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Abstract

This paper studies the effect of asset bubbles on economic growth in the presence of financial constraints and heterogeneous projects. I consider an economy with two sectors which differ in their productivity and the pledgeability of their output in financial markets. The first sector has low productivity and high levels of pledgeability (or low levels of financial constraints), whereas the second sector has higher productivity and lower levels of pledgeability. In this framework, asset bubbles raise interest rates and lower investment productivity by directing financial resources to the sector with lower financial constraints. Steady states in which asset bubbles lower investment productivity and consumption are termed *bubbly growth-traps*.

Key words: Asset bubbles, credit misallocation, inefficient investments, growth traps.

JEL classification: O11, O16, O41, O43.

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1 Introduction

There has been a surge in academic interest on asset bubbles since the last financial crisis. Many of these papers have focused on the role of financial constraints in the existence of bubbles (for example Farhi and Tirole (2011) and Martin and Ventura (2012)). In this paper, I explore how crowding-out of investments in the presence of bubbles may affect the composition of investments as well. In direct contrast to Martin and Ventura (2012), the results of my paper highlight a scenario in which asset bubbles lower the productivity of investments in an economy.

I study the impact of asset bubbles on economic growth in an economy with financial constraints and heterogeneous projects. Financial constraints are measured by the pledgeability of a project's output in financial markets. The economy has two sectors that absorb the net wealth of the economy every period. The economy has a traditional sector with low levels of productivity and, by virtue of being perceived as a stable, mature industry, low levels of financial constraints. There is also a modern sector which despite having a higher level of productivity faces a higher financial constraint.

Economic growth in this economy takes place in phases. At low levels of income (or wealth, the two are equivalent in the model), a large fraction of the financial resources are intermediated to the traditional sector since interest rates are high and the borrowing constraints bind tightly for the more financially constrained modern sector. As wealth increases and interest rates decline, the borrowing constraints of the modern sector are relaxed since projects need to borrow lower amounts from the credit markets. Thus at high levels of wealth, a larger fraction of financial resources is intermediated to the modern sector and the economy experiences a sharp pickup in economic growth driven by increases in investment productivity.

In my model, an increase in financial constraints lowers equilibrium interest rates and creates conditions for the existence of asset bubbles.[1] When they exist, asset bubbles raise interest rates and crowd-out financial resources allocated to investments, lowering future wealth. The combination of higher interest rates and lower wealth in the presence of asset bubbles further tightens the borrowing constraint faced by the modern sector. I show that in economies which otherwise naturally grow out of their low investment productivity phase, asset bubbles can impede wealth accumulation to the extent of ruling out the said transition. Such asset bubbles guide the economy to steady states with lower investment productivity and lower average consumption. These steady states, which only exist in the presence of bubbles, I call *bubbly growth-traps*.

[1] I find a necessary condition for the existence of bubbles similar to Farhi and Tirole (2011).

The lower investment productivity that is a hallmark of these *growth-traps* can account for long and deep recessions that have followed recent lending booms, particularly those in the housing sector. The key assumption necessary for the existence of such *misallocative* bubbles is the negative correlation between the productivity and financial pledgeability of the two sectors of the economy.[2] I argue that this assumption is also the empirically relevant case since if the productivity and financial pledgeability were positively correlated, the composition of credit or investments would be an irrelevant metric, with or without bubbles.

Related Literature. Samuelson (1958) and Tirole (1985) were among the first to show the potential for the existence of asset bubbles in the presence of dynamic inefficiency. They showed that asset bubbles help overcome dynamic inefficiency to boost consumption by crowding-out the inefficient investments. Tirole (1985) explored the notion of dynamic inefficiency in an economy where the return to capital being lower than the growth rate is a symptom of overinvestment. Cass (1972) and Zilcha (1990) are related contributions in the literature which suggest tests to identify capital overaccumulation in random capital chains given the lack of 'market signals or market adjustment mechanisms' which could suggest the existence of overinvestment or dynamic inefficiency.[3]

These papers predict a positive contribution of asset bubbles to welfare since they crowd-out inefficient investments to boost consumption and therefore welfare. Subsequent contributions have further bolstered the reputation of asset bubbles as a source of outside liquidity (Farhi and Tirole (2011)), as a means to facilitate transfer of resources from savers to financially constrained productive investors (Martin and Ventura (2012)) and as a source of collateral (Miao and Wang (2012) and Martin and Ventura (2016)). These papers have a largely sanguine view of the role of asset bubbles in alleviating financial constraints.[4]

To the best of my knowledge, my paper is the first to highlight the negative welfare consequences of asset bubbles through their effect on investment composition. I show that asset bubbles that exist in the presence of financial constraints not only crowd-out investments, but may also lower the average productivity of investments by crowding-out

[2] I also assume that every project requires a lumpy unit investment a la Matsuyama (2007) and that there is a finite measure of each project type.

[3] Cass (1972), page 220. Cass (1972) identifies overinvestment in a given capital chain as a situation where long-run inefficiency results from inter-temporal decisions that are short-run efficient.

[4] In Martin and Ventura (2012), bubbles crowd-in investments by transferring valuable investment resources from savers to constrained productive investors. Asset bubbles may crowd-out investments when they are sufficiently large, in which case they may or may not be welfare improving.

highly productive, albeit financially constrained, industries. The economy presented in my model neither benefits from the crowding-out of inefficient investments (a la Tirole (1985)) or crowding-in of productive investments (a la Farhi and Tirole (2011) or Martin and Ventura (2012)) in the presence of asset bubbles.

I am not unique in highlighting potential negative effects of asset bubbles on the long-run efficiency of investments, though the underlying source of inefficiency is unique to my paper. In endogenous growth models, as shown in Grossman and Yanagawa (1993) and Saint-Paul (1992), social returns to investments are higher than private returns owing to the positive externality of R&D on aggregate productivity. In these models, asset bubbles can exist when private returns are lower than the growth rate of the economy (a la Tirole (1985)) even though social returns to investments might be higher than the growth rate of the economy. While endogenous growth models rely on the externality effects of investments, my model relies on financial constraints to show how bubbles misallocate financial resources in credit markets.

I split the discussion of the model into three sections. I introduce the model in section 2 and discuss the endogenous credit composition resulting from heterogeneous projects in section 3. I introduce asset bubbles in the model in section 4.1 and discuss the role of financial constraints in creating theoretical conditions for the existence of bubbles. I discuss the effect of asset bubbles on credit composition and investment productivity in section 4.2. I conclude in section 5.

2 Model

I model a 2 period OLG (*overlapping generations*) economy where every agent derives utility only from second period consumption. The agents work in the first period and use their savings to invest in projects that convert their investments into capital. These projects are the only savings technology available to the agents. The capital resulting from a successful project is used for production in a Cobb-Douglas production function ($f(.)$) combining labour and capital. I assume that all agents have access to this production function and there is perfect competition in the goods and factor markets.

Every agent provides labor of measure 1 inelastically when young. The young of every generation work, earn wages w_t, and invest their wealth in capital projects mentioned above. There are heterogeneous projects and each agent can originate a project specific to their type, indicated by i. The measure of each type in the population is represented by Δ_i. Since the total population measure is 1, $\sum_i \Delta_i = 1$ holds.

The project initiated by an agent of type i is characterized by its, *Productivity* R_i, where R_i is the amount of capital resulting from each unit invested in the project. Without loss of generality, agents are numbered such that $R_1 > R_2 > ... > R_n$.

Pledgeability λ_i, where λ_i is the fraction of the project output that can be pledged to a potential lender. This parameter captures the tangibility of the capital when deployed in a firm. The distribution of this parameter among the agent types is a key driver of the results of the model.

Size 1, where 1 is the investment required to initiate a project.[5]

These capital projects are similar to the ones studied in Matsuyama (2007). Where as Matsuyama (2007) has a homogeneous agent who chooses from a set of possible projects, I embed project heterogeneity among the young of the population. While only one project is funded under equilibrium in Matsuyama (2007), the heterogeneity among the agents leads to an endogenously evolving credit composition where multiple project types can be funded under equilibrium.

Under general equilibrium, the market clearing rates are the credit market interest rate (r_t or $r(k_t)$) and the rental rates for capital (ρ_t or $\rho(k_t)$) and labour (w_t or $w(k_t)$). Since k_t is the level of capital resulting from investments in period $t-1$, the credit market interest rate and rental rate for capital at time t are r_{t+1} and k_{t+1} respectively. While the credit market interest rate is discussed in the next section, the latter two rates are determined competitively as[6],

$$\rho_t = \rho(k_t) = f'(k_t) \tag{1}$$

$$w_t = w(k_t) = f(k_t) - k_t f'(k_t) \tag{2}$$

Let $\{X_i\}_{i\epsilon\mathbb{N}}$ are the measures of the different projects initiated at time period t. The next period capital is a function of the individual project measures and given by,

$$k_{t+1} = \sum_i X_i R_i \tag{3}$$

, where X_i is a function of k_t.

[5] Matsuyama (2007) considers the effect of project size heterogeneity in project selection in the credit markets. In a credit market the qualitative effect of project size is the same as that of financial constraints and I assume that project size is the same across the agents for tractability.

[6] Since the labour supply equal measure 1 every period, the equilibrium conditions are reflected in terms of capital per labour k_t.

These capital projects are the only investment opportunities for the young. Hence, the total wealth of the young is allocated to these projects. More formally,

$$w(k_t) = \sum_i X_i(k_t) \tag{4}$$

At any time t, equilibrium is determined by the market clearing rates $(r_{t+1}, w_t, \rho_{t+1})$ and the measure of funded project types $\{X_i\}_{i\epsilon\mathbb{N}}$. The following section explains equilibrium conditions to derive these rates and discusses the resulting law of motion of capital.

3 Equilibria with heterogeneous projects

3.1 Equilibria without financial frictions

I first consider equilibria in an economy with two types of agents who are financially unconstrained, i.e. $\lambda_1 = \lambda_2 = 1$. Since the law of motion of capital is a direct outcome of the composition of investments in any given period, I begin with a discussion on the role of the credit market in determining the said composition.

As described in section 2, r_{t+1} is the credit market interest rate at period t. Every agent who borrows $1 - w_t$ to initiate a unit size capital project pays an interest rate of r_{t+1} on the borrowed amount. In return, the agent enjoys a return of $\rho_{t+1}R_i$ next period. An agent of type i would invest in a project if and only if investing in the project pays more than lending in the credit market. The participation constraint is thus,

$$\rho_{t+1}R_i - r_{t+1}(1 - w_t) \geq r_{t+1}w_t \tag{5}$$

The above condition reduces to,

$$\rho_{t+1}R_i \geq r_{t+1} \tag{6}$$

which implies that projects are funded solely on the basis of their productivity. This result is intuitive since the agents have been assumed to be financially unconstrained. I will relax this condition in the next sub-section when we consider specific financial constraints for each project type.

Equation 6 captures the simplicity of this case. At equilibrium, the more productive agent can always offer a higher return than the less productive type. Hence it is not possible that a less productive type can initiate a project profitably while there are unfunded projects with higher productivity. Even though the projects are funded in order of their productivity, the measure of each type funded under equilibrium depends on the net wealth of the economy. As an illustration, if the wealth of the economy (w_t) is less than the measure of available high productive projects(Δ_1), all the resources of the economy are channeled to type 1 projects. This imposes that equation 6 must hold with equality, or $\rho_{t+1}R_1 = r_{t+1}$. The project measures funded at equilibrium when $w_t < \Delta_1$ are given by $X_1 = w_t$, $X_2 = 0$. If on the other hand $w_t > \Delta_1$, the credit market interest rate r_{t+1} ensures that type 2 agents are indifferent between lending or initiating a project, i.e. $r_{t+1} = \rho_{t+1}R_2$. The project measures funded at equilibrium in this second case are given by $X_1 = \Delta_1$, $X_2 = w_t - \Delta_1$.

Thus, the measure of funded projects $\{X_1, X_2\}$ and credit market interest rate (r_{t+1}) can be characterized as a function of the level of capital (or wealth w_t) in the economy, as shown in the conditions below. A similar argument can be used to characterize the credit market interest rate in an economy with more than 2 agents where the level of financial constraints are in the same rank-order as project productivities.

$$(X_1, X_2) = \begin{cases} (w_t, 0) & \text{if } w_t \leq \Delta_1 \\ (\Delta_1, \Delta_1 - w_t) & \text{if } w_t > \Delta_1 \end{cases}$$

and

$$r_{t+1} = \begin{cases} \rho_{t+1} \cdot R_1 & \text{if } w_t \leq \Delta_1 \\ \rho_{t+1} \cdot R_2 & \text{if } w_t > \Delta_1 \end{cases}$$

3.2 Equilibria with financial frictions

3.2.1 Equilibrium conditions and credit composition

In section 3.1, I showed that when project productivity (R_i) is positively correlated with project pledgeability (λ_i), then higher productivity projects always prevail over lower productivity projects in the credit market. I relax this assumption in this section by studying an economy with two projects in which the more productive project has lower project pledgeability (i.e. $R_1 > R_2$ and $\lambda_1 < \lambda_2$).

Given the credit market interest rate r_{t+1} and return to capital ρ_{t+1}, an investor of type i will invest in a project rather then lend in the credit market if and only if

$$\rho_{t+1}R_i - r_{t+1}(1 - w_t) \geq r_{t+1}w_t$$

which reduces to equation 6, or $\rho_{t+1}R_i \geq r_{t+1}$.

Every agent also faces a borrowing constraint since the net amount they can borrow is restricted by the proportion of their next period wealth (λ_i) that can be pledged to the creditors. The borrowing constraint is given by

$$\lambda_i \rho_{t+1} R_i \geq r_{t+1}(1 - w_t)$$

$$\frac{\lambda_i \rho_{t+1} R_i}{(1 - w_t)} \geq r_{t+1} \tag{7}$$

The above two conditions are combined as a single expression in equation 8. A financially constrained project is funded if and only if the relation between its project parameters and the credit market interest rate is

$$\frac{\rho_{t+1} R_i}{max(1, \frac{1-w_t}{\lambda_i})} \geq r_{t+1} \tag{8}$$

A natural consequence of the above condition is that agents with the higher $R_i/max(1,(1-w_t)/\lambda_i)$ will be the first to initiate a project for a given level of k_t. $R_i/max(1,(1-w_t)/\lambda_i)$ is a function of k_t and gives rise to a unique ordering of this parameter for the different $i's$ for a given k_t. To clarify this point, in figure 1, I plot the pledgeability adjusted productivity factor for two agents whose parameters are shown in table 1.

Table 1: Parameter values for figure 1

R_1	1.3	R_2	0.9
λ_1	0.38	λ_2	0.8
Δ_1	0.58	Δ_2	0.35

The equilibrium for the case with financial constraints can be characterized in a similar fashion to the simpler case in section 3.1. The key difference is that, at each level of k_t, the credit market equilibrium is determined by the pledgeability adjusted productivity factor of the projects and not just their productivities.

With an increase in k_t the rank order of the pledgeability adjusted productivity indices can undergo changes. For example in figure 1, for low values of k_t project owners have to borrow larger amounts from credit market to initiate the projects. As a result, the less constrained type 2 projects are funded before the more productive type 1 projects. As k_t increases, agents have to borrow lesser amounts from the credit market and the individual project-specfic financial constraints play a lesser role in project selection. It is relatively straight forward to show that the economy transitions from primarily funding the less productive projects at low levels of capital to funding type 1 projects at the threshold value $\bar{k} = 1 - \lambda_1 R_1/R_2$ (see appendix 6.1).

Thus market clearing in the credit markets leads to an endogenous credit composition at different levels of wealth in the economy. When $\lambda_1 R_1 < \lambda_2 R_2$, the credit market funds the type 2 projects at low levels of capital and then switches to funding the higher productivity projects for $k_t > \bar{k}$ (right panel of figure 2). When $\lambda_1 R_1 > \lambda_2 R_2$[7], the difference in the pledgeabilities of the two project types is not high enough for type 2 projects to be preferred over type 1 projects at any level of capital (left panel of 2).

Assumption I - In the rest of the paper, I consider the case of an economy which satisfies $\lambda_1 R_1 < \lambda_2 R_2$ (right panel of figure 2).

3.2.2 Law of motion of capital

In this sub-section, I study the implications of the evolving credit market composition on capital accumulation in an economy with two types of agents which satisfied assumption I ($R_1 > R_2$ and $\lambda_1 R_1 < \lambda_2 R_2$).

Depending upon how the credit markets clear, agents find themselves playing one of 3 roles in the credit market. These roles -

Supra-marginal investors - agents who strictly prefer borrowing to lending in the credit market.

Marginal investors - agents who are indifferent between borrowing and lending at the market clearing interest rate.

Lenders - agents who provide the credit intermediated to supra-marginal and marginal investors.

In the simple set-up with two types of agents, there are 4 types of equilibria depending upon the supra-marginal and marginal investors. At low levels of capital ($k_t < \bar{k}$), the type 2 agents take precedence over type 1 agents in the credit market since they can can commit to a higher interest rate than type 1 agents at low levels of capital (note that when $k_t < \bar{k}$, $R_2/max(1,(1-w_t)/\lambda_2) > R_1/max(1,(1-w_t)/\lambda_1)$). In the region $k_t < \bar{k}$, we may have either $w_t < \Delta_2$ or $w_t > \Delta_2$, where Δ_2 is the measure of type 2 agents. In case $w_t < \Delta_2$, there are enough type 2 projects to absorb the net wealth of the economy and and only type 2 projects are funded in the credit market. In case $k_t < \bar{k}$ and $w_t > \Delta_2$, all type 2 agents and some type 1 agents are funded. Type 2 agents act as supra-marginal

[7]When $\lambda_1 R_1 > \lambda_2 R_2$, the more productive type 1 projects are funded before the type 2 projects for all values of k_t.

investors and type 1 agents act as either lenders or marginal investors. These are the two type of equilibria that can exist when $k_t < \bar{k}$. I refer to these equilibria as A and B respectively.

Another set of equilibria can exist when $k_t > \bar{k}$. In this region, $R_1/max(1,(1-w_t)/\lambda_1) > R_2/max(1,(1-w_t)/\lambda_2)$. If $w_t < \Delta_1$, type1 projects absorb all the financial resources intermediated in the credit market. If $w_t > \Delta_1$, type 2 agents enter the credit market as marginal investors after all type 1 projects have been funded. We refer to these equilibria as C and D respectively.

As described above, the law of motion of capital and the equilibrium credit market interest rate are determined as per the following conditions :

$$(X_1, X_2) = \begin{cases} (0, w_t) & \text{if } w_t \leq \bar{k} \text{ and } w_t \leq \Delta_1 \text{ (A)} \\ (w_t - \Delta_2, \Delta_2) & \text{if } w_t \leq \bar{k} \text{ and } w_t > \Delta_1 \text{ (B)} \\ (w_t, 0) & \text{if } w_t > \bar{k} \text{ and } w_t \leq \Delta_2 \text{ (C)} \\ (\Delta_1, w_t - \Delta_1) & \text{if } w_t > \bar{k} \text{ and } w_t > \Delta_2 \text{ (D)} \end{cases}$$

and

$$r_{t+1} = \begin{cases} \frac{\rho_{t+1} \cdot R_2}{max(1, \frac{1-w_t}{\lambda_2})} & \text{if } w_t \leq \bar{k} \text{ and } w_t \leq \Delta_1 \\ \frac{\rho_{t+1} \cdot R_1}{max(1, \frac{1-w_t}{\lambda_1})} & \text{if } w_t \leq \bar{k} \text{ and } w_t > \Delta_1 \\ \frac{\rho_{t+1} \cdot R_1}{max(1, \frac{1-w_t}{\lambda_1})} & \text{if } w_t > \bar{k} \text{ and } w_t \leq \Delta_2 \\ \frac{\rho_{t+1} \cdot R_2}{max(1, \frac{1-w_t}{\lambda_2})} & \text{if } w_t > \bar{k} \text{ and } w_t > \Delta_2 \end{cases}$$

Therefore, the economy goes through different phases of economic growth based on the changing composition of credit. I show these phases along the law of motion of capital in figure 3. At very low levels of capital, the economy finds itself in equilibria of type A or B. When the economy crosses the threshold $\bar{k}$, the credit composition switches towards type 1 projects and the economy transitions to higher investment productivity and faster economic growth (equilibria of type C or D).

3.2.3 Steady states

The economy can find itself in one of these 4 types of equilibria at steady state. For example, in figure 3, the economy has two possible steady states - one each of type B (both types of projects are funded, type 2 projects are the supra-marginal projects) and C (only type 1 projects are funded). The viable steady states depend upon the parameters of the economy - $\alpha, \{\lambda_i, R_i, \Delta_i\}_{i=1,2}$. It is the interaction of these parameters that determines the actual steady states of the economy.

For example a steady state in which the economy finds itself in equilibria of type A is characterized by a solution to the equation

$$k_A^* = R_2 \cdot (1-\alpha) \cdot k_A^{*\alpha}$$

Lemma 1A A steady state of type A can exist if and only if the candidate solution k_A^* satisfies $k_A^* < \bar{k}$ and $(1-\alpha)k_A^{*\alpha} < \Delta_2$.

Proof. The two conditions ensure that the steady state value when only type 2 projects are invested is indeed feasible. The first condition ensures that type 2 projects are selected before type 1 projects in the credit market. The second condition ensures that there are enough type 2 projects to absorb the total wealth of the economy at steady state[8].

The existence condition for a steady state in which the economy is in a type B equilibria (steady state in which both project types are funded, type 2 projects are supra-marginal) can be characterized in a similar way. k_B^* is a solution to the equation

$$k_B^* = R_2 \cdot \Delta_2 + [(1-\alpha) \cdot k_B^{*\alpha} - \Delta_2] \cdot R_1$$

Lemma 1B A steady state of type A can exist if and only if the candidate solution k_B^* satisfies $k_B^* < \bar{k}$ and $R_1(1-\alpha)\bar{k}^\alpha - \bar{k} < \Delta_2(R_1 - R_2)$.

We can similarly state the candidate solutions for steady states of type C and type D and the necessary conditions for their existence. Steady state of type C (k_C^*, where only type 1 projects are funded) is a solution to the equation

$$k_C^* = R_1 \cdot (1-\alpha) \cdot k_C^{*\alpha}$$

and k_C^* is feasible if and only if $k_C^* > \bar{k}$ and $(1-\alpha)k_C^{*\alpha} < \Delta_1$. Finally, the candidate steady state of type D (k_D^*, both project types are funded, type 1 projects are supra-marginal) is a solution to the equation

$$k_D^* = R_1 \cdot \Delta_1 + [(1-\alpha) \cdot k_D^{*\alpha} - \Delta_1] \cdot R_2$$

and exists if and only if $k_D^* > \bar{k}$ and $\bar{k} - R_2(1-\alpha)\bar{k}^\alpha < \Delta_1(R_1 - R_2)$.

In figure 4, I illustrate two economies which have multiple steady states of the types

[8]The two investment projects in the economy continue to satisfy $R_2\lambda_2 > R_1\lambda_1$, as in all other sections on the paper.

described in this subsection. On the left panel I show an economy which has two steady states - of type B and type C. The economy in the right panel has a unique steady state of type C. To structure an understanding of these steady states, proposition I encapsulates basic properties of these steady states.

Proposition I The same economy can have at most one steady state of each type and the candidate solutions can be ordered as $k_A^* < k_B^* < k_C^* < k_D^*$. Moreover, all the steady states can co-exist except C and D which preclude the existence of each other.

4 Equilibria with bubbles

In section 3.2, introducing financial frictions into the model added interesting dynamics in the process of capital accumulation, with endogenous credit composition as a function of wealth and multiple steady states. I introduce bubbles in this stylized model to study the impact of asset bubbles in the credit market. I generate two interesting theoretical results. Firstly, I show that financial constraints create conditions for the existence of asset bubbles by lowering equilibrium interest rates in the credit market. Hence the existence of asset bubbles in this economy do not necessarily suggest the presence of dynamic inefficiency in investments. Secondly, I characterize economies in which bubbles not only crowd-out investments but also lower the average investment productivity. The lowering of investment productivity results from the diversion of credit from high to low productivity projects (or sectors) of the economy. The reduction in investment productivity also lowers average consumption. To the best of my knowledge, my paper is unique in highlighting negative welfare consequences of asset bubbles through their effect on credit composition.

I study conditions under which the financial resources in the credit market may be intermediated to assets that are bought and sold for the sole purpose of a financial return without those resources being used for any productive activity. Such assets can exist in a credit market provided (i) there exists a buyer for the asset in the next period[9] and (ii) the asset promises a return at least as high as the credit market interest rate. As is customary in this literature, I call these assets *bubbles.* In fact, any difference in the credit market interest rate and returns on holding the bubble can not comprise a market equilibrium since it would lead to a rush towards the asset promising a higher return. This conditions helps to establish the expected return from the bubble at period t as specified below.

$$E_t \frac{b_{t+1}}{b_t} = r_{t+1}^b \tag{9}$$

where, b_t is the value of the bubbles in the current period and b_{t+1} is the value of the

[9]At any rate, buyers of such an asset in a given period must be convinced that future buyers will relieve them of the asset in the subsequent period.

bubbles in the next period. r^b_{t+1} is the credit market interest rate in the presence of the bubble. The rest of the equilibrium conditions are similar to the ones described for the equilibria without bubbles.

4.1 Bubbly Equilibria with one project type

I start with the case of an economy with only one type of agent who can invest in homogeneous projects of unit size which generate R units of capital. The pledgeability of these projects is given by $\lambda < 1$.[10] The discussion of the one project case helps to build intuition for the role of financial constraints in creating conditions in which asset bubbles can exist in this simple economy. I discuss the role of asset bubbles in the economy with heterogeneous projects in the next sub-section.

At time t, agents prefer to invest in capital projects provided $\rho_{t+1}R \geq r_{t+1}$. Further, the borrowing constraint (discussed in section 3.1) is given by $\lambda\rho_{t+1}R \geq r_{t+1}(1-w_t)$. Since there is only one type of agent in the economy, every agent must be indifferent between borrowing and lending in the credit market. Hence, the relation between the credit market interest rate and the rental rates of labour and capital is determined by which ever of the two constraints binds at equilibrium, i.e., $r_{t+1} = \rho_{t+1}R/max(1,(1-w_t)/\lambda)$.

The existence condition for bubbles when the projects are not financially constraint is equivalent to the one in Tirole (1985). In this paper I restrict the discussion regarding the effect of asset bubbles when the agents are financially constrained. I discuss this condition in assumption 2.

Assumption II. The borrowing constraint is binding at the natural steady state. This condition can be formally expressed as $max(1,(1-w^*)/\lambda) = (1-w^*)/\lambda \Rightarrow (1-\alpha)\cdot [R(1-\alpha)]^{\frac{\alpha}{1-\alpha}} < 1-\lambda$.

This condition ensures that the investors are financially constrained at steady state. This is more likely to hold true for low values of the pledgeability parameter λ. The steady state level of capital is given by $k^* = [R(1-\alpha)]^{\frac{1}{1-\alpha}}$ and the steady state credit market interest rate is given by $r^* = \rho^* R\lambda/(1-w^*)$. All the results in the subsequent sections apply in the parameter space which satisfy assumptions I and II.

4.1.1 Existence condition for bubbly steady states

Asset bubbles divert resources away from productive investments and are bought purely because they promise an interest rate equal to the credit market interest rate. They are indistinguishable from other assets in the credit market. In this section I dis-

[10] This economy is a special case of the economy presented in section 3.2, with $R_1 = R_2 = R$ and $\lambda_1 = \lambda_2 = \lambda$.

cuss the existence of steady state levels of capital k_b^*, credit market interest rate r_b^* and aggregate bubble size b^* in the economy described here[11]. I presented the governing equations for the natural steady-state levels of capital and credit market interest rate, k^* and r^* respectively, in the earlier sub-section. The governing equations for the steady state levels of $\{r_b^*, k_b^*, b^*\}$ in the presence of a bubble are given by,

$$r_b^* = R\lambda \cdot \frac{\alpha k_b^{*\alpha-1}}{1-(1-\alpha)\cdot k_b^{*\alpha}} = 1 \tag{10}$$

and

$$k_b^* = R \cdot [(1-\alpha)\cdot k_b^{*\alpha} - b^*] \tag{11}$$

The first condition ensures a constant bubble size given the economy under consideration does not have any economic growth at steady state[12]. The second equation is the law of motion of capital given that the net wealth of the economy held by wage earners is allocated into both bubbles and capital by the credit market. I characterize the existence of bubbly steady states in terms of the model parameters in proposition I.

Proposition II The economy with a homogeneous agent can have steady states with credit market asset bubbles if the capital projects satisfy $R\lambda < 1$. No asset bubbles can exist if $R\lambda > 1$. Further, $R\lambda < 1$ is a necessary, but not a sufficient condition for the existence of equilibria with asset bubbles. The sufficient conditions can be characterized based on the fundamental steady state values of capital and credit market interest rates, k^* and r^* in the following way -

a. The economy has at least one steady state in the presence of asset price bubbles if $r^* < 1$.

b. In case $r^* > 1$, the economy has at least two steady states in the presence of asset bubbles if and only if $k^* > 1$. There are no steady states in the presence of asset bubbles if $r^* > 1$ and $k^* < 1$.

Proof. At steady state, the growth rate of the bubble must equate the growth rate of the economy at steady state, which is 1 in the current model. The necessary condition of $R\lambda < 1$ follows directly from equation 10 which states this condition formally. The function $g(k) = \alpha k^{\alpha-1}/(1-(1-\alpha)k^{\alpha})$ has a global minima at $k = 1$, with $min(g(k)) = 1$. Similarly, $min(R\lambda \cdot \alpha k^{*\alpha-1}/(1-(1-\alpha)k^{*\alpha})) = R\lambda$. In case $R\lambda > 1$, the equations 10 and 11 do not have a real solution in k_b^* and b^* where the equilibrium credit market interest rate can equal 1. Therefore asset bubbles can exist at steady state only if $R\lambda < 1$.

[11]The sub-script $_b$ refers to the steady state values in the presence of bubbles. The steady state values in the absence of bubbles, the natural or fundamental values, are reflected without the sub-script.

[12]Since there is no exogenous or endogenous change in productivity or population in the model.

This result suggests a clear link between financial constraints and the existence of asset bubbles.

$R\lambda < 1$ is not a sufficient condition for the existence of bubbles. The steady state levels of capital and credit market interest rate under fundamental equilibria provide an intuition for the scenarios in which asset bubbles can exist at steady state. For example, there exists at least one bubbly steady state when $r^* < 1$. The existence of such a steady state can be shown using equation 11 where it can be shown that there exists a $k_b^* < k^*$ such that $R\lambda \cdot \alpha k_b^{*\alpha-1}/(1-(1-\alpha)\cdot k_b^{*\alpha}) = 1$ for a given b^*. Further, it is relatively straight forward to show that such a bubble b^* satisfies $0 < b^* < (1-\alpha)\cdot k_b^{*\alpha}$. Hence this pair $\{k_b^*, b^*\}$ qualifies as a feasible bubbly steady-state.

In case of $R\lambda < 1$, $r^* > 1$ and $k^* > 1$, there exist atleast two candidate solutions $k_{b_1}^* < k^*$ and $k_{b_2}^* < k^*$ that satisfy $R\lambda \cdot \alpha k_b^{*\alpha-1}/(1-(1-\alpha)\cdot k_b^{*\alpha}) = 1$ for two different values of bubbles b_1^* and b_2^* respectively. The two candidate steady state solutions are possible because in each case the condition that bubble can not be larger than the net wealth of the economy at steady state, i.e. $0 < b^* < (1-\alpha)\cdot k_b^{*\alpha}$, is shown to hold. Finally, in case $R\lambda > 1$, $r^* > 1$ and $k^* < 1$, there are no possible solutions $k_b^* < k^*$ which satisfy $R\lambda \cdot \alpha k_b^{*\alpha-1}/(1-(1-\alpha)\cdot k_b^{*\alpha}) = 1$. Since the steady state level of capital in the presence of bubbles can not be larger than the natural steady-state level of capital, the existence of asset bubbles can be ruled out in this final case. I discuss the stability property of these different steady states in Appendix 3 *q.e.d.*

Figure 5 plots the function $g(k) = \alpha k^{\alpha-1}/(1-(1-\alpha)k^{\alpha})$ with respect to k and highlights how a reduction in the level of financial constraints for the investment projects can give rise to asset bubbles in an economy. The homogeneous project in the economy in the represented in left panel does not satisfy $R\cdot\lambda < 1$. There is no level of asset bubbles which can lead the economy to a steady state in which the credit market interest rate can equal 1. The economy represented in the right panel of figure 5 has investment projects with the same productivity as the economy in the left panel but much higher levels of financial constraints. In particular, the projects in this economy satisfy $R\lambda < 1$. The credit market interest rate in the fundamental steady state for this economy is $r^* < 1$ (not shown in the figure). Crowding-out of investments in the presence of asset bubbles raises the interest rates until the credit market interest rate equals 1. Hence, asset bubbles can exist in the economy in the right panel of figure 5.

The above results clarify the crucial role that financial constraints play in the existence of bubbles in the economy with homogeneous projects. In the presence of financial frictions asset bubbles may exist even when the natural steady-state credit market interest rate is not less than 1, the standard condition for dynamic inefficiency.

4.2 Bubbly equilibria with two project types

I consider the possibility of asset bubbles in an economy with two types of projects, as were discussed in section 3.2. There exist projects of type 1 that are productive but financially constrained and projects of type 2 that are unproductive and financially unconstrained ($R_1\lambda_1 < R_2\lambda_2$). I study whether the presence of asset bubbles can affect the steady states of such an economy.

Bubbles play two roles under equilibrium. Firstly, they transfer resources costlessly across generations. This inter-generational transfer of resources is responsible for increased consumption and credit booms that bubble models in economic literature try to replicate. Bubbles crowd-out investments and lower the law of motion of capital, possibly affecting the steady state of the economy. It is the second role that is unique to this model. In the current model, bubbles can crowd-out productive investments to the extent that they can lead to steady states in which the unproductive investments are funded even though such a steady state is not possible in the absence of steady states. I call these steady states bubbly growth-traps. Before we study the growth traps, I present the equilibrium equations that govern the law of motion of capital in the presence of bubbles. Let at time t, the capital level of the economy is k_t^b and the value of bubbles is b_t. Equilibrium is given by $\{r_{t+1}^b, w_{t+1}^b, \rho_{t+1}^b\}$ such that r_{t+1}^b clears the credit market in period t and the following equations hold,

No arbitrage condition - $b_{t+1}/b_t = r_{t+1}$

Credit market interest rate - $r_{t+1} = R_1\rho_{t+1}/max(1, 1 - w(k_t)/\lambda_i)$ (where i represents the marginal investors)

Resource constraint - $b_t < w(k_t) \cdot Measure\ of\ lenders$ (note - this measure is determined endogenously)

Law of motion of capital - $k_{t+1}^b = \sum_i X_i^b R_i$, where X_i^b is the measure of type i that initiates capital projects under credit market equilibrium.

4.2.1 Existence condition for bubbly growth-traps

The steady states in the presence of bubbles can potentially be very different from the fundamental steady states of the economy in the presence of heterogeneous projects. For example, in the economy represented by the left hand panel of figure 4, there are no fundamental steady states of either type A or B, where type 2 agents (the unproductive agents with low levels of financial constraints) are the supra-marginal investors. These steady states may not occur because of various reasons. Steady states of type A can not exist if the theoretical value of k_A^* do not satisfy either $k_A^* < \bar{k}$ or $w(k_A^*) < \Delta_2$. Similarly a steady state of type B can not exist if $k_B^* < \bar{k}$ or $R_1(1-\alpha)\bar{k}^\alpha - \bar{k} < \Delta_2(R_1 - R_2)$ is not satisfied (details in section 3.2.3). First I discuss how asset bubbles relax the necessary conditions for the existence of steady states in my model in Lemma 1 and then discuss the existence condition for *bubbly growth-traps*.

Lemma II - Bubbles relax the necessary conditions for the existence of steady states in which the less productive agents are the investors. Bubbles can thus generate steady states of type A or B even though these steady states do not occur as the natural consequence of capital accumulation. I call such steady states *bubbly growth-traps*.

Proof. If steady states of type A do not exist as a fundamental equilibrium, then one or both of $w(k_A^*) > w(\bar{k})$ and $w(k_A^*) > \Delta_2$ must hold true. Since $k^{A*} > k_b^{A*}$, where k_b^{A*} is the level of capital when the economy is in a bubbly steady state of type A, bubbles relax the existence conditions for bubbly steady states of type A. For example, even if $w(k^*) > w(\bar{k})$, we can have $w(k_A^{b*}) < w(\bar{k})$ since $k_A^* > k_A^{b*}$.

A similar argument holds for the steady state of type B. I consider an economy without natural steady states of type B - i.e. either one or both of $R_1(1-\alpha)\bar{k}^\alpha - \bar{k} \leq \Delta_2(R_1 - R_2)$ and $w(k^{B*}) \geq \triangle_2$ are violated. Since $k_B^* > k_B^{b*}$, we again see that bubbles relax the condition for the existence of steady states of type B. Hence even if $R_1(1-\alpha)\bar{k}^\alpha - \bar{k} < \Delta_2(R_1 - R_2)$ is not satisfied, the condition $R_1(1-\alpha)\bar{k}^\alpha - \bar{k} < \Delta_2(R_1 - R_2) + R_1 b^*$ may be satisfied in the presence of bubbles.

I mention specific conditions for the existence of these bubbly growth-traps in Proposition III.

Proposition III I consider the existence of bubbly steady states of type A or B (in which type 2 projects are financed before type 1 projects) in an economy in which these steady states do not occur naturally[13].

III A In an economy without natural steady states of type A, bubbly steady states of type A can exist provided $R_2\lambda_2 < 1$ and the the candidate solutions for the capital level and credit market interest rate of the natural steady state of type A $\{k_A^*, r_A^*\}$ satisfy the conditions in *Proposition II.*

III B In an economy without natural steady states of type B, bubbly steady states of type B can exist provided $R_1\lambda_1 < 1$ and the the candidate solutions for the capital level and credit market interest rate of the natural steady state of type B $\{k_B^*, r_B^*$ defined in section 3.2.3$\}$ satisfy the conditions in *Proposition II.*

Proof. Only type 2 projects are funded in a steady state of type A. As per *Proposition II*, $R_2\lambda_2 < 1$ is a necessary condition for the existence of type A bubbly steady states.

[13] *Lemma 1A* and *Lemma 1B* establish the existence conditions for natural steady states of type A and B respectively

In addition the candidate solutions of the natural steady state of type A help us clarify whether there exist bubbles which can transition the economy to a steady state with a credit market interest rate equal to 1.

A similar argument holds for bubbly steady states of type B in which type 1 projects are the marginal investors. The necessary condition is $R_1\lambda_1 < 1$ and the sufficiency conditions are based on the candidate solutions of the natural steady state of type B.

These bubbly growth-traps not only have a lower level of capital, but also a lower investment productivity as compared with the natural steady states of the economy. This is a direct result of the increase in interest rates and switch to the less productive sector engendered by the bubble.

Figure 6 shows a simple case in which *Proposition III* can be seen in practice. In the left panel, I show the law of motion of capital in an economy which does not have a fundamental steady state of type A. I select parameters such that bubbly steady states of type A are feasible in this economy ($R_2\lambda_2 < 1$,$r^*_A < 1$ and $k^*_A < 1$, suggesting at least one bubbly steady state of type A). The right panel of figure 6 shows the level of the bubbly steady state of type A with respect to the original fundamental steady state of type C in which only the more productive agents of type 1 invest the net wealth of the economy. Indeed, the bubbly steady state capital level k^{b*}_A is far lower than the natural steady state level of capital for the economy k^*_C.

In this economy, bubbles crowd-out real investments from the more productive investors and raise interest rates until the more productive albeit financially constrained projects completely cease to be funded. The economy stays at this low level of capital as long as the bubble exists. In the absence of the bubble, the economy naturally accumulates enough wealth so that the productive agents are not financially constrained anymore. This transition does not take place in the presence of bubbles and therefore the bubbly steady state acts a growth trap.

5 Conclusion

I studied an economy with two sectors in which asset bubbles affect the composition of investments at steady state. The economy has a productive, albeit financially constrained sector, and an unproductive sector with lower levels of financial constraints. Such an economy grows in phases since the accumulation of wealth lowers the financial constraints for the high productivity, financially constrained sector. With economic growth, the economy transitions from a low investment productivity phase to one marked by higher investment productivity, higher consumption and faster growth rates. In this economy, asset bubbles not only crowd-out investments, but also absorb valuable financial resources that (if used in real economic activity) provide the wealth essential to initiate financially constrained, productive projects in the future. I showed how the loss of financial resources to asset bubbles can rule out the transition to the high investment productivity phase when the economy never acquires the wealth necessary to make the said transition. Such a steady state with low investment productivity, which exists only in the presence of asset bubbles, is a *bubbly growth-trap*.

The model presented in this paper is also quite general in the way it can be tweaked to replicate other important contributions to the literature. I showed that asset bubbles resulting from dynamic inefficiency (a la Tirole (1985)) are a sub-set of all the possible asset bubbles in this model. While the effect of asset bubbles on credit composition in my paper is in stark contrast to the one presented in Martin and Ventura (2012), I do not believe my results contradict theirs in any way. The results in my paper emphasize the need for a careful understanding of the resources that drive the growth of an asset bubble. If the bubbles drive growth in a sector by intermediating scarce resources to a productive, financially constrained sector, the effects are likely to be different from that of an asset bubble that crowd-outs the current and future financial resources of productive industries. My results also complement those of Farhi and Tirole (2011) regarding the effect of financial constraints on the existence of bubbles. Finally, the framework in my paper broadens our understanding of pareto destroying asset bubbles by highlighting a channel distinct from the one emphasized by the literature on endogenous growth models (such as Grossman and Yanagawa (1993)).

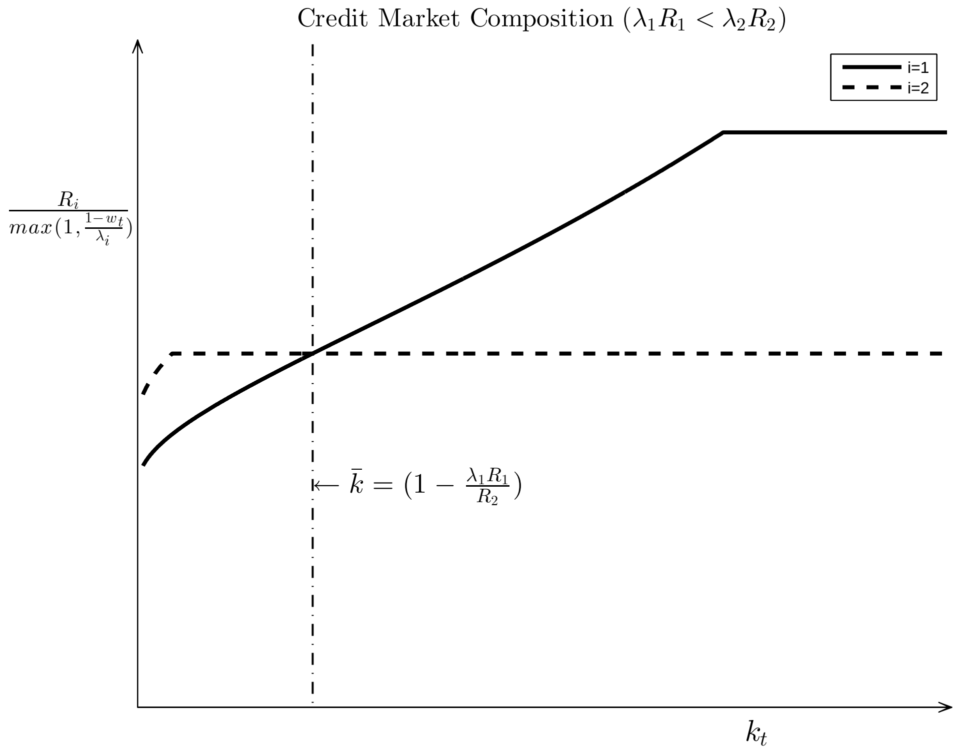

Figure 1: Pledgeability adjusted productivity parameters for the two possible investments The above figure shows the order in which projects (type 1 or type 2) are funded for different levels of capital, k_t in the economy. In the figure, the projects with a higher pledgeability adjusted parameter are always the first projects to be funded in the credit market. Type 1 agents are more productive $R_1 > R_2$ but more financially constrained. Hence at low levels of capital when agents have to borrow larger sums to fund their projects, the financial constraints matter more and type 2 projects are the first to be funded. This order changes as the economy grows richer and projects are funded in the order of their productivity and not pledgeability. The parameters used to generate the above figure are $\lambda_1 = 0.38, R_1 = 1.3, \lambda_2 = 0.8, R_2 = 0.9$.

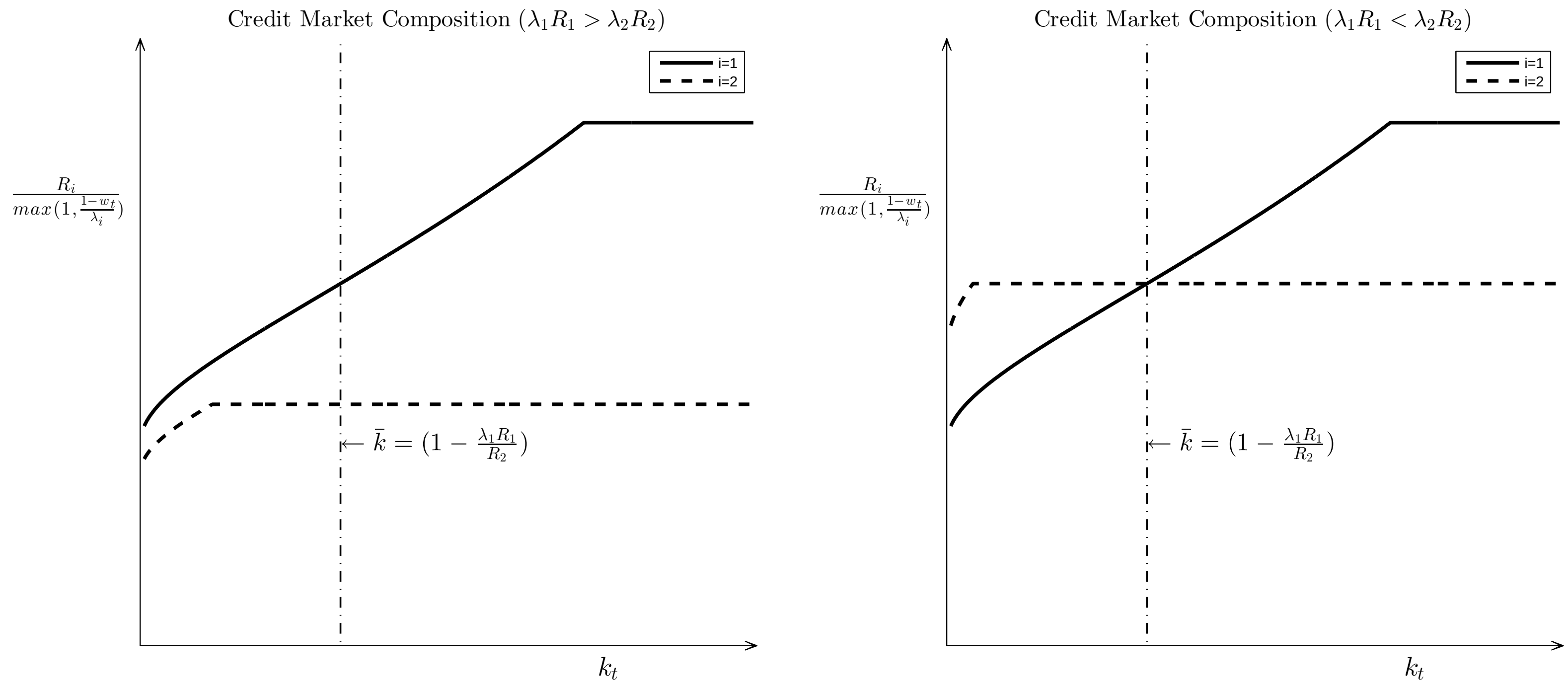

Figure 2: Benchmark cases of economies with financial frictions.

The above figure shows that in an economy with $R_1\lambda_1 > R_2\lambda_2$, the type 1 agents are not only more productive but also have a pledgeability such that projects of type 1 are always funded before type 2 projects (left panel). In such an economy, the financial constraint of individual projects does not matter even if $\lambda_1 < \lambda_2$. This is not the case for the economy with $R_1\lambda_1 > R_2\lambda_2$ (right panel) where the less productive type 2 projects are funded before the type 1 projects at low levels of capital. The parameters used to generate the above figure are $\lambda_1 = 0.38, R_1 = 1.3, \lambda_2 = 0.7, R_2 = 0.6.$ for the left panel and $\lambda_1 = 0.38, R_1 = 1.3, \lambda_2 = 0.8, R_2 = 0.9.$ for the right panel.

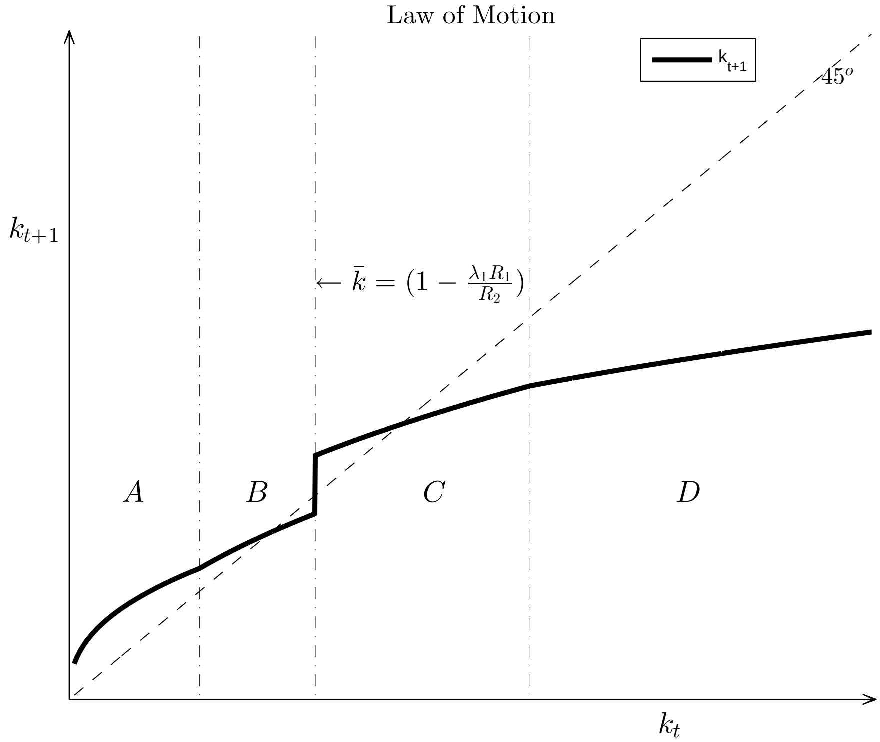

Figure 3: Capital accumulation without bubbles

The above figure shows the law of motion of capital for an economy with $R_1\lambda_1 < R_2\lambda_2$ (also figure 1). The economy can be in one of four types of equilibria based on the supra-marginal and marginal projects that are funded in the economy. At low levels of capital, only projects of type 2 are funded (A equilibria). After all type 2 projects are funded, type 1 projects are funded for slightly higher levels of capital (B equilibria), before the order in which projects are funded changes for $k > \bar{k}$. For levels of capital higher than $\bar{k}$, there are two types of equilibria - C equilibria in which only type 1 projects are funded or D equilibria in which both type 1 and type 2 projects are funded. The parameters used to generate the above figure are $\lambda_1 = 0.38, R_1 = 1.3, \Delta_1 = 0.58, \lambda_2 = 0.8, R_2 = 0.9, \Delta_2 = 0.35$ and $\alpha = 0.4$.

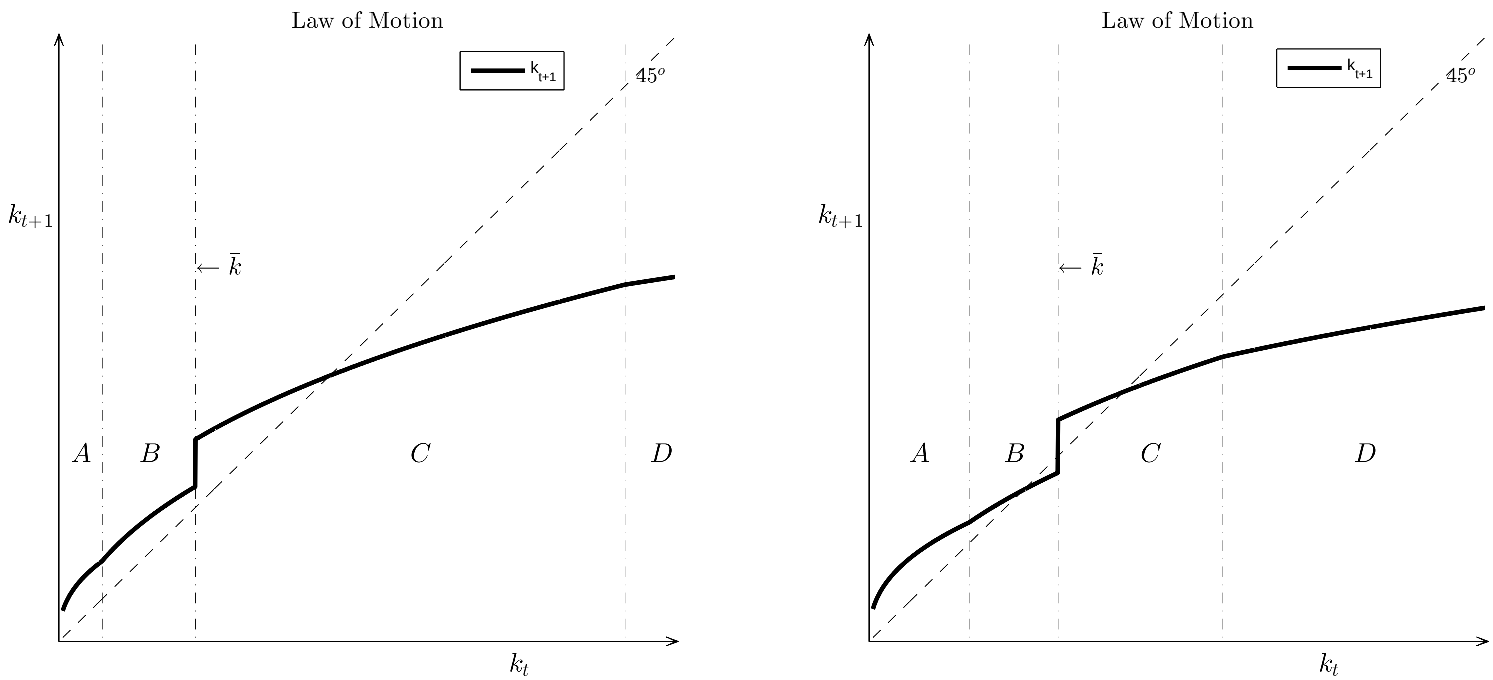

Figure 4: Capital accumulation without bubbles

The above figure compares the law of motion in an economy where credit is directed to the productive type 1 projects at steady state (left panel) against an economy where two steady states are possible (right panel). The economy on the right has a steady state in which the type 2 projects are the supra-marginal investors for levels of capital and another steady state for high levels of capital in which the type 1 projects take over that role. The parameters used to generate the above figure are $\lambda_1 = 0.38, R_1 = 1.35, \Delta_1 = 0.7, \lambda_2 = 0.8, R_2 = 0.85, \Delta_2 = 0.25$ for the left panel and $\lambda_1 = 0.38, R_1 = 1.3, \Delta_1 = 0.58, \lambda_2 = 0.8, R_2 = 0.9, \Delta_2 = 0.35$ for the right panel. For each law of motion, $\alpha = 0.4$.

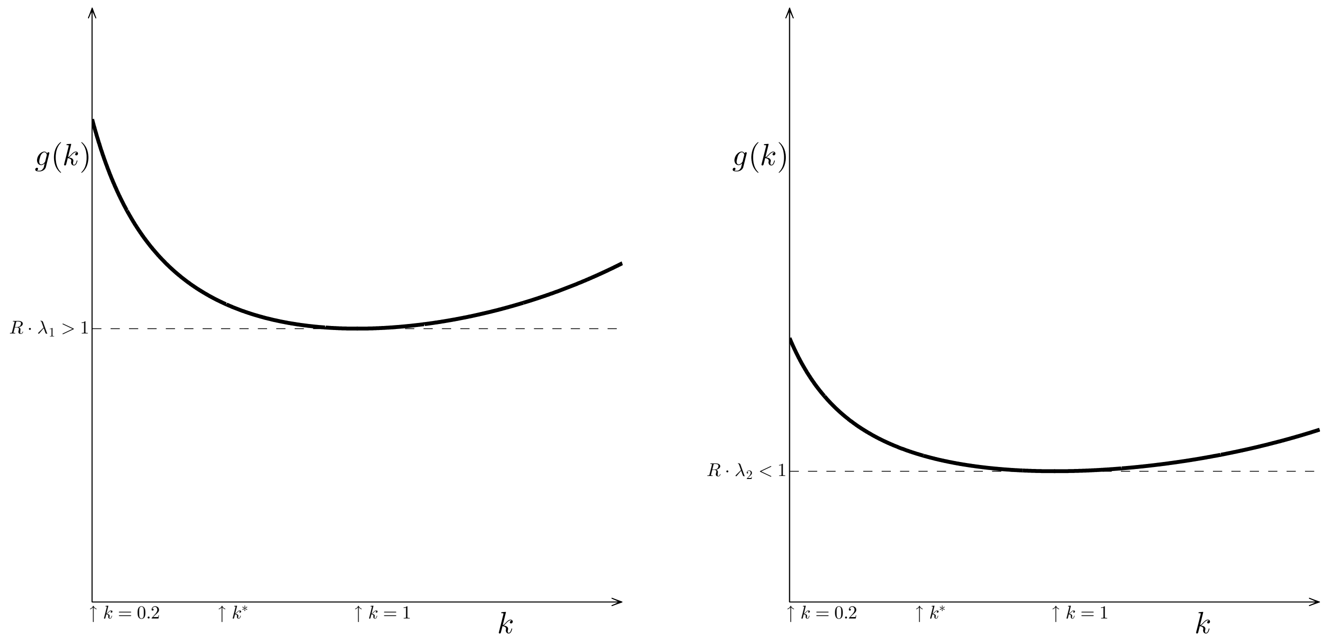

Figure 5: Global minima of $g(k)$

The above figure shows the global minima of the function $g(k) = \alpha k^{\alpha-1}/(1-(1-\alpha)k^{\alpha})$ at $k = 1$ for two separate cases. The global minima represents the lower bound of the credit market interest rate possible in the presence of investment projects $\{R, \lambda\}$. In the left panel, based on the parameters $R = 1.2, \lambda_1 = 1.1, \alpha = 0.4$, no asset bubbles are possible at steady state since the credit market interest rate can never be lower than 1 in the given economy ($R \cdot \lambda > 1$). In the right panel, based on the parameters $R = 1.2, \lambda_2 = 0.7, \alpha = 0.4$, the economy satisfies the conditions for the existence of asset bubbles as per Proposition 1 ($R \cdot \lambda < 1$ and the credit market interest rate in the fundamental steady state is less than 1). There exists a steady state bubble size for which the credit market interest rate in the economy represented in the right panel equals 1. The economies above highlight how an increase in financial constraints can create the conditions necessary for the existence of asset bubbles through the lowering of the credit market interest rate.

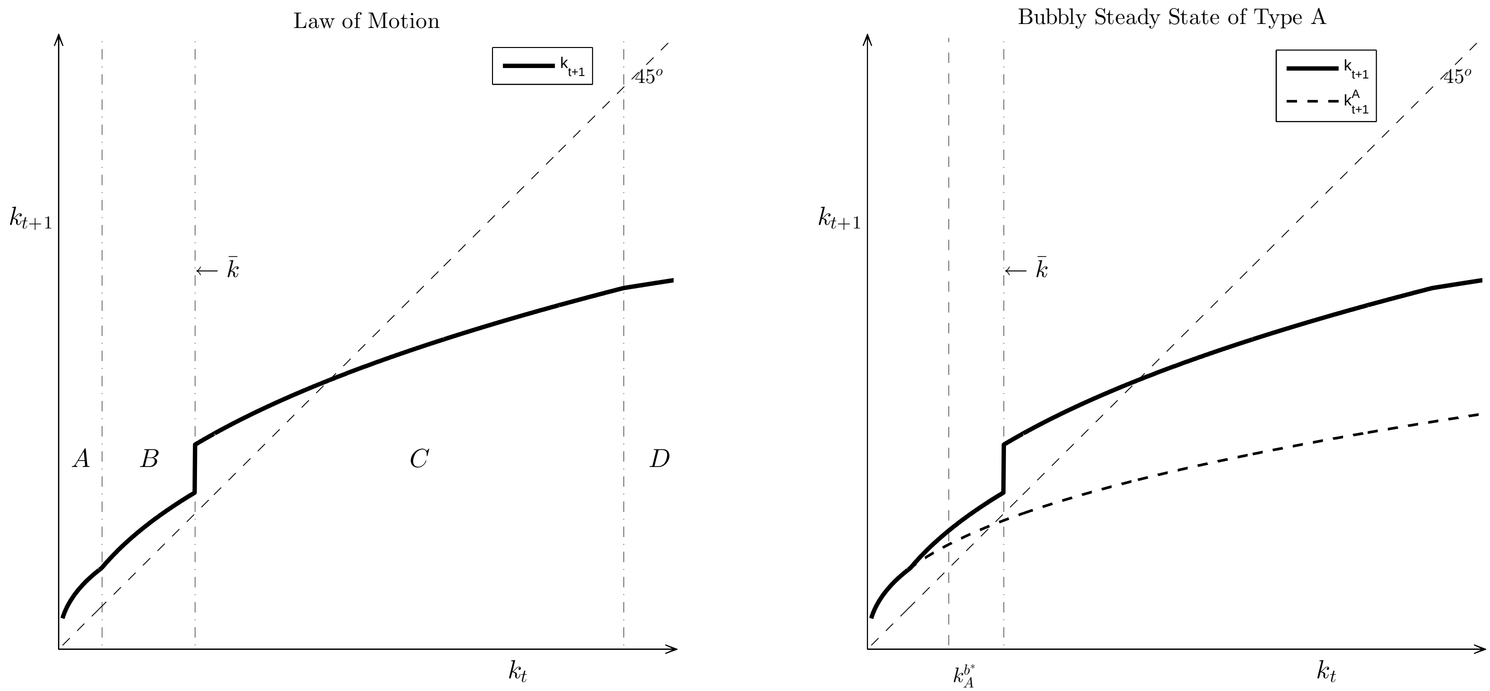

Figure 6: bubbly growth-traps

The above figure shows an economy that does not have any steady states in which only the type 2 projects are funded (or A-type equilibria) in the left panel. In the right panel, I show a steady state in the presence of bubbles in which only type 2 projects are funded (k^{A*}). This steady state exists at levels of capital higher than those in which A type equilibria generally exist in the absence of bubbles. Such bubbles crowd-out investments in the economy and lower the low of motion of capital to to an extent that the economy finds itself in a bubbly growth-trap.

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6 Appendix

6.1 Analytical expression for $\bar{k}$

$\bar{k}$ is the lowest level of capital at which the more productive investors of type1 are able to muscle out investors of type2 as the supra-marginal investors of the economy. Hence, it is the first level of capital at which $R_1/max(1,(1-w_t)/\lambda_1) = R_2/max(1,(1-w_t)/\lambda_2)$. It is easy to see that at this level of $\bar{k}$, type2 agents must be financially unconstrained, since otherwise the equality cant hold (note - $\lambda_1 > \lambda_2$ and $R_1\lambda_1 < R_2\lambda_2$). Hence $\bar{k}$ is obtained by solving $R_1/max(1,(1-w_t)/\lambda_1) = R_2$.

6.2 Necessary conditions for steady states of type B & D

For a steady state of type B, k_B^* must satisfy

$$k_B^* = R_2 * \Delta_2 + [(1-\alpha)k_B^{*\alpha} - \Delta_2] * R_1$$

and $k_B^* < \bar{k}$. The expression above can be written as

$$k_B^* + \Delta_2(R_1 - R_2) = R_1(1-\alpha)k_B^{*\alpha}$$

It can be then shown graphically that a solution to the above equation would reach a solution before $\bar{k}$ if $\bar{k} + \Delta_2(R_1 - R_2) < R_1(1-\alpha)\bar{k}^\alpha$. This reduces to the required expression. A similar condition can also be found for the steady state of type D.

6.3 Stability property of the bubbly steady states

I analyze the stability property of the bubbly steady states discussed in section 4 to check the behavior of the system close to the steady state of the following system of dynamic equations.

$$k_{t+1} = R \cdot [(1-\alpha) * k_t^\alpha - b_t]$$

$$\frac{b_{t+1}}{b_t} = R\lambda \cdot \frac{\alpha k_{t+1}^{\alpha-1}}{1-(1-\alpha)k_t^\alpha}$$

As I discussed in proposition II, when the fundamental equilibria steady state r^*satisfies

$r^* < 1$, there exists atleast one bubbly steady state with $k_b^* < k^*$. There are atleast two such bubbly steady states when $r^* > 1$ and $k^* > 1$.

The type of bubbly steady states that exist when $r^* < 1$ are similar to the ones that exist in Tirole (1985). $r^* < 1$ is indicative of dynamic inefficiency in the fundamental steady state in which the return to capital is less than the growth rate of the economy. As was the case in Tirole (1985), such bubbly steady state is saddle path stable. The behavior of the two candidate bubbly steady states identified in proposition 1 for the case $r^* > 1$ and $k^* > 1$ is done by analyzing the eigenvalues of the Jacobian matrix of the dynamic system described above. The Jacobian matrix, evaluated at the steady state is given by

$$\begin{bmatrix} R(1-\alpha)\alpha k_b^{*(\alpha-1)} & (-R) \\ R\alpha(1-\alpha)k^{*(\alpha-1)}b^*[\frac{\alpha-1}{k^*} + \frac{1}{R^2\lambda\alpha k_b^{*(\alpha-1)}}] & 1 + \frac{Rb^*(1-\alpha)}{k_b^*} \end{bmatrix}$$

I use standard parameters and find that of the two steady states one acts as a saddle point and the other (the one with a larger value of capital at steady state) acts as a focus. Further, simulation also show that such a focus can act as both a sink or a source, indicating the dependence of the exact nature of this particular type of steady state on the parameter values. Figure 7 presents some simulations to highlight the above results for economies which satisfy the necessary condition (for bubbly steady states) of $R\lambda < 1$. The top left quadrant is an economy in which the level of capital at the fundamental steady state, k^*, is less than 1, while $r^* > 1$. Such an economy does not have any bubbly steady states. The economy on the bottom quadrant has one steady state ($r^* < 1$) and the economy on the right quadrant has two steady states ($r^* > 1, k^* > 1$).

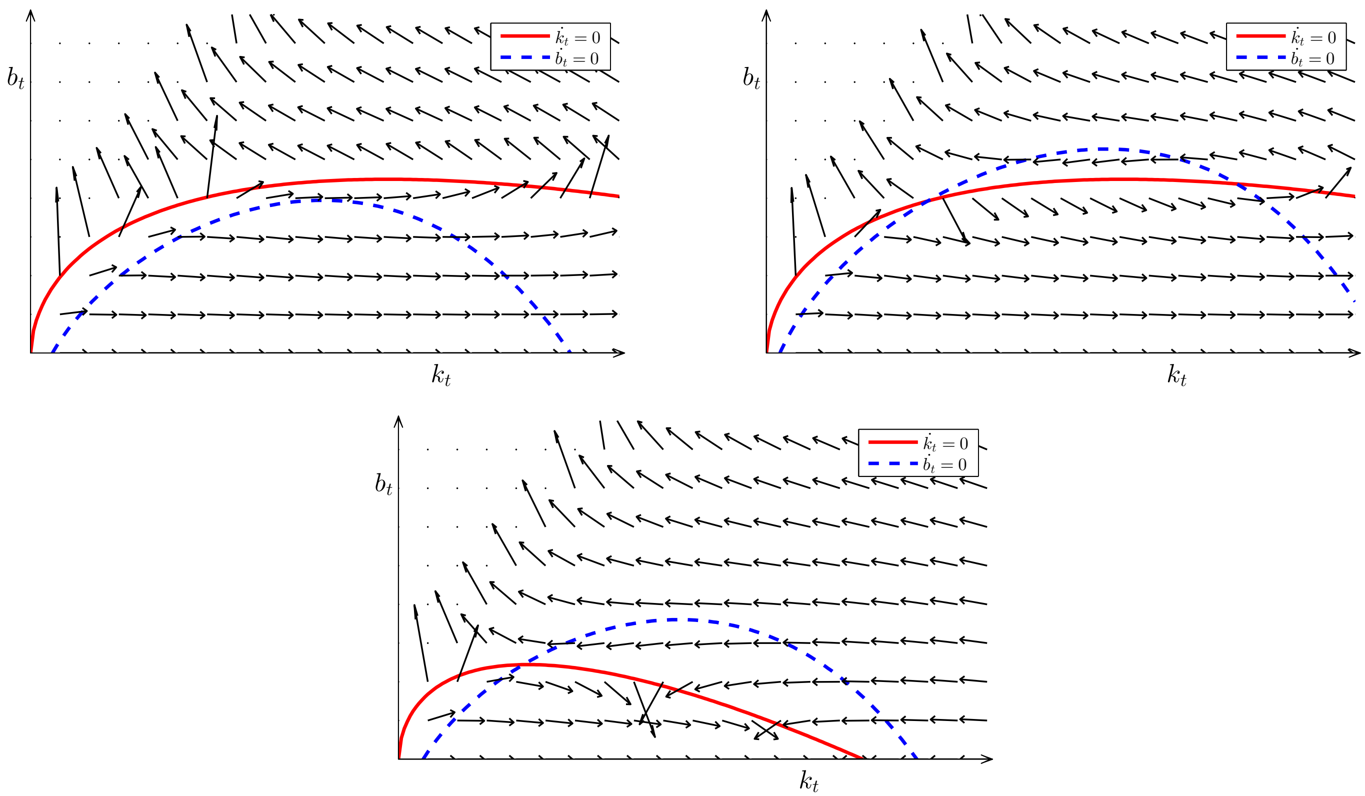

Figure 7: Different types of bubbly steady states

The above figure highlights the multiple equilibria possible in the economy described in this paper. I plot combinations of k_t and b_t that can be a potential equilibrium for the economy and identify the steady states of the economy. The economy may not have a steady state with bubbles (top left) or one unique bubbly steady state (bottom). A single steady state is always a saddle point. The economy may also have two steady states (top right) one of which acts as a saddle point and the other a focus. The steady state that acts as a focus can be a sink or source based on the parameters of the economy.