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Abstract

Motivated by the desire to probe macroeconomic tail events and to capture non-linear economic dynamics, we estimate two types of regime switching models: threshold VAR and Markov switching VAR. For each of the models, we estimate regimes which carry the interpretation of recessionary/normal and financially stressful/stable periods. Using the recursiveness assumption and conditional on shocks of one standard deviation, we show that (i) *financial shocks* hitting during times of recessions create disproportionately more severe contractions in output; (ii) *output growth shocks* hitting in financially stressful times result in disproportionately further financial stress; (iii) *monetary policy shocks* hitting in recessionary times create more severe contractions in output. We also demonstrate the power of a *feedback loop* between real and financial sectors when extremely large shocks hit the economy in normal/financially stable periods. Afterwards, we perform out-of-sample forecasting exercises, and find that the threshold VAR model has the potential to predict tail events in *conditional* forecasting. Overall, our findings provide strong evidence of nonlinearities and shock amplification mechanisms in the UK data, as well as empirical support to the theoretical findings of Brunnermeier and Sannikov (2014).

Key words: Macroeconomic tail events, nonlinear VARs, generalised impulse response functions, density forecasts.

JEL classification: C31, C53, E32, G01.

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1 Introduction

This paper seeks to enhance the understanding of macroeconomic tail events. We define macroeconomic tail events as high-impact economic outcomes which arise with a small probability. We adopt a regime switching modelling approach to capture potential nonlinearities in the data and consequently study tail events in the macroeconomy. Using regime switching vector autoregressive (VAR) models, in the spirit of Alessandri and Mumtaz (2014) and Hubrich and Tetlow (2015), we exploit changes in economic dynamics during stressful times. These non-linear VARs enable us to assign proper economic meanings to different states of the world and to perform structural analyses.

Economic dynamics during economically or financially stressful times are potentially different from normal times. Of particular importance are the nonlinearities induced by the switches in economic regimes and the existence of adverse feedback loops between real and financial sectors, as highlighted by Brunnermeier and Sannikov (2014). Various empirical papers also discuss the importance of nonlinearities, including Hamilton (1989), Kim and Nelson (1999a), Piger et al. (2005), Primiceri (2005), Mishkin (2010). In such situations, linear models lose appeal. As Drehmann et al. (2007) discusses, linear approximations might sufficiently work in the middle of the distributions but can behave badly in the tails. Consequently, linear models are not adequate for studying tail events and for capturing the possible impacts of adverse shocks. Our adoption of regime switching models explicitly addresses this issue.

We build on Hubrich and Tetlow (2015)'s seminal findings to explore different dynamics of the economy in different states. We construct a simple system of five variables summarising real economic activity along with its linkages to, and the linkages between, the banking sector and financial markets in the UK. Apart from a Markov switching VAR (MSVAR) model as in Hubrich and Tetlow (2015), we additionally consider threshold VAR (TVAR) models to study non-linear dynamics and a linear Bayesian VAR (BVAR) model as a benchmark. By employing generalized impulse response function analysis, we capture the powerful feedback loops between financial and real sectors, which would otherwise be missed with the conditionally linear impulse responses commonly used in the literature such as by Hubrich and Tetlow (2015), among others. We go one step further to show that out-of-sample *conditional* forecasting exercises performed with threshold VARs carry big potential in forecasting macroeconomic tail events.

We consider two different threshold variables in the TVAR exercise: real GDP growth rate and aggregate corporate bond spreads. The use of the real GDP growth rate as a threshold variable is motivated by the desire to study *macroeconomic* recessions. In our case, the estimated threshold value hovers around zero percent, consistent with the common definition of recessions. Hence we can discern *recessionary regimes* from *non-recessionary* ones. The use of corporate bond spreads as the threshold variable is motivated by the observation that not all recessions are related to financial stress, and our desire to study regimes which are *financially stressful*.

As for the MSVAR, we assume one Markov chain which governs the transition of regimes for both variance and coefficient regimes in a two regime economy. The latent regimes captured by this model are broadly similar to the stressful regimes picked up by our TVARs, on top of the periods characterised by high interest rate and inflation rates in the mid 1980s and early 1990s. Taken together, our regime switching models are able to identify periods of extreme stress in the UK economy.[1]

To study structural shock transmission under different regimes, we adopt the common recursiveness assumption. We study financial shocks (proxied by exogenous jumps in corporate bond spreads), negative output growth shocks and monetary policy shocks, and compute the generalised impulse response functions as described in Koop et al. (1996). The TVAR models successfully capture three valuable facts when small shocks hit during *bad* regimes. First, financial shocks hitting during recessionary periods create disproportionately more severe recessions and significant declines in aggregate bank excess returns. Second, negative output growth shocks occurring during times of financial stress lead to disproportionately higher financial stress. In particular, these shocks in financially stressful periods generate a surge in corporate bond spreads *seven times as large* as growth shocks of a similar size in the financially non-stressful regime. Third, we find some evidence that the drop in output growth induced by interest rate shocks is deeper, but much less persistent in recessions. Aggregate bank excess returns drop in recessions, as opposed to a rise, as predicted by the linear model. Our MSVAR model generates qualitatively similar results. These results, which the linear BVAR model fails to capture, point out the significance of acknowledging nonlinearities and provide useful information to investigate tail events conditional on structural shocks.

We also provide evidence for the existence of powerful feedback loops between real and financial sectors. Here, we shift our attention to shocks deep in tails (proxied by the size of three-standard-deviation shocks in our exercise) hitting during *normal* times, especially in booms.[2] For example, an extremely large credit spread shock during normal times leads to disproportionately large falls in output, which then feed back into the financial sector resulting in a further increase in

[1] An alternative approach is to consider smooth transition VARs, which do not allow for abrupt but instead smooth transition in regimes however still require an explicit specification of threshold variables. This class of model arguably falls between the classes of threshold VARs and MSVARs. In this paper, we do not consider smooth transition models for two reasons. First, we aim to preserve model parsimony by not estimating extra parameters in the transition function. Second, we also want to give a chance to the data to detect abrupt regime changes, which is possible with low-frequency quarterly data. As we show later in the paper, the threshold VARs and MSVARs produce qualitatively similar results. Hence, we conjecture that the adoption of smooth transition VARs also leads to reasonably similar conclusions.

[2] This is motivated by the belief that the probability of tail events occurring is time-varying and may depend on financial or business cycles. In the context of supervisory stress-testing, Bank of England (2015) postulates that 'the severity [of scenario] is likely to be greater in a boom, for example, when growth in credit is rapid and asset prices unsustainable high' and hence proposes a counter-cyclical approach in the design of stress scenario.

financial stress. Such feedback effects can be explained by the endogenous switches of regimes from *non-stressful* to *stressful* ones, which adds to the existing non-linear dynamics in the model.

The above findings also carry important empirical implications for the theoretical results highlighted in Brunnermeier and Sannikov (2014). Their prediction that small shocks can be amplified once the economy is already in crisis regimes is supported by our empirical results that the economic impact is more severe when small shocks hit during recessions or financially stressful times. Our results also speak to their prediction that the economy reacts to large exogenous shocks differently than to the small ones. In particular, our simulations exploiting tail shocks show that their impact on the real and financial sectors are disproportionately larger relative to smaller shocks, suggesting that large shocks are strongly amplified. On the whole, we present strong empirical evidence that reactions to shocks in an economic system can be highly nonlinear.

We then turn to a reduced-form out-of-sample forecasting analysis. Having re-estimated our models with data right before the Great Recession set in, we produce multi-step ahead predictive densities for the variables in the system and compare them against the outturns during the Great Recession period. We then try to answer the following two questions. First, do non-linear models produce *unconditional* density forecasts that are more informative in the tail than their linear counterparts? Second, could we produce more reliable predictive densities when we *condition* on variable paths consistent with stressful times? Notice that we are not trying to predict the timing of crises or any stress events, but rather assessing the capabilities of different models to generate a broad view on the occurrence and nature of tail events.

The answers to the two questions are sequentially ‘slightly more’ and ‘potentially substantial’. We find that unconditional density forecasts produced by the MSVAR and TVAR do not exhibit huge advantages in generating a broad view of tail event forecasts. On the other hand, when we follow Waggoner and Zha (1999) to perform *conditional forecasting* based on variable path of *corporate bond spreads* during the Great Recession, TVAR, and, to a lesser extent MSVAR, show substantial improvements in the tail density forecasts for the output growth. Our findings confirm the importance of incorporating nonlinearities in modelling macro data, and demonstrate the usefulness of such non-linear models to generate reasonable tail forecasts.

There are a range of studies examining tail events/risks from different perspectives. In finance, value-at-risk models are commonly used to measure the loss event on a specific portfolio of financial exposures. This concept has been recently adopted to macroeconomics. Boucher and Maillet (2015) estimate value-at-risk of US output using quantile regressions, and produces a fan chart for out-of-sample forecasts of industrial production growth. This use of quantile regression highlights the importance of outliers, which are associated with extreme events and undoubtedly provide valuable information for modelling and forecasting future tail events. This point is shown in Covas et al. (2014) who use fixed effect quantile autoregressive models to capture the dynamic of banks losses and revenues and to project capital shortfalls, and in Adrian and Brunnermeier (2014) who propose a measure of systemic risk conditional on an institution under stress with quantile regressions. Recently, White et al. (2015) provide a theoretical framework to estimate and make inference in multivariate, multiquantile models.

While acknowledging the usefulness of quantile regressions, we see two advantages of using non-linear VARs. First, we can explicitly identify different regimes of the economy. Second, we can refrain from making assumptions on which quantiles shocks have to originate in order to contribute to macroeconomic tail events. Our approach fully accommodates scenarios where small shocks can be amplified to create big economic impact.

Our paper is related to an active area of empirical research using non-linear VAR models. Balke (2000) examines the non-linear propagation of shocks through the role of credit with a threshold VAR. He shows that contractionary monetary shocks have a larger impact on output than the expansionary shocks when in a tight credit regime. Sims and Zha (2006) employ a multivariate regime switching model for US monetary policy in a structural VAR framework. Alessandri and Mumtaz (2014) construct a set of linear and non-linear econometric models to study predictive densities, especially by focusing on tails to assess the power of financial indicators for output and inflation in the US. Franta (2015) investigates the effects of the rare shocks in the US economy and associated nonlinearities using a threshold VAR and t-distributed shocks. Hubrich and Tetlow (2015) investigate the interaction between a financial stress index for the US and real activity, inflation, monetary policy using a Markov switching VAR model. The empirical findings support the inadequacy of single regime models to capture the dynamics of the economy. On a similar front, Galvão and Owyang (2014) identify financial stress regimes by using a factor augmented smooth transition VAR by considering a large panel of US financial and macro variables.

The remainder of this paper proceeds as follows. Section 2 presents the features of our data set. We introduce the econometric models used in this paper in Section 3. We illustrate the estimation results of the proposed procedure in Section 4. Section 5 provides our analysis on the structural shock transmission in our models. We present robustness analysis in 6. Section 7 follows with the forecasting results. We conclude with Section 8. Specific details on the data, model estimation and priors are given in the Appendix.

2 Data

We use quarterly data for the UK from 1965:Q2 to 2014:Q2. We have a 5 variable system including output growth (measured by quarterly change in real GDP), inflation (measured by the quarterly change in consumer price index), excess bank returns, corporate bond spreads (proxied by the difference between UK Corporate Bond Yield and UK 10-year Government Bond Yield) and the short term interest rate (proxied by the three month interest rate). Except for the short term interest rate and corporate bond spreads, all series are annualised.

Using corporate bond spreads as a measure of financial frictions follows mainly from Gilchrist and Zakrajšek (2012) and Philippon (2009). The former examines the relationship between corporate bond spreads and economic activity. They find the excess bond premium, which is one of the two components of GZ index representing the cyclical changes between measured default event and credit spreads, to be a predictor of economic activity. Philippon (2009) looks at a similar concept from the market's perspective. It shows that, similar to Tobin's q, a market-based measure of a q can be constructed from corporate bond prices and it performs much better than the traditional one at predicting investment and part of the economic activity. Furthermore, it is worthwhile to note that the Financial Stress Index used by Hubrich and Tetlow (2015) for the US economy includes corporate bond spreads. Our rationale of choosing corporate bond spreads as a proxy of financial stress is, in addition to the use in the literature, for the search of a consistent and long enough series reflecting financial stress. In a similar spirit, excess bank returns are used as a proxy of aggregate bank profitability.

The choice of variables reflects our goal to capture the overall dynamics in the economy and to link the real economic sector to the banking and financial sectors while maintaining a parsimonious model. Tables 1 and 2 in Appendix A describe the basic statistics of the data alongside the data sources, and Figure A.1 plots the five variables of interest.

3 Model Specifications

This section describes the three models we deploy: threshold VAR, the Markov switching VAR and the linear VAR. In a TVAR, the researcher has to *pre-define* a threshold variable. In other words, any switches of regimes in TVAR are solely determined by the dynamics of the chosen threshold variable, and the switch of regimes is abrupt. In contrast, regime switches in MSVAR are determined by the joint dynamics of the economic system (but the interpretation of such regimes might not be as straightforward), and the transition of regimes are governed by a probability transition matrix.

We give a brief overview of each of the models below. In all of the models, the Akaike and Bayesian Information criteria suggest two lags as the optimal choice of lag length, which we will adopt in our estimations.[3]

[3]Deviance information criterion suggests six lags as the optimal lag length. We estimate the models again with six lags and perform structural analyses. All of our results, including the the estimated regimes and shock amplification mechanisms, remain qualitatively similar. We therefore adopt two lags as our baseline models for parsimony.

3.1 Threshold VAR

A threshold VAR model comprises an explicit threshold variable which allows regimes to switch endogenously. The associated model is

$$Y_t = \left[c_1 + \sum_{j=1}^{P} \beta_{1,j} Y_{t-j} + v_t\right] R_t + \left[c_2 + \sum_{j=1}^{P} \beta_{2,j} Y_{t-j} + v_t\right] (1 - R_t)$$

where

$$R_t = 1 \Longleftrightarrow Z_{t-d} \leq Z^* \quad (\text{or } \; Z_{t-d} \geq Z^* \text{depending on the threshold variable})$$

$$R_t = 2 \text{ otherwise}$$

and $v_t \sim N(0, \Omega_{R_t})$.

The delay parameter, d, is also referred as threshold lag. We consider two different threshold variables: real GDP growth rate and the level of aggregate corporate bond spreads. We define $R_t = 1$ as the *recessionary regime* if and only if the real GDP growth is below an estimated threshold rate for $d = 0$ and $d = 1$ simultaneously, given the common definition of recessions associated with two consecutive periods of negative output growth. Otherwise, the regimes are defined as *non-recessionary.* Our estimated threshold rate hovers around zero percent at the posterior mode, which validates this definition. We denote this system as *TVAR-Y*.

The use of corporate bond spreads as the threshold variable is motivated by the observation that not all recessions are related to financial stress, and by our desire to explicitly define financially stressful regimes. We define $R_t = 1$ as the *financially stressful regime* if and only if the credit spreads rise beyond an estimated threshold value for $d = 1$. Otherwise, the regimes are *financially non-stressful.* Our estimated threshold value for the credit spreads is 290 basis points at the posterior mode. We denote this system as *TVAR-S*.

To estimate this model, we follow Alessandri and Mumtaz (2014) in using the Gibbs sampling algorithm which includes a Metropolis-Hastings step for sampling the threshold value in each simulation. The estimation procedures are explained in Appendix B.

3.2 Markov Switching Vector Autoregression Models

The Markov switching VAR Model (MSVAR) is written as

$$Y_t = c_{S_t} + B_{1,S_t} Y_{t-1} + B_{2,S_t} Y_{t-2} + \cdots + B_{l,S_t} Y_{t-L} + v_t, \tag{1}$$

where $v_t \sim N(0, \Omega_{S_t})$.

The regime switches follow a joint dynamic for coefficients and variance at the same time. The latent regimes S_t are assigned as $S = 1, 2$.[4] The switch between these latent states is governed by the transition matrix, P:

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

where $p_{ij} = prob(S_t = i|S_{t-1} = j)$ indicates regime i is followed by regime j. There are no restrictions on regime switches, i.e they are left unrestricted to jump back and forth. The columns sum up to 1. We follow the multi-step Gibbs sampling procedures proposed by Kim and Nelson (1999b), the details of which are discussed in Appendix C.

3.3 Benchmark Bayesian VAR

We use a Bayesian VAR model a benchmark against which to compare our non-linear VAR models, and it is given by

$$Y_t = c + B_1 Y_{t-1} + B_2 Y_{t-2} + \cdots + B_l Y_{t-L} + v_t, \tag{2}$$

where $v_t \sim N(0, \Omega)$.

3.4 Priors

For all models, we impose normal inverse Wishart priors following Bańbura et al. (2010), see Appendix D for more details.

4 Full sample estimation results

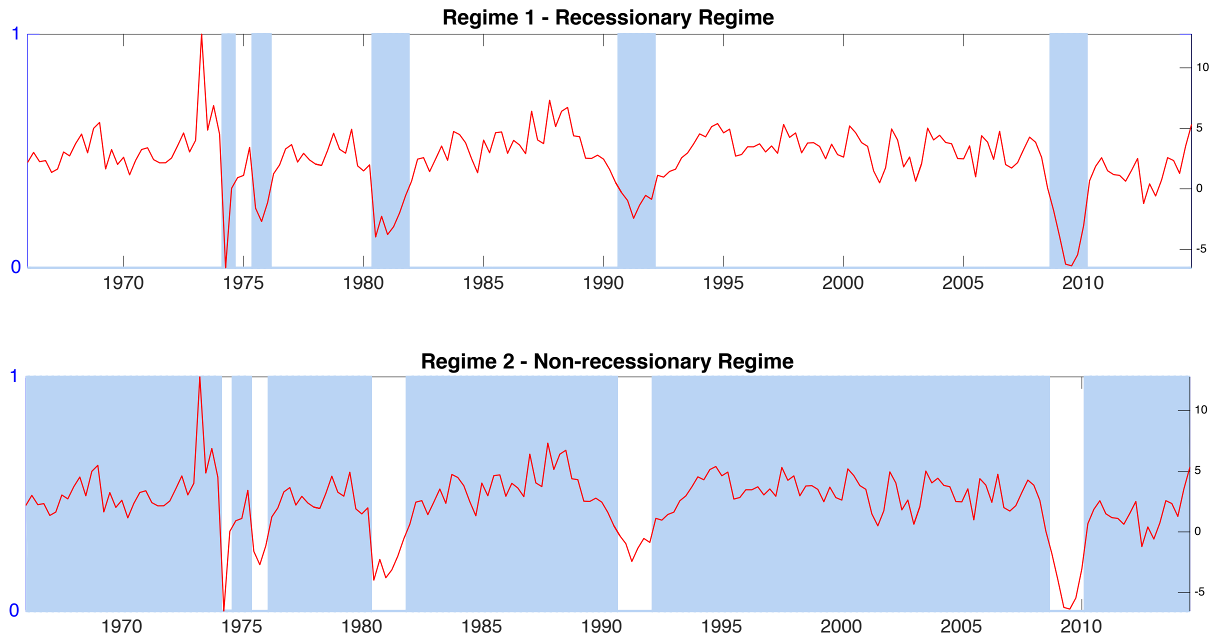
This section reports the estimation results for both non-linear models using the full sample from 1965:Q2 to 2014:Q2.[5] Figures 1 and 2 respectively illustrate the recessionary and non-recessionary regimes and financially stressful and non-stressful regimes as modelled by the

[4]According to the deviance information criterion, the data favor a two-regime rather than a three-regime MSVAR model.

[5]Information criteria do not provide statistical evidence that these non-linear models provide better in-sample fit to this dataset. This can be explained by non-linear models being more likely to be over-parameterized. Having taken note of the theoretical and empirical literature, as pointed out forcefully by Brunnermeier and Sannikov (2014) and Hubrich and Tetlow (2015), which lay emphasis on non-linear economic dynamic behaviour during economically stressful periods, we argue that these non-linear models still serve as a useful tool to investigate macroeconomic events and provide evidence to substantiate our claim.

TVAR-Y and TVAR-S models. Figure 3 reports the estimated regime probabilities addressing high and low stress states implied by our MSVAR model.

Figure 1: Full sample regimes for TVAR-Y

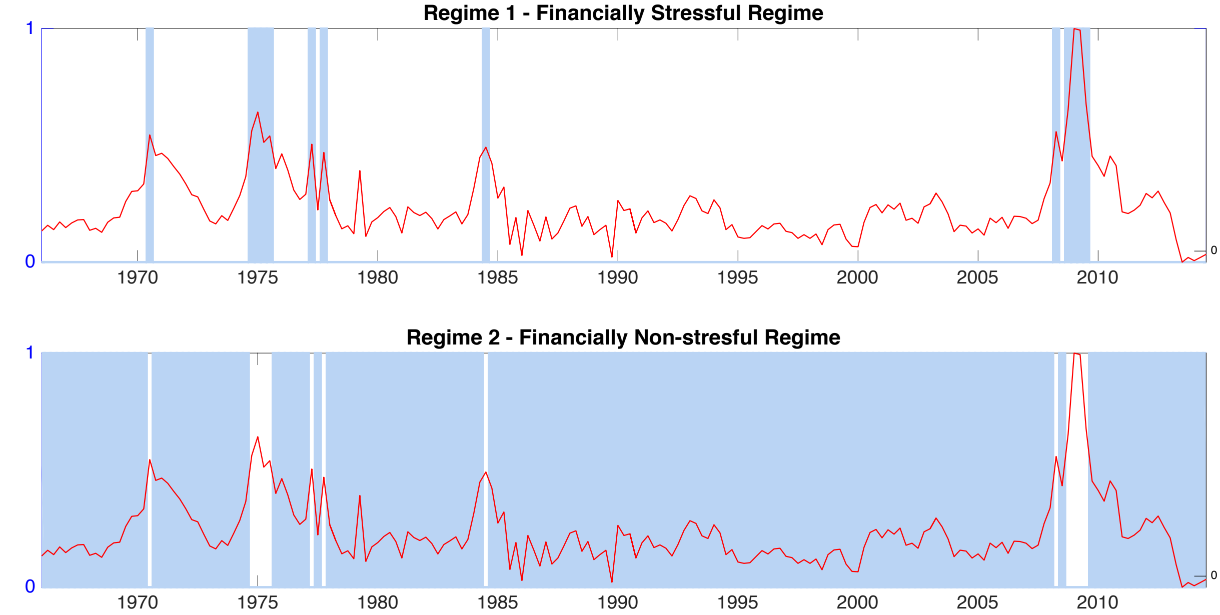


Notes: The threshold variable is the real GDP growth which is overlapped with the regimes.

In a TVAR model, the regime changes are abrupt and the economy is either in one regime or the other. Therefore the probabilities accompanying the regimes are either 0 or 1. The first regime estimated by the TVAR-Y model in Figure 1 is labelled as *recessionary* whereas the second regime as *non-recessionary* regime. The first two recessions coincide with the mid-1970s recessions. They are associated with the 1973 oil crisis and subsequent stagflation. The next is the recession in the early 1980s due to deflationary government policies including spending cuts, pursuance of monetarism to reduce inflation, and switches from a manufacturing economy to a services economy. The fourth is the recession in the early 1990s which started in the third quarter of 1990 and went on for five quarters. It was primarily caused by high interest rates as a result of membership of the Exchange Rate Mechanism, falling house prices and an overvalued exchange rate. The last recession is the Great Recession which followed the global financial crisis.

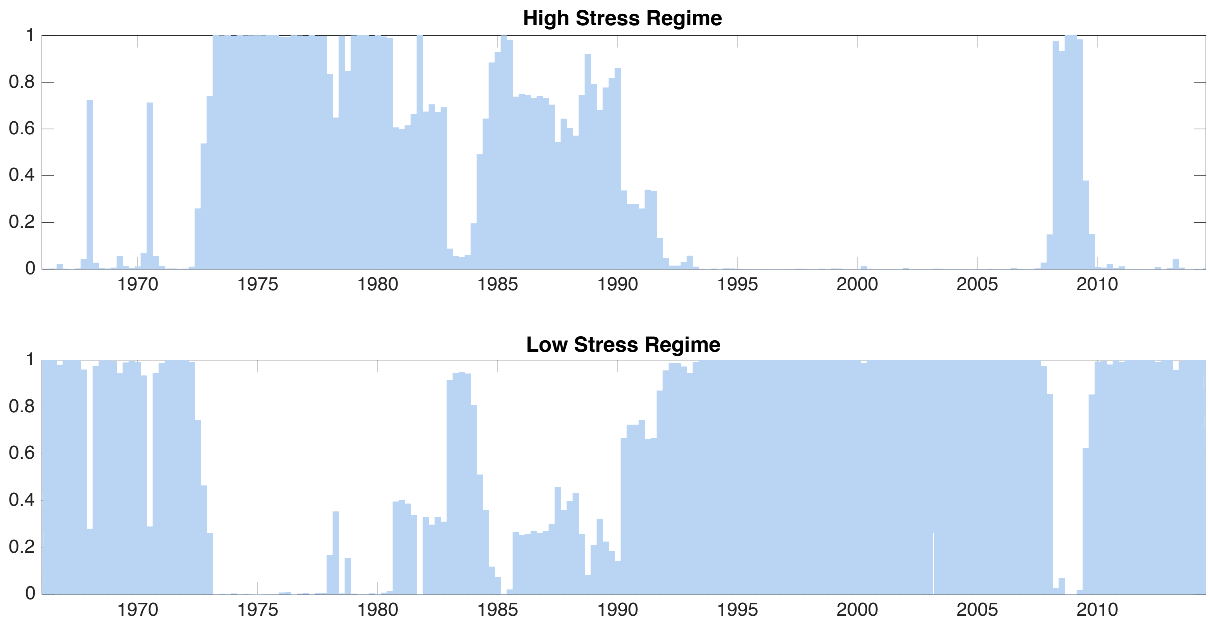
Similarly, Figure 2 presents the regimes where the corporate bond spreads are used as the threshold variable. The first financial stress period corresponds to the second quarter of 1970. The second reflects the impact of the oil crisis and stagflation on the corporate bond market over the periods 1974:Q3-1975:Q2. The third and fourth are associated with 1977:Q1 and 1984:Q2. The former may correspond to the period of introduction of small number of floating-rate issues. The last financial stress periods show the effects of the Great Recession onto the market and overlap with the recessionary regimes with an exception of the second quarter of 2008.

Figure 2: Full sample regimes for TVAR-S



Notes: The threshold variable is the corporate bond spreads series which is overlapped with the regimes.

Figure 3: Full sample regime probabilities for MSVAR



The regimes given by the MSVAR model in Figure 3 suggest a similar interpretation. In accordance with the data we use, the 'high stress regimes' do not only pick up recessionary or financially stressful periods, but also times when the inflation rate and short term interest rates were both high. High stress periods start around 1973 and mute by early 1990s with a significant low stress periods in between 1982 and 1985. The high stress period between 1985 and 1992 witnessed high interest rates and a recession in the early 1990s. Stable periods last for around fifteen years after 1992, followed by the stress periods during the Great Recession.

5 Impulse Response Analysis of Structural Shocks

Our setup of different models provides us a convenient platform to study structural shocks based on historical data. We are particularly interested in the potential differences in the shock transmission under specific regimes. This is crucial as the linear models are generally found to be inadequate to explore tail events and to investigate different shock transmission during stressful times.[6]

Broadly, there are two main findings. First, the transmission of shocks in recessionary or financially stressful periods is different than normal times. The distinction between the reaction of the economy in crises and normal times is highlighted by a theoretical model in Brunnermeier and Sannikov (2014). Our results stress the importance of acknowledging nonlinearities and distinguishing different states of the world. Second, we illustrate Brunnermeier and Sannikov (2014)'s theoretical point in application that the large shocks are strongly amplified by simulating our models with extremely big shocks. We discuss how these tail shocks cause disproportionately deeper stress and materialise feedback loops.

5.1 Generalised impulse response functions and shock identification

Koop et al. (1996) discuss that impulse responses in non-linear models are dependent on size, sign and history, which is in contrast to those computed by linear models. They introduce the generalised impulse response functions (GIRFs), which fully takes into account of the possibility of endogenous switches of regimes during simulations.

Following Koop et al. (1996), we calculate these impulse responses as

$$GIRF = E(y^{*,p}_{t+k}|y_t, \Upsilon, \triangle) - E(y^{*}_{t+k}|y_t, \Upsilon)$$

where k is the forecast horizon, Υ denotes the hyperparameters, $\triangle$ indicates the perturbed shocks while superscript p marks the forecasts with the perturbed path of errors. Appendix E reports the steps of generating non-linear impulse response functions for both TVAR and MSVAR models.

[6] Such a set-up offers insights on the impact of adverse shocks hitting certain *bad* states of the world, which can be potentially useful in calibrating macro scenarios where multiple adverse shocks hit an economy sequentially, for instance, in stress testing.

We adopt the Cholesky decomposition for our purposes, which is common in the empirical macroeconomic literature to identify macro structural shocks. As is well known, the order of variables matters while generating the linear impulse response functions. In our case, we order the variables as: (i) real GDP growth; (ii) inflation rate; (iii) aggregate bank excess returns; (iv) corporate bond spreads; (v) short term interest rate. This order is consistent with the monetary policy literature as real variables respond to monetary policy shocks with a time lag whereas monetary policy responds to shocks from the real sector and the financial sector contemporaneously. Our identification strategy is closely in line with Christiano et al. (1998) and Gilchrist and Zakrajšek (2012).[7]

In this section we consider three structural shocks: (i) corporate bond shocks which proxy for shocks in the financial market; (ii) output growth shocks; (iii) interest rate shocks. We seek to investigate the implications of one-standard-deviation shocks alongside three-standard-deviation shocks.

5.2 Shock propagation and macro tail events

Figures 4 to 13 depict the impulse response functions of all three models to three structural shocks of one-standard-deviation size. We first study *financial shocks*. Gilchrist and Zakrajšek (2012) explain that credit spreads can reflect the changes in the quality of corporate firms' balance sheet and their external finance as well as the capital position of financial intermediaries who supplies credit. Nevertheless, we interpret any exogenous rise in the corporate bond spreads as shocks to the financial intermediation process which is orthogonal to shocks to the real sector.

In our linear model given in Figure 4, this particular shock leads to a contraction in output growth, an initial drop in bank returns and inflation in the linear model. The responses of non-linear models, on the other hand, have noticeably different features. Figures 5, 6 and 7 show the impulse responses of of TVAR-Y, TVAR-S, MSVAR, respectively, corresponding to a one-standard-deviation shock to corporate bond spreads. In TVAR-Y (Figure 5), a one-standard-deviation financial shock in the recessionary world (70 basis point jump in bond spreads) generates a significant and deep decline in output growth with a maximum fall of 45 basis points 5 quarters after the shock. If such shocks happen in the non-recessionary world, the recessionary impact is comparatively much shallower (after factoring in the different size of the shocks). The financial shock also leads to a deeper and more persistent drop in the short-term interest rate in

[7] We checked alternative orderings of the last three variables. Results are mostly robust except when we order corporate spreads after the interest rate, i.e. when corporate bond spreads react to monetary policy shocks with a time lag but not the other way round. (Note that such an ordering is used by Hubrich and Tetlow (2015) where they construct a *monthly* system of US data.) We stress that our baseline ordering is in line with Gilchrist and Zakrajšek (2012) who place excess bond premium (a component of credit spreads) before the effective Federal funds rate, and is more convincing because monetary policy is able to respond to financial shocks within the same quarter.

the recessionary world. Interestingly, this shock leads to a rise in the inflation rate, which can be explained by the association of the identified recessionary regimes with high inflation rate rates in the 1970s. The most compelling implication is the response of the excess bank returns, a proxy for bank profits. In the recessionary regimes, financial shocks lead to almost 8 pp drop in aggregate bank excess returns two quarters after the shock, whereas there is a slight rise in the non-recessionary world. This reflects that aggregate excess returns of banks are seriously affected when financial shocks hit during recessions.

The GIRFs in TVAR-S, shown in Figure 6, point a similar picture. A financial shock in an initially financially stressful regime leads to a much greater contraction of output and greater drop in aggregate bank returns when compared to the financially non-stressful world. The responses of the MSVAR model in Figure 7 are qualitatively similar to those of the TVAR-Y and the TVAR-S models. Our observation related to excess bank returns rising in the good state also holds for the MSVAR.

These results are consistent with the predictions of the theoretical literature. During times where the real economy is in recession or the financial market is under stress, where balance sheets of the financial and nonfinancial sectors are weak, any shocks in the financial market further amplify the recessionary effects through the well-known financial accelerator mechanisms as described in Kiyotaki and Moore (1997) and Bernanke et al. (1999). This undoubtedly leads to an erosion of aggregate banks' profits. Our results are also in line with Brunnermeier and Sannikov (2014) whose theoretical model predicts that small shocks can be amplified once the economy is in crisis regimes.

We carry out a similar exercise by hitting in the system with a *negative output growth shock*. Since our identification scheme does not allow us to distinguish aggregate supply shocks from aggregate demand shocks[8], we consider this shock as proxying exogenous changes in the real economy that reduces output growth, which could originate domestically from productivity or demand shocks or internationally through the export-import channel.

Figures 9 and 10 display the impulse response functions of two threshold VAR variants. As inflation jumps on impact, the models seem to be picking up aggregate supply shocks. Relative to its impact in the non-recessionary/financially non-stressful regimes, this shock leads to a larger initial rise in the inflation rate in the recessionary/financially stressful regimes. This is again heavily driven by the stagflation experience in the UK. The rise in corporate bond spreads is also significant. In Figure 10, the spread can rise up to 25 basis points on impact in the financially stressful regimes as opposed to 3 basis points otherwise, even though the size of growth shocks is similar (about 1.7 pp) across the two states of the world. This shows that when

[8]Aggregate supply shocks are characterised by a fall in output growth and a rise in inflation rate, whereas aggregate demand shocks are characterised by both decrease in output growth and inflation. These two shocks can be further identified by sign restrictions, which is left for future research.

the economy is under financial stress, further bad shocks originating in the real economy lead to heightened stress in the financial system. This point is not picked up by the linear BVAR response as shown in Figure 8. GIRFs from the MSVAR model exhibit similar features.

We also consider the generalised impulse response functions of *short term interest rate shocks*, defined as any interest rate movement unexplained by the systematic responses of policy makers to variations in the state of the economy (see Christiano et al. (1998)). Examining the impulse responses of the linear model in Figure 12, we observe that the identified interest rate shock leads to a persistent decline of real output growth. Corporate spreads rise significantly two years after the shock. These results are consistent with the traditional monetary policy literature.[9]

Figure 13 shows the GIRFs of the TVAR-Y model to the interest rate shock under recessionary and non-recessionary regimes. There are several major differences relative to the linear impulse responses. First, the drop in output growth is less persistent in recessions. Second, aggregate bank returns drop by about 2 pp (annualised) in recessions as opposed to a rise otherwise. This may be an evidence that banks are particularly vulnerable to monetary policy shocks during recessions. Slightly puzzling is the small drop in corporate bond spreads in recessions.[10]

5.3 Tail shocks and the feedback loop

Our non-linear models allow us to investigate shock transmission mechanisms when extremely large shocks hit the economy, especially when the economy is in *normal* times. This is motivated by the belief that the occurence of tail events is time varying and may depend on financial or business cycles. This point is addressed by Bank of England (2013) in the context of scenario design in stress testing.[11]

We repeat our GIRF simulations with *three-standard-deviations* shocks, which arguably represent tail events. We first consider a tail shock in output growth during non-recessionary regimes in the TVAR-Y model, the results of which are reported in the grey shades in Figure 14. When compared with an output growth tail shock of the size 7 pp (annualised) in the recessionary regimes which leads to a maximum rise in credit spreads of 26 bp, a 4 pp shock in the non-recessionary regimes cause as a surge in spreads as high as 30 bps within the first five quarters.

[9] The inflation rate rises as a result of the shock. This constitutes the *price puzzle*, which is at odds with the theoretical literature which predicts a fall in prices with contractionary monetary policy shocks. A vast literature has discussed the puzzle and proposed solutions, see Sims (1992), Castelnuovo and Surico (2010), Hürtgen and Cloyne (forthcoming), Christiano et al. (1996) and Balke and Emery (1994).

[10] The TVAR-S model produces a boom conditional on an unexpected rise in short term interest rate. The results seem counterintuitive so we do not report them here.

[11] Bank of England (2013) states that one way to explore the severity of the scenarios in stress–testing is 'to recognise the variation in the probability and impact of systemic stresses over time'. For example, as credit conditions ease and leverage builds up, the banking system may be susceptible to more severe shocks. Conversely, in a downturn, with tightening credit conditions and lower leverage, a less severe scenario might be more appropriate. Our set-up also enables us to carry out an experiment in this regard.

Not only the magnitude but also the persistence in the rise in credit spreads becomes very severe. Particularly noticeable is its hump shape response starting from the third quarter. Such response is markedly different from those attributed to the standard one-standard-deviation shock shown in Figure 9. Such significant response in financial stress then feeds back to the real sector, giving rise to a far deeper and protracted recession as shown in the GDP growth response in Figure 14.

We perform similar simulations with credit spread shocks in the TVAR-S model. A tail shock comprising a 130 bp surge in credit spreads in financially non-stressful regimes, as shown in Figure 15, results in a very deep and persistent recession, with the trough occurring three years after the shock. The contraction reaches 1.4 pp, as opposed to the much shallower recession of a maximum contraction of 0.1 pp with the one-standard-deviation shock displayed in Figure 6. Again, due to the powerful feedback effects from the real sector, the persistence and rise in corporate spreads in the future horizon are more protracted. These responses are not only disproportionately larger when compared to the one-standard-deviation shock scenario, but also larger relative to the responses in the financially stressful regimes.

These results can be intuitively explained by the simulations which allow endogenous switches of regimes from *non-stressful* to *stressful* when such tail shocks hit, which further amplifies the existing non-linear dynamics. Our empirical findings provide strong support to non-linear general equilibrium models for financial stress as pioneered by Boissay et al. (2013) and Brunnermeier and Sannikov (2014), among others.

6 Robustness Analysis

6.1 Estimation of the delay parameter in TVAR-S

Unlike in TVAR-Y where we have a common definition of recessions, there may not be a commonly agreed definition for financially stressful regimes. We therefore relax the assumption of $d = 1$ for TVAR-S and estimate this parameter in conjunction with the rest of the parameters in the model. In particular, we allow the delay parameter to be estimated between one and six quarters. In an unreported figure, we find that the posterior distribution of d highly concentrates at $d = 1$ (which is accounted by 87% of the saved draws). Unsurprisingly, all of our results presented earlier carry through.

6.2 Normalising the initial size of the shocks

To address the concern that it may be difficult to compare the impulse responses caused by shocks of different initial sizes in different regimes, we re-estimate our GIRFs by controlling for the initial size of the shocks. In particular, for financial shocks, we normalise their size to be a 100 bp rise in corporate bond spreads for our TVAR models. Similarly, for real shocks, we normalise their size to be a 100 bp decrease in the annualised real GDP growth rate. To conserve space, we report the results for the impulse responses of financial shocks implied by TVAR-Y (Figure 16, to be compared with Figure 5) and the responses of growth shocks implied by TVAR-S (Figure 17, to be compared with Figure 10). The shock amplification mechanism described earlier remain robust.

6.3 Sensitivities to priors

We impose the normal inverse Wishart priors through the use of dummy observations, which is motivated by the fact that the number of observations can be sparse in stressful regimes. As noted in Appendix D, the hyperparameter τ governs the overall tightness of the prior. In the estimation above, its value is fixed at 0.1. In this subsection, we perform sensitivity checks by loosening this hyperparameter.

In the interest of space, we only reporte a couple of alternative hyperparameter values. For TVAR-Y, we relax τ to a value as far as 10. Figure 18 reports the GIRFs subject to financial shocks. It is obvious that the persistence of the variable responses is reduced under looser priors, which is especially so when the shock hits in recessions. However, the differences in responses across the two regimes remain statistically significant, and our qualitative conclusions that financial shocks cause deeper and more protracted recessions and decrease in bank returns described in Figure 5 are robust.

As for TVAR-S, we report the GIRFs when τ is loosened to a value of 2 in Figure 19. The dynamics of the system in financially stressful regimes become unstable when we further loosen this prior. Upon comparison with Figure 10, two observations are in order when the economy is hit by growth shocks. First, corporate bond spreads display a hump-shaped response with an even higher peak within the first three quarters when the priors are loosened. This is an interesting result because it suggests that the non-linear relationship between financial stress and recessionary shocks are strong and not caused by tight priors. Second, the responses of short-term interest rate is deeper and more protracted in both regimes when the priors are loosened.

To sum up, our evidence of nonlinearities and shock amplification mechanisms in the UK are robust to different choices of priors.

7 Forecasting analysis

In this section, we investigate how well these models are able to forecast tail events. In particular, we perform *unconditional* and *conditional* forecasting. We are interested in the following two questions. First, do non-linear models produce more informative unconditional density forecasts in the tail? Second, could we produce more reliable predictive densities when we condition on variable paths consistent with stressful times? Both exercises involve re-estimating the three models using data until 2007:Q2 and produce multi-step ahead, pseudo-out-of-sample forecasts for 12 quarters. We compare the predictive densities against the periods including the Great Recession in 2008-09.

The rationale for performing an unconditional forecasting exercise is to investigate whether the non-linear models are able to generate forecast densities which properly characterize tail events without conditioning on any information. Figure 20 reports the fan charts of the output growth and the credit spreads for all three models.[12] The black lines are the median forecasts whereas the shaded areas are the error bands from 20^{th} to 80^{th} percentiles with 5% increments. The charts are overlapped with the realisations of both series which are given by the red lines. Visually speaking, none of these models are capable of capturing the drop in output growth during the 2008-09 Great Recession, a truly extreme event in the post-war sample although there may be some slight improvement in the coverage of tails for MSVAR model.

We then study conditional forecasting to investigate whether our models are better at capturing tail events if we *ex ante* feed into specific paths of variables under stress.[13] This enables the models to exploit the correlational dynamics between the conditioning variable and the other variables being modelled. We employ the conditional forecasting techniques proposed by Waggoner and Zha (1999). The algorithm first involves sampling the residuals of the VAR system which are consistent with the exogenously imposed scenario paths for *all* equations. Then it constructs forecast densities of other variables conditional on the path of corporate bond spreads between 2007:Q3 and 2010:Q2, assuming that the economy is under recessionary regimes throughout the period. Figure 21 reports the corresponding fan charts.

We draw the following conclusions. First, the linear BVAR model consistently falls short of producing reasonable analysis of macro tail events, as shown in the first column of Figure 21. Second, the TVAR-Y model in the second column shows the most significant improvement in the forecast densities compared to other models. Third, the MSVAR model in the third column

[12] To keep the charts concise, we only report TVAR-Y results for the forecasting exercises.

[13] Conditional forecasting is a common exercise among policy makers. For example, in the Inflation Reports produced by the Bank of England, the fan chart projections for GDP growth and CPI inflation are generated conditional on 'market interest rate expectations' and 'the stock of purchased assets financed by the issuance of central bank reserves'.

shows some improvement in the forecast density of the output growth, although the improvement is not as significant as in TVAR-Y.

These results highlight the importance of incorporating nonlinearities in forecasting macro-financial variables, especially in the conditional forecasting exercise. This appears to support Clements and Smith (2000) that non-linear models can potentially perform better than the linear counterparts in terms of the density forecast precision, as long as the data contain non-linear features.

8 Concluding remarks

In this paper, we estimate a set of non-linear Bayesian VARs to study macroeconomic tail events. We utilise regime switching models to estimate the regimes governed by different time periods. Our estimated regimes are associated with recessionary/non-recessionary and financially stressful/stable periods. We obtain substantial evidence that financial shocks during recessionary periods cause disproportionately more severe contractions in the real sector. We also demonstrate the existence of a powerful feedback loop between the real and financial sectors as a result of tail shocks hitting the economy in non-recessionary/stable time periods. These findings serve as empirical support to the theoretical predictions in Brunnermeier and Sannikov (2014). Moreover, we check each model's out-of-sample forecasting power, and find that conditional predictive densities produced by TVARs hold a potential to explore downside events.

Future work involves extending our results to the literature of forecasting macroeconomic tail risks, as in Boucher and Maillet (2015) and De Nicolo and Lucchetta (2016), where the probabilities of the occurence of tail events are also under investigation. Our generalised impulse responses provide the densities of 'variables' at each forecast horizon conditional on a structural shock hitting at a particular economic or financial regime. With additional information on the densities of the 'regimes', conditional distributions of variables can be further transformed to marginal distributions to study tail risks. We leave this for future research.

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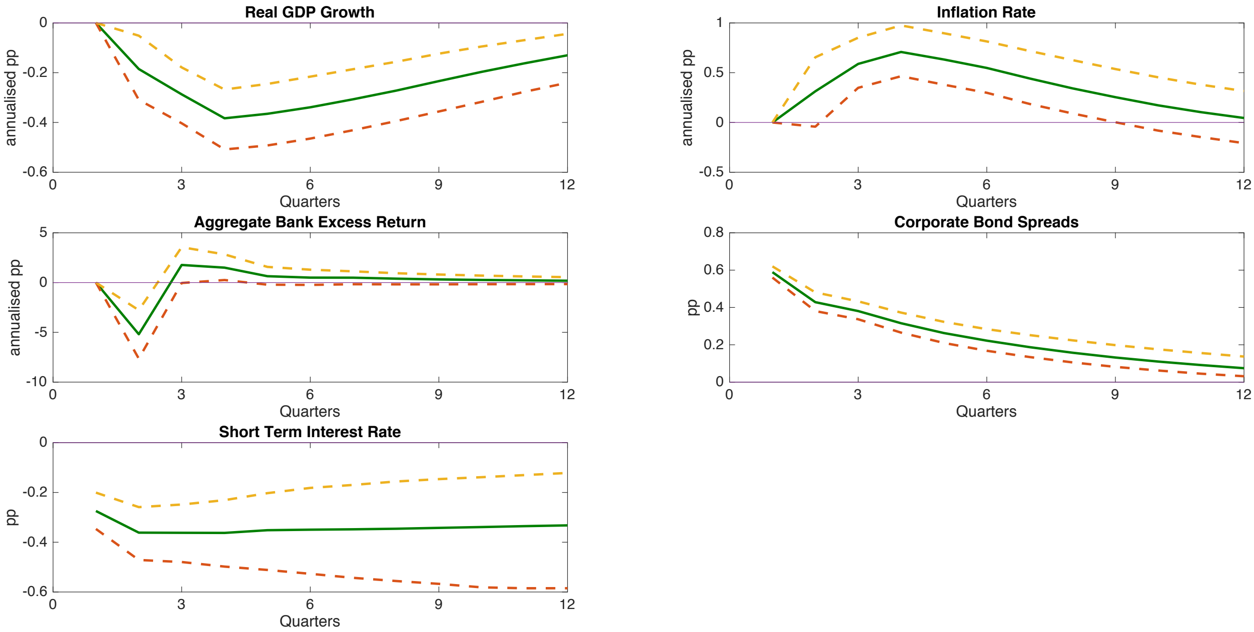
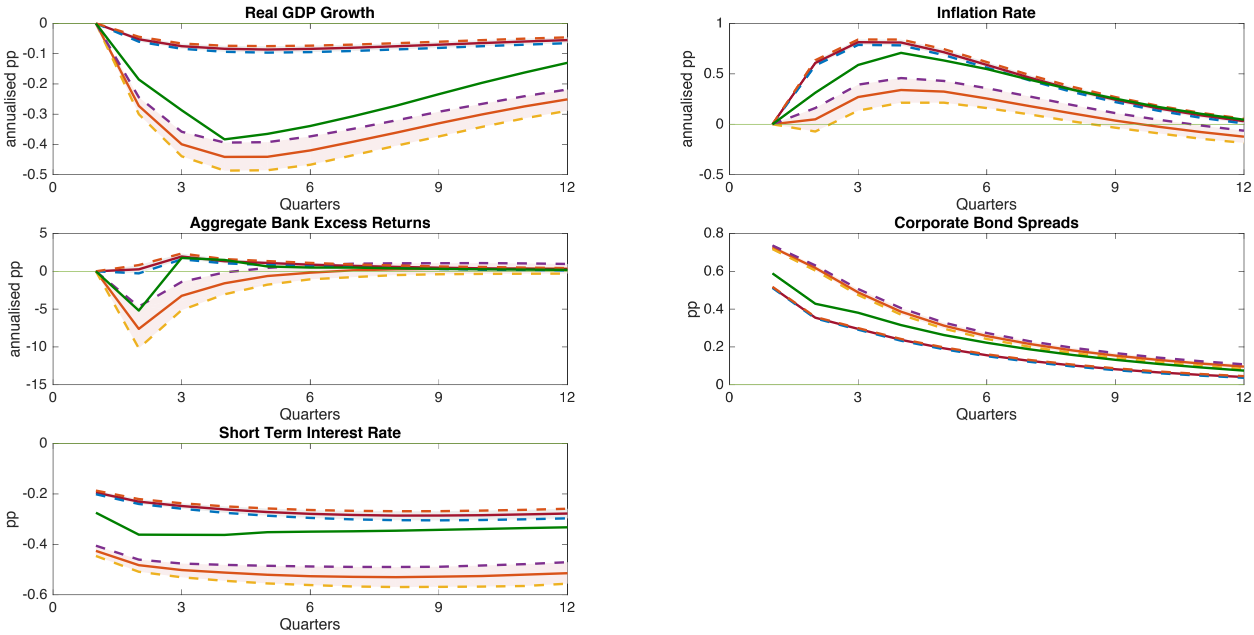
Figure 4: Impulse Responses to a 1 SD adverse shock to corporate bond spreads in BVAR model



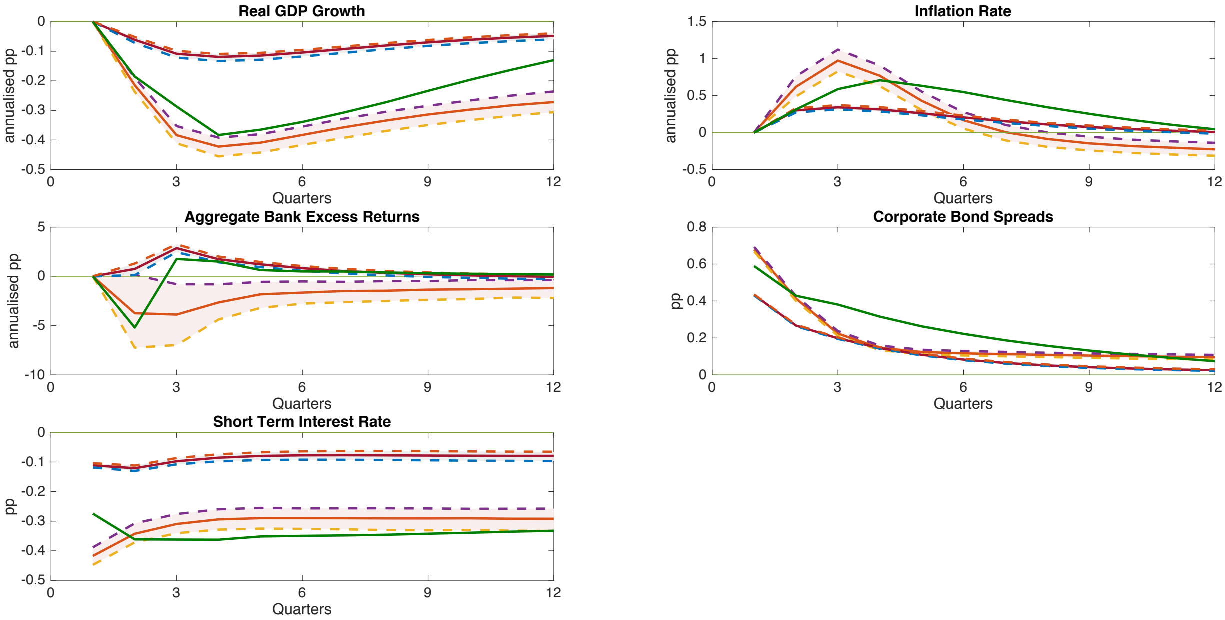
Notes: The error bands correspond to the 68% error bands.

Figure 5: Generalised Impulse Responses to a 1 SD adverse shock to corporate bond spreads in TVAR-Y model



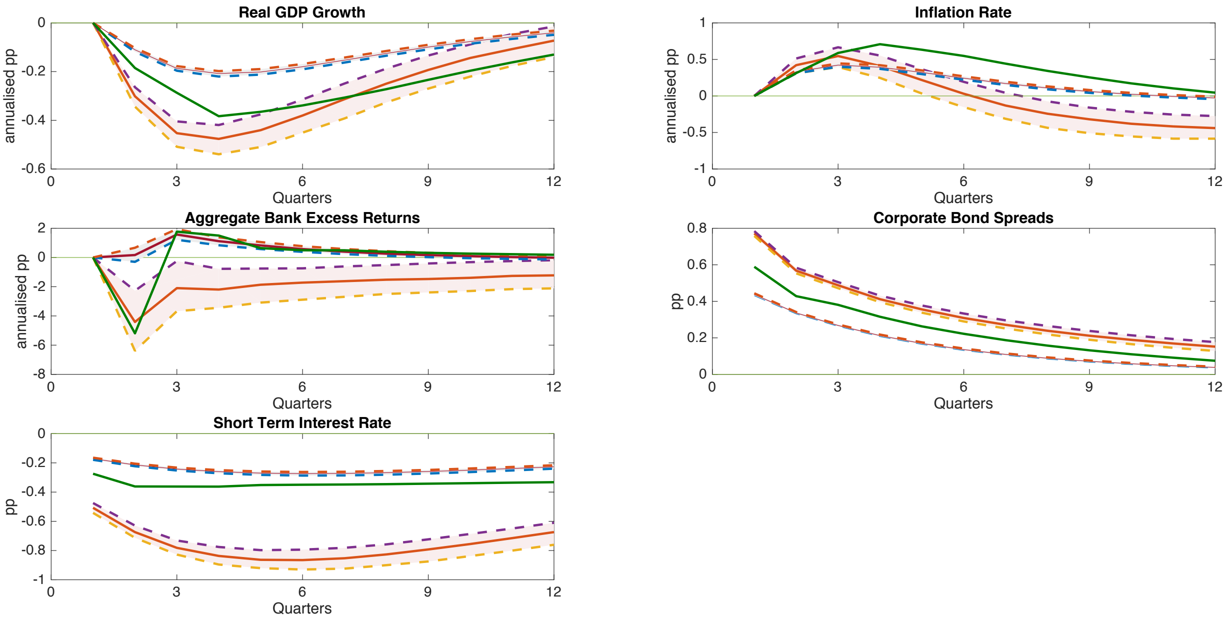
Notes: The error bands correspond to the 68% error bands. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes. The green line is the median response of the BVAR model to the same shock.

Figure 6: Generalised Impulse Responses to a 1 SD adverse shock to corporate bond spreads in TVAR-S model



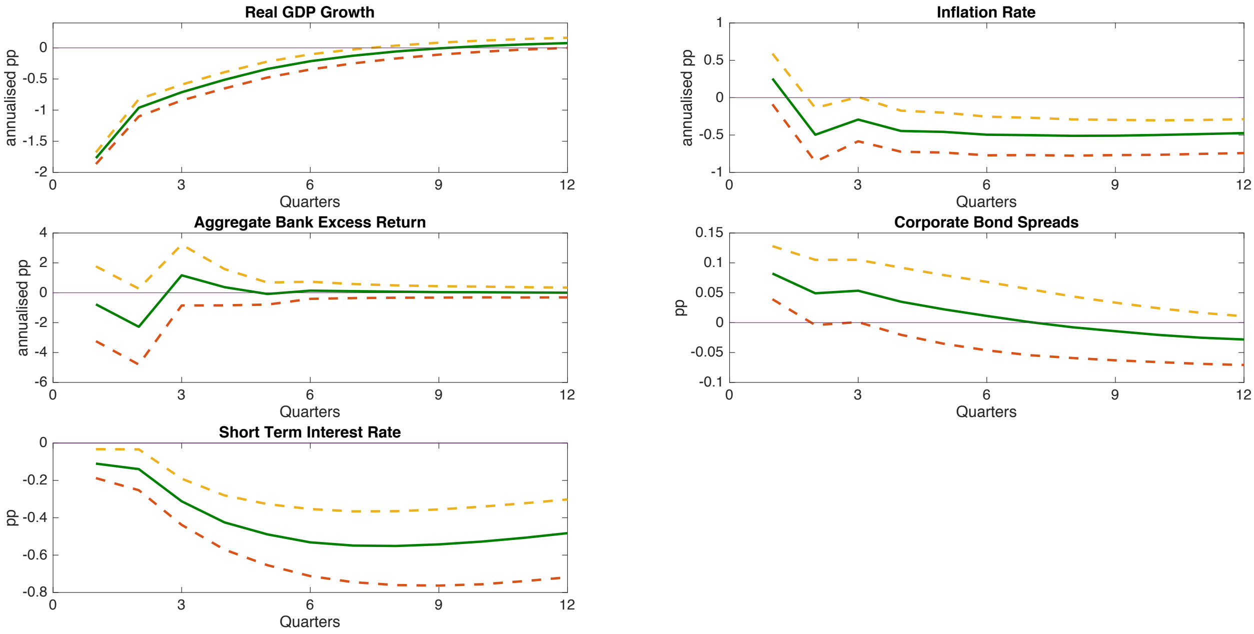
Notes: The error bands correspond to the 68% error bands. The red shaded area corresponds to the error bands of financial stress regimes. The grey shades correspond to those of financially non-stressful regimes. The green line is the median response of the BVAR model to the same shock.

Figure 7: Generalised Impulse Responses to a 1 SD adverse shock to corporate bond spreads in MSVAR model



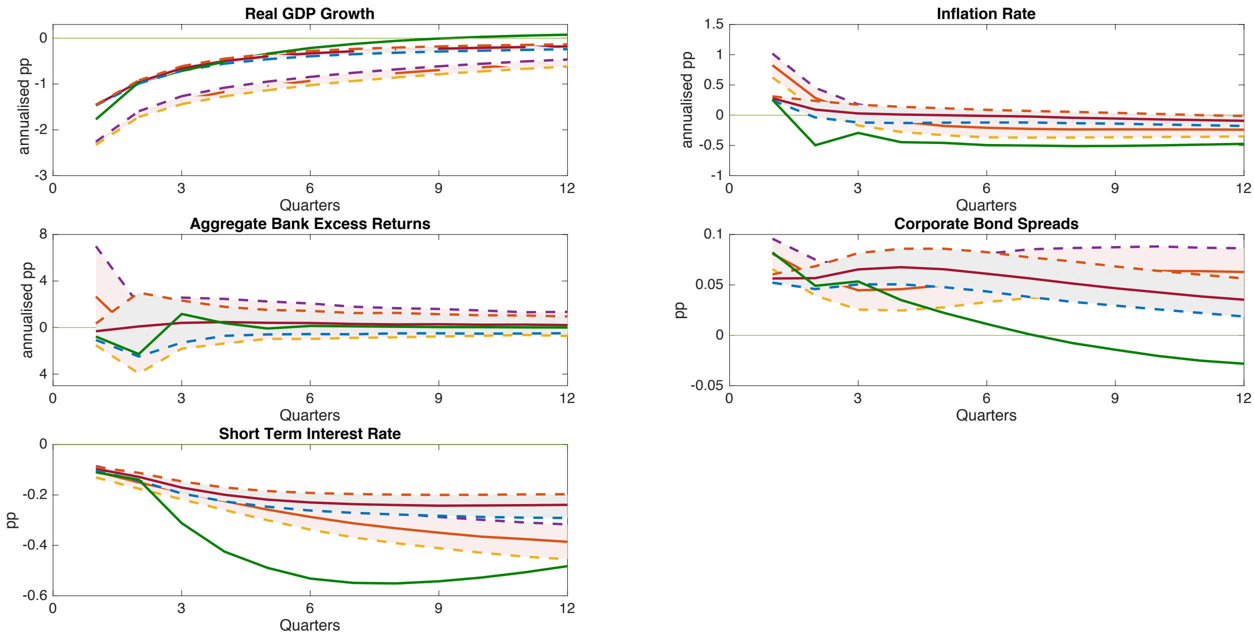
Notes: The error bands correspond to the 68% error bands. The red shaded area corresponds to the error bands of high stress regimes. The grey shades correspond to those of low stress regimes. The green line is the median response of the BVAR model to the same shock.

Figure 8: Impulse Responses to a 1 SD adverse shock to real GDP growth in BVAR model



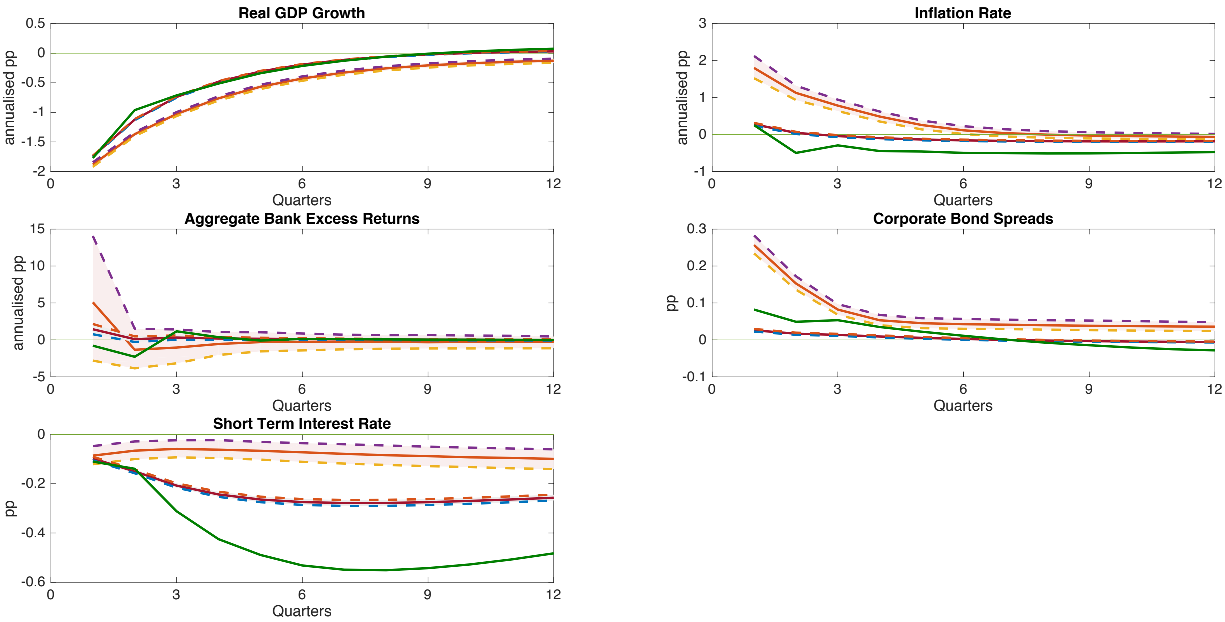
Notes: The error bands correspond to the 68% error bands.

Figure 9: Generalised Impulse Responses to a 1 SD adverse shock to real GDP growth in TVAR-Y model



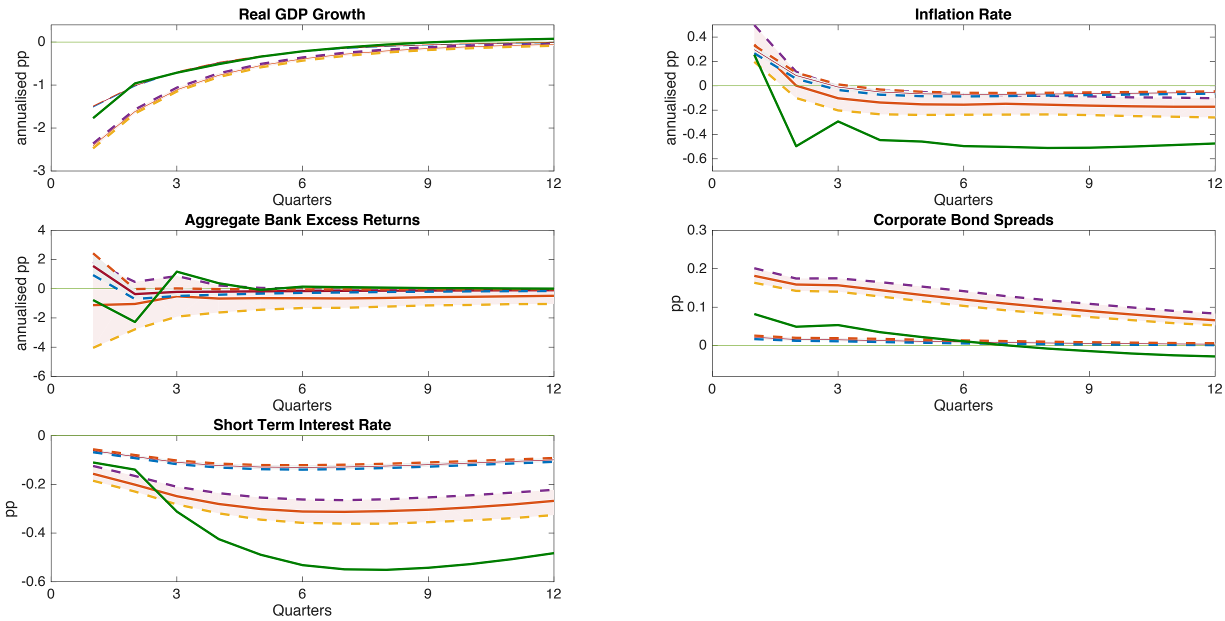
Notes: The error bands correspond to the 68% error bands. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes. The green line is the median response of the BVAR model to the same shock.

Figure 10: Generalised Impulse Responses to a 1 SD adverse shock to real GDP growth in TVAR-S model



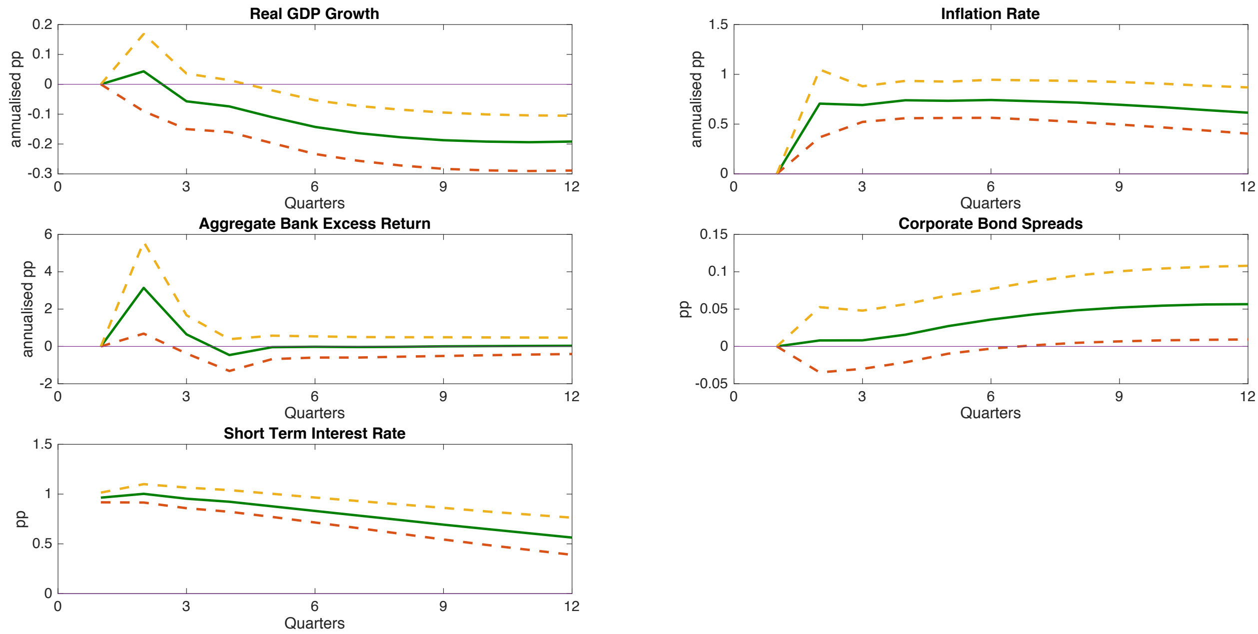
Notes: The error bands correspond to the 68% error bands. The red shaded area corresponds to the error bands of financial stress regimes. The grey shades correspond to those of financially non-stressful regimes. The green line is the median response of the BVAR model to the same shock.

Figure 11: Generalised Impulse Responses to a 1 SD adverse shock to real GDP growth in MSVAR model



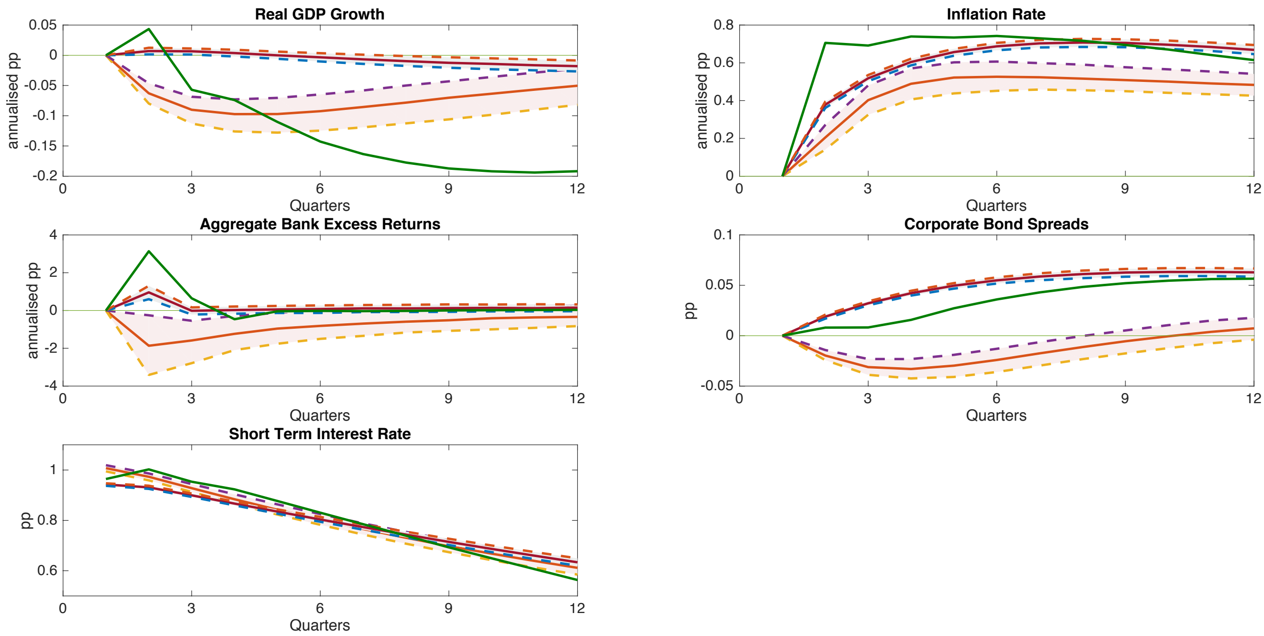
Notes: The error bands correspond to the 68% error bands. The red shaded area corresponds to the error bands of high stress regimes. The grey shades correspond to those of low stress regimes. The green line is the median response of the BVAR model to the same shock.

Figure 12: Impulse Responses to a 1 SD adverse shock to short term interest rate in BVAR model



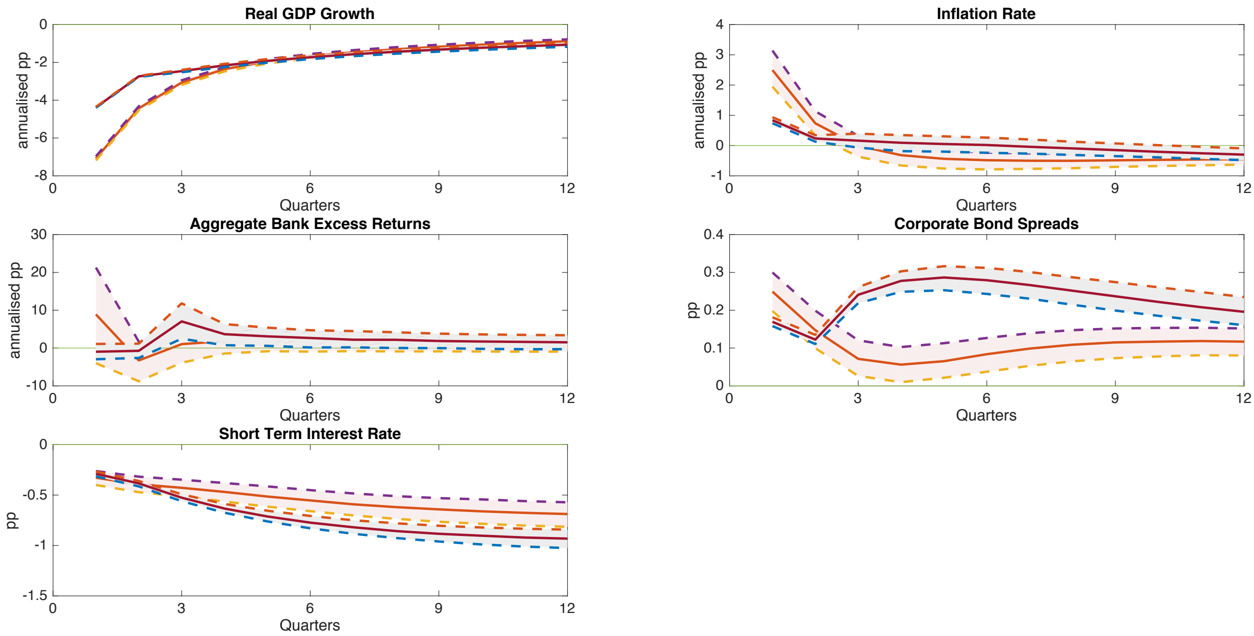
Notes: The error bands correspond to the 68% error bands.

Figure 13: Generalised Impulse Responses to a 1 SD adverse shock to short term interest rate in TVAR-Y model



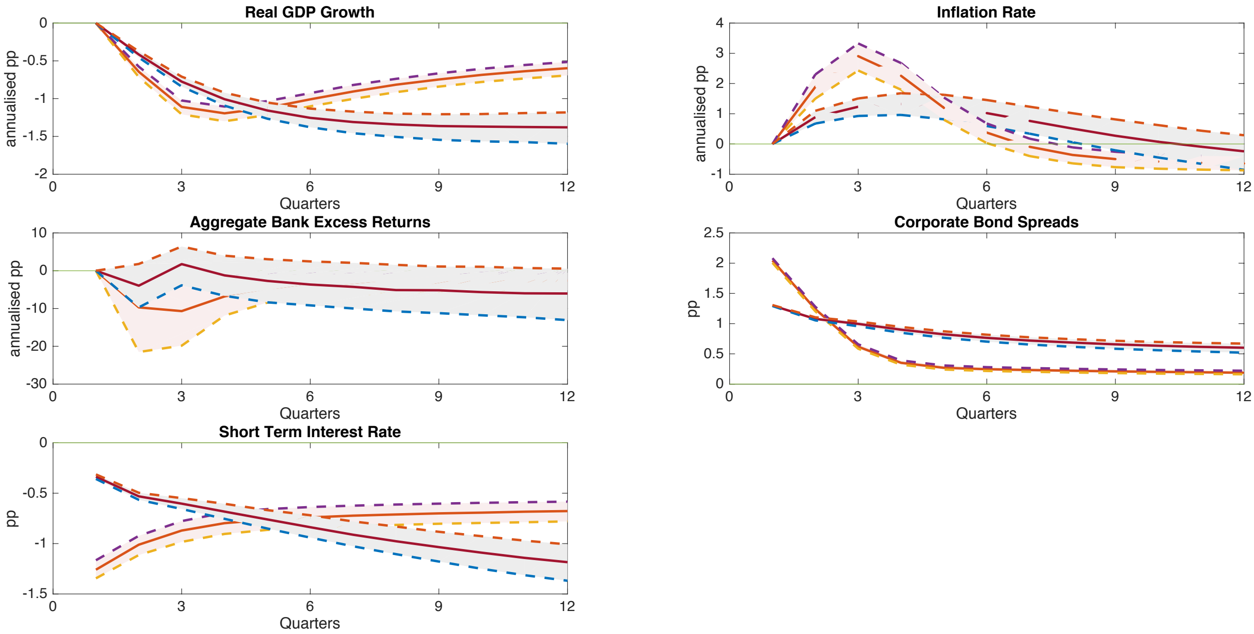
Notes: The error bands correspond to the 68% error bands. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes. The green line is the median response of the BVAR model to the same shock.

Figure 14: Generalised Impulse Responses to a **3 SD adverse shock to real GDP growth** in TVAR-Y model (to be compared with Figure 9)



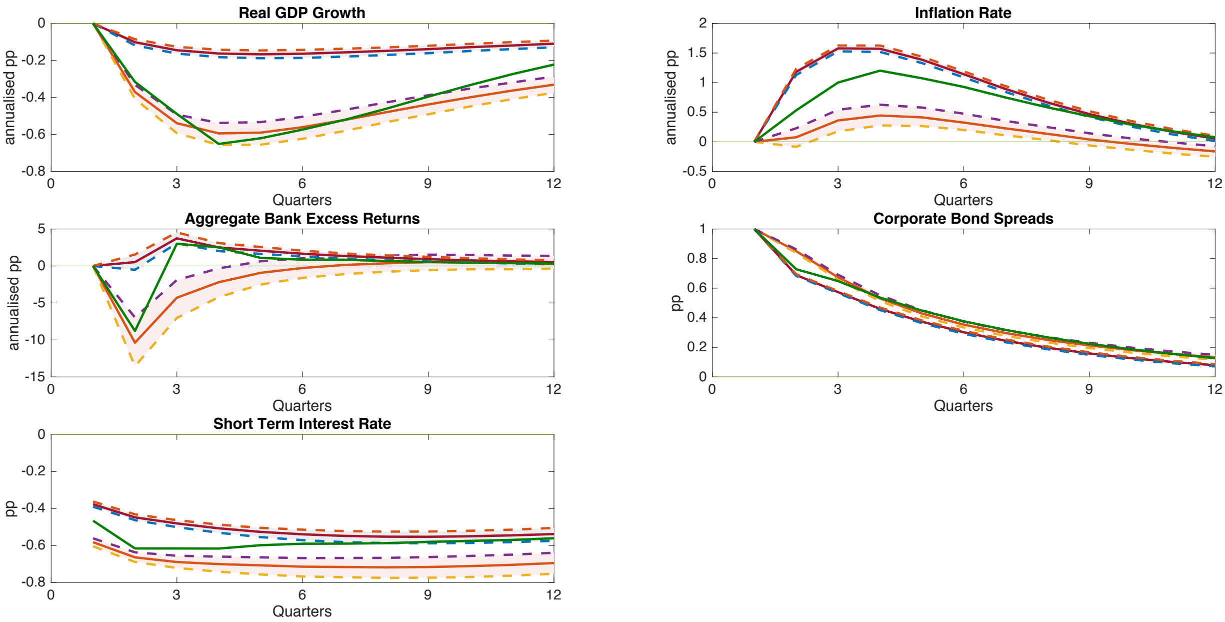
Notes: The error bands correspond to the 68% confidence intervals. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes.

Figure 15: Generalised Impulse Responses to a **3 SD adverse shock to corporate bond spreads** in TVAR-S model (to be compared with Figure 6)



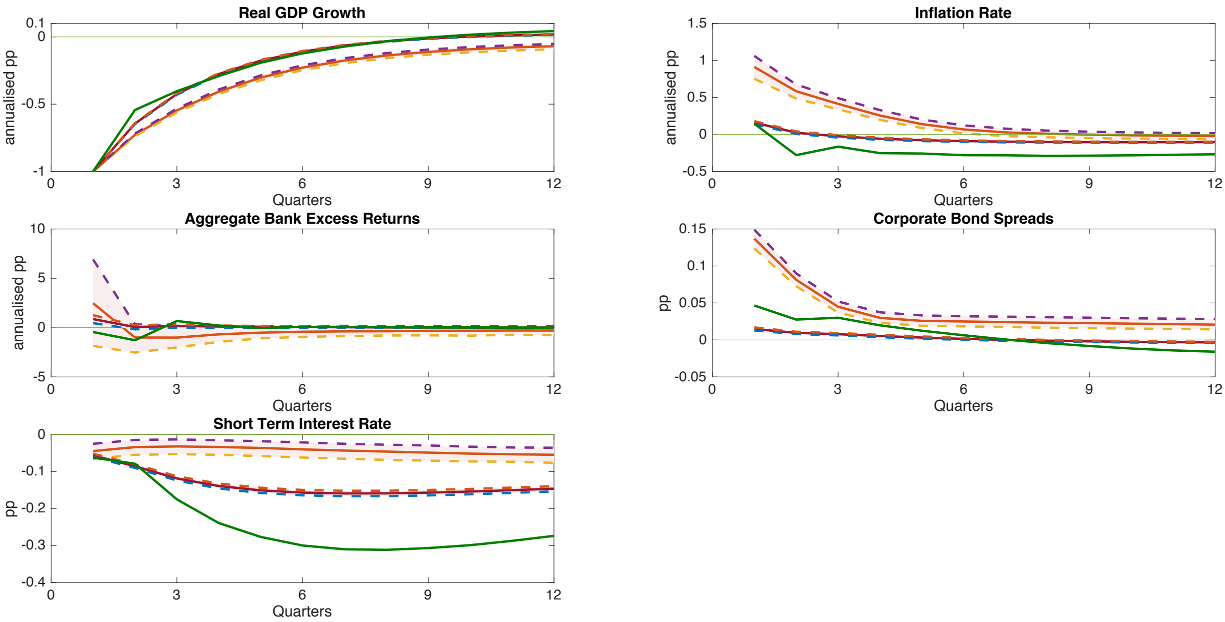
Notes: The error bands correspond to the 68% confidence intervals. The red shaded area corresponds to the error bands of financial stress regimes. The grey shades correspond to those of financially non-stressful regimes.

Figure 16: Generalised Impulse Responses to a 100bp adverse corporate bond spread shocks in TVAR-Y model (to be compared with Figure 5)



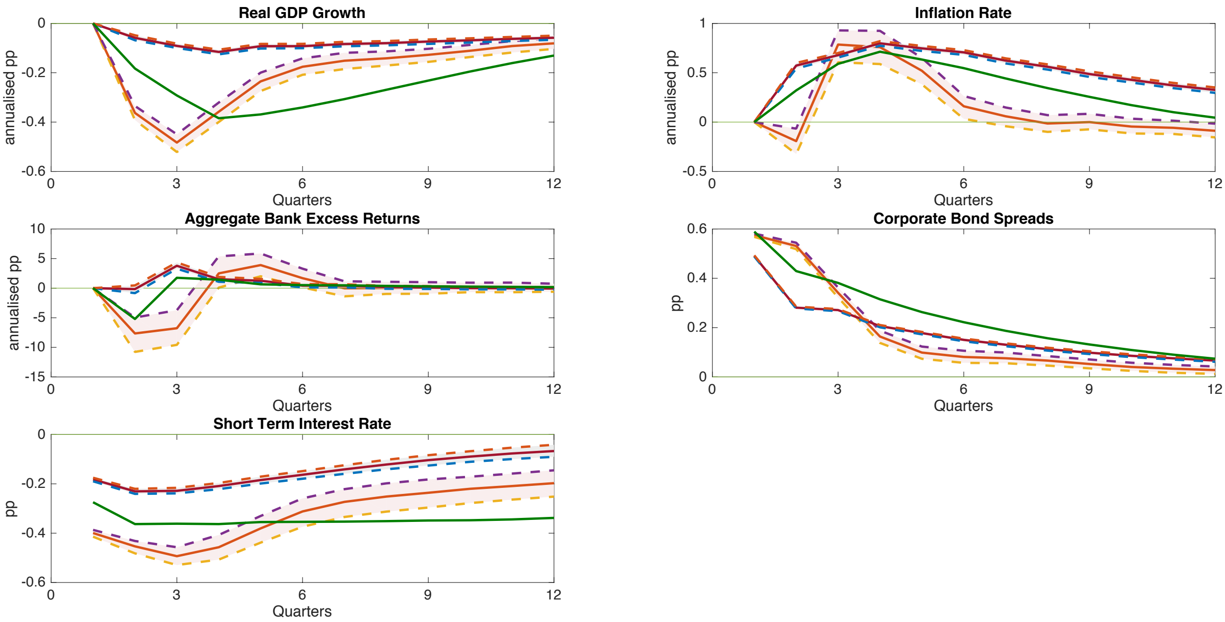
Notes: The error bands correspond to the 68% confidence intervals. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes.The green line is the median response of the BVAR model to the same shock.

Figure 17: Generalised Impulse Responses to a 100bp adverse shock to real GDP growth in TVAR-S model (to be compared with Figure 10)



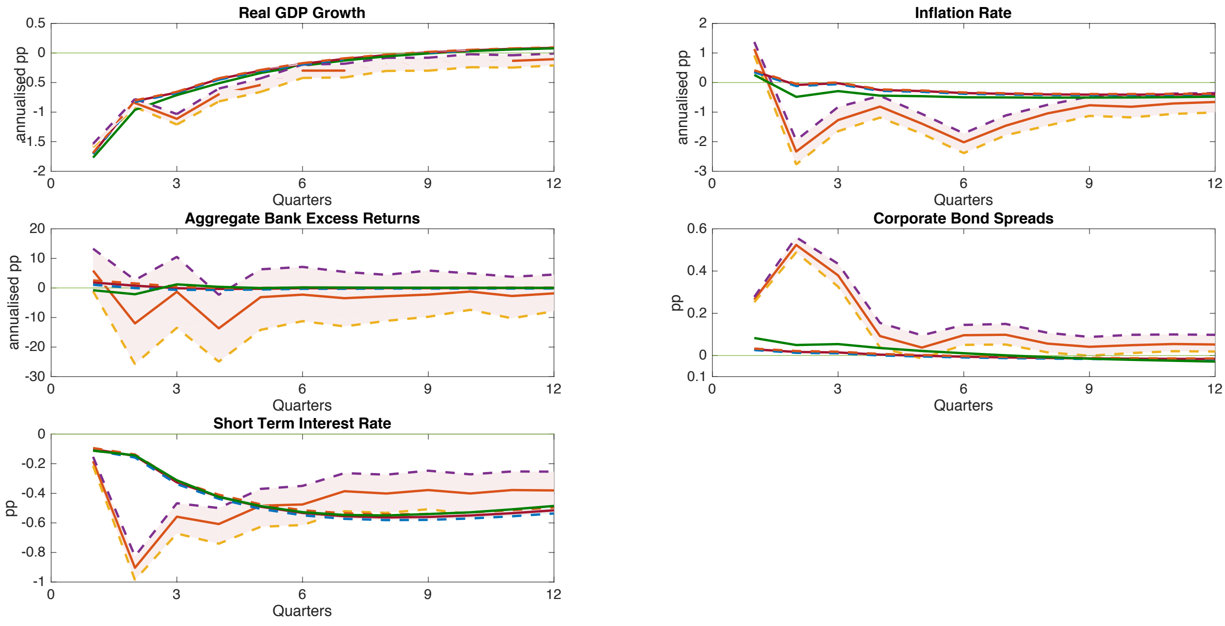
Notes: The error bands correspond to the 68% confidence intervals. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes. The green line is the median response of the BVAR model to the same shock.

Figure 18: Generalised Impulse Responses to a 1 SD adverse corporate bond spread shocks in TVAR-Y model with loose priors, $\tau = 10$, (to be compared with Figure 5)



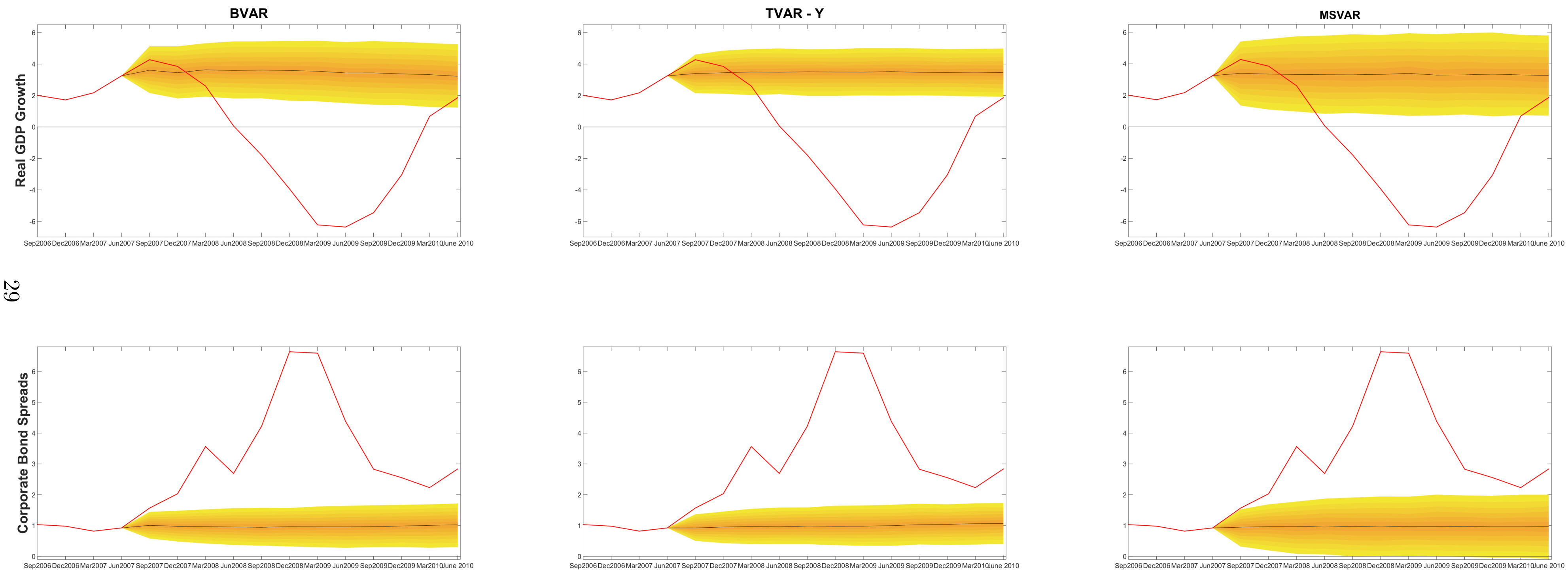
Notes: The error bands correspond to the 68% confidence intervals. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes. The green line is the median response of the BVAR model with the same priors to the same shock.

Figure 19: Generalised Impulse Responses to a 1 SD adverse shock to real GDP growth in TVAR-S model with loose priors, $\tau = 2$, (to be compared with Figure 10)



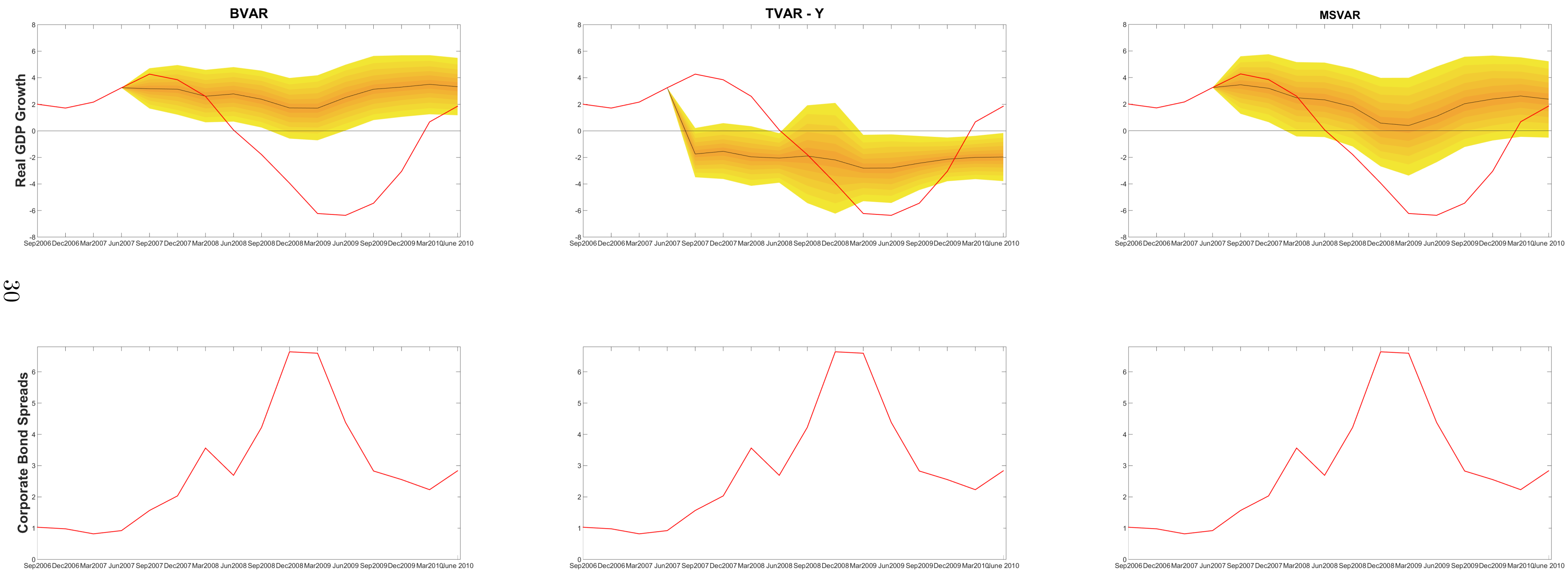
Notes: The error bands correspond to the 68% confidence intervals. The red shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of non-recessionary regimes. The green line is the median response of the BVAR model with the same priors to the same shock.

Figure 20: Unconditional predictive densities for the three models



Notes: Unconditional forecast densities of BVAR, TVAR-Y, and MSVAR models are respectively shown in the the first, second and third columns. Actual outturns are indicated by the red lines. Fan charts indicate forecast bands between 20^{th} and 80^{th} percentile with 5% increments. The densities correspond to the 12-horizon pseudo-out-of-sample forecasts between 2007:Q3 and 2010:Q2, generated based on data between 1965:Q2 and 2007:Q2.

Figure 21: Conditional predictive densities based on the path of Corporate Bond Spreads



Notes: Conditional forecast densities of BVAR, TVAR-Y, and MSVAR models are respectively shown in the the first, second and third columns. Actual outturns are indicated by the red lines. Fan charts indicate forecast bands between 20^{th} and 80^{th} percentile with 5% increments. The densities correspond to the 12-horizon pseudo-out-of-sample forecasts between 2007:Q3 and 2010:Q2, generated based on data between 1965:Q2 and 2007:Q2.

Appendix

A Data construction

We use *Global Financial Data* to construct the following variables: real GDP growth rate (mnemonic: GDPCGBR), inflation rate (mnemonic: CPGBRCM) and the short-term interest rate (mnemonic: ITGBR3D). To construct the aggregate credit bond spread series, we take the difference between UK corporate bond yield (mnemonic: INGBRW) and UK 10 year government bond yield (mnemonic: IGGBR10D).

Data for bank excess returns, UK equity index (mnemonic: TOTMKUK) and UK banks equity index (mnemonic: BANKSUK), are taken from *DataStream*.

The tables in this section give the descriptive statistics and the correlation matrix of these variables. The charts for these variables for the whole data span of 1965:Q2 to 2014:Q2 are also given below. The dashed vertical lines indicate the observation in 2007:Q2 which is the quarter when we separate the data for *conditional* forecasting purposes.

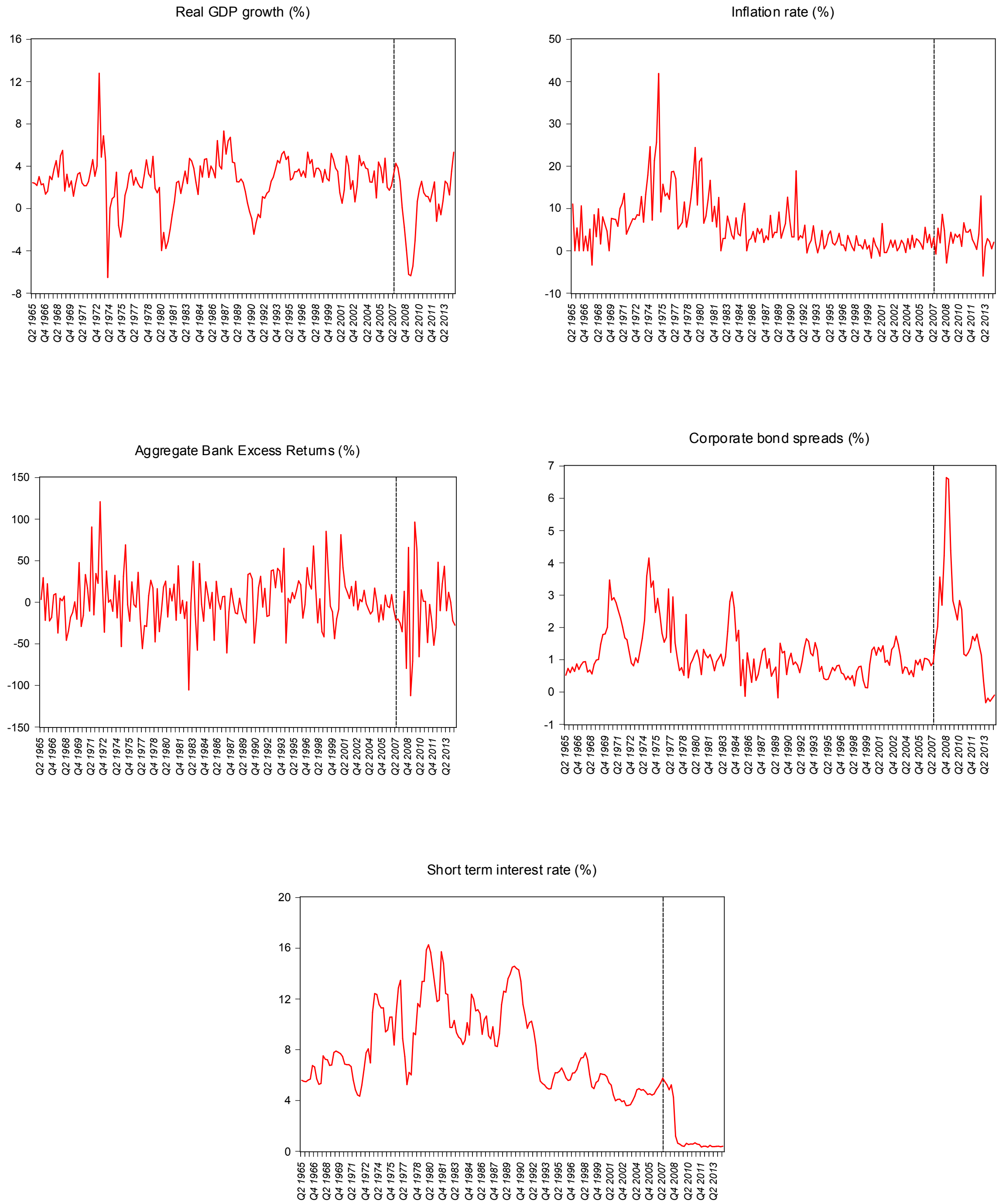
Table 1: Summary statistics

	Real GDP Growth	Inflation Rate	Agg. Bank Excess Returns	Corporate Bond Spreads	Short Term Interest Rate
Mean	2.39	5.55	1.26	1.30	7.19
Median	2.60	3.88	0.43	1.03	6.56
Maximum	12.78	41.90	120.85	6.64	16.27
Minimum	-6.52	-5.96	-112.36	-0.33	0.32
Std Deviation	2.49	6.11	33.34	1.02	3.88
Skewness	-0.82	2.06	0.11	2.07	0.15
Kurtosis	6.23	9.78	4.53	9.67	2.54
Observations	197	197	197	197	197

Table 2: Contemporaneous correlation coefficients of variables

	Real GDP Growth	Inflation Rate	Agg. Bank Excess Returns	Corporate Bond Spreads	Short Term Interest Rate
Real GDP Growth	1				
Inflation Rate	-0.2070	1			
Agg. Bank Excess Returns	0.0053	0.0223	1		
Corporate Bond Spreads	-0.4356	0.2302	-0.0151	1	
Short Term Interest Rate	0.0002	0.4850	0.0001	-0.1141	1

Figure A.1: Variables



B The Gibbs Sampling algorithm for TVAR

The following describes the Gibbs sampler for TVAR:

1. Given a value for the threshold variable, observations are separated into two regimes.

2. Given the observations in each regime, sample the VAR coefficients and covariances:

   - Sampling the VAR coefficients
     Given the regimes and the covariances, we sample $c_R, B_{1,R}, B_{2,R}, \ldots, \beta_{L,R}$.
     As discussed in Appendix D, the conditional posterior of the VAR coefficients is

     $$vec(\beta) \mid \Omega_R, Y \sim N(vec(\tilde{\beta}), \Omega_R \otimes (X^{*'} X^{*})^{-1}).$$

   - Sampling the covariances
     Given the regimes, we sample the covariance matrices from inverse Wishart distribution which will be discussed in Appendix D,

     $$\Omega_R \mid \beta \sim iW(\bar{H}_R, \varphi_R)$$

     where $R = 1, 2$. $\bar{H}_R$ refers to the covariance matrix computed by the VAR residuals in regime R, and the parameter φ_R refers to the number of the observations in each regime.

3. Given values for coefficients and covariances, sample the threshold value. Since the posterior distribution of the threshold value is not analytically tractable, we perform a Metropolis Hastings step, along with the Gibbs sampler:

   $$Z^{*}_{new} = Z^{*}_{old} + \Psi^{1/2}\varepsilon$$

   where $\Psi^{1/2}$ is abscaling factor and ε is distributed as $N(0, 1)$. The scaling factor is chosen to ensure that the acceptance rate is in 20–40% interval.

4. Conditional on the threshold value, we sample the delay parameter d. Chen and Lee (1995) showed that the conditional posterior density of this parameter is multinomial distribution with probability $\frac{L(Y_t)}{\sum^d L(Y_t)}$ where $L(.)$ is the likelihood function. We skip this step when fixing d to our desired values based on our definitions of recessionary regimes in TVAR-Y.

5. Run 100,000 draws and discard the first 60,000 to ensure convergence.

C The Gibbs Sampling algorithm for MSVAR

The following describes the Gibbs sampling procedure:

1. Sampling the regimes, S_t:

   Given values of VAR parameters and the covariances, we use multi–move Gibbs sampling proposed by Kim and Nelson (1999b). This method, conditional on data and the parameters, predicts the unobserved regime and then updates it by running a simulation smoother in order to obtain a draw from the joint posterior densities, namely $f(S_t|Y_t, c_S, B_{1,S}, \dots, B_{L,S}, P)$.

   - We first calculate $f(S_T|Y_T)$. Hamilton (1989) provides a filter to evaluate $f(S_t|Y_t)$ for $t = 1, 2, \dots, T$.
   - We then calculate $f(S_t|S_{t+1}, Y_t)$. Kim and Nelson (1999b) show that

     $$f(S_t|S_{t+1}, Y_t) \propto f(S_t|S_{t+1})f(S_t|Y_t) \tag{3}$$

     where the Hamilton filter provides $f(S_t|Y_t)$ and $f(S_t|S_{t+1})$.

2. Sampling the VAR coefficients

   Given the regimes and the covariances, we sample $c_S, B_{1,S}, B_{2,S}, \dots, B_{L,S}$.

   The conditional posterior of the VAR coefficients, as discussed in Appendix D, is

   $$vec(B) \mid \Omega_s, Y \sim N(vec(\tilde{B}), \Omega_S \otimes (X^{*'} X^*)^{-1}).$$

3. Sampling the covariances

   Given the regimes, we sample the covariance matrices from inverse Wishart distribution which will be discussed in Appendix D,

   $$\Omega_S \mid B \sim iW(\bar{H}_S, \varphi_S)$$

   where $S = 1, 2$. $\bar{H}_S$ refers to the covariance matrix computed by the VAR residuals in regime S, and the parameter φ_S refers to the number of the observations in each regime.

4. Sampling the transition probabilities, P:

   As the last step we sample the transition probabilities. We impose Dirichlet priors for the non–zero elements of the transition matrix, p_{ij}:

   $$p^0_{ij} = D(u_{ij})$$

where $D(.)$ represents the Dirichlet distribution. The posterior distributions of the transition probabilities are

$$p_{ij} = D(u_{ij} + \eta_{ij})$$

where η_{ij} denotes the number of times regime i is followed by regime j. The value of u_{ij} equals 20, which implies a prior belief that probability of staying in the same regime to be is 0.85.

5. Employ 50,000 iterations for the Gibbs sampling. We discard the first 10,000 draws as burn-in to ensure convergence.

D Normal Inverse Wishart priors

We impose normal inverse Wishart (natural conjugate) priors through dummy observations following Bańbura et al. (2010) for *each regime* of the regime-switching as well as for the linear VAR model. Consider artificial data denoted Y_D and X_D such that

$$b_0 = \left(X_D' X_D\right)^{-1} \left(X_D' Y_D\right)$$

$$S = (Y_D - X_D b_0)' (Y_D - X_D b_0)$$

A regression of Y_D on X_D will give b_0, the prior means for the VAR coefficients, and the sum of squared residuals give S, prior scale matrix of the error covariance matrix. The prior is of normal inverse Wishart form

$$p\left(\widetilde{B}|\widetilde{\Sigma}\right) \sim N\left(\widetilde{b}_0, \widetilde{\Sigma} \otimes \left(X_D' X_D\right)^{-1}\right)$$

$$p\left(\widetilde{\Sigma}\right) \sim IW\left(S, T_D - K\right)$$

where $\widetilde{b}_0 = vec\,(b_0)$, T_D is the length of the artificial data and K denotes the number of regressors in each equation. Given this artificial data, and denoting $Y^* = [Y; Y_D], X^* = [X; X_D]$, where Y and X respectively denote the regressands and regressors from the data, the conditional posterior distribution for the VAR parameters are given by

$$H\left(b|\Sigma, Y_t\right) \sim N\left(vec\left(B^*\right), \Sigma \otimes \left(X^{*\prime} X^*\right)^{-1}\right) \quad (4)$$

$$H\left(\Sigma|b, Y_t\right) \sim IW\left(S^*, T^*\right) \quad (5)$$

where $B^* = \left(X^{*\prime}X^*\right)^{-1}\left(X^{*\prime}Y^*\right)$, $S^* = (Y^* - X^*B^*)'(Y^* - X^*B^*)$ and T^* refers to the length of Y^*.

To implement the normal inverse Wishart priors, artificial data are created as follows:

$$Y_D = \begin{pmatrix} \frac{diag(\chi_1\sigma_1...\chi_N\sigma_N)}{\tau} \\ 0_{N\times(P-1)\times N} \\ .... \\ diag\left(\sigma_1...\sigma_N\right) \\ .... \\ 0_{1\times N} \end{pmatrix},\ X_D = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1...\sigma_N)}{\tau} & 0_{NP\times 1} \\ 0_{N\times NP} & 0_{N\times 1} \\ ..... & ...... \\ 0_{1\times NP} & c \end{pmatrix}$$

where N represents the number of variables in the VAR, P represents the number of lags, χ_i are the prior means of the coefficients of the first lag of the dependent variables and $J_P = diag(1, ..., P)$

The hyperparameters are listed as follows:

- τ controls the overall tightness of the prior
- c controls the tightness of the prior on constants
- σ_i are standard deviation of error terms from OLS estimates of AR regression for each variable

We choose our hyperparameters as $\tau = 0.1$ and $c = 10^5$ which are broadly similar to Canova (2007) and Blake and Mumtaz (2012).

E Generalized Impulse Response Functions

We compute the non-linear impulse response functions of MSVAR and TVAR models by following Koop et al. (1996), Baum and Koester (2011) and Afonso et al. (2011).

E.1 GIRFs for TVAR

The following steps are separately employed for each regime for both TVAR-Y and TVAR-S models .

1. Run the estimation and save all parameter draws.

2. Given a Gibbs draw, pick a random history from the set recessionary/financially stressful observations.

3. Draw random shocks and form a set of unconditional forecasts which are denoted as y_{t+k}^{Th} where Th indicates the TVAR model and k is forecast horizon. The output is a $(horizon \times N)$ matrix of forecasts for all N variables and these forecasts serve as a baseline.

4. Form another set of forecasts with the same random shocks except that a specific shock is perturbed at horizon 0. Refer these forecasts as $y_{t+k}^{Th,p}$. The output is a $(horizon \times N \times 1)$ matrix *for a given shock*. If one is interested in shocking all the variables, the resulting matrix is size of $(horizon \times N \times N)$.

5. Repeat steps 3 to 5 for $Simm = 500$.

6. Take the means of the forecasts over $Simm$ and calculate the difference between the means such that $\frac{1}{Simm}\sum_{Simm} y_{t+k}^{Th,p} - \frac{1}{Simm}\sum_{Simm} y_{t+k}^{Th}$.

7. Repeat steps 3 to 7 for all Gibbs draws and all histories. The result of this step is the time varying impulse response functions.

8. Take the mean of the resulting impulse response functions from all Gibbs draws. The output is the ultimate GIRFs of recessionary regime in TVAR model.

9. Repeat steps 3 to 9 for the non-recessionary/financially stressful regimes.

E.2 GIRFs for MSVAR

1. Run the estimation and save all parameter draws.

2. Given a Gibbs draw and at time t, project the ergodic probabilities and draw random shocks to form a set of unconditional forecasts y_{t+k}^{M} where k is the forecast horizon and superscript M marks the MSVAR model. The output is a $(horizon \times N)$ matrix of forecasts for all N variables and these forecasts serve as a baseline.

3. Form another set of forecasts with the same random shocks except with perturbed shocks at horizon 0. Refer these forecasts as $y_{t+k}^{M,p}$ where the additional superscript addresses the perturbed shocks. The output is a $(horizon \times N \times 1)$ matrix *for a given shock.* If one is interested in shocking all the variables, the resulting matrix is size of $(horizon \times N \times N)$.

4. Repeat both steps 2 and 3 for $Simm = 500$.

5. Take the mean of the resulting forecasts over Simm and the difference between the means such that $\frac{1}{Simm}\sum_{Simm} y_{t+k}^{M,p} - \frac{1}{Simm}\sum_{Simm} y_{t+k}^{M}$. This difference is for a given Gibbs draw and given time period.

6. Repeat steps 2 to 5 for all Gibbs draws and for all $t = 1, 2, 3, ..., T$.

7. Take the mean of the time varying impulse response functions from the previous step over the high stress regimes as identified by the model. The output of this step gives the GIRFs of MSVAR model.