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Abstract

In this paper we develop an index to monitor the intensity of financial stress in the UK over a period of 45 years. By aggregating various market-based indicators of financial stress from six major markets, we allow each indicator to be assessed in terms of its systemic importance. This enables the index to capture the interconnectedness of financial markets. The index successfully captures three episodes of heightened stress in UK financial history. We also attempt to determine how much a financial shock to the UK economy is amplified in a period of stress vis-à-vis a tranquil period. It involves exploring the dynamic relationship of the index with the UK real economy by two specifications of threshold vector auto-regression models. We find empirical evidence for the existence of feedback loops in the shock propagation between the real and the financial sector in the United Kingdom.

Key words: Financial stress index, AUROC, GARCH, threshold VAR.

JEL classification: C31, C54, G01, G15.

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1 Introduction

This paper develops a comprehensive financial stress index for the United Kingdom (UKFSI). Financial market stress periods are usually characterised by a larger co-movement of variables related to financial sector activity. The UKFSI has the advantage of capturing the interconnectedness of financial markets which also enables an indicator to be assessed in terms of its systemic importance. It reflects the functionality of the financial system and provides an aggregate measure of financial stress across six major markets. The UKFSI dates back to 1971 and is one of the longest UK financial stress indices available at a monthly frequency. It is able to depict well-known financial events over the course of UK financial history. Analysis involving regime-switching models shows that vector autoregressions which include the UKFSI are able to capture the feedback mechanisms in shock transmission that exist between the real and financial sectors.

Specifically, the UKFSI comprises 13 market-based indicators of financial stress that originate from the equity market, government bond market, foreign exchange market, corporate bond market, money market and housing market. Stress symptoms across these markets are measured on the basis of factors such as volatilities, valuation losses and risk spreads. The markets have been identified with a view to capturing a comprehensive description of the key features of financial stress. This includes heightened uncertainty about the value of assets and investor behaviour, as well as flights to quality and liquidity.

The individual market indicators are standardised using an empirical cumulative distribution function. In this way, the value of each indicator is replaced by its ranking number scaled by the sample size. We aggregate indicators within a market segment to form a sub-index, creating six sub-indices in the process. The market-specific sub-indices are then aggregated using a portfolio theory approach: each sub-index is weighted by its cross-correlation with the others. By aggregating correlated sub-indices, the resulting index reflects increased risk due to stronger co-movement with overall financial stress. This also ensures that the UKFSI accounts for the time-varying cross-correlations between the sub-indices and thereby focuses on the systemic dimension of financial stress.

The contribution of our paper is three-fold. First, we demonstrate a new methodology to construct a comprehensive financial stress index. Although we draw on some of the principles used in Duprey et al. (2017), there are major differences. To begin with, we incorporate three more markets into our analysis, namely the corporate bond market, money market and housing market. Increasing market coverage provides a more comprehensive measurement of stress. Second, we use known incidences of UK financial stress history to inform our analysis of potential future stress on financing conditions. This demonstrates the ability of the UKFSI to capture the timing and the magnitude of financial stress. Third, we adopt the 'Area Under the Receiver Operating Characteristic Curve' (AUROC) technique to characterise the usefulness of each raw indicator in signaling a binary crisis event. This enables a ranking of the usefulness of each indicator on the basis of its information content. The use of such information weights enhances the accuracy of our index.

The second contribution lies in the ability of the UKFSI to capture episodes of financial stress. The UKFSI successfully depicts three well-known episodes of financial stress in the UK: (i) between 1973 and 1975 when banks failed, caused by a period of excessive credit growth and leverage; (ii) between 1991 and 1994 when a quarter of small and medium sized UK banks failed, triggered by the closure of Bank of Credit and Commerce International in July 1991; (iii) the global financial crisis between 2007 and 2009 caused by excessive credit growth and leverage. In the UK context, the starting point was marked by the depositor run on Northern Rock in September 2007.

We present further evidence to demonstrate that using a wide range of markets, computing time-varying cross-correlations amongst markets and using information weights in the aggregation of sub-indices substantially improves the accuracy of the UKFSI.

Our third contribution is providing empirical evidence of feedback loops between the real and financial sectors in the UK during stress periods. It has been widely discussed that economic dynamics during stressful times are potentially different from normal times. For example, the Great Recession experienced extreme events where both the financial sector and the real economy suffered heightened stress. The UKFSI presents us with an opportunity to explore the 'regimes' that the UK economy experienced in the past.

In order to study economic dynamics we estimate regime-switching models. We adopt a threshold vector autoregression (TVAR) approach to exploit changes in economic dynamics during stressful times as compared to normal times in the UK. This is done by considering different threshold variables as in Alessandri and Mumtaz (2014). We construct a simple system summarising real economic activity along with its linkages to, and linkages between, the real sector and financial markets in the UK. By employing generalized impulse response function analysis (Koop et al. (1996)), we investigate the differences in the shock transmission in different states of the world.

There are two major results. First, our results stress the importance of acknowledging nonlinearities and distinguishing different states of the world. The transmission of shocks in recessionary or financially stressful periods is significantly different from what occurred during normal times. A linear model would have missed such a distinction. Second, we provide empirical evidence to illustrate Brunnermeier and Sannikov (2014)'s theoretical conclusion that the even small shocks are amplified during a crisis regime. In particular, the occurrence of output shocks during financial stress periods lead to more intense financial stress. Similarly, financial shocks that make their impact during recessionary periods create disproportionately more severe recessions. In other words, our models successfully capture the amplification mechanisms of small shocks under stressful regimes.

Although our paper is not designed as an early warning signal model, it draws on some of its principles. These models were initially used for forecasting currency crises in emerging markets (Reinhart et al. (1998), Kaminsky and Reinhart (1999)). They have, however, subsequently been extended to assess banking crises (Demirgüç-Kunt and Detragiache (1998)). Early warning systems identify indicators that signal economic vulnerabilities early enough to enable policy-makers to take mitigating action. A signal is extracted from an indicator variable when it breaches a pre-determined threshold. In other words, an indicator either stays below a threshold and issues no signal, or it breaches a threshold and issues a signal. Instead of signaling the build-up of vulnerabilities, we use similar tools to signal the contemporaneous materialisation of a crisis. With this in mind, our paper selects indicators based on the value of the Area under the Receiver Operator Characteristic Curve (AUROC) which summarises the information content of each indicator. The AUROC metrics literature has recently been introduced in economics, with examples such as Schularick and Taylor (2012) and Drehmann and Juselius (2014).

Recent studies (for example Duprey et al. (2017)) have constructed indices of financial stress across countries. However those studies focus on cross-country comparability without making adjustments to the methodology to capture the specifics of each country. We build on this work by including (a) more detailed UK data; (b) optimising the design of the index based on known events of stress in the UK, and (c) using a new method to weight each underlying stress indicator by its information content. On the incidence of financial crisis in the UK, it would have been useful to have a professional consensus like the business-cycle dates the National Bureau of Economic Research (NBER) produces for the US recessions. Owing to the non-availability of such a documented sequence, we rely on historical accounts of events in UK financial history.[1] In this regard, we follow Brave and Butters (2011) who, in the absence of a professional consensus, rely on historical accounts of events in US financial history.

Our paper also adds to the burgeoning literature commenting on the chronology of financial crisis. Duprey et al. (2017) is the first paper to develop model-based country-specific financial stress indices to capture the financial cycle in 27 European Union countries.

[1] Lo Duca et al. (2017) provides a list of episodes of financial crises for EU countries, including the UK, that combines both model-based analytics and expert judgment.

They use various regime switching models to identify financial stress regimes and use an algorithm to select those episodes with a substantial negative impact on the real economy. Hakkio and Keeton (2009) and Holló et al. (2012) study the link between financial stress and financial crises by looking at the response of economic activities with respect to changes in financial stress. Illing and Liu (2006), Oet et al. (2011) and Jing et al. (2015) construct a financial stress index that can predict or replicate a given sequence of expert-identified events.

Recently Kapetanios et al. (2017) construct a UK financial conditions index using partial least squares which takes into account the joint covariance of various financial indicators. They compare their measure with the Goldman Sachs financial conditions index by Dudley and Hatzius (2000). Similar papers both by academics and policy institutions draw attention to the importance of monitoring financial stress or financial conditions. Examples for the US include Hatzius et al. (2010), Kliesen and Smith (2010), Brave and Butters (2011), Carlson et al. (2014) and Oet et al. (2015), among others. IMF (2017) also provide similar indices for other countries including the US.

Our threshold autoregression results are also closely related to both the theoretical and empirical literature which have paid increasing attention to the potential nonlinearities existing between the real and financial sectors, as highlighted by Krishnamurthy and He (2011), Boissay et al. (2013) and Brunnermeier and Sannikov (2014). Our paper is similar in spirit to that of Hubrich and Tetlow (2015), who provide seminal empirical findings that shock transmission can be very different in financially stressful regimes in the United States using a US-based financial stress index.

Our paper is divided into the following sections. Section 2 discusses in detail the construction of the UKFSI. Section 3 presents the UKFSI and highlights the strengths of UKFSI methodology. Section 4 describes the threshold VAR models and the results of shock amplification in stressful regimes. Section 5 concludes.

2 The construction of the UKFSI

Although it may be difficult to come up with a precise definition of financial stress, there are certain phenomena that are typically associated with it. Hakkio and Keeton (2009) provide a comprehensive description on the key features of financial stress. These is increased uncertainty about fundamental values of assets; increased uncertainty about the behaviour of other investors; increased asymmetry of information; decreased willingness to hold risky assets (flight to quality) and decreased willingness to hold illiquid assets (flight to liquidity).

The UKFSI is designed to capture stress on asset class segments that are most significant for the UK economy. With this in mind, the indicators were drawn from markets where vulnerabilities would best characterise financial stress in the UK. The equations specifying the indicators can be found in Appendix A. Our methodology covers six market segments and aims to comprehensively capture risks in each of the segment as well as correlations among them. Stress symptoms across these markets are measured on the basis of factors such as volatilities, valuation losses and risk spreads. Financial market stress periods are usually characterised by a larger co-movement of variables related to financial sector activity. Stress indices of this nature have the advantage of capturing the interconnectedness of financial markets.

2.1 Market-based indicators of financial stress

The time span and frequency of financial stress indices are typically conditioned by the availability of data and the jurisdictions they are designed to cover. With a view to making them as comprehensive as possible, financial stress indices can be designed to include dozens of financial time series, but at the cost of shorter time spans and mixed frequencies. At the other end of the spectrum, one can be as parsimonious as possible in order to ensure cross-country comparability. Most indices of financial stress lie somewhere in-between applying judgment on which input is valuable enough to be added to the list of indicators.

The monthly UKFSI comprises 13 market-based indicators of financial stress that originate from the equity market (EM), government bond market (BM), foreign exchange market (FX), corporate bond market (CO), money market (MM) and housing market (HM). A key innovation in this paper is to identify the different inputs to the financial stress index based on a statistical selection of the most relevant indicators for the identification of financial crises in the UK. Table 1 summarises all the candidate indicators. The formulae used to define all the indicators are listed in Appendix A.1.

2.2 Episodes of financial stress in UK financial history

In order to select the indicators of stress that best line up with the sequence of crisis events discussed below, we select the most relevant set of indicators by ranking indicators depending on their ability to match a given sequence of crisis episodes. We try to get as close as possible to the following three stress periods that are widely accepted as episodes of financial crises in the academic literature on UK financial history.

Table 1: List of candidate market-based indicators of financial stress

	Market segment	Duprey et al. (2017)	Abbreviation	Stress indicators
EM	Equity market		ABS_EBR	Realised volatility of the excess bank returns over the broad stock index
			ABS_STX_BKS	Realised volatility of the bank sector stock market index returns
		•	**ABS_STX**	Realised volatility in the stock price index
			CMAX_BKS	Cumulative maximum loss in the bank sector stock market index over a two-year moving window
		•	CMAX_STX	Cumulative maximum loss in the real stock price index over a two-year moving window
			DIFF_STX	Difference in the monthly stock returns over monthly bond returns
			ROL_EBR	Monthly excess bank returns over the broad stock index
BM	Government bonds market		**ABS_SPR**	Realised volatility in the spread between UK 10-year government bonds and US 10-year government bonds
		•	ABS_R10	Realised volatility in the 10-year government bond yield
		•	**CDIFF_SPR**	Cumulative difference in the maximum increase of the spread of the real government bond spread relative to the US
			CMIN_R10	Increase in the 10-year yield compared to the minimum over a two-year rolling window
			SLOPE	Term spread: spread between the 10-year and 3-month government bond yield
FX	Foreign exchange market		CUMUL_CHF	Cumulative change in the bilateral exchange rate between the pound sterling and Swiss Franc
		•	CUMUL_EER	Cumulative change in the real effective exchange rate
			CUMUL_EUR	Cumulative change in the bilateral exchange rate between the pound sterling and the Euro
			CUMUL_JPY	Cumulative change in the bilateral exchange rate between the pound sterling and Japanese Yen
			CUMUL_USD	Cumulative change in the bilateral exchange rate between the pound sterling and US Dollar
			CMIN_CHF	Compare the Sterling-Swiss Franc exchange rate with its highest level over a two-year rolling window
			CMIN_EUR	Compare the Sterling-Euro exchange rate with its highest level over a two-year rolling window
			CMIN_JPY	Compare the Sterling-JapaneseYen exchange rate with its highest level over a two-year rolling window
			CMIN_USD	Compare the Sterling-USD exchange rate with its highest level over a two-year rolling window
			ABS_CHF	Monthly Realised volatility of the Swiss Franc
		•	ABS_EER	Monthly Realised volatility of the real effective exchange rate
			ABS_EUR	Monthly Realised volatility of the Euro
			ABS_JPY	Monthly Realised volatility of Japanese Yen
			ABS_USD	Monthly Realised volatility of the US Dollar
CO	Corporate bonds market		**SPREAD_CRP**	Yield spread between corporate bonds and government bonds
			CMIN_CRP	Compute the increase in the 10-year corporate bond yield $CORP$ compared to the minimum
			ABS_CRP	Realised volatility in the corporate bond market
MM	Money market		**ABS_IBS**	Realised volatility in the spread of the 3-month interbank rate and the 3-month Treasury bill rate
			CDIFF_IBS	The cumulative difference corresponding to the maximum increase of the interbank spread
HM	Housing market		**CMAX_HPI**	Cumulative maximum loss in the real house price index over a two-year moving window
			G_HPI	Monthly growth of the real housing price index
			RATIO_HPI	An affordability index given by the ratio of real house price index over real income per household
			HPO	A measure of overvaluation of house prices

Notes: This table lists all the raw indicators being considered. The dots refer to the data used in Duprey et al. (2017). The selected indicators are emphasised in bold; our methodology is detailed in Section 2.4.

1. 1973-1975 was a period characterised by excessive credit growth and leverage. Banks that subsequently failed tended to exhibit a pronounced boom and bust cycle in lending growth. The crisis was marked by a range of support measures affecting a number of individual institutions. It included the Bank of England's participation in what came to be known as the 'Lifeboat operation' (Reid (1982)).

2. In the early 1990s, about a quarter of the small and medium sized UK banks failed causing the 1991-1994 crises (Logan (2001)). The trigger was the closure of Bank of Credit and Commerce International in July 1991, due to fraud. This led to the withdrawal of wholesale funding to smaller banks, which coupled with pressure from bad loans associated with the recession, made weaker banks collapse.

3. The 2007-2009 crisis was caused by excessive credit growth and leverage. Crisis management policies included bank recapitalisation and the Special Liquidity Scheme to allow banks to swap temporarily their high-quality mortgage-backed securities for UK Treasury bills. See Bank of England (2008) for a detailed discussion on the causes of the crisis.

The period of 1979-1981 experienced excessive public expenditure which led to double-digit inflation. This, in turn, led to an economic slowdown and uncertainty. Intervention included a reduction of the fiscal deficit and a tightening of monetary policy. Notwithstanding the strained economic environment, we concur with Lo Duca et al. (2017) in not classifying this as a period witnessing financial stress.

2.3 Methodology used to capture crisis events

Having identified the financial crisis events, the next step would be to develop a technique that would be effective in signaling the onset of a crisis. In this paper, the criterion used to rank indicators in terms of their information content is referred to as 'partial AUROC'. This section introduces the concept of partial AUROC with a quick refresher on signal theory to justify our choice.

Early warning signals: noise and signal ratios Early warning systems identify indicators that signal economic vulnerabilities early enough to enable policy-makers to take mitigating action. A signal is extracted from an indicator variable when it breaches a pre-determined threshold. In other words, an indicator either stays below a threshold and issues no signal or it breaches a threshold and issues a signal. The different outcomes can be classified as follows: (A) if an indicator is above a threshold value and a crisis occurs, the observation is categorised as a good signal as the crisis is well predicted; (B) if an indicator is above a threshold value and no crisis occurs, it is a false alarm (Type II error); (C) if an indicator is below a threshold value and a crisis occurs, the observation is categorised as a missed signal (Type I error); (D) if an indicator is below a threshold value and a crisis does not occur, it is categorised as a good silence as the tranquil period is well predicted.

	crisis	no crisis
above threshold	A	B
below threshold	C	D

For each value of the threshold, the performance of an indicator can be assessed by ratios such as the percentage of missed crises (Type I errors) or of false alarms (Type II errors). The threshold should be set by assessing the cost linked to the two types of errors. The trade-off is between (i) missing too many crises (Type I errors) or (ii) wrongly predicting crises that do not exist (Type II errors). The lower the threshold, the more frequent the signal. Therefore, setting the threshold sufficiently low would help predict the whole set of crises, but may generate numerous false alarms. On the other hand, the higher the threshold, the less signals the indicator emits, at the risk of missing more crises. As the number of crisis and non-crisis observations are fixed, we can condense these summary statistics further into the signal ratio:

$$SR = \frac{A}{A + C}$$

and the noise ratio:

$$NR = \frac{B}{B + D}$$

For any given threshold, the policy maker would prefer an indicator with a high signal ratio and a low noise ratio. However, there will be a trade-off between these two desirable features. For low thresholds, both the signal ratio and the noise ratio are expected to be high as the indicator emits signals most of the time. The opposite scenario applies for high threshold values. The noise-to-signal ratio is a measure which can be used to compare the signaling qualities of different models for a given threshold.

A useful indicator is supposed to have a noise-to-signal ratio of less than 1. A value of 1 would result if an indicator provides purely random signals. A shortcoming of the noise-to-signal ratio is that it relies on a specific threshold and often reaches its minimum value at both very low noise and signal ratios. This sort of configuration would be associated with a high threshold. A high threshold implies that policy-makers are extremely averse to false alarms, but do not attach much significance to missing financial crisis. When viewed in the context of the global financial crisis, this is unlikely to reflect policy-makers true preferences. If costs of macroprudential interventions are low and benefits high, policymakers may prefer a low threshold to avoid Type I errors rather than Type II errors.

Area under receive operating characteristic curve (AUROC) The receiver operating characteristic curve (ROC) summarises the trade-off and plots the noise ratio (false positive rate) against the signal ratio (true positive rate) for every possible threshold value above which a signal is defined. The area under the ROC curve (AUROC) is a test of predictive ability of an indicator that is independent of a policy-maker's preferences.

Figure 1 shows the AUROC for an indicator of stress in the corporate bond market captured by the yield spread between corporate bonds and government bonds. Since the AUROC is a portion of the area of the unit square, its value will always be between 0 and 1. However, a random signal produces the diagonal line between (0,0) and (1,1), which has an area of 0.5. An AUROC value of 0.5 means that the indicator is uninformative. In order to be relevant, an indicator must have an AUROC greater than 0.5. As the threshold value falls, both the noise and the signal ratio rise, so the ROC curve slopes upwards. By progressively lowering the threshold from its maximum to its minimum value, we can continuously increase the number of emitted signals. The percentage of well predicted crises then goes from 0% (with 0% false alarms) to 100% (yielding also 100% of false alarms as all values emit a signal).

Figure 1 shows a ROC curve plotting a trade-off between the noise ratio and the signal ratio, for the corporate bond spread, as the threshold varies. As a relevant indicator it detects a high percentage of crises with few false alarms keeping the ROC well above the 45 degree line. High thresholds are close to the origin (as few signals will be issued under a high threshold, few crises are correctly identified and few incorrectly signaled) while low thresholds are close to the opposite end of the chart (as many signals will be issued under a low threshold, many crises will be correctly identified, but many false signals will also be issued.)

The AUROC value is an encompassing measure which takes into account the indicator's accuracy for each possible threshold value. Therefore, the AUROC per se does not require the identification of a threshold and is a summary statistic of the balance between Type I errors (missed crises) and Type II errors (false alarms). The AUROC value for the indicator in Figure 1 is 0.74. The advantage of the AUROC is that it is a 'threshold-free' measure of how good an indicator is in signaling a binary crisis event. With the AUROC there is no need to take a stand on policy maker preferences over Type I or Type II errors. If we were to account for policy makers' preferences between Type I and Type II errors, we could define a loss function to rank indicators and analyse their usefulness.

Partial AUROC as information weights Alessi and Detken (2014) argue that before the global financial crisis central bankers were generally less averse to missing a crisis than to receiving a false alarm and that after the crisis these preferences have become more balanced. This issue of a balanced perspective leads us to a situation where rather than considering the full AUROC we focus on a partial AUROC (pAUROC) that cuts off the areas associated with implausibly low and high values of a policy maker's aversion between Type I and II errors. The estimation of the pAUROC involves specification of the restricted range of false positives and the computation of the partial area under the ROC curve.

As described in Alessi and Detken (2014) the loss function for the policy maker can be written as follows:

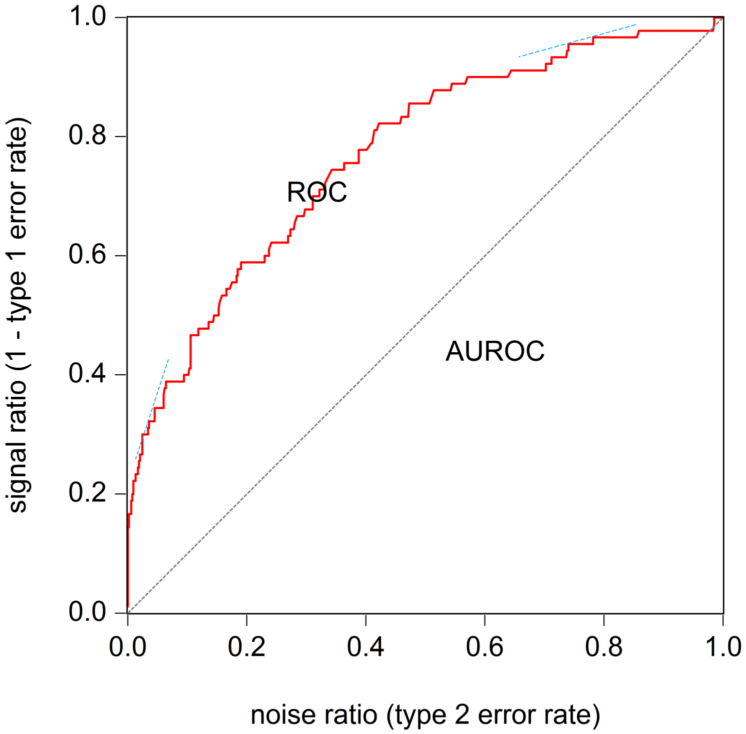
$$L(\theta) = \theta T_1 + (1 - \theta)T_2$$

T_1 is the Type I error rate and is given by $C/(A+C)$. Similarly, T_2 is the share of Type II errors $B/(B + D)$. Parameter θ reveals the policy maker's relative risk aversion between Type I and Type II errors. Therefore, the policy maker's loss function L is defined as the weighted average of the two types of errors generated by the indicator crossing a given threshold. The weighting parameter θ varying between 0 and 1 indicates the policy maker's preferences for avoiding Type I errors compared to those of Type II. If $\theta > 0.5$, it implies a larger aversion towards missing a crisis than towards receiving a false alarm.

In this paper we estimate pAUROC where the weighting parameter θ (the policy maker's relative aversion between Type I and Type II errors) ranges from 0.2 to 0.8. Figure 1 shows the two tangent points between the ROC curve and the restricted preferences of the regulator. The area under the ROC curve restricted to be between the two tangent lines is the partial AUROC for the corporate bond spread. The partial AUROC, in this case, restricts combinations of noise ratios and signal ratios that are outside of bounds $\theta = [0, 2, 0.8]$. In this paper, the criterion used to rank indicators in terms of their information content is the partial AUROC.

A major methodological contribution of the UKFSI compared to other financial stress indicators is the use of the pAUROC as weights in the aggregation of stress indicators across the different market segments into a composite index. In the construction of the UKFSI, we only consider individual stress indicators that are associated with a pAUROC above 0.5, that is to say with a meaningful coincident movement with the episodes of crises.

Figure 1: Area under the receiver operating curve for the corporate bond spreads



Notes: The plain line is the Receiver Operating Characteristic (ROC) curve. The Area Under the ROC curve (AUROC) assesses the value of the indicator to match the sequence of financial crises events in the UK. The 45 degree dotted line corresponds to an uninformative indicator. The two dashed tangent lines are the preferences of the regulators when the relative preferences between type I and type II errors are 0.2 or 0.8. The AUROC restricted to the area between the two tangent points is the partial AUROC with restricted policy-makers preferences.

2.4 Maximising the information content of indicators

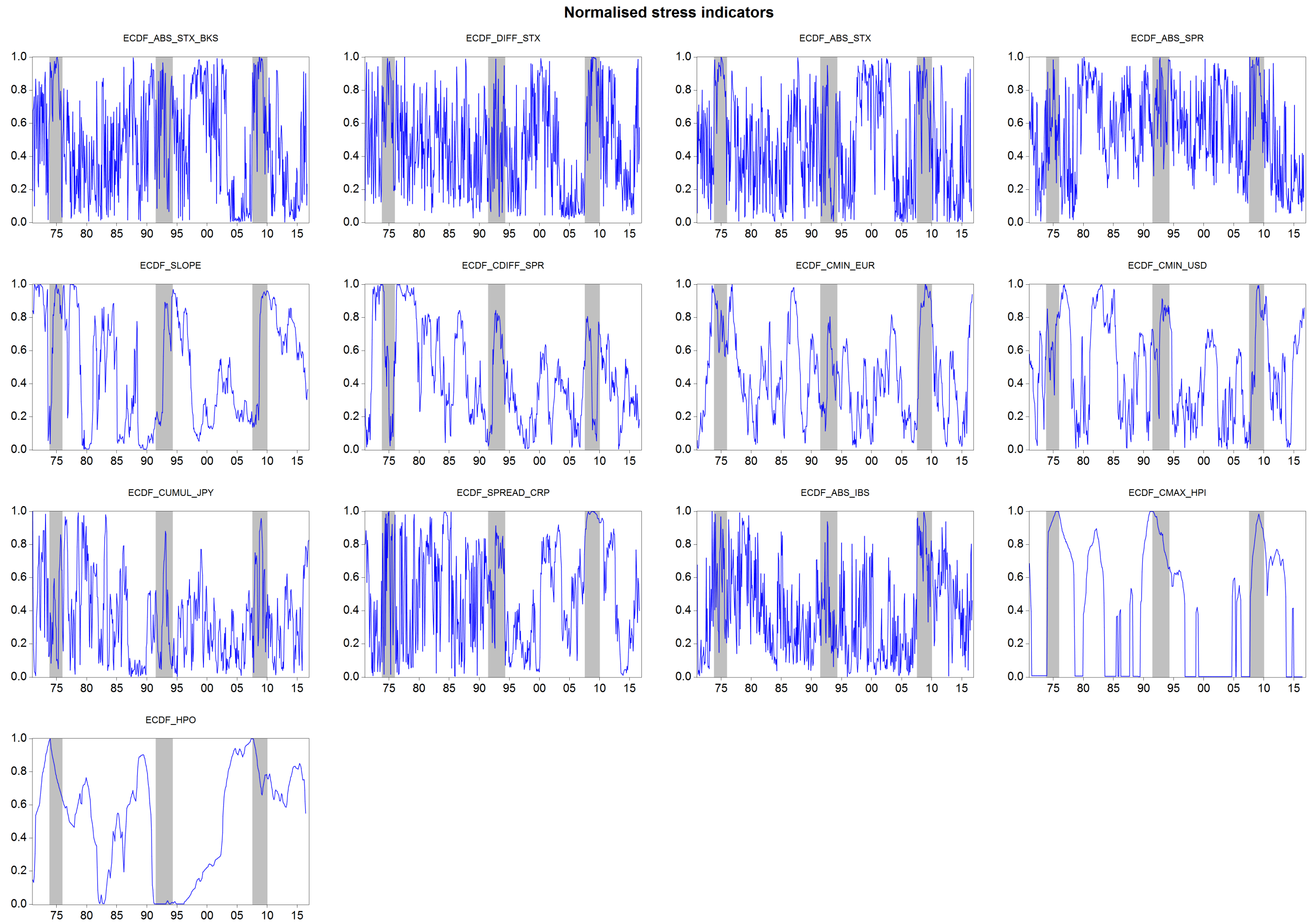
To preserve parsimony in the construction of UKFSI, we need to further narrow down the list of candidate market-based indicators of financial stress by selecting those candidates which keep adding more information to the overall ability of the market segment to match our sequence of crises events.[2] Indeed, additional data can provide marginal benefits in identifying stressful times although they may appear less powerful when looked at individually. It may nevertheless reflect a different aspect of the crisis, and, thus, further contribute to the identification of the crisis as a whole. We start with the candidate indicator with the largest pAUROC for each market segment. Then we keep adding other

[2]Expanding the list of candidate indicators of financial stress comes at the cost of a smaller time span or the risk of redundant information. This is an issue that has not been tackled directly by previous research. Indices constructed with principal component approach indirectly resolve this issue by reducing the data set to a smaller set of factors. However, adding only variables capturing similar facets of a crisis could give the false impression that the first principle component provides a sufficient representation of the evolution of financial stress. In addition, such a tool extracts the components that are best able to explain the overall variance of the dataset, but not necessarily the dichotomy between tranquil and stressful events.

candidate indicators with smaller pAUROC, only if it increases the overall informational content of the market segment as measured by the pAUROC. We present a more detailed explanation on our selection algorithm in Appendix A.2.

Table 2 displays the pAUROC values of all the indicators. It also reports the largest possible pAUROC obtained for each market segment by adding an indicator incrementally. As a result of this process, a total of 13 indicators (emphasised in bold) are selected.

Figure 2: Standardised stress indicators



Notes: This chart shows the empirical cumulative distribution function (ECDF) depicted in probability space of the 13 indicators selected based on the selection process described in Section 2.4. Detailed description of each acronym can be found in Table 1.

Having selected thirteen of the most useful indicators, we proceed to the aggregation of these indices. A defining characteristic of different financial stress indices is the method employed to aggregate individual stress indicators into a composite measure. When aggregating stress measures, we want to ensure that we give more weight to measures that most closely match with the sequence of financial stress episodes in the UK. We achieve

Table 2: Indicators, AUROC and Partial AUROC

Market Segment		Stress Indicator	AUROC	partial AUROC		
			of stress measure	of stress measure	of market segment	incremental
EM	Equity market	**ABS_STX_BKS**	0.757	0.743	0.743	0.000
		DIFF_STX	0.700	0.684	0.752	0.009
		CMAX_STX	0.686	0.673	0.746	−0.006
		ABS_STX	0.690	0.672	0.752	0.001
		CMAX_BKS	0.690	0.672	0.741	−0.012
		ABS_EBR	0.655	0.645	0.734	−0.019
		ROL_EBR	0.643	0.627	0.743	−0.009
BM	Government bonds market	**ABS_SPR**	0.677	0.652	0.652	0.000
		ABS_R10	0.637	0.624	0.645	−0.007
		SLOPE	0.633	0.619	0.693	0.041
		CDIFF_SPR	0.557	0.556	0.725	0.032
		CMIN_R10	0.499	0.501	0.721	−0.004
FX	Foreign exchange market	**CMIN_EUR**	0.754	0.760	0.760	0.000
		CMIN_CHF	0.732	0.728	0.760	−0.001
		CMIN_USD	0.730	0.675	0.805	0.045
		CMIN_JPY	0.690	0.668	0.752	−0.053
		ABS_EUR	0.669	0.657	0.757	−0.048
		CUMUL_CHF	0.618	0.603	0.801	−0.004
		CUMUL_USD	0.605	0.593	0.779	−0.026
		ABS_JPY	0.618	0.590	0.781	−0.024
		ABS_CHF	0.597	0.588	0.774	−0.031
		CUMUL_JPY	0.584	0.573	0.812	0.007
		ABS_USD	0.563	0.557	0.803	−0.008
		CUMUL_EER	0.555	0.548	0.790	−0.022
		CUMUL_EUR	0.558	0.546	0.797	−0.014
		ABS_EER	0.501	0.502	0.812	0.000
CO	Corporate bonds market	**SPREAD_CRP**	0.764	0.748	0.748	0.000
		ABS_CORP	0.662	0.647	0.725	−0.023
		CMIN_CRP	0.637	0.618	0.714	−0.034
MM	Money market	**ABS_IBS**	0.726	0.695	0.695	0.000
		CDIFF_IBS	0.629	0.614	0.691	−0.057
HM	Housing market	**CMAX_HPI**	0.890	0.872	0.872	0.000
		G_HPI	0.796	0.805	0.827	−0.045
		RATIO_HPI	0.500	0.536	0.852	−0.020
		HPO	0.525	0.522	0.887	0.015

Notes: This table lists the thirteen indicators (emphasised in bold) being selected based on our selection methodology detailed in Section 2.4. Details of each indicator can be found in Table 1. Indicators in each market segment are ranked based on their partial AUROC values, with the largest value being ranked the first. Partial AUROC of *market segment* is also provided, computed by adding one extra indicator one at a time on top of the ones with higher individual partial AUROC value. The last column 'incremental' computes the incremental change in *partial AUROC of the market segment* when an extra indicator is added.

this by using the pAUROC as information weights. More precisely, we use this method twice, as explained below.

2.5 Standardisation and creation of market sub-indices

The different raw stress indicators do not have the same unit, so additional normalisation is required before aggregating them into the six market stress components. Each raw stress indicator is transformed to lie in $[0, 1]$ by using the empirical cumulative distribution function (ECDF). Creating an ECDF involves replacing the value of each indicator by its ranking number scaled by the sample size. The normalisation is carried out over an initially fixed window of 10 years (i.e. 120 months), and then over an expanding window (see Holló et al. (2012)). Thus we ensure that each indicator is standardised against enough data while allowing for the distribution of stress to evolve over time. Figure 2 shows the standardised stress indicators for these selected indicators. The shaded vertical bars correspond to the years during which the UK economy experienced a financial crisis in our sample period.

When aggregating individual stress indicators into the market sub-indices (I_{EM}, I_{BM}, I_{FX}, I_{CO}, I_{MM}, I_{HM}), instead of computing the average of the raw measures, we take the average of the raw measures weighted by their information content as measured by the pAUROC. An individual stress measure that has better information content is given more weight in the market sub-index.

For instance, as shown in Table 2, three indicators are chosen to capture stress in the equity market. We denote these three indicators as $j = 1, 2, 3$. The equity market sub-index is constructed as:

$$I_{\text{EM}} = \frac{\sum_{j=1}^{3} \text{EM}_{j,t} \times (\text{pAUROC}_j - 0.5)}{\sum_{j=1}^{3} (\text{pAUROC}_j - 0.5)}$$

where $\text{EM}_{j,t}$ is the ECDF of the indicator j in the segment EM.

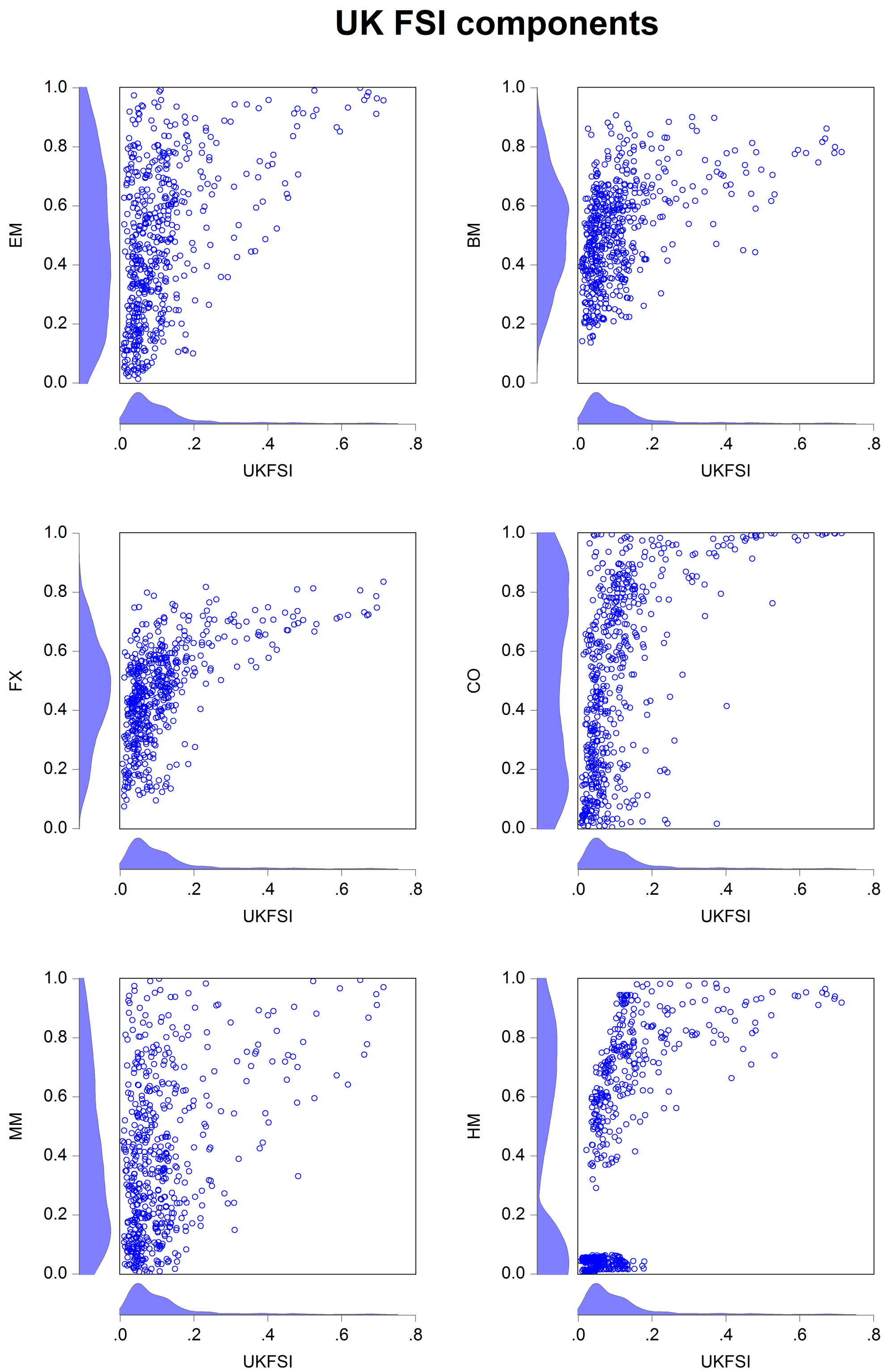
The Kernel density functions for the six sub-indices, plotted against the density function of the final UKFSI, are shown in Figure 3.[3]

2.6 Aggregation of market sub-indices into a composite

The individual stress indicators capture similar elements of risk (i.e volatility, large losses or spreads) for each market segment. As in Holló et al. (2012) and Duprey et al. (2017) we

[3] Most kernel densities of the sub-indices were characterized by a fat tail and, in some instances, a bi-modal distribution. This suggests the data can be approximated by a mixture of two distributions with different mean and variance parameters. It would call for a Markovian model where one observation can be drawn from one distribution or the other, but we do not know for sure which one. In addition, note that high levels of the UKFSI are never associated with low levels of an individual market stress measure. This means that individual market stress is indeed correlated and tends to be high when the overall stress is also high. This confirms the choice of a correlation-based aggregation measure.

Figure 3: Kernel density functions for six sub-indices



Notes: Kernel densities for each of the six market segments against UKFSI. 'EM' denotes equity market, 'BM' denotes government bond market, 'FX' denotes foreign exchange market, 'CO' denotes corporate bond market, 'MM' denotes money market, and 'HM' denotes housing market.

aggregate the sub-indices for the six market segments based on a portfolio theory approach that weights each sub-index by its cross-correlation with the others. When sub-indices are positively correlated, the overall financial stress index is *larger* than the mere sum of its sub-components. By aggregating correlated sub-indices, the resulting index reflects increased risk due to stronger co-movement with overall financial stress. Conversely, when the sub-indices are negatively correlated, the overall financial stress index is *smaller* than the sum of its sub-components.

Aggregation using the principles of portfolio theory enables the UKFSI to account for the time-varying cross-correlations between the sub-indices and thereby focus on the systemic dimension of financial stress. The efficacy of the UKFSI in detecting systemic stress episodes would depend on its ability to capture the co-movement across financial market segments.

To be specific, the UKFSI is computed as:

$$\mathrm{UKFSI}_t = \frac{(w_t \times I_t)' \times C_t \times (w_t \times I_t)}{26/36} \in [0, 1]$$

We discuss each term in detail.

Vector of six market sub-indices $I_t = (I_{\mathrm{EM,t}}, I_{\mathrm{BM,t}}, I_{\mathrm{FX,t}}, I_{\mathrm{CO,t}}, I_{\mathrm{MM,t}}, I_{\mathrm{HM,t}})$ is a vector of standardised sub-indices for the equity market (EM), government bond market (BM), foreign exchange market (FX), corporate bond market (CO), money market (MM) and housing market (HM) at each point in time.

Vector of information weights w_t is the 1×6 vector of information weights given by the pAUROC that emphasizes those sub-indices that are most relevant for identifying crises episodes. The weights are computed as pAUROC $- 0.5$ since the measure is more informative than a coin toss only when it is above 0.5. For instance, the weight for the equity market (EM) would be defined as:

$$w_{\mathrm{EM}} = \frac{\mathrm{pAUROC}_{\mathrm{EM}} - 0.5}{\sum_m \mathrm{pAUROC}_m - 0.5}$$

With $m = \{\mathrm{EM, BM, FX, CO, MM, HM}\}$.

Time-varying correlation matrix C_t, being the key term in the formula, is the 6×6 time-varying cross-correlation matrix that emphasizes the episodes of simultaneous stress across different market segments.

$$C_t = \begin{bmatrix} 1 & \rho_{\text{EM,BM},t} & \rho_{\text{EM,FX},t} & \rho_{\text{EM,CO},t} & \rho_{\text{EM,MM},t} & 0 \\ \rho_{\text{EM,BM},t} & 1 & \rho_{\text{BM,FX},t} & \rho_{\text{BM,CO},t} & \rho_{\text{BM,MM},t} & 0 \\ \rho_{\text{EM,FX},t} & \rho_{\text{BM,FX},t} & 1 & \rho_{\text{FX,CO},t} & \rho_{\text{FX,MM},t} & 0 \\ \rho_{\text{EM,CO},t} & \rho_{\text{BM,CO},t} & \rho_{\text{FX,CO},t} & 1 & \rho_{\text{MM,CO},t} & 0 \\ \rho_{\text{EM,MM},t} & \rho_{\text{BM,MM},t} & \rho_{\text{FX,MM},t} & \rho_{\text{CO,MM},t} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It can be seen that only 26 entries are different from zero, reflecting our choice of excluding the housing market in computing the cross-correlations. The time-varying cross-correlations $\rho_{m,m',t}$ with $m' \neq m$ are estimated using GARCH (the preferred method) and EWMA models, with details described in Appendix A.3.

Pair-wise cross correlations increase significantly during periods of financial stress and reached peak levels of above 0.9 between some markets in the months following the collapse of Lehman Brothers in September 2008. Table 3 provides a simple visual representation on the contrast in cross correlations between 2008 and 2015.

Table 3: A snapshot of cross-correlations between the five markets in 2008 and 2015

	2008				
	EM	BM	FX	CO	MM
EM	1.00				
BM	0.81	1.00			
FX	0.88	0.87	1.00		
CO	0.84	0.74	0.87	1.00	
MM	0.82	0.81	0.87	0.81	1.00
	2015				
	EM	BM	FX	CO	MM
EM	1.00				
BM	0.59	1.00			
FX	0.66	0.65	1.00		
CO	0.38	0.42	0.47	1.00	
MM	0.25	0.34	0.22	0.20	1.00

>0.75 <0.75 and >0.50 <0.50 and >0.25

Notes: 'EM' denotes equity market, 'BM' denotes government bond market, 'FX' denotes foreign exchange market, 'CO' denotes corporate bond market, and 'MM' denotes money market.

A scaling factor The term 26/36 is a scaling factor to account for the fact that cross-correlation coefficients between the housing market sub-index and the other market sub-indices is equal to zero. We exclude housing since it uses data that has been extrapolated to the monthly frequency, as opposed to other sub-indices that rely on daily data. Otherwise the monthly correlations are likely to be biased with respect to housing, as changes take more time to be reflected in the housing sub-index due to extrapolation.

Aggregation to overall UKFSI When arriving at a market-segment driven sub-index we account for the pAUROC of individual indicators. However when transitioning to overall UKFSI, the weights are determined by the time-varying cross-correlations between these sub-indices. Thus a sector that was more relevant than others in relation to the sequence of crisis episodes in the UK has been assigned more weight in the financial stress measure.

This strategy allows us to optimise the construction of the UKFSI given the incidence of known crisis events, and reduce the risk of combining informationally redundant data that would over-emphasize a given sector. It also brings more rigour in the choice of the components of the stress measure in its ability to capture specific characteristics of the UK economy, instead of implementing ad-hoc weighting schemes. Alternatives include simple averages of the various sub-components as in Duprey et al. (2017), an average weighted by the estimated elasticities of the component with respect to industrial production as in Holló et al. (2012), or an average weighting process using the volume of financial assets that relate to each market segment as in Illing and Liu (2006).

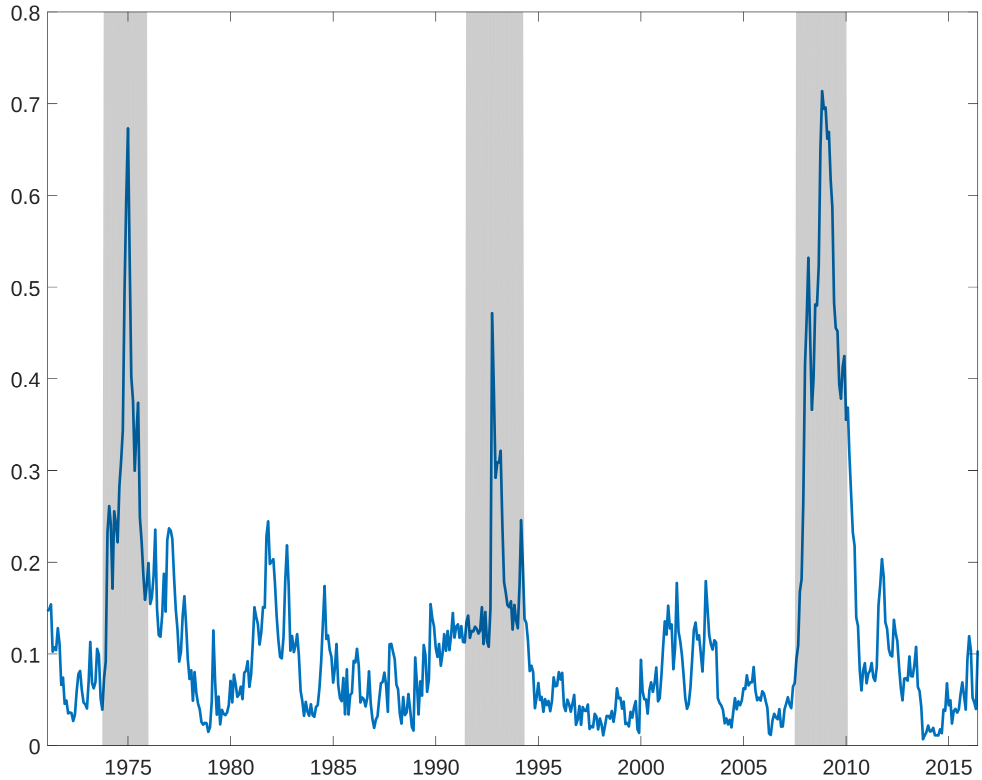
3 The UKFSI and the strengths of our methodology

3.1 The UKFSI

Figure 4 displays the UKFSI from March 1971 to June 2016, with shaded vertical bars corresponding to the years during which the UK economy experienced a financial crisis. As discussed in Section 2.1, there were three financial crises in UK financial history over the last 45 years. It can be seen that the sharpest spikes in the UKFSI occurred during periods of financial crisis.

Figure 5 depicts the decomposition of the UKFSI into contributions coming from each of the six sub-indices and the overall contribution from the cross-correlations. The chart is constructed such that the UKFSI is a weighted average of the contributions from each of the six sub-indices and the cross-correlations between them. The sum of the weights add up to 1. At a given point in time, for instance, the contribution to the UKFSI from the equity market is simply the fraction of UKFSI accounted for by the equity market sub-index alone. We repeat the same computation for the remaining five markets. The residual, the portion of the UKFSI unaccounted for by the six markets, reflects the contribution from the cross-market correlations described by the matrix C_t in Section 2.6. The strength of the correlation components during periods of financial crises confirms our choice of aggregation method based on time-varying cross-correlations.

Figure 4: The UKFSI, March 1971 – June 2016



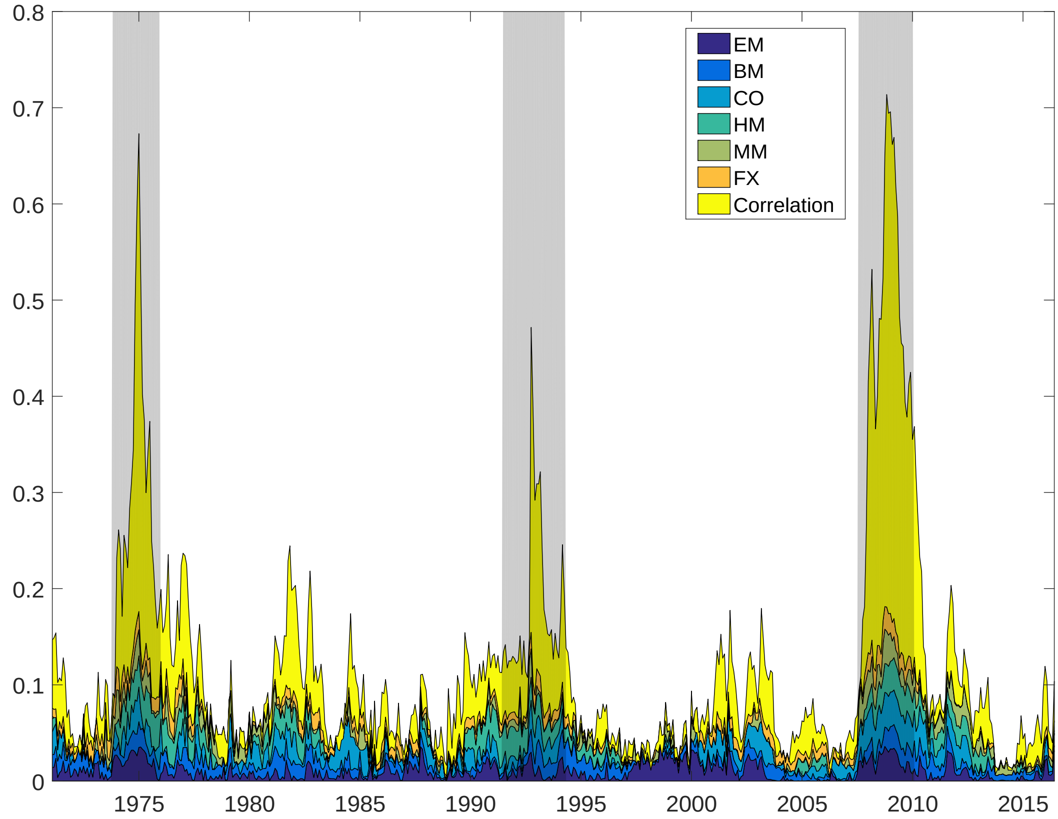
Notes: The construction methodology is detailed in Section 2. Based on the literature in UK financial history discussed in Section 2.2, the three periods of financial stress include (i) banks failed as a result of excessive credit growth and leverage during 1973-1975; (ii) a quarter of the small and medium sized UK banks failed causing the 1991-1994 crises triggered by the closure of Bank of Credit and Commerce International; (iii) the 2007-2009 crisis was caused by excessive credit growth and leverage. These three periods of financial stress periods are shown by the shaded areas.

3.2 Comparing UKFSI with alternative measures

Figure 6 provides a visual representation of the strength of the UKFSI over alternative measures of financial stress, as captured by the ROC curve. The alternative measures include using corporate bond spreads[4] (which, to the best of our knowledge, is the only alternative measure available for the past four decades allowing for a fair comparison), and

[4]This series is constructed using *Global Financial Data*, and is defined as the difference between UK corporate bond yield (mnemonic: INGBRW) and UK 10-year government bond yield (mnemonic: IGGBR10D).

Figure 5: Contributions of individual indices and overall correlation to the UKFSI



Notes: This figure shows decomposition of the UKFSI into contributions coming from each of the six sub-indices and the overall contribution from the cross-correlations. 'EM' denotes equity market, 'BM' denotes government bond market, 'FX' denotes foreign exchange market, 'CO' denotes corporate bond market, 'MM' denotes money market, and 'HM' denotes the housing market. 'Correlation' denotes the overall correlation from the cross-correlations. The three periods of financial stress periods, based on the literature in UK financial history discussed in Section 2.2, are given by the shaded areas.

constructing UKFSI without information weights w_t and without time varying correlation C_t. The figure clearly demonstrates that using the UKFSI generates more robust outcomes. Specifically,

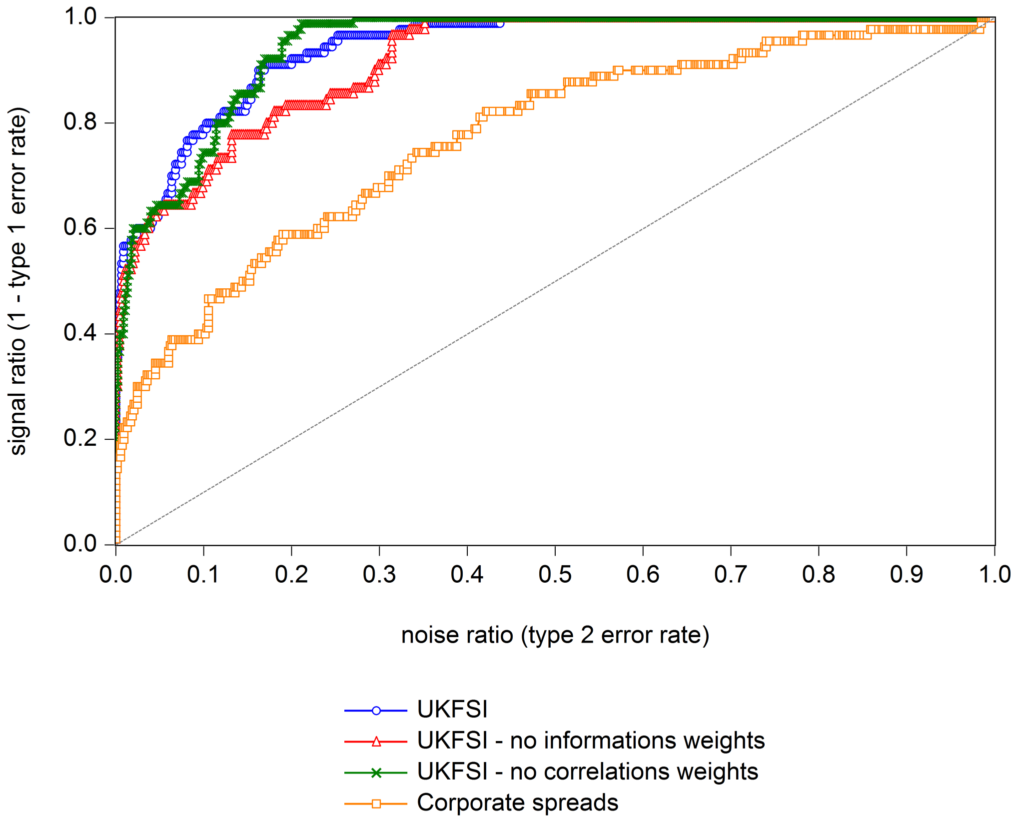
- Including indices from all six markets significantly improves the signal ratio, reflecting the fact that stress information extracted from the other five markets improves the information content of UKFSI;[5]
- Removing the time-varying correlation in our methodology (the crossed line) worsens the signal ratio, especially when the noise ratio is below 0.3. This provides evidence for the importance of including correlations in the construction of UKFSI;[6]

[5]Figure A.1 captures the superior information content of the UKFSI when compared with a simple proxy for financial stress like corporate bond spreads. The corporate bond spread overstates the intensity of financial stress particularly during non-crisis periods. It also falls short of identifying all the crises periods.

[6]Figure A.2 shows the UKFSI and its counterpart without using correlation weights. Calculating the

- Using information weights (the line with the circles instead of the triangles) further improves the signal ratio of UKFSI. This shows the value of adopting partial AUROC in constructing our weighting vector.

Figure 6: AUROC for UKFSI, its variants and corporate bond spreads



Notes: This figure shows the AUROC for UKFSI and its variants, as well as corporate bond spreads. The AUROC shows the ability of the UKFSI to line up with known crises events. The 45 degree line corresponds to uninformative measures. The closer to the North-West corner the better. Please refer to the main text for construction details.

Table 4 provides further evidence on the efficacy of our methodology. In particular, it compares the coincidence between different versions of the UKFSI and the set of financial crisis episodes for the UK with the AUROC and partial AUROC metrics. The UKFSI with the construction method favored in this paper has the largest AUROC and partial AUROC, and removing the correlation weights or information weights yields inferior results.

Similarly using the principal component analysis (PCA) on the sub-indices, or on the

index as a simple average of the six sub-indices implicitly assumes zero correlation across all sub-indices at all times (i.e. the off-diagonal entries of the matrix C_t are zeros). Both peak at the same time, but the simple average features a relatively high level of financial stress even during non-crisis periods. These characteristics are not consistent with narratives on the intensity of stress during crises/non-crises.

individual stress indices, yields lower information content. This table also shows that combining several sub-indices yields an improvement over individual markets, even if the housing market segment on its own already provides good results. By combining sub-indices that have an AUROC or partial AUROC ranging between 0.69 to 0.90, we obtain a UKFSI with AUROC or partial AUROC ranging between 0.71 to 0.93 depending on the aggregation method.

Table 4: The incidence of financial stress events with various financial stress measures

	information weights	equal weights	first principal component	correlation weights	AUROC	pAUROC
UKFSI (baseline)	•			•	0.93	0.86
UKFSI (variant)	•				0.93	0.85
UKFSI (variant)				•	0.91	0.84
UKFSI (variant)		•			0.91	0.84
PCA (six market segments)			•		0.86	0.83
SPREAD_CRP					0.77	0.74
CLIFS of Duprey et al. (2017)		•			0.71	0.71
HM	•				0.90	0.89
FX	•				0.79	0.81
FX		•			0.79	0.79
HM		•			0.82	0.77
EM	•				0.75	0.75
CO	•				0.76	0.75
CO		•			0.76	0.75
EM		•			0.75	0.75
BM	•				0.73	0.73
MM	•				0.73	0.70
MM		•			0.73	0.70
BM		•			0.71	0.69

Notes: For the six market segments, the column labelled 'information weights' refers to the use of partial AUROC in the selection algorithm and the aggregation of individual stress indicators into market stress indicators using partial AUROC weights. For the various UKFSI measures, the column labelled 'information weights' refers to the use (or not) of partial AUROC weights when combining the six market indicators into a single stress composite. 'EM' denotes equity market, 'BM' denotes government bond market, 'FX' denotes foreign exchange market, 'CO' denotes corporate bond market, 'MM' denotes money market, and 'HM' denotes housing market.

4 Feedback loop between UK financial and real sectors

In this section, we study economic dynamics using our newly developed UKFSI. It has been widely discussed that economic dynamics during stressful times are potentially different from normal times. Researchers and policy makers have paid increasing attention to the potential nonlinearities existing between the financial sector and the real economy in the aftermath of the Great Recession, as highlighted by Krishnamurthy and He (2011), Boissay et al. (2013) and Brunnermeier and Sannikov (2014). Hubrich and Tetlow (2015) provide seminal empirical findings that shock transmission can be very different in financially stressful regimes in the United States.

In order to study state-contingent dynamics we estimate regime-switching models. We adopt a threshold vector autoregression (TVAR) approach to exploit changes in economic dynamics during stressful times compared to normal times in the UK, as in Alessandri and Mumtaz (2014), by considering different threshold variables. We construct a simple system summarising real economic activity along with its linkages to, and the linkages between, the real sector and financial markets in the UK. By employing generalized impulse response function analysis (Koop et al. (1996)), we investigate the differences in the shock transmission in different states of the world.

Our models successfully capture the amplification mechanisms of small shocks under stressful regimes. In particular, output shocks hitting financial stress periods create larger financial stress. Similarly, financial shocks hitting during recessionary periods create disproportionately more severe recessions. These findings provide empirical support for the theoretical results highlighted in Brunnermeier and Sannikov (2014) by showing that the economic impact is more severe when small shocks hit during recessions or financial stress periods.

4.1 Model Specification and Data

A threshold VAR model comprises an explicit threshold variable which allows regimes to switch endogenously through the dynamics of the chosen threshold variable, and the switch of regimes is abrupt.

The associated model is given in the following equation,

$$Y_t = \left[c_1 + \sum_{j=1}^{P} \beta_{1,j} Y_{t-j} + v_t \right] R_t + \left[c_2 + \sum_{j=1}^{P} \beta_{2,j} Y_{t-j} + v_t \right] (1 - R_t)$$

where

$$R_t = 1 \iff Z_{t-d} \leq Z^* \quad (\text{or } \; Z_{t-d} \geq Z^* \text{depending on the threshold variable})$$

$$R_t = 0 \text{ otherwise}$$

and $v_t \sim N(0, \Omega_{R_t})$.

The delay parameter, d, is also referred as threshold lag. We consider two different threshold variables. The first one is our UKFSI, motivated by our interest in studying the potential differences in shock transmission mechanism under financial stress, consistent with the spirit of Hubrich and Tetlow (2015) – we denote this system as *TVAR-S*. We define $R_t = 1$ as the *financial stress regime* if and only if the UKFSI rises beyond an estimated threshold value, which we let the data decide. The resulting estimated financially stressful regimes can then be crossed checked with the regimes discussed in the UK financial literature. To be discussed below, we adopt Bayesian techniques in our estimation, and we find that the posterior mode of the threshold is at 0.3. The parameter d is freely estimated and is found to be equal to 1 at the posterior mode.

We also consider using the real GDP growth as our alternative threshold variable, and denote the system as *TVAR-Y*. This is motivated by the desire to study stress in the real economy as opposed to the financial stress. As the regime charts show in Section 4.2.1, periods of stress in the real economy do not necessarily overlap with periods of financial stress. We define $R_t = 1$ as the *recessionary regime* if and only if the real GDP growth is below an estimated threshold rate for $d = 0$ and $d = 1$ simultaneously, given the common definition of recessions associated with two consecutive periods of negative output growth. Our estimated threshold rate hovers around zero percent at the posterior mode, which validates this definition.

To estimate this model, we follow Alessandri and Mumtaz (2014) in using the Gibbs sampling algorithm which includes a Metropolis-Hastings step for sampling the threshold value in each simulation. The estimation procedures are explained in Appendix B. For both models, we impose normal inverse Wishart priors following Bańbura et al. (2010); see Appendix C for more details. The use of such priors is motivated by the fact that the sample can be relatively short in stressful regimes. In particular, we choose $\tau = 0.1$ to be the value of the hyperparameter controlling for the overall tightness of the prior, broadly following Canova (2007) and Bańbura et al. (2010).[7]

Our system comprises the following five variables: (i) annualised real GDP growth rate; (ii) annualised inflation rate; (iii) growth rate of the nominal effective exchange rate (NEER); (iv) short term interest rate; (v) UKFSI. The choice of these variables reflects our goal to capture the overall dynamics in a small open economy and to link the real economic sector to the financial sector while maintaining a parsimonious model. We use quarterly data for the UK from 1973Q3 to 2016Q2. We choose to work with quarterly data due to our desire to include real GDP growth into the system. We adopt four lags for estimation and this is elaborated in Section 4.2.3.

[7] In unreported robustness checks, we relax the value of this parameter to $\tau = 10$ and obtain qualitatively similar results. We pick uninformative priors for the constants with $c = 10^5$.

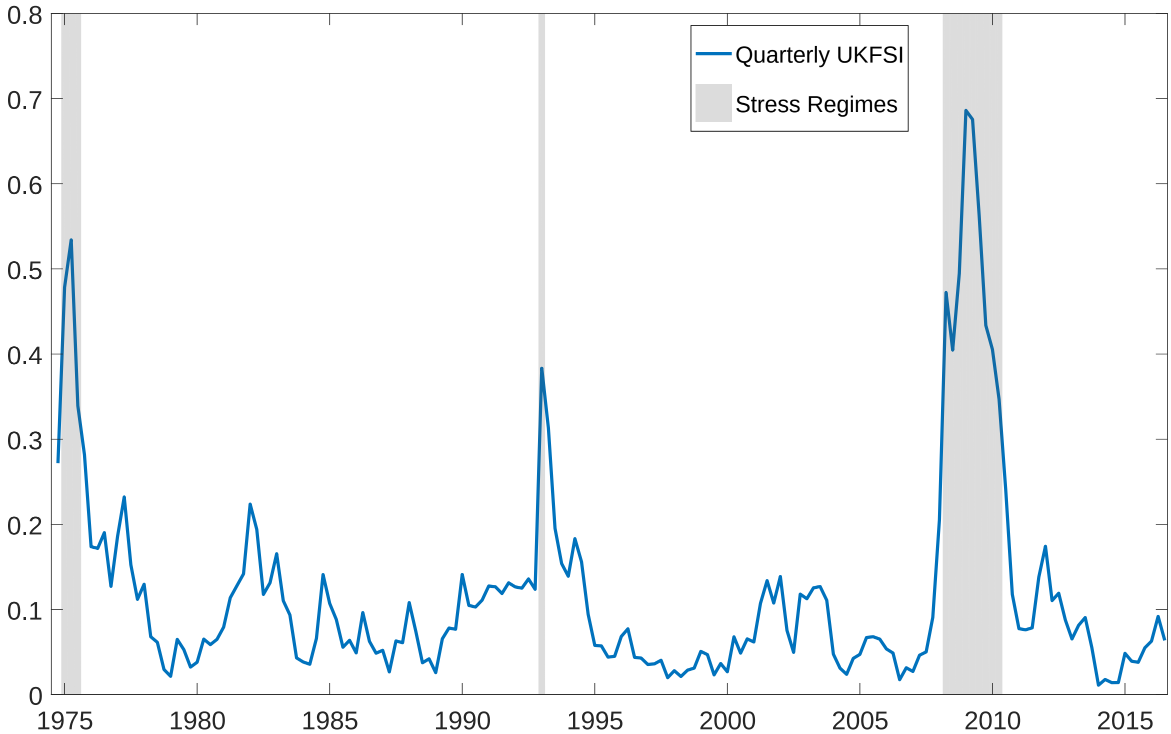
4.2 Results

This section reports the estimation results for both non-linear models. Figures 7 and 8 respectively illustrate the financial stress and the recessionary regimes as modelled by the TVAR-S and TVAR-Y models respectively. Figures 9 and 10 show the generalised impulse response functions of real GDP growth and UKFSI for both models. We also briefly discuss model fit which provides evidence on the suitability of non-linear models and optimal lag length.

4.2.1 Regimes

The regime changes in TVAR are abrupt and the economy is either in one regime or the other. Figure 7 presents the regimes where the UKFSI is used as the threshold variable. The TVAR-S is able to capture all the financial stress periods in the UK history, as summarised in Section 2.2. It is reassuring that the statistically estimated regimes are consistent with the three major financial crises periods identified in the UK financial history literature.

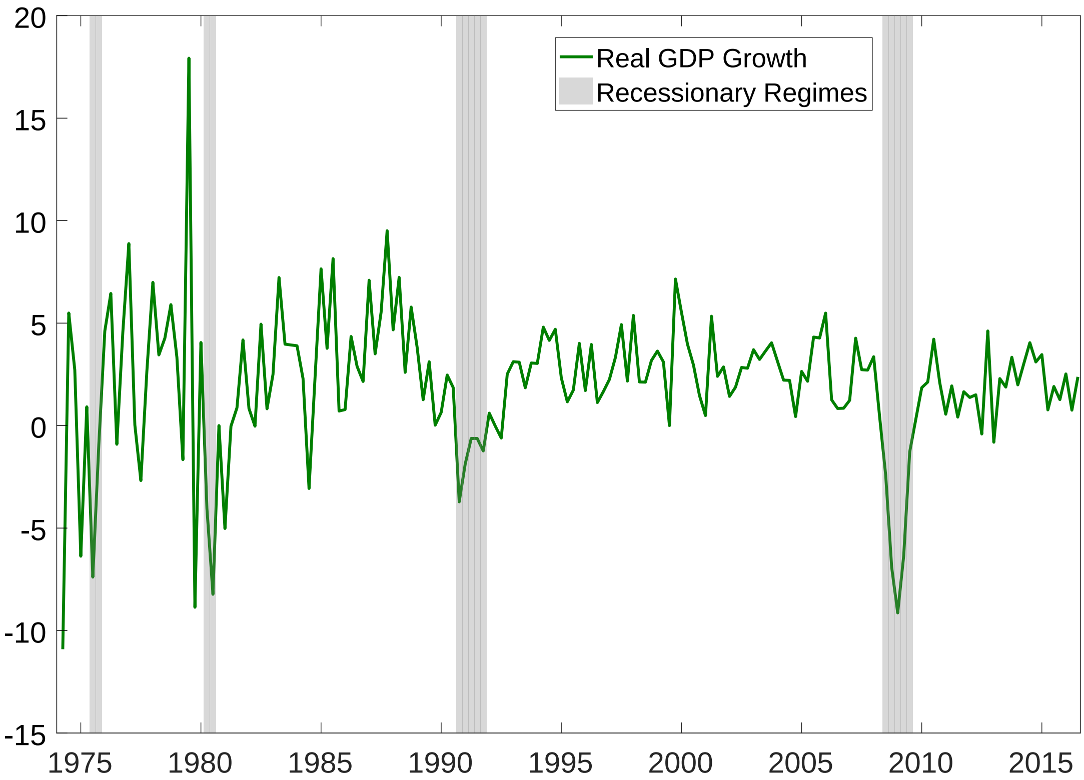
Figure 7: TVAR-S regimes overlapped with the quarterly UKFSI



Notes: Model-based financially stressful regimes estimated by TVAR-S using the quarterly system described in Section 4.1.

Similarly, the regime estimated by the TVAR-Y model in Figure 8 is labelled as *recessionary*. The model is able to detect the recessionary periods in the history of the UK economy. In addition to the three periods of financial stress summarised in Section 2.1, it also detects a short period of recession in early 80s. This period experienced excessive public expenditure which led to double-digit inflation leading to the tightening of monetary policy.

Figure 8: TVAR-Y regimes overlapped with the quarterly real GDP growth



Notes: Model-based recessionary regimes estimated by TVAR-Y using the quarterly system described in Section 4.1.

4.2.2 Impulse Response Analysis

To study the potential differences in the shock transmission under specific regimes, we adopt the the generalised impulse response functions (GIRFs) as introduced by Koop et al. (1996). Koop et al. (1996) discuss that impulse responses in non-linear models are dependent on size, sign and history, which is in contrast to those computed by linear models. The construction of GIRFs fully takes into account of the possibility of endogenous switches of regimes during simulations. This approach allows for feedbacks mechanisms between the real and financial sector, which might otherwise be suppressed by the use of regime-specific impulse responses.

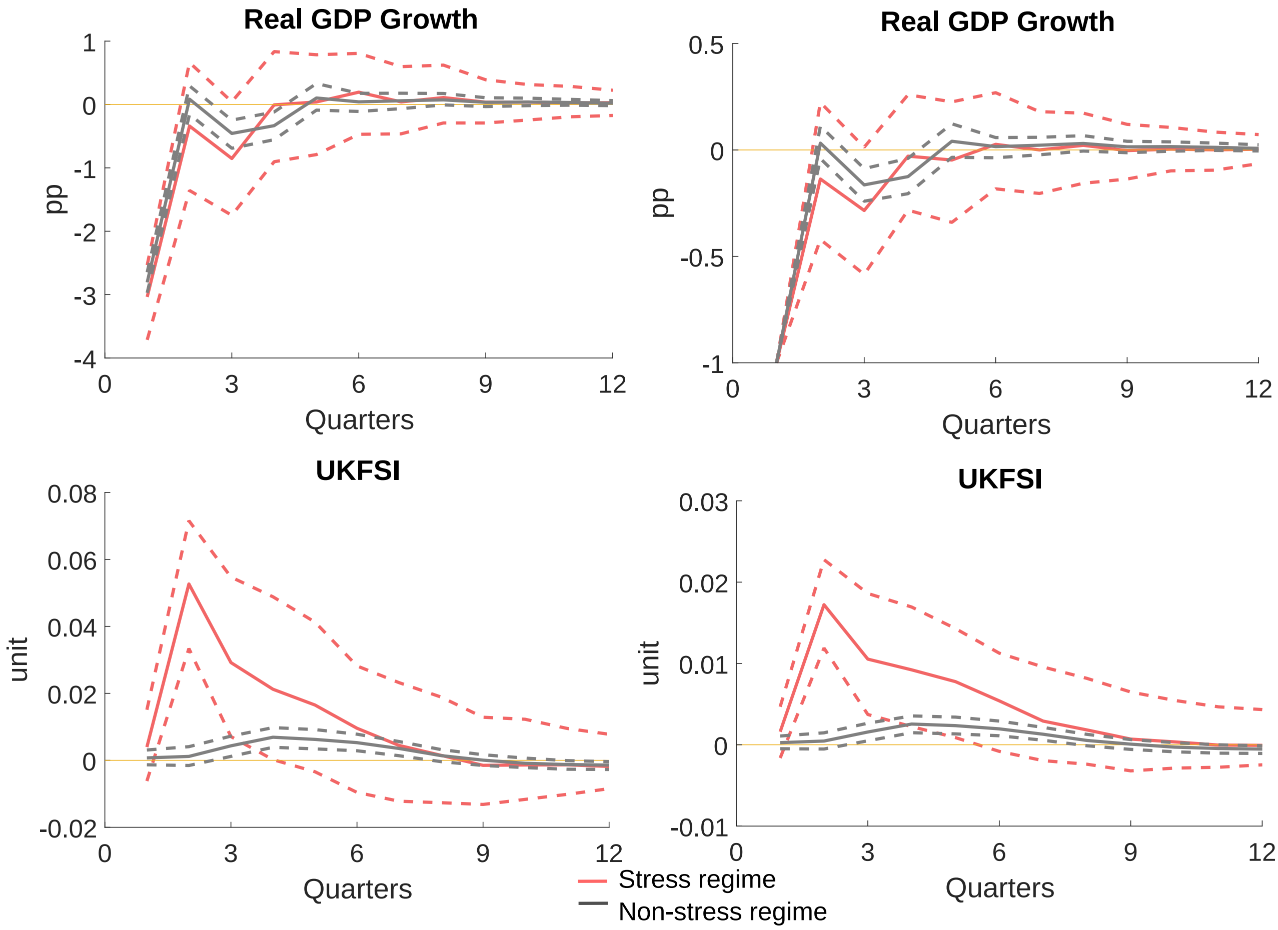
We adopt the Cholesky decomposition to identify structural shocks of interest. The use of 'recursiveness' assumption follows Hubrich and Tetlow (2015), Auerbach and Gorodnichenko (2012) and Caggiano et al. (2014), among others, and is common in the empirical macroeconomic literature. Consistent with Hubrich and Tetlow (2015), we order our UKFSI measure as the last variable, reflecting that it is a high frequency indicator which responds to shocks contemporaneously. The rest of the variables are ordered following the usual empirical macroeconomic literature such as Christiano et al. (1998) and Christiano et al. (2005): real and price variables are placed before the exchange rate and the short-term interest rate. We will focus on two types of structural shocks: *real* shocks (structural shocks causing GDP growth to fall) and *financial* shocks (structural shocks causing financial stress to rise).

We consider one-standard-deviation shocks, arguably small and probable shocks. Since the size of such shocks can be different across regimes, to facilitate comparison, we also consider shocks normalised to the same magnitude. In the interest of space and our focus is on the relationship between the real and financial sectors, we only report the responses of GDP and UKFSI in Figures 9 and Figure 10. The error bands around the median responses correspond to 68% confidence bands.

Figure 9 displays the impulse response functions of TVAR-S model. UKFSI's response to a one-standard-deviation shock to real GDP growth is approximately more than 10 times larger in magnitude in a stress regime than a non-stress regime in the second quarter, although the size of the growth shocks is more or less the same in both regimes (2.6 percentage points (pp) in non-stress regime versus 2.9pp stress regime). This observation still holds when the normalised shock hits. This provides evidence for the amplification of real shocks under financial stress: when the economy is already under financial stress, further bad shocks originating in the real economy lead to heightened stress in the financial system.

Turning to how financial shocks propagate in recessions, Figure 10 shows that in TVAR-Y, a one-standard-deviation financial shock in the recessionary state generates a mostly significant and deep decline in output growth with a maximum fall of approximately 0.5pp two quarters after the shock. A shock of similar magnitude in the non-recessionary world leads to a maximum drop of 0.5pp four quarters after the shock. For normalised shocks for the magnitude of 0.05, the response of the recessionary state is severe compared to that of non-recessionary state, with a maximum drop of 0.9pp in recessionary state two quarters after the shock vs 0.4pp in non-recessionary state four quarters after the shock. This provides evidence for the amplification of financial shocks under recession: when the economy suffers recession, further financial stress will lead to deeper recessions.

Figure 9: TVAR-S responses to real GDP growth shocks
one-standard-deviation shock (left) vs 1pp normalised shock (right)



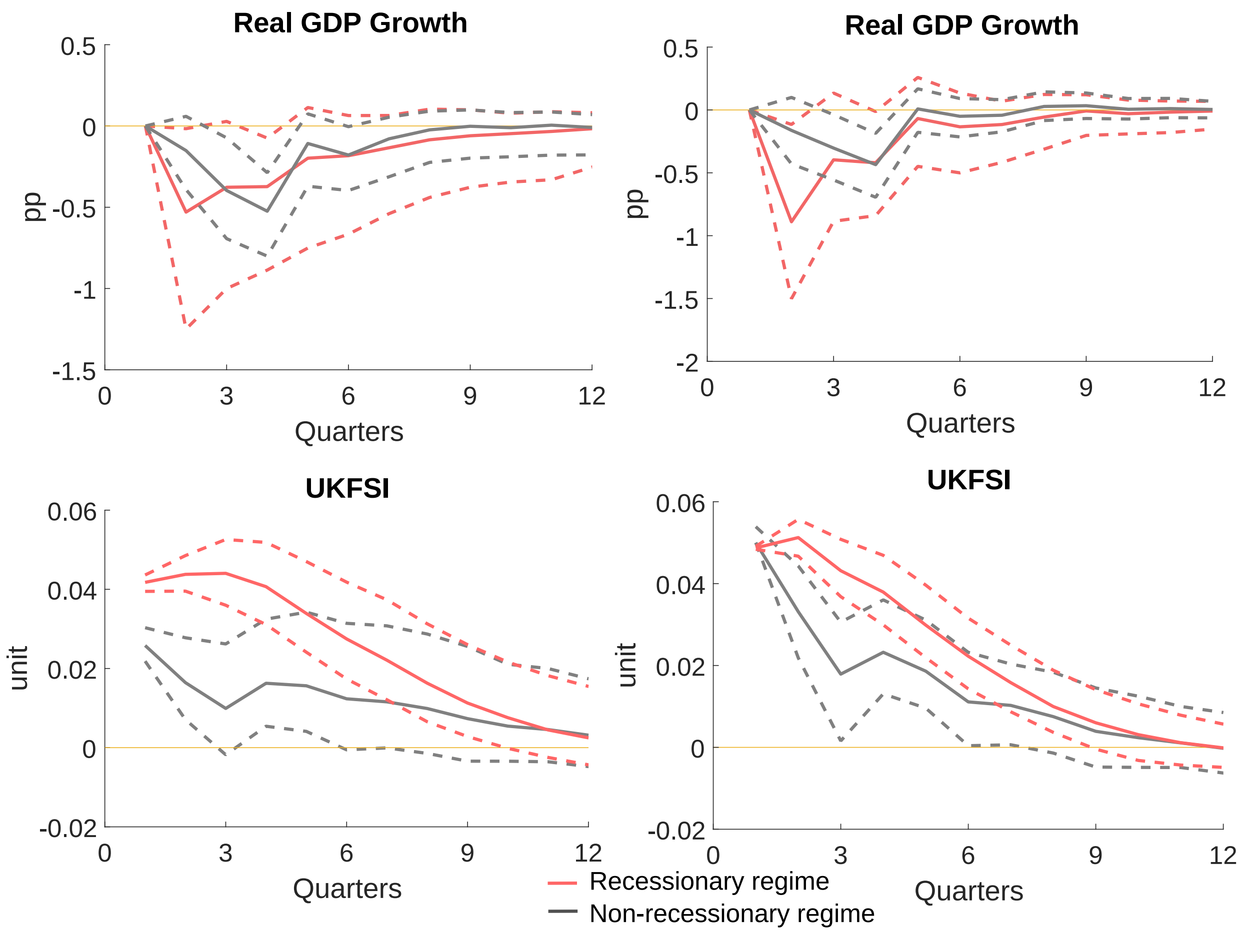
Notes: Details about the TVAR-S are described in Section 4.1.

To sum up, there are two major findings. First, the transmission of shocks in recessionary or financially stressful periods is significantly different from normal times. Our results stress the importance of acknowledging nonlinearities and distinguishing different states of the world. A linear model will have missed such a distinction. Second, we provide empirical evidence to illustrate Brunnermeier and Sannikov (2014)'s theoretical conclusion that the even small shocks are amplified once in a crisis regime. In both models, we observe the feedback loop between real and financial sector so exogenous shocks, even when they are small, cause heightened economic stress.

4.2.3 Lag Length and Model Fit

In our estimation, we adopt four lags in estimating our non-linear models. In this section, we provide the empirical justification for this choice.

Figure 10: TVAR-Y responses to UKFSI shocks
one-standard deviation shock (left) vs 0.5 unit normalised shock (right)



Notes: Details about the TVAR-Y are described in Section 4.1.

Table 5 reports the deviance information criterion (DIC), following Spiegelhalter et al. (2002) and Chan and Grant (2016), for the two non-linear models under study. Smaller information criteria indicate better model fit. The table suggests four lags both for TVAR-S and TVAR-Y models as the DIC drops as we use more lags.[8] Our qualitative results remain robust when we run the TVAR models with different lag lengths.

We also compare the model fit of our TVAR models relative to the linear VAR model. Table 5 presents evidence that TVAR-S consistently provide lower DIC value compared to the linear VAR model. TVAR-Y estimated with one and two lags provide marginally better fit to the data than the linear model. This provides statistical support for our choice of TVAR models in this paper.

[8]Both Akaike and Bayesian information criteria, in unreported results, suggest one lag for both models which does not seem sufficient to accurately model the dynamic relationship in a VAR model. Our results, however, are similar and robust to other choices of leg length.

Table 5: Deviance information criteria for different models and lag lengths

	DIC			
Lags	1	2	3	4
TVAR-S	1475	1443	1374	1305
TVAR-Y	1573	1519	1513	1462
BVAR	1575	1525	1490	1377

5 Conclusion

In this paper, we develop a comprehensive financial stress index for the United Kingdom. It allows us to make an assessment of how today's stress level compares to previous levels. Dating back to 1971, this index has the advantage of capturing the interconnectedness of six major financial markets which also enables an indicator to be assessed in terms of its systemic importance. We provide evidence of statistical precision relative to various measures of financial stress, and show that this index successfully captures the major financial events in the country.

A key policy question we address is how financial stress serves as an additional source of shocks to the UK economy, and how it can be an amplifying channel for other shocks. We estimate a threshold vector-autoregression with the view to capturing elements of macro-financial feedback in the UK. There are two major results. First, the transmission of shocks in recessionary or financially stressful periods is significantly different from normal times. Our results stress the importance of acknowledging nonlinearities and distinguishing different states of the world. Second, we provide empirical evidence to illustrate that the even small shocks are amplified once we are in a crisis regime. In particular, output shocks occurring during financial stress periods create larger financial stress. Similarly, financial shocks hitting during recessionary periods create disproportionately more severe recessions.

The UKFSI has important implications for monetary and macroprudential policy. First, it is a monthly index that could enable the real-time assessment of stress levels within the entire financial system. Our approach is transparent and the index can be easily updated as new observations become available. Second, the UKFSI can be decomposed into contributions from each market segment. Therefore, we can track how much each sector contributes to the build-up of stress at any given point in time. Third, our aggregation methodology allows each indicator to be assessed in terms of its systemic importance and ranked accordingly. This approach ensures parsimony and may be effective in analysing the efficacy of monetary and macroprudential policy interventions.

Practitioners and policy makers might also be interested in developing an index which would have the ability to predict financial crises. Therefore, a possible extension is to expand the scope of the index so that it has the capacity to predict. We leave this for future research.

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Appendix

A Further details of the methodology of UKFSI

A.1 Candidate market-based indicators of financial stress

In the UK, as in most other countries, inflation rates have varied substantially over time. To account for this, we take real stock prices ($rSTX$), real banking sector index ($rBKS$) and real government bond yields ($rR10$).

$$\begin{cases} rSTX_i = \frac{STX_t}{CPI_t} \\ rBKS_i = \frac{BKS_t}{CPI_t} \\ rR10_t = R10_t - \frac{CPI_t - CPI_{t-261}}{CPI_{t-261}}.100 \end{cases}$$

We list below the various candidate stress measures that are referred to in Table 1. The individual stress indicators capture similar elements of risk (i.e volatility, large losses or spreads) for each market segment. Stress in the equity market is captured by possibly up to seven useful candidates that line up well with crises episodes in the UK. The government bonds market has five, the foreign exchange market has fourteen, the corporate bonds market three, the money market two and the housing sector four candidates.

A.1.1 Equity market stress (EM)

Realised volatility in the stock price index (ABS_STX) Asset return volatilities tend to increase with investors' uncertainty about future fundamentals. Monthly realised volatility ($VSTX$) of the real stock price index ($rSTX$) is computed as the monthly average of absolute daily log-returns of the real stock price index. In order to take into account the possibility of long term changes in the volatility of the variables, we adjust the log returns by their 10 year volatility.

$$\begin{cases} \ln STX_t = \log(rSTX_t) - \log(rSTX_{t-1}) \\ ABS\_STX_t = \frac{\sum_{i=0}^{19} \left| \frac{\ln STX_{t-i}}{\sigma_{\ln STX_{t-i,t-2609-i}}} \right|}{20} \end{cases}$$

Cumulative maximum loss in the real stock price index over a two-year moving window (CMAX_STX) We compute the cumulative maximum loss ($CMAX$) that corresponds to the maximum loss compared to the highest level of the stock market over two years. Except for the first two years, the CMAX is computed over a rolling window of 522 days.

$$CMAX_STX_t = 1 - \frac{rSTX_t}{\max_{i=0}^{521}(rSTX_{t-i})}$$

Difference in the monthly stock returns over monthly bond returns (DIFF_STX)

$$DIFF\_STX_t = (\log(rSTX_t) - \log(rSTX_{t-20})) - (\log(rBKS_t) - \log(rBKS_{t-20}))$$

Monthly excess bank returns over the broad stock index (ROL_EBR) Monthly average excess bank returns over the broad stock index. Excess bank returns are computed by regressing the daily real stock price returns over daily bank returns over a two year rolling window (522 business days). We can then consider the residual term ε_t of the OLS regression as the excess returns of the bank stock index. We take the monthly average.

$$\begin{cases} \ln BKS_t = \log(rBKS_t) - \log(rBKS_{t-1}) \\ \ln STX_t = a + b \ln BKS_t + \varepsilon_t \\ ROL\_EBR_t = \frac{\sum_{i=0}^{19} \varepsilon_{t-i}}{20} \end{cases}$$

Realised volatility of the excess bank returns over the broad stock index (ABS_EBR)

$$\begin{cases} ABS\_EBR_t = \frac{\sum_{i=0}^{19} |\varepsilon_{t-i}|}{20} \end{cases}$$

Realised volatility of the bank sector stock market index returns (ABS_BKS)

$$\begin{cases} \ln BKS_t = \log(rBKS_t) - \log(rBKS_{t-1}) \\ ABS\_BKS_t = \frac{\sum_{i=0}^{19} \left| \frac{\ln BKS_{t-i}}{\sigma \ln BKS_{t,t-2609-i}} \right|}{20} \end{cases}$$

Cumulative maximum loss in the bank sector stock market index over a two-year moving window (CMAX_BKS)

$$CMAX\_BKS_t = 1 - \frac{rBKS_t}{\max_{t=0}^{521}(rBKS_{t-i})}$$

A.1.2 Bond market stress (BM)

Realised volatility in the spread between UK 10-year government bonds and US 10-year government bonds (ABS_SPR)

$$\begin{cases} SPR_t = rR10_t - rR10_{US,t} \\ ABS\_SPR_t = \frac{\sum_{i=0}^{19}\left|\frac{\ln SPR_{t-i}}{\sigma \ln SPR_{t-i,t-2609-i}}\right|}{20} \end{cases}$$

The cumulative difference corresponding to the maximum increase of the spread of the real government bond spread with respect to the US (CDIFF_SPR). This spread is calculated in basis points over a two-year rolling window. We prefer using the spread instead of the 10-year yield in order to disentangle changes in risk profiles from changes in the proxy for the risk-free rate. $CDIFF$ refers to the cumulative difference.

$$CDIFF\_SPR_t = rR10_t - rR10_{US,t} - \min_{i=0}^{521}(rR10_{t-1} - rR10_{US,t-i})$$

Realised volatility in the 10-year government bond yield (ABS_R10) The monthly realised volatility is computed as the monthly average of absolute daily changes on real 10-year government bonds. We prefer using changes and not growth rates since, for some periods, very low yields would create would create excessively large variations.

$$\begin{cases} chR10_t = rR10_t - rR10_{t-1} \\ ABS\_R10_t = \frac{\sum_{i=0}^{19}\left|\frac{chR10_{t-i}}{\sigma_{chR10_{t-i,t-2609-i}}}\right|}{20} \end{cases}$$

Increase in the 10-year yield compared to the minimum (CMIN_R10) We compute the increase in the 10-year yield compared to the minimum over a two-year rolling window.

$$CMIN\_R10_t = \frac{(100 + rR10_t)}{\min_{i=0}^{521}(100 + rR10_{t-i})} - 1$$

Term spread: spread between the 10-year and 3-month government bond yield (Slope)

$$Slope_t = R10_t - R3_t$$

A.1.3 Foreign exchange market stress (FX)

Stress in the foreign exchange market also relies on a number of variables with two possible transformations. We report only the transformations for the real effective exchange rate EER, but the same applies for the pound sterling against the euro EUR, the US Dollar USD, the Swiss Franc CHF or the Japanese Yen JPY. In order to obtain the daily real effective exchange rate, we use the broad daily nominal effective exchange rate of the BIS (backcasted with a more narrow measure), and we splice it to the monthly real effective exchange rate of the BIS. Within a month, the distinction nominal/real in the growth rate of the effective exchange rate should vanish.

Cumulative change in the real effective exchange rate (CUMUL_EER) Longer-lasting changes in the real effective exchange rate EER should be associated with more severe stress, owing to the necessary adjustment of the real economy. Therefore, we compute the cumulative change ($CUMUL$) over six months (130 days): if $CUMUL > 0$, then the real effecting exchange rate is volatile around a changing rate.

$$CUMUL\_EER_t = |EER_t - EER_{t-130}|$$

Monthly Realised volatility of the real effective exchange rate (ABS_EER)

$$\begin{aligned} chJPY_t &= GBP/JPY_t - GBP/JPY_{t-1} \\ ABS\_EER_t &= \frac{\sum_{i=0}^{19} \left| \frac{chJPY_{t-i}}{\sigma_{chJPY_{t-i,t-2609-i}}} \right|}{20} \end{aligned}$$

Compare the sterling-euro exchange rate with its highest level (CMIN_EUR) over a two-year rolling window. This is a measure of financial stress associated with exchange rate depreciation.

$$CMIN\_EUR_t = \frac{GBP/EUR_t}{\min_{t=0}^{521}(GBP/EUR_{t-i})} - 1$$

It will be likewise for the CMIN_CHF, CMIN_USD and CMIN_JPY.

Monthly realised volatility in the exchange rate between sterling and the Japanese Yen (ABS_JPY)

$$chJPY_t = GBP/JPY_t - GBP/JPY_{t-1} ABS\_JPY_t = \frac{\sum_{t=0}^{19} \left| \frac{chJPY_{t-1}}{\sigma_{chJPY_{t-i},t-2609-i}} \right|}{20}$$

Cumulative change in the exchange rate between the sterling and the Japanese Yen (CUMUL_JPY). Longer-lasting changes in the bilateral exchange rate with a major trading partner, such as Japan, would be associated with financial stress. Again if CUMUL>0, the Yen would be volatile around a changing rate.

$$CUMUL\_JPY_t = [GBP/JPY_t - GBP/JPY_{t-130}]$$

A.1.4 Corporate bond market stress (CO)

Yield spread between corporate bonds and government bonds (SPREAD_CRP) Corporate bond spreads computed as the difference in yield between a 10-year sterling-denominated corporate bond and a 10-year UK government bond.

Compute the increase in the 10-year corporate bond yield ***CRP*** **compared to the minimum (CMIN_CRP)** This is computed over a two-year rolling window.

$$CMIN\_CORP_t = \frac{(100 + CORP_t)}{\min_{i=0}^{521}(100 + CORP_{t-i})} - 1$$

Realised volatility in the corporate bond market (ABS_CORP) The monthly realised volatility is computed as the monthly average of the absolute daily changes in the corporate bond yield.

$$\begin{cases} \ln CORP_t = \log(CORP_t) - \log(CORP_{t-1}) \\ ABS\_CORP_t = \frac{\sum_{t=0}^{19} \left| \frac{\ln CORP_{t-i}}{\sigma_{\ln CORP_{t-i,t-2609-i}}} \right|}{20} \end{cases}$$

Monthly realised volatility in the yield spread between corporate and government bonds (ABS_CORP)

$$CorpSpr_t = Corp_t - R10_t ABS\_CorpSpr_t = \frac{\sum_{t=0}^{19} \left| \frac{CorpSpr_t}{\sigma_{CorpSpr_{t-i,t-2609-i}}} \right|}{20}$$

A.1.5 Money market stress (MM)

Realised volatility in the spread of the 3-month interbank rate and the 3-month Treasury bill rate (ABS_IBS) We first compute the spread of the between the 3-month Libor and 3-month UK T-bills. We then take a monthly average of the daily data.

$$\begin{cases} IBS_t = LIBOR_t - TBill_t \\ ABS\_IBS_t = \frac{\sum_{t=0}^{19} \left| \frac{IBS_{t-i}}{\sigma_{IBS_{t-i,t-2609-i}}} \right|}{20} \end{cases}$$

The cumulative difference corresponding to the maximum increase of the interbank spread

$$CDIFF_t = LIBOR_t - TBill_t - \min_{i=0}^{521}(LIBOR_{t-i} - TBill_{t-i})$$

A.1.6 Housing market stress (HM)

The data is available only at the quarterly frequency. We take a monthly linear interpolation.

Cumulative maximum loss in the real house price index over a two-year moving window (CMAX_HPI)

$$CMAX\_HPI_t = 1 - \frac{rHPI_t}{\max_{i=0}^{24}(rHPI_{t-i})}$$

Monthly growth of the real housing price index (G_HPI). We take the negative so that a larger drop corresponds to higher stress.

$$G\_HPI_t = \frac{rHPI_t - rHPI_{t-1}}{rHPI_{t-1}}$$

An affordability index given by the ratio of real house price index over real income per household (RATIO_HPI)

$$RATIO\_HPI_t = \frac{rHPI_t}{rI_{t-1}}$$

A measure of overvaluation of house prices (HPO) We regress the (log) value of the real house price index in the UK over (log) real per capital disposal income and the long-term 10-year interest rate.

$$\ln rHPI_t = \alpha_1 + \alpha_2 rI_t + \alpha_3 \ln rR10_t + \varepsilon_t HPO = \varepsilon_t$$

As in Bauer (2017) we use an asset-pricing approach where the value of a house is the expected discounted value of future cash flows. The cash flows would be the rents and the discount rate would be the mortgage interest rate. Owing to limitations in the data availability over a long period we use (log) real, per-capita disposable income under the assumption that rents are driven by per-capita economic growth over the long-run. We also assume that discount rates are proportional to long-term (10-year) government bond yields. The estimated residual captures the deviation of the actual house price from its predicted value and is used as an estimate of the 'overvaluation' of house prices.

A.2 Selection procedure of candidate stress indicators with partial AUROC

We use the following strategy to select the set of relevant stress indicators to be used in the construction of the UKFSI among all potential stress indicators.

We start by ranking all candidate stress indicators by their partial AUROC (Table 2).

1. For each of the six market segments, we discard candidate stress indicators with a partial AUROC below 0.5 as it means those indicators are doing worse than a coin flip in terms of matching our sequence of crises events.

2. For a given market segment, we always select the stress indicator with the highest partial AUROC. For the equity market segment, that means ABS_STX_BKS.

3. For a given market segment, we consider the inclusion of the candidate indicator that has the next largest partial AUROC. For the equity market segment, that means $DIFF\_STX$. We then compute the weighted average of the previously selected stress measures with this additional one. That is, we compute the temporary equity market segment I'_{EM}:

$$I'_{\mathrm{EM}} = \frac{ABS\_STX\_BKS * (\mathrm{pAUROC}_{ABS\_STX\_BKS} - 0.5)}{(\mathrm{pAUROC}_{DIFF\_STX} - 0.5) + (\mathrm{pAUROC}_{ABS\_STX\_BKS} - 0.5)} + \frac{DIFF\_STX * (\mathrm{pAUROC}_{DIFF\_STX} - 0.5)}{(\mathrm{pAUROC}_{DIFF\_STX} - 0.5) + (\mathrm{pAUROC}_{ABS\_STX\_BKS} - 0.5)}$$

   We then compute the partial AUROC of the temporary market segment, $\mathrm{pAUROC}_{I'_{\mathrm{EM}}}$ in our example. This is referred to in Table 2 as the column 'partial AUROC of market segment'. The (positive or negative) incremental partial AUROC corresponds to $\mathrm{pAUROC}_{I'_{\mathrm{EM}}} - \mathrm{pAUROC}_{ABS\_STX\_BKS}$.

- If the incremental partial AUROC is positive, i.e. $\text{pAUROC}_{I'_{\text{EM}}} > \text{pAUROC}_{ABS\_STX\_BKS}$, then adding the additional candidate stress indicator $DIFF\_STX$ improves the informational content of the overall market segment. So this stress indicator is also selected.
- Otherwise if the incremental partial AUROC is negative, this stress indicator, here $DIFF\_STX$, is not selected.

We repeat step 3 with the indicator with the next largest partial AUROC. The idea is that stress indicators with lower informational content (with lower partial AUROC when considered individually) can nevertheless add relevant information to the overall market stress measure (with higher partial AUROC when considered jointly) if it captures a different aspect of financial stress.

Table 2 lists the AUROC and partial AUROC values for each of the raw stress indicators initially considered. Those finally selected on the basis of their information content are in bold.

A.3 Computing cross-correlations in C_t

The time-varying cross-correlations $\rho_{m,m',t}$ are estimated using an exponentially weighted moving average (EWMA) specification as well as multivariate GARCH. Although more complex, the multivariate nature of the GARCH limits the risk of omitted variable bias when computing cross-correlations. The time varying cross-correlation terms are estimated employing the commonly used diagonal BEKK multivariate GARCH(1,1) process.[9] This specification requires as input the vector of demeaned normalised sub-indices (so that it has zero mean), which is simply $\overline{I} = I_t - 0.5$ because of the properties of the cumulative density function. The diagonal representation of the model makes it more parsimonious and is equivalent to assuming that variances are only a function of the past demeaned sub-indices while the covariance terms rely on the cross-product of the demeaned indices, that is to say the 5×5 variance-covariance matrix H_t takes the following form:

$$H_t = V_0 V_0' + A' \overline{I}_{t-1} \overline{I}'_{t-1} A + B' H'_{t-1} B$$

while A and B are 5×5 diagonal matrices to estimate. As mentioned above, entries for the housing market correlations are set to zero in the matrix C_t. The results are robust when compared with the simpler EWMA method, but at the cost of computing only pair-wise correlations.

[9] In all cases the GARCH(1,1) specification turns out to be superior in terms of log-likelihood and Schwartz criteria over models with more lags.

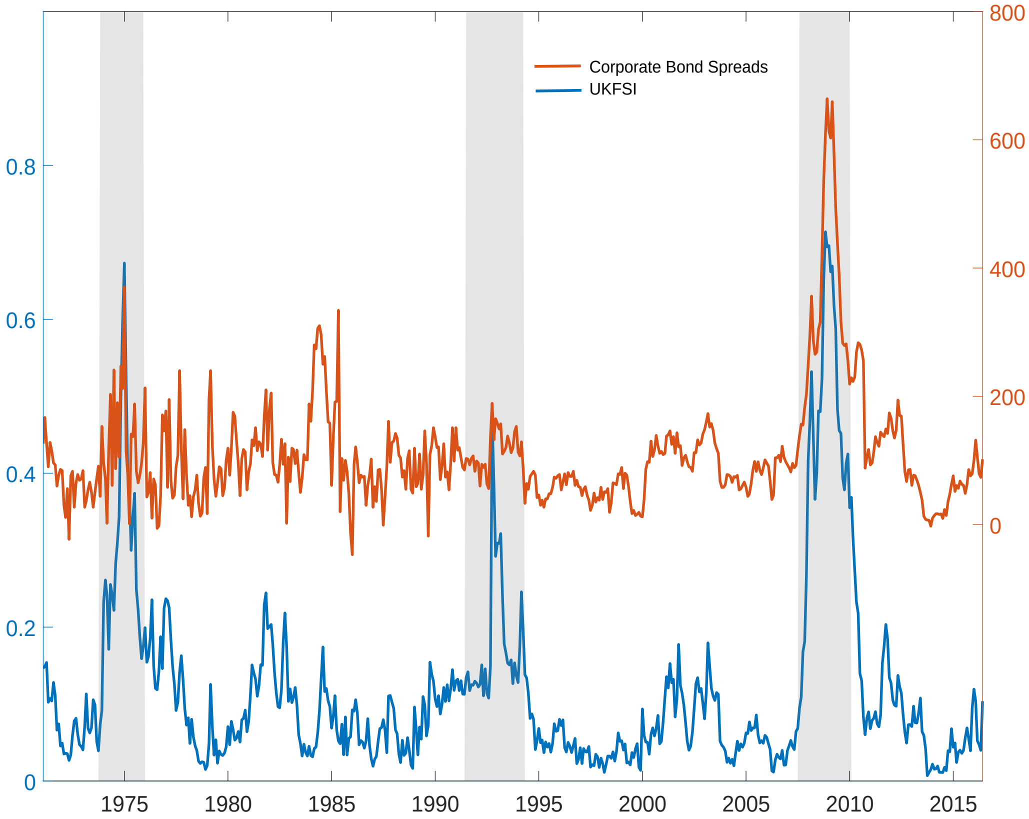
The GARCH method provided better results in terms of AUROC values and was the preferred approach for estimating the UKFSI.

A.4 Comparing UKFSI with alternative measures of financial stress

Figure A.1 captures the superior information content of the UKFSI when compared with a simple proxy for financial like corporate bond spreads. The corporate bond spread overstates the intensity of financial stress particularly during non-crisis periods. It also falls short of identifying all the crises periods.

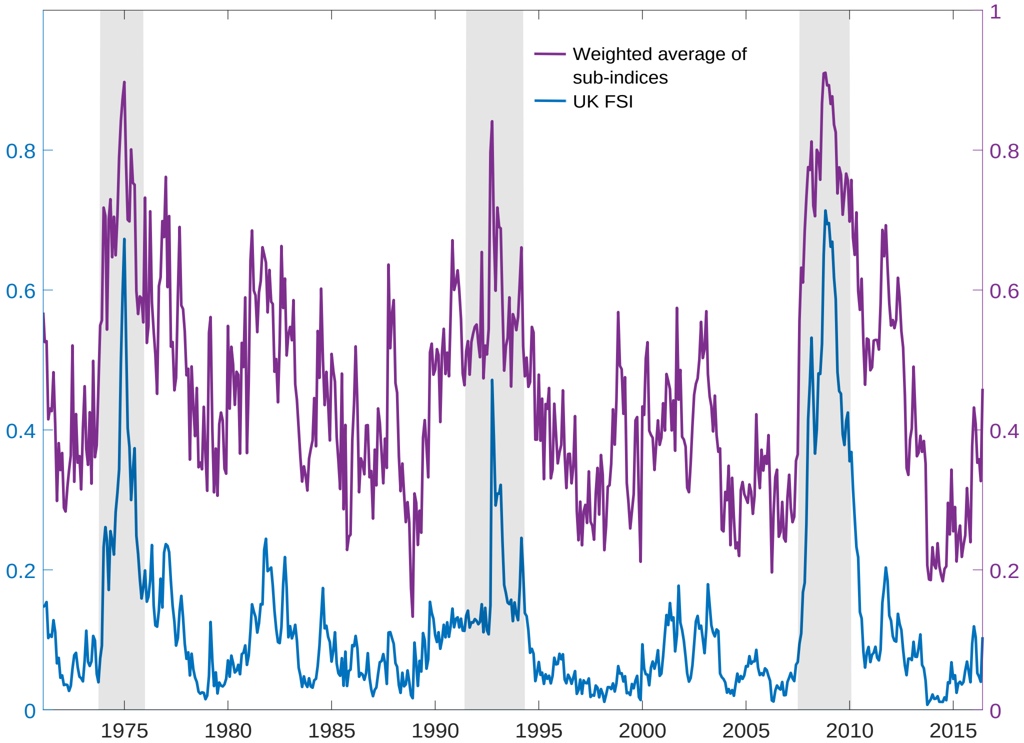
Figure A.2 shows the UKFSI and its counterpart without using correlation weights used in Duprey et al. (2017). Calculating the index as a simple average of the six sub-indices implicitly assumes perfect correlation across all sub-indices at all times. Both peak at the same time, but the simple average features a relatively high level of financial stress even during non-crisis periods. These characteristics are not consistent with narratives on the intensity of stress during crisis/non-crises.

Figure A.1: UKFSI and corporate bond spreads



Notes: This figure compares the UKFSI with the corporate bond spreads series. The left axis corresponds to the scale of the UKFSI (in blue), whereas the right refers to the scale of corporate bond spreads (in orange). The shaded vertical bars correspond to the years during which the UK economy experienced a financial crisis in our sample period.

Figure A.2: UKFSI versus simple weighted average of sub-indices



Notes: The figure compares the UKFSI with a variant of the index using simple weighted average of sub-indices. The shaded vertical bars correspond to the years during which the UK economy experienced a financial crisis in our sample period.

B The Gibbs Sampling Algorithm for TVAR

The following describes the Gibbs sampler for TVAR:

1. Given a value for the threshold variable, observations are separated into two regimes.

2. Given the observations in each regime, sample the VAR coefficients and covariances:

   - Sampling the VAR coefficients

     Given the regimes and the covariances, we sample $c_R, B_{1,R}, B_{2,R}, \ldots, \beta_{L,R}$.

     As discussed in Appendix C, the conditional posterior of the VAR coefficients is

     $$vec(\beta) \mid \Omega_R, Y \sim N(vec(\tilde{\beta}), \Omega_R \otimes (X^{*'} X^*)^{-1}). \tag{B.1}$$

   - Sampling the covariances

     Given the regimes, we sample the covariance matrices from inverse Wishart distribution which is discussed in Appendix C,

$$\Omega_R \mid \beta \sim iW(\bar{H}_R, \varphi_R)$$

where $R = 1, 2$. $\bar{H}_R$ refers to the covariance matrix computed by the VAR residuals in regime R, and the parameter φ_R refers to the number of the observations in each regime.

3. Given values for coefficients and covariances, sample the threshold value. Since the posterior distribution of the threshold value is not analytically tractable, we perform a Metropolis Hastings step, along with the Gibbs sampler:

$$Z^*_{new} = Z^*_{old} + \Psi^{1/2}\varepsilon \tag{B.2}$$

where $\Psi^{1/2}$ is a scaling factor and ε is distributed as $N(0, 1)$. The scaling factor is chosen to ensure that the acceptance rate is in 20–40% interval.

4. Conditional on the threshold value, we sample the delay parameter d. Chen and Lee (1995) showed that the conditional posterior density of this parameter is multinomial distribution with probability $\frac{L(Y_t)}{\sum^d L(Y_t)}$ where $L(.)$ is the likelihood function. We skip this step when fixing d to our desired values based on our definitions of recessionary regimes in TVAR-Y.

5. Run 100,000 draws and discard the first 60,000 to ensure convergence.

C Normal Inverse Wishart priors

We impose normal inverse Wishart (natural conjugate) priors through dummy observations following Bańbura et al. (2010) for *each regime* of the regime-switching as well as for the linear VAR model. Consider artificial data denoted Y_D and X_D such that

$$b_0 = \left(X'_D X_D\right)^{-1} \left(X'_D Y_D\right)$$

$$S = (Y_D - X_D b_0)' (Y_D - X_D b_0)$$

A regression of Y_D on X_D will give b_0, the prior means for the VAR coefficients, and the sum of squared residuals give S, prior scale matrix of the error covariance matrix. The prior is of normal inverse Wishart form

$$p\left(\widetilde{B}|\widetilde{\Sigma}\right) \sim N\left(\widetilde{b}_0, \widetilde{\Sigma} \otimes \left(X'_D X_D\right)^{-1}\right)$$

$$p\left(\widetilde{\Sigma}\right) \sim IW\left(S, T_D - K\right)$$

where $\widetilde{b}_0 = vec\left(b_0\right)$, T_D is the length of the artificial data and K denotes the number of regressors in each equation. Given this artificial data, and denoting $Y^* = [Y; Y_D], X^* = [X; X_D]$, where Y and X respectively denote the regressands and regressors from the data, the conditional posterior distribution for the VAR parameters are given by

$$H\left(b|\Sigma, Y_t\right) \sim N\left(vec\left(B^*\right), \Sigma \otimes \left(X^{*'}X^*\right)^{-1}\right) \tag{C.1}$$

$$H\left(\Sigma|b, Y_t\right) \sim IW\left(S^*, T^*\right) \tag{C.2}$$

where $B^* = \left(X^{*'}X^*\right)^{-1}\left(X^{*'}Y^*\right)$, $S^* = \left(Y^* - X^*B^*\right)'\left(Y^* - X^*B^*\right)$ and T^* refers to the length of Y^*.

To implement the normal inverse Wishart priors, artificial data are created as follows:

$$Y_D = \begin{pmatrix} \frac{diag(\chi_1\sigma_1...\chi_N\sigma_N)}{\tau} \\ 0_{N\times(P-1)\times N} \\ .... \\ diag\left(\sigma_1...\sigma_N\right) \\ .... \\ 0_{1\times N} \end{pmatrix}, \; X_D = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1...\sigma_N)}{\tau} & 0_{NP\times 1} \\ 0_{N\times NP} & 0_{N\times 1} \\ ..... & ...... \\ 0_{1\times NP} & c \end{pmatrix}$$

where N represents the number of variables in the VAR, P represents the number of lags, χ_i are the prior means of the coefficients of the first lag of the dependent variables and $J_P = diag(1, ..., P)$

The hyperparameters are listed as follows:

- τ controls the overall tightness of the prior
- c controls the tightness of the prior on constants
- σ_i are standard deviation of error terms from OLS estimates of AR regression for each variable

D Generalized Impulse Response Functions

We compute the non-linear impulse response functions of the TVAR models by following Koop et al. (1996). The following steps are separately employed for each regime for both

TVAR-Y and TVAR-S models.

1. Run the estimation and save all parameter draws.
2. Given a Gibbs draw, pick a history from the set of recessionary/financially stressful observations.
3. Draw random shocks and form a set of unconditional forecasts which are denoted as y_{t+k}^{Th} where Th indicates the TVAR model and k is forecast horizon. The output is a $(horizon \times N)$ matrix of forecasts for all N variables and these forecasts serve as a baseline.
4. Form another set of forecasts with the same random shocks except that a specific shock is perturbed at horizon 0. Refer these forecasts as $y_{t+k}^{Th,p}$. The output is a $(horizon \times N \times 1)$ matrix *for a given shock*. If one is interested in shocking all the variables, the resulting matrix is size of $(horizon \times N \times N)$.
5. Repeat steps 3 to 5 for $Simm = 500$ for all possible histories in Step 2.
6. Take the means of the forecasts over $Simm$ and calculate the difference between the means such that $\frac{1}{Simm}\sum_{Simm} y_{t+k}^{Th,p} - \frac{1}{Simm}\sum_{Simm} y_{t+k}^{Th}$.
7. Repeat steps 3 to 7 for all Gibbs draws and all histories. The result of this step is the time varying impulse response functions.
8. Take the mean of the resulting impulse response functions from all Gibbs draws. The output is the ultimate GIRFs of recessionary regime in TVAR model.
9. Repeat steps 3 to 9 for the non-recessionary/financially non-stressful regimes.