BANK OF ENGLAND

Appendix to Staff Working Paper No. 628
Fiscal consolidation in a low inflation environment: pay cuts versus lost jobs

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1 Model Derivations

1.1 Household's maximisation problem

The household's Lagrangean can be written as

$$\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{\infty}\beta^t\left\{\frac{(c_t-h_bc_{t-1})^{1-\eta}}{1-\eta}+\Phi\frac{(1-n_t^p-n_t^g-u_t)_t^{1+\varphi}}{1+\varphi}-\Upsilon\frac{x_t^{1+\xi}}{1+\xi}\right.\\
&- \lambda_{c,t}\left[(1+\tau_c)\,c_t+i_t^p+b_{g,t+1}+e_tr_{f,t-1}b_{f,t}-\left[r_t^p-\tau_k\left(r_t^p-\delta^p\right)\right]k_t^p\right.\\
&\quad \left.-r_{t-1}b_{g,t}-e_tb_{f,t+1}-(1-\tau_n)\left(w_t^pn_t^px_t+w_t^gn_t^g\right)-\mathrm{b}u_t-\Pi_t^p-T_t\right]\\
&- \lambda_{k,t}\left[k_{t+1}^p-\left[1-\frac{\omega}{2}\left(\frac{i_t^p}{i_{t-1}^p}-1\right)^2\right]i_t^p-\left(1-\delta^p\right)k_t^p\right]\\
&- \lambda_{n^p,t}\left[n_{t+1}^p-(1-\sigma^p)n_t^p-\psi_t^{hp}(1-s_t)u_t\right]\\
&- \left.\lambda_{n^g,t}\left[n_{t+1}^g-(1-\sigma^g)n_t^g-\psi_t^{hg}s_tu_t\right]\right\}
\end{aligned}$$

where the household's composition has been substituted into the utility function, and the household takes as given the probability of finding a job in each sector and does not internalise the effect of their choice of u_t^jon the number of matches. We assume external habits in consumption, meaning that c_{t-1} is taken as given in period t.

The choice variables are c_t, i_t^p, k_{t+1}^p, n_{t+1}^p, n_{t+1}^g, u_t, s_t, x_t, $b_{g,t+1}$ and $b_{f,t+1}$. The first order conditions are:

[wrt c_t]

$$\lambda_{c,t}\left(1+\tau_c\right) = \left(c_t-h_bc_{t-1}\right)^{-\eta} \tag{1}$$

[wrt i_t^p]

$$\lambda_{c,t}-\lambda_{k,t}\left\{1-\frac{\omega}{2}\left(\frac{i_t^p}{i_{t-1}^p}-1\right)^2-\omega\left(\frac{i_t^p}{i_{t-1}^p}-1\right)\frac{i_t^p}{i_{t-1}^p}\right\}=\beta\lambda_{k,t+1}\omega\left(\frac{i_{t+1}^p}{i_t^p}-1\right)\left(\frac{i_{t+1}^p}{i_t^p}\right)^2 \tag{2}$$

$\left[\text{wrt } k^p_{t+1}\right]$

$$\lambda_{k,t} = \beta\left\{\lambda_{k,t+1}\left(1-\delta^p\right)+\lambda_{c,t+1}\left[r^p_{t+1}-\tau_k\left(r^p_{t+1}-\delta^p\right)\right]\right\} \tag{3}$$

$\left[\text{wrt } n^p_{t+1}\right]$

$$\lambda_{n^p,t} = \beta\left[\lambda_{n^p,t+1}\left(1-\sigma^p\right)+\lambda_{c,t+1}\left(1-\tau_n\right)w^p_{t+1}x_{t+1}-\Phi l^{\varphi}_{t+1}\right] \tag{4}$$

$\left[\text{wrt } n^g_{t+1}\right]$

$$\lambda_{n^g,t} = \beta\left[\lambda_{n^g,t+1}\left(1-\sigma^g\right)+\lambda_{c,t+1}\left(1-\tau_n\right)w^g_{t+1}-\Phi l^{\varphi}_{t+1}\right] \tag{5}$$

$[\text{wrt } u_t]$

$$\Phi l^{\varphi}_t = \lambda_{c,t}\mathrm{b}+\lambda_{n^p,t}\psi^{hp}_t\left(1-s_t\right)+\lambda_{n^g,t}\psi^{hg}_t s_t \tag{6}$$

$[\text{wrt } s_t]$

$$\lambda_{n^g,t}\psi^{hg}_t = \lambda_{n^p,t}\psi^{hp}_t \tag{7}$$

$[\text{wrt } x_t]$

$$\Upsilon x^{\xi}_t = \lambda_{c,t}\left(1-\tau_n\right)w^p_t n^p_t \tag{8}$$

$[\text{wrt } b_{g,t+1}]$

$$1 = \beta\frac{\lambda_{c,t+1}}{\lambda_{c,t}}r_t \tag{9}$$

[wrt $b_{f,t+1}$]

$$1 = \beta \frac{\lambda_{c,t+1}}{\lambda_{c,t}} \frac{e_{t+1}}{e_t} r_{f,t} \tag{10}$$

We can define the marginal value to the household of having an additional member employed in the private sector, as follows:

$$\begin{aligned} V^h_{n^p t} \equiv \frac{\partial \mathcal{L}}{\partial n^p_t} &= \lambda_{ct} w^p_t x_t (1 - \tau_n) - \Phi l^{\varphi}_t + (1 - \sigma^p) \lambda_{n^p t} \\ &= \lambda_{ct} w^p_t x_t (1 - \tau_n) - \Phi l^{\varphi}_t + (1 - \sigma^p) \beta E_t (V^h_{n^p t+1}) \end{aligned} \tag{11}$$

where the second equalities come from equation (4).

1.2 Derivation of the private wage

The Nash bargaining problem is to maximize the weighted sum of log surpluses:

$$\max_{w^p_t} \left\{ (1 - \vartheta) \ln V^h_{n^p t} + \vartheta \ln V^f_{n^p t} \right\}$$

where $V^h_{n^j t}$ and $V^f_{n^j t}$ are defined as:

$$V^h_{n^p t} \equiv \frac{\partial \mathcal{L}}{\partial n^p_t} = \lambda_{ct} w^p_t x_t (1 - \tau^n_t) - \Phi l^{\varphi}_t + (1 - \sigma^p) \lambda_{n^p t} \tag{12}$$

$$V^F_{n^p t} \equiv \frac{\partial Q^p}{\partial n^p_t} = p_{x,t} (1 - \phi) \frac{y^p_t}{n^p_t} - w^p_t x_t + \frac{(1 - \sigma^p) \kappa}{\psi^{fp}_t} \tag{13}$$

The first order conditions of this optimization problem is:

$$\vartheta V^h_{n^p t} = (1 - \vartheta) \lambda_{ct} (1 - \tau^n_t) V^f_{n^p t} \tag{14}$$

Plugging the expressions for the value functions into the FOC, we can rearrange to find the

expression for the private wage. Using (12),(13) and (14) we obtain:

$$w_t^p x_t = (1-\vartheta)[p_{x,t}(1-\phi)\frac{y_t^p}{n_t^p} + \frac{(1-\sigma^p)\kappa}{\psi_t^{fp}}] + \frac{\vartheta}{(1-\tau_n)\lambda_{c,t}}(\Phi l_t^{\varphi} - (1-\sigma^p)\lambda_{n^p t}) \tag{15}$$

Finally, taking the time t expectation of (14) evaluated at time $t+1$, and using the FOCs of the household and firm, we obtain

$$\vartheta\lambda_{n^p t} = (1-\vartheta)\lambda_{ct}(1-\tau_t^n)\frac{\kappa}{\psi_t^{fp}}$$

which allows us to simplify (15) to obtain the final expression for the private wage

$$w_t^p x_t = (1-\vartheta)p_{x,t}(1-\phi)\frac{y_t^p}{n_t^p} + \frac{\vartheta}{(1-\tau_n)\lambda_{c,t}}\Phi l_t^{\varphi} \tag{16}$$

2 Additional Figures

Here we include the following figures:

- Consolidation in normal times without variable labor effort
- Consolidation at the ZLB without the shock to the relative price of investment
- Sensitivity of the results to open economy parameters
- Sensitivity to asymmetric wage rigidities and labor immobility
- Repeat of sensitivity analysis in the main text, showing results in normal times

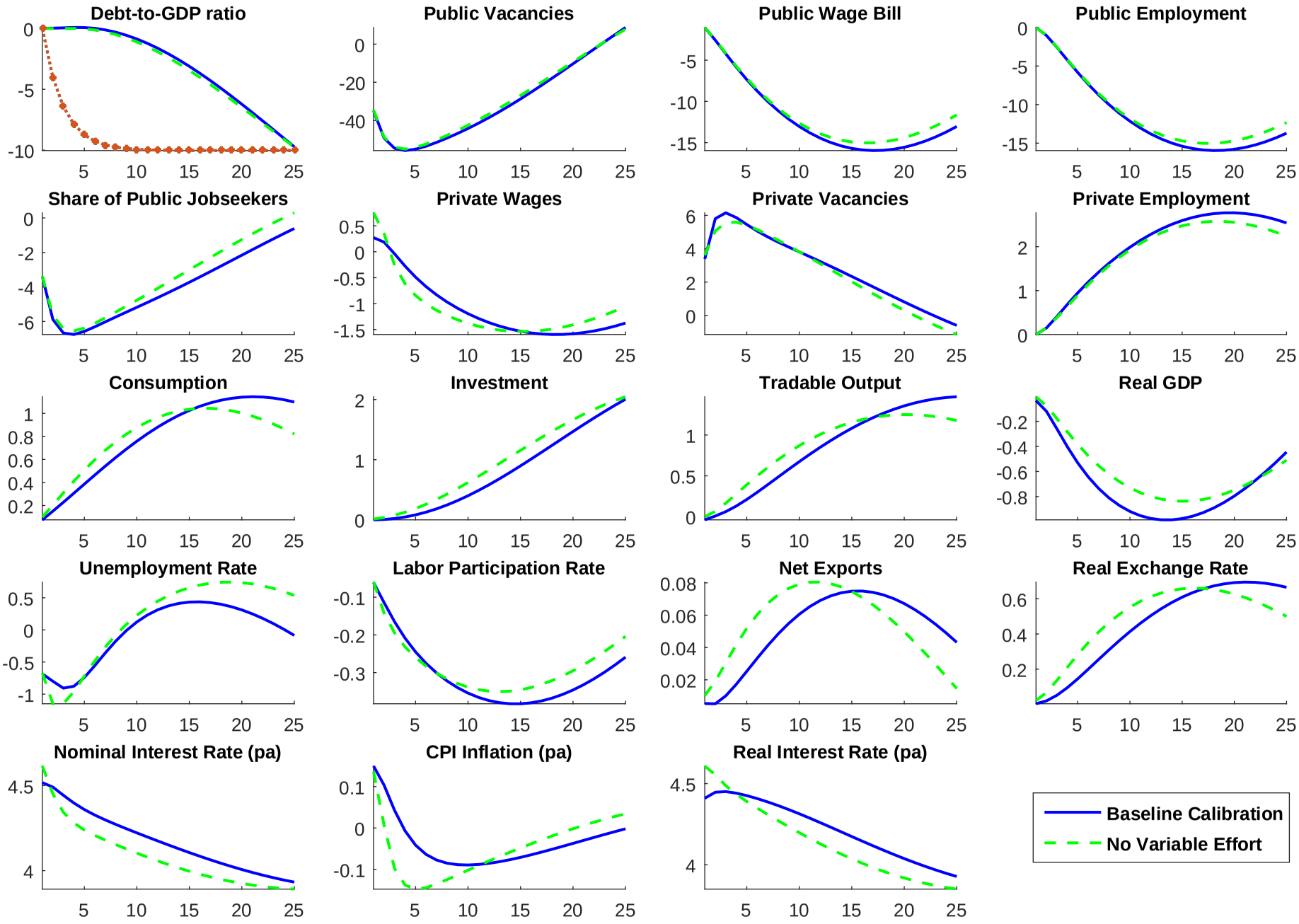


Figure 1: Fiscal Consolidation in Normal Times Without Variable Labor Effort: Vacancy Cuts

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.

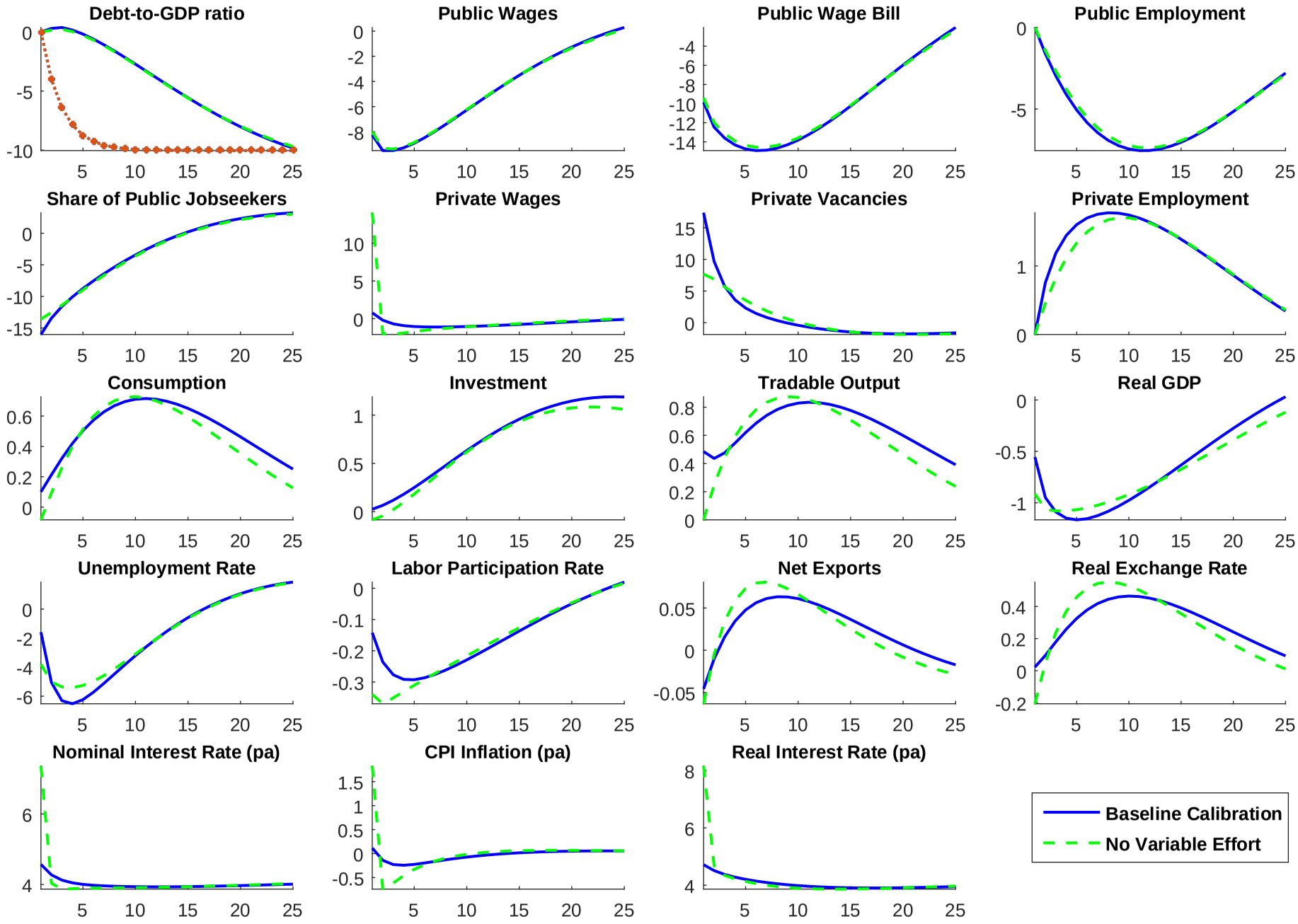


Figure 2: Fiscal Consolidation in Normal Times Without Variable Labor Effort: Wage Cuts

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.

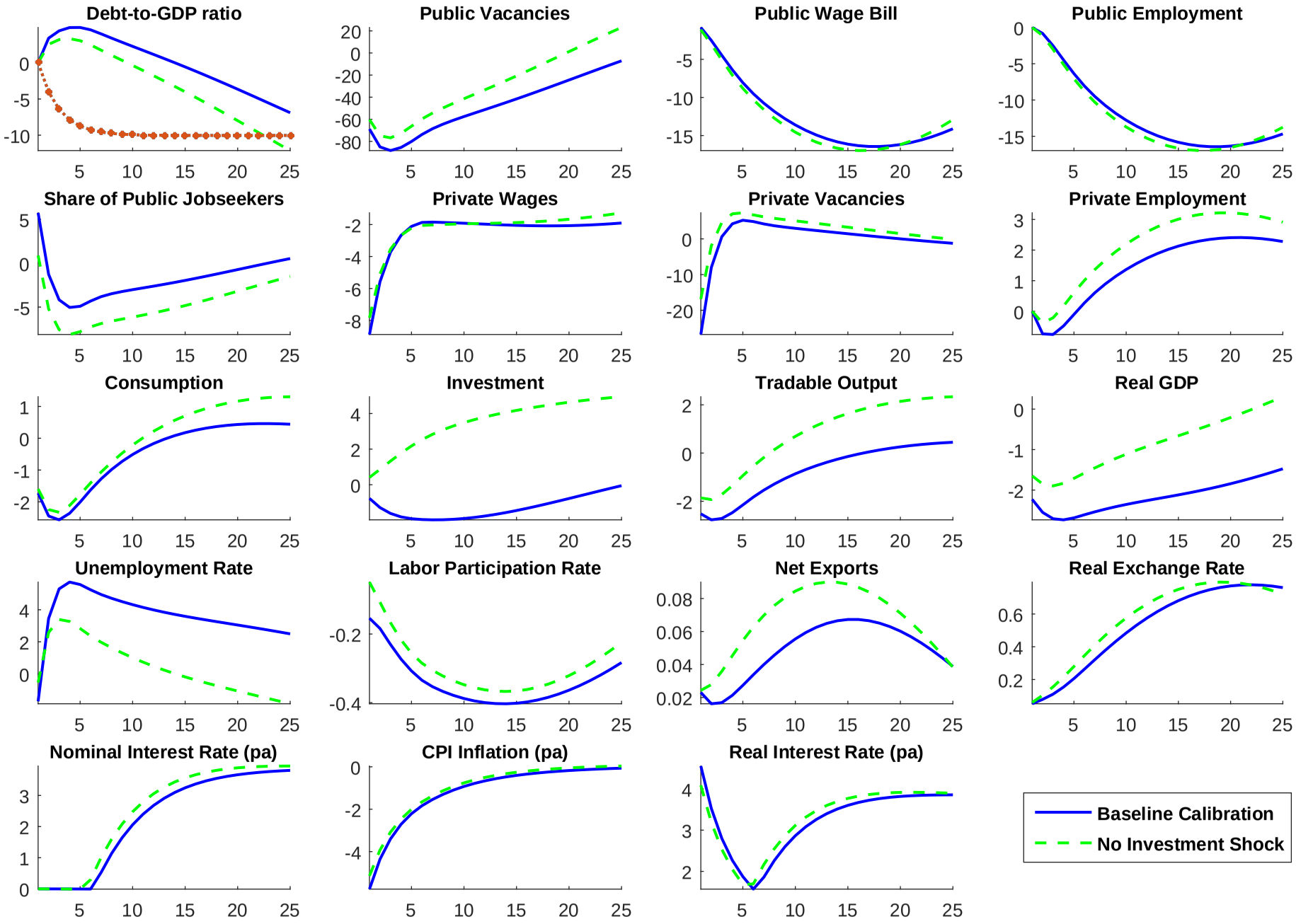


Figure 3: Fiscal Consolidation at the ZLB Without Relative Price of Capital Shock: Vacancy Cuts

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.

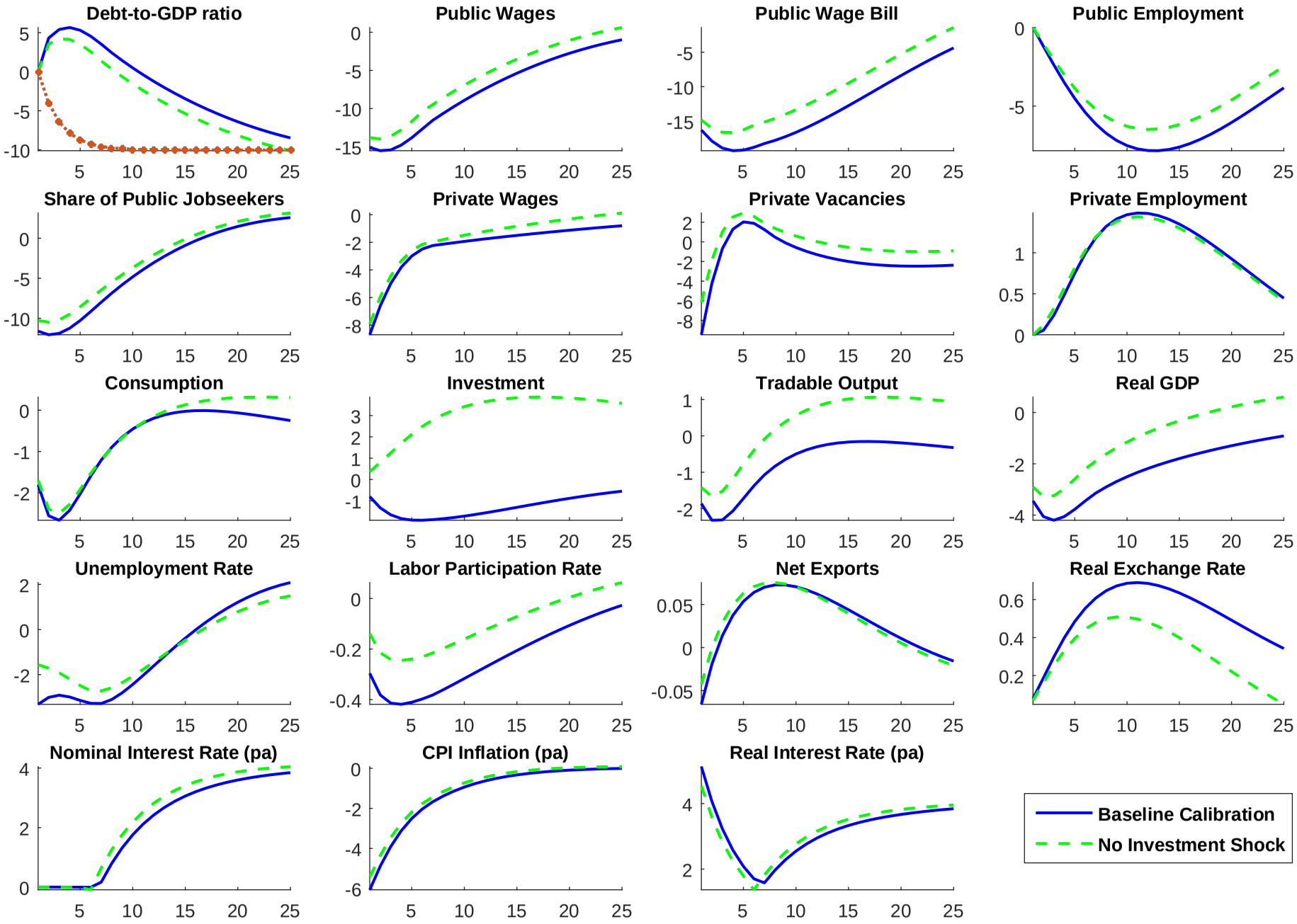


Figure 4: Fiscal Consolidation at the ZLB Without Relative Price of Capital Shock: Wage Cuts

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.

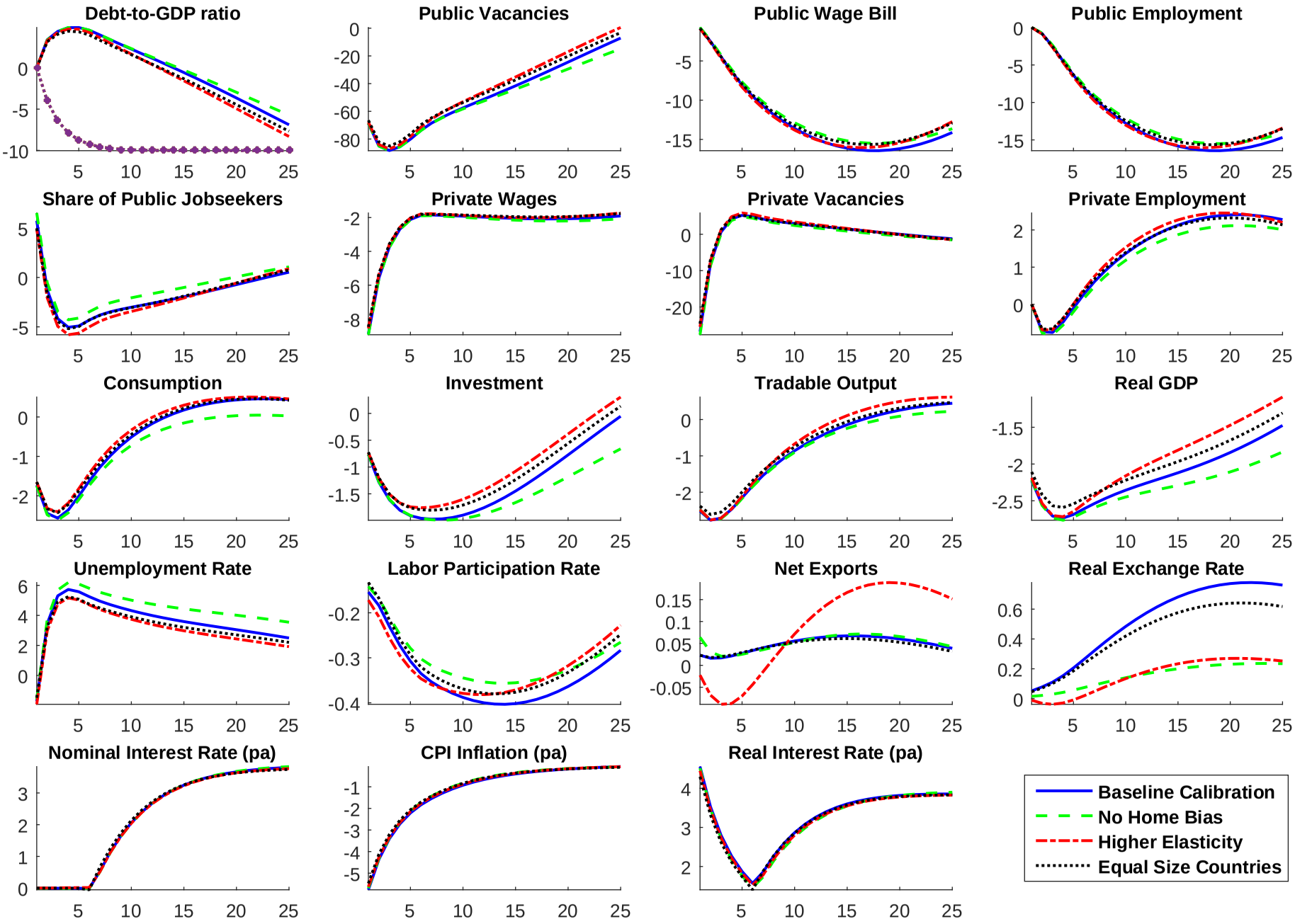


Figure 5: Sensitivity to Open Economy Parameters: Vacancy Cuts

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.

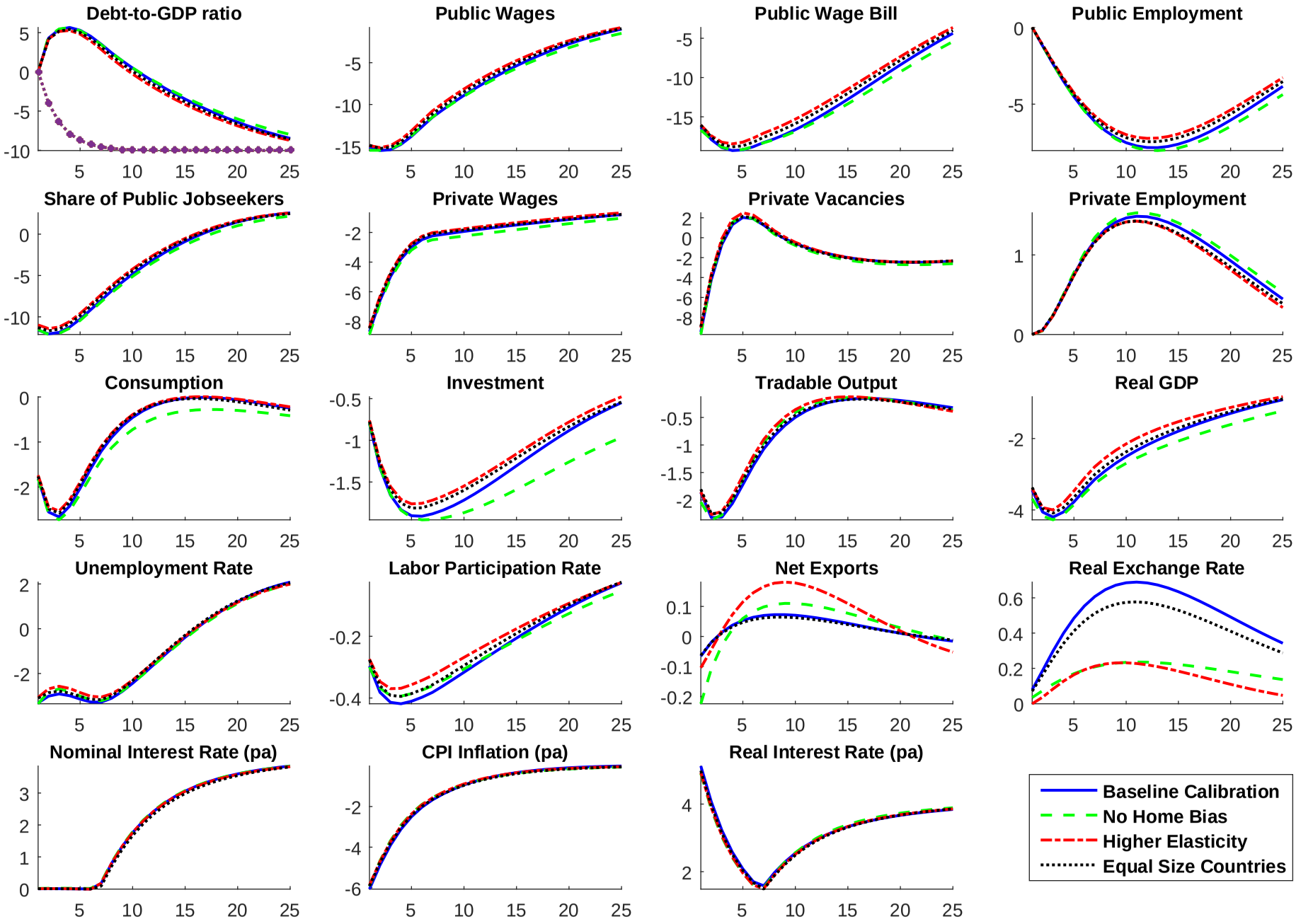
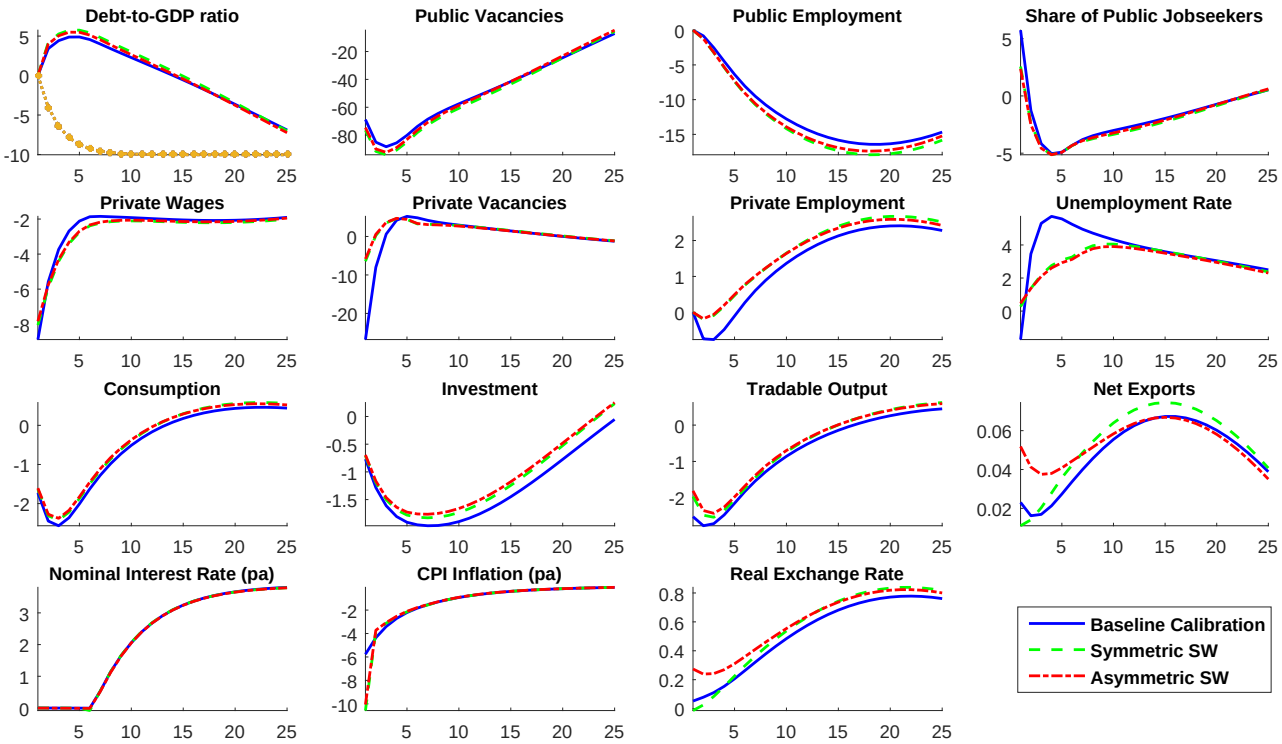
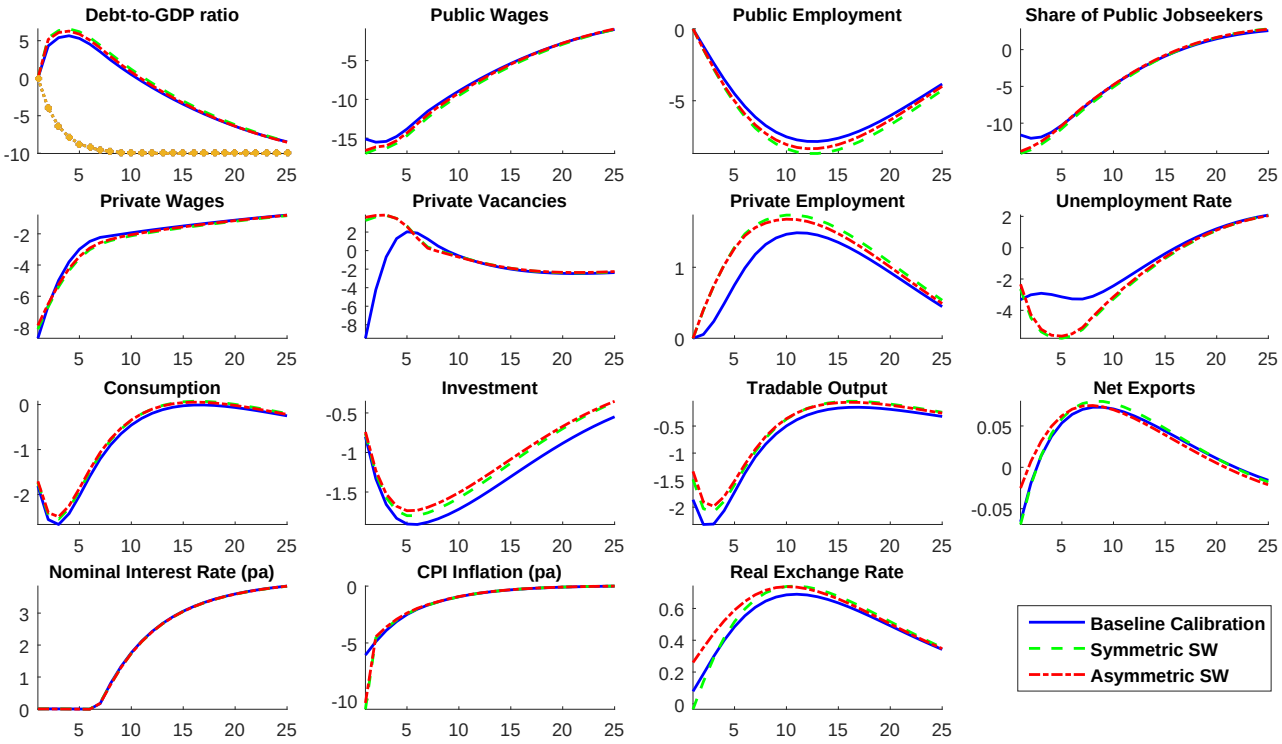


Figure 6: Sensitivity to Open Economy Parameters: Wage Cuts

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.



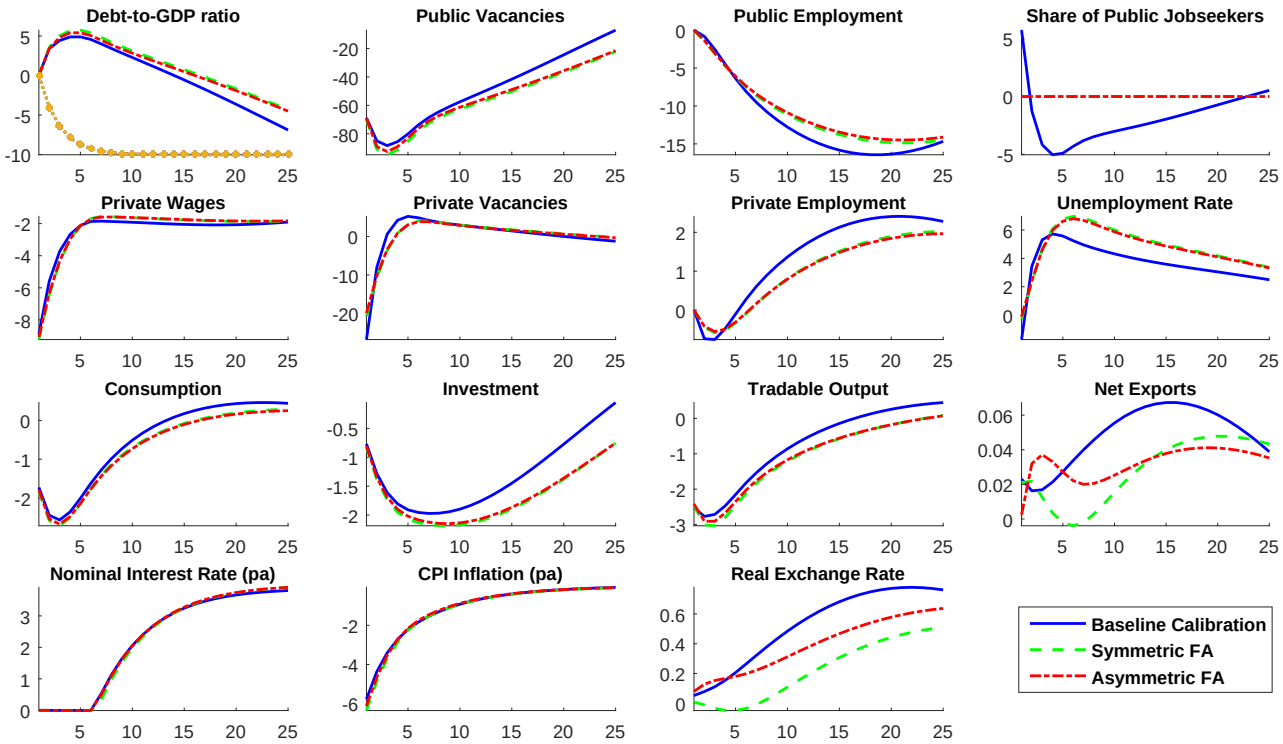
(a) Public Vacancy Cuts



(b) Public Wage Cuts

Figure 7: Sticky Wages (SW) at the Zero Lower Bound

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.



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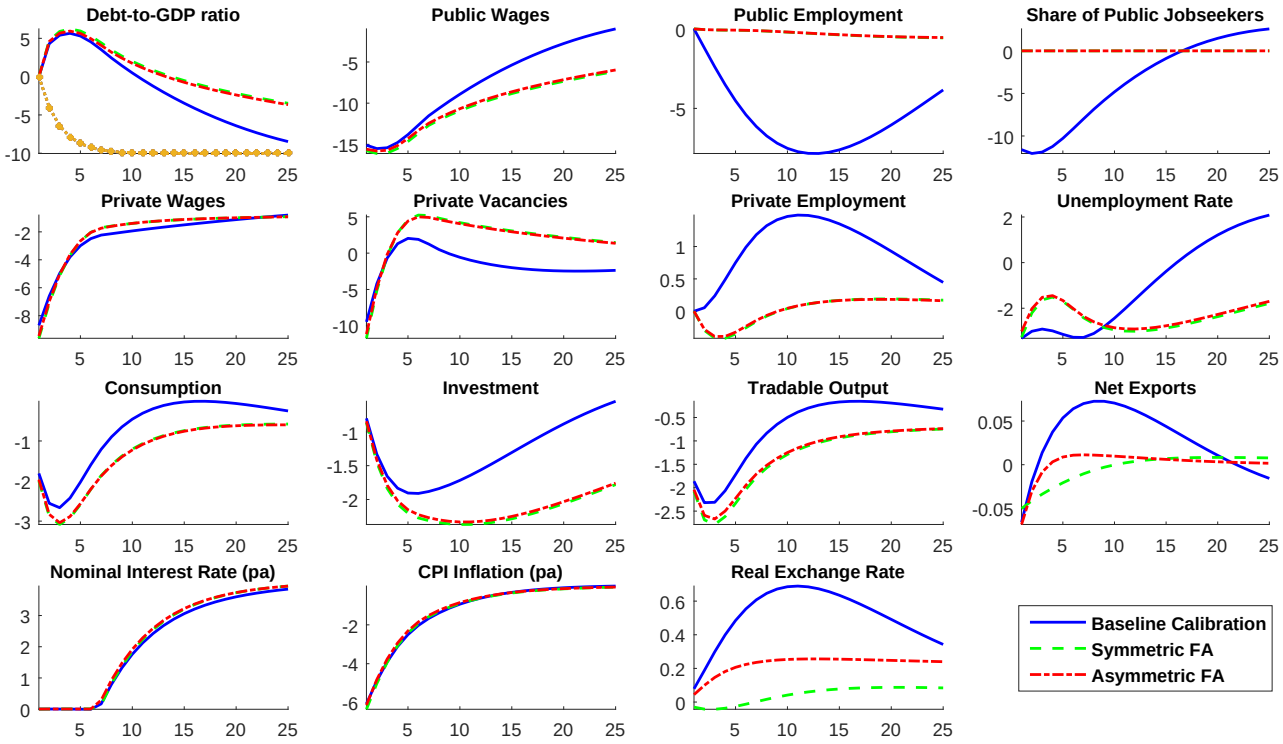
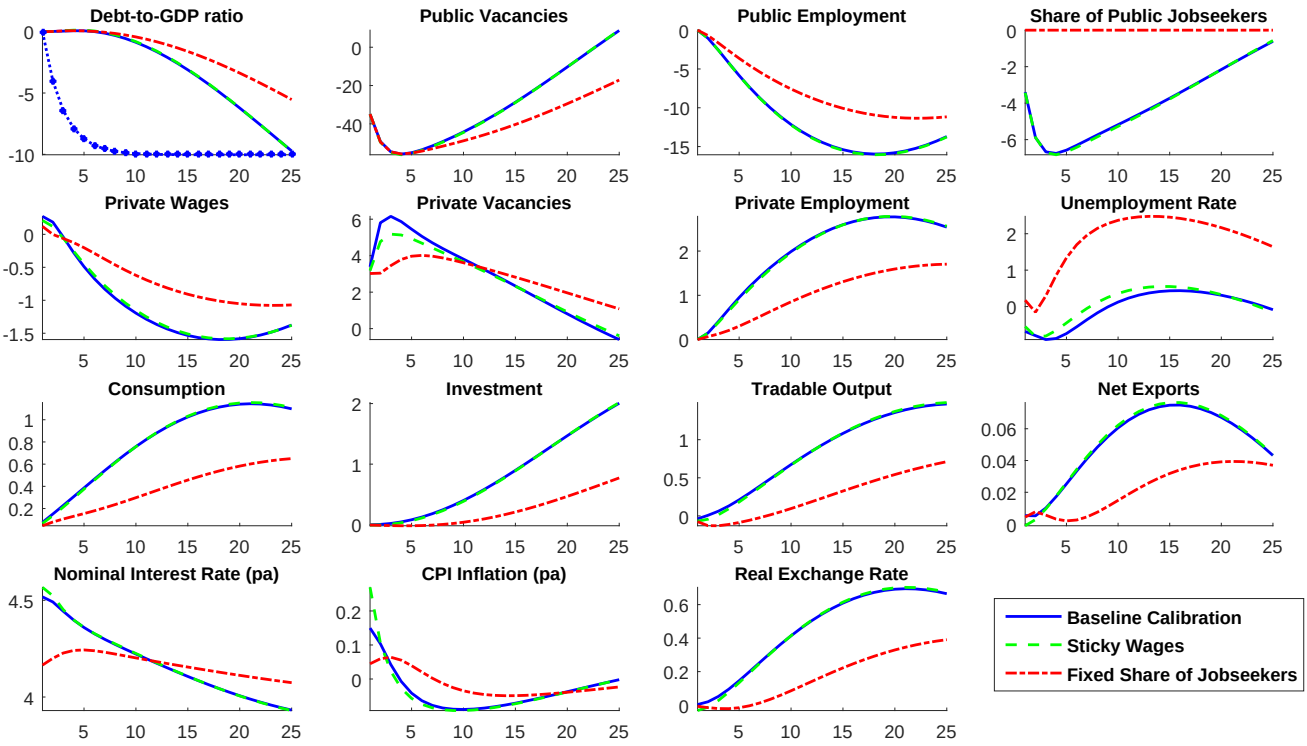


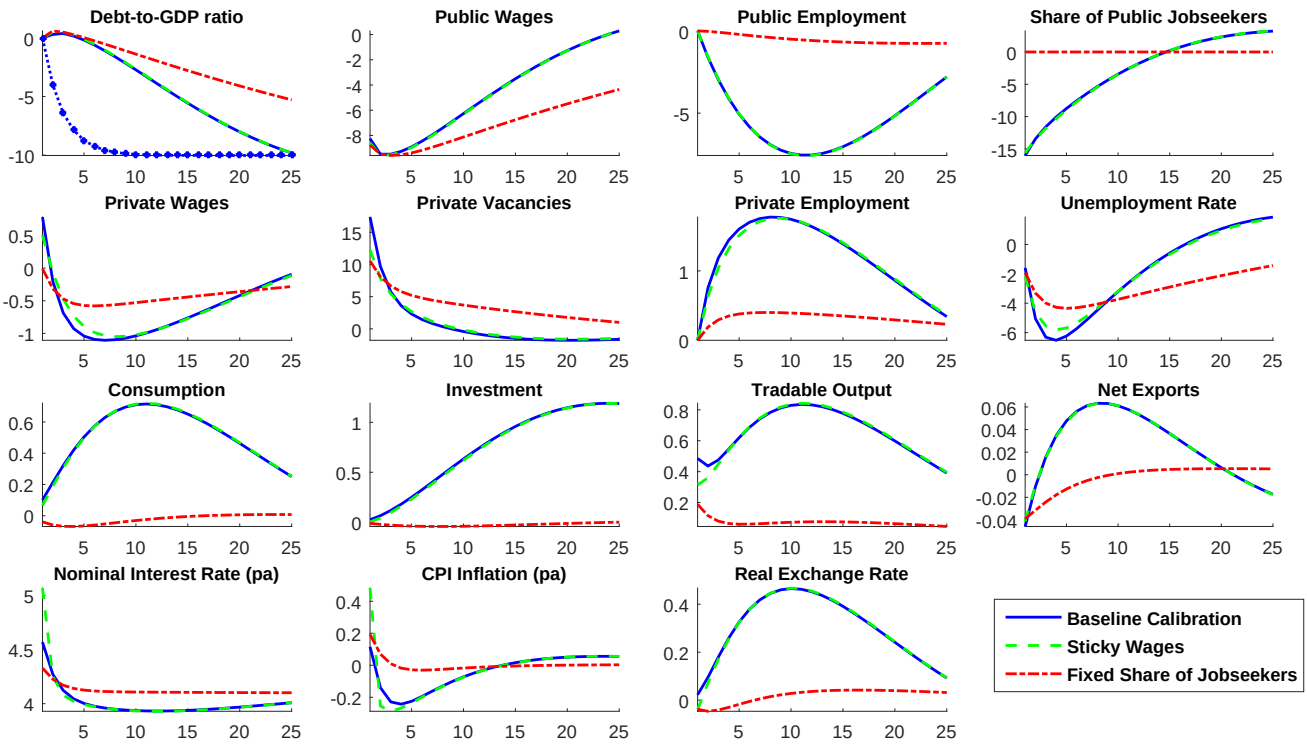
(b) Public Wage Cuts

Figure 8: Fixed Allocation of Jobseekers (FA) at the Zero Lower Bound

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.



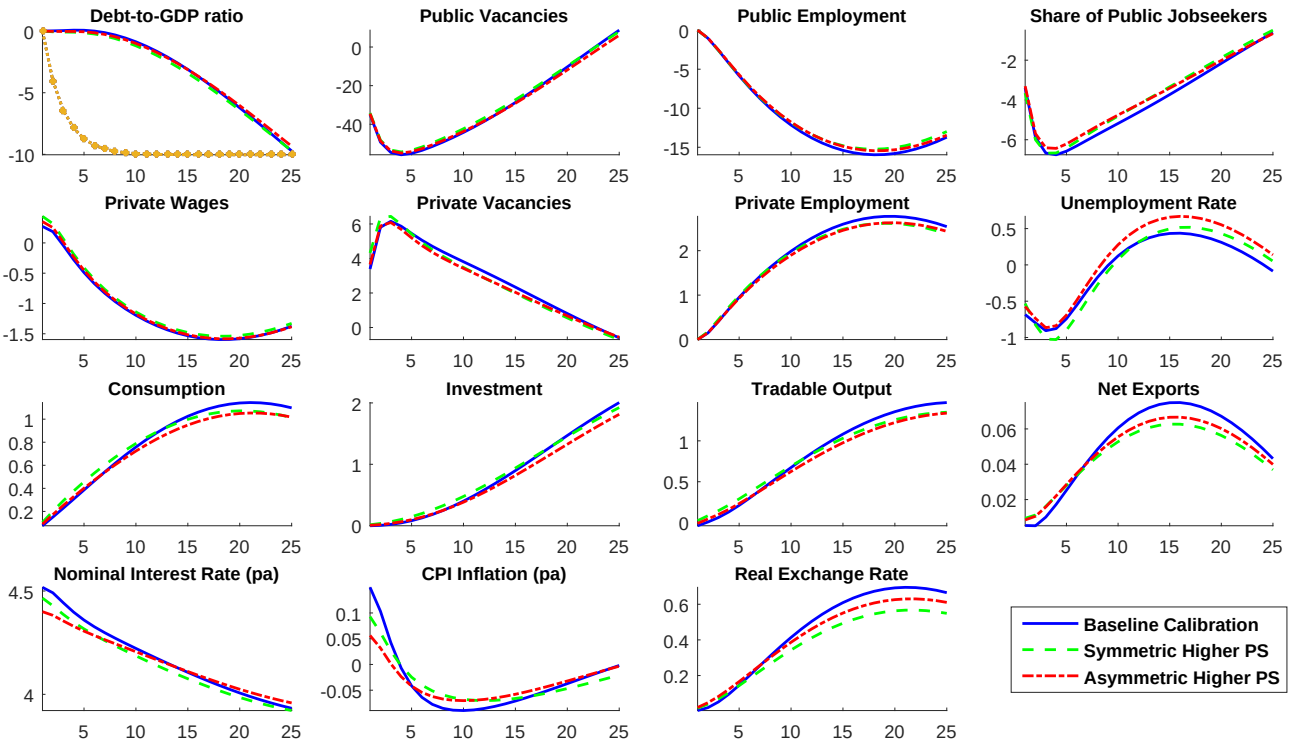
(a) Public Vacancy Cuts



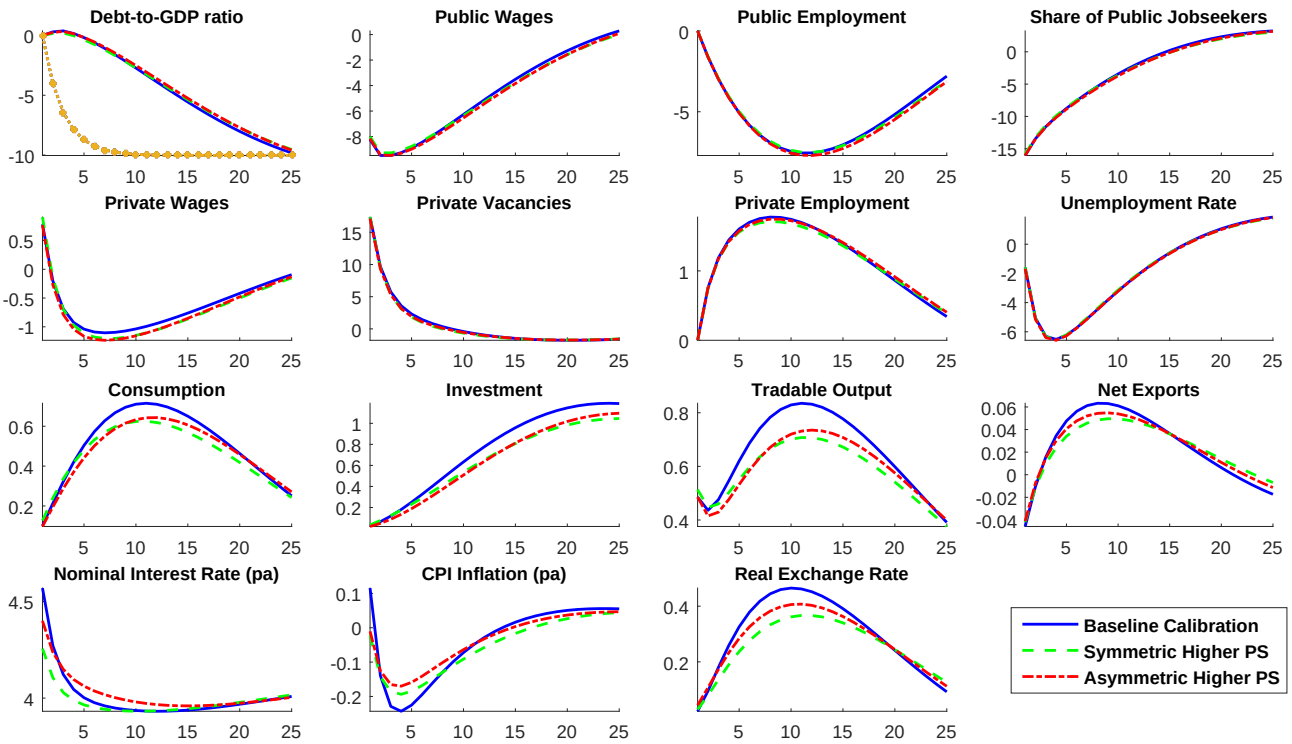
(b) Public Wage Cuts

Figure 9: Labour Market Rigidities in Normal Times

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.



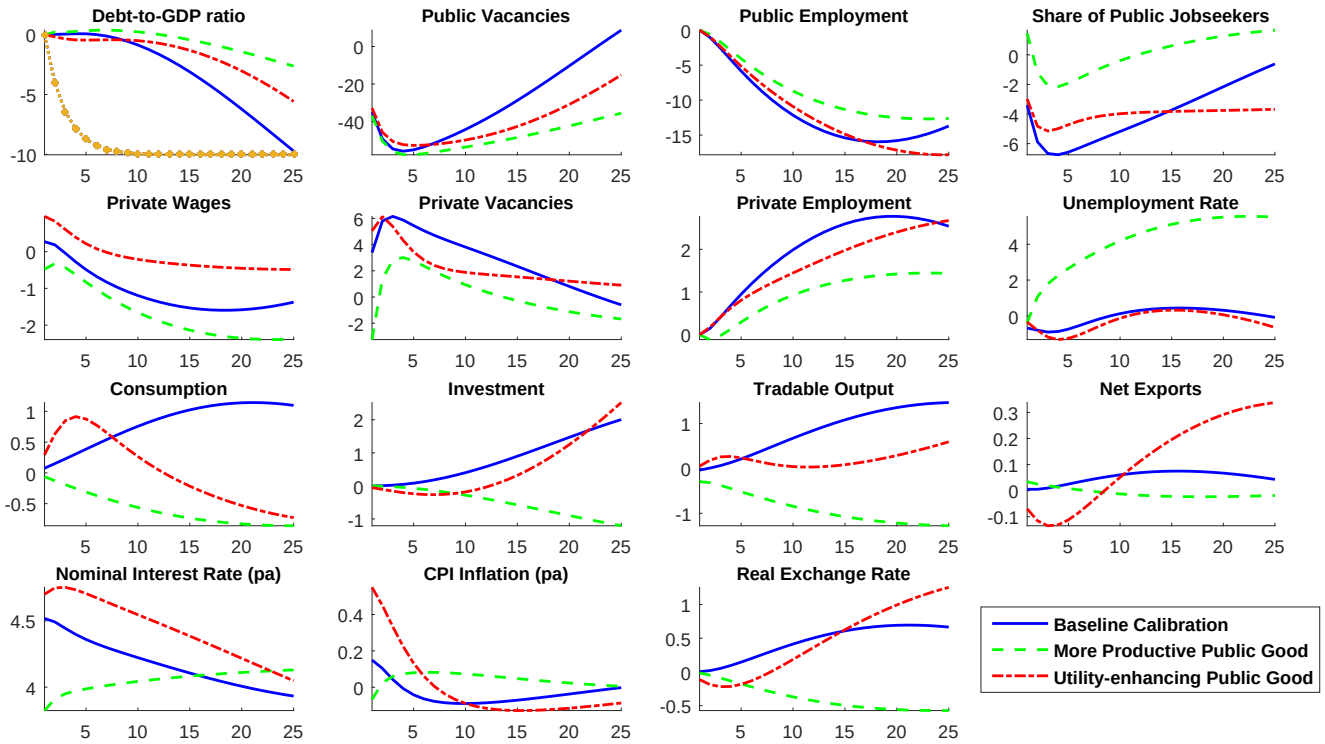
(a) Public Vacancy Cuts



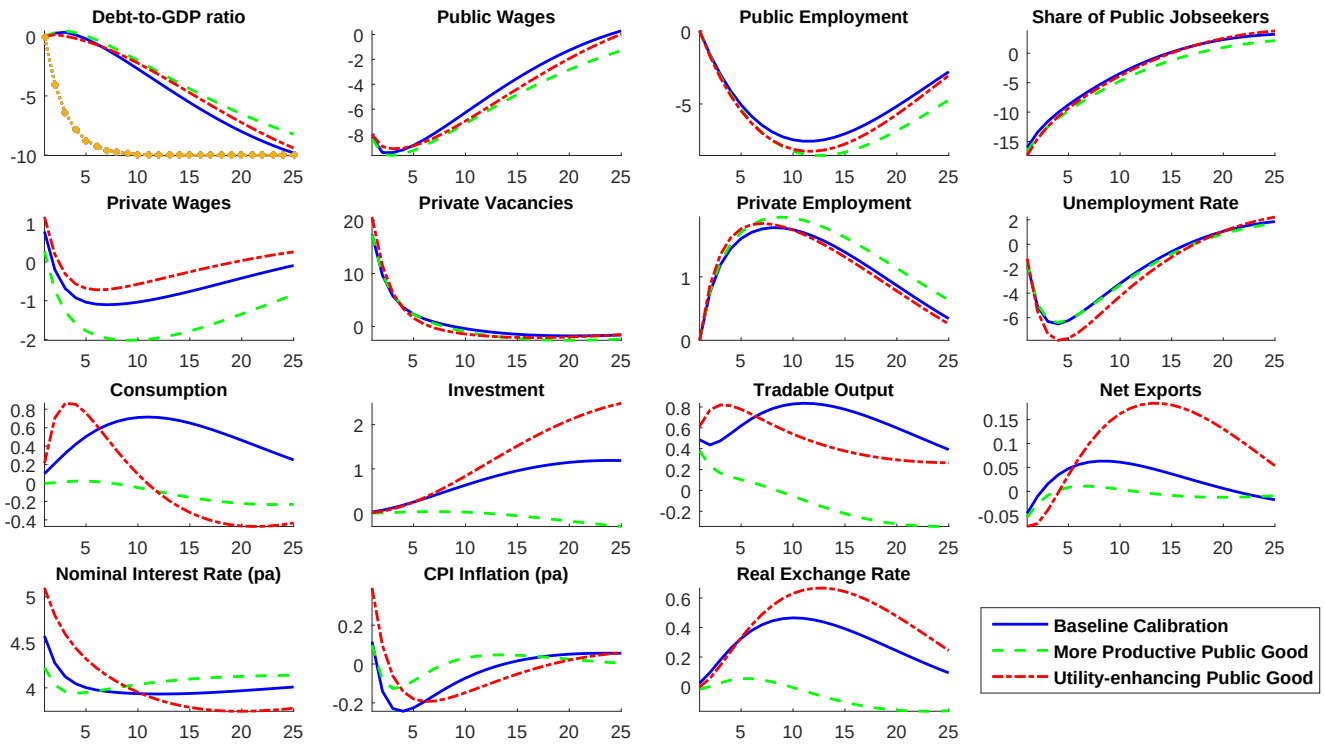
(b) Public Wage Cuts

Figure 10: Higher Price Stickiness (PS) in Normal Times

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.



(a) Public Vacancy Cuts



(b) Public Wage Cuts

Figure 11: The Role of the Public Good in Normal Times

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels.