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Overseas unspanned factors and domestic bond returns

Andrew Meldrum,[(1)] Marek Raczko[(2)] and Peter Spencer[(3)]

Abstract

Using data on government bond yields in Germany and the United States, we show that overseas unspanned factors — constructed from the components of overseas yields that are uncorrelated with domestic yields — have significant explanatory power for subsequent domestic bond returns. This result is remarkably robust, holding for different sample periods, as well as out of sample. Shocks to overseas unspanned factors have large and persistent effects on domestic yield curves. Dynamic term structure models that omit information about foreign bond yields are therefore likely to be misspecified.

Key words: Return-forecasting regressions, dynamic term structure models.

JEL classification: E43, G12.

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1 Introduction

Using data on government bond yields in Germany and the USA, this paper shows that a factor extracted from the part of overseas yields that is orthogonal to domestic yields can explain a substantial part of subsequent domestic bond returns. Moreover, this 'overseas unspanned factor' has significant *additional* predictive power for domestic bond returns relative to the information contained in the domestic yield curve. The result is remarkably robust, holding for different sample periods as well as out-of-sample.

A large number of studies have demonstrated that most of the variation in government bond yields at different maturities within a single country can be explained by the first three principal components of domestic yields (typically labelled as level, slope and curvature - e.g. Litterman and Scheinkman (1991)). Models of the term structure that specify bond yields as linear functions of three or more principal components are therefore likely to achieve a high in-sample fit to the cross section of yields. That does not, however, imply that three domestic principal components are sufficient for modelling the *time-series* behaviour of yields. Previous studies have shown that other variables, unspanned by level, slope and curvature, have significant explanatory power for US excess returns. These include other factors extracted from domestic bond yields (Cochrane and Piazessi (2005) and Duffee (2011b)) and macroeconomic variables (Joslin et al. (2014)). This paper extends this emerging literature on unspanned factors in the term structure by demonstrating that an 'overseas unspanned factor' extracted from *overseas* yields but unspanned by domestic yields is an important predictor of future domestic returns.

We use a simple two-stage regression-based method to construct our overseas unspanned factors. We first regress bond yields from the 'foreign' country on a cross-section of yields from the 'domestic' country, thereby obtaining the components of foreign yields that are orthogonal to domestic yields. We then construct our overseas unspanned factor as a linear combination of these orthogonal components at different maturities, with the weights chosen to maximise fit to excess bond returns averaged across maturities.

To assess the information content of this factor, we include it in two sets of empirical exercises: (i) return-forecasting regressions, both in- and out-of-sample; and (ii) dynamic factor models of bond yields. We highlight the following results from these exercises. First, in return-forecasting regressions with a twelve-month holding period, the overseas unspanned factor has a statistically significant coefficient for all maturity returns; and excluding it results in substantially worse in-sample fit. Second, these results are remarkably robust and do not appear to be a result of in-sample over-fitting: they hold for alternative samples, out-of-sample and if we extend the analysis to consider returns on UK bonds. Third, in the dynamic factor model for German yields, a one standard deviation shock to our overseas unspanned factor is followed by a decline in yields of up to 70 basis points; in the model of US yields, the largest reaction is somewhat smaller but still reasonably substantial, at around 40 basis points. And fourth, shocks to the overseas unspanned factors also account for a substantial portion of the unexpected variation in long-term bond yields - for example, they account for around 40-50% of forecast error variance of German yields over a five-year forecast horizon. This proportion is lower for the US but still non-negligible (10-20%).

Our approach to constructing our overseas unspanned factor is similar to that used by Cochrane and Piazessi (2005). They construct a 'return-forecasting factor' as a single linear combination of US forward rates and then show that this factor can explain a substantial part of US excess bond returns. Dahlquist and Hasseltoft (2013) find similar results for Germany, Switzerland and the UK (as well as for the US); and that a global factor constructed as a GDP-weighted average of the local return-forecasting factors raises the explanatory power of return-forecasting regressions relative to versions that only include the local return-forecasting factors for countries other than the US.[1]

There are, however, three important differences between Dahlquist and Hasseltoft (2013) and the present study. First, we show that there is information in foreign yields which is not reflected in *any* linear combination of domestic yields (not just the single linear combination they use as a domestic return-forecasting factor). Second, our overseas unspanned factor contains no information extracted from domestic yields, whereas the Dahlquist and Hasseltoft

[1] Zhu (2015) shows that such a global return-forecasting factor can predict returns out-of-sample for Germany, Japan, the UK and the US.

(2013) global factor is a weighted average of local factors from the different countries. So it is clear in our case that the return-forecasting ability of the overseas unspanned factor does not derive from its containing information about current domestic yields. And third, Dahlquist and Hasseltoft (2013) find that their global factor does not help to explain excess returns in the US, whereas we show that there is information in overseas yields that is relevant for explaining US returns. These three differences are particularly important when building dynamic term structure models, since our paper clearly demonstrate that we cannot capture all of the information relevant for modelling the time-series dynamics of yields simply by adding more factors extracted from domestic yield curves, even for the US.

Our dynamic factor models of yields - which we estimate separately for yields in each country - are broadly similar to the model of Diebold and Li (2006) in that they model the time-series dynamics of the factors driving bond yields using a Vector-Autoregression and have a simple cross-sectional mapping between factors and yields. The non-standard feature of our model is that we incorporate the respective overseas unspanned factor as a state variable alongside principal components of local yields. We can motivate this by appealing to a no-arbitrage term structure model with unspanned factors, similar to Joslin et al. (2014) (we provide further detail on this point in Appendix A). While we do not impose no-arbitrage restrictions on the cross section of yields,[2] this is unlikely to imply a materially different mapping between the factors and bond yields, however, so such an exercise would add little to the contribution of this paper (Duffee (2011a) provides a discussion of the impact of no-arbitrage restrictions on yield forecasts from dynamic term structure models).

Section 2 of this paper summarizes the US and German data sets we use and demonstrates the extent to which there is unspanned information in overseas yields. The return-forecasting regressions including several robustness checks are presented in Section 3 and the dynamic term structure model in Section 4. Section 5 concludes.

[2] For example, as the affine term structure models of Duffie and Kan (1996) and Duffee (2002). Dahlquist and Hasseltoft (2013) estimate no-arbitrage term structure models that include their global factor.

2 The unspanned component of overseas yields

2.1 Data

Our data set consists of estimates of German and US end-month zero-coupon yields from January 1990 until December 2014, with maturities of 6 months and 1, 2, 3, 5, 7 and 10 years. For the US, we use the estimates of Gürkaynak et al. (2007) using the Svensson (1994) parametric method, which are updated and published by the Federal Reserve Board.[3] For Germany, we use estimates published by the Bundesbank, also estimated using the Svensson method.[4] In Sections 4 and 5 we also report results of extensions to cover the UK; estimates of UK zero-coupon yields are published by the Bank of England and computed using the smoothed cubic spline method of Anderson and Sleath (2001).[5]

Table 1 reports summary statistics of the US and German yields at selected maturities. As is well known, the average term structures are upward sloping, the volatility of yields declines slowly with maturity and yields are highly persistent, with autocorrelation coefficients close to one for all maturities. For example, the average US six-month and ten-year yields are approximately 3.3% and 5.1% respectively; whereas the equivalent averages for Germany are 3.5% and 4.8%. The average German yield curve is therefore a little flatter than the average US yield curve (the average spread between the ten-year and six-month yield is 1.9 percentage points in the US and 1.4 percentage points in Germany). The standard deviation of the US six-month and ten-year yields are 2.3% and 1.8% respectively; with corresponding standard deviations of 2.6% and 2.0% in Germany.

Table 2 reports correlations of domestic yields across maturities for the two countries separately. As is well known, yields of nearby maturities within a single country are strongly correlated - for example, the seven- and ten-year yields have a correlation greater than 0.995 in both the US and Germany. The correlations between very short and very long maturity yields are somewhat weaker but are still positive - for example, the correlations between the

[3] Available at: http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html.

[4] Available at: http://www.bundesbank.de/Navigation/EN/Statistics/Money_and_capital_markets/Interest_rates_and_yields/Term_structure_of_interest_rates/term_structure_of_interest_rates.html.

[5] Available at: http://www.bankofengland.co.uk/statistics/pages/yieldcurve/default.aspx.

Table 1: Summary statistics of nominal zero-coupon yields

Maturity (months)	6	12	24	36	60	84	120
(a) United States							
Mean	3.275	3.406	3.673	3.923	4.365	4.729	5.135
Minimum	0.089	0.099	0.188	0.306	0.627	1.007	1.552
Maximum	8.382	8.568	8.780	8.863	8.909	8.919	8.924
Standard deviation	2.338	2.373	2.343	2.264	2.087	1.939	1.781
AR(1) coefficient	0.992	0.992	0.990	0.989	0.988	0.987	0.987
(b) Germany							
Mean	3.472	3.528	3.695	3.883	4.238	4.525	4.841
Minimum	-0.113	-0.125	-0.124	-0.095	0.032	0.231	0.615
Maximum	9.630	9.470	9.130	9.089	9.240	9.286	9.229
Standard deviation	2.623	2.572	2.506	2.444	2.317	2.193	2.027
AR(1) coefficient	0.994	0.995	0.994	0.995	0.996	0.997	0.998

All numbers except for the AR(1) coefficients are in annualized percentage points. The AR(1) coefficient reports the first-order autocorrelation coefficient from an AR(1) model including an intercept, estimated using OLS. The sample ranges from January 1990 to December 2014.

six-month and ten-year yields are 0.85 in the US and 0.90 in Germany.

Table 3 reports correlations of yields *across countries*. Cross-country correlations are strongly positive for all pairs of yields and are generally higher for longer maturity yields. For some maturities, we note that the foreign yield with the highest correlation does not necessarily have the same maturity. In particular, German yields are generally more highly correlated with longer maturity US yields than with the US yield of the corresponding maturity. This suggests that when we are analyzing the extent to which foreign and domestic yield curves contain the same information we cannot just focus on bivariate correlations between yields of the same maturity; rather, we should consider whether a given yield is spanned by the full set of maturities in the other country. We return to this issue in the following sub-section.

2.2 Unspanned overseas information

The simple correlation analysis above demonstrates a high degree of co-movement of bond yields across the two countries. But the fact that the cross-country correlations are less than one shows that there is nevertheless *some* information in yields that is specific to individual

Table 2: Correlations of yields across maturities within a single country

Maturity (months)	6	12	24	36	60	84	120
(a) United States							
6	1.000	0.997	0.983	0.965	0.927	0.890	0.845
12	0.997	1.000	0.994	0.981	0.948	0.915	0.874
24	0.983	0.994	1.000	0.996	0.976	0.951	0.916
36	0.965	0.981	0.996	1.000	0.991	0.973	0.945
60	0.927	0.948	0.976	0.991	1.000	0.995	0.979
84	0.890	0.915	0.951	0.973	0.995	1.000	0.995
120	0.845	0.874	0.916	0.945	0.979	0.995	1.000
(b) Germany							
	6	12	24	36	60	84	120
6	1.000	0.998	0.989	0.977	0.953	0.931	0.903
12	0.998	1.000	0.996	0.988	0.966	0.945	0.918
24	0.989	0.996	1.000	0.997	0.984	0.967	0.943
36	0.977	0.988	0.997	1.000	0.994	0.982	0.963
60	0.953	0.966	0.984	0.994	1.000	0.997	0.986
84	0.931	0.945	0.967	0.982	0.997	1.000	0.996
120	0.903	0.918	0.943	0.963	0.986	0.996	1.000

The table reports r-Pearson pairwise correlation coefficients computed for end-month values of the considered maturities for the period January 1990 to December 2014.

countries. To isolate the information in the yields of country j that is not (linearly) spanned by yields in country $i \neq j$, we regress yields in country j on yields from country i:

$$y^{(j)}_{n,t} = \beta_0 + \beta_6 y^{(i)}_{6,t} + \beta_{12} y^{(i)}_{12,t} + ... + \beta_{120} y^{(i)}_{120,t} + u^{(j)}_{n,t}, \tag{1}$$

for $n = 6, 12, 24, 36, 60, 84, 120$ and where $y^{(i)}_{n,t}$ is the time-t, n-period yield for country i.

Panel (a) of Table 4 reports the R^2 statistics for these regressions. These are consistent with the general pattern observed in the cross-country correlation analysis reported above. Yields in the foreign country can explain a large proportion of the variation in domestic long-term yields: the R^2s for the ten-year yields are both close to 0.95. At shorter maturities, the R^2s are lower: regressing the six-month US yield on German yields gives an R^2 of 0.66; and regressing the six-month German yield on US yields gives an R^2 of 0.81.

Panel (b) of Table 4 reports results from restricted versions of (1) in which the only re-

Table 3: Correlations of yields across countries

Germany \ United States							
Maturity (months)	6	12	24	36	60	84	120
6	0.711	0.731	0.767	0.797	0.840	0.864	0.880
12	0.733	0.754	0.790	0.820	0.861	0.884	0.899
24	0.758	0.780	0.819	0.848	0.889	0.911	0.926
36	0.771	0.796	0.836	0.866	0.907	0.930	0.944
60	0.781	0.808	0.850	0.882	0.924	0.947	0.964
84	0.780	0.807	0.851	0.884	0.927	0.952	0.969
120	0.771	0.800	0.845	0.878	0.923	0.948	0.968

The table reports r-Pearson pairwise cross-country correlations of monthly yields for US and Germany computed for end-month values of the considered maturities for January 1990 to December 2014. German yields are in rows and US yields are in columns. For example, the number 0.758 from the third row and first column reports the correlation between 24-month German yield and the 6-month US yield.

gressors are a constant and the matched maturity yield in country i (i.e. regressing $y_{n,t}^{(j)}$ on $y_{n,t}^{(i)}$). The R^2 statistics are substantially lower and F-tests of the implied zero restrictions suggest that they should be strongly rejected in all cases. Similar to the correlation analysis in the previous sub-section, this shows that when analyzing the common information in international term structures, we cannot necessarily just consider bivariate relationships between yields that have the same maturity.

3 Return regressions

3.1 An unspanned overseas return-forecasting factor

As discussed above, when specifying a dynamic term structure model, it may be important to include variables unspanned by the yield curve - and which therefore do not improve the cross-sectional fit of the model - but are nevertheless important for predicting future yields (Joslin et al. (2014)); and we can use simple reduced-form return-forecasting regressions to provide an indication of whether there are such unspanned factors in the yield curve (Appendix A provides future motivation for these regressions). In this section, we therefore turn to the question of whether the information in the foreign yield curve that is orthogonal to domestic

Table 4: Regressions of domestic yields on foreign yields

Maturity (months)		6	12	24	36	60	84	120
(a) Multivariate regressions								
United States	R^2	0.66	0.70	0.76	0.81	0.88	0.92	0.95
Germany	R^2	0.81	0.84	0.88	0.91	0.95	0.96	0.96
(b) Univariate regressions								
United States	R^2	0.50	0.57	0.67	0.75	0.85	0.90	0.94
	F-test (p-values)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Germany	R^2	0.50	0.57	0.67	0.75	0.85	0.90	0.94
	F-test (p-values)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Panel (a) of the table shows R^2 statistics for regressions of yields in the relevant country on a constant and yields with maturities of 6,12, 24, 36, 60, 84 and 120 months from the other country (equation (1)). Panel (b) shows the R^2 statistics for regressions of yields in the relevant country on a constant and the single yield from the other country with the same maturity. Figures in parentheses in panel (b) show the p-values of F-tests of the restrictions that all omitted regressors included in the regressions reported panel (a) are equal to zero. The sample ranges from January 1990 to December 2014.

yields is nevertheless useful for explaining domestic excess returns.

With seven different maturities for each country, the dimensions of the orthogonal information contained in the seven residuals $u_{n,t}^{(j)}$ for $n = 6, 12, 24, 36, 60, 84, 120$ from (1) is clearly large. But it turns out that the large majority of the information contained in those residuals that is relevant for forecasting country i returns can be summarised in a single 'overseas unspanned factor' (OUF).

Note first that the excess return from holding a country i n-month bond between times t and $t+12$ is defined as

$$rx_{n,t,t+12}^{(i)} = \log\left(P_{n-12,t+12}^{(i)}\right) - \log\left(P_{n,t}^{(i)}\right) - 12 \times y_{12,t}^{(i)}, \tag{2}$$

where $P_{n,t}^{(i)}$ is the time-t price of an n-period bond. To construct a single linear combination of the information in the residuals from (1), we regress the average excess return on country i bonds of different maturities between times t and $t+12$ on the time-t components of all foreign yields orthogonal to domestic yields (i.e. $u_{n,t}^{(j)}$):

$$\overline{rx}_{t,t+12}^{(i)} = \gamma_0 + \boldsymbol{\gamma}_1' \mathbf{u}_t^{(j)} + \varepsilon_{t,t+12}^{(i)}, \tag{3}$$

Here, $\overline{rx}_{t,t+12}^{(i)}$ denotes the average 12-month excess return on 2-, 3-, 5-, 7- and 10-year bonds and $\mathbf{u}_t^{(j)} = \left[u_{6,t}^{(j)}, u_{12,t}^{(j)}, u_{24,t}^{(j)}, ..., u_{120,t}^{(j)}\right]'$. Our return-forecasting factor, which we denote $z_t^{(j)}$ below, is given by the fitted value from this regression ($z_t^{(j)} = \widehat{\gamma}_0 + \widehat{\boldsymbol{\gamma}}_1' \mathbf{u}_t^{(j)}$). This is similar to the procedure in Cochrane and Piazessi (2005), although their regressors are domestic forward rates.

We can evaluate how well this single-factor specification explains excess returns on bonds across different maturities in a second step, by running separate regressions of the form

$$rx_{n,t,t+12}^{(i)} = \alpha_0 + \alpha_n z_t^{(j)} + \widehat{\varepsilon}_{n,t,t+12}^{(i)} \tag{4}$$

for $n = 24, 36, 60, 84, 120$. The R^2s from these regressions are in the region of 0.1-0.2 for the US and 0.2-0.4 for Germany (Table 5). In both cases, there is information in overseas yields, unspanned by domestic yields, which can explain a substantial part of the variation in domestic excess returns.

Table 5: R^2 of regression of excess bond returns on single and multiple unspanned factors

Maturity (months)	24	36	60	84	120
(a) United States					
Single-factor specification	0.107	0.138	0.166	0.171	0.161
Unrestricted	0.172	0.176	0.173	0.172	0.179
(b) Germany					
Single-factor specification	0.335	0.351	0.338	0.296	0.223
Unrestricted	0.357	0.363	0.340	0.296	0.228

The table reports R^2 statistics for two models. The ‘single-factor specification’ refers to regressions of excess bond returns on a constant and the overseas unspanned factor (4). The ‘unrestricted’ specification refers to regressions of excess bond returns on a constant and the components of all considered domestic yields orthogonal to overseas yields (5). The sample ranges from January 1990 to December 2014.

Fitting a model with a single-factor obtained from the two-step procedure of estimating (3) and then (4) does of course involve some loss of information. To evaluate how well our single factor captures the relevant information contained in all the residuals $u_{n,t}^{(j)}$, we can also estimate the unrestricted version of (4):[6]

$$rx_{n,t,t+12}^{(i)} = \gamma_{0,n} + \boldsymbol{\gamma}_n' \mathbf{u}_t^{(j)} + \varepsilon_{n,t,t+12}^{(i)} \tag{5}$$

for $n = 24, 36, 60, 84, 120$. The R^2s from these regressions are also shown in Table 5 (the rows headed 'unrestricted'). In almost all cases, these are very similar to those obtained from the single-factor model (4), i.e. there is little information lost by using the single-factor specification.

3.2 Does the overseas unspanned factor contain information for predicting returns relative to the domestic yield curve?

We next assess the extent of the *marginal* information in the unspanned portion of overseas yields - relative to the information contained in the domestic term structure - by estimating regressions of the form

$$rx_{n,t,t+12}^{(i)} = \kappa_0 + \boldsymbol{\kappa}' \mathbf{y}_t^{(i)} + \alpha_n z_t^{(j)} + \eta_{n,t+12}^{(i)} \tag{6}$$

where $\mathbf{y}_t^{(i)} = \left[y_{6,t}^{(i)}, y_{12,t}^{(i)}, ..., y_{120,t}^{(i)} \right]'$ denotes a vector of all considered yields for country i. Return-forecasting regressions usually have fewer explanatory variables than this, so it is worth emphasizing that the point we are making here is not necessarily that a model with so many variables is desirable for all purposes; the point of this particular exercise is to show that *no* linear combination of the considered domestic yields can replicate the information contained in the overseas unspanned factor - hence why we include all seven as explanatory variables.

Table 6 reports results from estimating (6) and from a version with α_n restricted to zero.

[6] This is similar to the approach taken by Cochrane and Piazessi (2005) when considering the return-forecasting information in domestic forward rates.

For both the US and Germany as the domestic country i, the increase in the explanatory power of the regression, measured by its R^2, is substantial - from about 0.35 to 0.5 for US returns and from about 0.2 to 0.5 for German returns. In both cases, the change in the R^2 is strongly significant based on the bootstrap procedure by Bauer and Hamilton (2015) (our implementation of this bootstrap is explained in Appendix B). And the coefficients on the overseas unspanned factor α_n are also individually strongly statistically significant. In summary, therefore, there is clearly statistically and economically significant information in overseas yield curves, unspanned by domestic yields, which is nevertheless important for predicting future domestic bond returns.

Table 6: Regression of excess bond returns on domestic yields and the unspanned overseas factor

Maturity (months)	24	36	60	84	120
(a) United States					
α_n	0.24	0.52	1.01	1.39	1.81
t-statistics	(7.5)	(8.6)	(9.8)	(10.0)	(9.4)
	[-4.3,4.1]	[-4.3,4.1]	[-4.3,4.1]	[-4.3,4.1]	[-4.2,4.1]
R^2 including OUF	0.47	0.49	0.52	0.52	0.50
R^2 restricted $\alpha_n = 0$	0.37	0.35	0.35	0.35	0.34
ΔR^2	0.10	0.14	0.16	0.17	0.16
	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]
(b) Germany					
α_n	0.30	0.61	1.08	1.36	1.58
t-statistics	(13.6)	(14.5)	(14.1)	(12.7)	(10.5)
	[-4.9,4.9]	[-4.9,4.9]	[-4.9,4.9]	[-4.9,4.8]	[-4.9,4.8]
R^2 including OUF	0.50	0.54	0.54	0.50	0.46
R^2 restricted $\alpha_n = 0$	0.17	0.20	0.21	0.21	0.24
ΔR^2	0.33	0.34	0.33	0.29	0.22
	[0.06]	[0.06]	[0.06]	[0.06]	[0.06]

The table reports estimated parameters from regressions of excess bond returns on a constant, seven domestic yields and the overseas unspanned factor (α_n), i.e. equation (6). Numbers in parentheses report the values of t-statisitcs and numbers in brackets refer to the 95% confidence interval for these t-statistics obtained using the Bauer and Hamilton (2015) bootstrap procedure. The final two rows of each section (a) and (b) report the R^2 statistics from models with and without the overseas unspanned factor ('Including OUF' and 'Restricted' respectively). Numbers in brackets refer to the 95% critical value for the change in the R^2. The sample ranges from January 1990 to December 2014.

3.3 Interpreting the overseas unspanned factors

In this section we attempt to provide some interpretation of the overseas unspanned factor. As Duffee (2011b) notes when discussing the interpretation of 'hidden' factors in the US term structure, "The Holy Grail of the term structure literature is a testable, inuitive model linking yields to macroeconomic forces... [but] we are not close to finding it." In summary, similar to Duffee's findings for hidden factors in domestic yield curves, our overseas unspanned factors are correlated with some plausible explanatory variables and these variables can themselves explain some of the variation in bond returns; but it would be too much of a stretch to claim that we can put forward a clear-cut and robust account of what drives the factor or use it to help identify the structural determinants of bond returns.

3.3.1 How do overseas unspanned factors relate to conventional yield curve factors?

Before turning to the question of the macroeconomic determinants of the overseas unspanned factor, we first attempt something less ambitious: to relate the overseas unspanned factors to conventional yield curve factors, in the form of principal components extracted from overseas yields (Section 4 provides more details on how these principal components are constructed). Specifically, we regress the US unspanned factor (i.e. the component of US yields that is uncorrelated with German yields) on principal components extracted from US yields; and the German unspanned factor on the principal components of German yields.

Table 7 shows the R^2 from regressions of the overseas unspanned factors on a constant and different combinations of overseas principal components ($\mathbf{x}_t^{(j)}$):

$$z_t^{(j)} = \delta_0 + \boldsymbol{\delta}' \mathbf{x}_t^{(j)} + v_t^{(j)} \tag{7}$$

The first column of panel (a) reports results from regressions of the factor constructed from the component of US yields that is uncorrelated with German yields on principal components of US yields, while the second column reports the reverse case. In each case, a single traditional principal component can explain a much larger proportion of the unspanned factor than any other. The slope of the US yield curve (i.e. the second principal component) explains about 30% of the variation in the US unspanned factor, with smaller contributions from the level factor (i.e. the first principal component) and curvature factor (i.e. the third principal component). Figure 1 shows that the broad movements in the US unspanned factor and the US slope are similar. The curvature of the German yield curve explains about 60% of the variation in the German unspanned factor, with other principal components of the German yield curve explaining almost nothing. Figure 2 shows a high degree of co-movement between the German unspanned factor the the curvature of the German yield curve. In both cases, the fourth to seventh principal components of overseas yields together explain little of the unspanned factor and the overall proportion of variation explained by all seven overseas principal components is around two thirds. That implies that although the *domestic* yields are uncorrelated with the overseas unspanned factor by construction, they play an important role in filtering out part of the information in overseas yields - for example, three domestic and three overseas principal components can collectively explain more than 70% of the variation in the overseas unspanned factor (and, trivially, including all seven overseas and domestic principal components raises the explained proportion to 100%).

In panel (b), we consider the extent to which traditional principal components capture the part of the overseas unspanned factor that is relevant for forecasting returns. On average across maturity, the increase in R^2 from including the US overseas unspanned factor in return regressions for German bonds is 0.30 (see also Table 6). To assess the extent to which this return-forecasting power is captured by traditional principal components, we can instead include the fitted overseas unspanned factor from (7), i.e. $\widehat{z}_t^{(j)} = \widehat{\delta}_0 + \widehat{\boldsymbol{\delta}}' \mathbf{x}_t^{(j)}$. If we consider only the slope of the US yield curve in $\mathbf{x}_t^{(j)}$, the average increase in R^2 associated with the fitted overseas unspanned factor from (7) is 0.10, i.e. the improvement in R^2 is about one third of what we obtain from including the overseas unspanned factor itself. If we include the first three principal components of US yields in $\mathbf{x}_t^{(j)}$ the proportion explained rises to 0.17, although this is still only about half of the total explanatory power of the overseas unspanned factor. But if we include the first three domestic principal components of *both* US *and* German yields, the average increase in R^2 rises to 0.26. Taken together, these results suggest that the explanatory power of the US unspanned factor for German returns does not just stem from the fact that it includes information about US yields and that German yields play an important role in the construction of the unspanned factor, by filtering out the correlated part of the variation in US yields.

Results for the German unspanned factor are quite different. Here, the average increase in R^2 is smaller (0.15), as discussed above. But all of this explanatory power appears to stem from the third principal component of the German yield curve, with other German and US principal components playing a relatively unimportant role.

<Figures 1 and 2 about here>

3.3.2 Macroeconomic determinants of the overseas unspanned factors

We next consider the extent to which certain macroeconomic and financial variables that might plausibly drive term premia can explain variation in our overseas unspanned factor, estimating equations of the form:

$$z_t^{(i)} = \theta_0 + \boldsymbol{\theta}_1' \mathbf{m}_t + \zeta_t^{(i)}, \tag{8}$$

where $z_t^{(i)}$ is one of the overseas unspanned factors; $\mathbf{m}_t$ is a vector of candidate explanatory variables; and $\eta_t^{(i)}$ is an error term. To try to avoid too much risk of data mining, we limit our focus to four types of explanatory variable: (i) the year-on-year percentage growth rate in industrial production in the two countries; (ii) the annual percentage CPI inflation rate in each country; (iii) the US and German one-year yields; and (iv) the year-on-year percentage change in the dollar-euro exchange rate (Δs_t).[7]

[7] The data source for industrial production and inflation is FRED St Louis; for exchange rates and equity implied volatilities it is Bloomberg.

Table 7: The relationship between the overseas unspanned factors and traditional principal components

	US factor unspanned by German yields	German factor unspanned by US yields
(a) Regressions of overseas unspanned factor on traditional principal components (R^2s)		
Overseas PC1	0.14	0.00
Overseas PC2	0.29	0.02
Overseas PC3	0.07	0.59
Overseas PC1-3	0.50	0.61
Overseas PC1-7	0.60	0.65
Overseas PC1-3 & domestic PC1-3	0.71	0.77
Overseas PC1-7 & domestic PC1-7	1	1
(b) Gains to return-forecasting regressions (increase in R^2 averaged across maturities)		
OUF	0.30	0.15
Fitted OUF, based on a single PC (PC2 for US and PC3 for DE)	0.10	0.13
Fitted OUF, based on first 3 PCs	0.17	0.13
Fitted OUF, based on 3 overseas and 3 domestic PCs	0.26	0.14

The table reports the R^2s from regressions of the overseas unspanned factors on the principal components (PCs) of overseas and domestic yield curves. Panel (a) refers to regressions of overseas unspanned factors on principal components of yields - equation (7). For example, the first column refers to regressions of the factor constructed from the component of US yields that is uncorrelated with German yields on principal components of US yields. Panel (b) refers to return-forecasting regressions using the overseas unspanned factor (first row) or fitted values of the overseas unspanned factor from (7).

Table 8 reports results from these regressions. We focus first on the US factor unspanned by German yields (i.e. the factor we use in the excess return regressions for Germany). Regressing this factor on all of the considered variables (Model I) suggests that these variables can explain around half of the variation in the factor (panel (a)). And including the fitted OUF from (8) in German return regressions (panel (b)) gives an average increase in R^2 of 0.19, suggesting that these macro variables can explain around two thirds of the ability of the US unspanned factor to explain German returns. In Model I, only the coefficients on the short-term interest rates and US inflation are significantly different from zero, so in Model II we consider a version that only includes the short-term interest rates. The results from this model - in terms of the R^2 of (8) and the average increase in German return regressions - are similar to those from Model I, which suggests that the ability of standard macro variables to explain the overseas unspanned factor is actually largely captured by the short-term interest rates. That is not to say that other macroeconomic variables are entirely unrelated to the unspanned factor, however: Model III shows that if we include all variables other than the short rates the reductions in R^2s are not too pronounced.

Attempts to link the German overseas unspanned factor to macro variables are much less encouraging, both in terms of the proportion of the factor that can be explained by macro variables - less than a third - and the extent to which the macro variables capture the return-forecasting power of the unspanned factor.

Table 8: Regressions of overseas unspanned factors on macroeconomic variables

	US factor unspanned by German yields			German factor unspanned by US yields		
	I	II	III	I	II	III
(a) Regressions of overseas unspanned factor on macro variables (estimated coefficients and R^2)						
Constant	4.27***	3.69***	3.59***	3.87***	2.86***	3.82***
DFX	-0.03	-	-0.07***	-0.06*	-	-0.06***
US industrial production	-0.08	-	-0.22**	-0.14*	-	-0.12**
US inflation	-0.32**	-	-0.84***	-0.51**	-	-0.32**
US one-year yield	-0.82***	-0.89***	-	0.07	0.00	-
German industrial production	0.06	-	0.10	0.13**	-	0.12**
German inflation	-0.06	-	0.87***	-0.16	-	-0.04
German one-year yield	0.67**	0.62***	-	0.12	-0.00	-
R^2	0.53	0.48	0.38	0.28	0.00	0.24
(b) Gains to return-forecasting regressions (increase in R^2 averaged across maturities)						
OUF		0.30			0.15	
Fitted OUF	0.19	0.22	0.14	0.05	0.02	0.06

Panel (a) of the table shows results from regressions of the overseas unspanned factors on macro variables. A */**/*** indicates that the estimated coefficient is significant at the 10/5/1% level. Panel (b) shows the gain in R^2 from including either the actual or model-implied overseas unspanned factor in a return-forecasting regression, averaged across 2-, 3-, 5-, 7- and 10-year maturities.

3.4 Robustness tests

Our paper is not the first to find a variable which appears to predict future bond returns. In general, of course, a problem in this literature is a lack of robustness: results are particular to the considered sample period or disappear out-of-sample. This may be a particular concern in our case, given the high colinearity of the regressors in the construction of the return-forecasting factor (3). Viewed in that light, however, our results appear to be remarkably robust. Most importantly, the overseas unspanned factor significantly improves forecasts of returns out-of-sample. But our results also hold across a number of different sub-samples and when we consider alternative domestic yield curve variables. While the results are weaker if we consider a six-month investment horizon, our overseas unspanned factors can still provide a statistically significant improvement in the predictability of domestic returns. Finally, we also show that very similar results apply if we extend our analysis to include the UK as a third country in our analysis.

3.4.1 Different sample periods

A potential concern about the results reported above is that the sample period we use contains two obvious potential structural breaks: the introduction of the euro in January 1999 and the fall in short-term nominal interest rates close to the zero lower bound during the recent financial crisis. Consequently we first consider three sub-sample periods: (i) the pre-euro period (January 1990-December 1998); (ii) the post-euro period (January 1999-December 2014); and (iii) the pre-lower bound period (January 1990-December 2007). Tables 9 and 10 report R^2s for models including and excluding the overseas unspanned factor for the different sub-samples. The goodness of fit varies across samples, yet the overall R^2s remain high for models including the overseas unspanned factor, ranging from 47% to 82%. Most importantly, in all cases the fit of the regressions that exclude the overseas unspanned factor are worse, particularly for German short-maturity returns. The coefficients on the overseas unspanned factor are strongly statistically significant in all cases.

Table 9: Regressions of excess returns on domestic yields and the overseas unspanned factor for different sub-samples

Maturity (months)	24	36	60	84	120	24	36	60	84	120
	(a) United States					(b) Germany				
(i) Full sample: 1990-2014										
α_n	0.24	0.52	1.01	1.39	1.81	0.30	0.61	1.08	1.36	1.58
t-statistics	(7.5)	(8.6)	(9.8)	(10.0)	(9.4)	(13.6)	(14.5)	(14.1)	(12.7)	(10.5)
	[-4.3,4.1]	[-4.3,4.1]	[-4.3,4.1]	[-4.3,4.1]	[-4.2,4.1]	[-4.9,4.9]	[-4.9,4.9]	[-4.9,4.9]	[-4.9,4.8]	[-4.9,4.8]
R^2 including OUF	0.47	0.49	0.52	0.52	0.50	0.50	0.54	0.54	0.50	0.46
R^2 restricted $\alpha_n = 0$	0.37	0.35	0.35	0.35	0.34	0.17	0.20	0.21	0.21	0.24
ΔR^2	0.10	0.14	0.16	0.17	0.16	0.33	0.34	0.33	0.29	0.22
	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.06]	[0.06]	[0.06]	[0.06]	[0.06]
(ii) Pre-ZLB sample 1990-2007										
α_n	0.22	0.49	0.98	1.41	1.93	0.31	0.63	1.09	1.33	1.50
t-statistics	(7.4)	(8.8)	(10.4)	(11.0)	(11.2)	(10.9)	(11.4)	(10.8)	(9.5)	(7.5)
	[-4.3,4.5]	[-4.3,4.4]	[-4.3,4.4]	[-4.4,4.4]	[-4.4,4.4]	[-5.2,5.1]	[-5.2,5.2]	[-5.1,5.2]	[-5.1,5.1]	[-5.1,5.0]
R^2 including OUF	0.67	0.68	0.69	0.67	0.64	0.58	0.62	0.62	0.59	0.54
R^2 restricted $\alpha_n = 0$	0.58	0.56	0.51	0.47	0.42	0.32	0.37	0.40	0.41	0.41
ΔR^2	0.09	0.12	0.18	0.20	0.22	0.26	0.25	0.22	0.18	0.13
	[0.05]	[0.05]	[0.05]	[0.05]	[0.05]	[0.08]	[0.08]	[0.08]	[0.08]	[0.08]

The table reports results from regressions of excess bond returns on a constant, the seven considered domestic yields and the overseas unspanned factor - i.e. equation (6), estimated for the indicated sample periods. Numbers in parentheses report the values of t-statisitcs and numbers in brackets refer to the 95% confidence interval for these t-statistics obtained using the Bauer and Hamilton (2015) bootstrap procedure. The final two rows of each part of the table report the R^2 statistics from models with and without the overseas unspanned factor ('Including OUF' and 'Restricted' respectively). Numbers in brackets refer to the 95% critical value for the change in the R^2.

Table 10: Regressions of excess returns on domestic yields and the overseas unspanned factor for different sub-samples

Maturity (months)	24	36	60	84	120	24	36	60	84	120
	(a) United States					(b) Germany				
(iii) Pre-euro sample: 1990-1998										
α_n	0.13	0.32	0.71	1.08	1.58	0.20	0.51	1.06	1.45	1.83
t-statistics	(5.8)	(7.0)	(8.1)	(8.6)	(8.9)	(4.3)	(5.8)	(7.1)	(6.9)	(5.8)
	[-4.3,4.4]	[-4.3,4.3]	[-4.2,4.3]	[-4.2,4.3]	[-4.2,4.3]	[-4.7,4.8]	[-4.7,4.8]	[-4.7,4.9]	[-4.7,4.8]	[-4.7,4.9]
R^2 including OUF	0.82	0.81	0.77	0.74	0.71	0.56	0.64	0.69	0.65	0.57
R^2 restricted $\alpha_n = 0$	0.75	0.70	0.60	0.52	0.44	0.47	0.51	0.50	0.46	0.41
ΔR^2	0.07	0.11	0.17	0.22	0.27	0.09	0.13	0.19	0.19	0.16
	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)
(iv) Post-euro sample: 1999-2014										
α_n	0.29	0.58	1.05	1.33	1.47	0.33	0.64	1.08	1.35	1.57
t-statistics	(7.0)	(8.2)	(9.2)	(8.6)	(6.6)	(27.2)	(25.9)	(21.9)	(18.4)	(14.7)
	[-4.0,4.0]	[-4.0,3.9]	[-3.9,3.9]	[-3.9,3.9]	[-3.8,3.9]	[-4.9,4.8]	[-5.0,4.8]	[-4.9,4.8]	[-4.9,4.9]	[-4.9,4.9]
R^2 including OUF	0.59	0.63	0.67	0.67	0.62	0.83	0.81	0.76	0.71	0.65
R^2 restricted $\alpha_n = 0$	0.47	0.48	0.51	0.53	0.53	0.10	0.08	0.09	0.13	0.21
ΔR^2	0.12	0.15	0.16	0.14	0.09	0.73	0.73	0.67	0.58	0.44
	(0.05)	(0.05)	(0.04)	(0.04)	(0.04)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)

The table reports results from regressions of excess bond returns on a constant, the seven considered domestic yields and the overseas unspanned factor - i.e. equation (6), estimated for the indicated sample periods. Numbers in parentheses report the values of t-statisitcs and numbers in brackets refer to the 95% confidence interval for these t-statistics obtained using the Bauer and Hamilton (2015) bootstrap procedure. The final two rows of each part of the table report the R^2 statistics from models with and without the overseas unspanned factor ('Including OUF' and 'Restricted' respectively). Numbers in brackets refer to the 95% critical value for the change in the R^2.

3.4.2 Out-of-sample performance

We next evaluate whether the increase in explanatory power from including our overseas unspanned factors holds out of sample. In our forecasting exercise we estimate the models using rolling windows of 120 monthly observations to generate 168 forecasts. More precisely, we start by estimating the model using the ten-year period January 1990-December 1999 and construct a twelve-month ahead forecast of returns for the period ending December 2000. We then move the estimation period on by one month (i.e. February 1999 to January 2000) and repeat. Table 11 reports root mean squared forecast error (RMSFE) statistics from this forecasting exercise for different maturities, computed across all the resulting 168 forecasts.

The RMSFE for the model including the unspanned overseas factor is lower than for the restricted model for all maturity returns in both countries. Giacomini and White (2006) tests of the statistical significance of the improvements in forecasting performance show that models including the unspanned overseas factor perform *significantly* better at forecasting returns, with the single exception of German ten-year bonds. The model including the overseas unspanned factor even out-performs a random walk for US seven- and ten-year bonds and for all maturities for Germany. In summary, therefore, our results are remarkably robust out of sample, which should substantially alleviate concerns that they are an artefact of in-sample over-fitting.

Table 11: Root Mean Squared Forecast Error of out-of-sample excess return predictions

Maturity (months)	24	36	60	84	120
(a) United States					
Random walk	1.16	2.35	4.70	7.08	10.62
Restricted $\alpha_n = 0$	2.15	4.18	7.26	9.63	12.54
Including OUF	1.78**	3.22**	5.24**	6.77**	8.79**
(b) Germany					
Random walk	1.34	2.60	4.80	6.60	8.85
Restricted $\alpha_n = 0$	1.41	2.73	4.79	6.18	7.55
Including OUF	0.78***	1.56***	3.12***	4.59**	6.51

The table reports Root Mean Square Forecast Errors for excess bond returns for three different forecasting models: a random walk i.e. a simple naive forecast; and our benchmark model both including the overseas unspanned factor and excluding it ('Restricted' and 'Including OUF' respectively). All model parameters, as well as the OUFs are computed using 10-year rolling samples (i.e. 120 months). All numbers reported are in annualized percentage points. Asterisks indicate significance levels from Giacomini-White test (see Giacomini and White (2006)) assessing the difference of forecasting power between the models excluding and including the overseas unspanned factor: ***,**, * denote significance at $p = 0.01$, $p = 0.05$ and $p = 0.1$ respectively for the best performing model. The sample ranges from January 1990 to December 2014, implying a forecasting period of January 2000 to December 2014.

3.4.3 Alternative domestic yield curve variables

As explained above, the primary purpose of our return-forecasting regression (6) is to demonstrate that there is information contained in the overseas unspanned factor which is not reflected in any linear combination of the considered domestic yields - i.e. it is not necessarily to show that this is the 'best' forecasting model of yields. Indeed, it is plausible that a more parsimonious model would deliver superior out-of-sample forecasts of returns to those presented in Section 3.4.2. In this sub-section we show that our specification nevertheless performs favourably out-of-sample compared with three more parsimonious alternatives.

All of the alternative models we consider here can be written as

$$rx^{(i)}_{n,t,t+12} = \widetilde{\kappa}_0 + \widetilde{\boldsymbol{\kappa}}'\mathbf{x}^{(i)}_t + \widetilde{\alpha}_n z^{(j)}_t + \widetilde{\eta}^{(i)}_{n,t+12}, \tag{9}$$

where $\mathbf{x}^{(i)}_t$ is a vector of variables constructed from domestic yields for country i. In all cases we also consider versions of the models that exclude the overseas unspanned factor ($z_t^{(j)}$).

The first alternative model uses the first three principal components of domestic yields, which is fairly standard number in the dynamic term structure literature. The second uses a purely domestic return-forecasting factor constructed a broadly similar way to Cochrane and Piazessi (2005) - i.e. regressing average excess returns on *ex ante* domestic forward rates. Specifically, we first regress average excess returns on bonds with 2, 3, 5, 7 and 10 years to maturity on a vector of domestic forward rates $\mathbf{f}_t^{(i)} = \left[f_{12,t}^{(i)}, f_{24,t}^{(i)}, f_{36,t}^{(i)}, f_{60,t}^{(i)}, f_{84,t}^{(i)}, f_{120,t}^{(i)}\right]'$:[8]

$$\overline{rx}_{t,t+12}^{(i)} = \theta_0 + \boldsymbol{\theta}\mathbf{f}_t^{(i)} + \epsilon_{t,t+12}^{(i)}. \tag{10}$$

The domestic return-forecasting 'CP factor' is the fitted value from this regression. The third alternative model includes both the first three domestic principal components and the domestic CP factor. Table 12 reports the results of out-of-sample forecasting exercises for these more parsimonious alternative models, reporting the RMSFE for different maturity excess returns from the different models. We adopt two coding schemes to assist in reading the table. First, a bold number indicates the best performing model out of our benchmark specification and the three alternatives. A box round a number indicates which is the best performing model if we also include a random walk in the set of considered models.

We highlight the following results. First, in most cases, our benchmark specification is actually the best performing model; the only exceptions are for German longer maturity returns, where the model with 3 local principal componets, the CP factor and the overseas unspanned factor performs best. Second, in almost all cases the versions of the models that include the overseas unspanned factor perform significantly better than the versions that exclude it, according to Giacomini and White (2006) tests of their comparative predictive ability. Here, the only exception is the model of US returns based on three domestic principal components, which performs slightly better if the overseas unspanned factor is excluded, although in this case the differences are *not* statistically significant. Third, our specification compares quite favourably with a random walk: for Germany, the benchmark model substan-

[8] The data sources for forward rates are the same as those described in Section 3.

tially out-performs a random walk at all maturities, whereas for the US it does so for the longer-maturity returns (seven and ten years).

3.4.4 Different investment horizons

In our analysis above, we have focused on twelve-month excess returns, in line with much of the literature on return predictability, including the related studies by Cochrane and Piazessi (2005) and Dahlquist and Hasseltoft (2013). In this section, we examine whether our results hold if we consider shorter holding periods. Specifically, we assess the information content of domestic and overseas unspanned factors for one- and six-month excess returns by estimating (6) with left-hand side variables changed to one- and six-month excess returns respectively.

Tables 13 and 14 report R^2 coefficients for models with different investment horizons. For the 6-month investment horizon, both domestic yields and unspanned overseas factors still contain substantial information about future excess returns, although the gain from including the unspanned overseas factor (in terms of the increase in R^2) is around half that for the 12-month horizon. At the one-month investment horizon return predictability is generally substantially lower and there is negligible gain from including the overseas unspanned factor. This clearly indicates that the information content of unspanned overseas factors is more substantial for longer horizons, which is consistent with previous studies showing that bond return predictability increases with the holding period (e.g. Fama and Bliss (1987)).

Table 12: Root mean squared forecast error of excess returns predictions from different models estimated over 10 years of data

Maturity (months)	24	36	60	84	120
(a) United States					
Random walk	1.16	2.35	4.70	7.08	10.62
7 Local factors	2.15	4.18	7.26	9.63	12.54
7 Local factors and z	**1.78****	**3.22****	**5.24****	**6.77****	**8.79****
3 Local factors	2.22	4.26	7.14	9.12	11.36
3 Local factors and z	2.37	4.50	7.38	9.16	10.84
CP factor	2.20	4.35	7.65	10.01	12.54
CP factor and z	1.80**	3.50**	6.09***	7.92***	9.89**
3 Local factors and CP factor	2.41	4.61	7.75	9.89	12.32
3 Local factors and CP factor and z	1.98**	3.67**	5.92**	7.41**	9.19**
(b) Germany					
Random walk	1.34	2.60	4.80	6.60	8.85
7 Local factors	1.41	2.73	4.79	6.19	7.55
7 Local factors and z	**0.78*****	**1.56*****	3.13***	4.59**	6.51
3 Local factors	1.31	2.52	4.41	5.72	7.10
3 Local factors and z	0.81***	1.62***	3.18***	4.60**	6.49
CP factor	1.26	2.48	4.55	6.17	8.13
CP factor and z	0.86**	1.64**	3.14***	4.57**	6.58**
3 Local factors and CP factor	1.39	2.71	4.76	6.15	7.49
3 Local factors and CP factor and z	0.80**	1.58***	**3.07*****	**4.43****	**6.23**

The table reports Root Mean Square Forecast Errors for excess bond returns for five forecasting models: (i) a random walk; (ii) the benchmark model including seven domestic yields and the overseas unspanned factor (z); (iii) a model with three domestic principal components and the overseas unspanned factor; (iv) a model with our 'CP' factor and the overseas unspanned factor; and (v) a model that includes three domestic principal components, our CP factor and the overseas unspanned factor. All model parameters, as well as the domestic principal components and overseas unspanned factors are computed using 10-year rolling samples (i.e. 120 months). All numbers reported are in annualized percentage points. Asterisks indicate significance levels from Giacomini-White test (see Giacomini and White (2006)) assessing the difference of forecasting power between the considered model and the version without the overseas unspanned factor: ***,**, * denote significance at $p = 0.01$, $p = 0.05$ and $p = 0.1$ respectively for the best performing model. The sample ranges from January 1990 to December 2014, implying a forecasting period of January 2000 to December 2014.

Table 13: Regression of excess bond returns on domestic yields and the unspanned overseas factor for 1-month holding period

Maturity (months)	24	36	60	84	120
(a) United States					
α_n	0.20	0.31	0.36	0.20	-0.15
t-statistics	(1.0)	(0.9)	(0.6)	(0.3)	(-0.1)
	[-2.0,2.0]	[-2.0,2.0]	[-2.0,2.0]	[-2.0,2.1]	[-2.0,2.1]
R^2 including OUF	0.10	0.09	0.08	0.08	0.07
R^2 restricted $\alpha_n = 0$	0.09	0.09	0.08	0.08	0.07
ΔR^2	0.01	0.00	0.00	0.00	0.00
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]
(b) Germany					
α_n	0.44	0.72	1.17	1.45	1.67
t-statistics	(3.0)	(3.1)	(3.1)	(2.9)	(2.4)
	[-2.1,2.0]	[-2.1,2.0]	[-2.1,2.0]	[-2.1,2.0]	[-2.1,2.0]
R^2 including OUF	0.12	0.09	0.07	0.06	0.04
R^2 restricted $\alpha_n = 0$	0.09	0.05	0.03	0.03	0.02
ΔR^2	0.03	0.04	0.04	0.03	0.02
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]

Table reports results from regressions of one-month excess bond returns on an intercept, seven domestic yields and the overseas unspanned factor - i.e. equation (6). For each holding period the table reports the estimate of the coefficient on the overseas unspanned factor (α_n). Numbers in parentheses report the values of t-statisitcs and numbers in brackets refer to the 95% confidence interval for these t-statistics obtained using the Bauer and Hamilton (2015) bootstrap procedure. The final two rows of each part of the table report the R^2 statistics from models with and without the overseas unspanned factor ('Including OUF' and 'Restricted' respectively). Numbers in brackets refer to the 95% critical value for the change in the R^2. The sample ranges from January 1990 to December 2014.

3.4.5 Incorporating the UK into the analysis

In this sub-section, we show that similar results hold if we extend the analysis to cover the excess returns on UK bonds. We first estimate two overseas unspanned factors using the procedure explained previously: one each from the components of US and German yields that are orthogonal to UK yields. More precisely, we first estimate (1) and then (3) with the US as country j and the UK as country i to obtain an overseas unspanned factor $z_t^{(US)}$. We then repeat the process with Germany as country j to obtain an overseas unspanned factor

Table 14: Regression of excess bond returns on domestic yields and the unspanned overseas factor for 6-month holding period

Maturity (months)	24	36	60	84	120
(a) United States					
α_n	0.41	0.76	1.36	1.81	2.34
t-statistics	(5.8)	(6.5)	(6.9)	(6.8)	(6.4)
	[-3.6,3.6]	[-3.6,3.6]	[-3.7,3.6]	[-3.6,3.7]	[-3.6,3.7]
R^2 including OUF	0.32	0.32	0.33	0.32	0.30
R^2 restricted $\alpha_n = 0$	0.24	0.22	0.22	0.21	0.20
ΔR^2	0.08	0.10	0.11	0.09	0.10
	[0.03]	[0.04]	[0.04]	[0.04]	[0.04]
(b) Germany					
α_n	0.48	0.83	1.37	1.71	2.00
t-statistics	(9.3)	(9.6)	(9.4)	(8.7)	(7.6)
	[-3.7,3.7]	[-3.8,3.8]	[-3.9,3.8]	[-3.9,3.9]	[-4.0,4.0]
R^2 including OUF	0.36	0.36	0.33	0.30	0.27
R^2 restricted $\alpha_n = 0$	0.17	0.15	0.12	0.11	0.11
ΔR^2	0.19	0.21	0.21	0.19	0.16
	[0.05]	[0.06]	[0.06]	[0.06]	[0.06]

Table reports results from regressions of six-month excess bond returns on an intercept, seven domestic yields and the overseas unspanned factor - i.e. equation (6). For each holding period the table reports the estimate of the coefficient on the overseas unspanned factor (α_n). Numbers in parentheses report the values of t-statisitcs and numbers in brackets refer to the 95% confidence interval for these t-statistics obtained using the Bauer and Hamilton (2015) bootstrap procedure. The final two rows of each part of the table report the R^2 statistics from models with and without the overseas unspanned factor ('Including OUF' and 'Restricted' respectively). Numbers in brackets refer to the 95% critical value for the change in the R^2. The sample ranges from January 1990 to December 2014.

$z_t^{(DE)}$. We then assess whether either of these factors contains information for predicting UK returns relative to the information contained in the UK term structure by estimating extended versions of (6):

$$rx_{n,t,t+12}^{(UK)} = \kappa_0 + \boldsymbol{\kappa}'\mathbf{y}_t^{(UK)} + \alpha_{n,US} z_t^{(US)} + \alpha_{n,DE} z_t^{(DE)} + \eta_{n,t+12}^{(UK)} \tag{11}$$

Table 15 reports R^2 coefficients from versions of this regression with different combinations of the overseas unspanned factors. Including either of the overseas unspanned factors causes the R^2 to rise substantially, particularly at short maturities, although the difference is greater when the US factor is added. For example, the model with no overseas unspanned factors has an R^2 of 0.23 for the excess return on the two-year bond; this rises to 0.51 for the model including the US unspanned factor; or 0.37 for the model including the German factor. Including both overseas factors raises the R^2 a little further.

Table 16 reports results from an out-of-sample forecasting exercise for UK returns, analogous to those reported in Section 3.4.2. The best performing model for all maturities is the one that includes both the US and German overseas unspanned factors and the improvement relative to a model that only includes domestic yields is strongly statistically significant according to Giacomini and White (2006) tests. The model with both overseas unspanned factors even out-performs a random walk for maturities longer than five years.

Table 15: United Kingdom excess bond returns regressions

Maturity (months)	24	36	60	84	120
$\alpha_{n,US}$	0.274	0.529	0.860	1.049	1.188
t-statistics	(10.4)	(11.3)	(10.5)	(9.0)	(7.1)
	[-4.5,4.6]	[-4.5,4.6]	[-4.5,4.6]	[-4.5,4.7]	[-4.6,4.7]
$\alpha_{n,DE}$	0.157	0.382	0.740	0.934	0.996
t-statistics	(4.9)	(6.8)	(7.4)	(6.6)	(5.0)
	[-3.9,3.9]	[-3.9,3.9]	[-3.9,3.9]	[-3.9,3.9]	[-4.0,3.9]
(a) Including z_t^{US} and z_t^{DE}	0.547	0.605	0.610	0.580	0.529
(b) Restricted $\alpha_{n,DE} = 0$	0.508	0.540	0.533	0.513	0.487
(c) Restricted $\alpha_{n,US} = 0$	0.372	0.422	0.457	0.457	0.442
(d) Restricted $\alpha_{n,DE} = 0$ and $\alpha_{n,US} = 0$	0.231	0.231	0.255	0.288	0.332
$\Delta R^2 = R^2_{(a)} - R^2_{(d)}$	0.316	0.374	0.355	0.292	0.197
	[0.066]	[0.071]	[0.075]	[0.076]	[0.075]

The table reports results from regressions of UK excess bond returns on a constant, seven domestic yields and two overseas factors for US and Germany - i.e. equation (11). The table reports the estimates of the coefficients on the overseas unspanned factors ($\alpha_{n,US}$ and $\alpha_{n,DE}$). Numbers in parentheses report the values of t-statisitcs and numbers in brackets refer to the 95% confidence interval for these t-statistics obtained using the Bauer and Hamilton (2015) bootstrap procedure. The final four rows report the R^2 statistics from models with different combinations of the two overseas unspanned factors. Numbers in brackets refer to the 95% critical value for the change in the R^2. The sample ranges from January 1990 to December 2014.

Table 16: Root mean squared forecast error of out-of-sample UK excess return predictions

Maturity (months)	24	36	60	82	120
Random walk	1.376	2.533	4.657	6.742	9.607
Restricted $\alpha_{n,DE} = 0$ and $\alpha_{n,US} = 0$	2.273	4.055	6.330	7.689	8.854
Including z_t^{DE}	1.890**	3.355**	5.269**	6.489**	7.732**
Including z_t^{US}	1.778***	3.122***	4.944***	6.203***	7.657**
Including z_t^{US} and z_t^{DE}	1.589***	2.767***	4.399***	5.591***	7.108***

The table reports Root Mean Square Forecast Errors for UK excess bond returns for five forecasting models: a random walk and four restricted and unrestricted versions of equation (11). All model parameters, as well as the OUFs are computed using 10-year rolling samples (i.e. 120 months). All numbers reported are in annualized percentage points. Asterisks indicate significance levels from Giacomini-White test (see Giacomini and White (2006)) assessing the difference of forecasting power between the considered model and the version without either overseas unspanned factor: ***,**, * denote significance at $p = 0.01$, $p = 0.05$ and $p = 0.1$ respectively for the best performing model. The sample ranges from January 1990 to December 2014, implying a forecasting period of January 2000 to December 2014.

4 A dynamic term structure model

4.1 Model

In this section, we use our preceding results to motivate a simple dynamic term structure model. Specifically, for each country we consider a first-order VAR of the form:

$$\begin{aligned} \mathbf{m}_t^{(i)} &= \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{m}_{t-12}^{(i)} + \boldsymbol{\Sigma}\mathbf{v}_t \\ \mathbf{v}_t &\sim i.i.d.\left(\mathbf{0},\mathbf{I}\right). \end{aligned} \tag{12}$$

Here, the 4×1 vector $\mathbf{m}_t = \left[\mathbf{x}_t^{(i)\prime}, z_t^{(j)}\right]^\prime$ collects the first three principal components of domestic yields ($\mathbf{x}_t^{(i)\prime}$) and the overseas unspanned factor ($z_t^{(j)}$); and $\boldsymbol{\Sigma}$ is a lower triangular matrix. We use a lag of twelve months in the VAR, rather than the more standard single month lag in the dynamic term structure literature. We justify this choice by appealing to the results in the previous section: return predictability is substantially stronger at lags of twelve months than one month. We estimate the model using our benchmark sample (i.e. January

1990-December 2014), which means that we have 288 overlapping sample points with which to estimate the model.

We can motivate the choice of three domestic principal components - which is standard in the term structure literature - by referring to a preliminary principal components analysis of domestic yields. In both countries the first three principal components collectively account for more than 99.9% of the variation in the considered bond yields (Table 15). As is standard, the loadings on the first ('level') principal component have the same sign and are relatively constant across maturities. For the second ('slope') principal component, the loadings are increasing with maturity, while for the third ('curvature'), the loadings are higher at very short and very long maturities.

Table 17: Principal component analysis of domestic bond yields

	Cum. prop. explained	PC loadings (maturities in months)						
	(Percentage)	6	12	24	36	60	84	120
(a) United States								
PC1	96.444	0.403	0.414	0.414	0.401	0.366	0.334	0.298
PC2	99.859	-0.506	-0.386	-0.153	0.034	0.288	0.435	0.546
PC3	99.985	0.551	0.069	-0.403	-0.480	-0.215	0.130	0.485
PC4	99.998	0.409	-0.408	-0.365	0.086	0.489	0.249	-0.474
PC5	100.000	-0.313	0.604	-0.168	-0.418	0.168	0.447	-0.328
PC6	100.000	0.112	-0.348	0.524	-0.215	-0.436	0.562	-0.200
PC7	100.000	-0.035	0.150	-0.456	0.616	-0.528	0.324	-0.072
(b) Germany								
PC1	97.604	0.412	0.407	0.400	0.391	0.368	0.345	0.313
PC2	99.852	-0.501	-0.387	-0.170	0.016	0.281	0.434	0.551
PC3	99.994	0.589	0.013	-0.421	-0.463	-0.208	0.101	0.456
PC4	99.999	-0.430	0.548	0.278	-0.201	-0.432	-0.158	0.431
PC5	100.000	-0.202	0.537	-0.323	-0.368	0.294	0.439	-0.390
PC6	100.000	0.080	-0.291	0.556	-0.344	-0.350	0.560	-0.213
PC7	100.000	0.022	-0.104	0.378	-0.583	0.589	-0.389	0.087

Table reports the cumulative proportion of the variation in the considered yields explained by successive principal components (PCs) and loadings of PC on different maturity bond yields. The sample used ranges from January 1990 to December 2014.

The model (12) specifies the time-series dynamics of the factors that drive bond yields, analogous to (A.9) in a standard no-arbitrage term structure model. Given that the domestic yield curve factors are principal components of yields, our model also has an affine cross-sectional mapping between the factors and current yields:

$$\mathbf{y}_t^{(i)} = \mathbf{A}^{(i)} + \mathbf{B}^{(i)}\mathbf{x}_t^{(i)} + \mathbf{w}_t^{(i)}. \tag{13}$$

Here, $\mathbf{y}_t^{(i)} = \left[y_{6,t}^{(i)}, y_{12,t}^{(i)}, ..., y_{120,t}^{(i)}\right]'$ is a 7×1 vector of bond yields observed at time t; the coefficients $\mathbf{A}^{(i)}$ and $\mathbf{B}^{(i)}$ are determined by the relevant principal component loadings; and $\mathbf{w}_t^{(i)}$ is a measurement error.

We identify the impact of shocks to the overseas unspanned factor using a Cholesky factorization, ordering it last in the VAR (12). While Cholesky identification is sensitive to the ordering of variables in the VAR, ordering the overseas unspanned factor last makes intuitive sense in this case, given that it is orthogonal to domestic yields by construction: the assumption that $\mathbf{\Sigma}$ is lower triangular means that a shock to the final element of $\mathbf{v}_t$ is one that has an impact on the overseas unspanned factor but no contemporaneous impact on domestic yields.

4.2 Results

Our results suggest that the impact of shocks to overseas unspanned factors on domestic bond yields can be substantial and persistent. Figure 3 shows impulse response functions for the local principal components from the model with Germany as the domestic country. Following a one standard deviation shock to the overseas unspanned factor the first principal component (i.e. the level of the yield curve) falls and the second and third principal components (i.e. the slope and curvature) rise, although the effect on the level is much larger and more persistent than on the other domestic principal components. Figure 4 translates this into the reaction of yields of different maturities. The shock is followed by a drop in domestic yields (as explained above, there is no contemporaneous reaction by construction). This reaction is largest for short maturity yields: six-month to three-year yields all fall by around 50 basis points twelve months after the shock, while the fall in the ten-year yield is only about 30 basis points. The peak impact on short maturity yields comes after two years but longer maturity yields continue to fall for four years after the shock. After about seven years, the remaining effect is roughly equal across the yield curve (as the impacts on the slope and curvature factors have largely died out).

<Insert Figure 3 about here>

<Insert Figure 4 about here>

Figure 5 decomposes the variance of forecast errors for selected maturity yields into the contributions from innovations to different factors, for different forecast horizons.[9] Panel (a) shows results for the one-year yield, panel (b) for the five-year yield and panel (c) for the ten-year yield. At short forecast horizons, the majority of the forecast errors are explained by the level factor, with a smaller contribution from the slope and a negligible contribution from the curvature. The contribution of the overseas unspanned factor grows with maturity; at the ten-year forecast horizon it accounts for more than 40% of the variance of forecast horizons, with the largest contribution at shorter maturities. At forecast horizons longer than three or four years (depending on the maturity) the overseas unspanned factor accounts for more of the forecast error variance than the level factor.

<Insert Figure 5 about here>

Figures 6 and 7 report the equivalent impulse response functions from the model with the US as the domestic country. Similar to the case of Germany, the US level factor falls following the shock to the overseas unspanned factor, although the impact is much less persistent. The peak response of all yields (around 30 to 40 basis points) comes twelve months following the shock. The forecast error variance decompositions are also somewhat different for the US (Figure 8): the proportion explained by the overseas unspanned factor is somewhat smaller than for Germany, although it still reaches about 15% for the ten-year forecast horizon (with shocks to all of the domestic yield curve factors playing a relatively more important role in explaining forecast errors).

[9] Appendix A explains how these are computed.

<Insert Figure 6 about here>

<Insert Figure 7 about here>

<Insert Figure 8 about here>

4.3 Robustness: a dynamic term structure model for the UK

In this sub-section, we examine whether we obtain similar results if we estimate a dynamic term structure model of UK yields that includes both US and German unspanned factors, constructed as described in Section 3.4.4. The US factor is the penultimate variable in the time-series VAR and the German variable is the final variable; this implies that shocks to the US factor can have a contemporaneous impact on the German factor but not vice versa. First, Figures 9 and 10 show the responses of UK yields following a one standard deviation shock to the US and German unspanned factors respectively. Similar to the results for the two-country models reported above, yields fall following the shock - in this case, by up to about 50 basis points - and the effect is persistent. The overseas unspanned factors explain a substantial part of the forecast error variance of yields (Figure 11). For example, for the ten-year yield each accounts for 10-20% of the variance of ten-year ahead forecast errors.

<Insert Figure 9 about here>

<Insert Figure 10 about here>

<Insert Figure 11 about here>

4.4 Do global factor models capture all of the relevant information for forecasting returns?

Our results suggest that modelling yields using only factors extracted from the same set of domestic yields is likely to omit important information for the dynamics of yields. The specification considered in this section - with an overseas unspanned factor added to a VAR of domestic principal components - is one way of incorporating the relevant information in overseas yields. Other studies that have attempted to model yields in multiplier jointly take different approaches to specifying the factor structure, with different numbers (and definitions) of 'global' and 'local' factors (e.g. Diebold et al. (2008), Bauer and Diez de los Rios (2012) and Kaminska et al. (2013)). What matters, however, is whether the factors include capture the relevant information for the time-series dynamics of yields. We therefore consider the extent to which different choices about the factor structure affect the extent to which we capture the relevant information as measured by the overseas unspanned factor.

One extreme approach to specifying a factor structure for a joint model of domestic and foreign yields would simply be to stack together principal components extracted from yields of each country separately. Above we showed that three domestic and three overseas principal components (i.e. six factors in total) together explain more than 70% of the variation in the overseas unspanned factors and together account for the large majority of their return-forecasting power. Reducing this to two principal components for each country (i.e. four factors in total) can have a substantial impact, with the four principal components explaining less than 10% of the variation in the German factor unspanned by US yields (Table 18). The other extreme is to specify a model that uses only principal components extracted from the pooled data set (i.e. 'global' factors). Two global factors are sufficient to explain just over half of the variation in the US unspanned factor, providing some evidence that a relatively parsimonious joint model could capture most of the important information in the factor, although five global factors still only explain about 70% of the variation, so there is clearly some information missing. The results are much less encouraging for the German unspanned factor, with the first four global principal components explaining just 6% of the variation in the factor. It is not until we get to the fifth global factor that we can start to explain a reasonable portion of the variation in the German unspanned factor. This suggests that while joint modelling of yields across countries has an obvious appeal, there is a risk that adopting parsimonious factor structures means that we would actually miss out the important information for forecasting returns.

Table 18: Regressions of overseas unspanned factors on principal components of yields

	US factor unspanned by German yields	German factor unspanned by US yields
(a) Principal components extracted separately from US and German yields (R^2)		
1 US, 1 German	0.53	0.00
2 US, 2 German	0.65	0.08
3 US, 3 German	0.71	0.77
4 US, 4 German	0.83	0.80
5 US, 5 German	0.94	0.85
(b) Principal components of pooled US and German yields (R^2)		
1 global	0.03	0.00
2 global	0.52	0.00
3 global	0.58	0.01
4 global	0.64	0.06
5 global	0.70	0.75

Panel (a) shows R^2s from regressions of overseas unspanned factors on a constant plus different numbers of principal components extracted from US and German yields separately. Panel (b) shows equivalent results using principal components of pooled US and German yields ('global' principal components).

5 Conclusions

The recent literature on unspanned factors in the term structure of interest rates argues that there is a non-trivial portion of information that is not contained in the yield curve, but helps to predict yields' dynamics. We argue that there is important information contained in foreign yields, which is not contained in (spanned by) domestic yields and that helps to predict future moves of domestic yields.

More specifically, we show that there is important information spanned by the German yield curve, but unspanned by the US yield curve, which helps forecasting future dynamics of US yields and vice versa. We use simple return-forecasting regressions to prove that the overseas unspanned factors matter, both in- and out-of-sample. We also show that this result is robust to different sample selections as well as to different specification of domestic yield curve factors. In addition, we find that it is not only a US-DE phenomenon. We also show that US and German factors unspanned by the UK yield curve have substantial predictive power for UK yields. An advantage of the modular structure of our approach for adding different countries mean that this analysis would be straightforward to extend to other countries.

Our results are especially important for dynamic factor models of bond yields. Current state of the art models focus only on domestic yields, hence, in the light of our findings, they lack important information and are potentially misspecified. In fact, when we enrich simple dynamic term structural model, consisting of the first 3 principal components, with overseas unspanned factor we find that shocks to this factor drive sizeable portions of future yields variation. This effect is especially pronounced for German and UK yields, but is also significant for US yields.

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Appendix

A Motivation for the dynamic term structure model

Although we do not estimate no-arbitrage term structure models in this paper, we can nevertheless motivate our empirical exercises by appealing to the standard Gaussian dynamic no-arbitrage affine term structure models (ATSM) of Duffie and Kan (1996) and Duffee (2002). These models have four basic building blocks. First, the assumption of no arbitrage implies that the price at time t of an n-period default-free zero-coupon bond ($P_t^{(n)}$) is given by

$$P_t^{(n)} = E_t^{\mathbb{Q}} \left[\exp\left(-r_t\right) P_{t+1}^{(n-1)} \right], \tag{A.1}$$

where r_t is the risk-free one-period interest rate and expectations are formed with respect to the risk-neutral probability measure, denoted $\mathbb{Q}$. Second, the short-term interest rate is an affine function of a $K \times 1$ vector of unobserved pricing factors ($\mathbf{x}_t$):

$$r_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t. \tag{A.2}$$

Third, the pricing factors follow a Gaussian Vector Autoregression (VAR) under $\mathbb{Q}$:

$$\begin{aligned} \mathbf{x}_{t+1} &= \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}} \mathbf{x}_t + \boldsymbol{\Sigma} \mathbf{v}_{t+1}^{\mathbb{Q}} \\ \mathbf{v}_{t+1}^{\mathbb{Q}} &\sim i.i.d.\mathcal{N}\left(\mathbf{0}, \mathbf{I}\right). \end{aligned} \tag{A.3}$$

Under these assumptions, n-period bond yields turn out to be affine functions of the state variables:

$$y_t^{(n)} = -\frac{1}{n} \log P_t^{(n)} = -\frac{1}{n} \left(a_n + \mathbf{b}_n' \mathbf{x}_t \right), \tag{A.4}$$

where the coefficients a_n and $\mathbf{b}_n$ follow the recursive equations

$$a_n = a_{n-1} + \mathbf{b}_{n-1}' \boldsymbol{\mu}^{\mathbb{Q}} + \frac{1}{2} \mathbf{b}_{n-1}' \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathbf{b}_{n-1} - \delta_0 \tag{A.5}$$

$$\mathbf{b}_n' = \mathbf{b}_{n-1}' \boldsymbol{\Phi}^{\mathbb{Q}} - \boldsymbol{\delta}_1. \tag{A.6}$$

Finally, the Radon-Nikodym derivative which relates the time-series and risk-neutral dynamics takes the form

$$\left(\frac{d\mathbb{P}}{d\mathbb{Q}} \right)_{t+1} = \exp \left[-\frac{1}{2} \boldsymbol{\lambda}_t' \boldsymbol{\lambda}_t + \boldsymbol{\lambda}_t' \mathbf{v}_{t+1} \right] \tag{A.7}$$

where the prices of risk ($\boldsymbol{\lambda}_t$) are affine in the pricing factors, as proposed by Duffee (2002):

$$\boldsymbol{\lambda}_t = \boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t\right). \tag{A.8}$$

This implies that the factors also follow a Gaussian VAR(1) under the time-series measure:

$$\begin{aligned} \mathbf{x}_{t+1} &= \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\Sigma}\mathbf{v}_{t+1} \\ \mathbf{v}_{t+1} &\sim i.i.d.\mathcal{N}\left(\mathbf{0}, \mathbf{I}\right), \end{aligned} \tag{A.9}$$

where

$$\boldsymbol{\mu} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Sigma}\boldsymbol{\lambda}_0 \tag{A.10}$$

$$\boldsymbol{\Phi} = \boldsymbol{\Phi}^{\mathbb{Q}} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1. \tag{A.11}$$

In a model with unobserved factors, we must impose additional identification restrictions. Here we consider the normalization of Dai and Singleton (2000),[10] where $\boldsymbol{\delta}_1 = \mathbf{1}$, $\boldsymbol{\mu}^{\mathbb{Q}} = \mathbf{0}$, $\boldsymbol{\Phi}^{\mathbb{Q}}$ is a diagonal matrix and $\boldsymbol{\Sigma}$ is lower triangular.[11] All other parameters are unrestricted.

To capture the case where we have some factors that are unspanned by current yields, we can partition the vector of factors into $K_s < K$ factors that are spanned by the yield curve ($\mathbf{x}_t^s$) and $K - K_s$ unspanned factors ($\mathbf{x}_t^u$), i.e. $\mathbf{x}_t = \begin{bmatrix} \mathbf{x}_t^{s\prime} & \mathbf{x}_t^{u\prime} \end{bmatrix}^\prime$, as in Joslin et al. (2014). Given the normalization that $\boldsymbol{\Phi}^{\mathbb{Q}}$ is diagonal, the assumption that $\mathbf{x}_t^u$ is unspanned implies zero restrictions on the elements of $\boldsymbol{\delta}_1$ corresponding to the unspanned factors, i.e. $\boldsymbol{\delta}_1 = \begin{bmatrix} \mathbf{1}^\prime_{K_s \times 1} & \mathbf{0}^\prime_{(K-K_s)\times 1} \end{bmatrix}^\prime$ (where we have also imposed that the elements of $\boldsymbol{\delta}_1$ corresponding to the spanned factors are normalized to one, as explained above). It is not possible to identify the prices of unspanned factors in such a model, so we can set the corresponding elements of the prices of risk to zero, i.e.:

$$\begin{aligned} \boldsymbol{\lambda}_0 &= \begin{bmatrix} \boldsymbol{\lambda}_0^s \\ \mathbf{0}_{(K-K_s)\times 1} \end{bmatrix} \\ \boldsymbol{\Lambda}_1 &= \begin{bmatrix} \boldsymbol{\Lambda}_1^{ss} & \boldsymbol{\Lambda}_1^{su} \\ \mathbf{0}_{(K-K_s)\times K_s} & \mathbf{0}_{(K-K_s)\times(K-K_s)} \end{bmatrix}. \end{aligned}$$

[10] Other normalizations are feasible, e.g. the scheme proposed by Joslin et al. (2011).

[11] Hamilton and Wu (2012) show that identification also requires an additional restriction on the ordering of the elements of $\boldsymbol{\Phi}^{\mathbb{Q}}$.

The one-period excess return on an n-period bond is defined as

$$rx_{t+1}^{(n-1)} = \log P_{t+1}^{(n-1)} - \log P_t^{(n)} - r_t. \tag{A.12}$$

Using (A.2), (A.4)-(A.6) and (A.9)-(A.11) in (A.12) gives[12]

$$rx_{t+1}^{(n-1)} = -\frac{1}{2}\mathbf{b}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathbf{b}_{n-1} + \mathbf{b}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\lambda}_0 + \mathbf{b}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\mathbf{x}_t + \mathbf{b}_{n-1}'\boldsymbol{\Sigma}\mathbf{v}_{t+1}. \tag{A.13}$$

The first two terms on the right-hand side of (A.13) are constant. The final term is the unexpected component of excess returns. The third term captures the time-variation in expected returns, which depends on the price of risk parameters ($\boldsymbol{\Lambda}_1$). Taking expectations of both sides of (A.13) gives

$$E_t\left[rx_{t+1}^{(n-1)}\right] = -\frac{1}{2}\mathbf{b}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathbf{b}_{n-1} + \mathbf{b}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\lambda}_0 + \mathbf{b}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\mathbf{x}_t \tag{A.14}$$

which is equivalent to equation (15) in Cochrane and Piazessi (2008). Our reduced-form regressions reported in Section 3 involve regressing excess returns of different maturities on a constant and various factors, some of which are extracted from domestic yields and some of which are unspanned by domestic yields by construction. We can motivate these regressions by appealing to (A.14): if an unspanned factor has a non-zero slope coefficient in these unrestricted regressions, this factor must affect the price of one or more of the spanned factors.[13] And if an unspanned factor enters the price of risk, it must also enter the time series dynamics of yields (A.9), which have an analogous specification to that of the dynamic factor model reported in Section 4 (as explained above, the only difference between an ATSM and our factor model is in the cross-sectional relationship between factors and yields, which is unlikely to make a material difference to our results).

B Bauer and Hamilton (2015) bootstrap procedure

In this appendix, we explain how we implement the procedure for computing confidence intervals for the return-forecasting regressions proposed by Bauer and Hamilton (2015). Our return-forecasting regressions take the general form

$$rx_{n,t,t+h}^{(i)} = \kappa_0 + \kappa'\mathbf{y}_t^{(i)} + \alpha_n z_t^{(j)} + \eta_{n,t+12}^{(i)} \tag{B.1}$$

[12] Abrahams et al. (2015) provide a fuller derivation of the following equation.

[13] We can illustrate this easily using a 2×2 example in which the second factor is unspanned (i.e. the loading on this factor is $b_{n-1,2} = 0$ for all n). In this case, the third term simplifies to $\mathbf{b}_{n-1}'\boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\mathbf{x}_t = b_{n-1,1}\sigma_{11}\left(\lambda_{11}x_{1,t} + \lambda_{12}x_{2,t}\right)$. A non-zero slope coefficient on $x_{2,t}$ in a regression of excess returns on a constant and the factors requires that $\lambda_{12} \neq 0$, i.e. that the unspanned factor affects the price of the spanned factor.

Bauer and Hamilton propose a bootstrap procedure to simulate the distribution of the coefficient on the unspanned factor (i.e. α_n) and the increase in the R^2 of (B.1) resulting from the inclusion of the unspanned factor $z_t^{(j)}$ under the null hypothesis that $\alpha_n = 0$. In our implementation of their procedure, we first estimate separate VAR(1) models for the spanned yield curve factors for country i (in our case, $\mathbf{y}_t^{(i)} = \left[y_{6,t}^{(i)}, y_{12,t}^{(i)}, y_{24,t}^{(i)}, ..., y_{120,t}^{(i)}\right]'$) and the unspanned factor for country j $z_t^{(j)}$:

$$\mathbf{y}_{t+1}^{(i)} = \boldsymbol{\mu}_y + \boldsymbol{\Phi}_y \mathbf{y}_t^{(i)} + \mathbf{v}_{t+1}^{(i)} \tag{B.2}$$
$$z_{t+1}^{(j)} = \mu_z + \phi_z z_t^{(j)} + w_{t+1}^{(j)} \tag{B.3}$$

We assume that yields with maturities other than those included in $\mathbf{y}_t^{(i)}$ (such as the six-year yield) are given by affine functions of $\mathbf{y}_t^{(i)}$, i.e.

$$y_{n,t}^{(i)} = a_n + b_n' \mathbf{y}_t^{(i)} + e_{n,t}^{(i)} \tag{B.4}$$

Given these assumptions, we use a residual bootstrap to produce 10,000 draws of the domestic yields for country i and the country-j unspanned factor. In each bootstrapped sample, the country-j unspanned factor has no predictive power for domestic returns by construction (consistent with the null hypothesis). For each bootstrapped sample, we compute 12-month returns on domestic yields with maturities of $1, 2, ..., 10$ years, obtaining the required yields not included in $\mathbf{y}_t^{(i)}$ using (B.4).[14] We then estimate (B.1) for each bootstrapped sample, as well as a restricted version with $\alpha_n = 0$. The critical values for α_n and the increase in R^2 reported in the text are the $97.5th$ percentile of the bootstrapped distributions.

C Forecast error variance decompositions

Forecast error variance decomposition is another useful tool to assess the impact of unspanned overseas factors. In order to compute variance decomposiotions of yields' forecast errors, we assume that there is no measurement error in (13). Using our factor specification we can then re-write (12) as:

$$\mathbf{y}_t^{(i)} = \mathbf{A}^{(i)} + \widetilde{\mathbf{B}}^{(i)} \mathbf{f}_t^{(i)}, \tag{C.1}$$

where $\widetilde{B} = \left[\begin{smallmatrix} B^i \\ 0 \end{smallmatrix}\right]$. This allows us to map forecast variance error decomposition of different factors into forecast variance decomposition of yields. As we look at annual forecasting horizons, for simplicity we drop monthly time notation and denote time in annual units, ex. t+1

[14] Unlike Bauer and Hamilton (2015), we ignore the measurement error on these yields in the bootstrap. Given that we are effectively estimating a seven-factor yield curve model, these measurement errors will be tiny.

means t plus 1 year. Taking into account (equation C.1), we can define h-year forecast Mean Squared Forecast Errors matrix as:

$$\Omega_y(h) = \sum_{i=0}^{h-1} \left(\widetilde{B}\Phi^i \Sigma \Sigma' (\Phi^i)' \widetilde{B}' \right). \tag{C.2}$$

Note that $\Phi^i\Sigma$ is simply the i-th parameters of the VMA representation of VAR(12). More importantly diagonal elements of $\Omega_y(h)$ are the h-year MSFE of the j-th yield - $\Omega_{y_j}(h)$. The contribution of innovations in factor k to the h-year MSFE of yield j is given by:

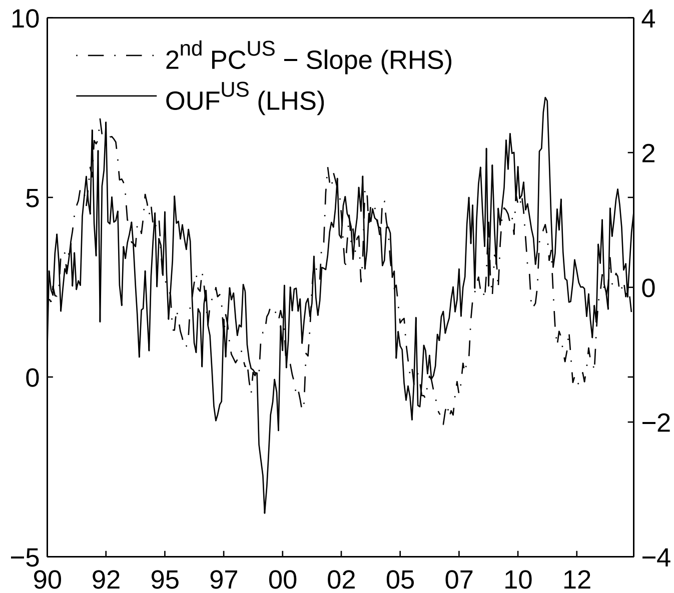
$$\sum_{i=0}^{h-1} \left(e_j' \widetilde{B}\Phi^i \Sigma e_k \right)^2, \tag{C.3}$$

where e_j is the j-th colum of the identity matrix. Dividing the contribution (C.3) by total h-year MSFE of j-th yields we obtain the proportion of the h-year ahead forecast error variance of yield j accounted by an innovation to the k-th factor:

$$\omega_{j,k,h} = \frac{\sum_{i=0}^{h-1} \left(e_j' \widetilde{B}\Phi^i \Sigma e_k \right)^2}{\Omega_{y_j}(h)} \tag{C.4}$$

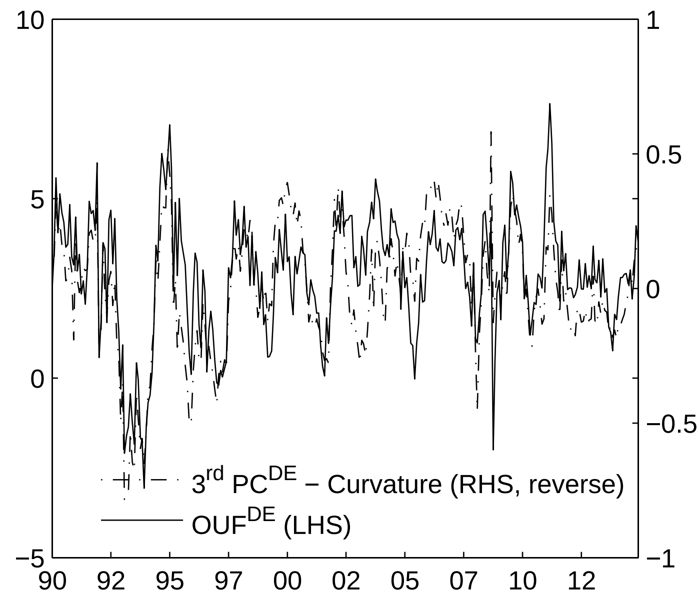
D Figures

Figure 1: Overseas unspanned factor extracted from US yield curve.



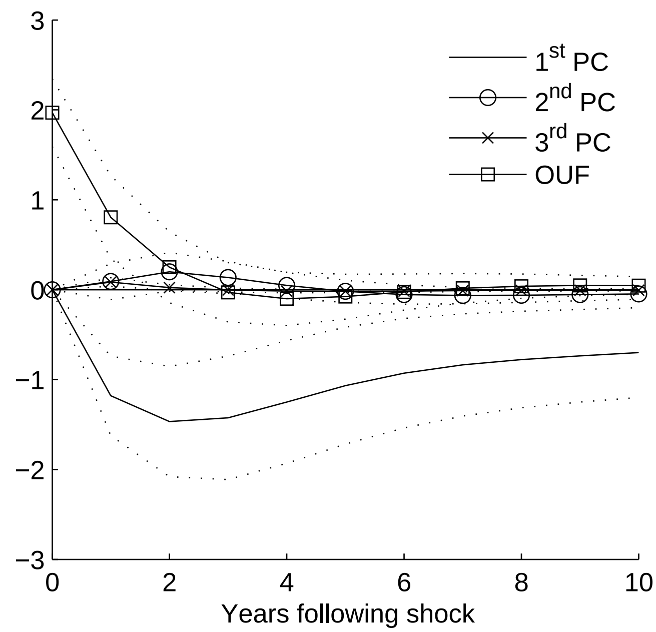
The figure depicts the overseas unspanned factor (OUF^{US}) extracted from US yield curve, i.e. the component of the US yield curve that is unspanned by German yields. $2^{nd}PC^{US}$ depicts the second principle component (slope) of US yield curve. Correlation between two series amounts to 54.1%.

Figure 2: Overseas unspanned factor extracted from German yield curve.



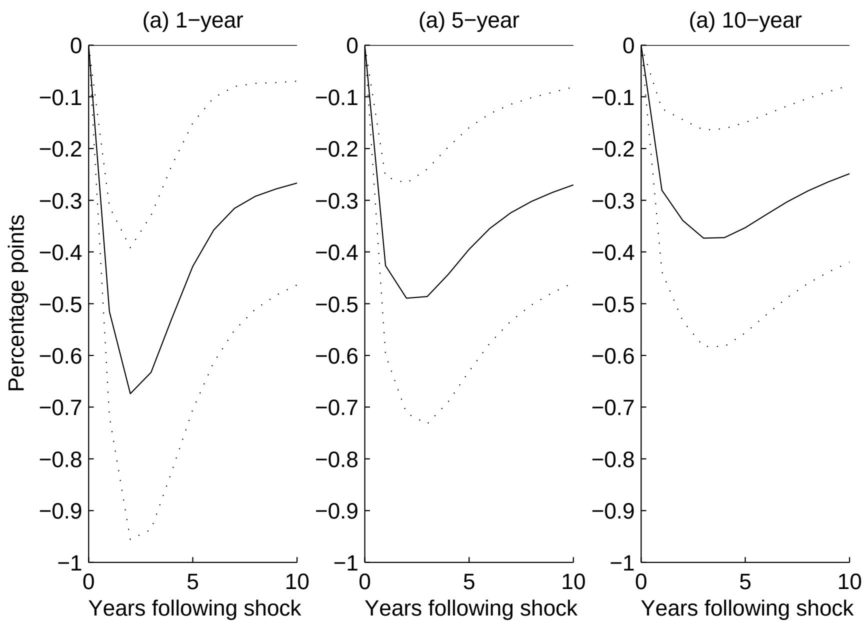
The figure depicts the overseas unspanned factor (OUF^{DE}) extracted from German yield curve, i.e. the component of the German yield curve that is unspanned by US yields. $3^{rd}PC^{DE}$ depicts the third principal component ('curvature') of German yield curve. Correlation between two series amounts to 77.1%.

Figure 3: German yield curve factors response to an innovation in the unspanned overseas factor.



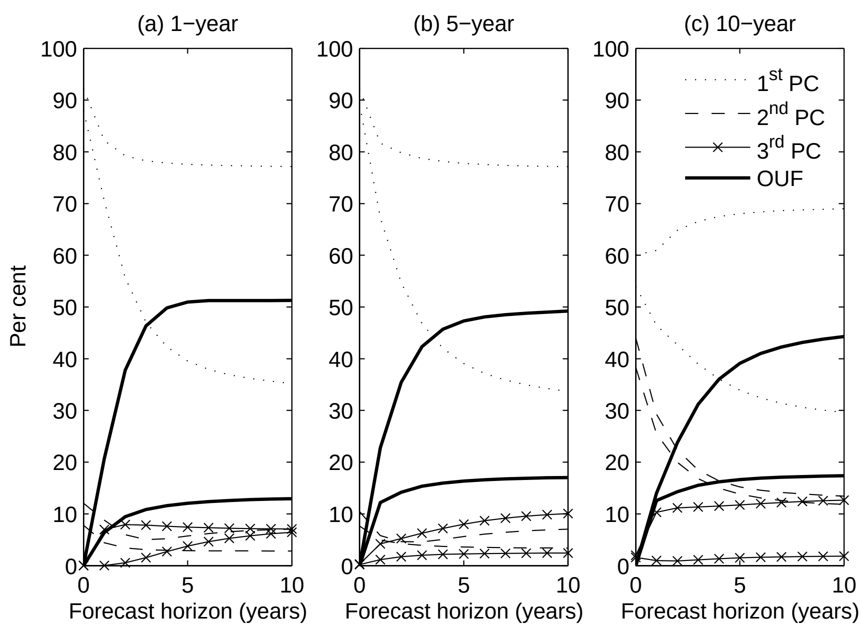
The figure depicts impulse responses of the first three principal components of German yield curve (1^{st} PC, 2^{nd} PC and 3^{rd} PC) and unspanned overseas factor (OUF) to a one standard deviation shock to the unspanned overseas factor.

Figure 4: German yields response to an innovation to the unspanned overseas factor.



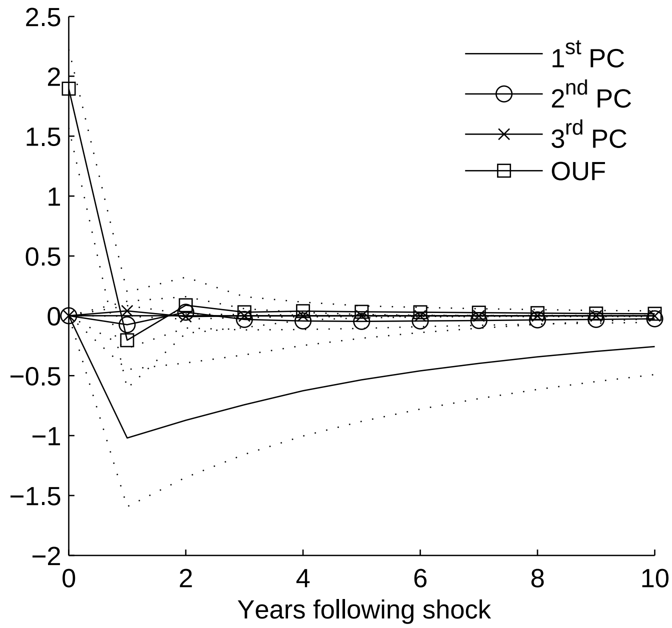
The figure depicts impulse response of different maturity German yields to a one standard deviation shock to the unspanned overseas factor (OUF).

Figure 5: Forecast error variance decomposition of German yields.



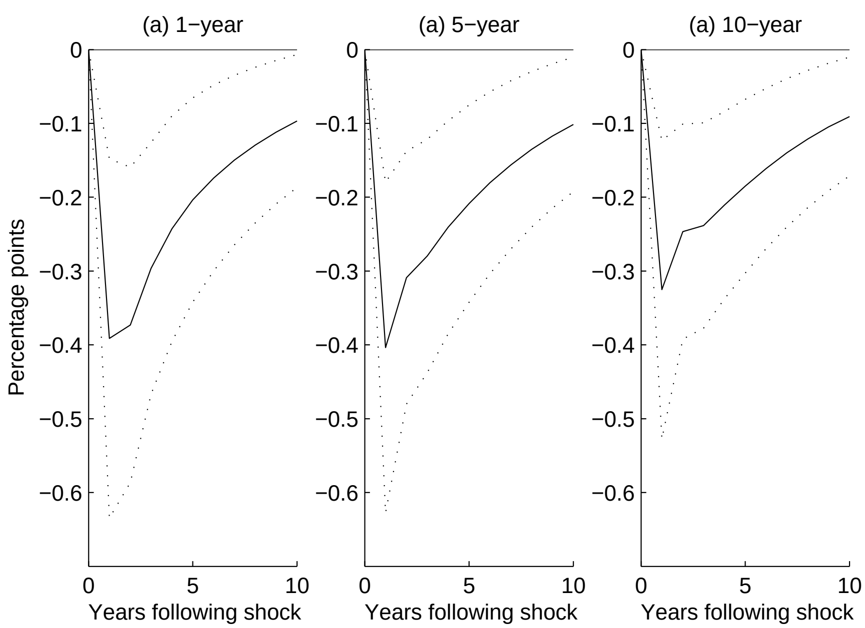
The figure depicts forecast error variance decompositions for three different yields for forecast horizons of up to 10 years. Panels (a), (b) and (c) report decompositions for 1-year, 5-year and 10-year German yields, respectively. Each panel shows the proportion of the yield forecast error variance accounted by the first three principal components (1^{st} PC, 2^{nd} PC and 3^{rd} PC) of the German yield curve and the unspanned overseas factor (OUF).

Figure 6: US yield curve factors response to an innovation in the unspanned overseas factor.



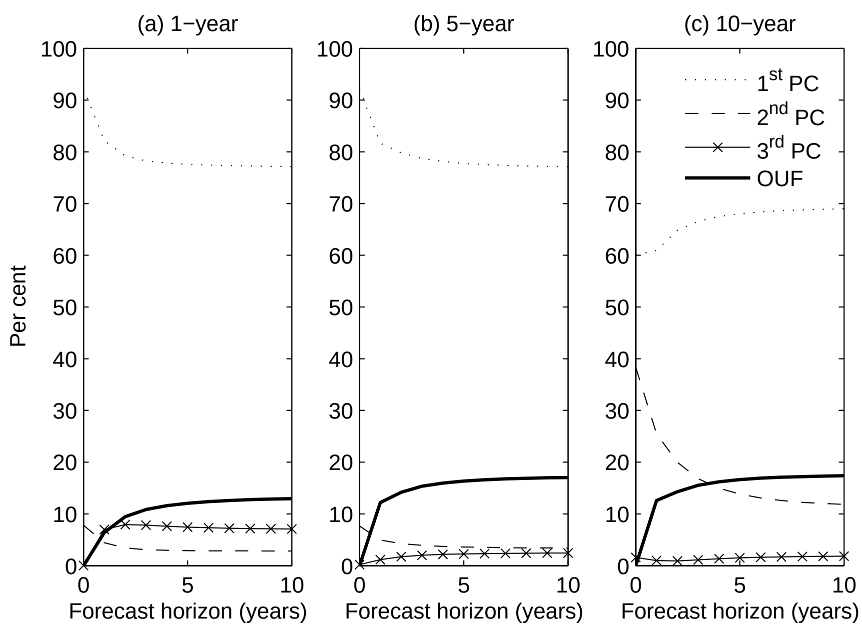
The figure depicts impulse responses of the first three principal components of US yield curve (1^{st} PC, 2^{nd} PC and 3^{rd} PC) and the unspanned overseas factor (OUF) to a one standard deviation shock to the unspanned overseas factor.

Figure 7: US yields response to an innovation to the unspanned overseas factor.



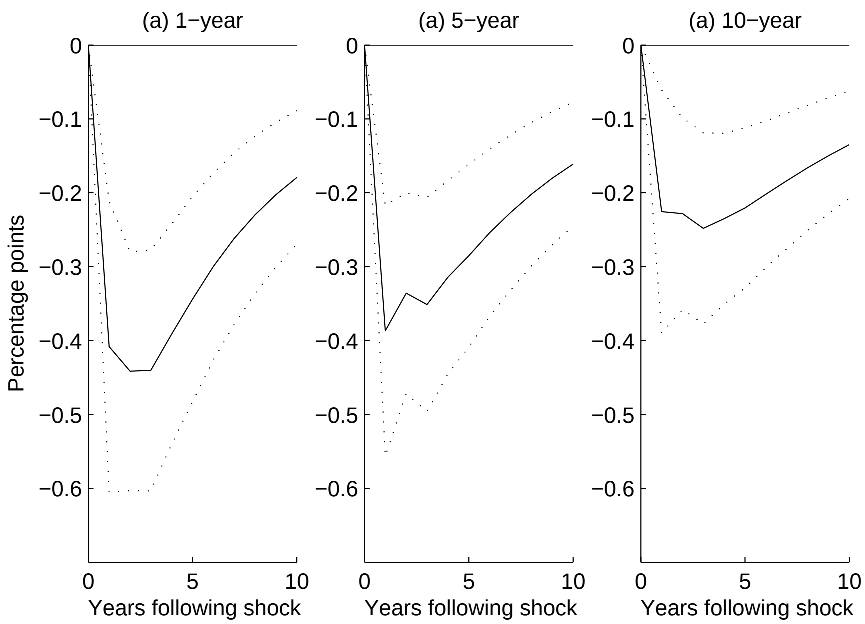
The figure depicts impulse response of different maturity US yields to a one standard deviation shock to the unspanned overseas factor (OUF).

Figure 8: Variance decomposition of US yields.



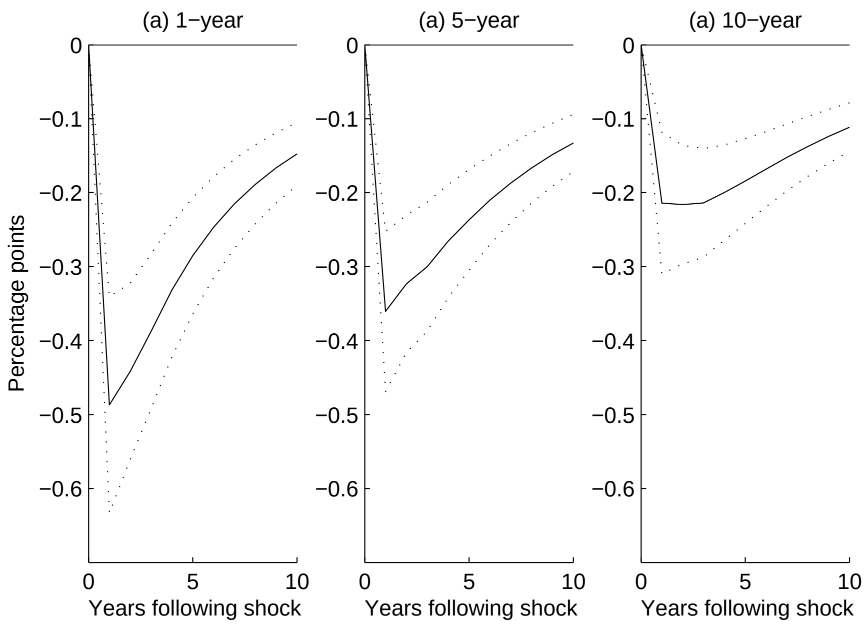
The figure depicts forecast error variance decompositions for three different yields for forecast horizons of up to 10 years. Panels (a), (b) and (c) report decompositions for 1-year, 5-year and 10-year US yields, respectively. Each panel shows the proportion of the yield forecast error variance accounted by the first three principal components (1^{st} PC, 2^{nd} PC and 3^{rd} PC) of the US yield curve and the unspanned overseas factor (OUF).

Figure 9: UK yields response to an innovation to the German unspanned overseas factor.



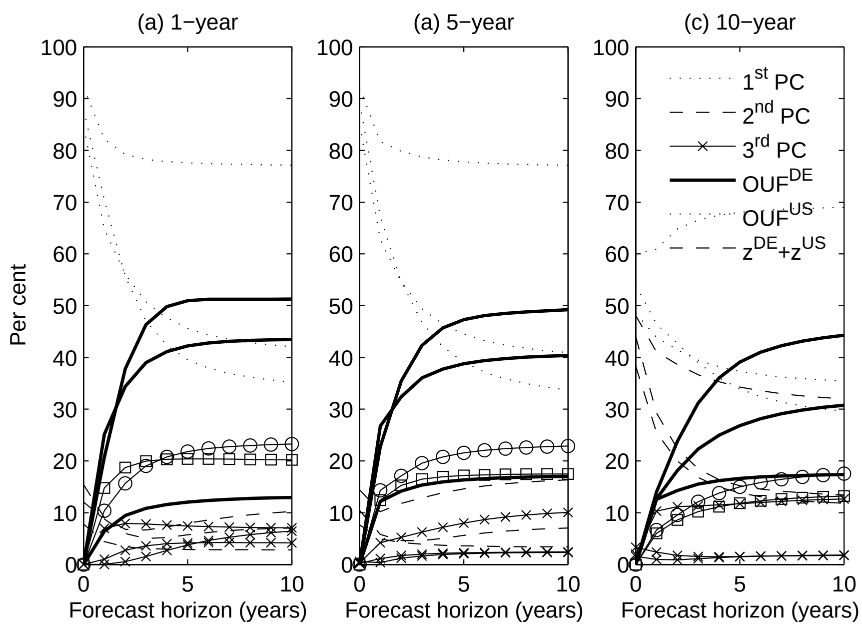
The figure depicts impulse response of different maturity UK yields to a one standard deviation shock to the German unspanned overseas factor (z_t^{DE}).

Figure 10: UK yields response to an innovation to the US unspanned overseas factor.



The figure depicts impulse response of different maturity UK yields to a one standard deviation shock to the German unspanned overseas factor (z_t^{US}).

Figure 11: Variance decomposition of UK yields.



The figure depicts forecast error variance decompositions for three different yields for forecast horizons of up to 10 years. Panels (a), (b) and (c) report decompositions for 1-year, 5-year and 10-year UK yields, respectively. Each panel shows the proportion of the yield forecast error variance accounted by the first three principal components (1^{st} PC, 2^{nd} PC and 3^{rd} PC) of the UK yield curve and the two unspanned overseas factors, namely: the German unspanned overseas factor (z_t^{DE}) and the US unspanned overseas factor (z_t^{US}).