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# Staff Working Paper No. 669

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## Central counterparty auction design

Gerardo Ferrara<sup>(1)</sup>, Xin Li<sup>(2)</sup> and David Marszalec<sup>(3)</sup>

### Abstract

We analyze the role of auctions in managing the default of a clearing member in a generic central counterparty (CCP). We first consider three established alternative sealed bid auction formats in which clearing members simultaneously submit bids for a defaulting clearing member's portfolio: first price without penalty, first price with penalty, and first price with budget constraints. Under our assumptions regarding bidders' behaviour, although the revenue of the portfolio by the CCP might be the same for these auction formats mentioned above, there could be significant differences in the externalities arising from each of them. Additionally, this paper considers how mechanisms to incentivize competitive bidding could, in some circumstances, have adverse implications for financial stability by inefficiently distributing losses to surviving clearing members. In response to these potential adverse implications, we propose a fourth auction type — second price with loss sharing — which takes into account a bidder's consideration that may bear part of the CCP's losses.

**Key words:** Auction, default management, central counterparty.

**JEL classification:** D44, G18.

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# 1 Introduction

CCPs are a type of financial market infrastructure that improve the efficiency and stability of financial markets by placing themselves in the middle of trades between buyers and sellers and assuming any financial risks associated with the default of either counterparty. By using a CCP, market participants replace their exposures to multiple counterparties with a single exposure to the CCP. Effective management of the default of a clearing member is therefore an important element of a CCP's risk management framework. To reduce the potential for financial contagion arising from the default of a clearing member, a CCP must have a clear process in place to liquidate (closeout) the defaulted member's proprietary positions and to either transfer to other members (port) or liquidate its client positions. The auction is an important part of the closeout process, particularly for over-the-counter (OTC) products.

For instance, when Lehman Brothers failed on September 15, 2008, it left behind a \$9 trillion notional portfolio of interest rate swaps and some 66,000 trades with SwapClear, the LCH.Clearnet's interest swap clearing service. The priority for LCH.Clearnet was to hedge and to auction off the portfolio of its defaulted clearing member in a timely manner by using a first price auction with a juniorization mechanism and inviting the 19 remaining members of SwapClear to participate in the auction.

Different auction types might be designed for different product types and positions.<sup>1</sup> Roughly speaking, an auction allows the CCP to find one or more clearing members to take on the positions of the defaulted member and thereby restore a matched book. At the same time, the outcome of the auction determines: whether any losses are incurred on the defaulted member's portfolio; whether these are covered by the defaulted member's margin (and/or default fund contributions); and whether residual losses must be allocated to CCP capital and/or surviving members. From the viewpoint of the CCP, an efficient auction design would be a mechanism in which

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<sup>1</sup>For instance, auctions of short positions implicitly allow for negative bids since bidders need to be compensated for taking on the additional risk of the defaulter's positions.

rules are stated to maximize the revenue (or liquidation value of the portfolio to be auctioned) with the least dispersed revenue distribution (Milgrom and Weber, 1982). The aim of this paper is to investigate the revenue outcomes of alternative auction designs, while also examining the stability implications of mechanisms intended to incentivize more competitive bidding.

Careful auction design is essential to optimize the default management process. In general, it is necessary to tailor the design of the auction to the particular setting in which it occurs. Unfortunately, the auction types that will be used in a potential default management process are not always clearly stated in the rulebook of CCPs, and bidders may not entirely understand what is the right framework to form their bids. As a result, the bidders' uncertainty regarding the type of auction in which they are participating could result in less revenue for a CCP.

This study addresses this potential concern by extending the analysis on auction theory to the framework of CCPs and provides conditions to ensure that the auction process is able to limit the CCP's losses, introducing more realistic assumptions regarding the constraints faced by clearing members. In particular, we develop a framework for comparing auctions with different characteristics including the possibility of budget constrained bidders. Analyzing different auction types is valuable because the bidding behavior can be affected differently by different factors such as a clearing member's budget constraint (e.g., leverage ratio, capital to support risk-weighted assets, liquidity available, etc.). In addition to investigating the revenue outcomes of alternative auctions designs, this paper aims to examine the stability implications of mechanisms intended to incentivize more competitive bidding and to propose a new type of auction that maximizes the revenue for the CCP.

Three types of established sealed bid auction types are initially considered in this paper – first price without penalty, first price with penalty, and first price with budget constraints – in which bidders simultaneously submit their bids. In its consideration of these auction types, this paper develops a theoretical framework for examining the first price auction with penalty, that was not previously present in the auction literature. The idea is to study those auctions for which clearing members face incentives similar to those in the real world, and to enhance this study, we propose a

new forth type of auction – the second price auction with loss sharing – which more efficiently meets the requirements of the CCP.

The remainder of the paper is organized as follows. Section 2 provides a brief description of the rulebooks for a generic CCP. Section 3 contains the revenue characterization for the three auction forms and their respective general payoffs for the bidders. We continue with an analysis of equilibrium solutions across different auction types and discuss the implications of a penalty mechanism in Section 4. Section 5 concludes.

## 2 A Review of the Rulebooks

If one or several members default, the variation margin payments no longer match each other. For this reason, the CCP must return to a matched portfolio<sup>2</sup>. One possible mechanism to achieve this is to auction the defaulted clearing member's portfolio. During this process the CCP continues to honor all variation margin payments to open positions of non-defaulted members, which may entail a loss for the CCP in case the positions are loss-making. Due to daily variation margin payments, the CCP is only exposed to losses which occur after the last variation margin payment made by the defaulting member. For this reason, a CCP must devise a sequence of actions to prepare the portfolio for an auction and to get rid of it. This procedure is known as the closeout process and is enforceable by the CCPs. In general, the closeout process is composed of: splitting<sup>3</sup>, hedging<sup>4</sup>, and auctioning. However, this paper focuses solely on the auctioning component of the closeout process and its

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<sup>2</sup>Any position taken on with one counterparty must be always offset by an opposite position taken on with a second counterparty

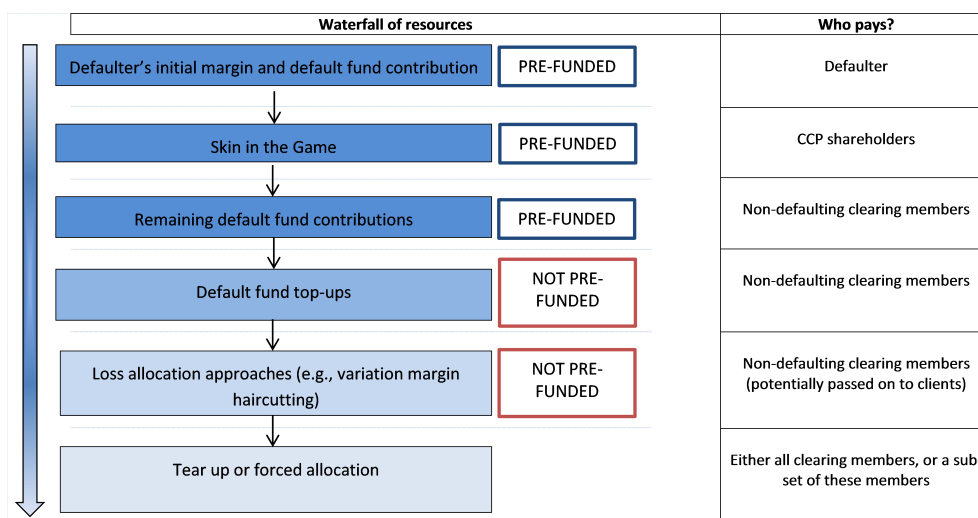
<sup>3</sup>Splitting is the process through which the original portfolio is segregated into smaller sets. The number of resulting portfolios can be done exogenously or endogenously to the closeout process, in accordance with the default management process.

<sup>4</sup>Hedging is considered as the sequence of market operations to reduce risk exposure in an investment portfolio. Portfolio hedging typically entails the use of financial derivatives to curtail losses related to non-cash portfolios. However, every hedge has a cost, so CCPs should weigh the costs of the hedge against its benefits.

potential unintended consequences.<sup>5</sup>

In contrast to most other financial firms, CCPs' obligations to their members, and vice versa, are governed by a central rulebook. CCPs have the ability to include in this rulebook rules setting out, among other things, how losses arising from the default of a member would be allocated between the CCP's participants, via the application of a default waterfall (see Figure 1). Such rules could have the advantage of offering transparency for an orderly and efficient allocation of losses and potentially allowing the continuity of systemically important clearing services. Given these considerations, authorities have an ongoing interest in the design of optimal loss allocation rules.

**Figure 1:** Example of a CCP default waterfall



The default waterfall typically starts with the collateral or margins provided by a defaulting member and proceeds to a mutualized default fund. These pre-funded resources can be called upon in the order set out in its rulebook. If these members' pre-funded contributions are depleted, clearing members are typically obliged (at least in OTC markets) to meet additional cash calls in order to replenish the unfunded resources. The rulebook also provides a complete description of how losses

<sup>5</sup>If the auction process is not successful in fully re-establishing a matched book, then alternative measures of forced position allocation may be used.

would be allocated to participants if the size of the losses exceeded the pre-funded resources by using un-funded, although capped, resources.

If the overall value of the default fund is less than a minimum floor following the completion of a default management process, the CCP may notify each surviving member that it is required to make a supplementary contribution to restore the required minimum fund.<sup>6</sup> In this case, these supplementary contributions must be paid within a few business days (e.g., two days for LCH.Clearnet) after notification.<sup>7</sup>

Within the default waterfall, the auction process determines the size of the losses, if any, the surviving clearing members will need to bear. Given the importance of the auction process for default management a number of CCPs also incentivize competitive bidding by penalizing non defaulting members who either do not bid or do not bid competitively. The most basic forms of incentivization attempt to encourage submission of bids with the juniorization of default funds for non-bidders. When a juniorization mechanism is in place, if a member's bid falls outside a pre-defined band, a CCP can juniorize the member's default fund contribution in case the auction revenue was not sufficient to cover the defaulter's losses, meaning its default fund would be used to mop up any losses after a defaulting firm's resources had been consumed, but before imposing mutualized losses on other participants.<sup>8</sup> More complicated versions also provide for a hierarchy of bids, whereby those unsuccessful bids far from the winning bid (i.e., low bidders) are juniorized relative to those of winning bids (BIS, 2014). The level of granularity and sophistication vary between clearing houses, and may include the establishment of categories of bidders (as in LCH and CMECE), whereby the default funds of clearing members in particular categories are used first on a pro-rata basis, or it may establish a reverse ladder (e.g., ICEU), where resources are used sequentially from worst bidder to best.

Juniorization of the default fund only acts as an incentive if clearing members have a large default fund at risk of being utilized. Potentially to overcome this

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<sup>6</sup>See CME Group, 2016; Eurex Group, 2016; ICE Group, 2014; LCH.Clearnet, 2016.

<sup>7</sup>ICEU under its rules can call for a replenishment of funds at any time after the default loss has been allocated (or the next business day for the CDS service).

<sup>8</sup>If losses are large enough even the highest bidder may incur some losses, since in some circumstances all the resources could be absorbed during the default management process.

shortcoming, Eurex also implements a fine (of up to €5 million) for non-bidders (when the auction disposes of the entire portfolio). When part of the auction is unsuccessful, non-bidders assume liability to dispose of the portfolio of up to €1 billion. These approaches are consistent with the CPMI-IOSCO (BIS, 2012) guidance on incentivizing participants during auctions.

### 3 Problem Formulation

We begin by describing a standard set of assumptions, and bidding mechanisms that the auctioneer (i.e., the CCP) uses to sell an item (i.e., a generic portfolio). Based on the chosen mechanism, we then define the bidding strategies and payoffs for the risk-neutral bidders (i.e., the clearing members) in the auction. It becomes clear that when bidders are risk-neutral their expected payoff functions are additively separable. To facilitate our analysis, some of our results will be established under the following two assumptions.

**Assumption 1.** *Each risk neutral bidder independently draws an independent private signal  $\varepsilon$  from a uniform distribution  $F$ , assuming all bidders have an identical value distribution.*

Thus, the bidder  $i$ 's overall valuation  $v_i$  of the auctioned portfolio is given by

$$v_i = V_0 + \varepsilon_i,$$

where  $V_0$  is a benchmark value calculated under publicly available information. Furthermore, bidder  $i$ 's value is independent of bidder  $j$ 's information – so bidder  $j$ 's information is independent in the sense that it doesn't affect the valuation process of anyone else. Let  $F(v)$  represent the distribution for the bidder's overall valuation, which is derived from the private value distribution with the additive term  $\varepsilon$ . Each bidder's overall valuation is a random draw from the distribution of  $F(v)$  with the range  $[\underline{v}, \bar{v}]$ . Furthermore, given the independence of the private information  $\varepsilon$ , the joint distribution function,  $F(v_1, \dots, v_n)$ , is calculated by the product  $\prod_{i=1}^N F_i(v_i)$  of separate distribution functions  $F_i(\cdot)$  for each of the  $v_i$ .



Additionally, we will make the following assumption about the information structure.

**Assumption 2.** *Payment is a function of bids alone.*

As is standard for any type of auction, each bidder  $i$  submits a bid  $b_i$ , and the price paid depends on which auction type we are considering. Additionally, it is not always true that only the winning bidder pays. If  $i$  wins the object and the value is  $v_i$ , his utility is given by  $v_i - b_i$ . When there is uncertainty about who wins, expected payoffs guide the bidders' behavior. In the following derivation, we will use both the risk neutrality and independence assumptions to yield a fairly simple solution.

In general, each bidder submits a sealed bid of  $b_i$ , and the payoffs are

$$u_i = \begin{cases} v_i - b_i, & \text{if } b_i > \max_{j \neq i} b_j \\ \zeta(b_i, b_{-i}), & \text{if } b_i < \max_{j \neq i} b_j, \\ \frac{1}{k}(v_i - b_i), & \text{if } b_i = \max_{j \neq i} b_j \end{cases} \quad (1)$$

where  $k$  and  $\zeta(b_i)$  represent the number of winners when multiple bidders offer the same (highest) bid and the payoff from failing to win the auction given the competitors' bids, respectively. Each bidder  $i$  submits a bid  $b_i$  and he gets the portfolio if and only if his bid is the highest among all bids. In this case he gets the portfolio and has to pay an amount equal to his submitted bid. In case of a draw (two or more bidders submitted the highest bid), some arbitrary rule may be chosen to determine the winner of the auction. For instance, in case of a draw between  $x$  bidders, each bidder could win the portfolio with probability  $1/x$ .

We focus on non-collusive mechanisms in which bidders make their participation decisions non-cooperatively. However, there are several studies which analyze the bidding strategies of cartels and compare them to collusive bidding strategies (Pensendorfer, 2000; Porter 2005). The extent to which incentives to collude vary under different auction formats can be of great practical concern when deciding which type of auction to use. Indeed, the indictment in the United States of a primary securities dealer in 1991 for fraudulent activities in the government securities market has

focused attention on the collusive potential of standard auction formats. It is important to keep in mind that all auctions are susceptible to collusive behavior. A basic hypothesis, first formulated in the literature by Mead (1987), is that sealed bid formats are less susceptible to collusion because bidders cannot observe rivals' bids until the auction is over. This belief may explain the popularity of sealed bidding, even though there are other formats with a superior revenue generating potential (Milgrom, 2004). Intuitively, Graham and Marshall (1987) suggest that the auctioneer should adopt a high reserve price if he suspects collusion.

Since participants are risk-neutral in evaluating their payoffs under uncertainty, each bidder seeks merely to maximize the mathematical expectation of his payoff. For example, if bidder  $i$  believes that he has a probability  $\beta(b_i)$  of winning the item for a payment of  $b_i$  and a probability  $(1 - \beta(b_i))$  of winning nothing and paying nothing, then his expected payoff equals  $\beta(b_i)(v_i - b_i)$ .

For simplicity, we shall assume throughout this paper that the clearing members are risk-neutral and have additively separable utility functions. Thus, if a bidder knows that his value estimate is  $v_i$ , then his expected utility from an auction mechanism described by  $\pi_i(v_i, b_i(v_i))$  is

$$\pi_i(v_i, b_i(v_i)) = (v_i - b_i(v_i))\beta(b_i(v_i)) - \zeta(b_i)(1 - \beta(b_i(v_i))), \quad (2)$$

where  $b_i(v_i)$  represents the bid submitted by bidder  $i$  when its valuation is  $v_i$  and the probability of winning the item is described by  $\beta(b_i(v_i))$ . If we don't know the values of the bidders, a CCP will not be able to determine in advance whether one type of auction will give it more revenue than another. However, given its beliefs about the probability distribution of bidders' values, the CCP can calculate its revenue from any type of auction.

### 3.1 First Price Auction without Penalty

The first format considered for auctioning a portfolio is the first price sealed bid auction without penalty. This type of auction is arguably the most common form of auction in use today. Although the results and their derivation are standard we

present them for completeness. Bidders submit their sealed bids in advance of a deadline, without knowledge of any of their opponents' bids. After the deadline, the highest bidder wins the item and pays the amount of his bid. Each bidder submits a sealed bid of  $b_i$ , and the possible payoffs are

$$u_i = \begin{cases} v_i - b_i, & \text{if } b_i > \max_{j \neq i} b_j \\ 0, & \text{if } b_i < \max_{j \neq i} b_j, \\ \frac{1}{k}(v_i - b_i), & \text{if } b_i = \max_{j \neq i} b_j \end{cases}$$

where a bidder gets a fraction of the auctioned portfolio (paying a fraction) when bidders get tied. Consequently, from a bidder's point of view, the expected payoff equals

$$\pi_i(v_i, b_i(v_i)) = (v_i - b_i(v_i))\beta(b_i(v_i)), \quad (3)$$

where  $b_i(v_i)$  represents the bid submitted by bidder  $i$  and the probability of winning the item is described by  $\beta(b_i(v_i))$ .

Considering an auction with  $N$  bidders, let the order statistic  $Y$  be the highest of the  $N - 1$  random draws from the distribution  $F(v)$ . Then the distribution of  $Y$  can be represented by  $F(v)^{N-1}$ , with its density function being  $(N-1)F(v)^{N-2}f(v)$ . Consider the bids submitted by bidders as a function of bidders' valuations,  $b(v)$ . Given the assumption of the symmetric bidding strategy used by all bidders, the probability for  $i$  to win the portfolio by bidding at the price  $b_i(v_i)$  becomes  $F[b^{-1}(b_i(v_i))]^{N-1}$  (i.e., the probability of the highest valuation among all the other  $N - 1$  being lower than the implied valuation from price  $b_i(v_i)$ ).

Bidder  $i$ 's expected payoff from bidding at  $b_i(v_i)$  becomes

$$\pi_i(v_i, b_i(v_i)) = (v_i - b_i(v_i))F[b^{-1}(b_i(v_i))]^{N-1} \quad (4)$$

while the optimal strategy from bidder  $i$ 's viewpoint is to choose a bidding price maximizing the expected payoff given the bidder's own valuation

$$\max_b (v - b(v))F[b^{-1}(b(v))]^{N-1}.$$

Vickrey (1961) provides a complete characterization of the equilibrium for a similar problem. Assume that the bidders' valuations are independent and identically distributed, and assume that the bidders are risk-neutral. Then Eq. (5) provides the unique equilibrium of the first price sealed bid auction without penalty

$$b(v) = v - \frac{1}{F(v)^{N-1}} \int_{\underline{v}}^v F(y)^{N-1} dy. \quad (5)$$

The highest bidder wins the portfolio as in conventional auctions. Then the expected payment collected from the closeout process is the estimation of the highest bidding price, which is submitted by the one with the highest overall valuation. Because the equilibrium is symmetric, then this is the optimal strategy for all bidders.

Let  $v_{(N)}$  denote the highest value for the portfolio. Then, given  $N$  random draws from the uniform distribution, the expected highest value is given by

$$E[v_{(N)}] = \underline{v} + \frac{N}{N+1}(\bar{v} - \underline{v}), \quad (6)$$

where the range for the valuation is denoted by  $[\underline{v}, \bar{v}]$ . Thus, with the estimated highest value from (6) and equilibrium bidding strategy from (5), the revenue from a CCP's point of view is the following

$$\begin{aligned} e^A &= E[b(v_{(N)})] \\ &= E\left[\frac{N-1}{N}v_{(N)} + \frac{\underline{v}}{N}\right] \\ &= \frac{N-1}{N}\left[\underline{v} + \frac{N}{N+1}(\bar{v} - \underline{v})\right] + \frac{\underline{v}}{N} \\ &= \frac{(N-1)\bar{v} + 2\underline{v}}{N+1}. \end{aligned} \quad (7)$$

See Appendix A for the complete proof.

Hence, when the number of participants increases, the CCP gets more chances to increase its revenue from liquidating the portfolio of the defaulted clearing member. Consider  $v$  is distributed uniformly between  $[0, 1]$ . Therefore, when 2 buyers partic-

ipate the revenue to the seller is  $1/3$ , when 3 bidders participate the revenue is  $1/2$ , when 4 bidders participate the revenue is  $3/5$ , and so on. In this case, as  $I \rightarrow \infty$ , we see that  $ER \rightarrow 1$ , which means the seller almost gets the maximum possible revenue. If our assumptions hold, then increasing the number of bidders in an auction should increase competitiveness, because a bidder's perceived probability of winning is lower and prompts more competitive bidding in a single-portfolio auction.

The result above is standard in the auction literature. The purpose of this section is not only to compare bid behavior to show that the first price auction is in general efficient, but also to show that the CCP has an incentive to increase the number of bidders participating in the auction. Therefore, CCPs make participation in auctions mandatory for clearing members who – in a member default event – have substantial positions in the same clearing service. However, we know that bidders could always submit a nil bid.

### 3.2 First Price Auction with Penalty

Research on standard first price auctions without penalty is quite standard. However, when we consider an auction managed by a CCP, bidders may face negative externalities deriving from their low level of competitiveness in the bidding process. The consequences of these externalities on bidding behavior and on revenue has not been previously studied for CCPs. For this reason, we develop here a new theoretical model for analyzing this type of auction.

We start by considering the auction in which all participants pay a percentage of their own bid as a bidding incentive. Each bidder submits a sealed bid, and the payoffs are

$$u_i = \begin{cases} v_i - b_i, & \text{if } b_i > \max_{j \neq i} b_j \\ -\left(1 - \frac{b_i}{V_0}\right)b_i, & \text{if } b_i < \max_{j \neq i} b_j, \\ \frac{1}{k}(v_i - b_i), & \text{if } b_i = \max_{j \neq i} b_j \end{cases}$$

where the clearing member will be punished for submitting uncompetitive bids. Since losing bids are wasted, clearing members have a strong incentive to bid competitively

if they bid at all.

Clearing members pay a higher percentage of their bid in function of the distance of their bid from the benchmark value  $V_0$ . However, it should be noted that the payoff from losing the auction is increasing for the clearing member's bid  $b_i$  only when  $b_i > V_0/2$ . As previously mentioned, some auctions also provide for a hierarchy of bids, whereby those unsuccessful bids far from a benchmark are ordered (BIS, 2014) and resources are used sequentially from worst bidder to best. Of course, there is no need to resort to this mechanism when losses are either completely absorbed by the defaulter's resources and the auction revenue or the default fund is used entirely. However, we want to analyze the incentive to bid when such a penalty needs to be used.

The expected payoff of the following auction form is the following

$$\pi_i(v_i, b_i(v_i)) = F[b^{-1}(b_i(v_i))]^{N-1}(v_i - b_i(v_i)) - \{1 - F[b^{-1}(b_i(v_i))]^{N-1}\} \left(1 - \frac{b_i(v_i)}{V_0}\right) b_i(v_i)$$

where the simplified expression is

$$\pi_i(v_i, b_i(v_i)) = F[b^{-1}(b_i(v_i))]^{N-1} \left( v_i - \frac{b_i(v_i)^2}{V_0} \right) - \left( b_i(v_i) - \frac{b_i(v_i)^2}{V_0} \right).$$

The bidder's problem is maximizing the expected payoff

$$\max_{b_i} \left\{ F[b^{-1}(b(v_i))]^{N-1} \left( v - \frac{b(v_i)^2}{V_0} \right) - \left( b(v_i) - \frac{b(v_i)^2}{V_0} \right) \right\}.$$

By taking the first order derivative, we have

$$\begin{aligned} \frac{\partial \pi}{\partial b(v_i)} &= (N-1)F[b^{-1}(b(v_i))]^{N-2}f[b^{-1}(b(v_i))]\frac{d[b^{-1}(b(v_i))]}{db(v_i)} \left( v - \frac{b(v_i)^2}{V_0} \right) + \\ &+ F[b^{-1}(b(v_i))]^{N-1} \left( -\frac{2b(v_i)}{V_0} \right) - \left( 1 - \frac{2b(v_i)}{V_0} \right), \end{aligned}$$

where the derivative of the cumulative distribution function  $F(\cdot)$  is the probability density function  $f(\cdot)$  for the distribution.



**Theorem 1.** *Assume that the bidders' valuations are independent and identically distributed with an overall value following the uniform distribution, and assume that the bidders are risk neutral. Then Eq. (8) provides the unique equilibrium of the first price auction with penalty*

$$b(v) = \frac{V_0 - V_0[1 - \frac{4(N-1)}{NV_0}v^N(1 - v^{N-1})]^{\frac{1}{2}}}{2(1 - v^{N-1})}. \quad (8)$$

*Proof.* The proof can be found in Appendix B. □

The auction is designed so that it is beneficial for each of the participants to submit a bid. According to the above bidding strategy, bidder  $i$  has the following estimated payoff

$$\begin{aligned} \pi_i(v_i, b(v_i)) &= F[v_i]^{N-1} \left[ v_i - \frac{b^2(v_i)}{V_0} \right] - \left[ b(v_i) - \frac{b^2(v_i)}{V_0} \right] \\ &= v_i^{N-1} \left[ v_i - \frac{b^2(v_i)}{V_0} \right] - \left[ 1 - v_i^{N-1} \right] \left[ b(v_i) - \frac{b^2(v_i)}{V_0} \right] \\ &= v_i^N - b(v_i) + \frac{1 - v_i^{N-1}}{V_0} b^2(v_i). \end{aligned}$$

The revenue from the auction is then the summation of the payment made by the winning bidder and the fees collected from the losing participants.

Considering the situation in which all bidders make payments for the amount of  $\omega(b_i)b_i$ , the winning bidder would get the refund of his fee after paying his winning bid. The estimated payment from one bidder is the following

$$E[b_i] = \int_{\underline{v}}^{\bar{v}} \left( 1 - \frac{b_i}{V_0} \right) b(v) dv.$$

Considering that the net amount of the fees is  $Nb_i - E[\omega[b(v_{(N)})]b(v_{(N)})]$ , the revenue becomes

$$e^B = E[b(v_{(N)})] + \left\{ NE[b^i] - E \left[ \left( 1 - \frac{b_N}{V_0} \right) b(v_{(N)}) \right] \right\}. \quad (9)$$

It should also be noted that, under the assumption of risk neutrality, the formula used in the third component of Eq. (9) could equally be used to determine the least competitive bidders that should be affected by a possible penalty. However, looking at the rulebooks, there is no clear rule or metric to distinguish the competitiveness of their bids.

The most important question to ask is whether bidders have the resources – and the risk taking capacity – to participate actively in the auction.

### 3.3 First Price Auction with Budget Constraints

We now consider a standard first price auction in which the members are budget constrained. The main motivation in considering this type of auction is that clearing members may face a shortage of liquidity due to wide market stress scenarios or due to limited access to capital markets. Additionally, leverage ratio also adds to the burden, by setting regulatory requirements in relation to notional numbers, rather than risk, so even a perfectly hedged book could have a significant cost.

As budget constraints are a common feature of a generic auction, neglecting their implications may lead to wrong expectations. In this type of auction, a generic bidder has an available budget  $B \in [\underline{B}, \bar{B}]$ , which is private information. Then, a generic bidder  $i$  is constrained if  $v_i \leq \underline{B}$ , since the lowest bound exceeds the valuation of the portfolio to be auctioned. Then the unconstrained auctions will generate the same revenue as the first price auction without penalty described in Section 2.1.

Now we suppose that  $v_i > \underline{B}$ . In this case, a generic bidder  $i$  with budget constraint  $B_i$  receives an expected surplus of  $\pi_i(v_i, b_i(v_i, B_i))$  in equilibrium given by

$$\pi_i(v_i, b_i(v_i, B_i)) = (v_i - b_i(v_i, B_i)) F[b^{-1}(b_i(v_i, B_i))]^{N-1},$$

while, for each  $B$ , the bidder will maximize the expected payoff given the bidder's own constrained valuation

$$\max_{\underline{B} \leq b \leq B} (v - b(v)) F[b^{-1}(b(v))]^{N-1}.$$



Compared to the simple first price auction without penalty described in Section 2.1, the budget constrained bidder needs to decrease his bid, thus potentially impacting the revenue of the auction. Roughly speaking, a bid  $b$  would win with at least the probability  $F(b)^{N-1}$  since other bidders cannot bid above their balance constraints.

Che and Gale (1996) provide an analytical framework to calculate the equilibria. Assume that the bidders are risk-neutral. Then Eq. (10) provides the equilibrium for the first price auction with budget constraints

$$b(v, B) = v - \frac{1}{F(v, B)^{N-1}} \max_{B \leq b \leq \bar{B}} (v - B) F(b, B)^{N-1}, \quad (10)$$

where a bidder with budget constraint  $B$  will win the auction with probability  $F(b, B)^{N-1}$  if it bids  $v(b, B)$ , because  $b(\cdot)$  is a strictly increasing function. The revenue is capped by the revenue in the first price auction without penalty since the surplus for a constrained bidder is weakly higher than an unconstrained case

$$(v(B) - b(B)) F(b, B)^{N-1} v F(b, B)^{N-1} - b \leq (v - b) F(b)^{N-1} v F(b)^{N-1} - b.$$

Thus, the CCP could receive a lower revenue when the members are budget constrained, and penalty measures could not be enough to force a clearing members to bid for a portfolio it does not want to own – the risks might outweigh the costs.

### 3.4 Second Price Auction with Loss Sharing

We now propose to the reader a novel modified version of the second price auction in which we incorporate a loss sharing mechanism. As part of this loss sharing mechanism, the CCP shares a portion of its losses with bidders and allocates the losses among the surviving members according to a predetermined formula. We assume the CCP has a loss sharing threshold,  $\psi$ . It will ask a portion  $P$  from,  $V_0 - \psi$ , of the CCP's losses to all clearing members whose bid is below  $\psi$ , where  $\psi < V_0$ . In summary, clearing members receive a penalty equal to  $P(V_0 - \psi)/N_\psi$  where  $N_\psi$  represents the number of members whose bid is lower than or equal to  $\psi$ .

Then, each bidder submits a sealed bid and the payoffs are

$$u_i = \begin{cases} v_i - \max_{j \neq i} b_j + \frac{(V_0 - \psi)P}{N_\psi}, & \text{if } b_i > \max_{j \neq i} b_j \\ \frac{(V_0 - \psi)P}{N_\psi}, & \text{if } b_i < \max_{j \neq i} b_j \text{ and } b_i < \psi \\ 0, & \text{if } b_i < \max_{j \neq i} b_j \text{ and } b_i > \psi \\ \frac{1}{k} \left( v_i - \max_{j \neq i} b_j \right), & \text{if } b_i = \max_{j \neq i} b_j \end{cases}$$

where the clearing member who is winning the auction has the highest payoff. Substantially, bidders have an incentive to bid more competitively in order to avoid the penalty. Then, the expected payoff from a clearing members' point of view is

$$\begin{aligned} \pi_i(v_i, b_i(v_i)) &= F[b^{-1}(b_i(v_i))]^{N-1} \left[ (v_i - \max_{j \neq i} b_j) + P \frac{V_0 - \psi}{N} \right] + \\ &+ \left[ 1 - F[b^{-1}(b_i(v_i))]^{N-1} \right] P \frac{V_0 - \psi}{N}. \end{aligned}$$

The bidder's problem is maximizing the expected payoff

$$\max_b \pi_i(v_i, b_i(v_i)).$$

**Proposition 1.** *The revenue is always positive since the portfolio will never be sold at a price below  $V_0$ .*

Assume that the bidders' valuations are independent and identically distributed with an overall value following the uniform distribution, and assume that the bidders are risk neutral. Then Eq. (11) provides the expected payment for the second price auction with loss sharing

$$b(v) = \begin{cases} \frac{1}{N} F^{N-1}(\psi) \psi, & \text{if } v < \psi \\ F^{N-1}(\psi) \psi + \int_\psi^v (N-1) v f(v) F^{N-2}(v) dv, & \text{if } v > \psi \end{cases}. \quad (11)$$

In summary, a bidder with a valuation below  $\psi$  has two possible choices: (i) bidding  $\psi$  to avoid the possible penalty, (ii) bidding its valuation and taking the risk

of getting the penalty.

**Theorem 2.** *Bidding one's valuation when it is greater than or equal to  $\psi$  is an equilibrium strategy if  $F(\psi)^N \leq P$ . Assume that the bidders' valuations are independent and identically distributed with an overall value following the uniform distribution, and assume that the bidders are risk neutral.*

*Proof.* If the clearing member has a valuation greater than  $\psi$  and bids its own valuation  $v$ , it is quite similar to the case of the standard second price auction (Vickrey, 1961). Basically, if bidders conform to this norm, the highest bid will always be made by the clearing member expecting the highest profit from the portfolio, so that the result will be pareto-optimal. We now consider a clearing member with  $v < \psi$ . Its expected utility of bidding  $\psi$  is the following

$$P \frac{V_0 - \psi}{N} + \frac{F(\psi)^N}{N} (v - \psi) > 0 \quad (12)$$

while the expected utility of bidding below  $\psi$  is equal to zero. Then, the clearing member cannot do better by bidding more than  $\psi$ .  $\square$

Increasing the number of bidders in the auction has two opposite effects: (i) it decreases the part of the risk shared losses received by every member who is bidding lower than  $\psi$ , and (ii) it increases the probability that at least one bidder has its valuation above  $\psi$ . This type of auction does not have the truth revelation property, as bidders with valuation below  $\psi$  overbid. This, however, does not change the winner's bid when its valuation is bigger than  $\psi$ . This is almost always the case when there is a reasonably large  $N$  and an appropriately chosen level of  $\psi$ . In general, the threshold  $\psi$  could be set at the break even point point between profit and loss from the CCP's point of view given by the mark-to market value of the portfolio minus the collateral left from the defaulted clearing member. When decreasing  $\psi$  the profit provided obviously decreases since the seller uses more collateral left by the defaulted clearing member in order to cover for losses.

As usual, the winner is the participant with the highest valuation, where the expectation of the highest value among  $N$  independent draws is  $v_{(N)} = \frac{N}{N+1}$ . Then,

according to equation (12), the expected bid submitted by the winning bidder is

$$E[b(v_{(N)})] = N(1 - F(\psi))F^{N-1}(\psi)\psi + E[Y^{(2)}|\psi \leq Y^{(2)}] + F^N(\psi)\psi \quad (13)$$

where  $Y^{(2)}$  is the random variable representing the second highest bid.

The revenue from the auction is then the summation of the payment made by the winning bidder plus the fees collected from the bidders who bid values lower of  $\psi$ . The estimated payment from one bidder is the following

$$\begin{aligned} E[b_i] &= \frac{F^N(\psi)}{N}\psi + (1 - F(\psi)) F^{N-1}(\psi)\psi + \\ &\quad + \int_0^\psi \left( \int_x^{\bar{v}} f v \, dv \right) (N-1) x f(x) F^{N-2}(x) \, dx \\ &= \frac{F^N(\psi)}{N}\psi + (1 - F(\psi)) F^{N-1}(\psi)\psi + \\ &\quad + \int_0^\psi x(1 - F(x)) f_1^{(2)}(x) \, dx \end{aligned}$$

where  $f_1^{(2)}$  represents the probability distribution representing other bidders' highest bid.

Then, the revenue from the CCPs' point of view becomes

$$\begin{aligned} e^D &= F^N(\psi)\psi + N(1 - F(\psi)) F^{N-1}(\psi)\psi + \\ &\quad + \int_0^\psi N x (1 - F(x)) f_1^{(2)}(x) \, dx. \end{aligned}$$

It should be noted that the expression for calculating the revenue is a monotonically increasing function of  $N$ . Therefore, if more bidders are obliged to participate because of the profit sharing mechanism, the revenue will increase. We believe that a second price auction with loss sharing, motivated by the goal of generating an efficient loss distribution, represents a more realistic framework to liquidate the portfolio of a defaulted clearing member.

## 4 Revenue Comparison of Auctions

This section uses our framework to compare the revenues from different auction types and to analyze their impact on default management. To the best of our knowledge, this paper is the first to provide this type of analysis in such a framework. First, we discuss the fact that, when clearing members are not budget constrained, all the first price auctions under consideration deliver the same revenue for the CCP. However, this revenue can change significantly in the second price auction with loss sharing since bidders should take into account that they may bear part of the CCP's losses. We then explore the potential unintended consequences of penalty mechanisms in the auction.

### 4.1 Revenues

Up to this point we have assumed that bidders are expected utility maximizers. We now analyze the results of the auctions and the bidding behaviors of clearing members throughout the auction process. While the framework is more complex for the first price auction with penalty, the revenue is actually the same for all the first price auction formats described in this study, when members' valuations are above their respective budget constraints. This result is a special case of the *Revenue Equivalence Theorem*,<sup>9</sup> as explained by Vickrey (1961). This is due to the fact that expected utilities are the same for both auction types. Therefore, under the assumption of complete information, the *Revenue Equivalence Theorem* holds, and the CCP could not recoup a higher revenue by simply changing the rules of the

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<sup>9</sup>One of the classic findings in auction theory is the *Revenue Equivalence Theorem* which provides a set of assumptions under which the seller's and buyers' expected payoffs are guaranteed to be the same under different auction formats (Vickrey, 1961). Consider any two auction formats satisfying both of the following properties: (i) the two auction formats assign the portfolio to the same bidder for every realization of random variables; and (ii) the two auction formats give the same expected payoff to the lowest valuation type,  $\underline{v}$ , of each bidder  $i$ . As a consequence of the *Revenue Equivalence Theorem*, the first price auctions with and without penalty give the CCP the same revenues. This is possible because portfolios are allocated efficiently in each of these auction formats. To understand this result, observe that clearing members in a first price auction either with or without penalty also have the same allocation rule in equilibrium (i.e., portfolios are allocated to the clearing member with the highest valuation).

auction.

**Proposition 2.** *The first price auction without penalty and the first price auction with penalty yield the same revenue.*

Rather than repeat the Vickrey (1961) proof of the *Revenue Equivalence Theorem*, we provide some intuition. Recall that when assumptions 1 and 2 hold, if clearing members do not know the bids of their competitors, then a CCP will not know in advance whether one type of auction will give him more revenue than another. In summary, the CCP's return from either type of auction is a random variable which can be calculated by using the probability distribution of bidders' values  $\varepsilon$ . The revenue from these auctions would be the same so long as the distributions of value  $\varepsilon$  are continuous and independent between bidders. That is, once we know who gets the portfolio in each possible situation, given a specified probability distribution of winning, and how much expected utility each bidder would get if his value estimate was at its lowest possible level, then the CCP's revenue from the auction does not depend on the required payment in function of the bids. Hence, the CCP must get the same revenue from any auction mechanisms which have the properties that (i) the portfolio always goes to the bidder with the highest value estimate, and (ii) every bidder would expect zero utility if his value estimate was at its lowest possible level. However, even though both first price auction types generate the same revenue, the variance is different for each of them (see Section 2.1 and 2.2).

**Proposition 3.** *The variance of the revenue in first price auction without penalty is smaller than the variance of the revenue in first price auction with penalty.*

In regards to variance, in the first price auction without penalty, conditional on the winning bidder's information, the revenue is constant. In a first price auction with penalty, however, conditional on the winning bidder's information, the revenue further depends on the penalties paid by other bidders. Thus, the expected revenue is the same in the two cases, but more variable in the auction with penalty. Also, unconditional on the winner's information, the revenue will be more variable in the penalized auction. This is an analogue of the example in Klemperer's (2004) where

he discusses differences between first and second price auctions. By Klemperer's argument we thus also know that the revenue from a second price auction would be more variable than the first price auction (though the author does not make this comparison).

Therefore, if we introduce an element of risk aversion for the purpose of evaluating these results (but not for calculating them), then the first price auction without penalty proves to be slightly superior to the first price auction with penalty due to the smaller dispersion of the revenues.

**Proposition 4.** *The revenue of the first price auction with budget constraints is lower than or equal to the first price auction with and without penalty.*

The equilibrium in all the auction formats presented in this study yields full efficiency. In the symmetric increasing equilibrium of the first price auction formats, the highest bid corresponds to the highest valuation, and so the item is assigned efficiently for every realization of the random variables. However, when the valuations are private but signals are distributed asymmetrically, Vickrey (1961) demonstrates that differential bid shading<sup>10</sup> in a discriminatory auction can lead to an inefficient allocation.

From a practical perspective, there could be many obstacles to achieving an efficient loss allocation. Klemperer (2002) highlights the fact that collusive behavior among bidders may be an example of such an obstacle. Moreover, he goes on to argue that open auctions may be more susceptible to collusive behavior than sealed bid auctions since bidders are able to monitor others' bids. Another potential obstacle to achieving an efficient loss allocation is that budget constraints may also reduce the bid of the member who has the highest valuation for the portfolio.

Under our assumptions regarding bidders' behavior, the revenue of the portfolio by the CCP might be the same for the first price without penalty, first price with penalty, and first price with budget constraints. However, compared to the first price auctions with penalty and without penalty, the first price auction with budget

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<sup>10</sup>This term describes the practice of a bidder placing a bid that is below what they believe a portfolio is worth.

constraints has the advantage of taking into account a potentially more realistic framework. The main motivation in considering this type of auction is that clearing members may face a shortage of liquidity due to wide market stress scenarios or due to limited access to capital markets.

This type of constraint, which is private information, does not preclude a CCP from using a juniorization mechanism to incentivize more competitive bidding and reduce its own losses (CME Group, 2016; Eurex Group, 2016; ICE Group, 2014; LCH.Clearnet, 2016). However, a juniorization mechanism in some circumstances can also increase the probability that the penalized members will not be able to meet their own future obligations, such as margin calls. Here we demonstrate, following Paddrik et al. (2016), that it is possible to estimate the juniorization's marginal contribution to stress by considering the reduction in the probability that the surviving members will be able to meet their obligations (e.g., margin calls). Thus, under the assumption of a generalized market stress scenario, despite the well-known advantage of a loss sharing mechanism to maintain the functionality of the financial infrastructure as a whole, we argue that a penalty mechanism can negatively impact surviving clearing members which are already under stress and thus are unable to bid competitively because of budget constraints.

**Proposition 5.** *The revenue of the second price auction with loss sharing is higher than or equal to the first price auctions with and without penalty.*

We already know from the *Revenue Equivalence Theorem* that the standard second price auction will be revenue equivalent to the first price auction with and without penalty; what remains to be considered is the second price auction with a loss sharing mechanism. Let's look at it in steps.

First, consider whether a bidder with a value above  $\psi$  would ever bid under value? No: since their value is above  $\psi$ , they will not pay a penalty if they bid their value – but they might, if they reduce their bid below  $\psi$ . Conditional on not reducing the bid below  $\psi$ , the standard truth-telling argument for the second price auction still holds. Thus, all bidders with values above  $\psi$  bid truthfully.

Now, suppose we know that all bidders have values above  $\psi$ . Then this is equiv-



alent to a standard second price auction, where truth-telling is an equilibrium. As a result, in the case when all bidders have values above  $\psi$ , the loss sharing auction will be revenue-equivalent to a standard second price auction.

Next consider a situation in which at least one bidder has a value below  $\psi$ . Would they ever bid below value? Since the value is already below  $\psi$ , if they bid even lower they still incur the penalty, but reduce their chance of winning. Is there an incentive to bid higher than value? For any continuous distribution that satisfies the assumptions specified in the paper, there will be a value close enough to  $\psi$  where it is worthwhile to bid  $\psi$  exactly, and avoid the penalty. This gives a further incentive to bid (strictly) above one's value. We can thus conclude that in any case, it is never optimal for a bidder to bid below their value, and it is optimal to bid strictly more if the value is below  $\psi$ .

Thus looking case-by-case, the loss sharing auction induces at least as aggressive bidding as the standard second price auction, and for a value of  $\psi$  greater than zero it will induce strictly higher bids in at least some cases. Since the cases above are jointly exhaustive, we can conclude that the revenue from the loss sharing auction will be higher than from the second price auction (and then by *Revenue Equivalence Theorem*, also higher than in the first price auction types). However, it also follows that this revenue will be more variable than the revenue in the first price auction types (by a similar argument used for Proposition 3).

All the comparisons here are done on the basis of case-by-case (payoff) dominance, and thus do not rely on a particular distribution (so asymmetries, non-equal supports, and risk aversion are all permitted). Recall that the proof of truthful bidding in the standard second price auction is also a dominance based argument, and doesn't rely on the *i.i.d.* assumptions or risk neutrality. The revenue-ranking and overbidding results in the second price auction with loss sharing (relative to the standard second price auction) will thus persist in a much broader setting than the assumptions of the *Revenue Equivalence Theorem*. Therefore, the second price auction with loss sharing incentivizes a level of bid values equal to or higher than first price auction types and, for reasonable settings of  $\psi$ , generates equal to or higher revenue for the CCP compared to other auctions types.

The arguments in this section highlight why auctions should play an important and recurring role in the rulebook of CCPs, since the simplified form of auctioneer-bidder interaction they embody is closely related to more complex forms of economic interaction as well. We think that a rulebook's clear description of auction types is particularly important when a CCP uses some form of penalty (such as a juniorization mechanism or sanction) as an incentive. In such a situation, a clearer description could help prevent bidders from either receiving a penalty for bidding too low or from potentially bidding unnecessarily high in order to avoid receiving a penalty.

## 4.2 Unintended Effect of Incentives

A fundamental difference between an auction type with penalty and without penalty is the risk that a penalty may add an unexpected stress to the balance sheet of a surviving clearing member. Additionally, a penalty can increase the risk that members may not be able to meet margin calls, default fund additional margin (DFAM),<sup>11</sup> and default fund replenishment.<sup>12</sup>

Additionally, since the default of a generic large financial institution will likely trigger the default management process in several CCPs at the same time, clearing members are likely to prioritize some clearing houses and some portfolios in order to reduce the impact of a potential penalty (Sourbes, 2015). However, an important question is whether members have the resources – and the risk-taking capacity – to participate in those multiple auctions.

Missing margin calls is not costless for an institution in terms of market reputation and compliance with CCP rules, and it can lead to a loss of access to centrally cleared markets. Additionally, a CCP might put a clearing member that delays a payment into default. For the purpose of this analysis, we summarize the sum of these costs  $\lambda$  as the marginal disutility from missing a unit of payment. To reflect the ability

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<sup>11</sup>Currently, a DFAM may be called in the event that any member's uncovered loss values breach the default fund amount.

<sup>12</sup>To equiparate these variables, we assume that the supplementary contributions that members have to make in order to replenish their default fund contribution must be paid within a few business days. Alternatively, they would be required to add even higher margin to compensate the lack of pre-funded resources.

of a CCP to endogenously adjust performance risk in response to losses, we assume that a CCP can optimize the performance probability  $q$ , as explained in Theorem 3.

When the direction of the portfolio  $\theta$  and the performance probability  $q$  of meeting margin calls are determined jointly, a member has two channels for optimizing its utility function in response to a possible penalty. All else held equal, the member can either reduce its future exposure by hedging/decreasing its position or it can increase its probability of not being able to meet its obligations. Theorem 3 shows that, in equilibrium, clearing members find it optimal to take the latter route of managing the exposure by increasing their performance risk.

**Theorem 3.** *When losses determined by the difference between the benchmark price and the revenue are allocated using a form of penalty, any increase in loss sharing is fully offset by the lower chance that obligations will be met.*

*Proof.* During an auction with a penalty mechanism, define  $p$  and  $q$  as the probability of being penalized and the probability of not being able to send their future payments, respectively. Assume that the clearing member maximizes with respect to  $q$  the Lagrangian

$$P = \frac{1}{2} \left[ (1-p)u(\theta) + pq u(\theta - L) + p(1-q)[u(\theta) - \lambda] \right],$$

resulting in the first order condition for  $q$

$$Q = p \left[ u(\theta - L(p, q, \theta)) - u(\theta) + \lambda + q u'(\theta - L(p, q, \theta)) \frac{-\partial L(p, q, \theta)}{\partial q} \right] = 0.$$

The first condition above provides an implicit link between the optimal choice of  $q^*$  and the probability  $p$ , according to which we can calculate the ratio

$$\frac{\partial q^*}{\partial p} = -\frac{\partial Q / \partial p}{\partial Q / \partial q} < 0,$$

which implies that a higher probability of allocating a loss through a penalty reduces the member's likelihood of meeting other obligations. □

Theorem 3 shows that, under our assumptions, ex-ante the portfolio  $\theta$  is perfectly inelastic with respect to the risk of getting a penalty because any increase in the loss probability  $p$  is fully offset by an equally large increase in the risk of not meeting its margin calls  $q$ . In summary, this framework suggests that a good default management process should be able to pre-empt scenarios where the penalty threatens the likelihood that surviving clearing members will be able to meet their obligations.

It is standard practice in auction theory to evaluate auction formats according to one of two criteria: revenue optimization and efficiency. Revenue optimization means maximizing the CCP's revenue or, in an alternative but equivalent framework, minimizing the expected loss in the default fund contributions of the surviving clearing members. However, common auctions by private parties where the explicit objective is often revenue optimization by implicitly assuming quasi-linear utilities, efficiency means putting the portfolios in the hands of those clearing members who value them the most. However, in our regulatory framework, efficiency must be a secondary target compared to the stability of the financial system as whole.

It should be noted that, in a period of generalized stress, the use of a penalty can potentially contribute to a number of serious consequences such as exposing participants to an unexpected liquidity stress and potentially creating disincentives for firms to clear through CCPs. As such, it is questionable whether the use of a penalty mechanism actually contributes to achieving the objective of continuing key services in the market.

## 5 Conclusions

This study provides a theoretical framework for investigating the design of an optimal auction type in which losses do not need to be allocated through a penalty mechanism. A good auction design promotes both higher revenue for the CCP and an efficient assignment of the portfolio to the bidder that is evaluating it the most. The structure of bidder preferences and the degree of competition are key factors in determining the best design. If a CCP is ready to bare a weak competition between bidders in exchange for a simple value structure, a first price sealed bid auction may

suffice. However, with more complex value structures, a more realistic auction design likely is needed to promote revenue objectives. In this case, a second price auction with loss sharing, rather than first price auctions with or without penalty, increases the liquidation value of the portfolio. If, instead, a CCP is aiming to reduce the variance of the revenue, it should prefer a first price auction without penalty.

This paper has drawn out a number of key messages, relevant to the ongoing policy debate on the implementation of reforms in the current regulatory environment for CCPs. As part of this, the paper has illustrated why it is important to also consider the consequences of the penalty mechanism itself. Given the stylized nature of the model, further work is needed to strengthen the conclusions, including: enriching the representation of agents' budget constraints; studying how sensitive our results are to assumptions such as risk neutral agents; and calibrating model parameters to real world observations so as to better establish the economic relevance of the results.

Increasing the transparency of CCPs' rulebooks and of other elements in the risk modelling framework would help to improve market participants' understanding and awareness of their exposure in the default management process. To the extent feasible, this could usefully be combined with regular fire-drills in which CCPs test their arrangements by auctioning representative portfolios in simulated severe market conditions that include possible bidders' constraints.

It should be noted that there are several additional strategic factors that might drive a bidder's valuation (e.g., estimate of other members' valuations; own and other bidders' potential exposure to other loss allocation tools later in the waterfall; knowledge of the maximum loss the CCP can bear before pre-funded resources are exhausted, etc.). However, we leave these possible extensions of our framework for future research. It would also be useful to conduct more research on the implications of alternative loss allocation mechanisms and auction types for efficiently increasing participant incentives.

# Appendix

## Appendix A: First Price Auction without Penalty

By taking the derivative of the expected payoff with respect to the bid  $B$ , the expression is the following

$$\frac{\partial \pi}{\partial b(v)} = -F[b^{-1}(b(v))]^{N-1} + (v - b(v))(N-1)F[b^{-1}(b(v))]^{N-2}f[b^{-1}(b(v))]\frac{d[b^{-1}(b(v))]}{db(v)}.$$

In the case of a symmetric equilibrium, bidder  $i$ 's optimal bidding strategy would also satisfy the bidding function  $b(v_i) = B$ . Thus, we have  $\frac{d[b^{-1}(b(v))]}{db(v)} = \frac{1}{b'(v)}$ . The optimization yields the first order condition

$$0 = -F(v)^{N-1} + [v - b(v)](N-1)F(v)^{N-2}f(v)\frac{1}{b'(v)}$$

$$F(v)^{N-1} = [v - b(v)](N-1)F(v)^{N-2}f(v)\frac{1}{b'(v)}$$

$$b'(v)F(v)^{N-1} = [v - b(v)](N-1)F(v)^{N-2}f(v)$$

$$b'(v)F(v)^{N-1} + b(v)(N-1)F(v)^{N-2}f(v) = v(N-1)F(v)^{N-2}f(v)$$

$$\frac{d[b(v)F(v)^{N-1}]}{dv} = \frac{d}{dv} \int_{\underline{v}}^v y(N-1)F(y)^{N-2}f(y)dy$$

$$b(v)F(v)^{N-1} = \int_{\underline{v}}^v y(N-1)F(y)^{N-2}f(y)dy$$

$$b(v) = \frac{1}{F(v)^{N-1}} \int_{\underline{v}}^v y(N-1)F(y)^{N-2}f(y)dy.$$

Additionally

$$\int_{\underline{v}}^v y(N-1)F(y)^{N-2}f(y)dy = yF(y)^{N-1}|_{\underline{v}}^v - \int_{\underline{v}}^v F(y)^{N-1}dy.$$

The bidding function thus becomes

$$b(v) = v - \frac{1}{F(v)^{N-1}} \int_{\underline{v}}^v F(y)^{N-1} dy .$$

Let's consider the overall valuation being uniformly distributed within the range  $[\underline{v}, \bar{v}]$ , where the bidding strategy becomes

$$\begin{aligned} b(v) &= v - \left[ \frac{\bar{v} - \underline{v}}{v - \underline{v}} \right]^{N-1} \int_{\underline{v}}^v \left[ \frac{y - \underline{v}}{\bar{v} - \underline{v}} \right]^{N-1} dy \\ &= v - \left[ \frac{\bar{v} - \underline{v}}{v - \underline{v}} \right]^{N-1} \left\{ \frac{(y - \underline{v})^N}{N[\bar{v} - \underline{v}]^{N-1}} \right\} \Big|_{\underline{v}}^v \\ &= v - \left[ \frac{\bar{v} - \underline{v}}{v - \underline{v}} \right]^{N-1} \frac{(v - \underline{v})^N}{N[\bar{v} - \underline{v}]^{N-1}} \\ &= v - \frac{v - \underline{v}}{N} . \end{aligned}$$

It follows that the expected bid to the CCP is

$$b(v) = \frac{N-1}{N}v + \frac{v}{N}.$$

## Appendix B: First Price Auction with Penalty

The expected payoff is the following,

$$\pi(v, b(v)) = F[b^{-1}(b(v))]^{N-1}(v_i - b(v)) - \{1 - F[b^{-1}(b(v))]^{N-1}\}\omega(b(v))b(v),$$

where the simplified expression is

$$\pi(v, b(v)) = F[b^{-1}(b(v))]^{N-1} \left( v_i - \frac{b(v)^2}{V_0} \right) - \left( b(v) - \frac{b(v)^2}{V_0} \right).$$



The bidder's problem is maximizing the expected payoff as follows

$$\max_b \left\{ F[b^{-1}(b(v))]^{N-1} \left( v - \frac{b(v)^2}{V_0} \right) - \left( b(v) - \frac{b(v)^2}{V_0} \right) \right\}.$$

By taking the first order derivative, we have

$$\begin{aligned} \frac{\partial \pi}{\partial b(v)} &= (N-1)F[b^{-1}(b(v))]^{N-2}f[b^{-1}(b(v))]\left(\frac{d[b^{-1}(b(v))]}{db(v)}\right)\left(v - \frac{b(v)^2}{V_0}\right) + \\ &+ F[b^{-1}(b(v))]^{N-1}\left(-\frac{2b(v)}{V_0}\right) - \left(1 - \frac{2b(v)}{V_0}\right). \end{aligned}$$

In equilibrium, a bidder follows the bidding strategy with  $b^{-1}(b(v)) = v$ , and the first order condition implies

$$\begin{aligned} (N-1)F[v]^{N-2}f(v)\frac{1}{b'(v)}\left(v - \frac{b(v)^2}{V_0}\right) + F[v]^{N-1}\left(-\frac{2b(v)}{V_0}\right) - \left(1 - \frac{2b(v)}{V_0}\right) &= 0 \\ (N-1)F[v]^{N-2}f(v)\left(v - \frac{b(v)^2}{V_0}\right) + F[v]^{N-1}\left(-\frac{2b(v)}{V_0}\right)b'(v) &= b'(v)\left(1 - \frac{2b(v)}{V_0}\right) \\ (N-1)F[v]^{N-2}f(v)\left(v - \frac{b(v)^2}{V_0}\right) + F[v]^{N-1}\left[1 - \frac{2b(v)}{V_0}b'(v)\right] - F[v]^{N-1} &= b'(v)\left(1 - \frac{2b(v)}{V_0}\right) \\ \frac{d\{F[v]^{N-1}(1 - \frac{b(v)^2}{V_0})\}}{dv} &= \frac{d}{dv} \int \left\{ F[y]^{N-1} + b'(y) - \frac{2b(y)}{V_0}b'(y) \right\} dy \\ F(v)^{N-1}\left(1 - \frac{b^2(v)}{V_0}\right) &= \int_{\underline{v}}^v F[y]^{N-1} dy + \int_{\underline{v}}^v b'(y) dy - \int_{\underline{v}}^v \frac{2b(y)}{V_0}b'(y) dy. \end{aligned}$$

Consider the simple case in which an overall value follows the uniform distribution and a normalized range of  $[0, 1]$ , then the bid satisfies the following equation

$$v^{N-1}\left(1 - \frac{b^2(v)}{V_0}\right) = \frac{1}{N}v^N + b(v) - \frac{b^2(v)}{V_0}$$



$$\frac{1 - v^{N-1}}{V_0} b^2(v) - b(v) + \frac{N-1}{N} v^N = 0.$$

Then, the solution for the bidding function is the following

$$b^*(v) = \frac{V_0 - V_0 \left[ 1 - \frac{4(N-1)}{NV_0} v^N (1 - v^{N-1}) \right]^{\frac{1}{2}}}{2(1 - v^{N-1})}.$$

For bidders in the auction, the auction is designed such that it is beneficial for each of them to bid. According to the above bidding strategy, bidder  $i$  has the following estimated payoff by playing the bidding strategy

$$\begin{aligned} \pi_i(v_i, b(v_i)) &= F[v_i]^{N-1} \left( v_i - \frac{b^2(v_i)}{V_0} \right) - \left( b(v_i) - \frac{b^2(v_i)}{V_0} \right) \\ &= v_i^{N-1} \left( v_i - \frac{b^2(v_i)}{V_0} \right) - \left( 1 - v_i^{N-1} \right) \left( b(v_i) - \frac{b^2(v_i)}{V_0} \right) \\ &= v_i^N - b(v_i) + \frac{1 - v_i^{N-1}}{V_0} b^2(v_i). \end{aligned}$$

By substituting the bidding expression, the expected payoff from participation becomes

$$\pi_i(v_i, b(v_i)) = v_i^N - \frac{N-1}{N} v_i^N = \frac{1}{N} v_i^N \geq 0.$$

The participation constraint therefore is satisfied for all bidders.<sup>13</sup> Thus, the best expected bid submitted by the winning bidder is

$$E[b(v_{(N)})] = \frac{V_0 - V_0 \left\{ 1 - \frac{4(N-1)}{NV_0} \left( \frac{N}{N+1} \right)^N \left[ 1 - \left( \frac{N}{N+1} \right)^{N-1} \right] \right\}^{\frac{1}{2}}}{2 \left[ 1 - \left( \frac{N}{N+1} \right)^{N-1} \right]}.$$

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<sup>13</sup>The expected utility for a bidder in the first price auction with penalty actually reaches the same amount as that in the first price auction without penalty. However, bidders' preferences for participating in those games are different, depending on the bidder's valuation.

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