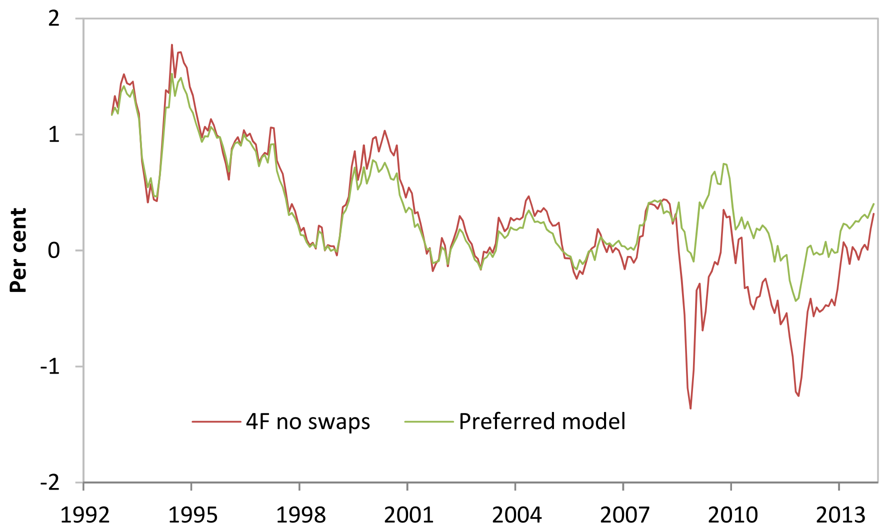
B 10-year CPI inflation expectation estimates without survey data – alternative model



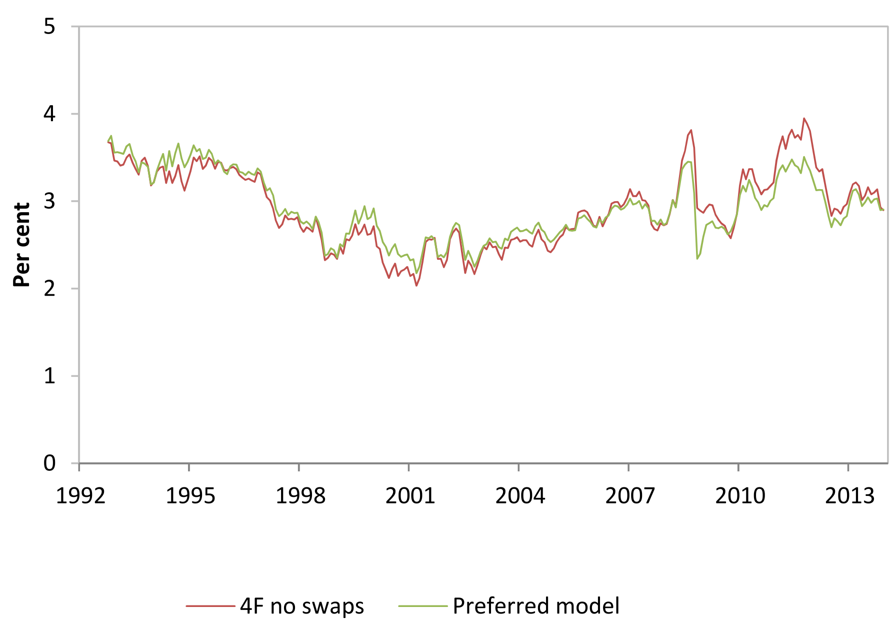
Note: The alternative model also includes 5 factors but does not include survey data.

Chart 12. Sensitivity analysis for the impact of liquidity assumption

A 10-year inflation risk premium



B 10-year RPI inflation expectations



Note: To test the impact of the liquidity assumption, we re-estimated the model without the inclusion of inflation swap data where liquidity premium is not modelled. This is a 4 factors model.