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Risk shocks close to the zero lower bound

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Abstract

Large risk shocks give rise to cost-push effects in the canonical New Keynesian model. At the same time, monetary policy becomes less effective. Therefore, stochastic volatility introduces occasional trade-offs for monetary policy between inflation and output gap stabilisation. The cost-push effects operate through expectational responses to the interaction between shock volatility and the zero lower bound (ZLB) on interest rates. Optimal monetary policy calls for potentially sharp reductions in the interest rate when risk is elevated, even if this risk never materialises. Close to the ZLB, small risk shocks become 'large' in this sense. If policy is initially constrained by the ZLB, lift-off is optimally delayed when risk increases.

Key words: Risk shocks, uncertainty, zero lower bound on interest rates, optimal monetary policy.

JEL classification: E52, E58.

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1 Introduction

Large risk shocks give rise to cost-push effects in the canonical New Keynesian model. At the same time, monetary policy becomes less effective. Consequently, stochastic volatility introduces occasional trade-offs for monetary policy between nominal and real stability. The cost-push effects operate through expectational responses to the interaction between shock volatility and the Zero Lower Bound on interest rates.

By risk shocks, I mean changes in the second moments of conventional level shocks that affect behavioral relations in the model. Throughout, I focus on the economy's response to such changes along the zero-shock path, i.e. the trajectory of the economy through the state space along which innovations to the level shocks do not actually occur. As risk never materialises along this path – as it were, nothing actually happens in this paper – the effects can be thought of as responses to the *perception* of risk, where risk is defined as the set of variances of the independent distributions from which innovations to level shock processes are drawn.[1]

Following the work of Bloom (2009), there has been a surge of interest in the responses of the economy to changes in risk. While the empirical literature has struggled to identify structural risk shocks from the volatility measures that are usually taken to be proxies for risk and uncertainty, the theoretical literature provides clear channels through which risk shocks may affect the economy as discussed in the recent survey by Bloom (2014).

Here, I emphasise a further 'bad news channel' (Bernanke, 1983) arising from the inability of monetary policy to respond to large adverse shocks because of the ZLB.[2] This follows closely the analyses of the implications for optimal discretionary monetary policy of the *presence* of risk in models with a ZLB by Adam and Billi (2007) and Nakov (2008) as well as more recently by Nakata and Schmidt (2014) and Evans et al. (2015). But further to these studies, I both analyse the stochastic steady state and explicitly allow for *variation* in risk. This allows me to trace out the dynamic responses of the economy to persistent movements in risk around both high and low risk steady states at and away from the ZLB. In addition, I show how such risk shocks may lead to trade-offs for monetary policy, even if risk does not actually materialise in innovations to level shocks.

[1] Even so, to remain loyal to the Knightian distinction between risk and uncertainty, I refer to the changes in second moments as *risk* rather than *uncertainty* shocks; within the context of the model, agents actually assign a number to the risk that they perceive, though it may well be that elevated risk in the model stands in for Knightian uncertainty in reality.

[2] The effective lower bound does not have to be exactly zero, but unconventional policy tools such as Quantitative Easing are assumed to be imperfect substitutes for conventional interest rate instruments.

The analysis complements recent work by Basu and Bundick (2015), who study risk shocks in a fully non-linear version of the New Keynesian model. The authors show that risk shocks have large adverse effects at the ZLB when monetary policy follows a simple instrument rule away from it. In line with the results in Eggertsson and Woodford (2003), Adam and Billi (2006) and Nakov (2008), a credible commitment to future stimulus according to an optimal policy plan is a powerful strategy to alleviate the negative effects of risk shocks when current policy is constrained by the ZLB in their model.

In this paper, by contrast, I focus on optimal discretion. While less potent at the ZLB, such a policy regime is arguably a more realistic description of actual monetary policy as it does not require policymakers to commit future incumbents to a time-inconsistent plan (see for example Bean, 2013). Neither does it require policymakers to follow an instrument rule mechanically. In addition, the simple quasi-linear version of the New Keynesian model used here allows for a clear separation of the effects stemming from constraints on policy and higher-order behaviorial effects such as precautionary saving (absent from the present analysis). The simple structures comes with the additional benefit that it can be solved without resorting to the complex algorithms of global policy function iteration. In this paper, moreover, I study the economy's stochastic steady state as well as fluctuations around it. I emphasise how variation in risk may also affect the economy away from the ZLB if agents worry it may bind in future. I also illustrate how the specific source of risk, be it supply or demand, is immaterial for the general results.

More broadly, the paper relates to a number of recent papers showing that risk considerations are more important at the ZLB. For example, Nakata (2013) and Basu and Bundick (2014) show how risk reduces key economic variables substantially more in non-linear models when the ZLB is a binding constraint on a simple policy rule. Johannsen (2014) and Fernandez-Villaverde et al. (2015) suggest that uncertainty about fiscal policy in particular has larger implications for the economy when monetary policy is constrained.

The model's prediction that the effect of a given risk shock is larger, the closer the economy is to the ZLB, is in line with the empirical evidence provided by Plante et al. (2014) and Castelnuovo et al. (2015). Similarly, the finding that monetary policy is less effective when risk is high is consistent with the evidence in Aastveit et al. (2013).

The paper is organised as follows. Section 2 describes the model and its solution. Section 3 presents the quantitative analysis. It describes the parameterisation and the numerical solution of the stochastic steady state, and it presents impulse responses to persistent shocks to risk both away from the ZLB and when the ZLB is binding. Section 4 concludes.

2 The model

The model is the canonical forward-looking New Keynesian model extended with a ZLB on interest rates. In addition to a specification of monetary policy, it consists of the equations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

$$x_t = E_t x_{t+1} - \frac{1}{\varsigma}\left(i_t - E_t \pi_{t+1} - r_t^*\right) \tag{2}$$

$$i_t + i^* \geq 0 \tag{3}$$

where E_t is the expectations operator, π_t is inflation at time t in deviation from its target π^*, x_t is the output gap defined as output in deviation from its efficient level, and i_t is the nominal interest rate in deviation from its normal deterministic steady-state value i^*. The first equation is the New Keynesian Phillips curve, the second is the forward-looking IS curve, and the third imposes the ZLB. The canonical model is derived from its microfoundations by Woodford (2003) and Galí (2008) among others.

There are two shock processes in the model. u_t is a cost-push process, and r_t^* is the efficient equilibrium real interest rate in deviation from its steady state level $r^* = i^* - \pi^*$. I assume that the latter is the sum of a deterministic but potentially time-varying component ρ_t and a stochastic process ϵ_t so that $r_t^* = \rho_t + \epsilon_t$. Both the stochastic component of the equilibrium real interest rate and the cost-push shock are given as first-order autoregressive processes with zero-mean Gaussian innovations:

$$\epsilon_t = \mu_\epsilon \epsilon_{t-1} + \nu_{\epsilon,t} \tag{4}$$

$$u_t = \mu_u u_{t-1} + \nu_{u,t} \tag{5}$$

where $\nu_{\epsilon,t} \sim N(0, \sigma_{\epsilon,t}^2)$ and $\nu_{u,t} \sim N(0, \sigma_{u,t}^2)$. Importantly, I allow the standard deviations of the innovations to vary over time as indicated by the time subscripts in $\sigma_{\epsilon,t}$ and $\sigma_{u,t}$.

I define a risk shock as a change in one or both of these standard deviations. Specifically, I let a baseline risk shock be such that $\varsigma^{-1}\sigma_{\epsilon,t} = \sigma_{u,t} = \sigma_t$ with

$$\sigma_t = \sigma + \mu_\sigma\left(\sigma_{t-1} - \sigma\right) + \nu_{\sigma,t} \tag{6}$$

where $\nu_{\sigma,t}$ is the innovation to risk, and σ is the underlying level of risk in the absence of risk disturbances. Stochastic volatility refers to a scenario where these innovations are drawn from a fixed distribution each period.

Under optimal policy under discretion, a policymaker, hypothetically unconstrained by the ZLB in (3), minimises the period loss function

$$L = \pi_t^2 + \lambda x_t^2 \tag{7}$$

each period subject to the Phillips curve in (1) while taking expectations as given. This gives rise to the targeting rule

$$\pi_t = -\frac{\lambda}{\kappa} x_t \tag{8}$$

stating the optimal policy trade-off between inflation and the output gap. The interest rate consistent with this optimal allocation can now be found from the IS curve in (2). Since the policymaker is, in fact, constrained by (3), the interest rate will be set to the maximum of this optimal level and zero. For comparison, under an alternative regime the policymaker follows the simple instrument rule

$$i_t = \max\{-i^*, \phi_\pi \pi_t + \phi_x x_t\} \tag{9}$$

I solve this quasi-linear version of the canonical model following the approach in the recent analysis of this exact model by Evans et al. (2015). I approximate the shock processes by independent Markov processes using the Rouwenhorst (1995) method. I then solve the model backwards from a distant future period T, beyond which there is no risk and all shocks are zero so that $E_t \pi_t = E_t x_t = 0$ for all $t > T$. In each step, I take expectations as given and calculate the unconstrained outcome under each policy regime for a state grid of values for the shock processes. I then check if this outcome is consistent with the ZLB in (3) for each node in the grid. If so, I take the unconstrained outcome as the solution for this particular node. If not, I calculate the outcome from the model equations with $i_t = -i^*$ imposed. I then update the *ex ante* expectations of inflation and the output gap using the Markov transition matrices before progressing to the previous period. See the Annex for details. The solution consists of the values for inflation, the output gap and the interest rate, to which this algorithm converges in the initial period $t = 0$.

I find impulse responses to a risk shock by running a double loop. The outer loop moves forward from period $t = 0$, while the inner loop solves the model backwards from period T to the period of the current iteration of the outer loop. For each iteration of the outer loop, I reduce the value of σ_t from an initial spike according to (6).[3]

[3] A limitation of this approach is that agents always expect risk to stay constant at a given point in time, though this assumption is the natural starting point for considering variation in general risk perceptions.

3 Quantitative analysis

Table 1 summarises the parametisation. The inflation target is assumed to be 2% and the normal or deterministic steady-state level of the efficient equilibrium real rate of interest is assumed to be constant at 1.75%. This gives a normal level for the nominal interest rate of approximately 3.75 and a value for the discount factor of $\beta = 0.995$. The slope of the Phillips curve is assumed to be $\kappa = 0.02$ and the inverse of the elasticity of intertemporal substitution $\varsigma = 1$ in line with values often used in the literature. Similarly, the Taylor rule parameters are set at the conventional values $\phi_\pi = 1.5$ and $\phi_x = 0.5/4$. The weight on the output gap in the monetary policy loss function is set to be fairly low at $\lambda = 0.02$ in keeping with derivations of the loss function from household preferences. The shock processes are assumed to be moderately persistent with $\mu_u = 0.25$ and $\mu_\epsilon = 0.75$. I consider values of risk in the interval $\sigma_t \in [0.1\%, 0.35\%]$. The grid size for the shock processes is $n_\epsilon \times n_u = 25 \times 25$. With this calibration, the solution converges for risk horizons much shorter than the assumed $T = 1000$.[4]

Table 1: Parameterisation

Parameter	Notation	Value
Inflation target	π^*	0.02
Normal real interest rate	r^*	0.0175
Discount factor	β	0.995
Slope of Phillips curve	κ	0.02
Coefficient of relative risk aversion	ς	1
Persistence of equilibrium rate	μ_ϵ	0.75
Persistence of cost-push shock	μ_u	0.25
Persistence of risk shock	μ_σ	0.75
Weight on output gap in loss function	λ	0.02
Weight on inflation in policy rule	ϕ_π	1.5
Weight on output gap in policy rule	ϕ_x	0.125
Grid size for shock processes	n_ϵ, n_u	25
Uncertainty horizon	T	1000

[4] Evans et al. (2015) set higher values for the persistence parameters and the weight on output in the loss function. Their parameterisation makes the solution explosive and results are highly sensitive to the choice of T. I can reproduce their results with an uncertainty horizon of $T = 51$. This possibility of explosive dynamics corresponds to the potential non-existence of equilibria analysed by Mendes (2011) under a simple instrument rule, and by Nakata and Schmidt (2014) for the case with optimal discretion.

3.1 Stochastic steady state

Figure 1 shows zero-shock paths of the model solution by backward induction from period $t = T$ to period $t = 0$ for different levels of risk and specifications of monetary policy. The zero-shock paths are conditional on the particular realisation of the two shock processes shown in the top-left panel in which no non-zero shocks actually occur. In this sense, the converged zero-shock paths at time $t = 0$ represent stochastic or *risky* steady states of the model as defined by Coeurdacier et al. (2011).[5]

The solid blue lines show the solution with a low level of risk ($\sigma = 0.1\%$). In this case, the ZLB is never a concern, and expectations never deviate from zero in the absence of disturbances. As a consequence, inflation is on target, the output gap is zero, and the interest rate is at its normal level independently of the monetary policy regime. The stochastic steady state coincides with the deterministic one in this case.

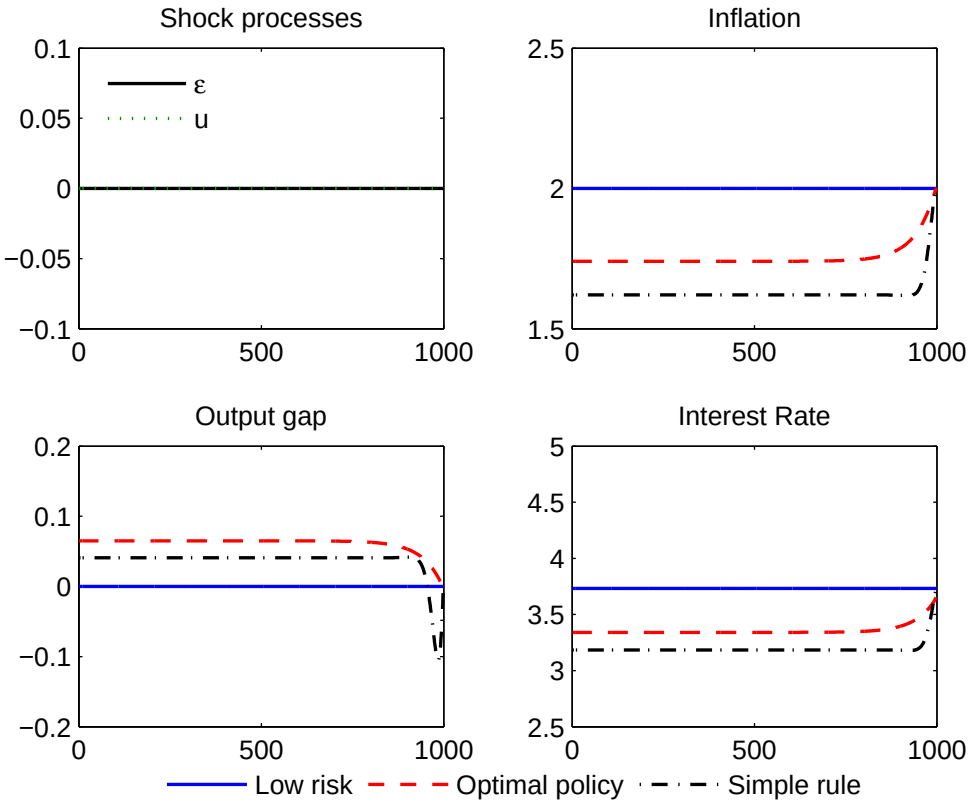

Figure 1: Zero-shock path of the solution to the canonical New Keynesian model with a ZLB on interest rates by backward induction with low risk ($\sigma = 0.01\%$; solid blue lines), high risk under optimal discretionary monetary policy ($\sigma = 0.35\%$; dashed red lines), and high risk with a simple monetary policy rule ($\sigma = 0.35\%$; dashed-dotted black lines), in per cent.

[5] Non-zero disturbances will of course drive the economy away from any of these steady states.

But when risk is perceived to be high enough that the monetary policymaker cannot respond sufficiently to some negative disturbances, here with $\sigma = 0.35\%$, expectations will be negatively skewed even in the absence of any actual shocks to the economy as emphasised by Adam and Billi (2007), Nakov (2008), Nakata and Schmidt (2014), Basu and Bundick (2015) and Evans et al. (2015). Low inflation expectations will weigh on the price-setting of firms and actual inflation will be below target. This deflationary effect from the ZLB results in a loosening bias in monetary policy. The high-risk steady state is therefore one with inflation below target, interest rates below normal deterministic levels, and a positive output gap. Under optimal policy (dashed red lines), inflation settles about 25 basis points below target with the baseline parameterisation. Despite a somewhat stronger loosening bias, inflation is lower still under the simple rule with standard parameters (dashed-dotted black lines).

Table 2 compares the outcomes for inflation, the output gap and the interest rate in the high-risk stochastic steady state under the baseline parameterisation (Case 1) with outcomes in nine alternative cases. Case 2 shows the effect of keeping the risk of cost-push shocks low so that risk is only high for shocks to the efficient equilibrium real interest rate. Case 3 shows the opposite case with low risk of r^* shocks and high risk of shocks to the Phillips curve. In both cases, the stochastic steady state deviates from the deterministic one under optimal policy with inflation settling below target. Also, the marginal contributions of the two shocks to the baseline are similar. But the deviations are much smaller with inflation rates of about 1.92% and 1.98%, respectively.

Table 2: High-risk stochastic steady state under optimal policy (in per cent)

	Parameters				Optimal policy			Simple rule		
Case	σ_ϵ	σ_u	r^*	π^*	π	x	i	π	x	i
1) Baseline	0.35	0.35	1.75	2.00	1.74	0.06	3.34	1.62	0.04	3.18
2) r^* shocks only	0.35	0.10	1.75	2.00	1.92	0.02	3.64	2.00	0.00	3.73
3) u shocks only	0.10	0.35	1.75	2.00	1.98	0.01	3.69	1.79	0.02	3.43
4) Large r^* shocks	0.42	0.10	1.75	2.00	1.74	0.06	3.41	2.00	0.00	3.72
5) Large u shocks	0.10	0.58	1.75	2.00	1.74	0.06	3.19	–	–	–
6) Low $r*$	0.35	0.35	1.50	2.00	1.60	0.10	2.86	1.44	0.06	2.67
7) Low π^*	0.35	0.35	1.75	1.75	1.37	0.10	2.89	1.20	0.06	2.69
8) High $r*$	0.35	0.35	2.00	2.00	1.82	0.05	3.71	1.73	0.03	3.59
9) High π^*	0.35	0.35	1.75	2.25	2.06	0.05	3.71	1.98	0.03	3.58
10) Very high π^*	0.35	0.35	1.75	5.50	5.50	0.00	7.15	5.49	0.00	7.14

These results suggest that agents are particularly concerned about the inability of policymakers to respond when large adverse disturbances to the cost-push process and the equilibrium rate coincide. Of course, higher risk for individual shocks results in larger biases. Increasing σ_ϵ to about 0.42% when the risk of cost-push shocks is low (Case 4) – or σ_u to about 0.58% when the risk of r^* shocks is low (Case 5) – leads to similar biases in inflation and the output gap when policy is optimal as under the baseline parameterisation. With a simple monetary policy rule, it takes very high values of σ_ϵ to induce a noticeable bias in the stochastic steady state when the risk of cost-push shocks is low. Even in Case 4, the stochastic steady-state effectively coincides with the deterministic one when rounded to two decimal places. In Case 5 by contrast, the risk of cost-push shocks is high enough that dynamics become explosive when monetary policy follows a simple rule.

Notice that, in all of these cases, inflation falls below target in the stochastic steady state only because agents worry that the ZLB may bind in future. With a policy rate above three per cent, monetary policy has a substantial distance between it and the ZLB. But when risk is perceived to be high, agents worry about policy's inability to respond to large adverse shocks in the future even when the policy rate is currently well above the ZLB.

For a given level of risk, the effect on expectations depends on the available monetary policy space. As illustrated by the remaining cases in Table 2, the closer the economy operates to the ZLB, the larger are the effects of risk on outcomes in the stochastic steady state. If the distance to the ZLB is reduced by about 25 basis points in the deterministic steady state, either because the equilibrium real rate of interest is lower (Case 6) or because monetary policy targets a lower inflation rate (Case 7), the stochastic steady-state inflation rate falls below target by a further 13 basis points or so. In Case 7, where inflation in the deterministic steady state itself is lower, this implies that inflation is about 37 basis points lower than in the baseline. By contrast, if i^* increases to about 4%, the negative bias in inflation is reduced by 7-8 basis points. With a higher inflation target, the component of i^* that can actually be chosen by policymakers, inflation settles around 2.1%. To fully eliminate the negative bias in inflation, however, policymakers will have to set a target for inflation well above 5% (Case 10) when risk is high.

The results presented in this section are fully consistent with the contemporaneous analysis by Hills et al. (2016), who study the stochastic steady state in a non-linear New Keynesian model with a ZLB. In an empirical application to the United States, the authors estimate that the level of inflation falls short of target by about 25 basis point in the risky steady state.

3.2 Impulse responses to a baseline risk shock

Now suppose that risk varies over time. Specifically, consider a baseline risk shock to an economy operating in a low-risk steady state so that the standard deviations of the innovations to the two shock processes jump from 0.1% to 0.35% with a gradual return to 0.1% according to the process in (6). The risk shock represents a scenario in which risk is temporarily elevated so that agents expect innovations to level shocks to be drawn from a distribution with fatter tails for some time in the future.

Figure 2 shows the impulse responses along the zero-shock path. That is, the economy is not actually hit by any shocks along this adjustment path; it is only the perception of risk that changes. When risk is high, agents worry about the monetary policymaker's inability to respond to large adverse shocks as a consequence of the ZLB. Therefore, inflation expectations fall short of the inflation target, and output expectations of potential. By (1), the risk shock has a negative cost-push effect: for any given level of the output gap, inflation falls in response to lower inflation expectations.

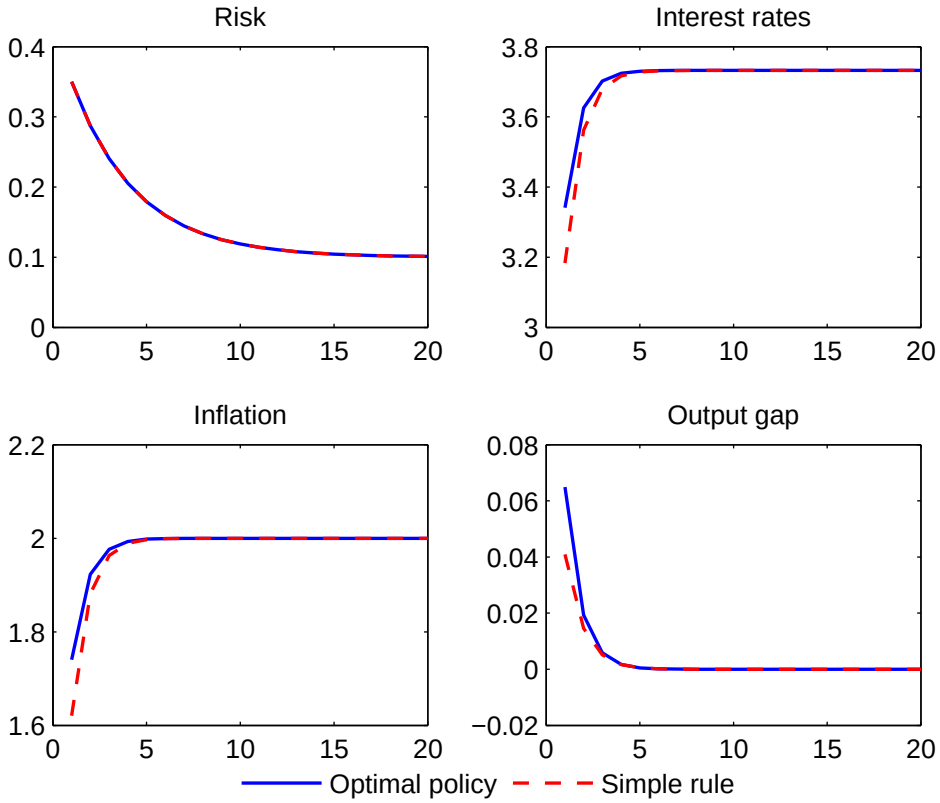

Figure 2: Impulse responses along the zero-shock path to a risk shock around a low-risk steady state ($\sigma = 0.01\%$) in the canonical New Keynesian model with a ZLB on interest rates under optimal discretionary monetary policy (solid blue lines) and with a simple interest rate rule (dashed red lines), in per cent.

This effect induces a trade-off for the policymaker as reflected in the targeting rule in (8). Under optimal discretion (solid blue lines), the policymaker loosens policy enough to bring output above its efficient potential. But compared to a conventional cost-push shock with the same impact effect through (1), the interest rate has to be reduced more to achieve the optimal balance between inflation and the output gap. There are two reasons for this. First, lower inflation expectations raises the real interest rate for a given level of the nominal rate. And second, since output expectations have also been adversely affected by the risk shock, policy needs to bring about a lower real interest rate to boost aggregate demand through (2). As risk falls back, the ZLB becomes less of a concern and the economy gradually returns to the low-risk steady state. Similarly with the simple policy rule (dashed red lines), the policymaker temporarily stimulates the economy in response to falling inflation.

Importantly, a trade-off arises in uncertain times even if shocks do not actually happen. The only prerequisite is that the risk shock is large enough that the ZLB becomes a concern. Small increases and reductions in risk around the low-risk steady state leave economic outcomes unaffected.[6] Around a high-risk steady state, by contrast, variation in risk have both positive and negative cost-push effects as illustrated in Figure 3 for $\sigma = 0.31\%$ with optimal policy. Responses to a positive shock (solid blue lines) are as before, except that the economy reverts to the high-risk steady state with a negative bias in inflation. But a negative risk shock (dashed red lines) now has a positive cost-push effect. As risk falls to low levels, agents stop worrying about the ZLB, and inflation expectations realign with the inflation target. Policymakers increase interest rates in response, while the output gap closes. Gradually, as risk returns, the economy reverts to the high-risk steady state.

The asymmetry in the responses to positive and negative risk shocks around the high-risk steady state reflect a non-linearity in the effect of risk on economic outcomes as illustrated in Figure 4. With low levels of risk, the economy operates in the deterministic steady state in the absence of level shocks. As risk increases, the ZLB eventually becomes binding in some conceivable states of the world. For small increases, the effects are small. But as risk increases further, the effects begin to accelerate. For levels of risk beyond those shown in the figure, dynamics become explosive with hyperdeflation and a collapse of output. In this unpleasant scenario, negative expectations – caused by a concern about the policymaker's inability to respond to adverse shocks – become self-fulfilling as the policymaker is, in fact, unable to respond sufficiently to these expectations because of the ZLB.

[6] Of course, if the economy operates close to the ZLB, more risk shocks become 'large' in this sense. Notice also that both inflation and the output gap will fall after a large increase in risk if monetary policy does not accommodate the risk shock with lower interest rates.

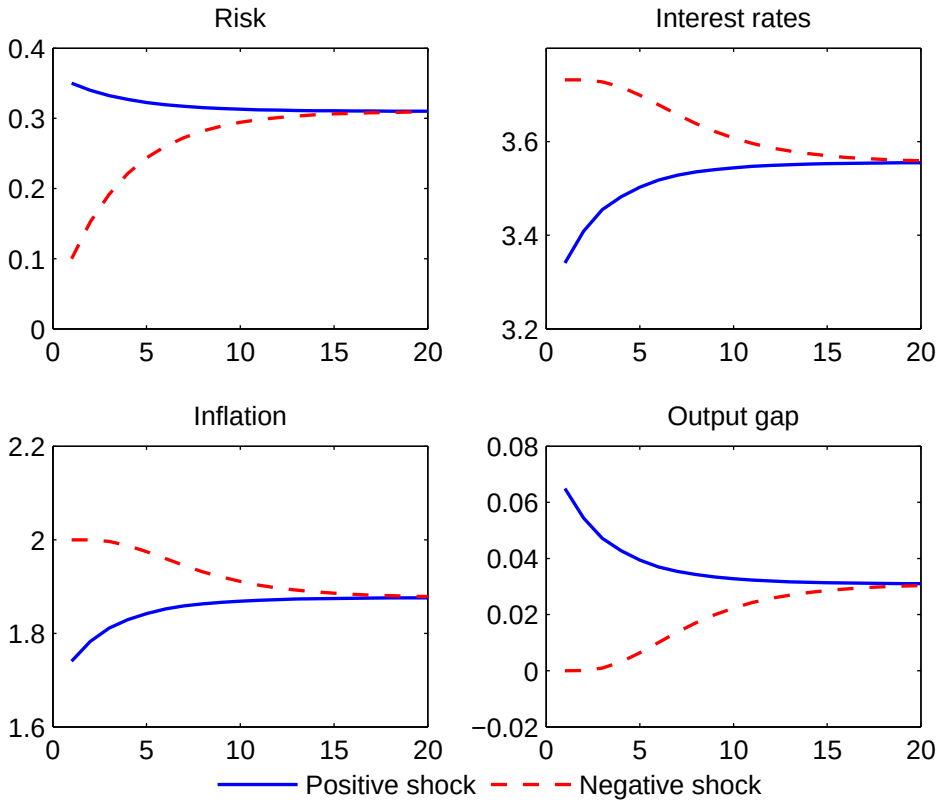

Figure 3: Impulse responses along the zero-shock path to a positive (solid blue lines) and a negative (dashed red lines) risk shock around a high-risk steady state ($\sigma = 0.31\%$) in the canonical New Keynesian model with a ZLB on interest rates under optimal discretionary monetary policy, in per cent.

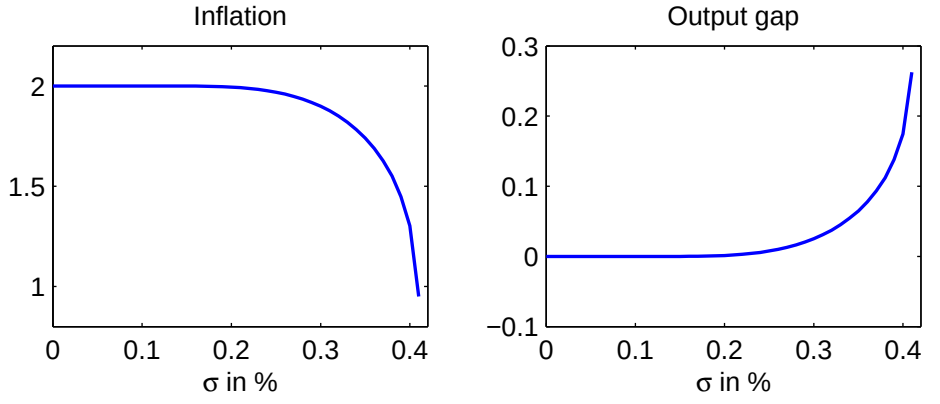

Figure 4: Economic outcomes as a function of risk in the canonical New Keynesian model with a ZLB on interest rate under optimal discretionary policy, in per cent.

3.3 On the sources of risk

The baseline risk shock considered so far affects the standard deviations of both shock processes in the model. Figure 5 shows the effects of a positive risk shock around a low-risk steady state for each of the two shocks in turn. Qualitatively, the economy is seen to respond in the same way to the two risk shocks along the zero-shock paths. Spikes in risk lead to cost-push effects both when risk is elevated for the shock to the equilibrium real rate of interest only (solid blue lines) and for the cost-push shock only (dashed red lines). In both cases, optimal monetary policy responds by stimulating the economy enough to push output above potential. It is only the numerical increases in risk required to induce similar quantitative dynamics that are different (top-left panel). The responses are driven by an increase in the likelihood that monetary policy cannot provide sufficient stimulus in response to adverse disturbances when risk is elevated. The sources of the potential adverse shocks are immaterial.

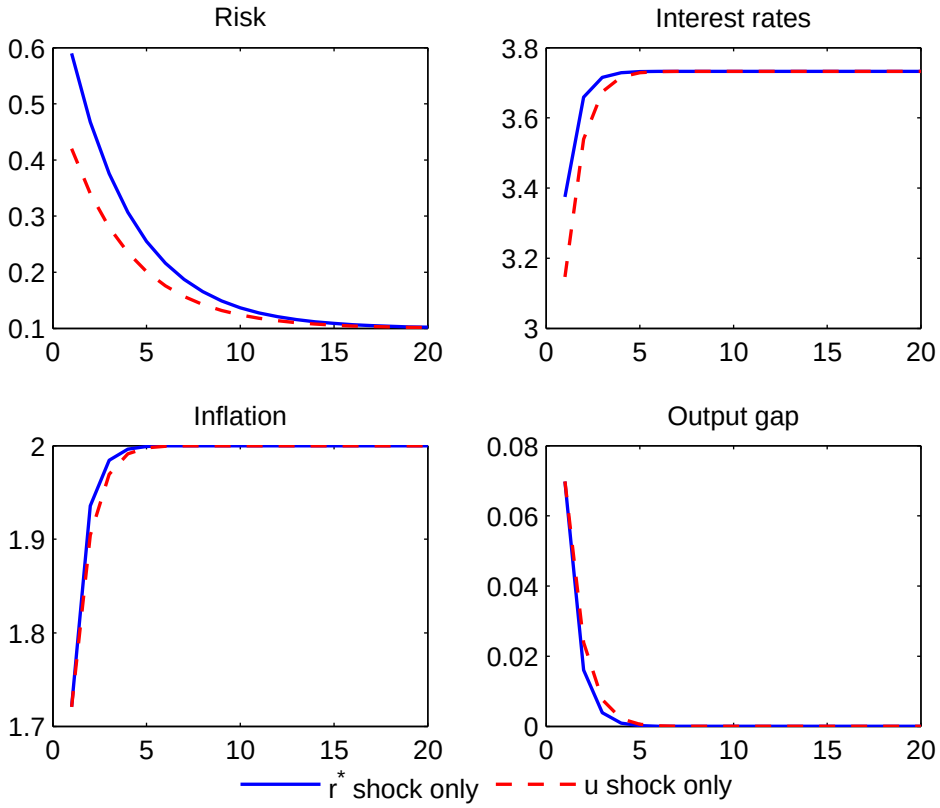

Figure 5: Impulse responses along the zero-shock path to a negative risk shock to the equilibrium real rate of interest only (solid blue lines) and to the cost-push shock only (dashed red lines) around a low-risk steady state ($\sigma = 0.01\%$) in the canonical New Keynesian model with a ZLB on interest rates under optimal discretionary monetary policy, in per cent.

Specifically, the trade-off for monetary policy does not rely on potential cost-push effects from the level shocks themselves. It arises also following spikes in risk to the efficient equilibrium real rate of interest alone. If monetary policy were unrestricted by the ZLB, shocks to r_t^* could always be perfectly offset by an appropriate stance of policy. In this case, the output gap would remain closed, and inflation would be on target by the divine coincidence (Blanchard and Galí, 2007). But with a binding ZLB, monetary policy cannot fully offset large negative shocks to r_t^*. With insufficient monetary stimulus, demand cannot keep up with potential output, and inflation falls below target. A trade-off arises for monetary policy as the prospect of such demand-driven recessions feed into inflation expectations when risk is elevated.[7] [8]

Notice, however, that shocks to r_t^* are not necessarily demand shocks in the traditional sense. In the canonical New Keynesian model, fluctuations in the efficient equilibrium real rate of interest are driven by changes in the expected growth rate of total factor productivity in addition to changes in preferences and exogenous spending, see e.g. Galí (2008). Heightened uncertainty about the future growth potential of the economy is therefore an example of a risk shock to r_t^*. A scenario in which such an increase in perceived risk is associated with a fall in expected future growth rates would correspond to a combination of a positive risk shock and a negative level shock to r^* in this framework.

Notice also that, in contrast to Basu and Bundick (2015), the analysis ignores the effect of risk on precautionary saving. Similarly, the simple New Keynesian model does not allow for negative demand effects from the option value associated with postponing irreversible investments when risk is high (Bernanke, 1983). The adverse effects from risk shocks arise solely because of adjustments to the expected mean paths for output and inflation when monetary policy is constrained. The advantage of this simplification is that the effects stemming from the constraints on policy are clearly separated from higher-order behavioural effects. But in reality, a risk shock of any kind is likely to be accompanied by what would be a negative level shock to r_t^* in this framework as households seek to build a buffer stock of savings while firms put investment projects on hold.[9]

[7] As illustrated by Adam and Billi (2007), a trade-off arises for persistent negative *level* shocks to r^* of an intermediate size for a similar reason: when the economy moves closer to the ZLB, more future shocks can potentially cause a recession for a given level of risk.

[8] For cost-push shocks, a negative bias in inflation expectations occurs because monetary policy cannot always achieve the appropriate balance between inflation and output following large negative cost-push shocks, while it can always achieve such a balance after positive shocks, see e.g. Evans et al. (2015).

[9] See Paoli and Zabczyk (2013) for an analysis of the effect of precautionary saving on the equilibrium real rate of interest.

Finally, I remark that the risk shocks considered here are very different from the cross-sectional shocks analysed by Christiano et al. (2014). In their paper, a 'risk shock' refers to a disturbance to the *ex post* realisation of the dispersion of the quality of capital acquired by entrepreneurs. When this dispersion widens, the agency problem associated with financial intermediation becomes more severe. As credit spreads increase, entrepreneurs demand less capital and aggregate demand contracts for a given stance of policy. Within the simple New Keynesian model, such a scenario corresponds to a negative shock to the level of r_t^*.

3.4 Risk shocks at the ZLB

Around a stochastic steady state, optimal discretionary monetary policy responds to risk shocks to mitigate their effects on the economy through expectations. Along the zero-shock path, monetary policy is not actually constrained by the ZLB – except following extreme spikes in risk that lead to hyperdeflations. Effectively, policymakers act now because they may be constrained in future. By contrast, if policy is constrained by the ZLB when a risk shock hits, policymakers are unable to provide further stimulus.

To illustrate the implications of a binding ZLB for the propagation of risk shocks, Figure 6 shows a scenario in which the economy is gradually recovering from a ZLB episode caused some time in the past by a large and persistent negative shock to the level of the equilibrium real interest rate. The nature of this initial shock, say a financial crisis, is well understood by agents in the economy by now. Specifically, the deterministic component is known to follow the path shown in the top-right panel of Figure 6 (dashed-dotted black line). Uncertainty surrounding this recovery is perceived to be low.

At around period $t = 4$, the efficient nominal interest rate turns positive and the policymaker, who operates under optimal discretion, is preparing to lift interest rates off the ZLB. In the absence of risk, the policymaker would simply follow the equilibrium interest rate on its trajectory back towards normal levels.[10] But as long as the equilibrium interest rate is this close to the ZLB, even small shocks are 'large', and the possibility that a shock drives the economy back to the ZLB in future is sufficient to optimally delay lift-off even when risk is low.[11]

[10] This corresponds to the perfect foresight case analysed by Adam and Billi (2007). The same response would follow if the ZLB had not been a constraint, either because negative nominal interest rates were possible or because unconventional monetary policy tools were available and effective.

[11] This is the argument made in Evans et al. (2015). But in Figure 6 the ZLB binds because of an initial level shock to the equilibrium real rate of interest, not because of an explosively high risk level that may keep the economy at the ZLB for an arbitrary length of time depending on the expectational horizon.

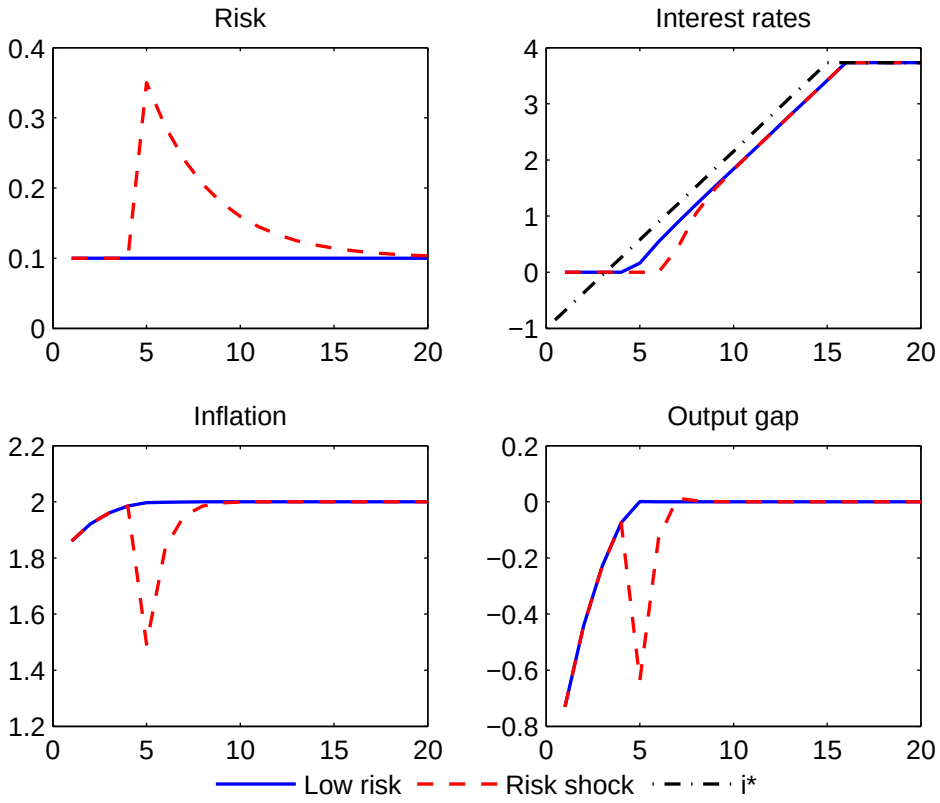

Figure 6: Recovery from a ZLB episode as reflected in the path of the equilibrium interest rate (dashed-dotted black lines in the top-right panel) under optimal discretionary policy in a low risk scenario (solid blue lines) versus a scenario with an unexpected risk shock at time $t = 5$ (dashed red lines), in per cent.

Now suppose that agents suddenly become more uncertain about economic prospects, perhaps reflected in turmoil across financial markets. Specifically, suppose the economy is hit by the baseline risk shock at time $t = 5$, just as lift-off was supposed to take place in the absence of any disturbances to the economy. Now that the economy is close to the ZLB, the impact effect of the risk shock on expectations is larger than before as the monetary policymaker is constrained by the ZLB in its response to the shock. As shown in Figure 6 (dashed red lines), inflation falls more as a consequence, and lift-off from the ZLB is further delayed. Now because of the binding ZLB, output also falls further below potential. Only as risk abates will the optimal interest rate path catch up with the equilibrium rate. The longer risk stays elevated, i.e. the more persistent the risk shock, the longer lift-off is optimally delayed even if the economy is not actually exposed to any shocks during the recovery.

In these simulations, the economy eventually returns to a low-risk steady state with inflation on target. If the initial shock had instead been associated with a permanent increase in underlying risk, the economy would of course revert to a high-risk steady state.

Similarly, if the initial shock is not fully reversed so that the deterministic component of r_t^* fails to return to its normal level before the crisis, the economy would settle in a high-risk steady state with inflation below target also for moderate levels of underlying risk.

4 Conclusion

Risk affects economic outcomes close to the ZLB in the canonical New Keynesian model. Because the ZLB impairs monetary policy's ability to respond to large adverse shocks with sufficiently stimulatory policy action – but never its ability to respond with contractionary action when needed – expectations of inflation and output will be negatively biased when risk is high given the available monetary policy space. In uncertain times, inflation may settle materially below the policymaker's target in the absence of disturbances even when the policy rate is well above the ZLB under optimal discretionary policy.

By implication, variation in risk has potentially large effects on economic outcomes. Even if nothing actually happens, changes in the perception of risk affect the economy through expectations. In the canonical New Keynesian model, risk shocks that are large relative to policymakers' room for manoeuvre give rise to cost-push effects regardless of the source of risk. Around a low-risk steady state, stochastic volatility introduces occasional trade-offs for monetary policy between nominal and real stability, and optimal discretionary monetary policy calls for potentially sharp reductions in the interest rate when risk is elevated. When the underlying risk is high and the economy evolves around a high-risk steady state, variation in risk has both negative and positive cost-push effects. If policy is initially constrained by the ZLB, risk shocks have larger effects on the economy and lift-off is optimally delayed for as long as risk is elevated.

The analysis is informative for monetary policy deliberations in inflation targeting countries faced with an effective lower bound on interest rates. While responses are likely to be too immediate in the highly stylised and purely forward-looking model presented here, they are indicative of the direction of the propagation of variations in risk in actual economies operating close to the lower bound. In particular, if the forces behind current low levels of interest rates persist, as e.g. Rachel and Smith (2015) argue that they will, in an economy that is no longer as greatly moderated as the one described by Stock and Watson (2003), the analysis suggests that the new normal for monetary policy may be one in which policymakers should respond to changes in the perception of risk, even as the economy escapes the lower bound.

A Solution for each step

In each state (ϵ, u) in the $n_\epsilon \times n_u$ state space in period t, expectations are taken as given so that $E_t x_{t+1} = \bar{x}^e_{t,t+1}(\epsilon, u)$ and $E_t \pi_{t+1} = \bar{\pi}^e_{t,t+1}(\epsilon, u)$. Combining (1) and (8) in the form

$$\pi_t(\epsilon, u) = \beta \bar{\pi}^e_{t,t+1}(\epsilon, u) + \kappa x_t(\epsilon, u) + u_t(\epsilon, u)$$
$$\pi_t(\epsilon, u) = -\frac{\lambda}{\kappa} x_t(\epsilon, u)$$

gives the unconstrained optimal allocation

$$\pi^{opt}_t(\epsilon, u) = \frac{\lambda}{\lambda + \kappa^2} \left[\beta \bar{\pi}^e_{t,t+1}(\epsilon, u) + u_t(\epsilon, u)\right]$$
$$x^{opt}_t(\epsilon, u) = -\frac{\kappa}{\lambda + \kappa^2} \left[\beta \bar{\pi}^e_{t,t+1}(\epsilon, u) + u_t(\epsilon, u)\right]$$

The interest rate consistent with this allocation follows from (2):

$$i^{opt}_t(\epsilon, u) = \bar{\pi}^e_{t,t+1}(\epsilon, u) + r^*_t(\epsilon, u) - \varsigma \left[x^{opt}_t(\epsilon, u) - \bar{x}^e_{t,t+1}(\epsilon, u)\right]$$

If $i^{opt}_t(\epsilon, u) \geq -i^*$, $\left\{x^{opt}_t(\epsilon, u), \pi^{opt}_t(\epsilon, u)\right\}$ is the solution for state (ϵ, u). If the ZLB is binding so that $i^{opt}_t(\epsilon, u) < -i^*$, the interest rate is set to $i^{zlb}_t(\epsilon, u) = -i^*$. Now from (2) and (1):

$$x^{zlb}_t(\epsilon, u) = \bar{x}^e_{t,t+1}(\epsilon, u) - \frac{1}{\varsigma} \left[-i^* - \bar{\pi}^e_{t,t+1}(\epsilon, u) - r^*_t(\epsilon, u)\right]$$
$$\pi^{zlb}_t(\epsilon, u) = \beta \bar{\pi}^e_{t,t+1}(\epsilon, u) + \kappa x^{zlb}_t(\epsilon, u) + u_t(\epsilon, u)$$

Hence, the solution for $\{x_t(\epsilon, u), \pi_t(\epsilon, u)\}$ for all nodes (ϵ, u) in the state grid is

$$\left\{x^{sol}_t(\epsilon, u), \pi^{sol}_t(\epsilon, u)\right\} = \begin{cases} \left\{x^{opt}_t(\epsilon, u), \pi^{opt}_t(\epsilon, u)\right\} & \text{if } i^{opt}_t(\epsilon, u) \geq -i^* \\ \left\{x^{zlb}_t(\epsilon, u), \pi^{zlb}_t(\epsilon, u)\right\} & \text{if } i^{opt}_t(\epsilon, u) < -i^* \end{cases}$$

Ex ante expectations across the state grid can now be found as

$$\bar{\mathbf{x}}^e_{t-1,t} = \mathbf{P}_\epsilon \mathbf{x}^{sol}_t \mathbf{P}'_u$$
$$\bar{\pi}^e_{t-1,t} = \mathbf{P}_\epsilon \pi^{sol}_t \mathbf{P}'_u$$

where $\mathbf{P}_\epsilon$ and $\mathbf{P}_u$ are Markov transition matrices of dimensions $n_\epsilon \times n_\epsilon$ respectively $n_u \times n_u$.

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