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Abstract

We propose new systematic tail risk measures constructed using two different approaches. The first extends the canonical downside beta and co-moment measures, while the second is based on the sensitivity of stock returns to innovations in market crash risk. Both tail risk measures are associated with a significantly positive risk premium after controlling for other measures of downside risk, including downside beta, co-skewness and co-kurtosis. Using these measures, we examine the relevance of the tail risk premium for investors with different investment horizons.

Key words: Asset pricing, downside risk, tail risk, co-moments, value at risk, systematic risk.

JEL classification: C13, C31, C58, G01, G10, G12.

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1. Introduction

The turbulence of financial markets over the last few decades has highlighted the importance of tail risk for asset pricing. Many studies have documented the significant impact of this type of risk on expected returns. Rietz (1988) shows that the inclusion of a crash state in a three-state model plays an important role in generating the observed risk premium for the US equity market for reasonable levels of risk aversion and consumption growth volatility. His findings are confirmed by Barro (2006) in a framework of consumption growth with a crash term and a specific measure of disaster risk. More recently, Gabaix (2012) and Wachter (2013) account for time-varying disaster risk as well as the relative performance of different assets in times of distress. However, the small number of actual economic disasters is a practical challenge for further development of measures of crash risk.

An alternative approach to examining the role of extreme downside risk is to directly investigate the impact of the tail of the return distribution on returns. Owing to the fact that asset returns are generally observable at high frequencies, measures corresponding to any aspect of the return distribution can be readily constructed, and moment-based risk measures such as variance, skewness and kurtosis, have all been shown to significantly influence stock returns. In a portfolio context, many studies have modeled the impact of systematic moment risks, such as CAPM beta, co-skewness and co-kurtosis, on returns. For example, Kraus and Litzenberger (1976) and Harvey and Siddique (2000) develop a three-moment CAPM and find that the co-skewness is significantly related to asset returns. Dittmar (2002) incorporates investor preferences over the first four moments of returns to derive a cubic form of the pricing kernel and finds that co-kurtosis also affects returns. The significance of the co-skewness and co-kurtosis premia is confirmed in many other studies (see, for example, Ang *et al.*, 2006a; Guidolin and Timmermann, 2008; Yang and Chen, 2009; Kostakis *et al.*, 2012). Moreover, Chung *et al.* (2006) demonstrate that higher comoments of up to order ten collectively capture the Fama and French (1993) factors.

In contrast with the findings for moment-based risk measures, which offer indirect evidence on the importance of tail risk in asset pricing, direct evidence on the role of tail risk is sparse, especially for systematic tail risk. Bali and Cakici (2004) examine the influence of tail risk on cross-sectional returns using Value at Risk (VaR). However, they do not decompose VaR into its systematic and idiosyncratic components. Huang *et al.* (2012) show that idiosyncratic tail risk is significant in determining stock returns, using a measure of idiosyncratic tail risk on the residuals of the Carhart (1997) four-factor model. However, they do not shed light on the importance of systematic tail risk for individual stock returns. A number of studies examine the relationship between tail risk and returns at the market level, thus bypassing the need for the decomposition of risk into its systematic and idiosyncratic components. Bali *et al.* (2009) find a positive relationship between the expected monthly market VaR and the corresponding monthly market returns. Bollerslev and Todorov (2011) estimate an 'Investor Fear Index' for the market, and show that it commands a significant premium. Kelly and Jiang (2014) develop a market tail risk measure based on the common component of the tail risk of individual stocks and show that it has significant predictive power for market returns. Ruenzi and Weigert (2013) propose a systematic tail risk measure, Left Tail Dependence (LTD), based on the estimated crash sensitivity of an individual stock to a market crash. The risk premium corresponding to this measure is positive since stocks with high LTD tend to offer low returns when investors' wealth is low. However, the LTD measure is subject to two shortcomings. First, the estimation procedure is data intensive. Second, it ignores the crash severity which is an important part of tail risk.

In this paper, we propose two new measures of systematic tail risk and investigate their relationship with stock returns. The first is based on the downside beta measure, while the second is based on the sensitivity of stock returns to market tail risk. We first investigate the relationship between expected returns and systematic tail risk by sorting all stocks in the NYSE, AMEX and NASDAQ into quintiles based on the systematic tail risk measures. We observe a monotonic increase in average returns from the lowest to the highest risk quintile, and the alphas of long-short strategies on these portfolios are significantly positive, confirming the economic significance of the systematic tail risk premium. We conduct a Fama and MacBeth (1973) cross-sectional analysis for the systematic tail risk measures, controlling for a large set of other risk measures including downside beta, upside beta, size, book-to-market, volatility, past returns and systematic higher moments. The cross-sectional analysis confirms the significant and positive tail risk premium.

We carry out extensive robustness checks. First, we find that the magnitude of systematic tail risk is unstable over time with very low persistence, and so past tail risk is not a reliable measure of future tail risk. This finding supports the use of realized tail risk measures in explaining contemporaneous returns, similar to the framework used by Ang *et al.* (2006a) to investigate downside risk. Second, we consider alternative measures of tail risk based on VaR and Expected Tail Loss. Third, we examine the sensitivity of our results to the choice of tail threshold and the estimation window length. We find a significantly positive tail risk premium in all cases, except in the case of extremely low tail threshold, where only the VaR-based measure works well. This is explained by the fact that the small sample undermines the performance of the downside beta-based measure, while the VaR-based measure relies on daily observations of VaR and hence a larger effective sample. This feature enables us to investigate the impact of systematic tail risk on returns over investment horizons of different lengths. Finally, we show that while the VaR-based measure is constructed in a similar way to the systematic volatility measure of Ang *et al.* (2006b), it nevertheless contains significant incremental information about expected returns.

The remainder of this paper is organized as follows. Section 2 introduces the new systematic tail risk measures. Sections 3 and 4 utilize these proposed measures to investigate the relationship between tail risk and expected returns. Section 5 presents the results of extensive robustness tests. Section 6 summarizes the main findings and offers some concluding remarks.

2. Systematic Tail Risk Measures

In this section, we propose two new approaches to estimate the systematic tail risk of an asset. The first approach adapts the downside-beta method, while the second is based on investors' demand to hedge against tail events and the argument that a systematic tail risk measure should capture the sensitivity of stock returns to market tail risk.

2.1. *Extreme Downside Beta and Extreme Downside Co-moment Measures*

An extensive literature emphasizes the role of downside beta in determining asset returns (see, for example, Harlow and Rao, 1989; Ang *et al.,* 2006a; Estrada, 2007). Bawa and Lindenberg (1977) derive a general asset pricing relationship between an asset's excess returns and its systematic lower partial moments. Similarly, Ang *et al.* (2006a) find evidence of a positive downside beta risk premium using a cross section of US stock returns. They find that this premium is significant, after controlling for a number of other risk factors including size, book-to-market, volatility and higher co-moments, among others. The importance of downside beta has also been confirmed internationally. Estrada (2007) finds that downside beta has higher explanatory power than the CAPM beta in explaining stock returns using data from 50 developed and developing markets.

We generalize the downside beta measure by varying the threshold that is used to partition returns. The commonly used measures of downside beta are those of Bawa and Lindenberg (1977) (β_{BL}^{D}), Ang *et al.* (2006a) (β_{ACY}^{D}) and Estrada (2007) (β_{ES}^{D}), given by:

$$\beta_{BL,i}^{D} = \frac{E\{(R_i - \mu_i) \times min(R_m - \mu_m, 0)\}}{E\{min(R_m - \mu_m, 0)^2\}} \quad (1)$$

$$\beta_{ACY,i}^{D} = \frac{E\{\tilde{R}_i^- \times \tilde{R}_m^-\}}{E\{\tilde{R}_m^{-\,2}\}} \quad (2)$$

$$\beta_{ES,i}^{D} = \frac{E\{min(R_i - \mu_i, 0) \times min(R_m - \mu_m, 0)\}}{E\{min(R_m - \mu_m, 0)^2\}} \quad (3)$$

where R_i and R_m are the excess returns of asset i and the market, μ_i and μ_m are the mean of R_i and R_m, R_i^- and R_m^- are the values of R_i and R_m conditional on R_m being lower than μ_m, and $\tilde{R}_i^-$ and $\tilde{R}_m^-$ are the demeaned values of R_i^- and R_m^-.

Changing the threshold for $(R_i - \mu_i)_\alpha$ from zero to some low α-quantile, we have the corresponding measures of Extreme Downside Beta (hereafter EDB):

$$EDB_{BL,i} = \frac{E\{(R_i - \mu_i) \times (R_m - \mu_m)_\alpha\}}{E\{(R_m - \mu_m)_\alpha^2\}} \quad (4)$$

$$EDB_{ACY,i} = \frac{E\{\tilde{R}_i^\alpha \times \tilde{R}_m^\alpha\}}{E\{\tilde{R}_m^{\alpha\,2}\}} \quad (5)$$

$$EDB_{ES,i} = \frac{E\{(R_i - \mu_i)_\alpha \times (R_m - \mu_m)_\alpha\}}{E\{(R_m - \mu_m)_\alpha^2\}} \quad (6)$$

where $(R_i - \mu_i)_\alpha$ is equal to $(R_i - \mu_i)$ if R_i is smaller than its α-quantile and 0 otherwise, $(R_m - \mu_m)_\alpha$ is equal to $(R_m - \mu_m)$ if R_m is smaller than its α-quantile and 0 otherwise, R_i^α, R_m^α are R_i, R_m conditional on R_m being smaller than its α-quantile and $\tilde{R}_i^\alpha$ and $\tilde{R}_m^\alpha$ are the demeaned values of R_i^α and R_m^α.

More generally, using the same approach, we can also extend the definitions of the co-moments of returns to obtain the corresponding Extreme Downside Co-moment (hereafter EDC) measures. Starting from the formula of the co-moment:

$$k^{th}\ comoment\ of\ asset\ i = \frac{E\left[(R_i - \mu_i)(R_m - \mu_m)^{k-1}\right]}{var(R_i)^{1/2} var(R_m)^{(k-1)/2}} \quad (7)$$

the corresponding Extreme Downside Co-moment measures are given as:

$$EDC_{BL,i} = \frac{E[(R_i - \mu_i)(R_m - \mu_m)_\alpha]}{var(R_i)^{1/2} E\left((R_m - \mu_m)_\alpha^2\right)^{1/2}} \quad (8)$$

$$EDC_{ACY,i} = \frac{E\left[\tilde{R}_i^\alpha \times \tilde{R}_m^\alpha\right]}{var\left(\tilde{R}_i^\alpha\right)^{1/2} var(\tilde{R}_m^\alpha)^{1/2}} \quad (9)$$

$$EDC_{ES,i} = \frac{E[(R_i - \mu_i)_\alpha (R_m - \mu_m)_\alpha]}{E\left((R_i - \mu_i)_\alpha^2\right)^{1/2} E\left((R_m - \mu_m)_\alpha^2\right)^{1/2}} \quad (10)$$

where the notation is as in (4) – (6). We use the second moment ($k = 2$) corresponding to the correlation coefficient in each EDC since we are interested in how asset returns relate to the market return in an extreme market event.

Each of these extreme downside measures captures a different aspect of the performance of an asset in periods of distress. The Bawa and Lindenberg (1977) type measures (hereafter the BL measures) capture the tendency of an asset to offer worse returns when the market is in distress. The Estrada (2007) type measures (hereafter the ES measures) capture an asset's tendency to crash in times of market distress, and thus they are similar to the LTD measure of Ruenzi and Weigert (2013). The Ang *et al.* (2006a) measures (hereafter the ACY measures) capture the tendency of an asset to move with the market in periods of distress. Since stocks with higher systematic tail risk are undesirable to investors, such assets should command a positive risk premium.

A problem with these measures is that they are estimated with a small number of observations. Moreover, the ACY measures suffer from another problem. We rarely observe market crashes on consecutive days and therefore the measured comovement between a stock return and the market return during the period of a market crash is unreliable. For example, consider an extreme case when there is a stock that always experiences the same, lowest return on the days that the market crashes. Suppose further that the market never crashes on two consecutive days. In this case, this stock has a zero beta across market crashes and thus the ACY measure would suggest it is a safe investment. However, its return during market crashes is always the lowest return that it offers and thus the stock is a highly risky investment. This problem does not arise for the ACY downside beta since there are frequently consecutive days when the market offers returns lower than the mean return, and thus there is a measureable comovement between a stock and the market during these periods.

2.2. *Extreme Downside Hedge measures*

The second group of systematic tail risk measures that we develop is the Extreme Downside Hedge (EDH) measures. These measures rely on the argument that investors are able to hedge against extreme downside risk, and that any asset that acts as a hedge for this type of risk should be in high demand and thus command a premium. These measures can be estimated by simply regressing asset returns on a measure of market tail risk. However, since these measures are obtained from a regression, we need a large number of observations on market tail risk. One solution would be to use high frequency intraday data. However, high frequency returns are only available for a relatively short period and for a limited number of stocks. Instead, we use daily data, which we use to estimate market value at risk (VaR).

In the tail risk literature, VaR is one of the most commonly used measures (see Alexander, 2009; Bali et al., 2009; Adrian and Brunermeier, 2011; among others). Many studies have developed efficient methods to estimate daily VaR (for a review see, for example, Kuester et al., 2006; Nieto and Ruiz, 2016) . We estimate daily market VaR, which is then used in the regression of asset returns to obtain a systematic tail risk measure for each asset. However, we do not directly regress the day t asset return on the day t market VaR (VaR_t), since VaR_t is just the expected tail risk of day t based on information up to day $t-1$. Thus, the change in market VaR from day t to day $t+1$, ΔVaR_{t+1}, is determined by the new information revealed to the market on day t. Therefore, we regress the day t asset return on this market tail risk innovation.[1] This approach is analogous to that of Ang et al. (2006b), who use the change in market volatility to capture the systematic volatility risk of the market. We use an AR(1)-GJR GARCH(1,1) location-scale filter to obtain *i.i.d.* residuals for the VaR estimation. This filtering is essential since the fitted distribution from which VaR is estimated assumes *i.i.d.* observations. Specifically, the day t market VaR is estimated from its day t excess return $R_{m,t}$ as follows:

$$R_{m,t} = \mu_{m,t} + \varepsilon_{m,t} = \mu_{m,t} + \sigma_{m,t} z_{m,t} \tag{11}$$

$$\mu_{m,t} = a_0 + a_1 R_{m,t-1} \tag{12}$$

[1] We also examine systematic tail risk calculated from directly regressing asset returns of day t on the market VaR of day t. However, this measure is not associated with any significant risk premium in any of our tests.

$$\sigma_{m,t}^2 = c_0 + c_1\sigma_{m,t-1}^2 + c_2\varepsilon_{m,t-1}^2 \tag{13}$$

We assume the residual $\varepsilon_{m,t}$ follows a Skewed Student-t distribution. This accommodates the fact that stock returns exhibit fat tails and skewness even after autocorrelation and volatility clustering are accounted for. The day t market VaR is then estimated from the theoretical Skewed Student-t distribution of the standardized residuals, which is then unstandardized using the estimated mean and variance in (12) and (13). This approach is common in the literature (see, for example, Berkowitz and O'Brien, 2002; Bao *et al.*, 2006; Kuester *et al.*, 2006; among others). Additionally, we use a five year rolling window (1,250 daily observations) and a five percent threshold (i.e. a 95 percent confidence level) for the estimation of VaR. To examine the robustness of our results, we investigate the performance of our measures in alternative settings, including using the GARCH and EGARCH models for the conditional volatility, assuming a Gaussian distribution for the residuals, and different rolling estimation windows. We proxy the market excess return by the difference between the CRSP all stock index return and the daily risk free rate obtained from Kenneth French's online database.

After estimating the daily market VaR, the systematic tail risk measure of an asset is estimated by the following regression:

$$R_{i,t} = c_i + EDH_i \times \Delta VaR_{m,t+1} + \varepsilon_{i,t} \tag{14}$$

where $R_{i,t}$ is the excess return of stock i on day t, $\Delta VaR_{m,t+1}$ is the innovation in daily market VaR from day t to day $t+1$, and c_i and $\varepsilon_{i,t}$ are the intercept and error term, respectively. The estimated EDH coefficient directly shows how an asset performs as market risk changes. In this setting, we do not take the absolute value of market VaR, so a lower VaR means a larger loss. Therefore, a lower VaR implies a higher risk and a lower ΔVaR implies that tail risk is increasing. An asset with a high EDH offers low returns when the tail risk increases, and so investors would demand a positive premium to hold it. On the other hand, an asset with a low EDH provides a hedge against increases in tail risk, and so investors will be willing to pay a premium to hold it.

3. Portfolio Sorting Analysis

To examine the risk-return relationship, we first measure the average returns of portfolios sorted by the different systematic tail-risk measures. Following Ang *et al.* (2006a) we sort portfolios using stocks' postformation risk. In other words, stocks are sorted into portfolios based on the realization of their tail risk during the period when the portfolio returns are calculated. Hereafter, we denote the investigation of the relationship between stock returns and the contemporaneous risk measure as the postformation setting. This is to distinguish it from the preformation setting, where we examine the relationship between stock returns and lagged tail risk. We use the postformation setting because systematic tail risk (and downside risk in general) is not stable over time and therefore, past tail risk is not a good proxy for current tail risk. We investigate this finding further in the robustness tests in the following section.

To carry out the postformation sort, at the beginning of every year from 1973 to 2012, we calculate the tail risk measures for all stocks in the NYSE, AMEX and NASDAQ markets using daily data during each year.[2] We then sort the stocks into quintiles based on each risk measure and calculate the equally weighted and value weighted monthly excess returns of these quintiles for the same year. The equally-weighted and value-weighted average returns of these portfolios over the entire sample period are reported in Tables 1 and 2, respectively. We also calculate the return of the long-short strategy which takes a long position in portfolio 5 (the quintile with the highest risk measure) and a short position in portfolio 1 (the quintile with the lowest risk measure). We report the alphas of Fama-French's (1993) (hereafter FF) three-factor model and Carhart's (1997) four-factor model in explaining the returns of this long-short strategy.

Table 1 shows a significant positive relationship between systematic tail risk and equally weighted average returns using all of our risk measures. Specifically, the average excess return increases monotonically from quintile 1 to quintile 5 and the returns from the long-short strategies are highly positive, even after controlling for the systematic risks in the FF and Carhart models. The Newey-West t-statistics of the long-short strategy returns and alphas are significant at the one percent level. Table 1 also shows that stocks with higher systematic tail risk tend to be larger. Consequently, the relationship between tail risk and value weighted returns reported in Table 2 is somewhat weaker since larger stocks have lower returns on

[2] We eliminate stocks with fewer than 125 observations.

average, offsetting the effect of higher systematic tail risk (see Ang et al., 2006a). Nevertheless, the long-short portfolios still have significantly positive excess returns using both the FF and Carhart models in the majority of cases.

4. Cross-Sectional Analysis

The portfolio sorting analysis does not allow us to account for multiple risk factors simultaneously. In order to uncover the influence of tail risk on returns beyond that of other, related risk factors, we now turn to the cross sectional regression analysis. Since the majority of our measures require at least one year of daily data to estimate, we follow the approach of Ang *et al.* (2006a) and use overlapping annual samples to yield a sufficient number of observations. Specifically, at the beginning of every month, we calculate the excess return of each stock relative to the T-bill rate over the following year. We then estimate the risk measures of the stock using daily returns over the same year. We refer to these measures as realized risk measures. Following Ang *et al.* (2006a), we also use various lagged measures including the lagged one-year return, size (measured by the natural logarithm of market capitalization) and book-to-market (calculated using book value from the last fiscal year and market value at the end of the calendar year). Again following Ang *et al.* (2006a) we separate beta into downside and upside beta in order to asses the significance of tail risk beyond that of general downside risk. Our full set of variables therefore comprises realized downside beta, realized upside beta, realized standard deviation, realized co-skewness, realized co-kurtosis, lagged one-year return, realized systematic tail risk, size and book-to-market. The risk premia associated with these variables are estimated using the Fama and MacBeth (1973) approach. In particular, each month we estimate a cross-sectional regression of realized excess returns over the following year on the set of variables using all of the stocks in the market, and calculate the time series average estimated slope coefficient for each variable. We use Newey-West's (1987) HAC estimator with 12 lags to estimate the standard errors of the estimated risk premiums allowing for the overlapping estimation window. To reduce the effects of outliers (particularly in book-to-market) we winsorize all independent variables using a threshold of one percent. The sample period is from January 1973 to December 2012, yielding 468 monthly cross-sectional regressions. Table 3 presents the results of the Fama-MacBeth analysis.

We first examine the performance of the standard risk measures in Models I to III, and confirm the established findings in the literature. In particular, size and book-to-market are associated with a significant negative and positive risk premium, respectively. We find a strongly positive downside beta risk premium and a much weaker upside beta risk premium, similar to Ang *et al.* (2006a). We also account for leverage and volatility feedback effects (see, for example, Black, 1976; Christie, 1982; Campbell and Hentschel, 1992) and find a marginally significantly negative coefficient on realized volatility. Further, the risk premium is negative for co-skewness and positive for co-kurtosis. Finally, our results regarding past returns tend to support the reversal effect rather than the momentum effect, although the average coefficient is statistically insignificant.

In models IV to X, we incorporate the new systematic tail risk measures. Our results reveal a positive and statistically significant relationship between tail risk and average returns for all measures with the exception of the ACY measure. As discussed in the previous section, the ACY measure is likely to be a poorer measure of systematic tail risk given the negligible probability of market crashes in consecutive days. The inclusion of the systematic tail risk measures reduces the significance of downside beta and co-skewness, but the other risk factors are unaffected. This is not surprising since downside beta, co-skewness and systematic tail risk all reflect downside risk, albeit at different levels of severity. Our results suggest that tail risk contains additional important information beyond general downside risk.

5. Robustness Checks

5.1 *Tail Risk Persistence*

We investigate the persistence of both downside risk and tail risk over time. In particular, we examine the tendency for a stock to remain in the same risk quintile in consecutive years. For each quintile in each year, we take the number of securities that are still in the same quintile in the following year, divided by the average number of securities in the quintile across the two years. We compute this fraction for each pair of consecutive years from 1973 to 2012, and then calculate the average value of this ratio for each quintile. In Table 4, we report this measure of persistence for the postformation tail risk measures and the commonly used preformation measures of size, book-to-market and idiosyncratic volatility. We use the downside beta of Ang et al. (2006a) as a benchmark for the postformation tail risk measures.

Table 4 shows that downside risk measures, including both downside beta and the new tail risk measures, are not persistent over time. Fewer than half of the stocks in the highest and lowest quintiles remain in these quintiles in the following year. In contrast, the preformation risk measures are relatively persistent. The difference in persistence between the preformation and postformation measures is greatest in the lowest and the highest quintiles. In these quintiles, the common fraction between two consecutive years is about 30-40 percent for the downside risk measures, but about 60-80 percent for the preformation risk measures. This suggests that the use of the preformation setting is unlikely to provide valid inference about the systematic tail risk premium.

Next, following Ruenzi and Weigert (2013) we examine how the risk level of each quintile changes over time. For each risk measure, we identify the constituent stocks of the five quintiles in every year. We then calculate the equally weighted average risk for these stocks in that year as well as in the following four years. This gives the five-year evolution of the average tail risk measure for each quintile. We then take the average of this five year pattern starting every year from 1973 to 2012. The evolution of these measures is shown in Figures 1 and 2. As before, we use Ang *et al.*'s (2006a) downside beta as the benchmark for the postformation measures

The figures show that our systematic extreme downside risk measures are much less persistent than the preformation measures. For the postformation measures, the differences in the level of risk between quintiles reduce significantly after just one year, while those of preformation measures are relatively stable over time. These patterns are similar to that of LTD measure in Ruenzi and Weigert (2013). This finding is further evidence against the use of the preformation setting for analyzing the effects of the downside risk on returns.

5.2 *Different Tail Risk Threshold Levels*

In this section, we examine how altering the quantile levels affects the performance of the systematic tail risk measures. Above, we use the five percent quantile of the return distribution for all measures. Table 5 reports results using both the 10 percent and one percent thresholds. The risk premium associated with the EDH measures is consistently significantly positive, while those of the EDB and EDC measures are significantly positive in most cases. Reducing the quantile level tends to reduce both the size of the coefficient, and its statistical significance. This is partly explained by the fact that the EDB and EDC measures rely on a small number of observations. For example, with one year of daily return data, the one percent quantile measures are based on only two observations. Thus, these measures are extremely sensitive to the size of the estimation window as well as the quantile level. The EDH measures do not suffer from this problem since we can obtain VaR for any significance level at the daily frequency. This enables us to examine the impact of systematic tail risk on returns at very short investment horizons.

5.3 *Different Value-at-Risk Measures*

We examine the robustness of the EDH measures by using different VaR models for the daily market tail risk. Specifically, we examine the performance of the VaR model employing the GARCH, GJR GARCH and EGARCH models. We also allow the residual distribution to be either Gaussian or Skewed Student-t. Further, we use alternative estimation windows of five years (1,250 days) and two years (500 days). Finally, we use Expected Tail Loss (ETL) as an alternative to VaR. We estimate ETL under the assumption that the location-scale filtered residual follows a Gaussian distribution. The ETL of a normally distributed variable is defined as:

$$ETL_\alpha = \frac{1}{\alpha}\varphi\left(\Phi^{-1}(\alpha)\right)\sigma - \mu \tag{15}$$

where returns are assumed to be $N(\mu,\sigma)$, $\Phi^{-1}(\alpha)$ is the α-percent quantile of the $N(0,1)$ distribution and φ is the $N(0,1)$ density function. The results with five percent VaR and ETL are summarized in Table 6. We obtain similar results using the 10 percent and one percent tail risk measures and these are available upon request. All of the EDH measures have a consistently positive relationship with returns and are significant at the one percent level.

5.4 *EDH and Systematic Volatility Risk*

Our EDH measures are closely related to the systematic volatility risk measure proposed by Ang *et al.* (2006b), constructed by regressing excess stock returns on the changes of the Chicago Board Options Exchange's VIX index. Ang *et al.* (2006b) show that this systematic volatility risk measure is significantly related to returns. Moreover, this risk is associated with a negative premium as stocks with high sensitivity to the VIX index tend to offer high returns when aggregate risk increases. Since VaR is significantly influenced by volatility, it is not clear whether systematic tail risk contains significant information beyond that contained in systematic volatility.

The correlation between Ang *et al.*'s (2006b) ΔVIX_t measure[3] and ΔVaR_{t+1} is about 60 percent. In contrast, the contemporaneous correlation between ΔVaR_t and ΔVIX_t is very modest. This supports our use of ΔVaR_{t+1} in the EDH regression (14). To examine the marginal contribution of tail risk, we include the systematic volatility (SV) risk measure in our cross sectional regressions. Ang *et al*. (2006b) estimate SV via a multivariate regression of each stock's excess returns on market excess returns and the change in the VIX index. In our model, we already include betas in the cross sectional regression, and so we estimate SV by regressing the stock's excess return on only the change in the VIX index. This also ensures that SV is constructed in the same way as our EDH measure. Table 7 reports the results of the cross-sectional analysis of the EDH measure constructed using different market VaR models. Owing to the short horizon of the VIX data, the sample covers the period from January 1986 to December 2012. The table confirms the significance and the negative sign of the systematic volatility risk premium. Importantly, however, the systematic tail risk premium is also significant and positive. Thus, systematic tail risk contains important information about asset returns, even after controlling for systematic volatility risk (see also Bali *et al.,* 2009; Harris *et al.,* 2015).

5.5 *The Performance of the EDH Measures Using Short Investment Horizons*

The EDH measures can be estimated with a very short sample and therefore enable us to examine how stock returns are affected by systematic tail risk at different investment horizons. In Table 8, we report the results of estimating the cross-sectional regressions for investment horizons from one to twelve months, where the EDH measures are estimated by VaR with a five percent quantile using the $AR(1) - GRJ\ GARCH(1{,}1)$ Skewed Student-t model and a five year estimation window. The results for alternative specifications of the EDH measures are similar and are available upon request. The table shows that the tail risk premium is always positive and becomes more significant at longer horizons. Indeed, it becomes statistically significant from the five month horizon onwards, which suggests that tail risk is more important to investors with longer investment horizons. Only market-to-book and co-kurtosis exhibit a consistent relationship with returns at all horizons.

[3] Similar to Ang et al (2006b), we use the old index VXO to expand the data beyond January 1986.

6. Conclusion

In this paper, we introduce new systematic tail risk measures using two approaches. The first (the EDB and EDC measures) follow naturally from existing downside beta and co-moment measures, respectively, while the second (the EDH measure) is based on the demand of investors to hedge against extreme downside risk. Using all three measures, we find evidence of a significantly positive tail risk premium. Moreover, a significant advantage of the EDH measure is that it can be estimated with a sample as short as one month, offering a solution to the problem of small samples often encountered when studying extreme downside risk. Using the EDH measure for horizons of one to 12 months, we show that tail risk is more relevant for longer horizons. An interesting direction for future research would be to examine the role of idiosyncratic tail risk in explaining the cross-section of stock returns. Owing to the fact that, in practice, investors are heterogeneous and not fully diversified, idiosyncratic risk has been shown to be important in many contexts. In particular, in relation to downside risk, many studies have confirmed the nontrivial influence of idiosyncratic skewness on asset returns (see Mitton and Vorkink, 2007; Boyer *et al.*, 2010; Conrad *et al.*, 2013; among others). It would be natural to explore whether a similar finding holds for idiosyncratic tail risk

References

Adrian, T., Brunnermeier, M.K., 2011. CoVaR. National Bureau of Economic Research

Alexander, C., 2009. Market Risk Analysis: Value at Risk Models. John Wiley & Sons.

Ang, A., Chen, J., Yuhang, X., 2006a. Downside Risk. Review of Financial Studies 19, 1191-1239

Ang, A., Hodrick, R.J., Yuhang, X., Xiaoyan, Z., 2006b. The Cross-Section of Volatility and Expected Returns. Journal of Finance 61, 259-299

Artzner, P., Delbaen, F., Eber, J.M., Heath, D., 1999. Coherent measures of risk. Mathematical Finance 9, 203-228

Bali, T.G., Cakici, N., 2004. Value at Risk and Expected Stock Returns. Financial Analysts Journal 60, 57-73

Bali, T.G., Demirtas, K.O., Levy, H., 2009. Is There an Intertemporal Relation between Downside Risk and Expected Returns? Journal of Financial and Quantitative Analysis 44, 883-909

Bao, Y., Lee, T.H., Saltoglu, B., 2006. Evaluating predictive performance of value-at-risk models in emerging markets: a reality check. Journal of Forecasting 25, 101-128

Barro, R.J., 2006. Rare disasters and asset markets in the twentieth century. Quarterly Journal of Economics 121, 823-866

Bawa, V.S., Lindenberg, E.B., 1977. Capital market equilibrium in a mean-lower partial moment framework. Journal of Financial Economics 5, 189-200

Berk, J.B., 2000. Sorting out sorts. The Journal of Finance 55, 407-427

Berkowitz, J., O'Brien, J., 2002. How Accurate Are Value-at-Risk Models at Commercial Banks? Journal of Finance 57, 1093-1111

Black, F., 1976. Studies of stock price volatility changes. In: The 1976 Meetings of the Business and Economic Statistics Section pp. 177-181. American Statistical Association

Bollerslev, T.I.M., Todorov, V., 2011. Tails, Fears and Risk Premia. Journal of Finance 66, 2165-2211

Boyer, B., Mitton, T., Vorkink, K., 2010. Expected Idiosyncratic Skewness. Review of Financial Studies 23, 169-202

Brockett, P.L., Kahane, Y., 1992. Risk, return, skewness and preference. Management Science 38, 851-866

Campbell, J.Y., Hentschel, L., 1992. No news is good news: An asymmetric model of changing volatility in stock returns. Journal of Financial Economics 31, 281-318

Carhart, M.M., 1997. On persistence in mutual fund performance. The Journal of Finance 52, 57-82

Carvalhal da Silva, A., 2006. Modeling and estimating a higher systematic co-moment asset pricing model in the Brazilian stock market. Latin American Business Review 6, 85-101

Chi-Hsiou Hung, D., Shackleton, M., Xinzhong, X., 2004. CAPM, Higher Co-moment and Factor Models of UK Stock Returns. Journal of Business Finance and Accounting 31, 87-112

Christie, A.A., 1982. The stochastic behavior of common stock variance: Value, Leverage and Interest Rate Effects. Journal of Financial Economics 10, 407-432

Chung, Y.P., Johnson, H., Schill, M.J., 2006. Asset Pricing When Returns Are Nonnormal: Fama-French Factors versus Higher-Order Systematic Comoments. Journal of Business 79, 923-940

Daniel, K., Titman, S., 1997. Evidence on the Characteristics of Cross Sectional Variation in Stock Returns. Journal of Finance 52, 1-33

Dittmar, R.F., 2002. Nonlinear Pricing Kernels, Kurtosis Preference and Evidence from the Cross Section of Equity Returns. Journal of Finance 57, 369-403

Estrada, J., 2007. Mean-semivariance behavior: Downside risk and capital asset pricing. International Review of Economics and Finance 16, 169-185

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3-56

Fama, E.F., MacBeth, J.D., 1973. Risk, Return and Equilibrium: Empirical Tests. Journal of Political Economy 81, 607

Friend, I., Westerfield, R., 1980. Co-skewness and capital asset pricing. The Journal of Finance 35, 897-913

Gabaix, X., 2012. Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance. Quarterly Journal of Economics 127, 645-700

Galagedera, D.U., Maharaj, E.A., 2008. Wavelet timescales and conditional relationship between higher-order systematic co-moments and portfolio returns. Quantitative Finance 8, 201-215

Goyal, A., 2012. Empirical cross-sectional asset pricing: a survey. Financial Markets and Portfolio Management 26, 3-38

Guidolin, M., Timmermann, A., 2008. International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences. Review of Financial Studies 21, 889-935

Harlow, W.V., Rao, R.K., 1989. Asset pricing in a generalized mean-lower partial moment framework: Theory and evidence. Journal of Financial and Quantitative Analysis 24, 285-311

Harris, R.D.F., Nguyen, L.H., Stoja, E., 2015. Extreme downside risk and financial crisis. Bank of England Staff Working Paper No.547.

Harvey, C.R., Siddique, A., 2000. Conditional Skewness in Asset Pricing Tests. Journal of Finance 55, 1263-1295

Huang, W., Liu, Q., Ghon Rhee, S., Wu, F., 2012. Extreme downside risk and expected stock returns. Journal of Banking and Finance 36, 1492-1502

Kaplanski, G., 2004. Traditional beta, downside risk beta and market risk premiums. Quarterly Review of Economics and Finance 44, 636-653

Kimball, M.S., 1993. Standard risk aversion. Econometrica 61, 589-611

Kostakis, A., Muhammad, K., Siganos, A., 2012. Higher co-moments and asset pricing on London Stock Exchange. Journal of Banking and Finance 36, 913-922

Kraus, A., Litzenberger, R.H., 1976. Skewness preference and the valuation of risk assets. Journal of Finance 31, 1085-1100

Kuester, K., Mittnik, S., Paolella, M.S., 2006. Value-at-risk prediction: A comparison of alternative strategies. Journal of Financial Econometrics 4, 53-89

Mehra, R., Prescott, E.C., 1985. The equity premium: A puzzle. Journal of Monetary Economics 15, 145-161

Mishra, S., DeFusco, R.A., Prakash, A.J., 2008. Skewness preference, value and size effects. Applied Financial Economics 18, 379-386

Mitton, T., Vorkink, K., 2007. Equilibrium Underdiversification and the Preference for Skewness. Review of Financial Studies 20, 1255-1288

Newey, W.K., West, K.D., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703-708

Nieto, M. R., Ruiz, E., 2016. Frontiers in VaR forecasting and backtesting. International Journal of Forecasting, 32, 475-501

Pástor, Ľ., Stambaugh, R.F., 2003. Liquidity Risk and Expected Stock Returns. Journal of Political Economy 111, 642-685

Post, T., Van Vliet, P., Levy, H., 2008. Risk aversion and skewness preference. Journal of Banking and Finance 32, 1178-1187

Rietz, T.A., 1988. The equity risk premium: a solution. Journal of Monetary Economics 22, 117-131

Ruenzi, S., Weigert, F., 2013. Crash Sensitivity and the Cross-Section of Expected Stock Returns. Working Paper. Available at SSRN: http://ssrn.com/abstract=2011746

Wachter, J.A., 2013. Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? The Journal of Finance 68, 987-1035

Yang, C.-Y., Chen, M.-C., 2009. The role of co-kurtosis in the pricing of real estate. Journal of Real Estate Portfolio Management 15, 185-195

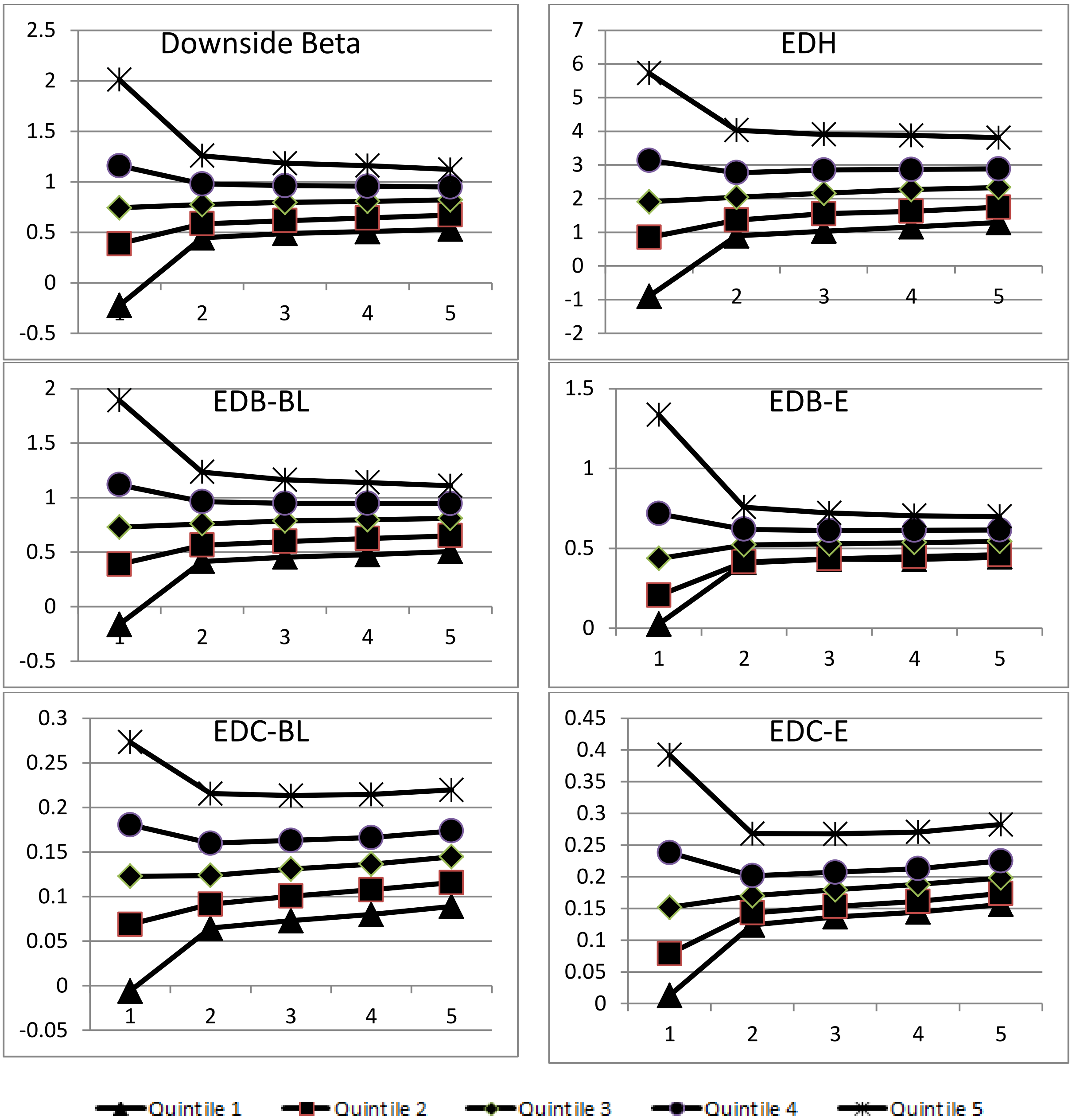

Figure 1: Persistence analysis of postformation measures. This figure displays the evolution over 5 years of average postformation measures of stocks in quintiles constructed at the beginning of the 5 year period. These quintiles are constructed by sorting all stocks in NYSE, AMEX, NASDAQ markets on the corresponding risk measure of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

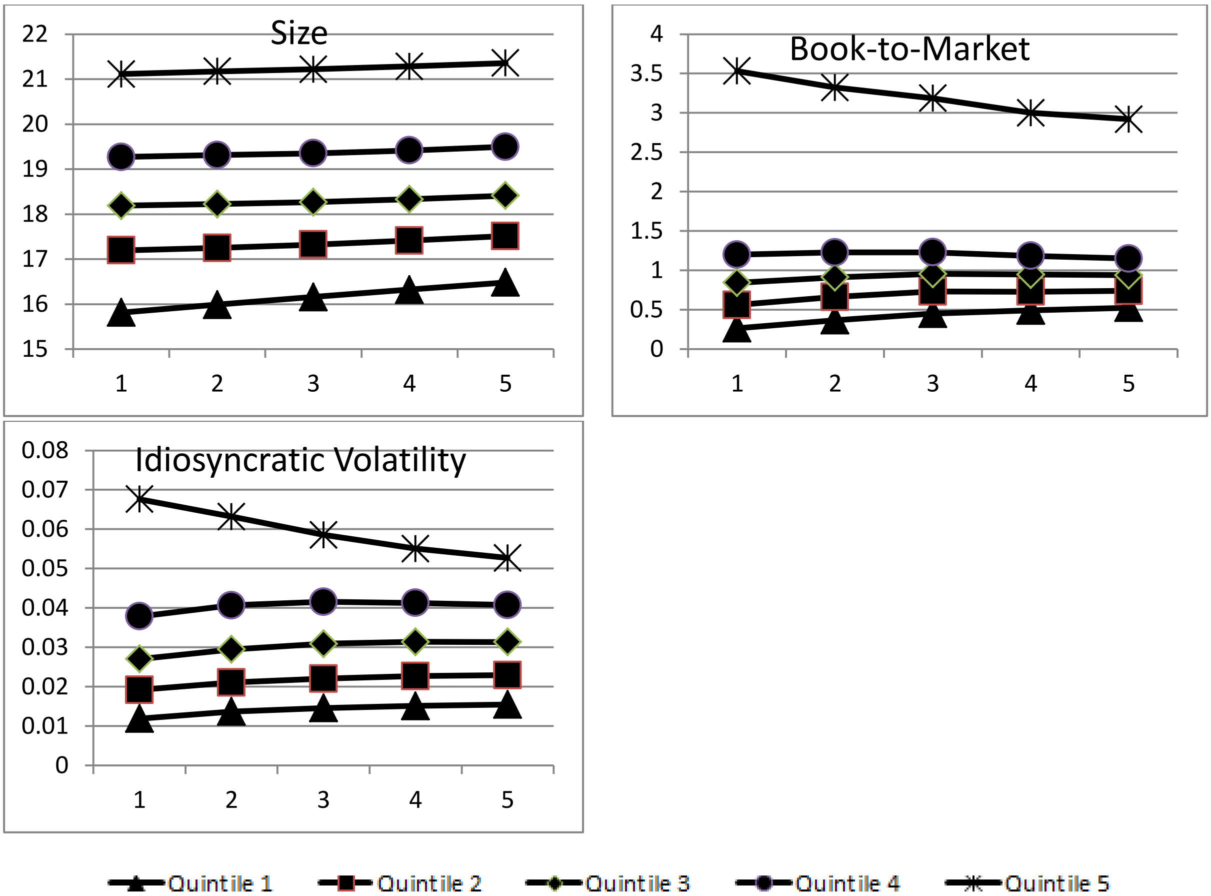

Figure 2: Persistence analysis of preformation measures. This figure displays the evolution over 5 years of average preformation measures of stocks in quintiles constructed at the beginning of the 5 year period. These quintiles are constructed by sorting all stocks in NYSE, AMEX, NASDAQ markets on the corresponding risk measure of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

Table 1: Average excess returns of equally weighted quintile portfolios sorting on systematic extreme downside risk measures

This table shows the average yearly excess returns and sizes of equally weighted quintile portfolios sorted on different systematic extreme downside risk measures. These quintiles are sorted using 1 year postformation measures, which are concurrent with the quintile returns. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns on the corresponding first row. The last three columns are the average excess return of the long-short strategy which buys quintile 5 and sells quintile 1, its alphas in Fama and French (1993) three factor model and Carhart (1997) four factor models. The overall sample period is from January 1973-December 2012.

Quintiles	1	2	3	4	5	5 - 1	FF	Carhart
				EDH				
Average returns	5.684	8.444	9.745	11.249	16.975	11.291	11.227	12.607
t-statistics	(1.758)	(3.047)	(3.565)	(3.874)	(3.910)	(3.622)	(3.464)	(3.620)
Average size	17.187	18.155	18.622	18.877	18.961			
				EDB - BL				
Average returns	4.301	8.010	9.722	12.069	18.611	14.310	13.824	11.725
t-statistics	(1.429)	(3.175)	(3.785)	(3.949)	(4.038)	(4.035)	(3.736)	(3.366)
Average size	17.333	18.342	18.774	18.887	18.611			
				EDB - ACY				
Average returns	6.837	8.577	10.162	12.238	14.753	7.915	10.154	6.993
t-statistics	(1.879)	(3.498)	(4.118)	(4.457)	(3.686)	(3.156)	(4.254)	(3.506)
Average size	17.703	18.544	18.831	18.790	18.076			
				EDB - E				
Average returns	4.604	7.573	10.237	12.555	17.586	12.982	12.672	11.186
t-statistics	(1.633)	(3.273)	(3.769)	(3.921)	(4.025)	(4.245)	(4.119)	(3.886)
Average size	17.532	18.453	18.691	18.756	18.511			
				EDC - BL				
Average returns	4.015	8.238	10.532	13.044	16.770	12.755	15.803	9.421
t-statistics	(1.164)	(2.419)	(3.430)	(4.740)	(5.191)	(3.784)	(6.993)	(4.717)
Average size	17.004	17.617	18.375	19.070	19.874			
				EDC - ACY				
Average returns	6.044	7.908	11.111	12.900	14.673	8.629	12.194	7.445
t-statistics	(1.813)	(2.540)	(3.587)	(4.609)	(4.902)	(3.282)	(6.268)	(4.127)
Average size	17.819	18.156	18.370	18.630	18.984			
				EDC - E				
Average returns	2.680	5.242	10.141	13.920	20.523	17.843	20.563	14.900
t-statistics	(0.888)	(1.603)	(3.238)	(4.824)	(5.815)	(6.065)	(9.280)	(8.099)
Average size	17.341	17.821	18.359	18.871	19.551			

Table 2: Average excess returns of value weighted quintile portfolios sorting on systematic extreme downside risk measures

This table shows the average yearly excess returns and sizes of value-weighted quintile portfolios sorted on different systematic extreme downside risk measures. These quintiles are sorted using 1 year postformation measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the return in the corresponding first row. The last three columns are for the average excess return of the long-short strategy which goes long quintile 5 and goes short quintile 1, its alphas in Fama and French (1993) three factor model and Carhart (1997) four factor models. The overall sample period is from January 1973-December 2012.

Quintiles	**1**	**2**	**3**	**4**	**5**	**5 - 1**	**FF**	**Carhart**
				EDH				
Average returns	4.950	6.113	5.577	5.354	6.984	2.034	3.115	4.171
t-statistics	(2.531)	(3.507)	(3.088)	(2.888)	(2.082)	(0.580)	(1.016)	(1.179)
				EDB – BL				
Average returns	4.547	4.796	6.350	7.078	9.707	5.161	4.492	3.300
t-statistics	(2.566)	(3.065)	(3.551)	(2.647)	(2.467)	(1.339)	(1.247)	(0.865)
				EDB – ACY				
Average returns	2.219	4.554	6.019	8.439	7.442	5.223	7.284	3.884
t-statistics	(1.009)	(2.167)	(2.715)	(3.467)	(2.347)	(2.490)	(2.732)	(1.599)
				EDB – E				
Average returns	0.638	4.505	5.304	7.878	10.803	10.165	9.584	8.417
t-statistics	(0.354)	(2.497)	(2.703)	(3.046)	(2.963)	(3.024)	(2.477)	(2.209)
				EDC – BL				
Average returns	2.217	0.186	1.452	3.498	8.725	6.508	9.121	7.139
t-statistics	(1.184)	(0.104)	(0.734)	(1.694)	(3.337)	(2.742)	(3.336)	(3.019)
				EDC – ACY				
Average returns	2.943	3.032	3.651	6.845	8.784	5.841	8.929	7.102
t-statistics	(1.414)	(1.343)	(1.673)	(3.181)	(3.568)	(3.965)	(4.629)	(3.674)
				EDC – E				
Average returns	-3.755	-2.053	0.972	4.065	10.531	14.286	17.005	16.117
t-statistics	(-1.716)	(-1.052)	(0.476)	(1.795)	(4.189)	(7.803)	(8.646)	(8.696)

Table 3: Cross-sectional analysis of systematic extreme downside risk

This table shows the Fama and MacBeth (1973) average risk premiums of standard risk measures and of the proposed systematic tail risk measures, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measures of downside beta, upsize beta, volatility, co-skewness, co-kurtotis and systematic tail risk; its past excess return of last year; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1973-December 2012 (468 monthly observations) and covers all stocks in NYSE, AMEX and NASDAQ markets.

Model	I	II	III	IV	V	VI	VII	VIII	IX	X
Intercept	0.047	0.525	1.030	1.079	1.032	1.046	1.026	1.058	1.045	1.033
	(2.032)	(3.571)	(8.175)	(8.681)	(8.205)	(8.232)	(8.135)	(8.312)	(8.173)	(8.309)
β-	0.059	0.076	0.039	0.005	0.010	0.037	0.031	0.034	0.034	0.045
	(3.412)	(3.777)	(1.897)	(0.255)	(0.754)	(1.827)	(1.605)	(1.704)	(1.732)	(2.307)
β+	-0.002	0.010	0.003	-0.005	-0.001	0.004	0.000	0.000	0.004	-0.007
	(-0.250)	(1.270)	(0.295)	(-0.459)	(-0.117)	(0.504)	(0.013)	(-0.009)	(0.445)	(-0.867)
Log-size		-0.026	-0.054	-0.056	-0.054	-0.056	-0.054	-0.056	-0.055	-0.055
		(-3.703)	(-8.911)	(-9.247)	(-8.899)	(-8.946)	(-8.782)	(-9.104)	(-8.860)	(-9.123)
B/M		0.019	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.012
		(3.138)	(2.421)	(2.403)	(2.464)	(2.389)	(2.425)	(2.378)	(2.407)	(2.336)
Past			-0.002	-0.002	-0.004	-0.002	-0.003	-0.003	-0.003	-0.001
Return			(-0.133)	(-0.157)	(-0.220)	(-0.129)	(-0.167)	(-0.154)	(-0.152)	(-0.037)
Volatility			-1.205	-1.575	-1.509	-1.130	-1.389	-1.147	-1.189	-1.295
			(-1.332)	(-1.803)	(-1.779)	(-1.247)	(-1.614)	(-1.278)	(-1.303)	(-1.464)
Coskew			-0.121	-0.097	-0.065	-0.157	-0.098	-0.036	-0.166	0.136
			(-2.432)	(-2.058)	(-1.159)	(-3.013)	(-1.938)	(-0.594)	(-3.127)	(2.814)
Cokurt			0.108	0.095	0.095	0.117	0.104	0.089	0.121	0.036
			(7.480)	(6.862)	(6.551)	(7.427)	(6.958)	(5.568)	(7.039)	(2.372)
EDH				0.028						
				(3.534)						
EDB-BL					0.052					
					(2.684)					
EDB-ACY						-0.006				
						(-2.451)				
EDB-E							0.032			
							(2.151)			
EDC-BL								0.279		
								(2.969)		
EDC-ACY									-0.035	
									(-2.262)	
EDC-E										0.581
										(12.376)

Table 4: Persistency analysis-Common fraction of quintiles in two consecutive years

This table shows the percentage of common fraction between two consecutive years of sorted quintiles sorted on different risk measures. These values are averaged over the whole sample period from 1973 to 2012.

Quintile	1	2	3	4	5	average
Panel 1: Postformation measures						
Downside beta	37.38%	28.13%	25.92%	27.36%	39.67%	31.69%
EDH	37.55%	29.15%	26.27%	28.15%	39.11%	32.05%
EDB-BL	39.27%	28.54%	26.32%	28.14%	41.22%	32.70%
EDB-E	30.20%	26.01%	22.44%	24.27%	32.89%	27.16%
EDC-BL	36.36%	24.60%	22.92%	26.71%	47.84%	31.69%
EDC-E	28.39%	22.50%	20.28%	22.22%	38.31%	26.34%
Panel 2: Preformation measures						
Size	79.20%	60.44%	62.07%	70.69%	86.71%	71.82%
B/M	60.11%	42.08%	38.52%	42.25%	63.42%	49.28%
Idiosyncratic volatility	72.22%	50.60%	43.02%	43.08%	59.59%	53.70%

Table 5: Cross-sectional analysis of systematic extreme downside risk captured using different extreme downside thresholds

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures and of the systematic tail risk measures captured under different extreme downside thresholds, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measure of downside beta, upsize beta, volatility, co-skewness, co-kurtotis and systematic tail risk; its past excess return of last year; and its and Book-to-Market available at the time of the regression. The overall sample period is from January 1973-December 2012 and covers all stocks in NYSE, AMEX and NASDAQ markets.

Model	10 percent tail quantile					1 percent tail quantile				
	I	II	III	IV	V	I	II	III	IV	V
Intercept	1.075	1.057	1.029	1.109	1.053	1.083	1.031	1.018	1.041	1.022
	(8.642)	(8.353)	(8.218)	(8.349)	(8.511)	(8.721)	(8.174)	(8.119)	(8.262)	(8.171)
β-	0.013	-0.027	0.029	0.024	0.030	-0.005	0.037	0.037	0.035	0.048
	(0.695)	(-2.316)	(1.591)	(1.215)	(1.564)	(-0.233)	(1.865)	(1.825)	(1.727)	(2.351)
β+	-0.003	-0.002	0.000	-0.002	-0.011	-0.006	0.003	0.000	0.004	0.000
	(-0.307)	(-0.253)	(0.052)	(-0.198)	(-1.357)	(-0.763)	(0.350)	(0.028)	(0.474)	(-0.013)
Log-size	-0.056	-0.055	-0.054	-0.060	-0.060	-0.056	-0.055	-0.053	-0.055	-0.053
	(-9.232)	(-9.017)	(-8.868)	(-9.121)	(-9.963)	(-9.272)	(-8.912)	(-8.766)	(-9.079)	(-8.793)
B/M	0.013	0.013	0.013	0.013	0.012	0.013	0.013	0.013	0.013	0.013
	(2.398)	(2.527)	(2.446)	(2.347)	(2.288)	(2.412)	(2.443)	(2.455)	(2.459)	(2.435)
Past	-0.003	-0.006	-0.003	-0.003	0.000	-0.003	-0.003	-0.003	-0.003	-0.003
Return	(-0.160)	(-0.328)	(-0.167)	(-0.158)	(-0.013)	(-0.179)	(-0.168)	(-0.193)	(-0.174)	(-0.158)
Volatility	-1.529	-1.802	-1.500	-1.019	-1.153	-1.622	-1.234	-1.303	-1.149	-1.336
	(-1.735)	(-2.102)	(-1.822)	(-1.121)	(-1.306)	(-1.871)	(-1.394)	(-1.448)	(-1.266)	(-1.487)
Coskew	-0.101	-0.062	-0.104	0.065	0.267	-0.092	-0.123	-0.093	-0.179	-0.054
	(-2.146)	(-1.158)	(-2.022)	(1.123)	(5.729)	(-1.931)	(-2.274)	(-1.826)	(-3.114)	(-1.062)
Cokurt	0.097	0.084	0.104	0.048	-0.020	0.092	0.108	0.102	0.125	0.090
	(7.040)	(6.223)	(7.160)	(3.441)	(-1.255)	(6.613)	(7.110)	(7.093)	(6.972)	(6.219)
EDH	0.022					0.043				
	(3.594)					(3.374)				
EDB-BL		0.112					0.004			
		(3.522)					(0.601)			
EDB-E			0.033					0.029		
			(1.353)					(5.129)		
EDC-BL				0.667					-0.137	
				(5.982)					(-2.117)	
EDC-E					1.053					0.147
					(15.212)					(8.738)

Table 6: Cross-sectional analysis of EDH measures captured using different models for market tail risk

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures and of the EDH measures captured using different market tail risk models, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measure of downside beta, upsize beta, volatility, co-skewness, co-kurtotis and EDH; its past excess return of last year; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1973-December 2012 and covers all stocks in NYSE, AMEX and NASDAQ markets. The name of each regression model specifies whether the corresponding EDH measure utilizes market VaR or ETL model, Gaussian (G) or Skewed Student-t (S) residual term distribution assumption, GARCH or EGARCH or GJR GARCH conditional volatility and 5 year (1250 days) or two year (500 days) estimation period.

Model	**VaR_EGARCH G_1250**	**VaR_EGARCH S_1250**	**ETL_EGARCH G_1250**	**VaR_EGARCH G_500**	**VaR_EGARCH S_500**	**ETL_EGARCH G_500**
Intercept	1.076	1.080	1.078	1.078	1.083	1.081
	(8.611)	(8.654)	(8.627)	(8.592)	(8.634)	(8.606)
β-	0.017	0.013	0.013	0.017	0.016	0.014
	(0.858)	(0.683)	(0.636)	(1.019)	(0.973)	(0.796)
β+	-0.002	-0.004	-0.003	-0.003	-0.003	-0.003
	(-0.231)	(-0.471)	(-0.327)	(-0.294)	(-0.249)	(-0.277)
Log-size	-0.056	-0.057	-0.057	-0.056	-0.057	-0.057
	(-9.234)	(-9.269)	(-9.241)	(-9.202)	(-9.234)	(-9.208)
B/M	0.013	0.013	0.013	0.013	0.013	0.013
	(2.407)	(2.402)	(2.411)	(2.403)	(2.407)	(2.407)
Past	-0.003	-0.003	-0.003	-0.004	-0.004	-0.003
Return	(-0.159)	(-0.200)	(-0.172)	(-0.227)	(-0.240)	(-0.206)
Volatility	-1.441	-1.471	-1.464	-1.489	-1.524	-1.503
	(-1.606)	(-1.641)	(-1.636)	(-1.683)	(-1.732)	(-1.699)
Coskew	-0.119	-0.119	-0.119	-0.115	-0.115	-0.114
	(-2.499)	(-2.461)	(-2.482)	(-2.429)	(-2.410)	(-2.433)
Cokurt	0.098	0.097	0.097	0.096	0.093	0.095
	(7.242)	(7.153)	(7.155)	(7.156)	(6.968)	(7.117)
EDH	0.024	0.022	0.028	0.028	0.025	0.033
	(3.018)	(3.123)	(3.007)	(3.207)	(3.109)	(3.153)

(continued)

Table 6: Continued

Model	VaR_GARCH G_1250	VaR_GARCH S_1250	ETL_GARCH G_1250	VaR_GARCH G_500	VaR_GARCH S_500	ETL_GARCH G_500
Intercept	1.067	1.067	1.068	1.066	1.064	1.066
	(8.565)	(8.560)	(8.571)	(8.555)	(8.540)	(8.558)
β-	0.021	0.019	0.016	0.018	0.016	0.014
	(1.216)	(1.095)	(0.943)	(1.070)	(1.011)	(0.857)
β+	0.009	0.012	0.013	0.014	0.015	0.018
	(0.624)	(0.806)	(0.827)	(1.025)	(1.113)	(1.232)
Log-size	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
	(-9.213)	(-9.214)	(-9.203)	(-9.210)	(-9.196)	(-9.204)
B/M	0.013	0.013	0.013	0.013	0.013	0.013
	(2.388)	(2.387)	(2.390)	(2.393)	(2.395)	(2.395)
Past Return	-0.003	-0.003	-0.003	-0.004	-0.004	-0.004
	(-0.197)	(-0.181)	(-0.189)	(-0.229)	(-0.221)	(-0.228)
Volatility	-1.416	-1.421	-1.441	-1.424	-1.427	-1.442
	(-1.573)	(-1.585)	(-1.604)	(-1.579)	(-1.578)	(-1.602)
Coskew	-0.098	-0.089	-0.088	-0.082	-0.080	-0.073
	(-1.959)	(-1.815)	(-1.775)	(-1.666)	(-1.544)	(-1.484)
Cokurt	0.102	0.103	0.102	0.106	0.106	0.105
	(7.564)	(7.584)	(7.531)	(7.756)	(7.791)	(7.740)
EDH	0.019	0.018	0.024	0.021	0.020	0.027
	(3.864)	(4.004)	(3.904)	(4.073)	(4.082)	(4.047)

(continued)

Table 6: Continued

Model	VaR_GJR G_1250	VaR_GJR S_1250	ETL_GJR G_1250	VaR_GJR G_500	VaR_GJR S_500	ETL_GJR G_500
Intercept	1.077	1.079	1.079	1.076	1.077	1.079
	(8.677)	(8.681)	(8.707)	(8.632)	(8.609)	(8.671)
β-	0.010	0.005	0.004	0.008	0.006	0.002
	(0.480)	(0.255)	(0.176)	(0.455)	(0.371)	(0.141)
β+	-0.003	-0.005	-0.005	-0.002	-0.001	-0.002
	(-0.332)	(-0.459)	(-0.483)	(-0.205)	(-0.062)	(-0.240)
Log-size	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
	(-9.253)	(-9.247)	(-9.271)	(-9.195)	(-9.176)	(-9.217)
B/M	0.013	0.013	0.013	0.013	0.013	0.013
	(2.399)	(2.403)	(2.403)	(2.402)	(2.407)	(2.403)
Past	-0.002	-0.002	-0.002	-0.004	-0.004	-0.004
Return	(-0.149)	(-0.157)	(-0.148)	(-0.253)	(-0.253)	(-0.233)
Volatility	-1.538	-1.575	-1.567	-1.603	-1.610	-1.644
	(-1.748)	(-1.803)	(-1.792)	(-1.874)	(-1.886)	(-1.944)
Coskew	-0.103	-0.097	-0.100	-0.090	-0.091	-0.084
	(-2.214)	(-2.058)	(-2.141)	(-1.894)	(-1.885)	(-1.788)
Cokurt	0.097	0.095	0.095	0.094	0.093	0.092
	(6.993)	(6.862)	(6.893)	(6.700)	(6.616)	(6.527)
EDH	3.027	0.028	0.037	3.239	0.030	0.041
	(3.338)	(3.534)	(3.327)	(3.526)	(3.527)	(3.521)

Table 7: Cross-sectional analysis of extreme downside measure given systematic volatility

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures, SV and EDH. In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measure of downside beta, upsize beta, volatility, co-skewness, co-kurtotis, SV and EDH; its last year excess return; and its size and Book-to-Market available at the time of the regression.The overall sample period is from January 1986-December 2012 and covers all stocks in NYSE, AMEX and NASDAQ markets. The VaR and ETL model of the market use GJR GARCH conditional volatility filtering. The name of each regression model specifies whether the corresponding EDH measure utilizes market VaR or ETL model, Gaussian (G) or Skewed Student-t (S) residual term distribution, 5 year (1250 days) or two year (500 days) estimation period and tail threshold is of 5 percent or 1 percent.

Model	**VaR_G 1250_5**	**VaR_G 1250_1**	**VaR_S 1250_5**	**VaR_S 1250_1**	**ETL_G 1250_5**	**ETL_G 1250_1**	**VaR_G 500_5**	**VaR_G 500_1**	**VaR_S 500_5**	**VaR_S 500_1**	**ETL_G 500_5**	**ETL_G 500_1**
Intercept	1.126	1.128	1.127	1.128	1.127	1.128	1.125	1.128	1.128	1.131	1.127	1.129
	(7.404)	(7.414)	(7.389)	(7.386)	(7.409)	(7.413)	(7.320)	(7.360)	(7.318)	(7.364)	(7.354)	(7.377)
β-	-0.008	-0.016	-0.014	-0.024	-0.013	-0.019	-0.010	-0.019	-0.016	-0.028	-0.016	-0.022
	(-0.242)	(-0.464)	(-0.407)	(-0.721)	(-0.390)	(-0.551)	(-0.354)	(-0.623)	(-0.574)	(-0.962)	(-0.531)	(-0.702)
β+	-0.004	-0.005	-0.004	-0.005	-0.005	-0.005	-0.003	-0.002	-0.002	-0.001	-0.003	-0.002
	(-0.355)	(-0.421)	(-0.386)	(-0.449)	(-0.398)	(-0.456)	(-0.279)	(-0.191)	(-0.129)	(-0.087)	(-0.224)	(-0.189)
Log-size	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058
	(-7.961)	(-7.970)	(-7.936)	(-7.933)	(-7.965)	(-7.971)	(-7.875)	(-7.899)	(-7.857)	(-7.886)	(-7.896)	(-7.908)
B/M	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
	(0.865)	(0.874)	(0.873)	(0.889)	(0.870)	(0.878)	(0.874)	(0.880)	(0.881)	(0.891)	(0.879)	(0.883)
Past	-0.030	-0.030	-0.030	-0.030	-0.030	-0.030	-0.031	-0.031	-0.031	-0.031	-0.031	-0.030
Return	(-1.583)	(-1.580)	(-1.596)	(-1.592)	(-1.581)	(-1.578)	(-1.573)	(-1.593)	(-1.597)	(-1.623)	(-1.584)	(-1.597)
Volatility	-1.028	-1.035	-1.072	-1.081	-1.034	-1.037	-1.055	-1.089	-1.091	-1.144	-1.070	-1.090
	(-0.909)	(-0.917)	(-0.952)	(-0.960)	(-0.916)	(-0.920)	(-0.963)	(-1.002)	(-1.003)	(-1.059)	(-0.983)	(-1.005)
Coskew	-0.048	-0.045	-0.039	-0.036	-0.045	-0.044	-0.035	-0.028	-0.032	-0.021	-0.030	-0.025
	(-0.777)	(-0.728)	(-0.629)	(-0.569)	(-0.740)	(-0.701)	(-0.565)	(-0.456)	(-0.509)	(-0.340)	(-0.490)	(-0.420)
Cokurt	0.091	0.089	0.089	0.086	0.090	0.088	0.090	0.088	0.088	0.084	0.089	0.087
	(5.701)	(5.545)	(5.515)	(5.249)	(5.609)	(5.491)	(5.505)	(5.342)	(5.269)	(5.017)	(5.376)	(5.254)
SV	-8.939	-6.444	-7.635	-4.671	-7.221	-5.530	-10.036	-8.261	-9.748	-7.477	-8.915	-7.596
	(-2.399)	(-1.879)	(-2.030)	(-1.321)	(-2.066)	(-1.641)	(-2.523)	(-2.174)	(-2.235)	(-1.822)	(-2.319)	(-2.048)
EDH	2.402	3.997	0.024	0.046	0.034	0.049	2.653	4.319	0.027	0.049	0.036	0.051
	(1.708)	(1.945)	(1.930)	(2.148)	(1.881)	(2.025)	(1.912)	(2.096)	(1.899)	(2.097)	(2.029)	(2.137)

Table 8: Cross-sectional analysis of extreme downside risk in different investment horizon

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures, SV and EDH, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, monthly excess return of a stock is regressed against its realized risk measure of CAPM beta, co-skewness, co-kurtotis, SV and EDH; its lagged excess return; and its size and Book-to-Market available at the time of the regression. These measures are constructed using daily data within a specific investment horizon ranging from 1 month to 12 months, except for size and Book-over-Market. The overall sample period is from January 1986-December 2012 and covers all stocks in NYSE, AMEX and NASDAQ markets. The VaR model in EDH estimation use $AR(1) - GJR\ GARCH(1,1)$ Skewed Student-t, 5 percent tail threshold and 5 years of daily return observations.

	1 month	2 month	3 month	4 month	5 month	6 month	7 month	8 month	9 month	10 month	11 month	12 month
Intercept	-0.082	-0.051	0.007	0.090	0.185	0.300	0.420	0.543	0.680	0.798	0.940	1.127
	(-7.508)	(-2.617)	(0.258)	(2.436)	(4.078)	(5.583)	(6.770)	(7.607)	(8.283)	(8.575)	(8.961)	(7.389)
β-	0.001	0.003	0.003	0.003	0.003	0.003	0.004	0.003	0.001	-0.003	-0.008	-0.014
	(2.285)	(2.109)	(1.328)	(0.730)	(0.511)	(0.448)	(0.367)	(0.213)	(0.068)	(-0.163)	(-0.352)	(-0.407)
β+	-0.003	-0.005	-0.005	-0.007	-0.008	-0.009	-0.009	-0.008	-0.007	-0.006	-0.005	-0.004
	(-4.504)	(-4.206)	(-2.899)	(-2.404)	(-2.093)	(-1.895)	(-1.574)	(-1.351)	(-1.033)	(-0.753)	(-0.554)	(-0.386)
Log-size	0.003	0.001	-0.002	-0.006	-0.011	-0.017	-0.023	-0.029	-0.036	-0.041	-0.048	-0.058
	(6.315)	(1.590)	(-1.388)	(-3.808)	(-5.495)	(-7.008)	(-8.095)	(-8.748)	(-9.322)	(-9.403)	(-9.616)	(-7.936)
B/M	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.004
	(3.662)	(2.844)	(2.399)	(2.032)	(1.815)	(1.708)	(1.624)	(1.522)	(1.420)	(1.366)	(1.250)	(0.873)
Past	-0.037	-0.014	-0.003	0.001	0.008	0.013	0.014	0.009	0.000	-0.009	-0.020	-0.030
Return	(-10.618)	(-2.768)	(-0.517)	(0.142)	(0.731)	(0.929)	(0.924)	(0.563)	(0.013)	(-0.579)	(-1.233)	(-1.596)
Volatility	1.121	1.361	1.432	1.430	1.311	1.091	0.820	0.500	0.166	-0.166	-0.576	-1.072
	(8.573)	(5.569)	(4.152)	(3.232)	(2.510)	(1.817)	(1.233)	(0.700)	(0.212)	(-0.204)	(-0.681)	(-0.952)
Coskew	0.016	0.016	0.006	0.003	-0.001	0.000	0.001	-0.004	-0.012	-0.022	-0.033	-0.039
	(5.320)	(3.238)	(0.679)	(0.248)	(-0.033)	(0.016)	(0.059)	(-0.144)	(-0.339)	(-0.561)	(-0.736)	(-0.629)
Cokurt	0.007	0.016	0.024	0.034	0.042	0.051	0.057	0.063	0.069	0.072	0.079	0.089
	(6.200)	(8.278)	(8.144)	(8.439)	(8.016)	(8.114)	(8.033)	(7.561)	(7.206)	(6.609)	(6.331)	(5.515)
SV	0.993	1.390	1.714	1.883	1.895	1.690	1.307	0.203	-1.223	-3.040	-4.838	-7.635
	(4.041)	(3.983)	(3.112)	(2.226)	(1.788)	(1.268)	(0.769)	(0.100)	(-0.517)	(-1.126)	(-1.622)	(-2.030)
EDH	0.000	0.000	0.001	0.002	0.004	0.006	0.008	0.011	0.013	0.016	0.020	0.024
	(0.492)	(0.316)	(0.936)	(1.384)	(1.803)	(1.980)	(2.157)	(2.137)	(2.145)	(2.250)	(2.344)	(1.930)