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Appendix to Staff Working Paper No. 640

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Lena Boneva and Oliver Linton

January 2017

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A discrete choice model for large heterogeneous panels with interactive fixed effects with an application to the determinants of corporate bond issuance

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Supplementary material for: A Discrete Choice Model For Large Heterogeneous Panels with Interactive Fixed Effects with an Application to the Determinants of Corporate Bond Issuance*

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1 Proofs

Proof of Lemma 4.1

We have

$$\max_{1 \leq t \leq T} \|\hat{h}_t - \bar{h}_t\| = \max_{1 \leq t \leq T} \|\bar{u}_t\| = \max_{1 \leq t \leq T} \left| \frac{1}{N} \sum_{i=1}^N u_{it} \right|.$$

Define the event

$$B = \{|u_{it}| \leq \tau_{N,T} \text{ for all } i \leq N, t \leq T\}, \quad (1)$$

where $\tau_{N,T} < \infty$ is to be determined below. We have for any $x > 0$

$$\Pr \left(\sqrt{N} \max_{1 \leq t \leq T} \|\hat{h}_t - \bar{h}_t\| > x \right) \leq \Pr \left(\left\{ \sqrt{N} \max_{1 \leq t \leq T} \|\hat{h}_t - \bar{h}_t\| > x \right\} \cap B \right) + \Pr(B^c)$$

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Then by Bonferroni and Bernstein's inequality (Van der Vaart, 1998, p285)

$$\Pr \left(\left\{ \sqrt{N} \max_{1 \leq t \leq T} \|\hat{h}_t - \bar{h}_t\| > x \right\} \cap B \right) \leq 2T \exp \left(-\frac{1}{2} \frac{x^2}{\sigma_u^2 + x\tau_{N,T}/\sqrt{N}} \right),$$

where we use that u_{it} are i.i.d with mean zero and finite variance σ_u^2 . Then, taking $x = \log T$ we have

$$\Pr \left(\left\{ \sqrt{N} \max_{1 \leq t \leq T} \|\hat{h}_t - \bar{h}_t\| > \log T \right\} \cap B \right) \leq 2T \exp \left(-\frac{1}{2} \frac{\log^2 T}{\sigma_u^2 + \tau_{N,T} \log T / \sqrt{N}} \right) = o(1)$$

provided $\tau_{N,T} \log T / \sqrt{N} \rightarrow 0$. Furthermore, we note that with $\tau_{N,T} = (NT)^\pi$ for some $\pi > 0$, we have

$$\Pr(B) = F_{|u|}(\tau_{N,T})^{NT} \geq \left(1 - \frac{c}{(NT)^{\pi\alpha}} \right)^{NT},$$

where $F_{|u|}$ denotes the c.d.f. of the random variable $|u_{it}|$. The moment conditions imply that $F_{|u|}(x) \geq 1 - cx^{-\alpha}$ for x large for $\alpha \geq 4$. If $\pi\alpha > 1$, then $\Pr(B) \rightarrow 1$. Therefore, provided $\pi > 1/\alpha$ and $N^{\pi-1/2}T^\pi \log T \rightarrow 0$ the result is established.

Proof of Theorem 4.1

Because the infeasible estimator $\tilde{\theta}_i$ is consistent, it suffices to show that estimating the unobserved factors does not affect the criterion function. We have

$$\begin{aligned} & \Pr \left(\sup_{\theta \in \Theta} \left| \hat{Q}_T^i(\theta) - Q_T^i(\theta) \right| \geq \epsilon \right) \\ & \leq \Pr \left(\sup_{\|h - \bar{h}\|_{\mathcal{H}} \leq \delta_T} \sup_{\theta \in \Theta} \frac{1}{T} \sum_{t=1}^T |q_t^i(\theta, h_t) - q_t^i(\theta, \bar{h}_t)| \geq \epsilon \right) + \Pr \left(\|\hat{h} - \bar{h}\|_{\mathcal{H}} > \delta_T \right) \quad (2) \\ & \rightarrow 0, \quad (3) \end{aligned}$$

Proof of Theorem 4.2

To show that $\hat{\theta}_i$ is asymptotically normal, it suffices to show that estimating the unobserved factors does not affect the limiting distribution, that is,

$$\sqrt{T} \left(\hat{\theta}_i - \tilde{\theta}_i \right) = o_P(1). \quad (4)$$

By the Mean Value Theorem, we have

$$0 = \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\hat{\theta}_i, \hat{h}_t)}{\partial \theta}$$



$$\begin{aligned}
&= \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\tilde{\theta}_i, \hat{h}_t)}{\partial \theta} + \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^*)}{\partial \theta \partial \theta^\top} (\hat{\theta}_i - \tilde{\theta}_i) \\
&= \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\tilde{\theta}_i, \bar{h}_t)}{\partial \theta} + \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^{**})}{\partial \theta \partial h^\top} (\hat{h}_t - \bar{h}_t) + \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^*)}{\partial \theta \partial \theta^\top} (\hat{\theta}_i - \tilde{\theta}_i) \\
&= \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^{**})}{\partial \theta \partial h^\top} (\hat{h}_t - \bar{h}_t) + \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^*)}{\partial \theta \partial \theta^\top} (\hat{\theta}_i - \tilde{\theta}_i),
\end{aligned}$$

where h_t^* and h_t^{**} are intermediate values. Then, provided

$$\begin{aligned}
&\liminf_{N,T \rightarrow \infty} \left\| \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^*)}{\partial \theta \partial \theta^\top} \right\| > 0 \\
&\left\| \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^{**})}{\partial \theta \partial h^\top} (\hat{h}_t - \bar{h}_t) \right\| = o_P(T^{-1/2}).
\end{aligned}$$

the result (4) follows. These properties follow from the uniform convergence of $\hat{h}_t - \bar{h}_t$, the Cauchy-Schwarz inequality and conditions D. For example, with probability tending to one

$$\begin{aligned}
\left\| \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^{**})}{\partial \theta \partial h^\top} (\hat{h}_t - \bar{h}_t) \right\|^2 &\leq \frac{1}{T} \sum_{t=1}^T \left\| \frac{\partial^2 q_t^i(\tilde{\theta}_i, h_t^{**})}{\partial \theta \partial h^\top} \right\|^2 \times \|\hat{h} - \bar{h}\|_{\mathcal{H}}^2 \\
&\leq \sup_{\|h - \bar{h}\|_{\mathcal{H}} < \delta_T} \sup_{\|\theta - \theta_{0i}\| \leq \delta_T} \frac{1}{T} \sum_{t=1}^T \left\| \frac{\partial^2 q_t^i(\theta, h_t)}{\partial \theta \partial \theta^\top} \right\|^2 \times \|\hat{h} - \bar{h}\|_{\mathcal{H}}^2 \\
&= O_P\left(\frac{\log^2 T}{N}\right) = o_P(T^{-1})
\end{aligned}$$

Proof of Theorem 4.3

It suffices to show that the feasible objective functions are uniformly close to the infeasible ones, see CJL (2016). Thus

$$\begin{aligned}
&\Pr \left(\max_{1 \leq i \leq N} \sup_{\theta \in \Theta} \left| \widehat{Q}_T^i(\theta) - Q_T^i(\theta) \right| \geq \epsilon \right) \\
&\leq \Pr \left(\max_{1 \leq i \leq N} \sup_{\|h - \bar{h}\|_{\mathcal{H}} \leq \delta_T} \sup_{\theta \in \Theta} \left| \frac{1}{T} \sum_{t=1}^T q_t^i(\theta, h_t) - q_t^i(\theta, \bar{h}_t) \right| \geq \epsilon \right) + \Pr \left(\|\hat{h} - \bar{h}\|_{\mathcal{H}} > \delta_T \right) \\
&\leq \Pr \left(\max_{1 \leq i \leq N} \sup_{\|h - \bar{h}\|_{\mathcal{H}} \leq \delta_T} \sup_{\theta \in \Theta} \frac{1}{T} \sum_{t=1}^T \left| q_t^i(\theta, h_t) - q_t^i(\theta, \bar{h}_t) \right| \geq \epsilon \right) + \Pr \left(\|\hat{h} - \bar{h}\|_{\mathcal{H}} > \delta_T \right) \\
&\rightarrow 0. \tag{5}
\end{aligned}$$



Proof of Theorem 4.4

We show that

$$\widehat{\theta} - \widetilde{\theta} = o_P(N^{-1/2}). \quad (6)$$

The estimators $\widetilde{\theta}_i$ and $\widehat{\theta}_i$, $i = 1, \dots, N$ satisfy the first order conditions

$$\frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\widetilde{\theta}_i, \bar{h}_t)}{\partial \theta} = 0 = \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\widehat{\theta}_i, \widehat{h}_t)}{\partial \theta}.$$

We first work with a linear approximation to $\widehat{\theta}_i$. Define

$$L_{Ti}(\theta) = \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\widetilde{\theta}_i, \widehat{h}_t)}{\partial \theta} + M_i(\theta - \widetilde{\theta}_i) \quad (7)$$

from which we obtain for $\widehat{\theta}_i^*$ such that $L_{Ti}(\widehat{\theta}_i^*) = 0$,

$$\widehat{\theta}_i^* - \widetilde{\theta}_i = -M_i^{-1} \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\widetilde{\theta}_i, \widehat{h}_t)}{\partial \theta}.$$

We first establish the result for this linear approximation. By the Mean-Value Theorem, we have for $r = 1, \dots, p$

$$\begin{aligned} \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\widetilde{\theta}_i, \widehat{h}_t)}{\partial \theta_r} &= \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\theta_{0i}, \bar{h}_t)}{\partial \theta_r} + \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\theta_{0i}, \bar{h}_t)}{\partial \theta_r \partial h^\top} (\widehat{h}_t - \bar{h}_t) + \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\theta_{0i}, \bar{h}_t)}{\partial \theta_r \partial \theta^\top} (\widetilde{\theta}_i - \theta_{0i}) \\ &\quad + \frac{1}{2T} \sum_{t=1}^T (\widehat{h}_t - \bar{h}_t)^\top \frac{\partial^3 q_t^i}{\partial \theta_r \partial h \partial h} (\theta_i^*, h_t^*) (\widehat{h}_t - \bar{h}_t) + (\widetilde{\theta}_i - \theta_{0i})^\top \frac{1}{2T} \sum_{t=1}^T \frac{\partial^3 q_t^i(\theta_i^*, \bar{h}_t^*)}{\partial \theta_r \partial \theta \partial \theta^\top} (\widehat{h}_t - \bar{h}_t) \\ &\quad + (\widetilde{\theta}_i - \theta_{0i})^\top \frac{1}{2T} \sum_{t=1}^T \frac{\partial^3 q_t^i(\theta_i^*, \bar{h}_t^*)}{\partial \theta_r \partial \theta \partial \theta^\top} (\widetilde{\theta}_i - \theta_{0i}) \\ &= \sum_{k=1}^6 J_{rk;i}, \end{aligned}$$

where h_t^* and θ_i^* are intermediate values. It follows that

$$\frac{1}{N} \sum_{i=1}^N (\widehat{\theta}_i^* - \widetilde{\theta}_i) = \sum_{k=1}^6 \frac{1}{N} \sum_{i=1}^N M_i^{-1} \times J_{k;i} \equiv \sum_{k=1}^6 R_{rk}, \quad (8)$$

where $J_{k;i}$ denotes the vector with r^{th} element $J_{rk;i}$. We consider in sequence the vector random variables $R_1 - R_6$.



We have $E(R_1) = 0$ and $\partial q_t^i(\theta_{0i}, \bar{h}_t)/\partial\theta$ is i.i.d. across i and t conditional on X, d, f , so that

$$R_1 = \frac{1}{N} \sum_{i=1}^N M_i^{-1} \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\theta_{0i}, \bar{h}_t)}{\partial\theta} = \frac{1}{N} \frac{1}{T} \sum_{i=1}^N \sum_{t=1}^T M_i^{-1} \frac{\partial q_t^i(\theta_{0i}, \bar{h}_t)}{\partial\theta} = O_P(N^{-1/2}T^{-1/2}).$$

Consider

$$R_2 = \frac{1}{T} \sum_{t=1}^T \left(\frac{1}{N} \sum_{i=1}^N M_i^{-1} \frac{\partial^2 q_t^i}{\partial\theta\partial h}(\theta_{0i}, \bar{h}_t) \right) (\hat{h}_t - \bar{h}_t). \quad (9)$$

We have $\partial\Phi_{it0}/\partial\bar{h}_t = \phi_{it0}\bar{\kappa}_i$ and $\partial\phi_{it0}/\partial\bar{h}_t = -\theta_{it0}^\top z_{it}\phi_{it0}\bar{\kappa}_i$, whence

$$\begin{aligned} \frac{\partial^2 q_t^i}{\partial\theta\partial h}(\theta_{0i}, \bar{h}_t) &= \frac{1}{T} \frac{-\phi_{it0}^2 z_{it}\bar{\kappa}_i}{\Phi_{it0}(1-\Phi_{it0})} \\ &\quad - \frac{1}{T} \frac{Y_{it} - \Phi_{it0}}{(\Phi_{it0}(1-\Phi_{it0}))^2} (1-2\Phi_{it0}) \phi_{it0}^2 z_{it}\bar{\kappa}_i \\ &\quad - \frac{1}{T} \frac{Y_{it} - \Phi_{it0}}{\Phi_{it0}(1-\Phi_{it0})} (\theta_{it0}^\top z_{it}) \phi_{it0}^2 \bar{\kappa}_i z_{it}. \end{aligned}$$

We decompose (9) into three terms: the second and third terms are just linear combinations of the random variables $Y_{it} - \Phi_{it0}$, which are i.i.d. mean zero conditional on the factors; the first term is different and we treat this more carefully. This term can be written as

$$W_{NT} = \frac{1}{N^2 T} \sum_{i=1}^N \sum_{l=1}^N \sum_{t=1}^T r_i(d_t, f_t, \bar{u}_t, u_{it}) u_{lt}$$

for some function $r_i(\cdot)$. Write for each $l = 1, \dots, N$

$$r_i(d_t, f_t, \bar{u}_{it}, u_{it}) = r_i(d_t, f_t, \bar{u}_t^{-l}, u_{it}) + r_{i;3}(d_t, f_t, \bar{u}_t^{-l}, u_{it}) \frac{u_{lt}}{N} + \frac{1}{2} + r_{i;33}(d_t, f_t, \bar{u}_t^{-l*}, u_{it}) \frac{u_{lt}^2}{N^2}$$

for some intermediate value \bar{u}_t^{-l*} , where $\bar{u}_t^{-l} = \sum_{j \neq l} u_{jt}/N$ so that $\bar{u}_t - \bar{u}_t^{-l} = u_{lt}/N$. We have for $l \neq i$, $E(r_{i;3}(d_t, f_t, \bar{u}_t^{-l}, u_{it})u_{lt}) = 0$ and $E(|r_{i;33}(d_t, f_t, \bar{u}_t^{-l*}, u_{it})| u_{lt}^2) < \infty$. From this we obtain that

$$E(W_{NT}) = O(N^{-1}).$$

By similar arguments we obtain

$$E(W_{NT}^2) = \frac{1}{N^4 T^2} \sum_{i=1}^N \sum_{l=1}^N \sum_{i'=1}^N \sum_{l'=1}^N \sum_{t=1}^T \sum_{t'=1}^T E[r_i(d_t, f_t, \bar{u}_{it}, u_{it}) r_{i'}(d_{t'}, f_{t'}, \bar{u}_{i't'}, u_{i't'})^\top u_{lt} u_{l't'}] = O(N^{-1}T^{-1}),$$

because whenever either $t \neq t'$ or all four indices in $\{i, i', l, l'\}$ are distinct, then the expectation is zero for the leave out case, or small otherwise. Therefore,

$$W_{NT} = O_P(N^{-1}) + O_P(N^{-1/2}T^{-1/2}). \quad (10)$$

In conclusion, $R_2 = O_P(N^{-1}) + O_P(N^{-1/2}T^{-1/2})$. We have

$$\begin{aligned}
R_3 &= \frac{1}{N} \sum_{i=1}^N M_i^{-1} \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\theta_{0i}, \bar{h}_t)}{\partial \theta \partial \theta^\top} (\tilde{\theta}_i - \theta_{0i}) \\
&= \frac{1}{N} \sum_{i=1}^N \left(M_i^{-1} \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\theta_{0i}, \bar{h}_t)}{\partial \theta \partial \theta^\top} \right) (\tilde{\theta}_i - \theta_{0i}) \\
&= \frac{1}{N} \sum_{i=1}^N (\tilde{\theta}_i - \theta_{0i}) + \frac{1}{N} \sum_{i=1}^N \left(M_i^{-1} \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\theta_{0i}, \bar{h}_t)}{\partial \theta \partial \theta^\top} - I_p \right) (\tilde{\theta}_i - \theta_{0i}) \\
&= o_P(N^{-1/2}) + O_P(T^{-1}) = o_P(N^{-1/2}),
\end{aligned}$$

by Cauchy-Schwarz and the assumption that $T^2/N \rightarrow 0$. The remaining terms, $R_4 - R_6$ are also treated by crude bounding. For example, with probability tending to one

$$\begin{aligned}
&\max_{1 \leq i \leq N} \left\| \frac{1}{N} \sum_{i=1}^N M_i^{-1} \frac{1}{2T} \sum_{t=1}^T (\hat{h}_t - \bar{h}_t)^\top \frac{\partial^3 q_t^i}{\partial \theta_r \partial h \partial h} (\theta_{0i}, h_t^*) (\hat{h}_t - \bar{h}_t) \right\| \\
&\leq \|\hat{h} - \bar{h}\|_{\mathcal{H}}^2 \times \frac{1}{2NT} \sum_{t=1}^T \sum_{i=1}^N \|M_i^{-1}\| \sup_{\|h - \bar{h}\|_{\mathcal{H}} < \delta_T} \left\| \frac{\partial^3 q_t^i}{\partial \theta_r \partial h \partial h} (\theta_{0i}, h_t) \right\| \\
&= O_P \left(\frac{\log^2 T}{N} \right) = o_P(N^{-1/2}).
\end{aligned}$$

$$\begin{aligned}
&\left\| \frac{1}{N} \sum_{i=1}^N M_i^{-1} (\tilde{\theta}_i - \theta_{0i})^\top \frac{1}{2T} \sum_{t=1}^T \frac{\partial^3 q_t^i(\theta_{0i}^*, \bar{h}_t^*)}{\partial \theta_r \partial \theta \partial \theta^\top} (\tilde{\theta}_i - \theta_{0i}) \right\|^2 \\
&\leq \max_{1 \leq i \leq N} \|\tilde{\theta}_i - \theta_{0i}\|^2 \times \frac{1}{2NT} \sum_{i=1}^N \sum_{t=1}^T \|M_i^{-1}\| \sup_{\|h - \bar{h}\|_{\mathcal{H}} < \delta_T} \sup_{\|\theta - \theta_{0i}\| \leq \delta_T} \left\| \frac{\partial^3 q_t^i(\theta, h_t)}{\partial \theta_r \partial \theta \partial \theta^\top} \right\|^2 \\
&= O_P \left(\frac{\log^2 N}{T} \right) = o_P(N^{-1/2}).
\end{aligned}$$

Finally, we show that the linear approximation is very close to the actual score function of the feasible estimator

$$\begin{aligned}
&\sqrt{N} \max_{1 \leq i \leq N} \sup_{\|\theta - \theta_{0i}\| \leq \delta_N} \left\| \frac{1}{T} \sum_{t=1}^T \frac{\partial q_t^i(\theta, \hat{h}_t)}{\partial \theta} - L_{Ti}(\theta) \right\| \\
&= \sqrt{N} \max_{1 \leq i \leq N} \sup_{\|\theta - \theta_{0i}\| \leq \delta_T} \left\| \frac{1}{T} \sum_{t=1}^T (\theta - \tilde{\theta}_i)^\top \frac{\partial^2 q_t^i(\theta, \hat{h}_t)}{\partial \theta \partial \theta^\top} (\theta - \tilde{\theta}_i) \right\| \\
&\leq \sqrt{N} \times \max_{1 \leq i \leq N} \|\tilde{\theta}_i - \theta_{0i}\|^2 \times \max_{1 \leq i \leq N} \sup_{\|\theta - \theta_{0i}\| \leq \delta_T} \sup_{\|h - \bar{h}\|_{\mathcal{H}} < \delta_T} \left\| \frac{1}{T} \sum_{t=1}^T \frac{\partial^2 q_t^i(\theta, h_t)}{\partial \theta \partial \theta^\top} \right\|
\end{aligned}$$



$= o_P(1)$.

The argument follows as in CJL (2016, p68).

2 Additional tables for the simulation study

Tables 1-8 report the results for experiments 2-5 that are discussed in Section 5 of the main paper.

3 Tables for the robustness checks

Tables 9 to 12 report the results of our robustness checks that are discussed in Section 6.3 of the main paper.



Table 1: Small sample properties of the mean group estimator $\hat{\beta}$: Experiment 2

T/N	BIAS ($\times 1000$)			RMSE ($\times 1000$)			POWER			SIZE			COVERAGE PROBABILITY			
	50	100	200	300	50	100	200	300	50	100	200	300	50	100	200	
INFEASIBLE ESTIMATOR																
50	2.588	3.869	2.846	3.258	36.37	25.64	17.93	14.53	0.2875	0.556	0.851	0.9605	0.0545	0.0449	0.0442	
100	0.107	1.415	0.8318	0.9228	23.31	16.49	11.33	9.243	0.551	0.882	0.995	0.9995	0.0465	0.0455	0.0375	0.039
200	0.1966	0.2106	0.03057	0.03981	15.79	11.19	8.039	6.47	0.874	0.9935	1	1	0.044	0.0465	0.0475	0.0475
300	-0.4161	-0.04835	0.3275	0.1518	12.74	9.074	6.46	5.131	0.965	1	1	0.0405	0.036	0.0445	0.039	0.9595
CCEMG ESTIMATOR																
50	-1.487	0.6606	-0.0882	0.6682	36.15	25.12	17.64	14.18	0.2525	0.508	0.805	0.945	0.0565	0.0435	0.0485	0.0435
100	-4.385	-2.043	-2.021	-1.73	23.74	16.48	11.53	9.341	0.4755	0.837	0.9905	0.999	0.047	0.0505	0.0455	0.0475
200	-4.581	-3.372	-2.91	-2.7	16.53	11.72	8.584	7.009	0.7905	0.984	1	1	0.0535	0.061	0.07	0.0655
300	-5.233	-3.586	-2.63	-2.578	13.71	9.784	6.978	5.745	0.925	0.999	1	1	0.063	0.064	0.0675	0.0665
NAIVE ESTIMATOR																
50	158.7	159.4	158.7	158.1	162.5	161.9	160.4	159.6	1	1	1	1	0.9985	1	1	1
100	157.2	158.6	158.3	158.2	159	159.8	159.1	158.9	1	1	1	1	1	1	1	1
200	158.1	157.9	157.9	157.9	159	158.7	158.3	158.2	1	1	1	1	1	1	1	1
300	157.8	158	158.3	158.4	158.3	158.4	158.6	158.6	1	1	1	1	1	1	1	1

Notes: The data generating process is defined in (17)-(21) (in the main paper) except that $\beta_{1i} = 0.5$, for all i . The nominal size is 5% and power is computed under the alternative $\beta_1 = 0.45$. The number of replications is set to 2000.

Table 2: Small sample properties of the mean group estimator $\hat{\beta}$: Experiment 3

T/N	BIAS ($\times 1000$)			RMSE ($\times 1000$)			POWER			SIZE			COVERAGE PROBABILITY			
	50	100	200	300	50	100	200	300	50	100	200	300	50	100	200	
INFEASIBLE ESTIMATOR																
50	1.604	2.332	2.829	2.293	34.93	25.5	17.7	14.86	0.2645	0.5355	0.862	0.954	0.045	0.05	0.955	
100	1.35	0.4999	0.4462	0.686	23.16	16.77	11.83	9.817	0.567	0.8585	0.9925	1	0.046	0.0525	0.0475	0.948
200	0.8779	0.1244	-0.1658	0.4899	15.91	10.92	7.969	6.4	0.8725	0.994	1	1	0.039	0.036	0.0425	0.9635
300	0.3728	0.01739	0.1115	0.09328	12.83	9.209	6.444	5.294	0.9675	1	1	1	0.0365	0.0415	0.039	0.9585
CCEMG ESTIMATOR																
φ_{50}	-47.5	-46.79	-45.93	-45.71	57.33	52.61	49.07	48.35	0.0345	0.049	0.0485	0.084	0.276	0.477	0.7375	0.8565
100	-49.81	-49.47	-49.06	-48.69	54.28	52.06	50.51	49.76	0.0245	0.033	0.049	0.06	0.512	0.815	0.975	0.997
200	-50.81	-50.54	-50.53	-49.83	53	51.62	51.16	50.29	0.01	0.0155	0.016	0.0215	0.784	0.989	1	1
300	-51.54	-50.86	-50.3	-50.2	53.01	51.63	50.74	50.52	0.0075	0.0085	0.012	0.0205	0.9055	0.998	1	1
NAIVE ESTIMATOR																
50	56.46	56.73	56.68	57.78	66.45	63.91	61.78	62.62	0.9115	0.9855	0.9995	0.9995	0.428	0.679	0.847	0.907
100	56.55	55.25	56.1	55.67	61.54	58.81	58.53	58.02	0.997	1	1	1	0.6635	0.8775	0.974	0.987
200	56.32	55.77	55.1	55.82	58.73	57.34	56.29	56.87	1	1	1	1	0.891	0.9915	1	1
300	55.23	55.31	55.67	55.55	56.93	56.39	56.49	56.26	1	1	1	1	0.958	0.999	1	1

Notes: The data generating process is defined in (17)-(21) (in the main paper) except that $k_{j12} \sim NID(0, 0.1)$. The nominal size is 5% and power is computed under the alternative $\beta_1 = 0.45$. The number of replications is set to 2000.

Table 3: Small sample properties of the mean group estimator $\hat{\beta}$: Experiment 4

T/N	50	BIAS ($\times 1000$)		RMSE ($\times 1000$)			POWER			SIZE			COVERAGE PROBABILITY			
		100	200	300	50	100	200	300	50	100	200	300	50	100	200	
INFEASIBLE ESTIMATOR																
50	2.221	1.969	2.262	2.177	38.46	26.95	19.07	15.65	0.2515	0.466	0.7935	0.929	0.0495	0.045	0.043	
100	-0.06905	0.765	0.8125	0.9479	24.85	17.22	12.25	10.08	0.505	0.8375	0.985	0.999	0.0535	0.045	0.0425	0.0475
200	0.251	0.58	0.43	0.3177	16.47	11.69	8.278	6.73	0.838	0.9905	1	1	0.0365	0.039	0.037	0.0445
300	0.1099	-0.1997	-0.03769	-0.17	13.23	9.397	6.761	5.432	0.9575	0.9995	1	1	0.039	0.039	0.041	0.0335
CCEMG ESTIMATOR																
50	-52.04	-51.04	-50.59	-50.92	62.59	56.95	54.08	53.54	0.0595	0.058	0.1005	0.344	0.548	0.7995	0.915	0.656
100	-55.59	-54.52	-53.57	-53.44	60	56.86	54.99	54.55	0.049	0.0565	0.0875	0.1135	0.6535	0.9035	0.99	0.998
200	-56.03	-55.2	-54.65	-54.54	58.13	56.31	55.33	55.02	0.061	0.07	0.1085	0.141	0.926	0.9985	1	1
300	-56.61	-55.81	-55.47	-55.12	57.94	56.56	55.88	55.43	0.065	0.0955	0.1475	0.181	0.9895	1	1	1
NAIVE ESTIMATOR																
50	102.3	103.1	102.5	101.3	109.3	108	106.7	105.1	0.9955	1	1	1	0.8765	0.9715	0.995	0.9995
100	100.4	100.7	101.3	101.5	103.6	102.9	103.1	103.1	0.9995	1	1	1	0.9915	0.9995	1	1
200	101.1	100.6	101.1	101.1	102.7	101.6	102	101.9	1	1	1	1	1	1	0	0
300	100.7	100.4	100.5	100.7	101.8	101.1	101.1	101.3	1	1	1	1	1	1	0	0

Notes: The data generating process is defined in (17)-(21) and (22) (in the main paper). The nominal size is 5% and power is computed under the alternative $\beta_1 = 0.45$. The number of replications is set to 2000.

Table 4: Small sample properties of the mean group estimator $\hat{\beta}$: Experiment 5

T/N	50	BIAS ($\times 1000$)			RMSE ($\times 1000$)			POWER			SIZE			COVERAGE PROBABILITY		
		100	200	300	50	100	200	300	50	100	200	300	50	100	200	300
INFEASIBLE ESTIMATOR																
50	2.741	2.228	2.859	2.14	33.89	23.81	17.09	13.41	0.3005	0.5715	0.8825	0.9705	0.041	0.0465	0.0475	0.038
100	1.931	0.5019	0.5716	0.4187	22.8	15.45	11.26	9.172	0.5995	0.8905	0.996	1	0.053	0.0365	0.043	0.0505
200	0.1784	0.1349	0.1921	0.1349	15.37	10.74	7.521	6.425	0.8875	0.9935	1	1	0.0495	0.032	0.04	0.0405
300	-0.1834	-0.2191	0.0583	0.1146	12.55	8.898	6.36	5.097	0.9695	1	1	1	0.041	0.0435	0.0435	0.046
CCEMG ESTIMATOR																
-50	0.9652	1.81	2.908	2.42	34.33	24.39	17.38	13.64	0.271	0.5495	0.87	0.973	0.041	0.0465	0.044	0.037
100	-0.2707	-0.532	0.088339	0.1874	22.99	15.63	11.32	9.253	0.549	0.866	0.9945	0.9995	0.0495	0.04	0.046	0.047
200	-2.298	-1.033	-0.3968	-0.2565	15.65	10.82	7.547	6.443	0.8455	0.99	1	1	0.047	0.035	0.041	0.0415
300	-2.6	-1.443	-0.5476	-0.3022	12.94	9.048	6.409	5.125	0.949	1	1	1	0.051	0.0475	0.0445	0.046
NAIVE ESTIMATOR																
50	113.4	113.8	113.5	112.5	118.4	116.8	115.5	114.1	1	1	1	1	0.941	0.998	1	1
100	113.5	112.3	112.4	112.1	115.8	113.6	113.3	112.9	1	1	1	1	0.9995	1	1	1
200	112.6	112.6	112.6	112.3	113.6	113.3	113	112.6	1	1	1	1	1	1	1	1
300	112.6	112.3	112.8	112.5	113.3	112.7	113.1	112.7	1	1	1	1	1	1	1	1

Notes: The data generating process is defined in (17)-(21) (in the main paper) except that $k_{j,i2} = 0 \forall i, j$ and $\kappa_{2i} = 0 \forall i$. The nominal size is 5% and power is computed under the alternative $\beta_1 = 0.45$. The number of replications is set to 2000.

Table 5: Small sample properties of the marginal effect \widehat{ME} : Experiment 4

T/N	BIAS ($\times 1000$)				RMSE ($\times 1000$)			
	50	100	200	300	50	100	200	300
INFEASIBLE ESTIMATOR								
50	-4.939	-4.988	-4.945	-4.932	10.07	7.935	6.572	6.071
100	-2.63	-2.426	-2.451	-2.412	6.627	4.859	3.864	3.451
200	-1.231	-1.146	-1.193	-1.216	4.334	3.157	2.4	2.087
300	-0.8167	-0.9001	-0.8656	-0.9018	3.458	2.563	1.922	1.651
CCEMG ESTIMATOR								
50	-5.156	-5.015	-4.912	-4.96	10.06	7.908	6.482	6.087
100	-2.663	-2.493	-2.457	-2.43	6.666	4.881	3.875	3.449
200	-1.264	-1.176	-1.204	-1.222	4.411	3.19	2.416	2.108
300	-0.9125	-0.9155	-0.903	-0.9022	3.542	2.617	1.949	1.672
NAIVE ESTIMATOR								
50	54.03	54.25	54.07	53.73	55.17	55.17	54.87	54.46
100	57.75	57.95	58.01	58.21	58.32	58.37	58.38	58.55
200	59.99	59.9	59.97	59.97	60.27	60.1	60.15	60.14
300	60.52	60.38	60.53	60.49	60.7	60.52	60.64	60.61
LINEAR PROBABILITY ESTIMATOR								
50	4.402	4.511	4.652	4.57	10.16	7.901	6.498	5.96
100	2.369	2.574	2.565	2.575	6.82	5.066	4.039	3.636
200	1.296	1.401	1.365	1.368	4.527	3.356	2.553	2.232
300	0.812	0.8319	0.8303	0.8355	3.631	2.657	1.95	1.664

Notes: The mean group estimator of the average marginal effect is reported. The data generating process is defined in (17)-(21) and (22) (in the main paper). The number of replications is set to 2000.



Table 6: Small sample properties of the marginal effect \widehat{ME} : Experiment 5

T/N	BIAS ($\times 1000$)				RMSE ($\times 1000$)			
	50	100	200	300	50	100	200	300
INFEASIBLE ESTIMATOR								
50	-4.699	-4.763	-4.657	-4.821	10.04	7.882	6.44	5.971
100	-2.127	-2.531	-2.511	-2.549	6.709	5.037	4.044	3.619
200	-1.262	-1.289	-1.274	-1.286	4.599	3.35	2.509	2.242
300	-0.9358	-0.9462	-0.8643	-0.8519	3.759	2.741	2.037	1.703
CCEMG ESTIMATOR								
50	-5.962	-5.954	-5.863	-6.014	10.76	8.716	7.395	6.977
100	-2.843	-3.216	-3.225	-3.244	7.028	5.451	4.532	4.148
200	-1.672	-1.66	-1.652	-1.662	4.76	3.517	2.723	2.477
300	-1.195	-1.2	-1.119	-1.11	3.872	2.847	2.162	1.849
NAIVE ESTIMATOR								
50	40.28	40.56	40.45	40.03	41.5	41.38	41.04	40.56
100	44.19	43.91	43.92	43.82	44.74	44.25	44.19	44.05
200	45.81	45.84	45.82	45.75	46.06	46.01	45.94	45.85
300	46.49	46.36	46.53	46.41	46.65	46.47	46.61	46.48
LINEAR PROBABILITY ESTIMATOR								
50	4.937	4.867	4.945	4.805	10.63	8.276	6.815	6.093
100	2.921	2.508	2.51	2.492	7.253	5.199	4.134	3.663
200	1.263	1.279	1.294	1.28	4.745	3.441	2.586	2.275
300	0.7918	0.791	0.857	0.87	3.859	2.752	2.082	1.734

Notes: The mean group estimator of the average marginal effect is reported. The data generating process is defined in (17)-(21) (in the main paper) except that $k_{ji2} = 0 \forall i, j$ and $\kappa_{2i} = 0 \forall i$. The number of replications is set to 2000.



Table 7: Small sample properties of the marginal effect \widehat{ME} : Experiment 2

T/N	BIAS ($\times 1000$)				RMSE ($\times 1000$)			
	50	100	200	300	50	100	200	300
INFEASIBLE ESTIMATOR								
50	-4.741	-4.475	-4.7	-4.618	10.2	7.754	6.463	5.827
100	-2.622	-2.282	-2.425	-2.409	6.707	4.909	3.868	3.424
200	-1.268	-1.253	-1.313	-1.304	4.476	3.286	2.546	2.194
300	-0.9855	-0.8788	-0.7807	-0.8276	3.626	2.627	1.937	1.633
CCEMG ESTIMATOR								
50	-4.727	-4.542	-4.78	-4.672	10.25	7.779	6.516	5.86
100	-2.629	-2.333	-2.454	-2.432	6.764	4.935	3.903	3.445
200	-1.336	-1.308	-1.35	-1.338	4.575	3.336	2.589	2.219
300	-1.064	-0.9193	-0.8168	-0.851	3.673	2.665	1.964	1.648
NAIVE ESTIMATOR								
50	58.65	58.76	58.74	58.45	59.42	59.3	59.14	58.82
100	62.27	62.64	62.62	62.55	62.65	62.91	62.81	62.73
200	64.44	64.45	64.38	64.34	64.62	64.57	64.47	64.43
300	64.96	65.03	65.12	65.12	65.07	65.11	65.18	65.17
LINEAR PROBABILITY ESTIMATOR								
50	5.321	5.5	5.257	5.382	11.03	8.676	7.041	6.571
100	2.726	2.936	2.806	2.847	7.057	5.369	4.222	3.827
200	1.328	1.407	1.34	1.355	4.684	3.468	2.626	2.264
300	0.7718	0.9124	0.992	0.9621	3.68	2.719	2.082	1.742

Notes: The mean group estimator of the average marginal effect is reported. The data generating process is defined in (17)-(21) (in the main paper) except that $\beta_{1i} = 0.5$, for all i . The number of replications is set to 2000.



Table 8: Small sample properties of the marginal effect \widehat{ME} : Experiment 3

T/N	BIAS ($\times 1000$)				RMSE ($\times 1000$)			
	50	100	200	300	50	100	200	300
INFEASIBLE ESTIMATOR								
50	-4.954	-4.816	-4.75	-4.867	10.01	7.908	6.449	6.085
100	-2.297	-2.499	-2.527	-2.469	6.54	5.096	4.029	3.582
200	-1.088	-1.267	-1.356	-1.176	4.451	3.223	2.554	2.094
300	-0.7677	-0.8723	-0.8377	-0.8485	3.592	2.663	1.949	1.685
CCEMG ESTIMATOR								
50	-6.268	-6.303	-6.2	-6.24	10.68	8.915	7.572	7.232
100	-3.381	-3.574	-3.594	-3.515	7.047	5.732	4.77	4.367
200	-1.975	-2.096	-2.229	-2.034	4.813	3.652	3.115	2.682
300	-1.55	-1.643	-1.594	-1.618	3.955	3.071	2.393	2.204
NAIVE ESTIMATOR								
50	36.45	36.53	36.5	36.91	38.17	37.86	37.56	37.9
100	40.32	39.82	40.22	39.99	41.16	40.46	40.69	40.48
200	42.07	41.96	41.72	42	42.47	42.25	41.96	42.22
300	42.32	42.34	42.47	42.46	42.59	42.54	42.63	42.61
LINEAR PROBABILITY ESTIMATOR								
50	4.269	4.249	4.381	4.296	10.02	7.887	6.366	5.808
100	2.259	2.009	1.987	2.088	6.83	5.049	3.796	3.399
200	0.9125	0.7905	0.649	0.8506	4.57	3.164	2.33	1.986
300	0.4143	0.2925	0.3499	0.3362	3.726	2.657	1.858	1.56

Notes: The mean group estimator of the average marginal effect is reported. The data generating process is defined in (17)-(21) (in the main paper) except that $k_{ji2} \sim NID(0, 0.1)$. The number of replications is set to 2000.



Table 9: The effect of yields on bond issuance for US firms: controlling for liquidity

	Pre-crisis			Post-crisis		
	All	Financial	Other	All	Financial	Other
Coefficient estimates						
Yield	-0.127 (-1.727)	-0.062 (-0.394)	-0.184 (-2.116)	-0.05 (-1.327)	-0.06 (-1.046)	-0.09 (-1.887)
Size	-0.073 (-0.403)	-0.049 (-1.475)	-0.09 (-0.35)	-0.077 (-0.512)	-0.051 (-1.206)	-0.068 (-0.324)
Liquidity	2.474 (4.57)	2.398 (2.257)	2.604 (3.712)	15.561 (1.062)	-3.419 (-0.724)	21.484 (1.072)
Marginal effects						
Yield	-0.016	-0.001	-0.022	-0.009	-0.004	-0.015
Size	-0.004	-0.01	-0.002	-0.011	-0.009	-0.01
Liquidity	0.39	0.456	0.389	4.79	-0.82	6.627
Observations	321	62	225	375	72	270

Notes: The dependent variable is 1 if a firm issues a bond in a particular month and zero otherwise. Yield is the firm-specific corporate bond yield, size is measured by assets/1000 and liquidity is the share of current debt among total debt. All specifications include a measure of credit supply (leverage in the broker-dealer market), the federal funds rate (pre-crisis only) and the change in Federal Reserve Holdings of Treasury Notes (post-crisis only) as common factors. Columns (1) and (4) use all firms, (2) and (5) use financial sector firms and (3) and (6) use all other firms (excluding mining and agriculture). t-statistics are shown in parenthesis.



Table 10: The effect of yields on bond issuance for US firms: corporate spreads instead of yields

	All	Pre-crisis		Post-crisis		
		Financial	Other	All	Financial	Other
Coefficient estimates						
Spread	-0.115 (-1.452)	-0.261 (-1.334)	-0.137 (-1.56)	-0.044 (-1.049)	-0.013 (-0.224)	-0.098 (-1.845)
Size	0.085 (0.337)	-0.011 (-0.38)	0.13 (0.363)	-0.23 (-3.18)	-0.027 (-0.473)	-0.29 (-2.965)
Marginal effects						
Spread	-0.016	-0.029	-0.018	-0.008	0.002	-0.016
Size	0.02	-0.004	0.032	-0.027	-0.006	-0.034
Observations	321	62	225	378	72	273

Notes: The dependent variable is 1 if a firm issues a bond in a particular month and zero otherwise. Spread is the firm-specific corporate bond yield minus the federal funds rate and size is measured by assets/1000. All specifications include a measure of credit supply (leverage in the broker-dealer market) and the change in Federal Reserve Holdings of Treasury Notes (post-crisis only) as common factors. Columns (1) and (4) use all firms, (2) and (5) use financial sector firms and (3) and (6) use all other firms (excluding mining and agriculture). t-statistics are shown in parenthesis.



Table 11: The effect of yields on bond issuance for US firms: alternative sample choices

a) Firms with at least 20 time series observations

	Pre-crisis			Post-crisis		
	All	Financial	Other	All	Financial	Other
Coefficient estimates						
Yield	-0.166 (-1.972)	-0.118 (-0.691)	-0.216 (-2.114)	-0.057 (-1.382)	-0.025 (-0.405)	-0.109 (-2.099)
Size	0.043 (0.199)	0.063 (0.935)	0.031 (0.103)	-0.22 (-2.921)	-0.009 (-0.117)	-0.278 (-2.755)
Marginal effects						
Yield	-0.02	-0.001	-0.026	-0.01	-0.002	-0.017
Size	-0.028	-0.004	-0.035	-0.028	-0.004	-0.035
Observations	337	64	237	387	74	279

b) Firms with at least 40 time series observations

	Pre-crisis			Post-crisis		
	All	Financial	Other	All	Financial	Other
Coefficient estimates						
Yield	-0.163 (-1.995)	-0.193 (-1.149)	-0.216 (-2.245)	-0.034 (-0.869)	-0.022 (-0.349)	-0.083 (-1.695)
Size	0.105 (0.454)	0.022 (0.722)	0.12 (0.368)	-0.174 (-2.369)	-0.014 (-0.274)	-0.222 (-2.219)
Marginal effects						
Yield	-0.02	-0.016	-0.026	-0.006	0.001	-0.013
Size	0.023	0	0.031	-0.02	-0.002	-0.026
Observations	301	58	212	366	70	263

Notes: The dependent variable is 1 if a firm issues a bond in a particular month and zero otherwise. Yield is the firm-specific corporate bond yield and size is measured by assets/1000. All specifications include a measure of credit supply (leverage in the broker-dealer market), the federal funds rate (pre-crisis only) and the change in Federal Reserve Holdings of Treasury Notes (post-crisis only) as common factors. Columns (1) and (4) use all firms, (2) and (5) use financial sector firms and (3) and (6) use all other firms (excluding mining and agriculture). t-statistics are shown in parenthesis.



Table 12: The effect of yields on bond issuance for US firms: firms with at least 2 issuances

	All	Pre-crisis		Post-crisis		
		Financial	Other	All	Financial	Other
Coefficient estimates						
Yield	-0.137 (-1.195)	-0.252 (-1.175)	-0.139 (-0.981)	-0.066 (-1.291)	0.034 (0.391)	-0.152 (-2.424)
Size	0.108 (0.392)	-0.025 (-0.695)	0.137 (0.353)	-0.181 (-2.312)	-0.096 (-2.351)	-0.216 (-1.999)
Marginal effects						
Yield	-0.02	-0.019	-0.021	-0.011	0.009	-0.024
Size	0.021	-0.007	0.03	-0.023	-0.015	-0.027
Observations	240	48	170	250	50	180

Notes: The dependent variable is 1 if a firm issues a bond in a particular month and zero otherwise. Yield is the firm-specific corporate bond yield and size is measured by assets/1000. All specifications include a measure of credit supply (leverage in the broker-dealer market), the federal funds rate (pre-crisis only) and the change in Federal Reserve Holdings of Treasury Notes (post-crisis only) as common factors. Columns (1) and (4) use all firms, (2) and (5) use financial sector firms and (3) and (6) use all other firms (excluding mining and agriculture). t-statistics are shown in parenthesis.

